

Online CryoEM Study Group

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1 Dates and Topics

Meetings are generally on Thursdays (morning Pacific time, afternoon Eastern time, evening Europe)

Date	Time	Topic
Fri 21 Jan 2022	9 AM PST	Mathy cryoem paper: VESPER (Han et al., 2021)
Fri 28 Jan 2022	9:15 AM PST	Guest: Qinwen (Wendy) Huang - Weakly Supervised Learning for Joint Image Denoising and Protein Localization in Cryo-EM
Fri 4 Feb 2022	9 AM PST	Rotations - encoding of $S0(3)$
Fri 11 Feb 2022	9 AM PST	Mathy cryoem paper.: Transformations in SPARX and EMAN2 (Baldwin & Penczek, 2007)
Fri 18 Feb 2022	9 AM PST	no meeting
Fri 25 Feb 2022	9 AM PST	Rotating Averaging (Hartley et al., 2013)
Fri 4 Mar 2022	9 AM PST	Mathy cryoem paper: End-to-end orientation estimation from 2D cryo-EM images (Lian et al., 2022)
Fri 11 Mar 2022	9 AM PST	Guest: Lys Sanz Moreta - Deep Probabilistic Programming and 3D Protein Structure (Moreta et al, 2019; Moreta et al 2020)
Fri 18 Mar 2022	9 AM PST	advanced rotations
Fri 25 Mar 2022	9 AM PST	Mathy cryoem paper: Cryofold (Zhong et al., 2021)
Fri 25 Mar 2022	9 AM PST	advanced rotations

2 General Information

2.1 Archived material

Meetings from 2020-2021 are archived here. The audience was a mix of beginners and advanced practitioners, and computational methods developers.

2.2 Audience and Streams

Feel free to share this document and direct people to sign up at <https://forms.gle/BUeUW14vV4pyQbDDA> so I have the emails in one place. Online meeting links are emailed to those on this list. **Please join the Slack group and ask questions there, rather than emailing me.**

2.3 Audience

In 2022, I am catering to a computational methods development audience. I see this group as a way for computational methods developers to get together in a "pre-competitive" learning environment.

Practitioners → computational methods developers: You are a structural biologist, or biochemist, and perhaps an advanced cryo-EM practitioner. You would like to train in computational methods development, either to do very advanced data processing, or develop your own methods.

Pure computational discipline → cryo-EM computational methods developers: You have a background in computer science (computer vision, deep learning, statistics, electrical/computer engineering) and would like to develop methods for the "killer application" of cryo-EM.

2.4 Pre-requisites

The bar is quite high, and this group is not for all. There are very good resources out there for self-study; see this annotated bibliography. If you have done an undergraduate degree in an advanced computational program (physics, chemistry, computer science, statistics, applied math) or are a PhD student in a computational field, then you are in good company in this group.

2.5 Scope

2.5.1 Math / Computer science

1. Amortized inference, model learning
2. Physics aware/inspired/infused deep learning

3. Deep learning of the image formation model (rotation, etc)
4. Computationally modelling uncertainty in the image formation model
5. Geometric deep learning and invariance/equivariance in cryo-EM
6. Computational optimal transport
7. Computational differential geometry
8. Optimization
9. Custom GPU kernels, including gradient for backprop/autodiff

2.5.2 Physics

1. Electron optics
2. Higher order CTF aberrations
3. Multi-slice
4. Sample damage
5. Detector physics
6. Solvation
7. Poisson-Boltzmann equation
8. Modelling choices to encode electrostatic / Coulombic density

2.6 Meeting Format

The meetings are meant to be more informal that is typical in research talks. The point is to learn and discuss with other learners, experienced practitioners, and experts. They are also more comprehensive than typical journal clubs. We may stick with a papers or series of papers for multiple weeks to sufficiently learn the material.

2.7 Slack

We will use the Slack channel 'cryoem_study_group' for asynchronous chat. Please join the Slack group and ask questions there, rather than emailing me. You can request a link to join by emailing me.

