ML_PS2

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Part 1

```
import pandas as pd
import numpy as np
import statsmodels.formula.api as smf
import seaborn as sns
import matplotlib.pyplot as plt
from matplotlib import pyplot
import os
import statsmodels.formula.api as smf
from sklearn.preprocessing import MinMaxScaler
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_absolute_error, r2_score, mean_squared_error
```

1.a. QDA is expected to perform better on the training set/ This is because the QDA's greater flexibility yields a closer fit, but does have greater variance.

LDA is expected to do better on the test. While the QDA is flexible, this means it runs into the problem of overfitting the linear Bayes decision boundary.

- 1.b.(Non-linear) While QDA's flexibility increases its variance compared to LDA, when the Bayes boundary is non-linear, this flexibility is actually as good thing because it can be offset by a larger reduction in bias, thus better test performance.
- 1.c. We expect the tesst predition accuracy of the QDA to improve relative to the LDA as the sample size increases since is flexibility will yield a better fit, especially when we have more samples and this also helps deal with the problem of variance. We expect the test prediction accuracy of QDA relative to LDA to improve.

1.d. False.Flexible methods like QDA require more data to prevent overfitting, which happens due to the model's sensitivity to the noise in the training sets. Overfitting would make the QDA have a higher test error rate than the LDA, which already approximates the Bayes decision boundary accurately.

Part 2

2.a. X = [40 hours, 3.5 GPA] From the logistic regression model, we can fill in the formula: $P(Y=1)|X = \exp(-6 + 0.05 * X1 + X2) / (1 + \exp(-6 + 0.05 * X1 + X2))$

```
X1 = 40

X2 = 3.5

probability_A = np.exp(-6 + 0.05 * X1 + X2) / (1 + np.exp(-6 + 0.05 * X1 + \times X2))

print(f"Probability of getting an A is {probability_A * 100:.1f}%")
```

Probability of getting an A is 37.8%

2.b. Same student, 50%, how many hours (X1)

```
P(Y=.5)|X is .5 = \exp(-6 + 0.05 * X1 + 3.5) / (1 + \exp(-6 + 0.05 * X1 + 3.5)) solving for x1 gives us: \exp(0.05X1 - 2.5) = 1
```

```
X1 = 2.5 / 0.05 print(f"Student who wants a 50% probability needs to study {X1} hours")
```

Student who wants a 50% probability needs to study 50.0 hours

Part 3

Ave profit with dividends(X bar) =10 Ave profit 1/0 dividends(X bar) =0

Variance =36

80% issued dividends.

Using Bayes' theorem:

- A. Using normal distribution: Get likelihood for a company with profits X=4 to be in with dividend group
- B. Use 80% with dividend and 20% without dividend to wieigh likelihoods.
- C. Calculate posterior probability (weighted likelihood of with dividend group divided by the weighted likelihoods of with and without dividend)

Probability of issuing a dividend: 0.7519 or 75.19%

Part 4

```
# Load the dataset
directory = r"C:\Users\clari\OneDrive\Documents\Machine Learning\ps2"
auto_path = os.path.join(directory, "Data-Auto.csv")
auto_df = pd.read_csv(auto_path)
print(auto_df.dtypes)
print(auto_df.shape)
```

Unnamed: 0 int64 float64 mpg cylinders int64 displacement float64 horsepower int64 weight int64 float64 acceleration int64 year int64 origin name object

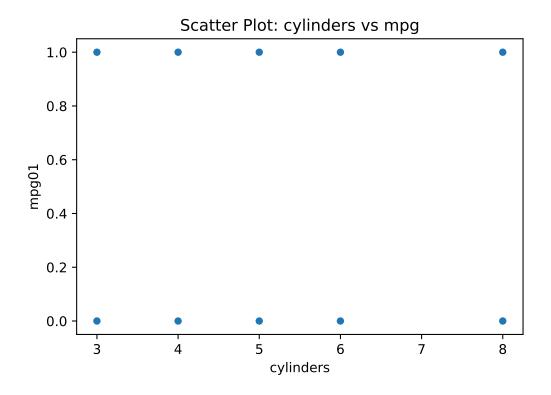
```
dtype: object (392, 10)
```

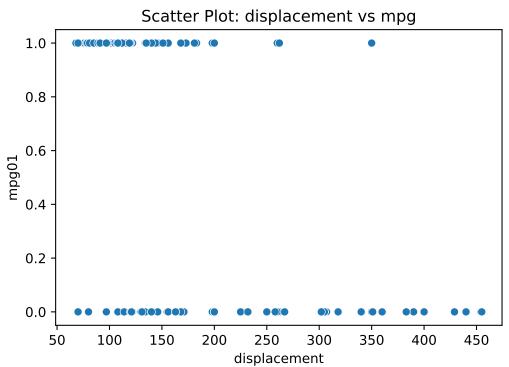
4.a. Making dummy variable

```
auto_df["mpg01"] = np.where(auto_df["mpg"] > auto_df["mpg"].median(), 1, 0)
```

