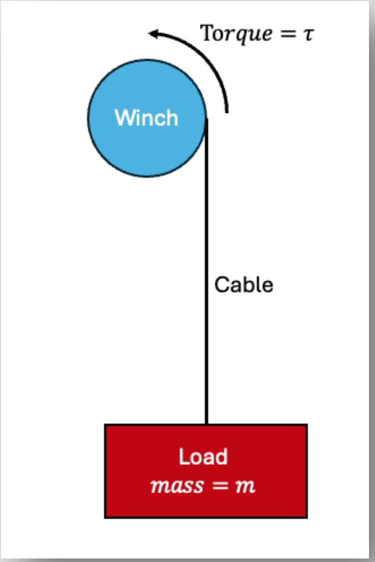
Simulating Continuous Systems Part 3

CMSC 326 Simulations

Simulating Continuous Systems

Ricker Model of Population Growth

Rotational Motion



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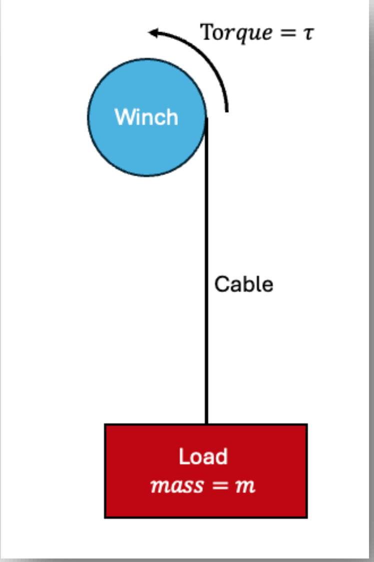

Ricker Model of Population Growth

Time is modeled by discrete time steps:

$$\mathrm{x}(n) = \text{population at step } n$$

Ricker Model of Population Growth

The Ricker model is:

$$\mathrm{x}(n+1) = r\, x(n)\, e^{-x(n)}$$

Ricker Model of Population Growth

$$\underbrace{\mathrm{x}(n+1)}_{\text{The population time step } n+1} = r\, x(n)\, e^{-x(n)}$$

Ricker Model of Population Growth

$$\mathrm{x}(n+1) = \underbrace{r\, x(n)\, e^{-x(n)}}_{\text{A calculation including the current population } x(n)}$$

Ricker Model of Population Growth

$$\mathrm{x}(n+1) = \underbrace{r}_{\text{Growth rate constant}}\, x(n)\, e^{-x(n)}$$

Ricker Model of Population Growth

$$\mathrm{x}(n+1) = \underbrace{r\,x(n)}\,e^{-x(n)}$$

The **population growth** part of the model

Ricker Model of Population Growth

$$\mathrm{x}(n+1) = r\,x(n)\,\underbrace{e^{-x(n)}}_{\text{The }\textbf{population decline}\text{ part of the model}}$$

Ricker Model of Population Growth

One might expect the sequence

$$\mathrm{x}(1), x(2), x(3), \ldots$$

to converge to a “stable” population

called an attractor

Ricker Model of Population Growth

For each r, it seems plausible that we will get a different stable population for large enough n