2.8 Testimonials

- *Suvrajit Maji, Associate Research Scientist at Columbia University Irving Medical Center & Joachim Frank, Chemistry Nobel Laureate (cryo-EM for biomolecules 2017), Professor of Biochemistry and Molecular Biophysics and Professor of Biological Sciences at Columbia University.* Fortunately, scripts of a tutorial for math underlying single-particle cryo-EM processing are already available in a github repository (15) [https://github.com/geoffwoollard/learn_cryoem_math, Geoffrey Woollard & Ricardo Righetto]. These include notebooks on numerous subtopics on the physics of the electron microscope and image formation (electron optics, electron diffraction patterns, contrast-transfer function), on mathematical concepts (Fourier transforms, convolution, coordinate systems and rotations, interpolations, Fourier slice theorem, 3D reconstruction), on Bayesian inference and estimation theory (maximal likelihood, 2D and 3D image classification, covariance), and on some important image processing tools (alignment via cross-correlation, 3D reconstruction via weighted back-projection, resolution estimation via Fourier shell correlation, detector noise modeling). There are simple object models to experiment with and learn about the basics of phase contrast image formation and data processing.
- *Shayan Shekarforoush, PhD student with Marcus Brubaker and David Fleet, Jan 2021.* I joined this reading group in mid October and I wish I would have done so much earlier. Although I joined when the group was in the middle of reading a fascinating, recently published book in Single-Particle cryoEM, everyone was so welcome that I did not feel I am way behind others. My background is in CS and I do research as a method developer in this field. With that said, I learned a lot from discussions of people with expertise in experimental side of this area. I believe that this group helped me to build a better intuition and now I feel more comfortable with the underlying math and physics of this topic. This group also provided the opportunity to attend talks of prominent researchers in cryoEM where anyone could openly ask their questions and have clear discussions. Looking forward to having more collaborations with the members of this group.

3 Learning Resources

1. I have made an annotated bibliography organized thematically here.
2. Coding notebooks to play around with are here. If there is incompatibility between the notebook and the code base in the repo, that is because the code base has been updated. Older version of the code are available via past commits.

4 Upcoming Meetings

4.1 4 March 2021- End-to-end orientation estimation from 2D cryo-EM images

–Pre-reading

- Lian, R., Huang, B., Wang, L., Liu, Q., Lin, Y., & Ling, H. (2022). End-to-end orientation estimation from 2D cryo-EM images. *Acta Crystallographica Section D Structural Biology*, 78(2), 174?186. <http://doi.org/10.1107/s2059798321011761>

–Questions

1. Introduction: Discuss the problem setup in Figure 1. What are the inputs and output of their method? What sort of specimens (flexibility, symmetry, etc) would it be suitable for?
2. 2.4 Network architecture: Do you understand the network architecture? Draw a schematic (in more details than Figure 2) of the architecture and point out the "256 feature maps with spatial dimensions 16 x 16" and "2048 feature maps with spatial dimensions 4x4".
3. 2.5. Loss functions: engage with this claim: "A straightforward loss function is the L1/L2 distance between the predicted normalized quaternion and the ground truth". Consider the distances for rotations defined in Hartley et al (see past meeting on 24 Feb 2021, 6.5)
4. 2.5: What is the connection between Eqs 11 and 12? Consider the Taylor expansions of cos and arccos
5. 2.5: The authors cite *Manhardt et al., 2019* in support of the claim "Naively regressing the annotated orientations often ends up with predicting an orientation that is closest to all results in the symmetry group". Look up this reference, and learn how it supports their claim.
6. 2.6. Data sets: Given the datasets they used to demonstrate the feasibility of their method, in what scenarios could the method be applied?
7. 2.7: How does the differentiable projection process enter into their method?
8. 2.8: How is the RMSE in Eq 16 different from the reprojection loss in Eq. 13 and 14?
9. 3.1 Orientation estimation from synthetic cryo-EM images: Consider Figures 3 and 4. What are the main take-homes?

10. 3.1: How does the surrounding area (nearby particles) in Figure 4d (real experimental data) affect the optimization, given the formulation of the RMSE and loss?
11. 3.3. Cross-validation tests for w : Study Figures 6 and 7. What story do they tell?
12. 3.4. How is SNR defined here (cf. method section on cryoSPARC)?
13. : 3.4: What is the influence of different noise levels? How much is too much noise? Is the noise in Figures 8 and 9 realistic to empirical data? How close is the sim-to-real gap here?
14. 3.5. Comparison of different representations of 3D orientations: why do you think quaternions performed better than axis-angle?
15. 3.6. 3D reconstruction using network estimation: What is your impression of the visual results shown in Figure 11a-d?
16. 4.3. Evaluation metric and reconstruction quality: How could we "modify the training objective by adding another loss term based on FSC"?
17. 4.4. Handling real cryo-EM images: how could we "efficiently process larger network inputs without significantly increasing the computational cost"?

