

CS11-711 Advanced NLP

Models w/ Latent Random Variables

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Site

<https://cmu-anlp.github.io/>

With Slides from Graham Neubig and Chunting Zhou

Discriminative vs. Generative Models

- **Discriminative model:** calculate the probability of output given input $P(Y|X)$
- **Generative model:** calculate the probability of a variable $P(X)$, or multiple variables $P(X,Y)$
- Which of the following models are discriminative vs. generative?
 - BERT-based POS tagger with independent predictions
 - Encoder-decoder based POS tagger
 - Language model

Types of Variables

- Observed vs. Latent:
 - **Observed:** something that we can see from our data, e.g. X or Y
 - **Latent:** a variable that we assume exists, but we aren't given the value
- Deterministic vs. Random:
 - **Deterministic:** variables that are calculated directly according to some deterministic function
 - **Random (stochastic):** variables that obey a probability distribution, and may take any of several (or infinite) values

Quiz: What Types of Variables?

- In the an attentional sequence-to-sequence model using MLE/teacher forcing, are the following variables observed or latent? deterministic or random?
 - The input word ids **f**
 - The encoder hidden states **h**
 - The attention values **a**
 - The output word ids **e**

Latent Variable Models

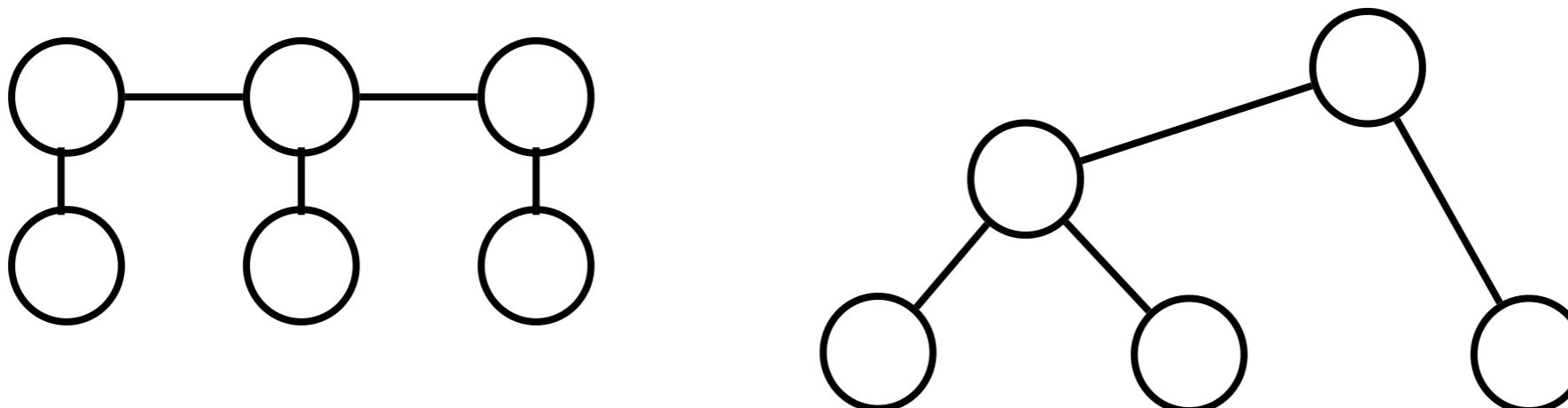
- A latent variable model (LVM) is a probability distribution over two sets of variables x, z :

$$p(x, z; \theta)$$

where the x variables are observed at learning time in a dataset and z are latent variables.

What Types of Latent Variables?

- Latent continuous vector (e.g. variational auto-encoder)
- Latent discrete vector (e.g. topic model)
- Latent structure (e.g. HMM or tree-structured model)