$$\mathrm{x}(n+1) = r\, x(n)\, e^{-x(n)}$$

Ricker Model of Population Growth

```python
# Population grown factor
r = 2

# Initialize population x(0)
x = 1

for n in range(500):

    # Compute the Ricker model formula
    x = r * x * np.exp(-x)

    print(f"n = {n},  r = {r},  x = {x:0.4f}")
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Ricker Model of Population Growth

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n = 490,  r = 4,  x = 1.3863
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Ricker Model of Population Growth

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n = 489,  r = 14,  x = 0.4270
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n = 492,  r = 14,  x = 5.1239
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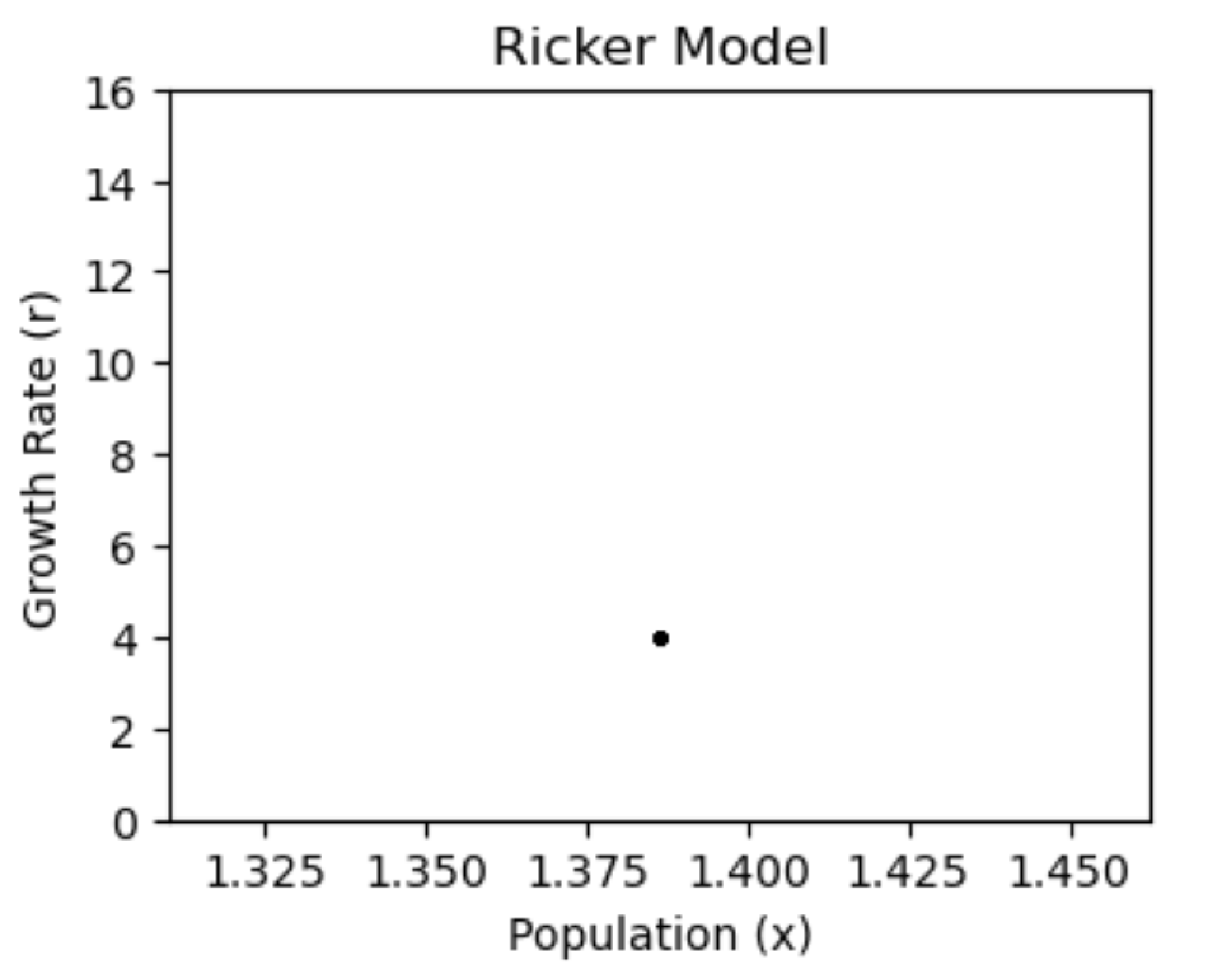
Ricker Model of Population Growth

What we'll do:

1. For a given r, run the sequence and compute a histogram of all the $\mathrm{x}(\mathrm{n})$ values obtained
2. If the sequence does in fact stabilize, we should see a single convergence point show up in the histogram
3. Plot any convergence point that shows up more than once, after ignoring $x(1), \ldots, x(50)$
(to allow convergence)

Ricker Model of Population Growth



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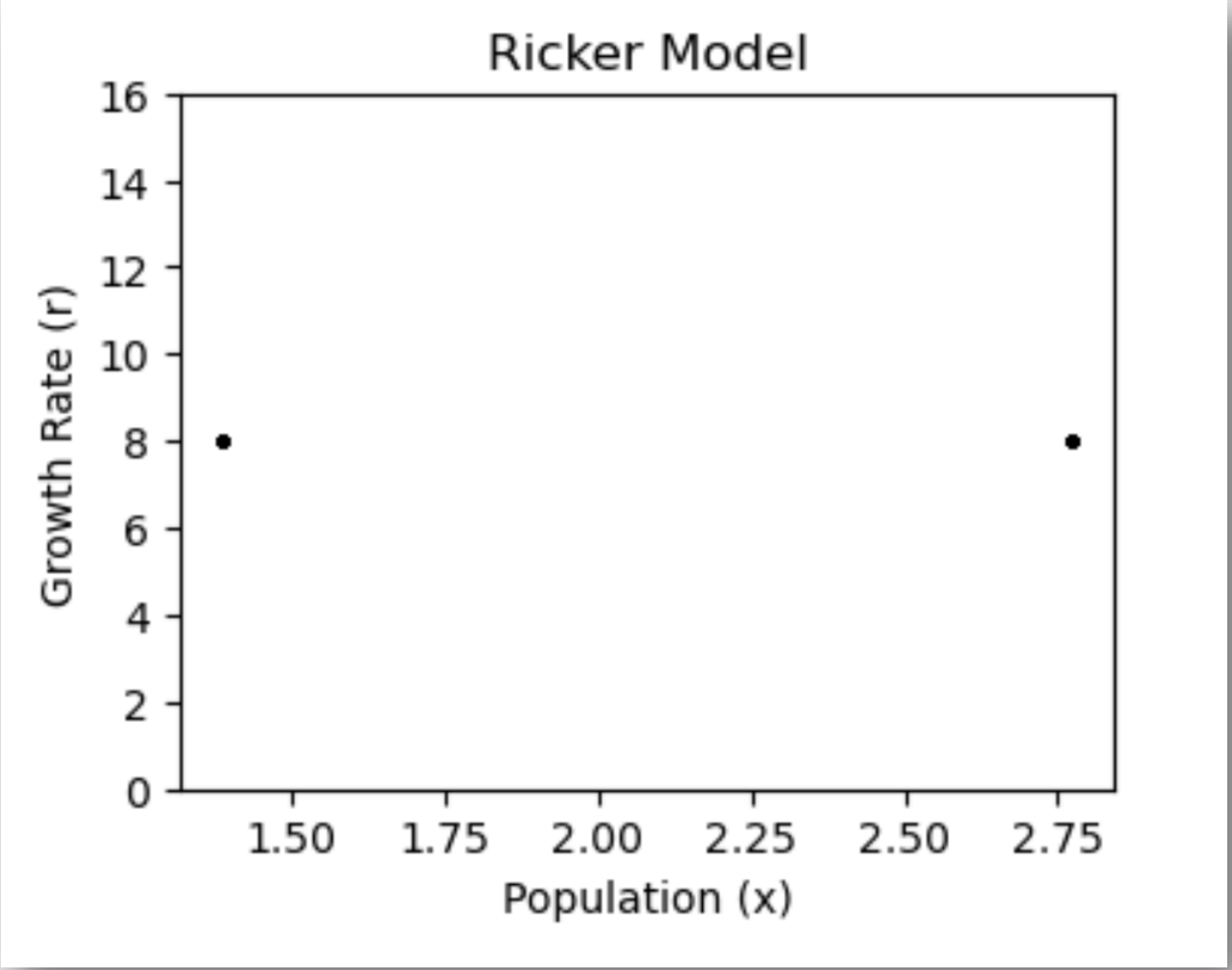

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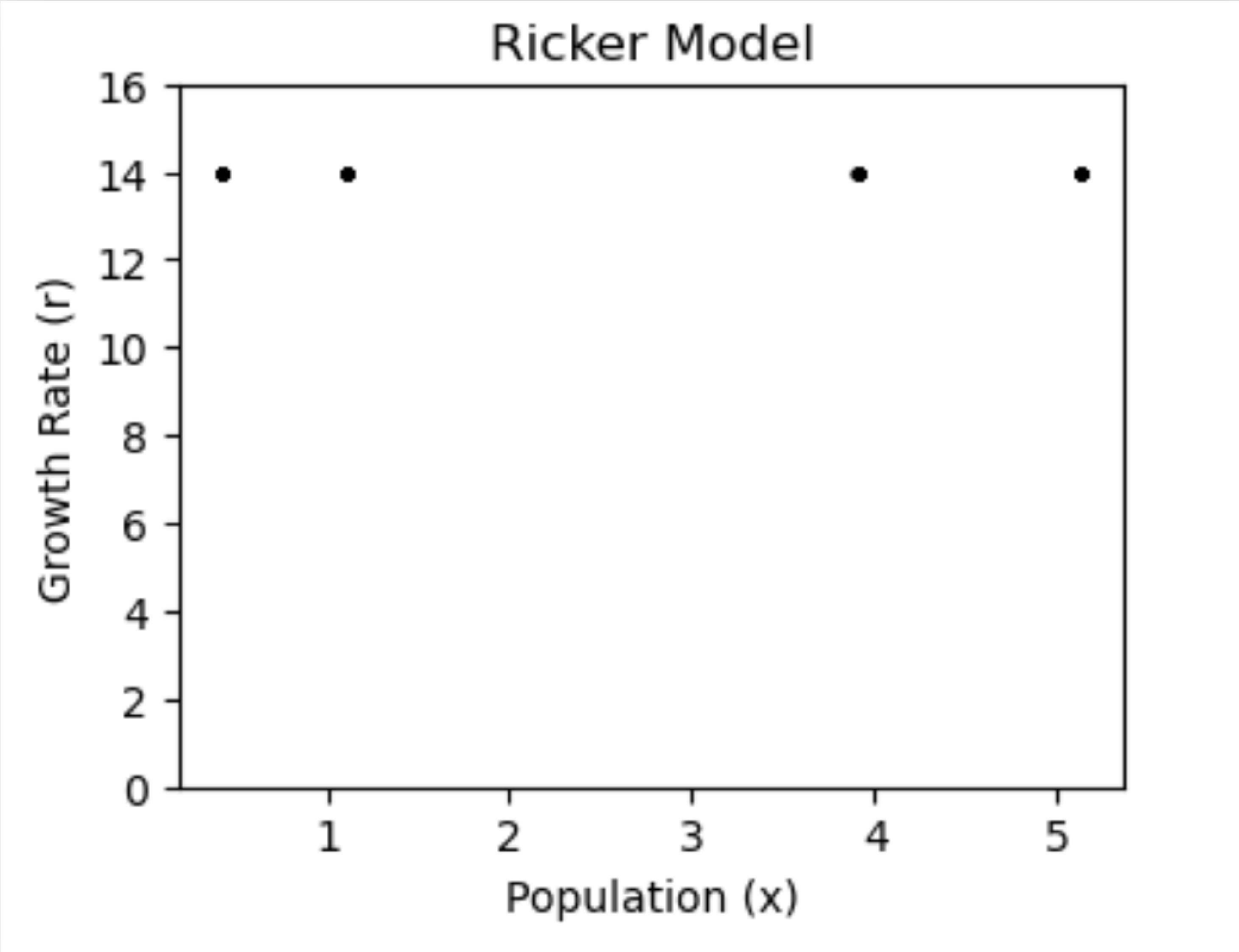

Ricker Model of Population Growth

Thus, we will have a collection of attractor points for any fixed r

Plot the attractors for different values of r

Ricker Model of Population Growth

