

Recitation November 10

1. Three people line up to board a flight and everyone has a ticket with an assigned seat. However, the first person in line has lost his ticket and takes a random seat. After that, each person takes the assigned seat if it is unoccupied, and one of unoccupied seats at random otherwise. What is the probability that the 3rd person gets to sit in their assigned seat?
2. There are 4 people at a party, and everyone has checked in their coats. The coats get mixed up and after the party, everyone gets a random coat. What is the expected number of people who get their own coat?
3. (Modification of HW9 Q5) Design a probabilistic algorithm that takes an array A of length $2n$ with n X 's and n Y 's as an input and finds any two distinct i, j such that $A[i] == X$ and $A[j] == X$. The expected number of operations should be $\Theta(1)$ for every input array A . The algorithm and its complexity should be explained and justified.
4. Let's suppose we have a function that takes a array of size n as input, and gives back the $n/4$ th smallest number in the array in $O(n)$ time. What would be the time complexity of quicksort if we use this function to choose the pivot?
(Hint 1: If we use this function to find the pivot, the array would get divided into 2 arrays of sizes $n/4$ and $3n/4$. So, the question boils down to solving the recursion $T(n) = T(n/4) + T(3n/4) + O(n)$. This is a bit different from the recurrences we've seen so far. However, you can still use a recursion tree to see how this recurrence works. How much work is done at each level? How many levels are there?)
(Hint 2: The answer is $T(n) = O(n \log_{4/3} n)$.)
5. (**Hard, Optional*) Consider the following modification to the binary search algorithm that searches for an input x in a sorted array A .

1. Initialize $L = 0, H = n - 1$
2. Randomly choose an index m between L and H
3. if $A[m] == x$: return i
4. if $A[m] < x$: $L = m$
5. else: $H = m$
6. Go back to step 2

What's the expected running time of this algorithm?

6. (**Hard, Optional*) Try solving problems 1 and 2 for n instead of 3 and 4.