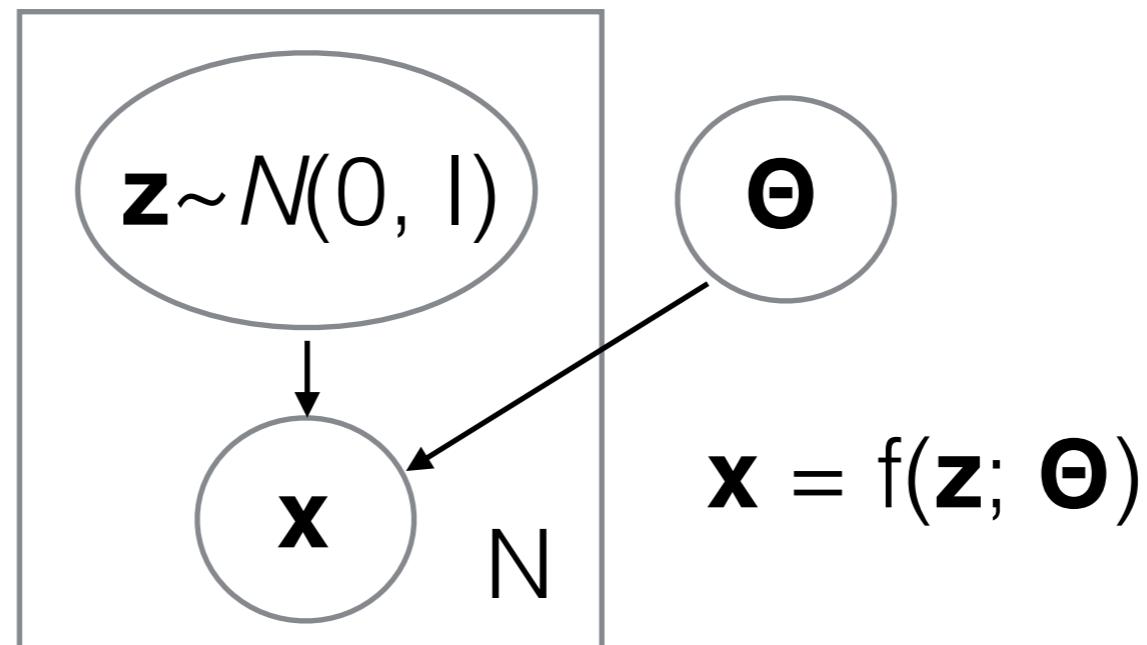


Variational Auto-encoders

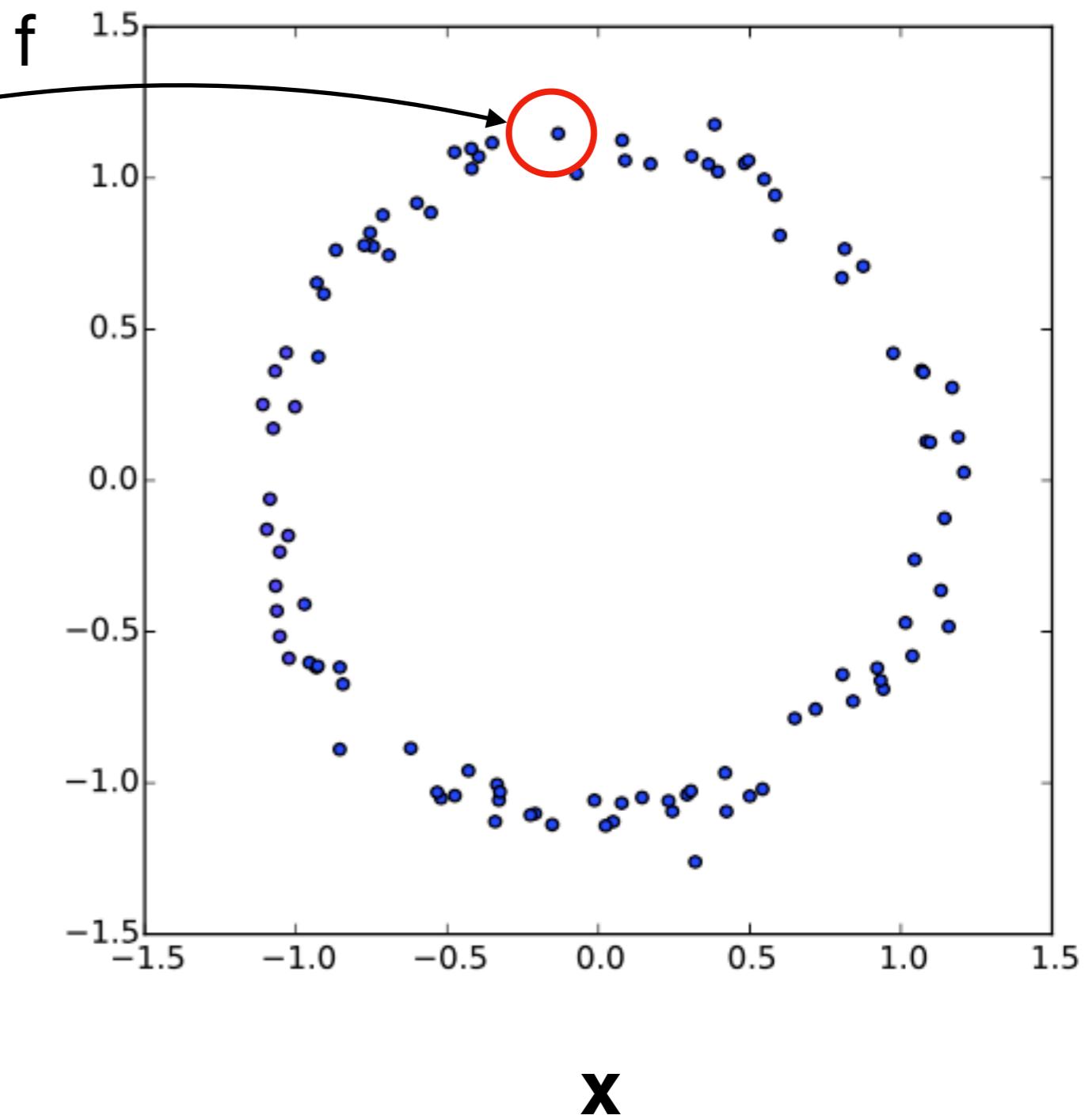
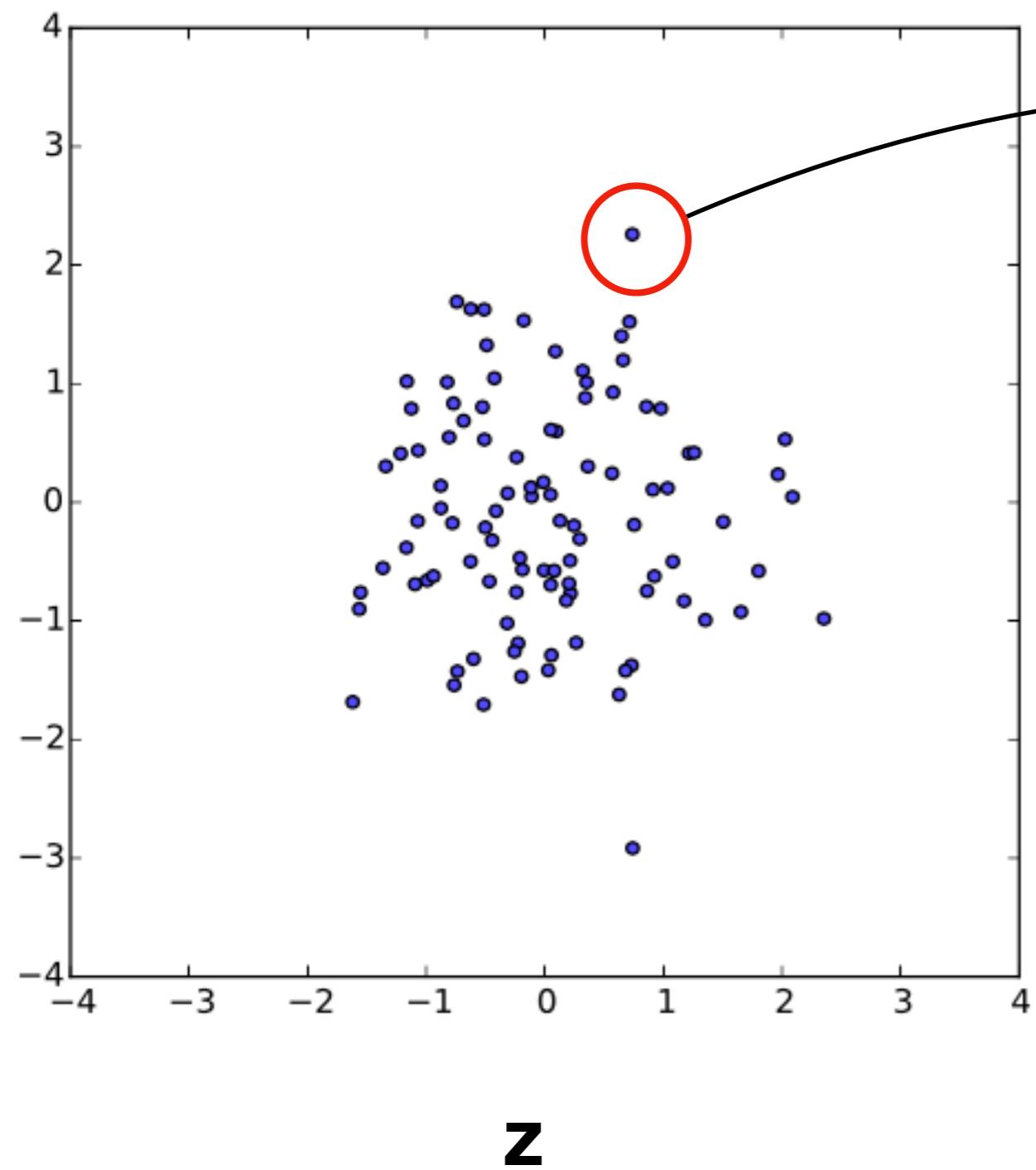
(Kingma and Welling 2014)

