Ogilvie 1999 equation 124:

$$\Sigma r^2 \Omega \left(\frac{\partial \vec{l}}{\partial t} + \bar{v}_r \frac{\partial \vec{l}}{\partial r} \right) = Q_1 \mathcal{I} r \Omega^2 \frac{\partial \vec{l}}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left(Q_2 \mathcal{I} r^3 \Omega^2 \frac{\partial \vec{l}}{\partial r} \right)$$
$$+ Q_2 \mathcal{I} r^2 \Omega^2 \left| \frac{\partial \vec{l}}{\partial r} \right|^2 \vec{l} + \frac{1}{r} \frac{\partial]}{\partial r} \left(Q_3 \mathcal{I} r^3 \Omega^2 \vec{l} \times \frac{\partial \vec{l}}{\partial r} \right) - \vec{l} \times \left(\vec{l} \times \vec{T} \right)$$

Linearizing:

$$\vec{l} = (\lambda_x, \lambda_y, 1)$$
$$\lambda^2 \ll 1$$

Units:

$$\mathcal{G} = c = 1$$

Keplerian:

$$\Omega^2 r^3 = M$$

Terms:(linear in λ)

$$\begin{aligned}
\vec{l} \times \frac{\partial \vec{l}}{\partial r} &= -\frac{\partial \lambda_y}{\partial r} \hat{x} + \frac{\partial \lambda_x}{\partial r} \hat{y} \\
-\vec{l} \times \left(\vec{l} \times \vec{T} \right) &= \Sigma \Omega r^2 \left(\vec{\Omega}_{LT} \times \vec{l} \right) \\
&= \Sigma \Omega r^2 \left(-\Omega_{LT} \lambda_y \hat{x} + \Omega_{LT} \lambda_x \hat{y} \right) \\
\left| \frac{\partial \vec{l}}{\partial r} \right|^2 &= 0
\end{aligned}$$

where

$$\Omega_{LT} = \frac{2 \mathcal{G} M a}{c^2 r^3} = \frac{2 M^2 a'}{r^3}, \quad 0 \geq a' = a/M \leq 1$$

taking $a' = 1$ the x and y components of the equation with time derivative equals to 0:

$$\Sigma\sqrt{Mr}\bar{v}_r \frac{\partial \lambda_{x,y}}{\partial r} = Q_1 \mathcal{I} \frac{M}{r^2} \frac{\partial \lambda_{x,y}}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(Q_2 \mathcal{I} M \frac{\partial \lambda_{x,y}}{\partial r}\right)$$
$$\mp \frac{1}{r}\frac{\partial}{\partial r}\left(Q_3 \mathcal{I} M \frac{\partial \lambda_{y,x}}{\partial r}\right) \mp \Sigma\sqrt{Mr}\Omega_{LT}\lambda_{y,x}$$

multiplying second equation with i and adding to the first one with $w = \lambda_x + i\lambda_y$

$$\Sigma\sqrt{Mr}\bar{v}_r \frac{\partial w}{\partial r} = Q_1 \mathcal{I} \frac{M}{r^2} \frac{\partial w}{\partial r} + \frac{1}{r}\frac{\partial}{\partial r}\left(Q_2 \mathcal{I} M \frac{\partial w}{\partial r}\right)$$
$$+ i\frac{1}{r}\frac{\partial}{\partial r}\left(Q_3 \mathcal{I} M \frac{\partial w}{\partial r}\right) + i\Sigma\sqrt{Mr}\Omega_{LT} w$$

with $Q_4 = Q_2 + iQ_3$, and isothermal $\mathcal{I} = \Sigma H^2$(Ogilvie 1999 page 18)

$$\bar{v}_r \frac{\partial w}{\partial r} = \frac{Q_1 H^2}{\sqrt{Mr}} \frac{M}{r^2} \frac{\partial w}{\partial r} + \frac{1}{\Sigma\sqrt{Mr}} \frac{1}{r}\frac{\partial}{\partial r}\left(Q_4 \mathcal{I} M \frac{\partial w}{\partial r}\right) + i\Omega_{LT} w$$

using α-disk model(Papaloizou, Pringle 1983):

$$\Sigma = \frac{\dot{M}\left[1 - \left(\frac{R_*}{r}\right)^{1/2}\right]}{3\pi\alpha H^2 \Omega}$$
$$H = H_* \left(\frac{r}{R_*}\right)^g \left[1 - \left(\frac{R_*}{r}\right)^{1/2}\right]^{1/2}$$
$$\bar{v}_r = -\frac{\dot{M}}{2\pi\Sigma r} = -\frac{3\alpha H_*^2 \left(\frac{r}{R_*}\right)^{2g} \sqrt{M}}{2r^{5/2}}$$

Up to first order in $|\psi|$, Q_1, Q_4 only depend on α, $Q_1 = Q_{10}$, $Q_4 = Q_{40}$

$$\frac{1}{\Sigma\sqrt{Mr}} \frac{1}{r}\frac{\partial}{\partial r}\left(Q_4 \mathcal{I} M \frac{\partial w}{\partial r}\right) = \frac{3\pi\alpha H^2 \Omega}{\dot{M}\left[1 - \left(\frac{R_*}{r}\right)^{1/2}\right]} \frac{Q_{40}\sqrt{M}}{r^{3/2}} \frac{\partial}{\partial r}\left(\Sigma H^2 \frac{\partial w}{\partial r}\right)$$
$$= H_*^2 \left(\frac{r}{R_*}\right)^{2g} \frac{Q_{40}\sqrt{M}}{r^3} \frac{\partial}{\partial r}\left(r^{3/2}\left[1 - \left(\frac{R_*}{r}\right)^{1/2}\right] \frac{\partial w}{\partial r}\right)$$

with $x = \left(\frac{R_*}{r}\right)^{1/2}$

$$\frac{1}{\Sigma\sqrt{Mr}} \frac{1}{r}\frac{\partial}{\partial r}\left(Q_4 \mathcal{I} M \frac{\partial w}{\partial r}\right) = \frac{Q_{40}}{2} \frac{\sqrt{M} H_*^2}{R_*^{5/2}} x^{6-4g} \frac{d}{dx}\left((1-x)\frac{dw}{dx}\right)\frac{dx}{dr}$$

Then plugging in $\bar{v}_r$, multiplying the whole equation with $\frac{dr}{dx}$, and letting $v = (1-x)\,\frac{dw}{dx}$

$$\frac{3\alpha}{2}x^{5-4g}\frac{dw}{dx} + Q_{10}x^{5-4g}v - \frac{Q_{40}}{2}x^{6-4g}\frac{dv}{dx} - i\frac{4Ma'}{H_*^2}\sqrt{MR_*}x^3 w = 0$$

Let $g = 9/8$ (Kumar Pringle 1985)

$$\frac{3\alpha}{2}\frac{1}{x\,(1-x)}\frac{dv}{dx} + Q_{10}\frac{1}{x}v - \frac{Q_{40}}{2}\frac{dv}{dx} - i\frac{4Ma'}{H_*^2}\sqrt{MR_*}x^{3/2}w = 0$$

Approximately $R_* \approx 3M$ and $H_* \approx 0.1R_* \approx 0.3M$. Then,

$$\frac{dv}{dx} = \frac{3\alpha}{Q_{40}}\frac{1}{x\,(1-x)}\frac{dv}{dx} + \frac{2Q_{10}}{Q_{40}}\frac{1}{x}v - i\frac{150}{Q_{40}}x^{3/2}w$$