```
# Loop through values of r
r = 1.0
while r < r_lim:
    # Initial population x(0)
    x = 1

    # Iterate
    for n in range(500):
        x = r * x * np.exp(-x)
        if n > 50:  # Ignore transient behavior
            x_vals.append(x)
            r_vals.append(r)

    r = r + delta_r  # Increment r
```

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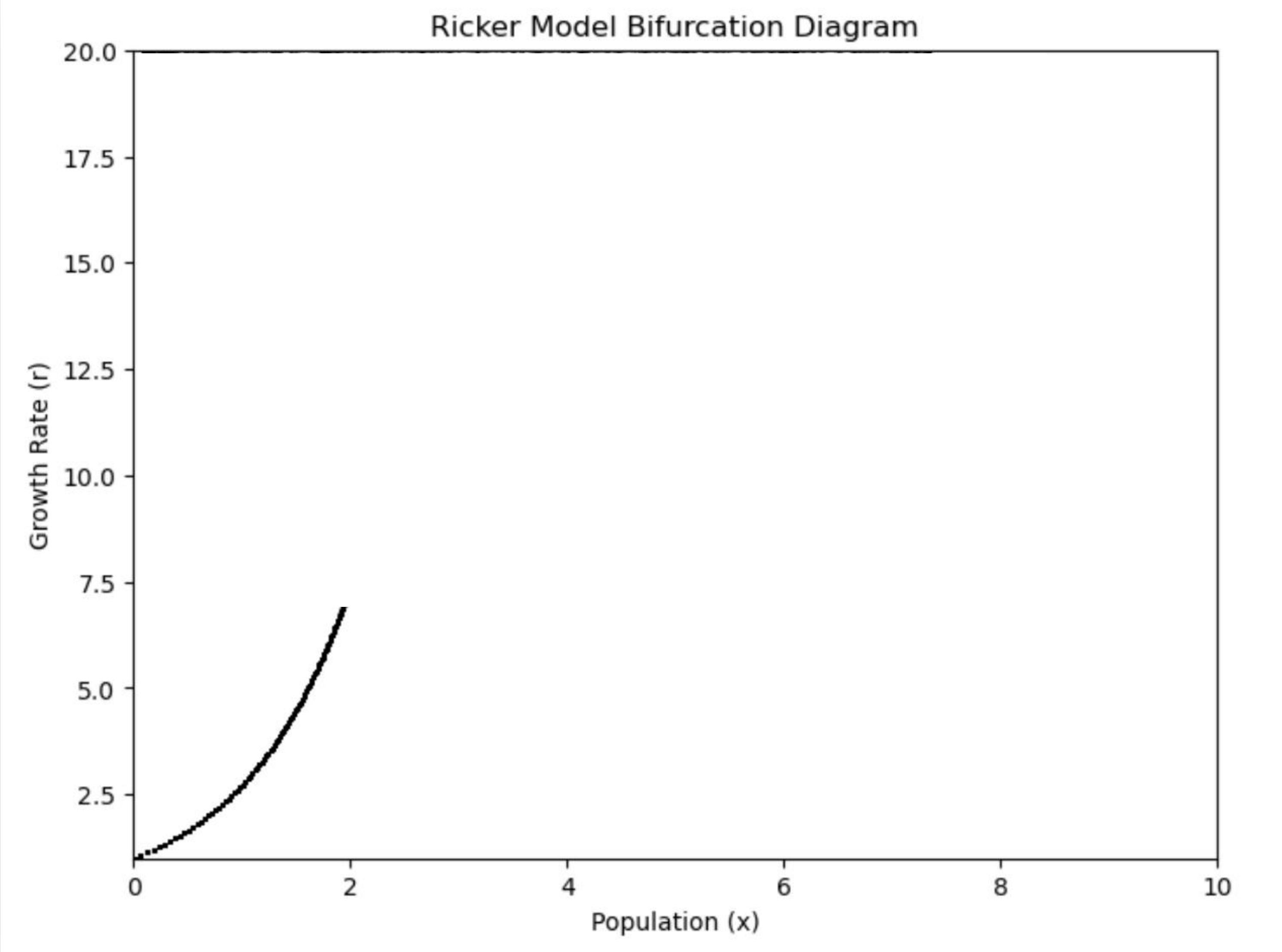

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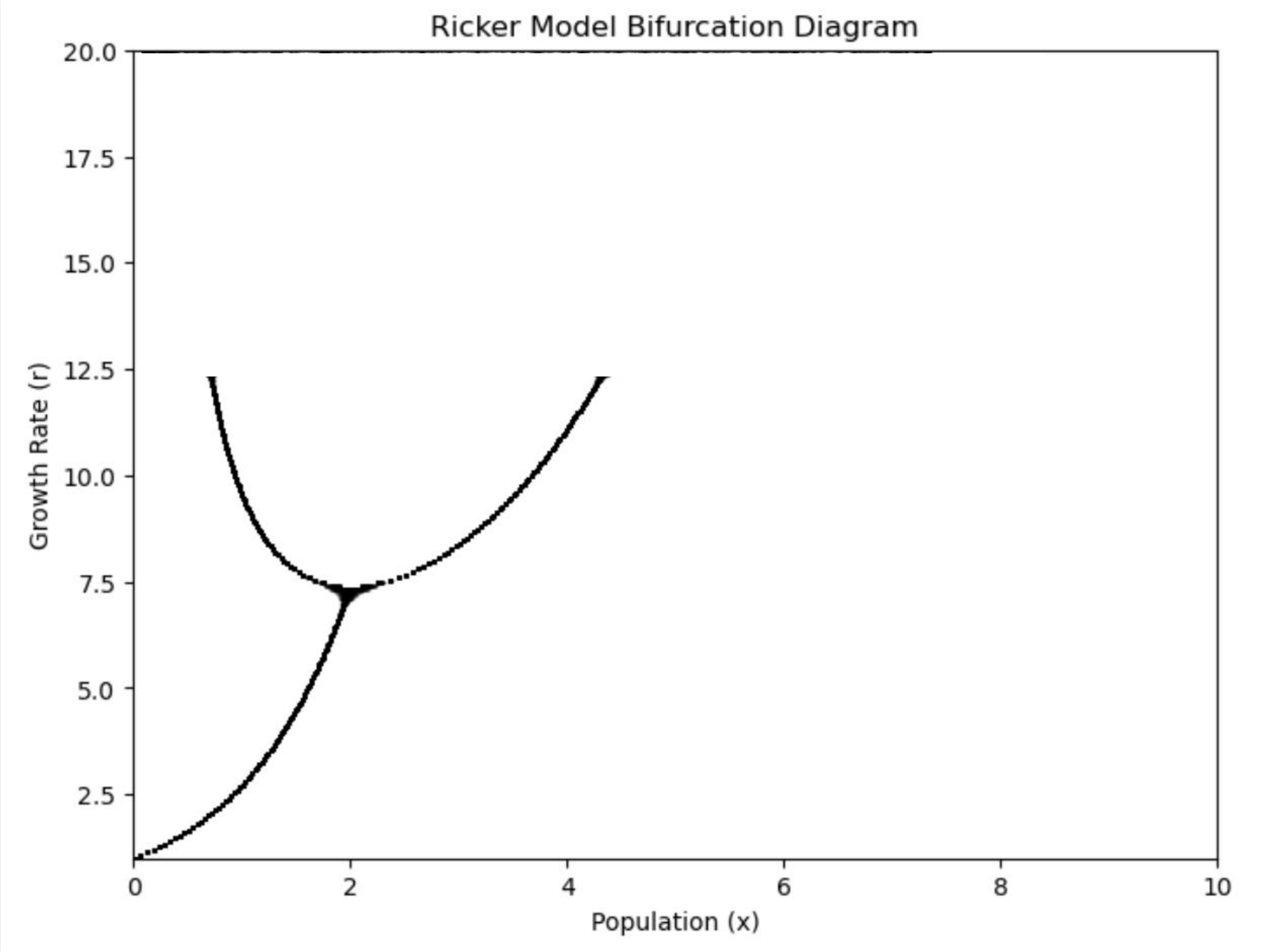

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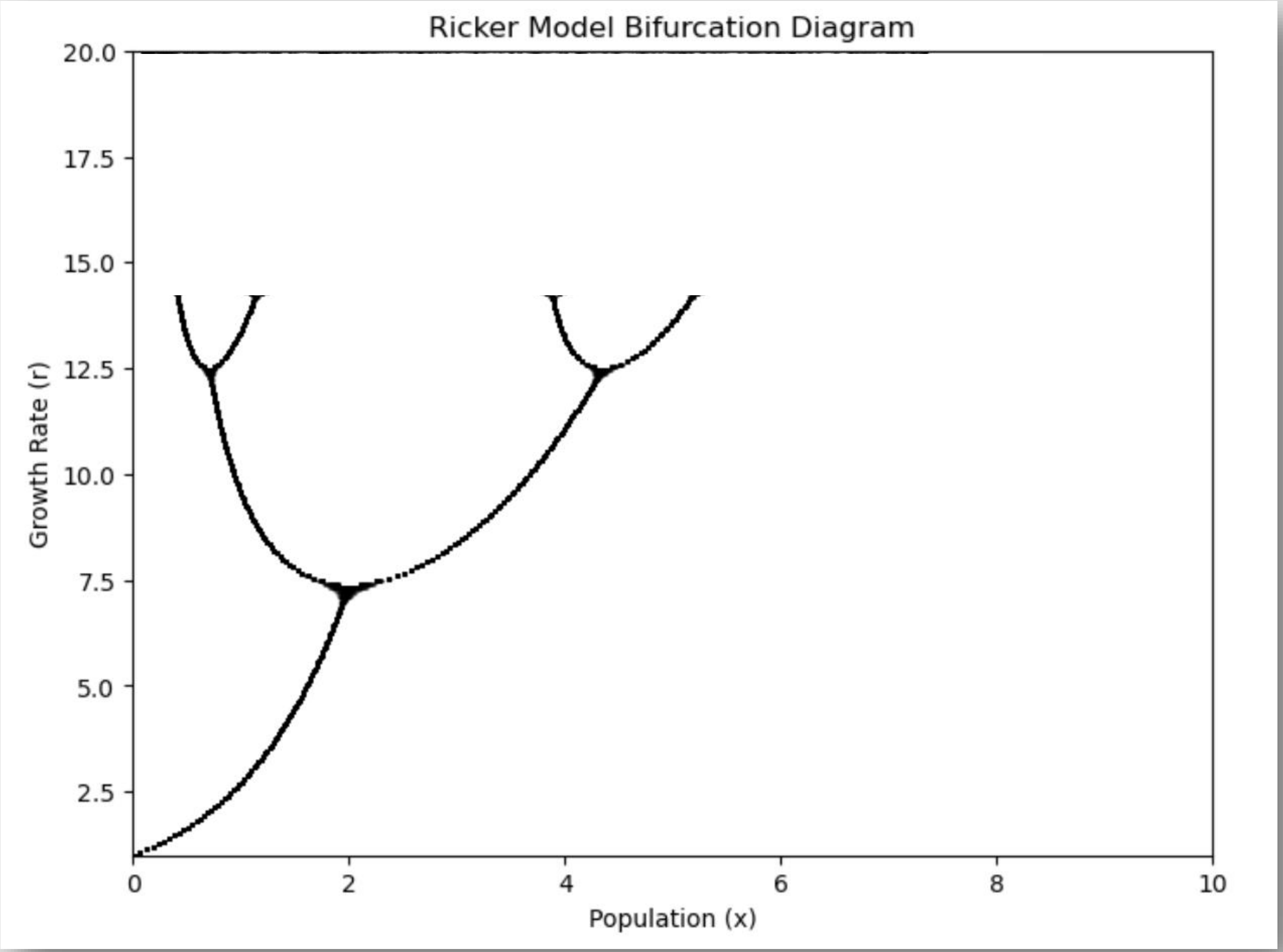

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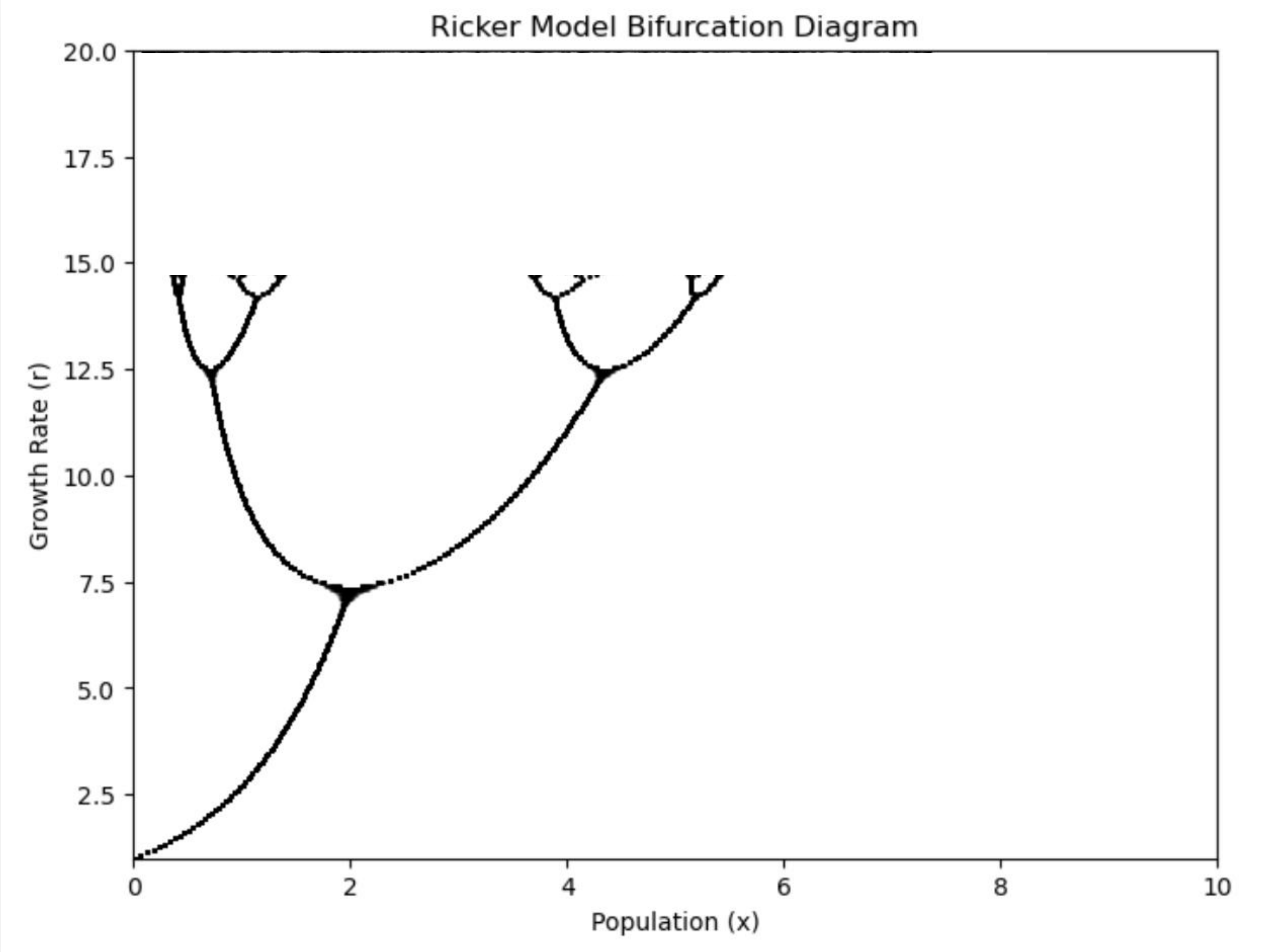

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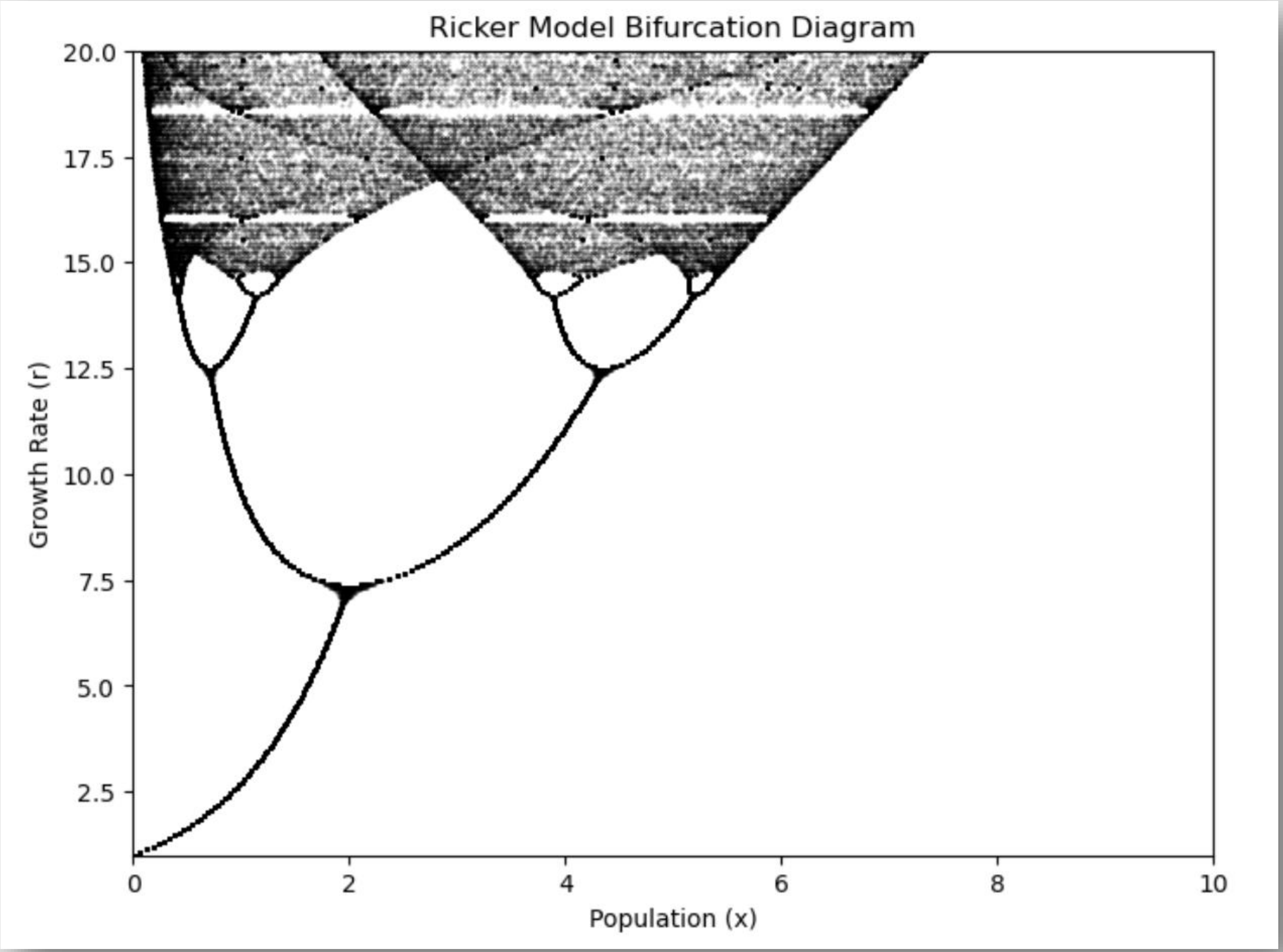


What do we learn from these examples?

A **changing system** can be mathematically modeled in many ways, the most common of which are:

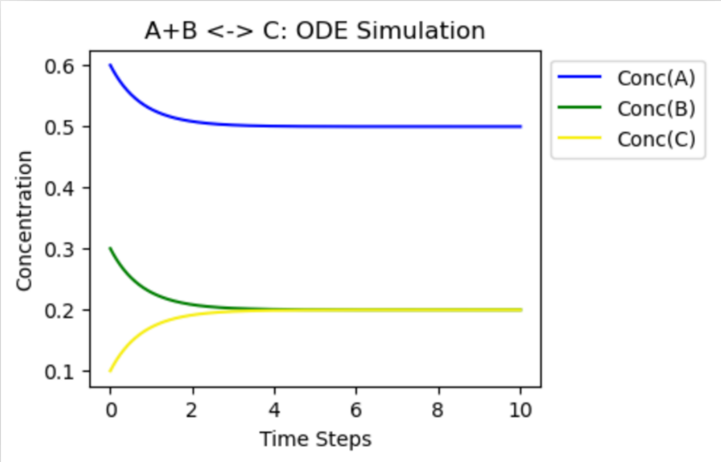
1. **Fully continuous** (differential equations)
2. **Fully discrete**