A Latent Variable Model

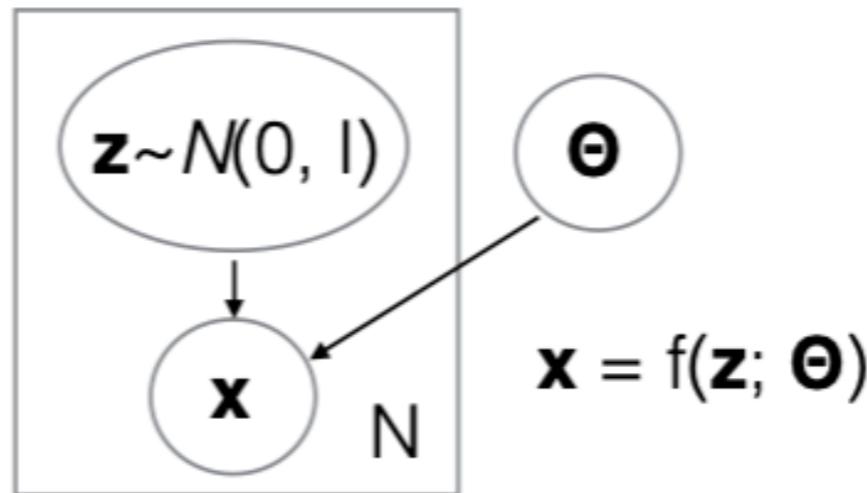
- We observed output \mathbf{x} (assume a continuous vector for now)
- We have a latent variable \mathbf{z} generated from a Gaussian
- We have a function f , parameterized by Θ that maps from \mathbf{z} to \mathbf{x} , where this function is usually a neural net



An Example (Goersch 2016)



A probabilistic perspective on Variational Auto-Encoder



- For each datapoint i :
 - Draw latent variables $z_i \sim p(z)$ (prior)
 - Draw data point $x_i \sim p_\theta(x|z)$
- Joint probability distribution over data and latent variables:

$$p(x, z) = p(z)p_\theta(x|z)$$

What is Our Loss Function?

- We would like to maximize the corpus log likelihood

$$\log P(\mathcal{X}) = \sum_{\mathbf{x} \in \mathcal{X}} \log P(\mathbf{x}; \theta)$$

- For a single example, the marginal likelihood is

$$P(\mathbf{x}; \theta) = \int P(\mathbf{x} | \mathbf{z}; \theta) P(\mathbf{z}) d\mathbf{z}$$

- We can approximate this by sampling **zs** then summing

$$P(\mathbf{x}; \theta) \approx \frac{1}{N} \sum_{\mathbf{z} \in S(\mathbf{x})} P(\mathbf{x} | \mathbf{z}; \theta) \quad \text{where} \quad S(\mathbf{x}) := \{\mathbf{z}' ; \mathbf{z}' \sim P(\mathbf{z})\} \\ N := |S(\mathbf{x})|$$

Variational Inference

Two tasks of interest:

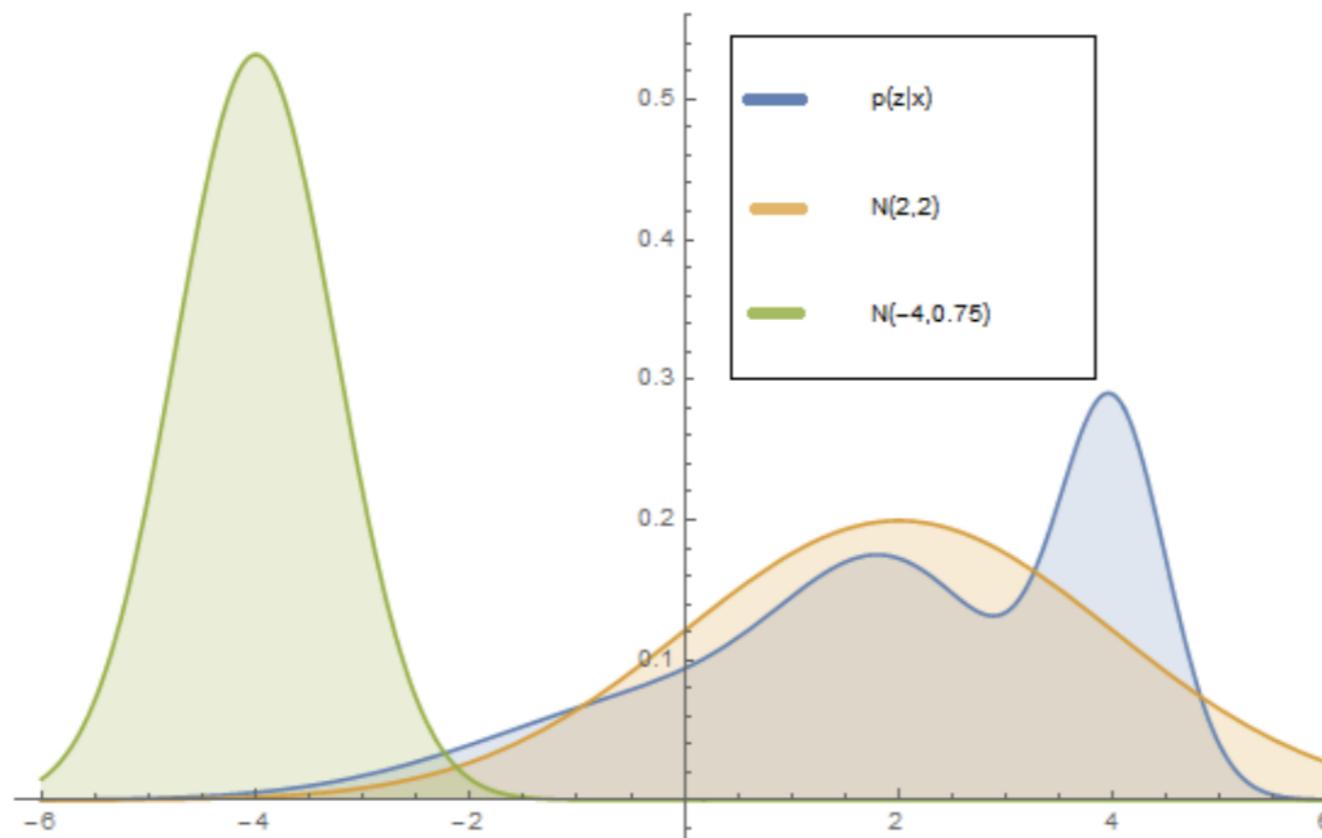
- Learn the parameters θ of $p_\theta(x|z)$
- Inference over z with the *posterior* distribution: $p_\theta(z|x)$ given input x , what are its latent factors?

$$p_\theta(z|x) = \frac{p_\theta(x|z)p(z)}{p(x)}$$

$$p(x) = \int p(z)p_\theta(x|z)dz \text{ <- intractable}$$

- Variational inference approximates the posterior with another model $q_\phi(z|x)$ that's easier to compute

Variational Inference



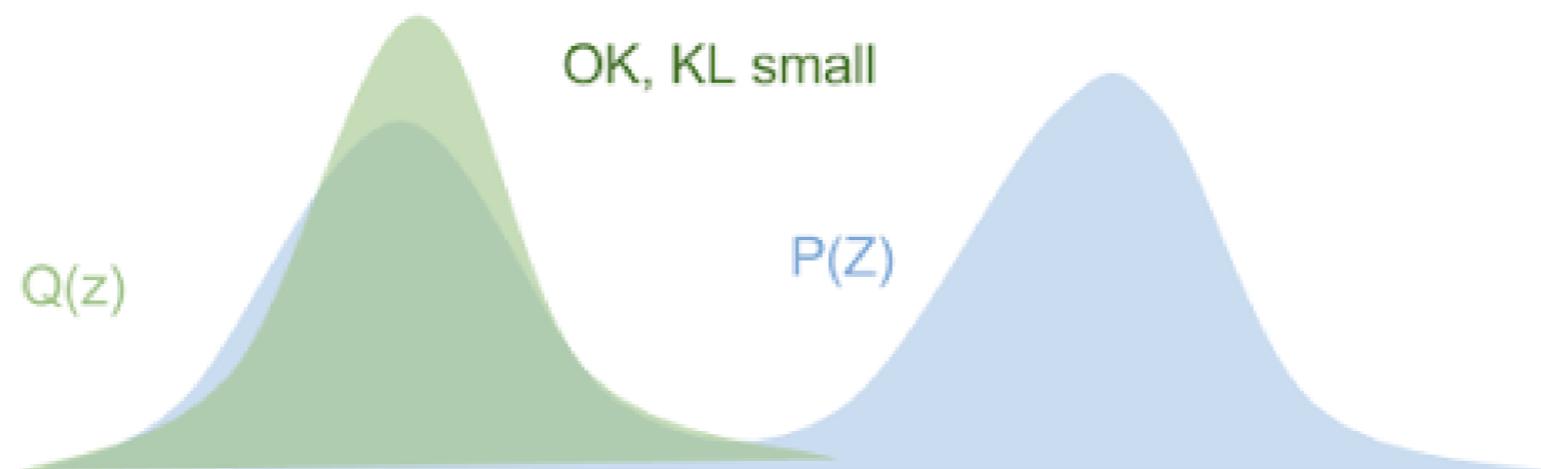
- Variational inference: optimize variational parameters so that $q(\mathbf{z}; \phi)$ is as close as possible to $p(\mathbf{z}|\mathbf{x}; \theta)$ while being simple to compute
- E.g. in figure, posterior (in blue) is better approximated by orange Gaussian than green

