Dynamical Systems and Chaos

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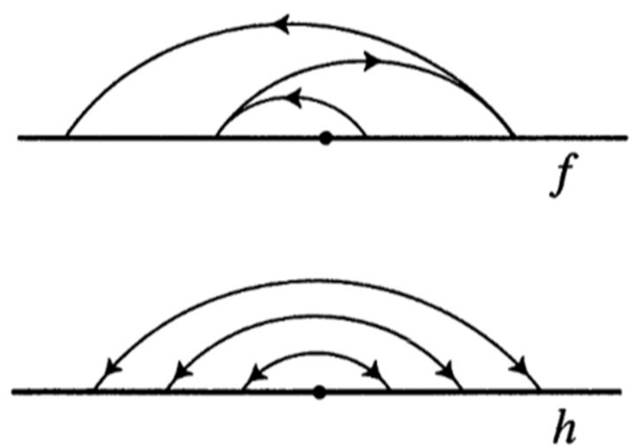
Dynamical Systems

A Dynamical system is a family of functions mapping a topological space X onto itself, a set $\{f^n\}_{n \, \epsilon \, \mathbb{Z}_+}$ with every f^n a mapping X to X

Applications

We think of applications of dynamical systems as each $f(x)$ representing a slight change in the conditions to give us a new system, if we were looking at a population changing over time, we could have a new $f^n(x)$ each hour to show change.

We depict these systems in **Phase Diagrams**



DEFINITION OF CHAOS

- Let X be a topological space. A function $f : X \to X$ is said to be Chaotic or have chaos if

  i. The set of periodic points of f is dense in X

  ii. For every U, V open in X, there exists $x \epsilon U \, and \, n \epsilon \mathbb{Z}_{+}$such that $f^{n}(x) \epsilon$ V

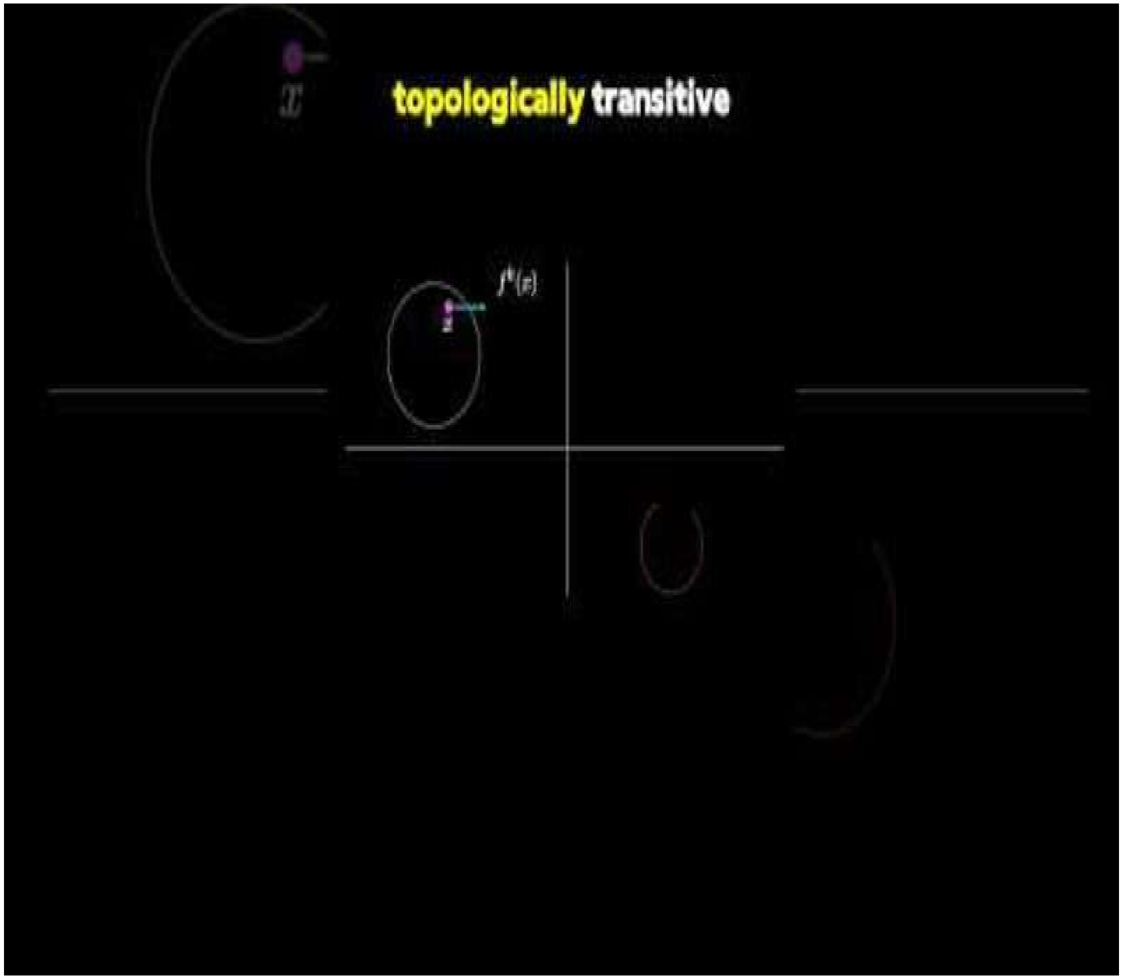
  - Referred to as topologically transitive

CONDITION i.

