Fish Sustainability

Modeling the population of chinook salmon in Lake Ontario

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Introduction

Overview:

I am considering the smallest of the great lakes, Lake Ontario, and I am interested in examining the change in population of the chinook salmon over time. The population depends on four conditions, the New York State Department of Environment stocking the lake with salmon, natural birth of salmon, natural death of salmon, and the harvest of the salmon. I am assuming a negligible overcrowding rate in this situation as the NYSED closely monitors the chinook salmon population in order to see how many fish to stock each year, based on several factors, such as how much food there may be for them to eat and how many of last year's stock is left that may have been breeding, so there is very low likelihood of overcrowding in Lake Ontario[1]. By looking at the stocking, birth, death, and harvest rates I determined that the overall population of chinook salmon in Lake Ontario could be determined through a differential equation model. A comparison can be made to measure the population of salmon at time t in years by the total population $y(t)$. A net rate of change of the population of fish can also be determined by denoting it as $y'(t)$ at any time, t in years. This net rate of change can be written in terms of stocking, birth, death, and harvest rates which can be culminated into a singular equation,

y'(t)= (Stocking Rate + Birth rate) - (Death rate) - (Harvest rate),

which measures the change in overall population each year. Through observations made over the years it has been suggested that stocking, birth, death, and harvests rates are closely proportional to the population size:

Stocked population: $sy(t)$

Birth rate at time t: $by(t)$

Death rate at time t: $my(t)$,

Harvest rate at time t: $hy(t)$,

where s, b, m, and h are definable non-negative constants of proportionality.
The s-constant represents the number of fish stocked into the lake, the b-constant represents the number of fish births, the h-constant represents the number of fish harvested, the m-constant is represented as a natural mortality coefficient. And I am assuming that the rate of natural mortality has nothing to do with the rate of harvest coefficient. By combining all these terms, I developed a differential equation which describes the total population, $y'(t)$, of the change in the chinook salmon population each year:

$$\frac{dy}{dt} = (sy(t) + b)y(t) - my(t) - hy(t),$$

where the quantity, $r = (s + b) - m$, is assumed to be positive with the stocked population plus the number of natural births being greater than the number of natural deaths, or there would be no fish. Based on New York State monitoring of the population of chinook salmon in Lake Ontario I could easily determine stocking, birth, and death rates of the chinook salmon population. So, we can assume that *s, b, m, and h* are known constants. Harvesting of salmon is considered to be a portion of the total-current population where *s* is a constant of proportionality. $y(t)$ will be determined from equation (2) for which the function can satisfy the equation for all t-values will be a solution of the Ordinary Differential Equation. The value $y(t_0)$ of $y(t)$ at t_0, the initial value of the problem, can be estimated and will be a critical factor in predicting later values of $y(t)$. Finally, the initial condition, $\boldsymbol{y(t_0) = y_0}$, can be found which will give us the final equation defining the carrying capacity of chinook salmon. For this project I will assume the initial condition to be the population of salmon in Lake Ontario in the year 1984, the first year that the state began stocking salmon in the lake.

Specific Results:

Approximately 26% of the chinook salmon in Lake Ontario are natural born[4], rather than reared in a hatchery, so if the stocked about, around 2 million per year, constitutes 74% of the total population, we can say that the total population of salmon born in Lake Ontario and stocked into the lake is around 2.65 million salmon each year.

The previous stocking rates have had a variable increase in the past couple of years ranging from 6 – 23% increase each year[3], in both 2022, and 2023 the state determined that there would be a 10% increase in population. So, I am going to assume a stocking of 110% of the previous year's population, which would make the *s*-constant equal to 10% of the population of salmon.

In calculating the natural birth and death rate I utilized data specifically regarding behavior of chinook salmon in open water, rather than in captivity which is how most of the lakes salmon population is raised, in hatcheries.

In open waters each female Chinook salmon will lay approximately 5000 eggs, of which only about 50 will survive to enter the ocean. Of those 50, only approximately 2 will return to breed[4]. So, I will be utilizing a natural birth rate of 1% of the total population. Almost all chinook salmon die after spawning in open water, and approximately 4% of adult chinook salmon will spawn[4]. The only other cause of death in Lake Ontario for chinook salmon is fishing, which will be accounted for in the harvest constant, so I am utilizing a 4% natural death rate.