Variational Inference

- The KL divergence for variational inference is:

$$\mathbf{D}_{KL}(q(z) \| p(z|x)) = \int q(z) \log \frac{q(z)}{p(z|x)} dz$$

- Intuitively, there are three cases
 - a. If q is low then we don't care (because of the expectation).
 - b. If q is high and p is high then we are happy.
 - c. If q is high and p is low then we pay a price.
- Note that p must be > 0 wherever $q > 0$



Variational Inference

- Variational inference approximates the true posterior $p_\theta(z|x)$ with an “inference” model $q_\phi(z|x)$

$$\text{minimize : } \text{KL}(q_\phi(z|x) || p_\theta(z|x))$$

- Variational Lower Bound (ELBO)

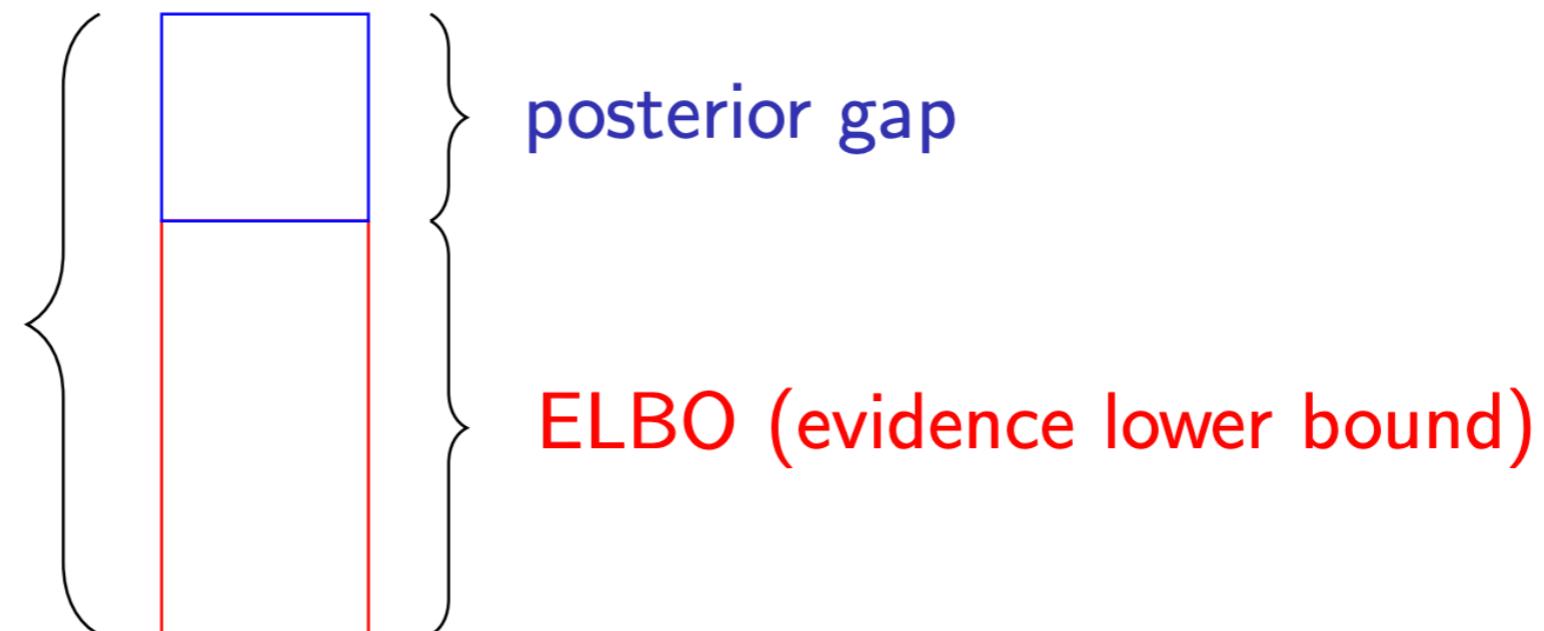
$$\log p(x) = \text{ELBO} + \text{KL}(q_\phi(z|x) || p_\theta(z|x))$$

$$\text{KL}(q||p) \geq 0 \Rightarrow \log p(x) \geq \text{ELBO}$$

Variational Inference

For any¹ distribution $q(z | x; \lambda)$ over z ,

$$\log \prod_{\mathbf{x} \in \mathcal{D}} p(\mathbf{x}; \theta) = L(\theta) = \mathbb{E}_q \left[\log \frac{p(x, z; \theta)}{q(z | x; \lambda)} \right] + \text{KL}[q(z | x; \lambda) \| p(z | x; \theta)]$$



Since KL is always non-negative, $L(\theta) \geq \text{ELBO}(\theta, \lambda)$.