- In a dynamical system, a periodic point is a point that eventually returns to its original position after a certain number of iterations of the system
  - Essentially there must exist some integer, k, such that application of our function, *f*, onto a point $\mathrm{x} \epsilon X$, yields $f^k(x) = x$
- a set of periodic points is considered dense if, every open neighborhood within the space contains at least one periodic point

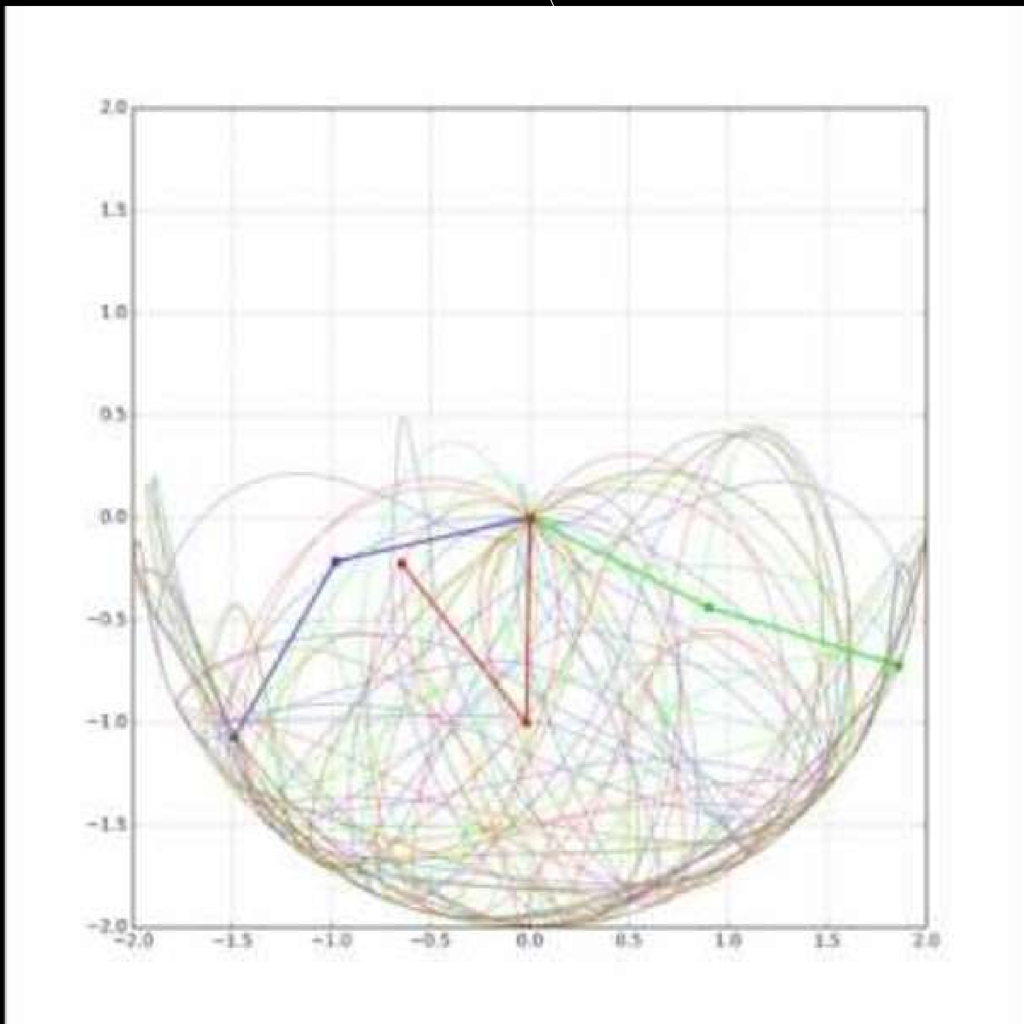
Condition ii) Topological Transitivity

- A dynamical system is topologically transitive if points can eventually move from any small open set to any other under iteration
- Essentially there is a path from any point to another via iterations of our function f