The harvest rate I computed utilizing the fact that around 700,000 NYS fishing licenses are issued each year, with around 86% of fishing trips being successful [5], and fishers catching an average of 3 fish each trip [5], so we can say the expected harvest rate of fish each season is 68% of the fish population.

Recommendations:

The C-constant can be found utilizing an initial value, I will use the population in the year 1984 as y(t) for t_0 as this is the first year that the state began stocking salmon. Utilizing the initial condition of y(0) = 1,100,000 salmon we can solve for C, and get $1{,}100{,}000 = C + \frac{k}{r}$. So, we can say $C = 1{,}100{,}000 - \frac{1.1y(0)}{0.04-0.01-0.68}$ $= 1{,}100{,}000 - \frac{1{,}210{,}000}{-0.65}$. Giving us a value of 2.96 for C.

Using this value our final equation to determine the population of chinook salmon, in millions, in Lake Ontario is $y = \frac{1.1y(0)}{(b-m-h)} + 2.96e^{(b-m-h)t}$, which if we input the found constants is equal to $y = (-1.7) + 2.96e^{(-0.71)t}$.

Discussion:

We can see that if the harvesting rate of chinook Atlantic salmon was a more sustainable, 47% of the yearly population, the species would be able to flourish better on it's own and would likely need less human intervention in restocking the lake. This point is made in comparing Figure 1 below to Figure 3 and noting that the graph in Figure 1 seems to approach carrying capacity quicker than the graph in Figure 3. Figure 2 shows an increase in population if there wasn't harvesting of salmon in Lake Ontario, and with this I also cancelled out the stocking rate, assuming that the population might be sustainable on its own here. If there was no human involvement in the population of chinook salmon in Lake Ontario it would take about 3 years for the population to have natural growth, rate 0.03, equal to what the government stocks each season, 0.1 increase of the previous population.

Proof of Linear Model:

1) Total Change:

Let r denote the total change in the population,

$$r = (b - m - h)$$

2) Maximum Sustainable Harvest Rate

Let k denote the total population: (sy(t) + b(y(t))

$$\text{kh} - \text{mh} - \text{hh} = 0$$

$$((k-m)h - h^2)' = 0$$

$$k - m - 2h = 0$$

$$-h = \frac{m-k}{2}$$

$$h = \frac{-m + \left(sy(t) + by(t)\right)}{2}$$

Which says that the maximum harvest rate should be equal to ½ of the total population minus the population death for that year. Which would be a sustainable approach to fishing leaving enough left for reproduction and the population to survive without human interaction.

3) <u>Proof of Linear Model</u>

$$y'(t) = \left[(Stoking\ rate)(initial\ population) + (Birth\ rate)y\right] - (Death\ rate)y - (Harvest\ rate)y$$

$$y'(t) = [1.10y(0) + 0.71y(t)] - 0.04y(t) - 0.68y(t)$$

Let r be given above,

$$y'(t) = 1.10y(0) + ry(t)$$

$$y'(t) - ry(t) - 1.10y(0) = 0$$

Solving LDE by Separation: I'm letting k = 1.10y(0) as this is just a constant

$$y'(t) = ry(t) - k$$

$$\frac{dy}{dx} = -k + ry(t)$$

$$\int \frac{dy}{-k + ry(t)} dy = \int 1\, dt$$

$$\frac{1}{r}\ln|-k + ry(t)| = t$$

$$\ln|-k + ry(t)| = rt + C$$

$$-k + y(t) = Ce^{rt}$$

$y = Ce^{rt} + \frac{k}{r}$, where C is the constant of integration

Solving Utilizing the Integrating Factor:

The integrating factor $u(t) = e^{\int P(t)dt}$

Here, $u(x) = e^{\int -r\, dt} = e^{-rt}$

$$e^{-rt}(y'(t) - re^{-rt}y = -ke^{-rt}$$

$$(ye^{-rt})' = -ke^{-rt}$$

$$ye^{-r} = \frac{k}{r}e^{-rt} + C$$

$y = \frac{k}{r} + Ce^{rt}$, where C is the constant of integration.