¹Technical condition: $\text{supp}(q(z)) \subset \text{supp}(p(z | x; \theta))$

Variational Inference

$$\begin{aligned}\log p(x; \theta) &= \mathbb{E}_q \log p(x) \quad (\textit{Expectation over } z) \\ &= \mathbb{E}_q \log \frac{p(x, z)}{p(z|x)} \quad (\textit{Mult/div by } p(z|x), \textit{ combine numerator}) \\ &= \mathbb{E}_q \log \left(\frac{p(x, z)}{q(z|x)} \frac{q(z|x)}{p(z|x)} \right) \quad (\textit{Mult/div by } q(z|x)) \\ &= \mathbb{E}_q \log \frac{p(x, z)}{q(z|x)} + \mathbb{E}_q \log \frac{q(z|x)}{p(z|x)} \quad (\textit{Split Log}) \\ &= \mathbb{E}_q \log \frac{p(x, z; \theta)}{q(z|x; \lambda)} + \text{KL}[q(z|x; \lambda) \parallel p(z|x; \theta)]\end{aligned}$$

q: like an encoder
network:
data -> latent

Huge number of
algorithms
from choices of q and
decompositions of
ELBO

Evidence Lower Bound
(ELBO)

We'll choose q , and
parameters θ, λ ,
to maximize this

posterior gap
Typically uncomputable,
but hopefully small
if we chose q well

We'll ignore this term
from now on

Variational Autoencoders

$$\log p_{\theta}(\mathbf{x}) \geq \text{ELBO}$$

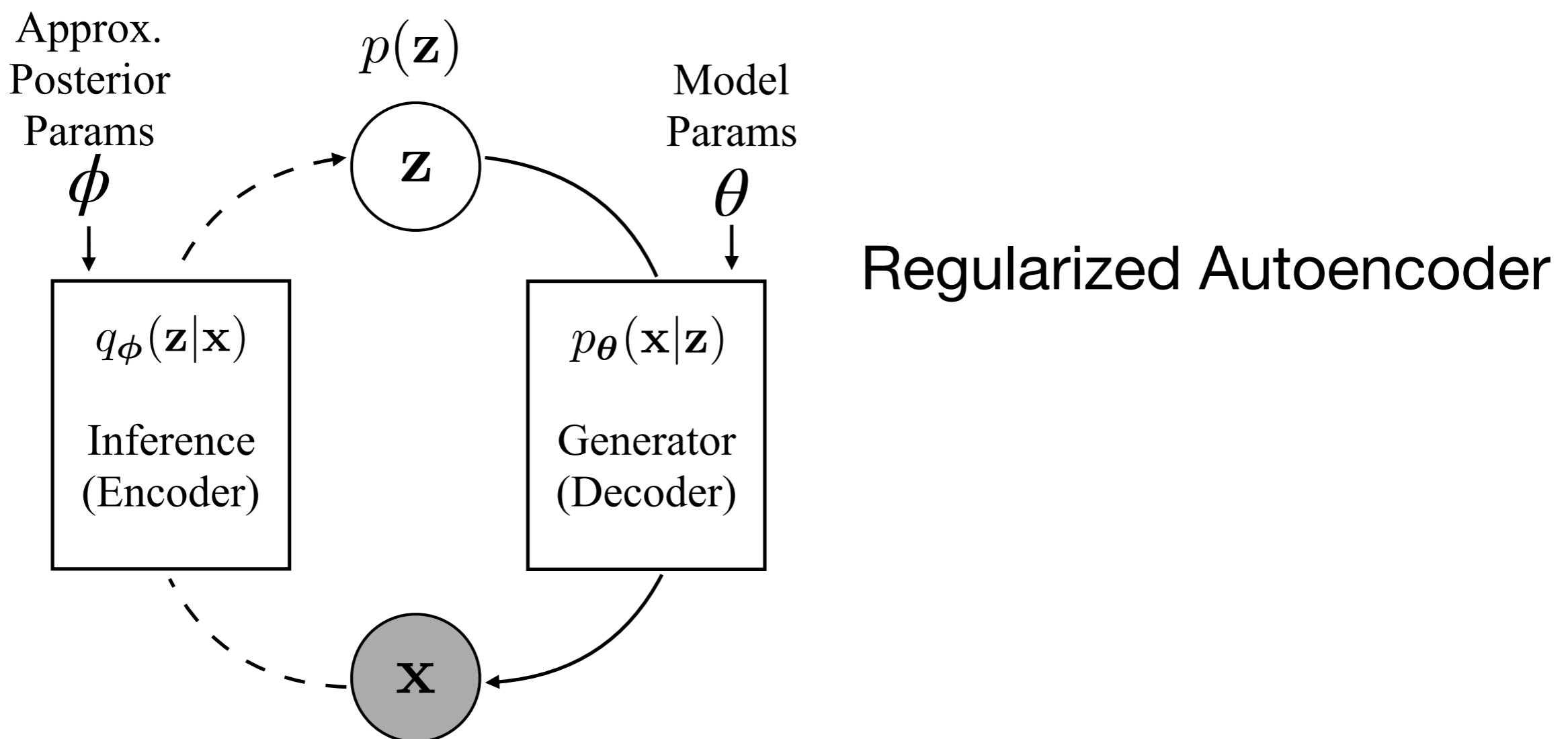
$$\begin{aligned}\text{ELBO} &= \mathbb{E}_{z \sim q_{\phi}(z|x)} \log \frac{p_{\theta}(x, z)}{q_{\phi}(z | x)} \\ &= \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})} [\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) || p(\mathbf{z}))}_{\text{KL Regularizer}}\end{aligned}$$