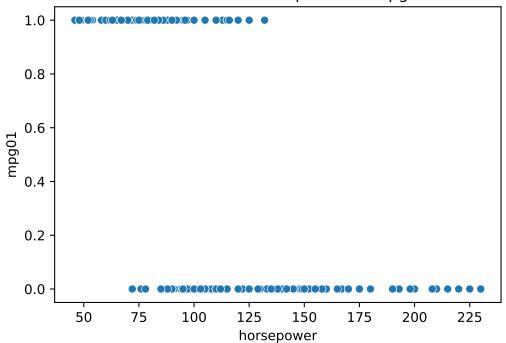
4.b. It looks like horsepower, acceleration, and weight could be useful in predicting mpg01 because the number of observations and the value of the variable tends to increase or decrease based on if mpg01 is 0 or 1 (although there are some overlaps)

Scatterplot

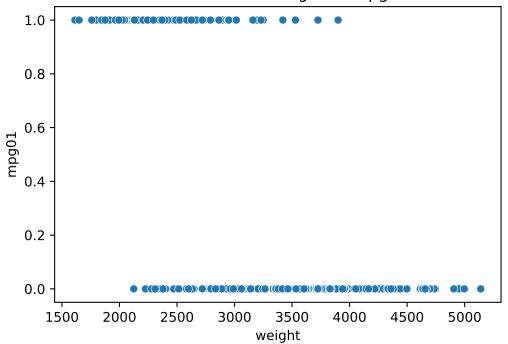


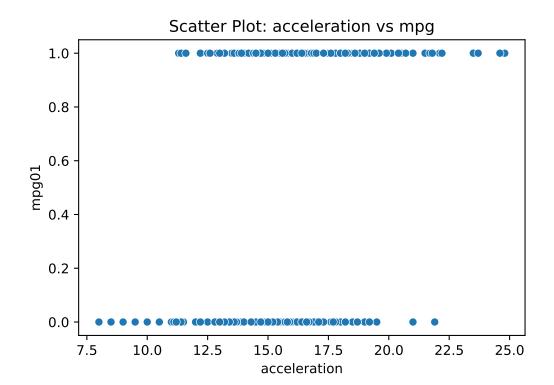


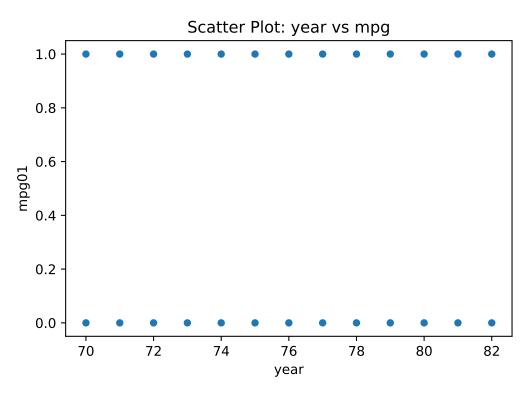


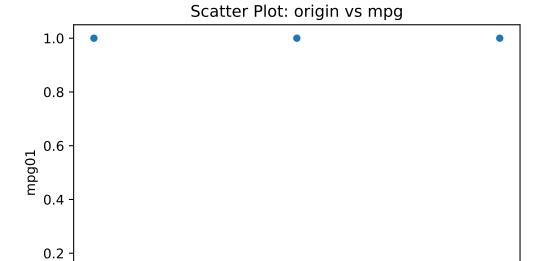


Scatter Plot: weight vs mpg









Boxplot

0.0

1.00

1.25

1.50

```
for var in auto_vars:
    plt.figure(figsize=(10, 6))
    sns.boxplot(x='mpg01', y=var, data=auto_df)
    plt.title(f'{var} Distribution by MPG Category', size=14)
    plt.xlabel('High MPG (1) vs Low MPG (0)')
    plt.ylabel(var)
    plt.show()
```

