59) 1) The randomly chosen die is rolled 7 times, with the following outcomes:

5, 3, 9, 3, 8, 4, 7

What is the probability that the die is die A?

We want $P(A|O)$ where O is the observation sequence

$$P(A|O) = \frac{P(O|A)\,P(A)}{P(O)} \Rightarrow P(O|A) = \frac{1}{20}\cdot\frac{3}{20}\cdot\frac{1}{20}\cdot\frac{3}{20}\cdot\frac{1}{20}\cdot\frac{2}{20}\cdot\frac{1}{20}$$

$$= \frac{18}{20^7}$$

$$P(A|O) = \frac{P(O|A)\,P(A)}{P(O|A)P(A) + P(O|B)P(B)} \qquad P(A) = 1/2 \quad P(B) = 1/2$$

$$P(O|B) = \frac{2}{20}\cdot\frac{2}{20}\cdot\frac{1}{20}\cdot\frac{2}{20}\cdot\frac{2}{20}\cdot\frac{2}{20}\cdot\frac{2}{20} = \frac{64}{20^7}$$

$$= \frac{\frac{18}{20^7}\cdot\frac{1}{2}}{\frac{18}{20^7}\cdot\frac{1}{2} + \frac{64}{20^7}\cdot\frac{1}{2}} = \frac{\frac{18}{20^7}}{\frac{82}{20^7}} = \frac{18}{82} = \boxed{\frac{9}{41}}$$

2) Assume that there is a third twenty-faced die, die C, on which the symbols 1-20 are written once each. As above, one of the three dice is selected at random and rolled 7 times, giving the outcomes 3, 5, 4, 8, 3, 9, 7.

What is the probability that the die is die A, die B or die C?

$$P(A) = P(B) = P(C) = 1/3$$

$$P(O|A) = \frac{3}{20}\cdot\frac{1}{20}\cdot\frac{2}{20}\cdot\frac{1}{20}\cdot\frac{3}{20}\cdot\frac{1}{20}\cdot\frac{1}{20} = \frac{18}{20^7}$$

$$P(O|B) = \frac{2}{20}\cdot\frac{2}{20}\cdot\frac{2}{20}\cdot\frac{2}{20}\cdot\frac{2}{20}\cdot\frac{1}{20}\cdot\frac{2}{20} = \frac{64}{20^7}$$

$$P(O|C) = \frac{1}{20}\cdot\frac{1}{20}\cdot\frac{1}{20}\cdot\frac{1}{20}\cdot\frac{1}{20}\cdot\frac{1}{20}\cdot\frac{1}{20} = \frac{1}{20^7}$$

$$P(A|O) = \frac{P(O|A)\,P(A)}{P(O|A)P(A) + P(O|B)P(B) + P(O|C)P(C)} = \frac{\frac{1}{3}\cdot\frac{18}{20^7}}{\frac{1}{3}\cdot\frac{18}{20^7} + \frac{1}{3}\cdot\frac{64}{20^7} + \frac{1}{3}\cdot\frac{1}{20^7}} = \frac{18}{83}$$

$$P(B|O) = \frac{P(O|B)\,P(B)}{P(O|A)P(A) + P(O|B)P(B) + P(O|C)P(C)} = \frac{\frac{1}{3}\cdot\frac{64}{20^7}}{\frac{1}{3}\cdot\frac{18}{20^7} + \frac{1}{3}\cdot\frac{64}{20^7} + \frac{1}{3}\cdot\frac{1}{20^7}} = \frac{64}{83}$$

$$P(C|O) = \frac{P(O|C)\,P(C)}{P(O|A)P(A) + P(O|B)P(B) + P(O|C)P(C)} = \frac{\frac{1}{3}\cdot\frac{1}{20^7}}{\frac{1}{3}\cdot\frac{18}{20^7} + \frac{1}{3}\cdot\frac{64}{20^7} + \frac{1}{3}\cdot\frac{1}{20^7}} = \frac{1}{83}$$