4.2 11 March 2022 - Guest: Lys Sanz Moreta

–Pre-reading

1. Moreta, L. S., Al-Sibahi, A. S., Theobald, D., Bullock, W., Rommes, B. N., Manoukian, A., & Hamelryck, T. (2019). A Probabilistic Programming Approach to Protein Structure Superposition. 2019 IEEE Conference on Computational Intelligence in Bioinformatics and Computational Biology, CIBCB 2019. <http://doi.org/10.1109/CIBCB.2019.8791469>
2. Moreta, L. S., Al-Sibahi, A. S., & Hamelryck, T. (2020). Bayesian protein superposition using Hamiltonian Monte Carlo. Proceedings - IEEE 20th International Conference on Bioinformatics and Bioengineering, BIBE 2020, 1, 1-11. <http://doi.org/10.1109/BIBE50027.2020.00009>

4.3 TBD - Cryofold

–Pre-reading

1. Zhong, E. D., Lerer, A., Davis, J. H., & Berger, B. (2021). Exploring generative atomic models in cryo-EM reconstruction, (Figure 1), 1-13.

4.4 March/April 2022 - Guest: Ellen Zhong. CryoDRGN2

–Pre-reading

1. Zhong, E. D., Lerer, A., Davis, J. H., & Berger, B. (2021). CryoDRGN2 : Ab initio neural reconstruction of 3D protein structures from real cryo-EM images. Iccv, 4066-4075.

–Questions

1. Write out a schematic (computational graph) of CryoDRGN2. What goes into and out of the encoder and decoder?
2. How could cross validation be incorporated into CryoDRGN? What would count as "out of distribution" for various computational workflows, and how would CryoDRGN perform?

4.5 March/April 2022 - Guest: Roy Lederman.

–Pre-reading

1. Lederman, R. R. (2017). Numerical Algorithms for the Computation of Generalized Prolate Spheroidal Functions. ArXiv, (1), 1-23.
2. Lederman, R. R., & Singer, A. (2017). Continuously heterogeneous hyper-objects in cryo-EM and 3-D movies of many temporal dimensions, 1-33.

4.6 May/June 2022 - Guest: Nina Miolane.

–Pre-reading

1. Donnat, C., Levy, A., Poitevin, F., & Miolane, N. (2022). Deep Generative Modeling for Volume Reconstruction in Cryo-Electron Microscopy, 1-26.

5 Reading list

5.1 Advanced treatment of rigid-body rotations ($\text{SO}(3)$)

–Pre-reading

1. (2022). Bingham Policy Parameterization for 3D Rotations in Reinforcement Learning. <https://arxiv.org/pdf/2202.03957.pdf>
 - $\text{SO}(3)$ distribution
2. Orthogonal Procrustes problem https://en.wikipedia.org/wiki/Orthogonal_Procrustes_problem
3. (2021). Brofos, J. A., Brubaker, M. A., & Lederman, R. R. Manifold Density Estimation via Generalized Dequantization. <https://arxiv.org/pdf/2102.07143.pdf>
4. (2017). State Estimation for Robotics. State Estimation for Robotics. Cambridge: Cambridge University Press. <http://doi.org/10.1017/9781316671528>
5. (2021). Implicit-PDF: Non-Parametric Representation of Probability Distributions on the Rotation Manifold <https://implicit-pdf.github.io/>
6. (2020). A Smooth Representation of Belief over $\text{SO}(3)$ for Deep Rotation Learning with Uncertainty. <https://arxiv.org/pdf/2006.01031.pdf>
7. (2021). Eliminating Topological Errors in Neural Network Rotation Estimation Using Self-selecting Ensembles. <https://dl.acm.org/doi/pdf/10.1145/3450626.3459882>
8. (2021). On the Continuity of Rotation Representations in Neural Networks. <https://arxiv.org/pdf/1812.07035.pdf>
9. (2021). Learning Rotation Invariant Features for Cryogenic Electron Microscopy Image Reconstruction. <https://arxiv.org/pdf/2101.03549.pdf>
10. (2020). $\text{SE}(3)$ -Transformers: 3D Roto-Translation Equivariant Attention Networks. <https://arxiv.org/pdf/2006.10503.pdf>
11. (2020). Falorsi, L., de Haan, P., Davidson, T. R., & Forr, P. Reparameterizing distributions on Lie groups. AISTATS 2019 - 22nd International Conference on Artificial Intelligence and Statistics, 89. <https://arxiv.org/pdf/1903.02958.pdf>
12. (2020). Dmitry Kostyaev. Hands on Tutorials: Better rotation representations for accurate pose estimation. Towards Data Science. <https://towardsdatascience.com/better-rotation-representations-for-accurate-pose-estimation-e890a7e1317f>