2.00

origin

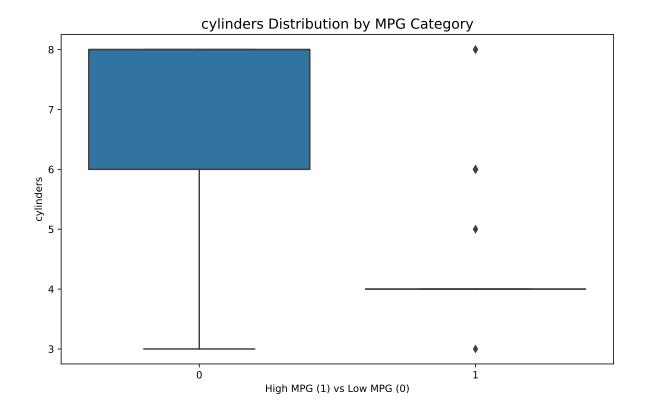
1.75

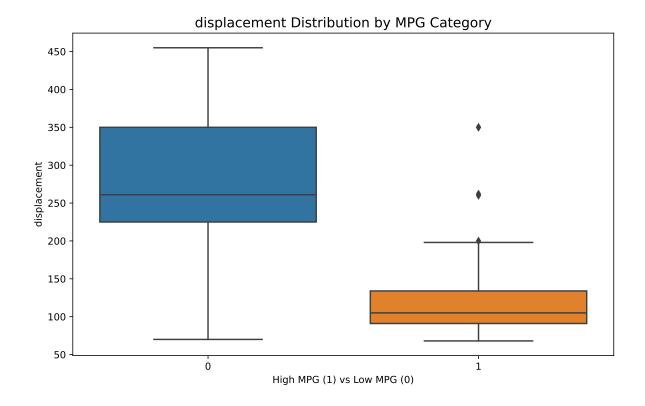
2.25

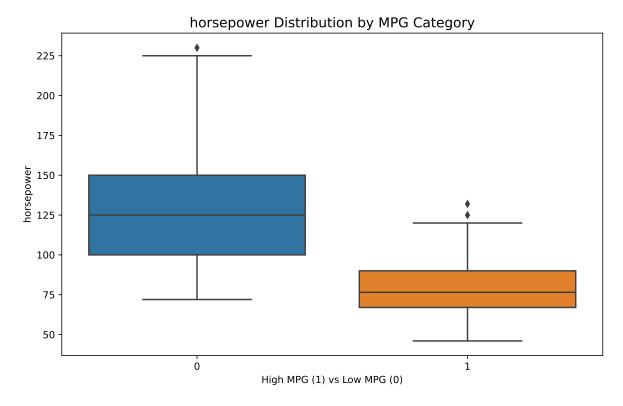
2.50

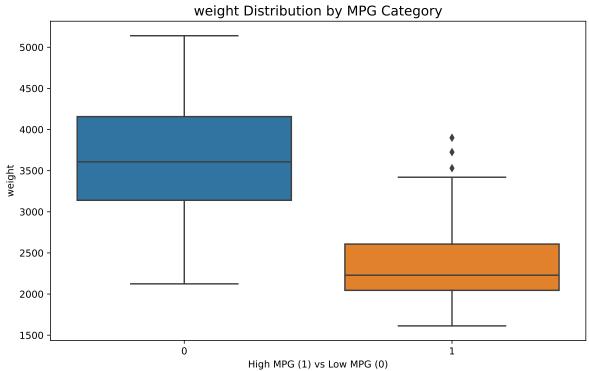
2.75

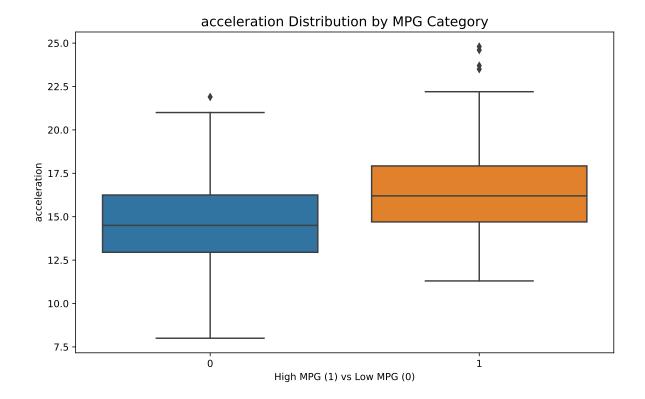
3.00

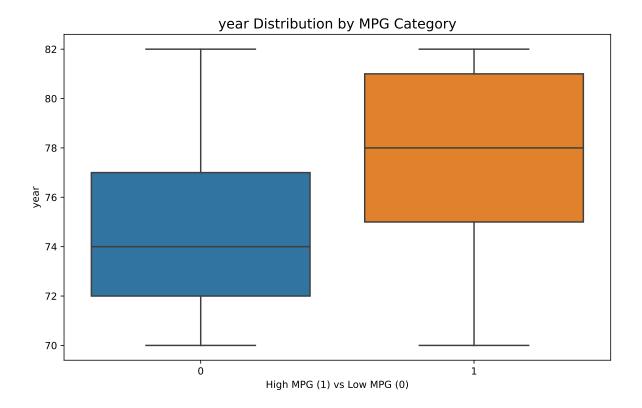




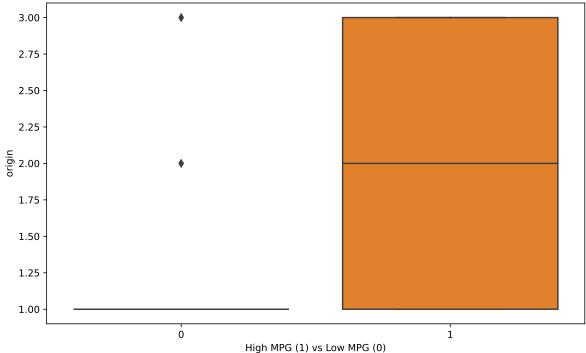












From the scatterplots, we see a bit of a pattern from horsepower, acceleration, and weight, where mpg01 being equal to 0 or 1 is more likely based on whether these variables have higher or lower values. ALthough there is some overlap, it at least shows greater distinctions compared to the other variables.

From the boxplots, we can see that the median weight of mgp01 = 1 cars much lower than that of mpg01 = 0 cars. So it may suggest heavier cars have lower mpg. We also see that mpg01=1 cars have much lower horsepower as mpg01 = 0 cars (though they have more variance in values) While acceleration shows less clear separation than weight/horsepower in the plots, it still shows a trend where faster acceleration (lower values) tends to be associated with lower mpg(mpg01=0)

Meanwhile, the other vairables don't show as clear patterns or have more overlaps in terms of the values of mpg01=0 or =1.

4.c. Splitting to training and test set

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
176	23.0	4	120.0	88	2957	17.0	75	2
