Tarea N° 2

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Curso: Segundo "A"

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Averiguar si las siguientes funciones son solución de la correspondiente ecuación diferencial. Ejercicios seleccionados

9. $y = 1 + c\sqrt{1-x^2}$ de $(1-x^2)y' + xy = x$

Solución:

$$y' = 1 + c \cdot \frac{-2x}{2\sqrt{1-x^2}}$$

$$y' = 1 - \frac{2c}{2\sqrt{1-x^2}}$$

$$\boxed{y' = 1 - \frac{c}{\sqrt{1-x^2}}}$$

Reemplazo

$$(1-x^2)y' + xy = x$$

$$(1-x^2)\frac{(\sqrt{1-x^2} - \sqrt{1-x^2}\,c)}{\sqrt{1-x^2}} + x(1 + c\sqrt{1-x^2}) = x$$

$$(1-x^2)(\sqrt{1-x^2} - c\sqrt{1-x^2}) + x(1 + c\sqrt{1-x^2}) = x$$

$$(1-x^2) - c(\sqrt{1-x^2}) + x + xc\sqrt{1-x^2} = x$$

$$xc\sqrt{1-x^2} = c(1-x^2) + x^2 - 1$$

$$(1-c)(\sqrt{1-x^2}) + x = 0 \,//$$

$\therefore$ $y = 1 + c\sqrt{1-x^2}$ no es solución de $(1-x^2)y' + xy = x$ //

13. $\left.\begin{array}{l} x = \cos t \\ y = e^t \end{array}\right\}$ de $y' + \frac{y}{\sqrt{1-x^2}} = 0$

Si: $y' + \frac{y}{\sqrt{1-x^2}} = 0$

$y' = e^t$

$x^2 = \cos^2 t$

$$e^t + \frac{y}{\sqrt{1-\cos^2 t}} = 0$$

$$e^t + \frac{y}{\sqrt{\operatorname{sen}^2 t}} = 0$$

$$e^t + \frac{y}{\operatorname{sen} t} = 0$$

$$y = -e^t \operatorname{sen} t \,//$$

$\therefore$ $\left.\begin{array}{l} x = \cos t \\ y = e^t \end{array}\right\}$ no es una solución de la ecuación diferencial $y' + \frac{y}{\sqrt{1-x^2}} = 0$ //

Hallar la solución general de las siguientes ecuaciones diferenciales.

10. $\frac{ds}{dt} = \ln t + 4t$

$$ds = (\ln t + 4t)\,dt$$

$$\int ds = \int (\ln t + 4t)\,dt$$

$$s + C_1 = \int \ln t \, dt + 4\int t\,dt$$

$$s + C_1 = t\ln(t) + 2t^2 - t + C_2$$

$$s + C_1 - C_2 = t\ln(t) + 2t^2$$

$$s + C = t(\ln t + 2t - 1) \,//$$

Escaneado con CamScanner

19. $y' = \frac{\cos^2 x}{y} \longrightarrow \frac{dy}{dx} = \frac{\cos^2 x}{y}$

$$\int y\, dy = \int \cos^2 x\, dx$$

$$\frac{y^2}{2} = \frac{\cos(x)\,\text{sen}(x)}{2} + \frac{1}{2}\int 1\, dx$$

$$\frac{y^2}{2} = \frac{\cos(x)\,\text{sen}(x)}{2} + \frac{x}{2}$$

$$\frac{y^2}{\cancel{2}} = \frac{\frac{\text{sen}\,2x}{2} + x}{2} + C$$

$$y^2 = x + \frac{1}{2}\,\text{sen}\,2x + C \quad //$$

20. $\frac{dy}{dx} = \frac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$

$$\frac{dy}{dx} = \frac{xy - y + 3x - 3}{xy - 2x + 4y - 8}$$

$$\frac{dy}{dx} = \frac{y(x-1) + 3(x-1)}{x(y-2) + 4(y-2)}$$

$$\frac{dy}{dx} = \frac{(y+3)(x-1)}{(x+4)(y-2)}$$

$$\frac{(y-2)}{(y+3)}\,dy = \frac{(x-1)}{(x+4)}\,dx$$

$$\int \frac{y-2}{y+3}\,dy = \int \frac{x+1}{x+4}\,dx$$

$$\int \frac{y - 2 + 5 - 5}{y+3}\,dy = \int \frac{x - 1 + 5 - 5}{x+4}\,dx$$

$$\int \frac{y+3-5}{y+3}\,dy = \int \frac{x+4-5}{x+4}\,dx$$

$$\int \left(1 - \frac{5}{y+3}\right) dy = \int \left(1 - \frac{5}{x+4}\right) dx$$

$$\int dy - 5\int \frac{dy}{y+3} = \int dx - 5\int \frac{dx}{x+4}$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$y - 5\ln(y+3) = x - 5\ln(x+4) \quad //$$

Hallar la solución particular correspondiente a las condiciones iniciales dadas.

22. $y' = 4 - 9x^2 - 6x^5 \qquad y(1) = 0 \longrightarrow$ Reemplazo

$$y' = \frac{dy}{dx} = 4 - 9x^2 - 6x^5$$

$$dy = (4 - 9x^2 - 6x^5)\,dx$$

$$\int dy = \int (4 - 9x^2 - 6x^5)\,dx$$

$$y = 4x - 3x^3 - \frac{6x^6}{6} + C$$

$$y(1) = 0 = 4(1) - 3(1)^3 - \frac{6(1)^6}{6} + C$$

$C = 0$.

$$y = 4x - 3x^3 - \frac{\cancel{6}x^6}{\cancel{6}}$$

$$y = 4x - 3x^3 - x^6 \quad //$$

Verificar si la EDO es de tipo homogenea. Encontrar la solución (cualquier método).

4. $y' = \frac{dy}{dx} = \frac{y}{[x + (xy)^{1/2}]} \quad f(x,y)$

método

a) $y' = f(x,y) = f(tx, ty)$

Si $f(x,y) = f(tx, ty) \longrightarrow$ ED HOMOGENEA

$$\frac{y}{[x + (xy)^{1/2}]} = \frac{(ty)}{[(tx) + (tx)(ty)^{1/2}]}$$

$$= \frac{y}{x + (xy)^{1/2}}$$

$\hookrightarrow$ si es homogenea.