

1. Give decision trees to represent the following Boolean expressions

- a.  $A \wedge \neg B$
- b.  $A \vee [\neg B \wedge C]$
- c.  $A \otimes B$
- d.  $[A \vee B] \wedge [\neg C \wedge D]$

2. Consider the following set of training examples

Instance	Classification	A1	A2
1	+	T	T
2	+	T	T
3	-	T	F
4	+	F	F
5	-	F	T
6	-	F	T

- a) What is the entropy of this collection of training examples with respect to the target function classification?
  - b) What is the information gain of A2 relative to these training examples?
3. Suppose we generate a training set from a decision tree and then apply decision-tree learning to that training set. Is it the case that the learning algorithm will eventually return the correct tree as the training-set size goes to infinity? Why or why not?
4. Suppose that an attribute splits the set of examples  $E$  into subsets  $E_k$  and that each subset has  $p_k$  positive examples and  $n_k$  negative examples. Show that the attribute has strictly positive information gain unless the ratio  $p_k/(p_k + n_k)$  is the same for all  $k$ .
5. A *decision graph* is a generalization of a *decision tree* that allows nodes (i.e., attributes used for splits) to have multiple parents, rather than just a single parent. The resulting graph must still be acyclic. Now, consider the XOR function of three binary input attributes, which produces the value 1 if and only if an odd number of the three input attributes has value 1.
- a. Draw a minimal-sized *decision tree* for the three-input XOR function.
  - b. Draw a minimal-sized *decision graph* for the three-input XOR function.