<u>Model Design</u>:

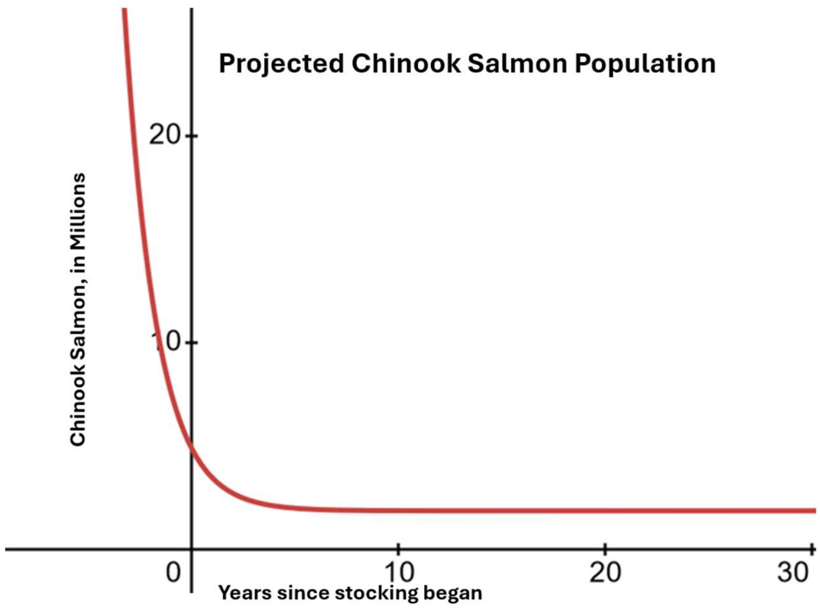

Figure 1: This graph shows the projected population of the chinook salmon, with the y-axis representing salmon, in millions, and the x-axis, which is years since 1984, our initial year.

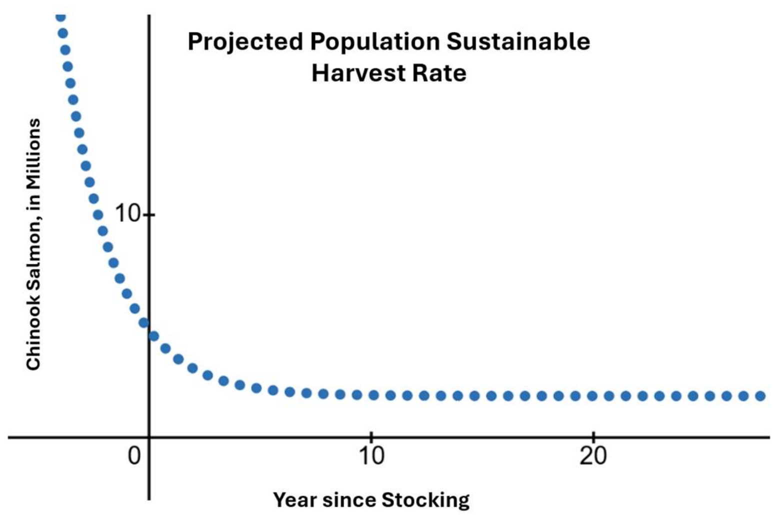

Figure 2: Represents the growth of the salmon population if there was no yearly harvesting and no human involvement. The population increases gradually and would eventually reach carrying capacity but as we can see it will take a long time.

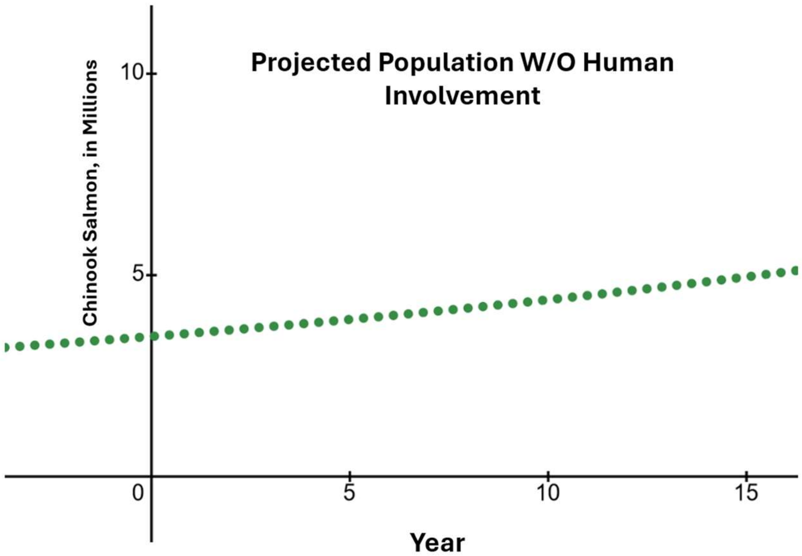

Figure 3: Represents the growth of the salmon population if the population was left alone, I removed both the harvest and stocking rates to see how the salmon population would fare on its own.

