# Q4.1. Define a CFG that describes the language

$a^x b^y c^z$ where $z \leq x + 2y$.

$S \to aSU \mid T$
$T \to bTUU \mid \epsilon$
$U \to c \mid \epsilon$

→ For every a there can be at most one c

→ for every b there can be at most two c

Q4.2. Given the following ambiguous CFG, modify it so that it produces the same strings but is not ambiguous

$S \to SaS \mid T$
$T \to bT \mid V$
$V \to c \mid \epsilon$

$\boxed{\begin{array}{l} S \to TaS \mid T \\ T \to bT \mid \end{array}}$

$S \to SaS \mid T$

$V \to c \mid \epsilon$

$SaS$

$S \to SA$

$S \to \cancel{S} \mid AS$ $S \to TaS \mid VaS \mid T \mid V$ $SaS$.

# Q4.3. Is the below CFG right recursive?

$S \to N + S \mid N * S \mid N$
$N \to 1 \mid (S)$

Yes!

Q5.1. Can the below grammar be parsed by a recursive-descent parser?

$S \rightarrow S * S \mid T$
$T \rightarrow 1 \mid 2 \mid 3 \mid (S)$

S → T * S * T
T → aT

1 * 2 * 3

No!

S → Sa | bS     S → T b S

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Q5.3. Can the below grammar be parsed by a recursive-descent parser?

$S \rightarrow Sa \mid U$
$U \rightarrow Uu \mid \epsilon$

No!

**Q7.2** Ambiguous CFG

4 Points

Prove that the following grammar is ambiguous by showing two distinct derivations of the same string.

$S \rightarrow \mathbf{a}S \mid S\mathbf{b} \mid T$
$T \rightarrow \mathbf{c}T \mid \mathbf{c}V$
$V \rightarrow V\mathbf{b} \mid \varepsilon$

~~acb~~     acb

S → aS → aT → acV → acVb → acb

S → Sb → aSb → aTb → acVb → acb.

construct CFG's for the following:

iii) $a^x b^y a^z$ where, $y = x+z$ and $x, y, z \geq 0$

→ empty string is accepted

$y = x$ when $z = 0$
$y = z$ where $x = 0$

$C \rightarrow ST$
$S \rightarrow aSb \mid \epsilon$
$T \rightarrow bTa \mid \epsilon$

$y = x+z$

iv) $S \rightarrow S+S \mid A$
$A \rightarrow A*A \mid B$
$B \rightarrow n \mid (S)$

Convert the following into right recursive grammar

$S \rightarrow A+S \mid A$
$A \rightarrow B*A \mid B$
$B \rightarrow n \mid (S)$

Q8.3. Consider the following CFG.

$$
\begin{aligned}
S &\rightarrow SaA \mid a \\
A &\rightarrow aS \mid bST \mid \epsilon \\
T &\rightarrow cT \mid \epsilon
\end{aligned}
$$

Show two ways by which the string "aaaa" can be derived from the CFG.

$S \rightarrow SaA \rightarrow aaA \rightarrow aaaS \rightarrow aaaa$

$S \rightarrow SaA \rightarrow SaAaA \rightarrow SaAaAaA \rightarrow aaAaAaA \rightarrow aaaAaA$
$\rightarrow aaaaA \rightarrow aaaa$

Q8.4. Define a CFG that describes the language.

$a^x b^y c^z$ where $y = 2x + 3z$, $x \geq 0$ and $z \geq 0$.

**Note:** To represent $\epsilon$ in the CFG, you can either copy and paste the symbol $\epsilon$, type the word epsilon or just type the letter e.

$a^x b^y c^z$

where, $y = 2x + 3z$

both $x$ as well as $z$ can be 0.

$\Rightarrow$ the CFG must accept an empty string

$y = 2x$ $\Rightarrow$ for every occurrance of a there will be 2 b's

$y = 3z$ $\Rightarrow$ for every occurrance of c there will be 3 b's

$\therefore$ CFG = $S \rightarrow AB$
$A \rightarrow aAbb \mid \epsilon$
$B \rightarrow bbbBc \mid \epsilon$

## Q9.1 Proof

8 Points

Using the following rules:

(1) $$\dfrac{}{A; n \to n}$$

(2) $$\dfrac{A(x) = v}{A; x \to v}$$

(3) $$\dfrac{A; e_1 \to v_1 \qquad A, x : v_1; e_2 \to v_2}{A;\ \textbf{let } x = e_1 \textbf{ in } e_2 \to v_2}$$

(4) $$\dfrac{A; e_1 \to n_1 \qquad A; e_2 \to n_2 \qquad n_1 < n_2}{A; e_1 < e_2 \to \textbf{true}}$$

(5) $$\dfrac{A; e_1 \to n_1 \qquad A; e_2 \to n_2 \qquad n_1 \geq n_2}{A; e_1 < e_2 \to \textbf{false}}$$

(6) $$\dfrac{A; e_1 \to n_1 \qquad A; e_2 \to n_2 \qquad v = n_1 - n_2}{A; e_1 - e_2 \to v}$$

(7) $$\dfrac{A; e_1 \to n_1 \qquad A; e_2 \to n_2 \qquad v = n_1 + n_2}{A; e_1 + e_2 \to v}$$

Fill in the blanks in the proof below:

$3<4$

$$\dfrac{\dfrac{}{A; 8 \to 8} \qquad \dfrac{\dfrac{}{\boxed{3}} \qquad \dfrac{\dfrac{\boxed{1}}{A, x : 8; x \to 8} \qquad \dfrac{}{\boxed{2}} \qquad 4 = 8 - 4}{A, x : 8; \boxed{4}} \qquad \boxed{5}}{A, x : 8; \boxed{6} \to \textbf{true}}}{A;\ \textbf{let } x = 8 \textbf{ in } 3 < x - 4 \to \textbf{true}}$$