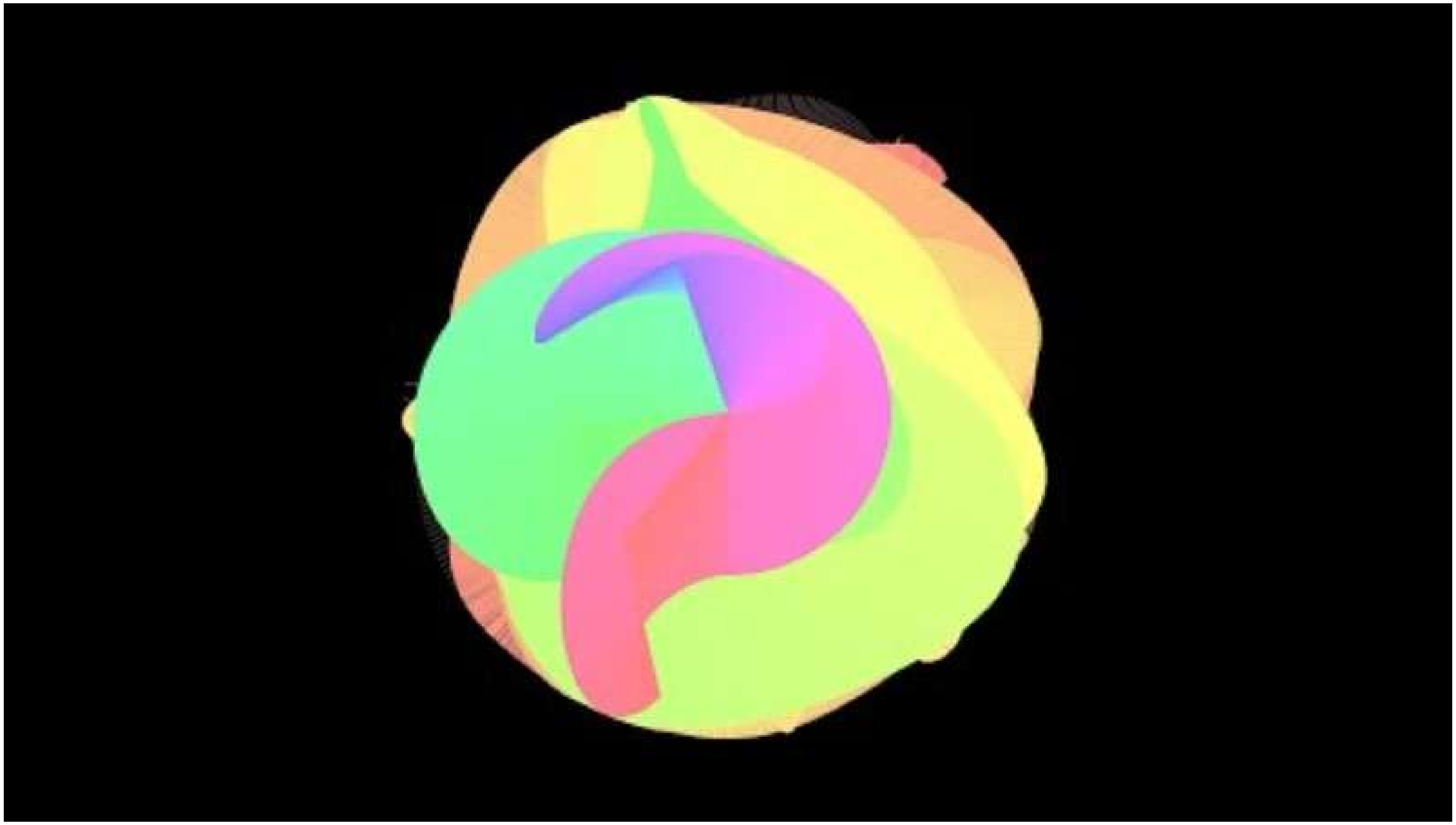
“The Butterfly Effect”

- In 1961 Edward Lorenz, a meteorologist, at MIT was attempting to simulate weather patterns on his computer with 20 fixed equations, one day rather that using the full six decimal point expansion of the equations, he only used the equations to three decimal points and continued his calculations from the previous day finding that the new results of running the equations produced entirely different results than what he had before.
- Even thought the equations were fixed just a minor change in the input conditions caused a major change in the outcome of the system. Which led him to the notion that chaos in a system directly implies sensitive dependence on initial conditions
- This is why long-term weather prediction is so difficult, and most are inaccurate after a couple days



Iterations of 3
Double Pendulums

Time To Get Really Crazy...



THANK YOU

Any Questions ?

Resources:

- https://math.jhu.edu/~brown/courses/s10/LectureNotes/421Week11Lecture1.pdf
- http://www.scholarpedia.org/article/Chaos_topology