3. **Mixed discrete and continuous** (Ricker model)

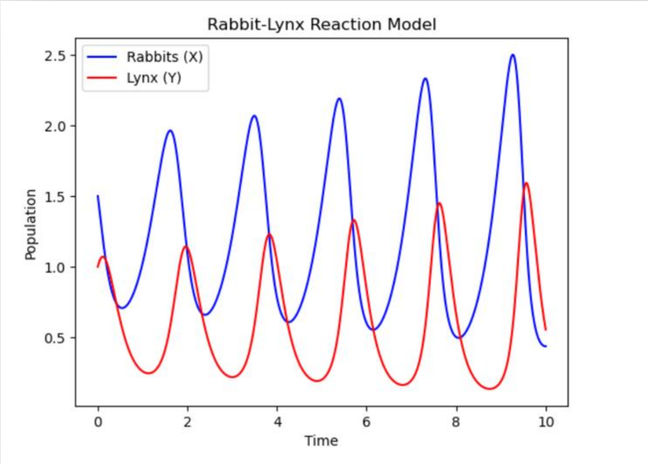
What do we learn from these examples?

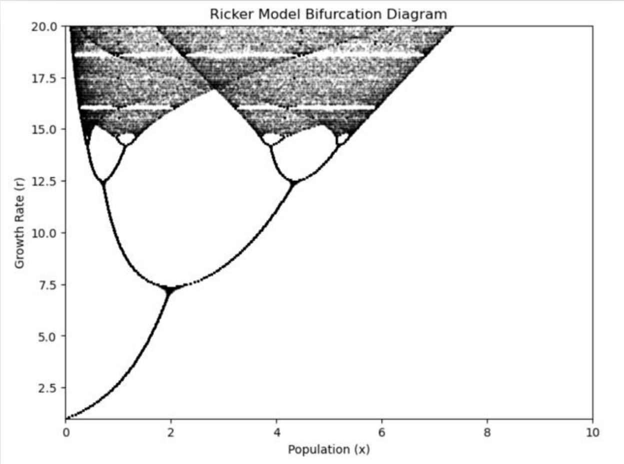
It is very hard to predict the behavior of dynamical systems just by staring at the governing equation

- For some parameters, a system might behave well
- For others, we might get wild oscillations or chaos

What do we learn from these examples?







$$A(t+s) = A(t) + s\,(K_c\,C(t) - K_{ab}\,A(t)\,B(t))$$

$$B(t+s) = B(t) + s\left(K_c\,C(t) - K_{ab}\,A(t)\,B(t)\right)$$

$$C(t+s) = C(t) + s\,(K_{ab}\,A(t)\,B(t) - K_c\,C(t))$$

$$X(t+s) = X(t) + s\,(k_1\,X(t) - k_2\,X(t)\,Y(t))$$

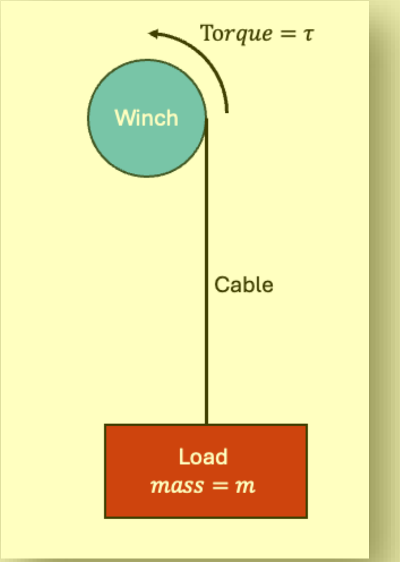
$$Y(t+s) = Y(t) + s\,(k_2\,X(t)Y(t) - k_3\,Y(t))$$

$$\mathrm{x}(n+1) = r\,x(n)\,e^{-x(n)}$$

Simulating Continuous Systems

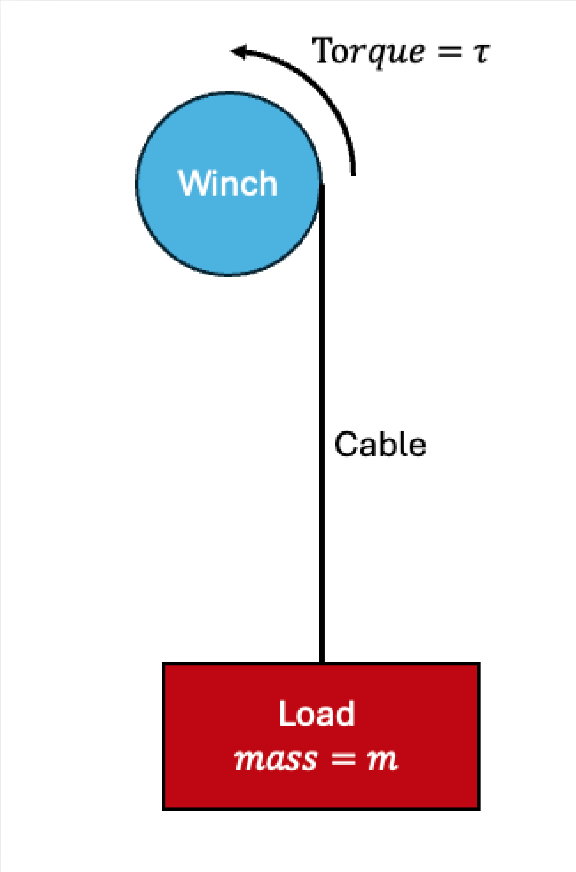
Ricker Model of Population Growth

Rotational Motion



Rotational Motion

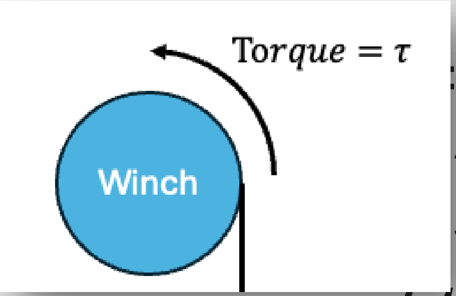
Consider a weight attached to a winch:



Rotational Motion

Define the following variables for the wheel:

$R =$ radius of wheel



= torque

) = angular displacement at time t

$\omega(t) =$ angular velocity

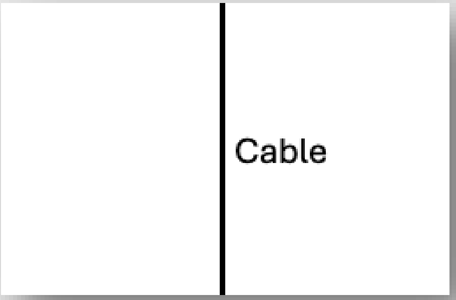
$\mu(t) =$ angular acceleration

$m_w =$ mass of wheel

Rotational Motion

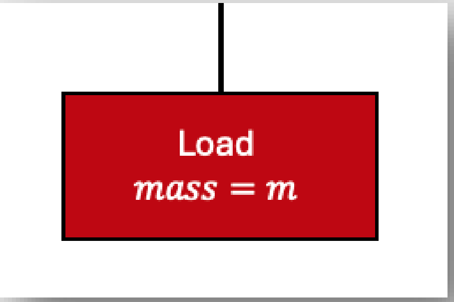
Define the following variables for the cable:

$$T = \text{tension in the cable}$$



Rotational Motion

Define the following variables for the wheel:



m = mass of load

$y(t)$ = vertical displacement of load

$v(t)$ = vertical velocity of load

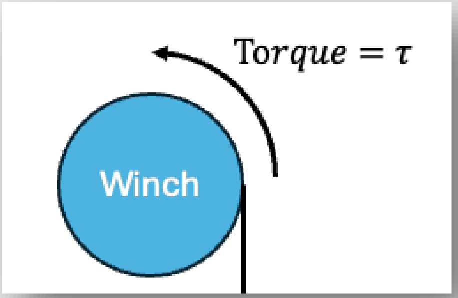
$a(t)$ = vertical acceleration of load

Apply Newton's Laws to Each Component

Let's start with the wheel:

1. The wheel has torque generated from the winch's motor: τ
2. The tension in the cable pulls the other way, and creates a reverse torque of $T\ R$
3. The difference is what determines the angular motion of the wheel:

$$m_w\, R^2\, \mu = \tau - TR$$

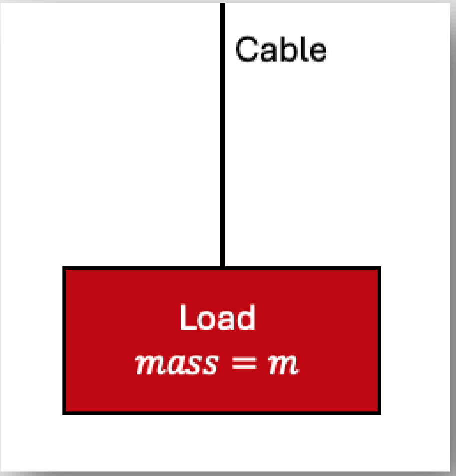


Apply Newton’s Laws to Each Component

Now the load:

1. The upward force is the cable's tension: T