$A, x:8(x) = 8$ $\qquad$ $4 = 8 - 4$

$A, x:8; x \Rightarrow 8$ $\qquad$ $A, x:8\ 4 \Rightarrow 4$

$A, x:8; 3 \Rightarrow 3$ $\qquad$ $A, x:8; x-4 \Rightarrow 4$ $\qquad$ $3<4$

$A; 8 \Rightarrow 8$ (1) $\qquad$ $A, x:8; 3 < x-4 \to$ true

(3) $A;\ \text{let } x = \underbrace{8}_{e_1} \text{ in } \underbrace{3 < x-4}_{e_2} \to \underbrace{\text{true}}_{v_2}$

## Q9. Operational Semantics

Using the given rules, fill in the blanks the complete the derivation below.

$$\frac{}{A;\ \text{true} \Rightarrow \text{true}} \qquad \frac{}{A;\ \text{false} \Rightarrow \text{false}}$$

$$\frac{A;\ e_1 \Rightarrow \text{true}}{A;\ (\text{not } e_1) \Rightarrow \text{false}} \qquad \frac{A;\ e_1 \Rightarrow \text{false}}{A;\ (\text{not } e_1) \Rightarrow \text{true}}$$

$$\frac{A;\ e_1 \Rightarrow \text{true} \qquad A;\ e_2 \Rightarrow v_1}{A;\ (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) \Rightarrow v_1} \qquad \frac{A;\ e_1 \Rightarrow \text{false} \qquad A;\ e_3 \Rightarrow v_1}{A;\ (\text{if } e_1 \text{ then } e_2 \text{ else } e_3) \Rightarrow v_1}$$

$$\frac{A;\ e_1 \Rightarrow v_1 \qquad A;\ e_2 \Rightarrow v_2 \qquad v_3 \text{ is } v_1 \mid\mid v_2}{A;\ (e_1 \mid\mid e_2) \Rightarrow v_3}$$

true is true || false

$$\frac{A;\ e_1 \Rightarrow v_1 \qquad A;\ e_2 \Rightarrow v_2 \qquad v_3 \text{ is } v_1 \,\&\&\, v_2}{A;\ (e_1 \,\&\&\, e_2) \Rightarrow v_3}$$

$e_1$?

$$\dfrac{\dfrac{A;\ \text{false} \Rightarrow \text{false}}{A;\ ((\#1)) \Rightarrow \text{true}} \qquad \dfrac{\dfrac{A;\ \text{true} \Rightarrow \text{true} \qquad A;\ \text{false} \Rightarrow \text{false} \qquad (\#3)}{A;\ ((\#2)) \Rightarrow \text{true}} \qquad \dfrac{A;\ \text{false} \Rightarrow \text{false} \qquad A;\ \text{true} \Rightarrow \text{true} \qquad (\#5)}{A;\ ((\#4)) \Rightarrow \text{false}}}{A;\ (\text{if } ((\#2)) \text{ then } ((\#4)) \text{ else true}) \Rightarrow \text{false}}}{A;\ (\text{if } ((\#1)) \text{ then } (\text{if } ((\#2)) \text{ then } ((\#4)) \text{ else true}) \text{ else false}) \Rightarrow \text{false}}$$

not false

true || false

(#1) not false

(#2) true || false

(#3) true is true || false

(#4) false && true

(#5) false is false && true

**Q9.2** myst Operator

4 Points

Given the following operational semantics rules for **myst**, which logical operator does **myst** represent?

$$\frac{A; e_1 \to \textbf{true} \qquad A; e_2 \to \textbf{true}}{A; e_1 \textbf{ myst } e_2 \to \textbf{false}} \qquad \frac{A; e_1 \to \textbf{true} \qquad A; e_2 \to \textbf{false}}{A; e_1 \textbf{ myst } e_2 \to \textbf{true}}$$

$$\frac{A; e_1 \to \textbf{false} \qquad A; e_2 \to \textbf{true}}{A; e_1 \textbf{ myst } e_2 \to \textbf{true}} \qquad \frac{A; e_1 \to \textbf{false} \qquad A; e_2 \to \textbf{false}}{A; e_1 \textbf{ myst } e_2 \to \textbf{false}}$$

You can write the name of the operator, or briefly describe what it does.

XOR or !=

$$\frac{}{\Gamma \vdash \Gamma(x) = T} \qquad \frac{\Gamma, x : T_1 \vdash e : T_2}{\Gamma \vdash (\text{fun } x : T_1 \to e) : T_1 \to T_2}$$

$$\frac{\Gamma \vdash f : T_1 \to T_2 \quad \Gamma \vdash x : T_1}{\Gamma \vdash f\ x : T_2} \qquad \frac{\Gamma \vdash x : \text{bool}}{\Gamma \vdash \text{not } x : \text{bool}}$$

$$\dfrac{\dfrac{\dfrac{\Gamma, x : \text{bool} \vdash \Gamma(x) = \text{bool}}{\Gamma, x : \text{Bool} \vdash \text{not } x : \text{bool}}}{\Gamma \vdash \text{fun } x \to \text{not } x : \text{bool} \to \text{bool}} \quad \dfrac{}{\Gamma \vdash \text{true} : \text{bool}}}{\Gamma \vdash (\text{fun } x : \text{Bool} \to \text{not } x)\ \text{true} : \text{Bool}}$$