Reconstruct
the input from
a “hint”, \mathbf{z}

Prior prevents hint
from being too
informative

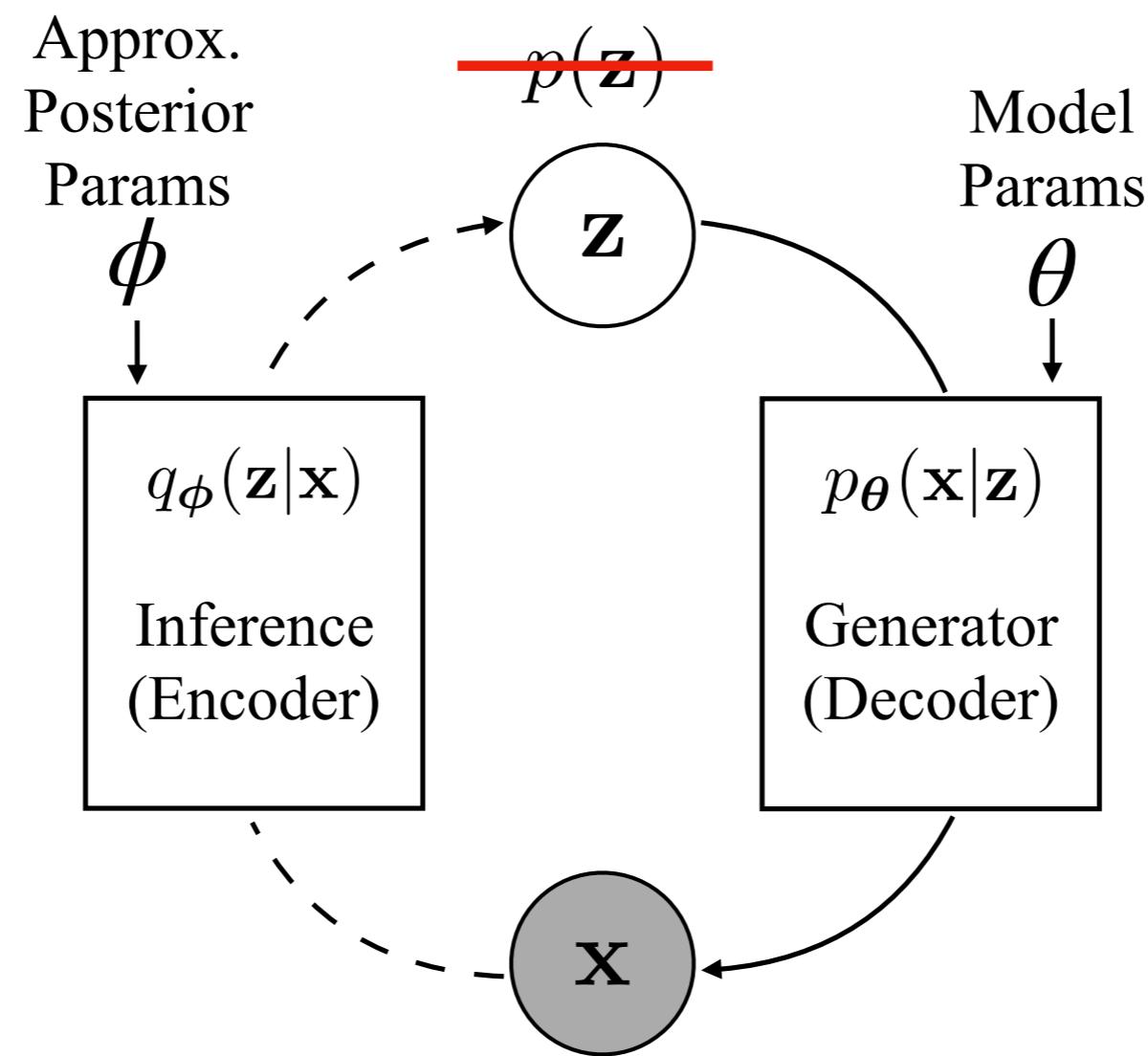
Variational Autoencoders

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$



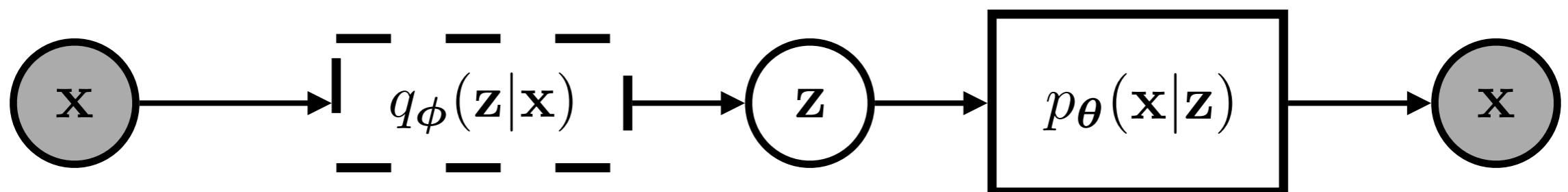
Why prior ?

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})} [\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$