2. The downward force is the weight $m\ g$
3. The difference explains the motion:

$$m\ a = T - m\ g$$

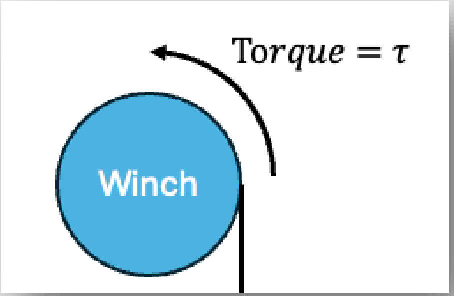


What are the differential equations?

First, the ones relating displacement, velocity, and acceleration for the wheel:

$$\theta'^{(t)} = \omega(t)$$

$$\omega'(t) = \mu(t)$$

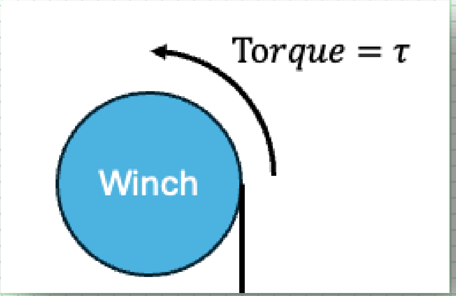


What are the differential equations?

The change in angular displacement over time

$$\theta'(t) = \omega(t)$$

$$\omega'(t) = \mu(t)$$

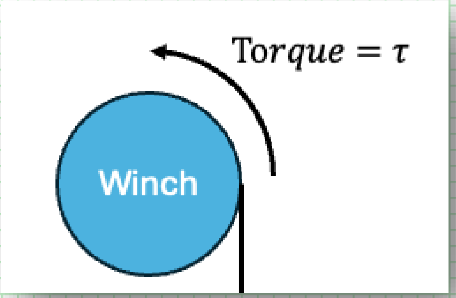


What are the differential equations?

Angular velocity

$$\theta'(t) = \omega(t)$$

$$\omega'(t) = \mu(t)$$

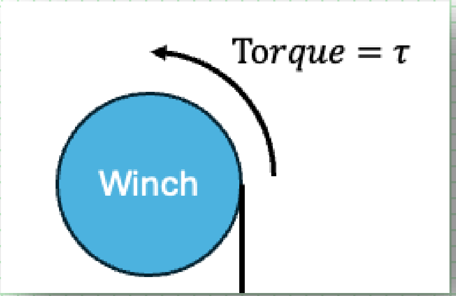


What are the differential equations?

$$\theta'^{(t)} = \omega(t)$$

The change in angular velocity over time

$$\omega'(t) = \mu(t)$$



What are the differential equations?

$$\theta'^{(t)} = \omega(t)$$

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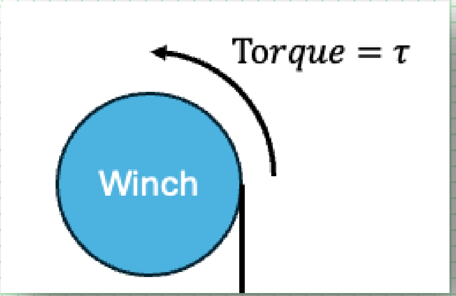
Angular acceleration

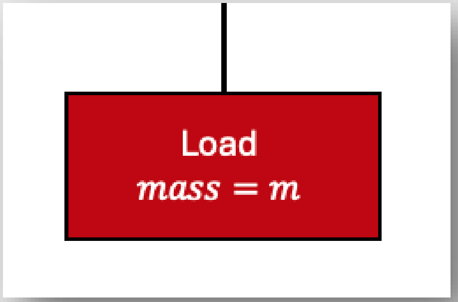


