Euler's totient function φ(n)

"Euler phi" function gives the number of integers between 0 … (n-1), which are coprimes with n:

In other words integers 0 < a ⦿ n-1 with GCD(a, n)

Properties:

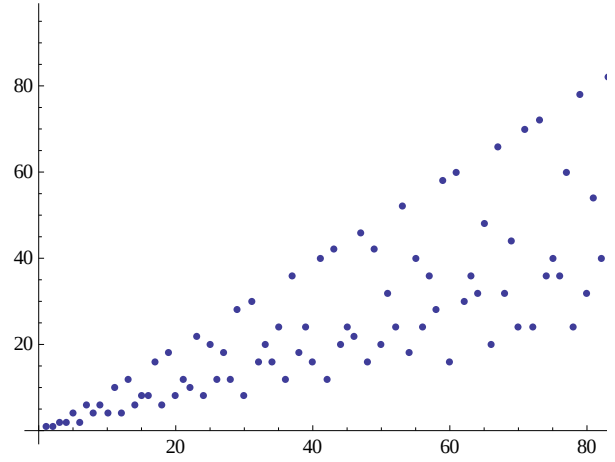
The size of multiplicative group Z_n^* is φ(n)

Calculation rules:

1. For primes p, φ(p) = p - 1
2. For n = pq (product of two primes) : φ(n) = (p-1)(q-1)
3. Generally, if p_i:s are the distinct prime factors of n

   $\varphi(n) = n\,(1-1/p_1)\,(1-1/p_2)\cdots$

In the plot of φ(n) of values n<100, the values at top come from prime numbers. The lowest values come from integers with lots of prime factors.



Examples of calculation of φ(n)

1. φ(127) = 127 – 1 = 126 (127 is a prime)
2. φ(221) = (13-1)(17-1) = 192 (prime factors: 221 = 13*17)
3. φ(54) = 54 (1-1/2)(1-1/3) = 18 (prime factors: 54 = 3*3*3*2)

In[12]:= Mathematica command

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EulerPhi[54]
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Out[12]= 18