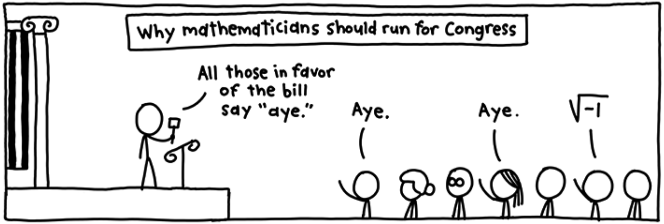
[1] The Field

The Field: Introduction to complex numbers

Solutions to $x^2 = -1$?

Mathematicians invented $\mathbf{i}$ to be one solution



Guest Week: Bill Amend (excerpt, `http://xkcd.com/824`)

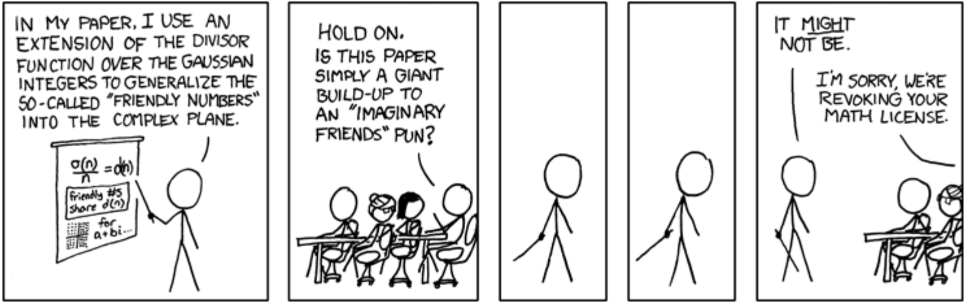
Can use $\mathbf{i}$ to solve other equations, e.g.:

$$x^2 = -9$$

Solution is $x = 3\mathbf{i}$

Introduction to complex numbers

Numbers such as $\mathbf{i}$, $-\mathbf{i}$, $3\mathbf{i}, 2.17\mathbf{i}$ are called *imaginary* numbers.



Math Paper (`http://xkcd.com/410`)

The Field: Introduction to complex numbers

- Solution to $(x-1)^2 = -9$?
- $x = 1 + 3\mathbf{i}$.

- A real number plus an imaginary number is a *complex number*.
- A complex number has a *real part* and an *imaginary part*.

$$\text{complex number} = (\text{real part}) + (\text{imaginary part})\,\mathbf{i}$$

The Field: Complex numbers in Python

Abstracting over *Fields*

- *Overloading:* Same names ($+$, etc.) used in Python for operations on real numbers and for operations complex numbers
- Write procedure `solve(a,b,c)` to solve $ax + b = c$:

  ```
  >>> def solve(a,b,c): return (c-b)/a
  ```

  Can now solve equation $10x + 5 = 30$:

  ```
  >>> solve(10, 5, 30)
  2.5
  ```
- Can also solve equation $(10 + 5\mathbf{i})x + 5 = 20$:

  ```
  >>> solve(10+5j, 5, 20)
  (1.2-0.6j)
  ```
- Same procedure works on complex numbers.

Abstracting over *Fields*

Why does procedure works with complex numbers?

Correctness based on:

- / is inverse of *
- - is inverse of $+$

Similarly, much of linear algebra based just on $+$, -, *, / and algebraic properties

- / is inverse of *
- - is inverse of $+$
- *addition is commutative:* $a + b = b + a$
- *multiplication distributes over addition:* $a * (b + c) = a * b + a * c$
- etc.

You can plug in any collection of "numbers" with arithmetic operators $+$, -, *, / satisfying the algebraic properties— and much of linear algebra will still "work".

Such a collection of "numbers" with $+$, -, *, / is called a *field*.
Different fields are like different classes obeying the same interface.