- Optimization
- $SO(3)$ encoding

5.2 Assorted mathy cryoem papers

–Pre-reading

1. Tagare, H. D., Kucukelbir, A., Sigworth, F. J., Wang, H., & Rao, M. (2015). Directly reconstructing principal components of heterogeneous particles from cryo-EM images. *Journal of Structural Biology*, 191(2), 245?262. <http://doi.org/10.1016/j.jsb.2015.05.007>
2. Zivanov, J., Nakane, T., & Scheres, S. H. W. (2019). A Bayesian approach to beam-induced motion correction in cryo-EM single-particle analysis. *IUCrJ*, 6(1), 5?17. <http://doi.org/10.1107/S205225251801463X>
3. Katsevich, E., Katsevich, A., & Singer, A. (2015). Covariance matrix estimation for the cryo-em heterogeneity problem. *SIAM Journal on Imaging Sciences*, 8(1), 126-185. <http://doi.org/10.1137/130935434>
4. Penczek, P. A. (2010). Resolution Measures in Molecular Electron Microscopy. In *Methods in Enzymology* (1st ed., Vol. 482, pp. 73?100). Elsevier Inc. [http://doi.org/10.1016/S0076-6879\(10\)82003-8](http://doi.org/10.1016/S0076-6879(10)82003-8)
5. Ede, J. M. (2020). Review: Deep learning in electron microscopy. *ArXiv*. <http://doi.org/10.1088/2632-2153/abd614>
6. Zhu, D., Wang, X., Fang, Q., Etten, J. L. Van, Rossmann, M. G., Rao, Z., & Zhang, X. (n.d.). Pushing the resolution limit by correcting the Ewald reconstructions. *Nature Communications*, (2018), 1-7. <http://doi.org/10.1038/s41467-018-04051-9>
7. Maji, S., Liao, H., Dashti, A., Mashayekhi, G., Ourmazd, A., & Frank, J. (2020). Propagation of Conformational Coordinates across Angular Space in Mapping the Continuum of States from Cryo-EM Data by Manifold Embedding. *Journal of Chemical Information and Modeling*, 60(5), 2484?2491. <http://doi.org/10.1021/acs.jcim.9b01115>
8. Zhong, E. D., Lerer, A., Davis, J. H., & Berger, B. (2021). CryoDRGN2 : Ab initio neural reconstruction of 3D protein structures from real cryo-EM images. *Iccv*, 4066-4075.

5.3 Advanced microscopy

–Pre-reading

1. Glaeser, R. M., Hagen, W. J. H., Han, B. G., Henderson, R., McMullan, G., & Russo, C. J. (2021). Defocus-dependent Thon-ring fading. *Ultramicroscopy*, 222(October 2020), 113213. <http://doi.org/10.1016/j.ultramicro.2021.113213>
2. Russo, C. J., & Egerton, R. F. (2019). Damage in electron cryomicroscopy: Lessons from biology for materials science. *MRS Bulletin*, 44(12), 935?941. <http://doi.org/10.1557/mrs.2019.284>
3. Russo, C. J., & Henderson, R. (2018). Ewald sphere correction using a single side-band image processing algorithm. *Ultramicroscopy*, 187, 26?33. <http://doi.org/10.1016/j.ultramicro.2017.11.001>
4. Electron optics textbook chapters (Hawkes and Kasper; Spence; Reimer and Kohl)

6 Past Meetings

6.1 21 Jan 2022 - VESPER: global and local cryo-EM map alignment using local density vectors.

–Pre-reading

1. Han, X., Terashi, G., Christoffer, C., Chen, S., & Kihara, D. (2021). VESPER: global and local cryo-EM map alignment using local density vectors. *Nature Communications*, 12(1). <http://doi.org/10.1038/s41467-021-22401-y>