307	41.5	4	98.0	76	2144	14.7	80	2
137	14.0	8	302.0	140	4638	16.0	74	1
18	27.0	4	97.0	88	2130	14.5	70	3
285	16.5	8	351.0	138	3955	13.2	79	1

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
280	22.3	4	140.0	88	2890	17.3	79	1
57	25.0	4	97.5	80	2126	17.0	72	1
46	19.0	6	250.0	100	3282	15.0	71	1
223	17.5	6	250.0	110	3520	16.4	77	1
303	28.4	4	151.0	90	2670	16.0	79	1

Name: mpg01, dtype: int32

Name: mpg01, dtype: int32

```
# Make a copy of the training and test data
X_train_dummy = X_train.copy()
X_test_dummy = X_test.copy()

## Insert the dummy variable in each set.
## df.insert(column #, 'column name', value)
X_train_dummy.insert(0, 'test', 0)
X_test_dummy.insert(0, 'test', 1)

X_full = pd.concat([X_test_dummy, X_train_dummy], axis = 0)

display(X_full)
print(X_full['test'].value_counts())
```

	test	mpg	cylinders	displacement	horsepower	weight	acceleration	year	origin
280	1	22.3	4	140.0	88	2890	17.3	79	1
57	1	25.0	4	97.5	80	2126	17.0	72	1
46	1	19.0	6	250.0	100	3282	15.0	71	1
223	1	17.5	6	250.0	110	3520	16.4	77	1
303	1	28.4	4	151.0	90	2670	16.0	79	1
•••									
358	0	22.4	6	231.0	110	3415	15.8	81	1
356	0	25.4	6	168.0	116	2900	12.6	81	3
300	0	34.5	4	105.0	70	2150	14.9	79	1
132	0	16.0	6	258.0	110	3632	18.0	74	1
373	0	36.0	4	98.0	70	2125	17.3	82	1

test

1 196
 1 196

Name: count, dtype: int64

Run regression

OLS Regression Results

Dep. Variable: test R-squared:

0.007

Model: OLS Adj. R-squared:

-0.014

Method: Least Squares F-statistic:

0.3430

Date: Fri, 07 Feb 2025 Prob (F-statistic):

0.949

Time: 21:51:49 Log-Likelihood:

-283.11

No. Observations: 392 AIC:

584.2

Df Residuals: 383 BIC:

620.0

Df Model: 8
Covariance Type: nonrobust

	coef 0.975]	std err	t	P> t	[0.025	:======
Intercept	0.1527	0.716	0.213	0.831	-1.255	
mpg 0.016	0.0009	0.008	0.119	0.905	-0.014	
cylinders 0.124	0.0277	0.049	0.563	0.574	-0.069	
displacement 0.001	-0.0012	0.001	-1.057	0.291	-0.003	
horsepower 0.004	-0.0004	0.002	-0.187	0.852	-0.005	
weight 0.000	0.0001	0.000	1.165	0.245	-8.87e-05	
acceleration 0.023	-0.0067	0.015	-0.448	0.654	-0.036	
year 0.021	0.0024	0.010	0.248	0.804	-0.017	
origin 0.079	-0.0065	0.044	-0.149	0.882	-0.092	

Omnibus: 1741.652 Durbin-Watson:

0.027

```
Prob(Omnibus): 0.000 Jarque-Bera (JB): 63.402
Skew: 0.003 Prob(JB): 1.71e-14
Kurtosis: 1.030 Cond. No. 8.74e+04
```

Notes:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 8.74e+04. This might indicate that there are

strong multicollinearity or other numerical problems.

Since all the p-values are larger than 0.1 we aren't as worried that the train and test set are significantly different, but we do want to check distribution of the training and test sets to see if they are balanced on characteristics we haven't included or are unobservable

Percentage of positive labels in the test set: 48.47 Percentage of positive labels in the training set: 51.53

4.d. LDA

```
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
# Choosing predictors related to mpg01
X_train_rel = X_train.copy()[["horsepower", "weight", "acceleration"]]
X_test_rel = X_test.copy()[["horsepower", "weight", "acceleration"]]
# Fit the LDA model
lda_model = LinearDiscriminantAnalysis()
lda_model.fit(X_train_rel, y_train)
# view the predicted test values
y_pred_lda = lda_model.predict(X_test_rel)
y_pred_lda
```

Testing error rate: #cite how to get this

```
from sklearn.metrics import accuracy_score, confusion_matrix, roc_curve
error_rate_lda = 1 - accuracy_score(y_test, y_pred_lda)
print(f"The error rate is: {round(error_rate_lda, 4)*100}%")
```

The error rate is: 11.2199999999999%

4.e. QDA model

```
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
qda_model = QuadraticDiscriminantAnalysis()
qda_model.fit(X_train_rel, y_train)

# view the predicted test values
y_pred_qda = qda_model.predict(X_test_rel)
y_pred_qda
```

```
# Get the error rate
error_rate_qda = 1 - accuracy_score(y_test, y_pred_qda)
print(f"The QDA model's error rate is: {round(error_rate_qda, 4)*100}%")
```

The QDA model's error rate is: 12.76%

4.f. Logistic regression

```
# Fit the model
from sklearn.linear_model import LogisticRegression
logisticRegr = LogisticRegression(max_iter=500)
logisticRegr.fit(X_train_rel, y_train)

# Predict the test set
y_pred_log = logisticRegr.predict(X_test_rel)
```

Get test error

The logistic regression model's error rate is: 9.69%

4.g.