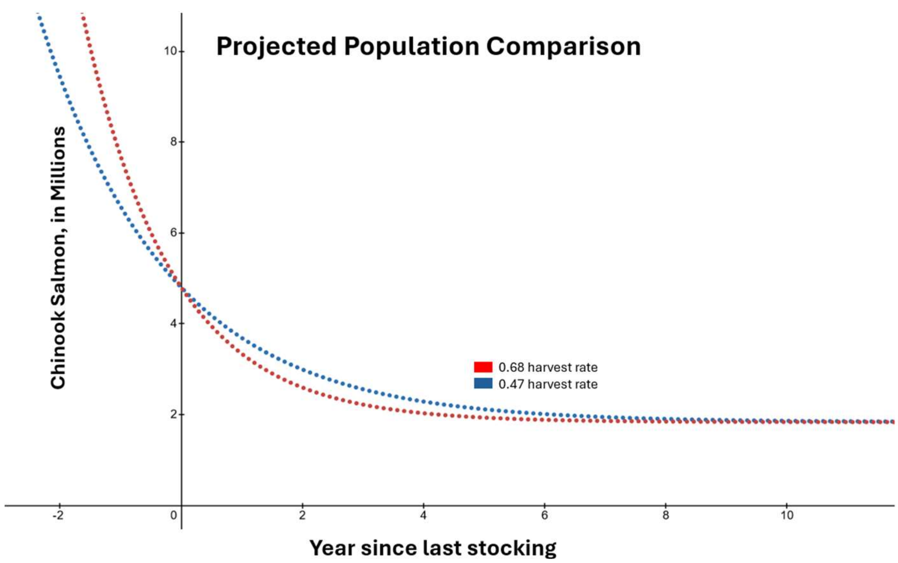

Figure 4: Shows the comparison between the current rate of harvest and a more sustainable rate of harvest, we can clearly see that the population decreases slower with a more sustainable rate, and it takes a couple years longer for the more sustainable rate to reach the higher rate, about 2 years longer to plateau.

Assumptions:

- $ky(t) \sim 0$, I am assuming a negligible overcrowding rate due to how closely the state monitors this population
- $my(t)$ has no relation to $hy(t)$
- $\mathrm{r} = (\mathrm{s} + \mathrm{b}) - \mathrm{m} > 0$

- $b \geq m$, or else there are no fish
- $h = \frac{-m+(sy(t)+by(t))}{2}$, is a sustainable rate of harvest allowing the population to flourish on its own

Conclusion

Based on the information discovered from the figures, I approached models in two ways, in terms of current human involvement and a high harvest rate compared to a more sustainable harvest rate that would allow the population to flourish on its own. Constants s, b, m, and h were determined through research of recorded population data and trends done by the New York State Department of Environmental Conservation. The values of these constants were determined to be $sy(0) = 1.10y(0),\ b = 0.01, m = 0.04,$ $h = 0.68$. Utilizing my maximized harvest rate equation, $h = \frac{-m+(sy(t)+by(t))}{2}$, I was able to determine a more sustainable harvest rate $h_1 = 0.47$. The C-constant was defined at time, t = 0, with a starting population of 1.1 million chinook salmon, giving us a value of 2.96 million.

As evident by a comparison of figures 1 and 2, a more sustainable harvest rate would allow the chinook salmon population to grow on its own, even with human stocking, the population would be healthier if we caught just 60% of what we do now. This point is illustrated well in figure 4, where we can clearly see that the population with the current 0.68 harvest rate decreases much faster than the 0.47 harvest rate projection. And we can also see that it takes about 2 years longer for the lower harvest rate to 'catch up' to the higher rate. In figure 3 I decided to examine how the population of chinook salmon in Lake Ontario would change if humans ceased our stocking of fish from hatcheries and our catching of the salmon, we can see that the population would slowly increase and may eventually approach carrying capacity for the population in the lake. I recognize that it is very unlikely that fishing for chinook salmon in the lake would cease, as it is a source of food for many people as well as a huge tourism activity benefiting New York state and Ontario, Canada. So I am not suggesting that we entirely stop harvesting salmon, but rather consider the number of salmon we have to raise and stock into the lake, just for them to be caught before breeding age and not replenish their own population, and consider how a more sustainable amount of harvesting could benefit both the population of chinook salmon in Lake Ontario and the people who depend on these fish as a valuable resource.

References:

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