–Questions

1. Consider Figure 1 and the Z-score results. When aligning two different maps, and getting DOT scores, how would one re-scale to a Z-score? And to what end?
2. Have you developed any algorithms that start from aligned maps? where did the initial alignment arise from?
3. How do you compare aligning ?by eye? with the algorithms mentioned in the paper, such as the popular fitmap in Chimera. what do you think you are doing in your mind, and how would you break it down into algorithmic steps?
4. Figure 2i (and text on last paragraph of p. 4) shows a case where VESPR did not perform well. two different proteins were predicted to be close in shape ?because they have an overall similar shape and also because these maps are largely hollow inside, and thus inconsistency inside the maps were not much penalized?. Relate this to the DOT score. What other approaches would not suffer from this?
5. In Table 2 there are some cases where the RMSD goes up (worse alignment) with finer grained sampling (rotation angle and translation). Why might this be happening?
6. In the Discussion, the authors explain intuitively how cross correlation (CC) and their DOT score result in different alignments: *In CC, positions with large absolute density values, such as those in a high-density region in a map, influence more to the overall CC value. On the other hand, for the DOT score the contribution of each aligned position pair is essentially the same because the vectors are normalized to the same length. But this also means that the DOT score can be affected by changes in local gradient caused by small structure variations.*

Consider the following cases and think through how the result might be different.

- (a) Continuous conformational heterogeneity: Blurred out flexible region with partial density.
- (b) Discrete conformational heterogeneity: Mixed discrete states in one map vs one of the partners.

- (c) Hinge like displacement (local rigid-body transformation of a domain).
 - (d) Two maps +/- domain.
7. What do you think of the opinion in the Discussion: *Note that, in general, the optimal parameter setting for a method differs for each map and the purpose of the computation. Thus, a perfectly fair comparison is not possible, and the comparison shown in this work is to characterize the performance of VESPER but not to rank the methods.* Hint: see **Responses to Comments by Reviewer #2**, p. 10.
 8. The authors chose a 7\AA grid spacing. Why? How do you suspect the performance of VESPER to change with other grid spacings? Does the method require the same grid spacing when comparing maps? Hint: read the section **Exploration of parameter combinations**.
 9. Reviewer #1 asked the authors to justify what map-to-map alignment is useful. Why would a researcher want to align maps (vs model-to-model and model-to-map)?

6.2 28 Jan 2022 - Guest: Qinwen (Wendy) Huang - Weakly Supervised Learning for Joint Image Denoising and Protein Localization in Cryo-EM

–Pre-reading

1. Qinwen Huang, Ye Zhou, Hsuan-Fu Liu, & Alberto Bartesaghi (2021). Weakly Supervised Learning for Joint Image Denoising and Protein Localization in Cryo-EM https://www.mlsb.io/papers_2021/MLSB2021_Weakly_Supervised_Learning_for.pdf

–Questions

1. This paper exploits analytical likelihood-prior-posterior conjugacy between gaussian distributions. How could we extend this to Poisson noise with Poisson-Gamma conjugacy? What would be Poisson, and what would be Gamma?

6.3 4 Feb 2022 - SO(3) - encoding of S0(3)

–Pre-reading

1. https://en.wikipedia.org/wiki/Euler_angles#Rotation_matrix
2. Gregory G. Slabaugh. Computing Euler angles from a rotation matrix
3. Axis/Angle Representations for Rotations

4. Quine, JR. 4. Orthogonal Transformations and Rotations in Mathematical Techniques in Structural Biology, pp. 25-31
5. Visualizing quaternions (4d numbers) with stereographic projection
6. Quaternions and 3d rotation, explained interactively
7. Visualizing quaternions An explorable video series
8. Wiki: Quaternions and spatial rotation
9. Maxime Tournier's Research Notes. Quaternions

–Questions

1. Problems (e.g. problem 7) from Quine, JR. 4.5 Problems in *Mathematical Techniques in Structural Biology*, pp. 30-31
2. Implement the pseudo-code in Figure 1 of Gregory G. Slabaugh. Computing Euler angles from a rotation matrix, and use it to compute the Euler angles from a rotation matrix.
3. Go to the doublecover interactive, and select the 4D tab on the bottom. Find the quaternion that corresponds to a rotation of 90 deg in the cw direction about the z axis.
4. Quaternions can be used to describe the rotation of the point $r = \langle r_x, r_y, r_z \rangle \in \mathbb{R}^3$ about angle θ in the direction of vector $\langle q_x, q_y, q_z \rangle$ as follows. $p_{rotated} = q * p * q^{-1}$, where $q = \langle \cos(\theta/2), q_x \sin(\theta/2), q_y \sin(\theta/2), q_z \sin(\theta/2) \rangle$ and $q^{-1} = \langle \cos(-\theta/2), q_x \sin(-\theta/2), q_y \sin(-\theta/2), q_z \sin(-\theta/2) \rangle$ encode the rotation; \vec{r} is represented as the quaternion $p = (0, r_x, r_y, r_z)$. Question: using pencil and paper, compute the rotation of the point $r = \langle 1, 0, 0 \rangle$ about the z-axis by $\theta = 180^\circ$ (ccw or cw, it's the same for this angle). Do you get $r = \langle -1, 0, 0 \rangle$? You will have to use the quaternion multiplication table ($ij = k$, etc.), and know how to interconvert quaternions to cartesian 3D vectors.
5. Study the sections Conversion to and from the matrix representation and Performance comparisons on the wiki page for Quaternions and spatial rotation. If we want to rotate many xyz points in space with a 3D rotation, what is the efficient way to do this? Taking into account interconversions between different rotation conventions (Euler angles, rotation matrices, axis-angle, and quaternions) how should we efficiently store rotations versus do rotations?
6. Besides storing and doing rotations, we have to search over rotations. Wikipedia has a section on Differentiation with respect to the rotation quaternion. Unpack the