Part 5

5.a.

```
# Load the dataset
directory = r"C:\Users\clari\OneDrive\Documents\Machine Learning\ps2"
default_path = os.path.join(directory, "Data-Default.csv")
default_df = pd.read_csv(default_path)
print(default_df.dtypes)
print(default_df.shape)
default_df.head(5)
```

default object student object balance float64 income float64 dtype: object (10000, 4)

	default	student	balance	income
0	No	No	729.526495	44361.625074
1	No	Yes	817.180407	12106.134700
2	No	No	1073.549164	31767.138947
3	No	No	529.250605	35704.493935
4	No	No	785.655883	38463.495879

Logistic Regression of income and balance on default

```
# Cchange default into a dummy variable
default_df["default01"] = default_df["default"].map({"Yes": 1, "No": 0})

print(default_df["default01"].value_counts(normalize=True) * 100)
# Checking if it worked
yes_rows = default_df[default_df["default"] == "Yes"]
print(yes_rows.head(5))
no_rows = default_df[default_df["default"] == "No"]
print(no_rows.head(5))
```

```
default01 0 96.67
```

```
1 3.33
```

```
Name: proportion, dtype: float64
   default student
                        balance
                                              default01
                                      income
136
       Yes
               Yes 1486.998122 17854.397028
173
       Yes
               Yes 2205.799521 14271.492253
                                                      1
201
       Yes
               Yes 1774.694223 20359.506086
                                                      1
206
       Yes
               No 1889.599190 48956.171589
                                                      1
209
       Yes
               Yes 1899.390626 20655.200003
 default student
                                    income default01
                      balance
                   729.526495 44361.625074
0
      No
              No
1
                   817.180407 12106.134700
                                                    0
      No
             Yes
2
              No 1073.549164 31767.138947
                                                    0
      No
3
                   529.250605 35704.493935
                                                    0
      No
              No
                                                    0
4
      No
                   785.655883 38463.495879
              No
```

```
# Defining X and y
X_default = default_df[["income", "balance"]]
y_default = default_df["default01"]

# Logisitc regression model
default_logit_reg = LogisticRegression(max_iter=500)
default_logit_reg.fit(X_default,y_default)
```

LogisticRegression(max_iter=500)

5.b.Spit data, random seed 42, .7

	income	balance
9069	41239.020510	0.000000
2603	37073.192381	961.999353
7738	19039.168273	655.611221
1579	27690.113535	864.047198
5058	57561.411261	1306.832034

	income	balance
6252	31507.089277	1435.662933
4684	42139.070269	771.789347
1731	21809.218509	0.000000
4742	32803.832648	113.571264
4521	49903.597081	1358.132472

```
9069
        0
2603
        0
7738
        0
1579
        0
5058
        0
Name: default01, dtype: int64
6252
        0
4684
        0
1731
        0
4742
        0
4521
        0
Name: default01, dtype: int64
# Fit the training data into logistic regression
default_logit_train= LogisticRegression(max_iter=500)
default_logit_train.fit(X_train,y_train)
```

LogisticRegression(max_iter=500)

```
# Predict the validation set
y_pred_log = default_logit_train.predict_proba(X_validation)[:, 1]
print("Predicted probabilities above 0.5:", y_pred_log[y_pred_log >= 0.5])
print("Count of values >= 0.5:", len(y_pred_log[y_pred_log >= 0.5]))
```

Predicted probabilities above 0.5: [0.54364253] Count of values >= 0.5: 1

```
# Classifying to default category if porbablity is > 0.5
y_pred_log = np.array(y_pred_log)
y_pred_log = y_pred_log.astype(float)
y_default_category = np.where(y_pred_log >= 0.5, 1, 0)
print(y_pred_log[:10]) # First 10 predictions
print(type(y_pred_log)) # Type check
print(pd.Series(y_default_category).value_counts())
```

```
[0.03355695 0.00641997 0.05732826 0.01540883 0.00314391 0.02882375
0.00609724 0.05234981 0.01991224 0.09350953]
<class 'numpy.ndarray'>
     2999
1
        1
Name: count, dtype: int64
# Compute the error rate
error_valid = 1 - accuracy_score(y_validation, y_default_category)
# Print the error
print(f"The validation set error is {round(error_valid, 4)*100}%")
The validation set error is 3.17%
5.c.
X = default_df[["income", "balance"]]
y = default_df["default01"]
random_states = [2, 6, 9]
error_rates = []
for state in random_states:
    # Split data with current random state
    X_train, X_validation, y_train, y_validation = train_test_split(
        X, y,
        train_size=0.7,
        random_state=state
    # Train model
    default_logit_train = LogisticRegression(max_iter=500)
    default_logit_train.fit(X_train, y_train)
    # Predict and calculate error
    y_pred_log = default_logit_train.predict_proba(X_validation)[:, 1]
    y_default_category = np.where(y_pred_log > 0.5, 1, 0)
    error_rate = 1 - accuracy_score(y_validation, y_default_category)
    error_rates.append(error_rate)
    print(f"Random state {state}: validation error = {error_rate:.2%}")
```

```
# Analyze results
print("\nSummary:")
print(f"Average error rate: {np.mean(error_rates):.2%}")
print(f"Standard deviation: {np.std(error_rates):.2%}")
```

```
Random state 2: validation error = 2.37%
Random state 6: validation error = 2.47%
Random state 9: validation error = 3.07%
```