What are the differential equations?

The same for the load:

$$y'(t) = v(t)$$

$$v'(t) = a(t)$$

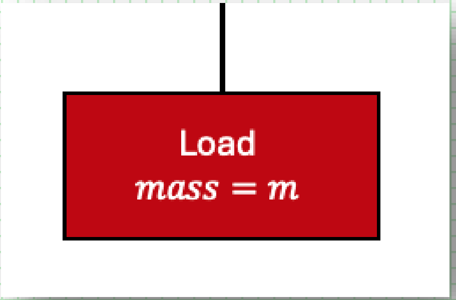


What are the differential equations?

The change in height over time

$$y'^{(t)} = v(t)$$

$$v'(t) = a(t)$$

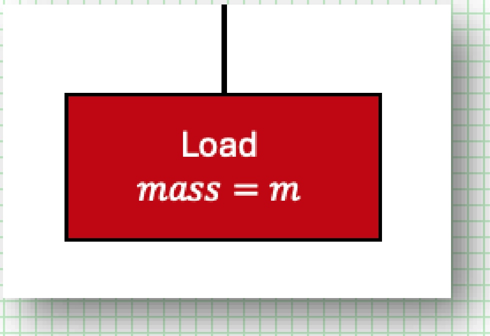


What are the differential equations?

$$y'^{(t)} = v(t)$$

The velocity of the load

$$v'(t) = a(t)$$

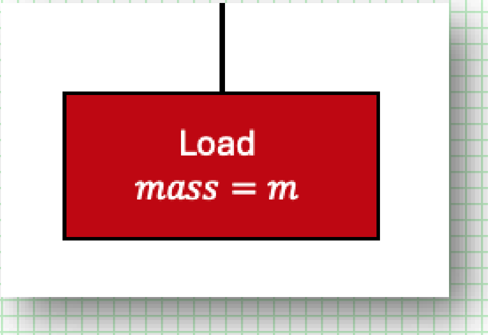


What are the differential equations?

$$y'^{(t)} = v(t)$$

$$v'(t) = a(t)$$

The change in velocity over time

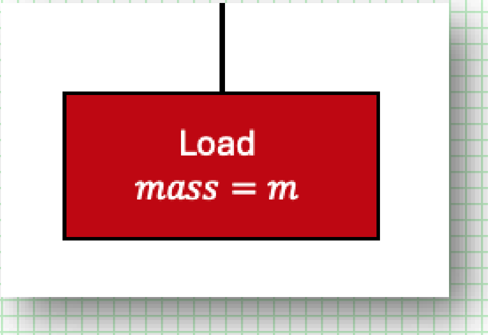


What are the differential equations?

$$y'^{(t)} = v(t)$$

$$v'(t) = a(t)$$

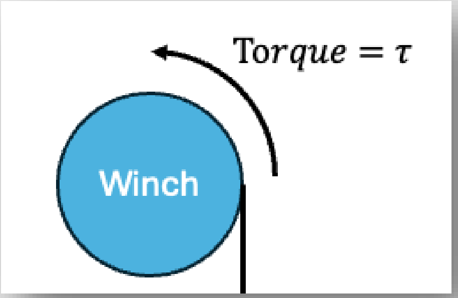
The acceleration of the load



Relate the Equations Through Tension

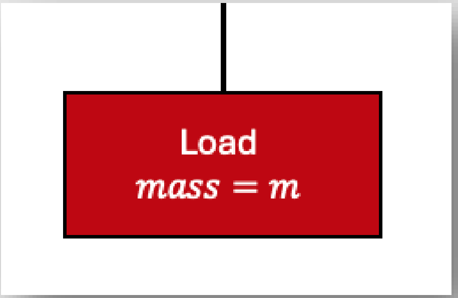
From the wheel:

$$T = \frac{\tau - m_w\, R^2\, \mu}{R}$$



From the load:

$$T = m\, R\, \mu - m\, g$$



Relate the Equations Through Tension

Equating the two and solving for μ:

$$\mu(t) = \frac{\tau - m\,g\,R}{m\,R^2 + m_w\,R^2}$$