equation for $\frac{\partial \mathbf{p}'}{\partial \mathbf{q}}$. Would this equation ever be useful in cryoEM? When would we want to estimate the rotation by optimization, and how would we do so with this equation?

6.4 11 Feb 2022 - Transformations in SPARX and EMAN2

–Pre-reading

1. (2007). Baldwin, P. R., & Penczek, P. A. The Transform Class in SPARX and EMAN2. Journal of Structural Biology, 157(1), 250-261. <http://doi.org/10.1016/j.jsb.2006.06.002>
 - (a) 2.1. Eulerian angles in different conventions
 - (b) 2.1.1. Internal representations and usage
 - (c) 2.2. Composition of rotations, distance between rotations, volume element for rotation parameters
 - (d) 2.2.1. Composition of rotations
 - (e) 2.2.2. The similarity between rotations; metric
 - (f) 2.2.3. Volume element
 - (g) 2.3. Quasi-uniform sampling of rotation parameters

–Questions

1. 2.1: Compare eq. 7 to the treatment in https://en.wikipedia.org/wiki/Rotation_matrix#Rotation_matrix_from_axis_and_angle. What is $\hat{n}\hat{n}$?
2. 2.1: Show that eq. 8 follows from eq. 7.
3. 2.1.1: Compute the RHS of eq 17. Multiply two 4x4 matrices of the form eq. 16 and show they are still of the form in eq. 16. What is the new translation of \bar{M} ?
4. 2.2: Derive eqs. 18,19,20,21
5. 2.2: Derive eq. 23 and the accompanying equation for $(d\hat{n})^2$ in the paragraph that follows.
6. 2.3: Unpack eq. 27. $\delta\theta = \arctan(1/R)$.
7. 2.3: Connect Figure 1 to eqs. 28,29.
8. What other ways are there to do uniform sampling on the sphere?

6.5 25 Feb 2022 - $SO(3)$ - encoding of S^3

–Pre-reading

- (2013). Hartley et al., Rotation Averaging. <https://link.springer.com/content/pdf/10.1007/s11263-012-0601-0.pdf>
 1. Introduction
 2. Previous Work on Rotation Averaging
 3. Alternative Pictures of Rotation Space
 4. Distance Measures on $SO(3)$
 5. Single Rotation Averaging

–Questions

1. See accompanying notebook I made https://github.com/geoffwoollard/learn_cryoem_math/blob/master/nb/rotation_averaging.ipynb
2. Does the rotation averaging problem arise in cryo-EM? If so, where? What about the conjugate and multiple rotating averaging problems?
3. 4.1.5: Table 2 Relationship between the different metrics on $SO(3)$: Code up the various metrics (d_{chord} , d_{quat}) and numerically verify these relationships on some (arbitrary) rotations
4. A ball in $SO(3)$ is defined in the appendix. To build some intuition, compute the angular distance between R_1 and R_2 that are (1) about the same axis and differ by angle θ ; (2) rotate the same angle (e.g. 5 degrees), but about axes that sweep from the Z-axis, down to the X-axis, through the ZX plane. Relate (1) and (2) to the ball.
5. Draw a picture illustrating **Proposition 2**.
6. 5.3: Code up Algorithm 1 for several rotations. How do you do the log and exp in step 3 and 7? Does your mean seem "in the middle"?
7. We can average together quaternions to get a new quaternion. This is a solution to the rotation averaging problem using what distance?