

OpSem

Problem 1: Show $1 + (2+3) \Rightarrow 6$.

$$\frac{\frac{2 \Rightarrow 2}{\boxed{1 \Rightarrow 1}} \quad \frac{3 \Rightarrow 3}{(2+3) \Rightarrow 5} \quad 5 \text{ is } 2+3}{1 + (2+3) \Rightarrow 6}$$

Problem 2: Show $A; \text{let } y=1 \text{ in let } x=2 \text{ in } x \Rightarrow 2$

$$\frac{\frac{\frac{A; y:1; x:2(x)=2}{A; y:1; 2 \Rightarrow 2} \quad A; y:1; x:2; x \Rightarrow 2}{A; 1 \Rightarrow 1} \quad A; y:1; \text{let } \underline{x=2 \text{ in } x} \Rightarrow 2}{A; \text{let } \underline{y=1 \text{ in let } x=2 \text{ in } x} \Rightarrow 2}$$

A is our "environment"

Problem 3: Show $A; \text{let } x=3 \text{ in let } y=x+6 \text{ in } x \Rightarrow 9$

$$\frac{\frac{\frac{A; x:3(x)=3}{A; x:3; x \Rightarrow 3} \quad \frac{A; x:3; 6 \Rightarrow 6}{A; x:3; x+6 \Rightarrow 9} \quad 9 \text{ is } 3+6}{A; x:3; x+6 \Rightarrow 9} \quad A; x:3; x+9(x)=9}{A; x:3; x+9; x \Rightarrow 9}$$

Take note of shadowing!

$\boxed{A; 3 \Rightarrow 3} \quad \boxed{A; x:3; \text{let } x=x+6 \text{ in } x \Rightarrow 9}$

$e_1 \quad e_2$

$A; \text{let } \underline{x=3 \text{ in let } x=x+6 \text{ in } x} \Rightarrow 9$

Problem 4: Show $A; \text{let } x=2 \text{ in let } y=3 \text{ in let } x=x+2 \text{ in } x+y \Rightarrow 7$

$$\frac{\frac{\frac{A; x:2, y:3(x)=2}{A; x:2, y:3; x \Rightarrow 2} \quad \frac{A; x:2, y:3; 2 \Rightarrow 2}{A; x:2, y:3; x+2 \Rightarrow 4} \quad 4 \text{ is } 2+2}{A; x:2, y:3; x+2 \Rightarrow 4} \quad A; x:2, y:3; x+4(x)=4}{A; x:2, y:3; x+4; x \Rightarrow 4} \quad A; x:2, y:3; x+4(y)=3}$$

Take note of shadowing here!

$\boxed{A; 2 \Rightarrow 2} \quad \boxed{A; x:2; \text{let } y=3 \text{ in let } x=x+2 \text{ in } x+y \Rightarrow 7}$

$e_1 \quad e_2$

$A; \text{let } \underline{x=2 \text{ in let } y=3 \text{ in let } x=x+2 \text{ in } x+y} \Rightarrow 7$

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Type Checking

Problem 1: Show eq0 if true then 0 else 1

$$G \vdash \text{true:bool} \quad G \vdash 0:\text{int} \quad G \vdash 1:\text{int}$$

$$G \vdash \text{if true then 0 else 1:bool}$$

$$G \vdash \text{eq0} \quad \text{if true then 0 else 1:bool}$$

G is our "context"

Problem 2: let $x=5$ in $\text{eq0 } x$ and false

$$G, x:\text{int} \vdash x:\text{int}$$

$$G, x:\text{int} \vdash \text{eq0 } x:\text{bool} \quad G, x:\text{int} \vdash \text{false:bool}$$

$$G \vdash 5:\text{int} \quad G, x:\text{int} \vdash \text{eq0 } x \text{ and } \text{false:bool}$$

$G \vdash \text{let } x=5 \text{ in eq0 } x \text{ and false:bool}$ and takes precedence.