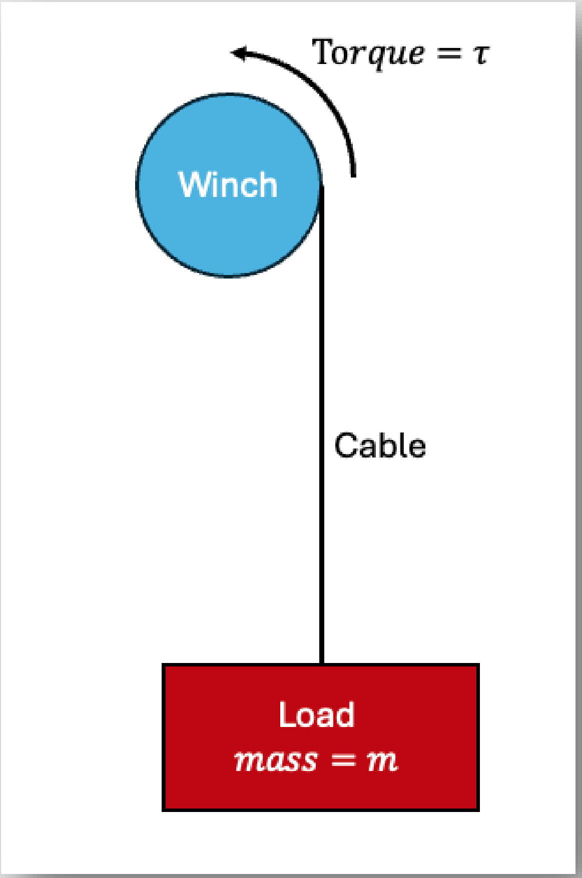
Equations That Describe the System

$$\mu(t) = \frac{\tau - m\,g\,R}{m\,R^2 + m_w\,R^2}$$

$$\theta'(t) = \omega(t)$$

$$\omega'(t) = \mu(t)$$

$$y(t) = R\,\theta(t)$$



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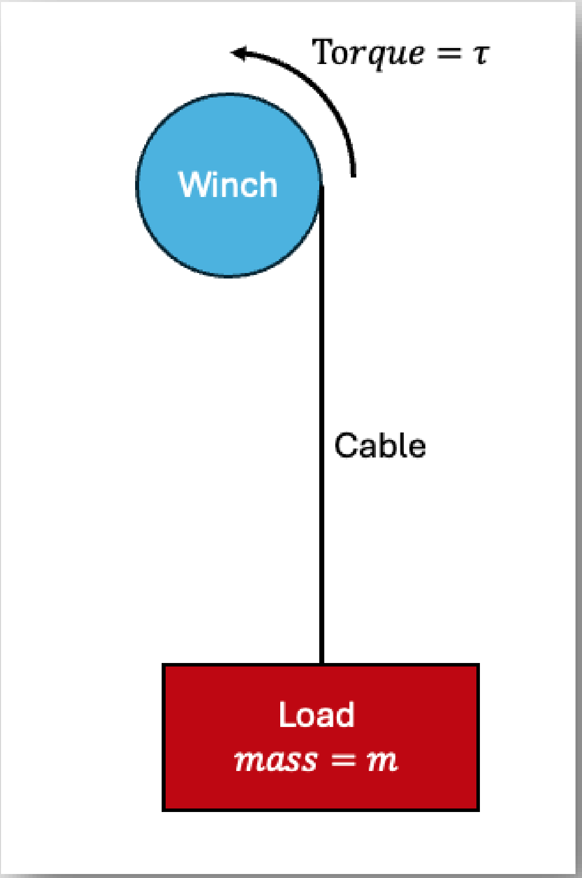
The change in height of the load

$$y(t) = R\,\theta(t)$$



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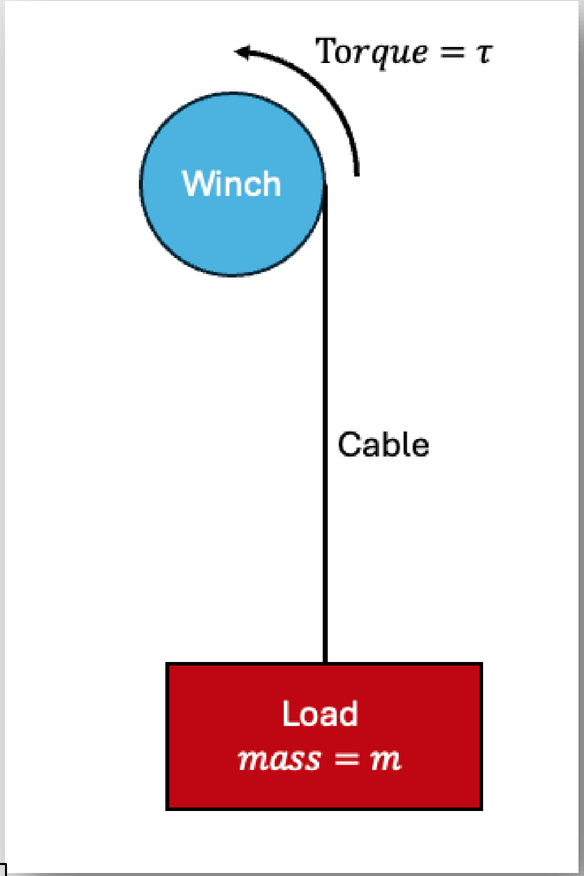
Radius of the winch times angle of displacement



Computing Variables in a Simulation

```
def step():
    # Advance the simulation by one time step
    t = t + delta_t

    # Calculate the angular acceleration based tension
    angular_acceleration = (torque - m * g * R) / m * R**2 + mw * R**2

    # Calculate the angular velocity
    angular_velocity = angular_velocity + angular_acceleration * delta_t

    # Convert angular velocity to linear distance
    delta_angle = angular_velocity * delta_t
    y = y + delta_angle * R
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