Summary:

Average error rate: 2.63% Standard deviation: 0.31%

- Consistency: The error rates across the three relatively close, ranging from 2.37% to 3.07%, meaning it's not overly sensitive to how the data is split
- Low Error Rates: They all have lower error rates than the random state 42 split. Slight Variability: There is some variability in the results, with a standard deviation of 0.31%. This variability is expected due to the random nature of the splits and demonstrates the importance of using multiple splits to assess model performance.

These results give us some level of confidence in the model's performance and its ability to generalize to new data. Butmaybe doing the k-fold cross-validation will get us an even more robust estimate of the model's performance.

5.d.

```
# Create dummy variable for student
default_df["student01"] = default_df["student"].map({"Yes": 1, "No": 0})

# Checking if it worked
yes_rows = default_df[default_df["student"] == "Yes"]
print(yes_rows.head(5))
no_rows = default_df[default_df["student"] == "No"]
print(no_rows.head(5))
```

	default	student	balance	income	default01	student01
1	No	Yes	817.180407	12106.134700	0	1
5	No	Yes	919.588530	7491.558572	0	1
7	No	Yes	808.667504	17600.451344	0	1
10	No	Yes	0.000000	21871.073089	0	1
11	No	Yes	1220.583753	13268.562221	0	1
(default s	student	balance	income	default01	student01

```
0
              No 729.526495 44361.625074
      No
                                                   0
                                                             0
2
      No
              No 1073.549164 31767.138947
                                                   0
                                                             0
3
                  529.250605 35704.493935
                                                             0
      No
              No
                                                   0
4
      No
              No
                  785.655883 38463.495879
                                                   0
                                                             0
                  825.513331 24905.226578
                                                             0
6
      No
                                                   0
              No
```

```
# Deine X, y
X = default_df[["income", "balance", "student01"]]
y = default_df["default01"]

# Logisitc regression model
student_logit_reg = LogisticRegression(max_iter=500)
student_logit_reg.fit(X,y)
```

LogisticRegression(max_iter=500)

	income	balance	student01
9069	41239.020510	0.000000	0
2603	37073.192381	961.999353	0
7738	19039.168273	655.611221	1
1579	27690.113535	864.047198	0
5058	57561.411261	1306.832034	0

	income	balance	student01
6252	31507.089277	1435.662933	0
4684	42139.070269	771.789347	0
1731	21809.218509	0.000000	0
4742	32803.832648	113.571264	0
4521	49903.597081	1358.132472	0

9069 0 2603 0 7738 0

```
1579
        0
5058
        0
Name: default01, dtype: int64
6252
        0
4684
        0
1731
        0
4742
        0
4521
        0
Name: default01, dtype: int64
# Fit the training data into logistic regression
student_logit_train= LogisticRegression(max_iter=500)
student_logit_train.fit(X_train,y_train)
LogisticRegression(max_iter=500)
# Predict the validation set
student_y_pred_log = student_logit_train.predict_proba(X_validation)[:, 1]
student_y_pred_log[:5]
array([0.21470633, 0.00421135, 0.00278732, 0.00100267, 0.01614738])
# Classifying to default category if porbablity is > 0/5
student_y_default_category = np.where(student_y_pred_log > 0.5, 1, 0)
# Compute the error rate
error_valid = 1 - accuracy_score(y_validation, student_y_default_category)
# Print the error
print(f"The validation set error is {round(error_valid, 4)*100}%")
print(pd.Series(student_y_default_category).value_counts())
The validation set error is 3.17%
     2961
       39
1
Name: count, dtype: int64
```

Adding in student dummy variable didn't change the test error rate of the validation set. This can be interpreted as: being a student doesn't affect one's probability of default, all else equal. This doesn't match with our expectations because being a student probably affects default. Maybe if we added in the other variable like balance and income into the model, this may lower the error rate.