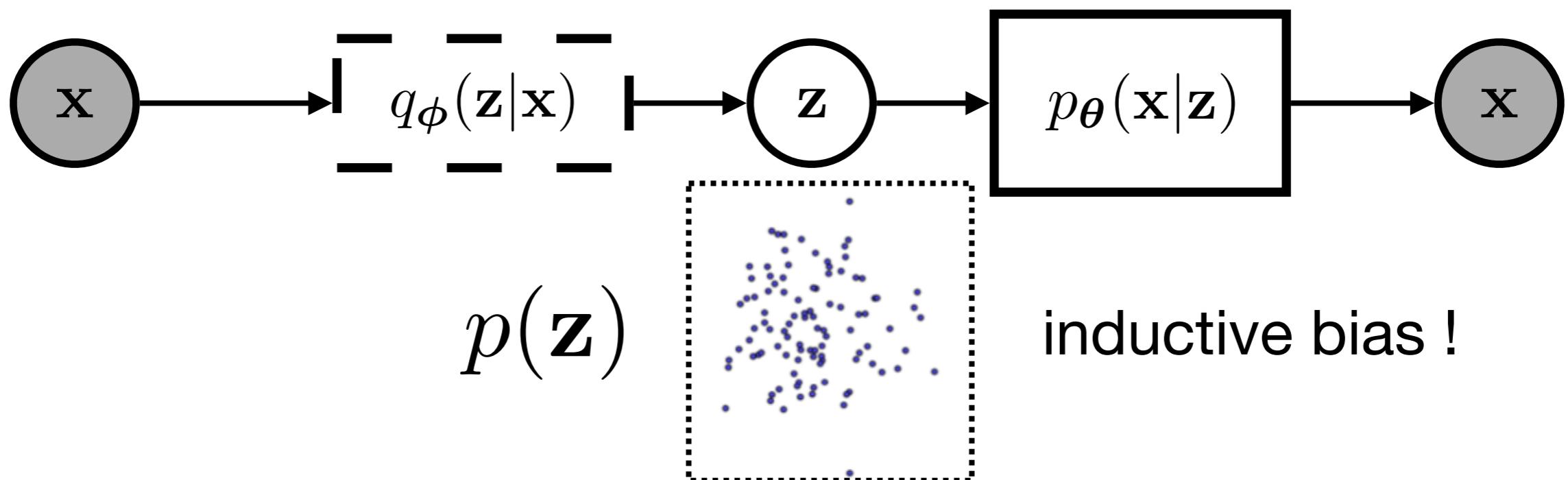
VAE vs. AE

VAE



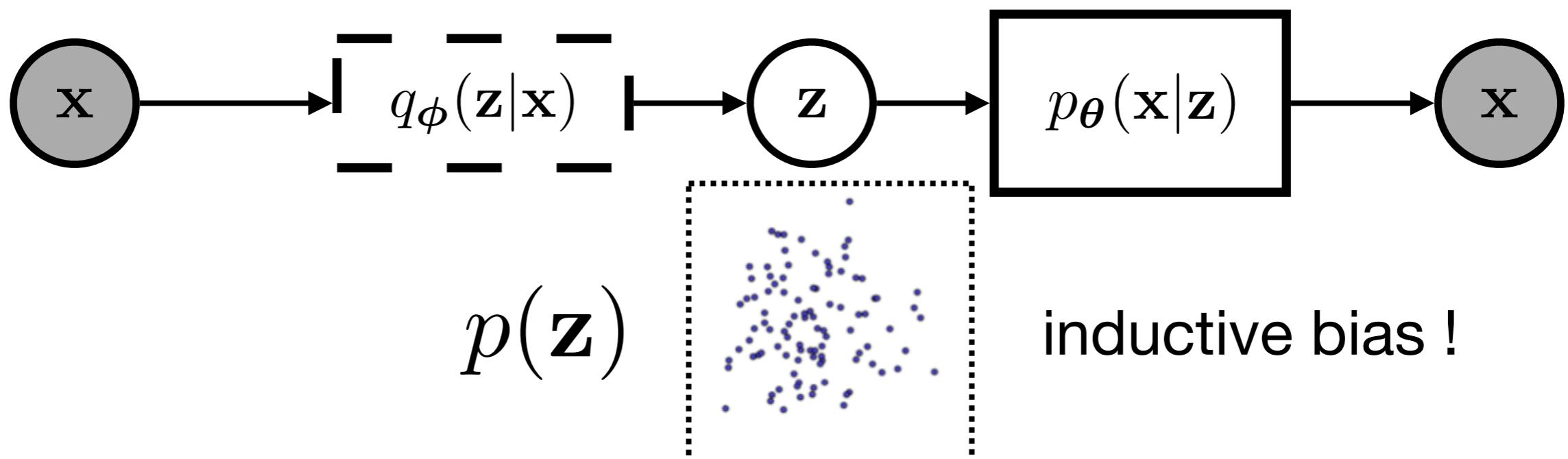
VAE vs. AE

VAE



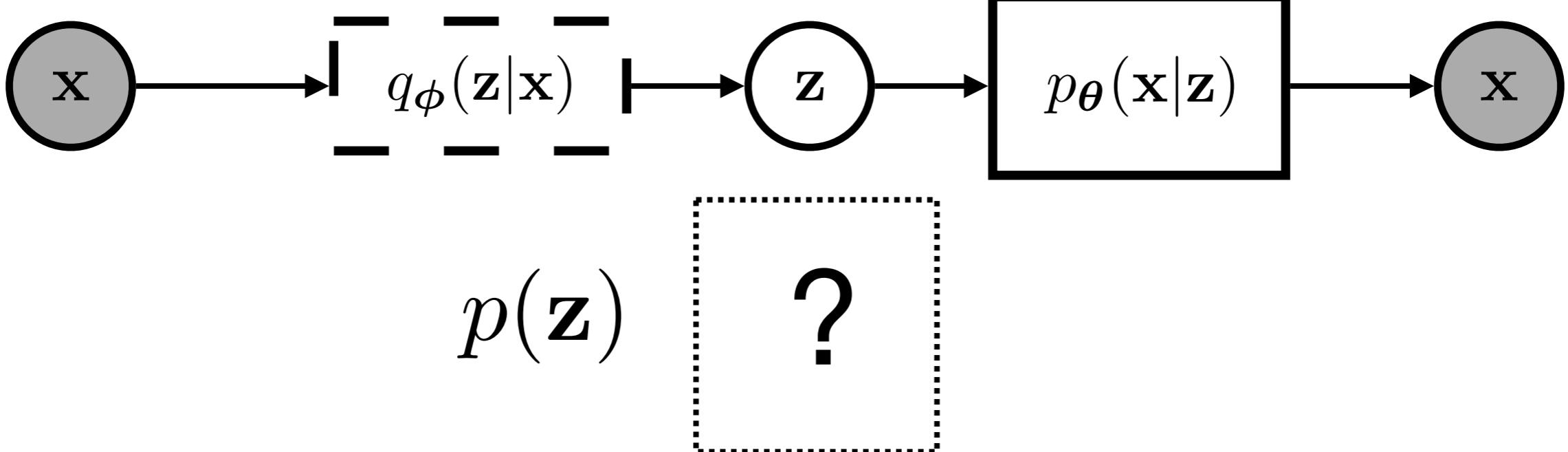
VAE vs. AE

VAE



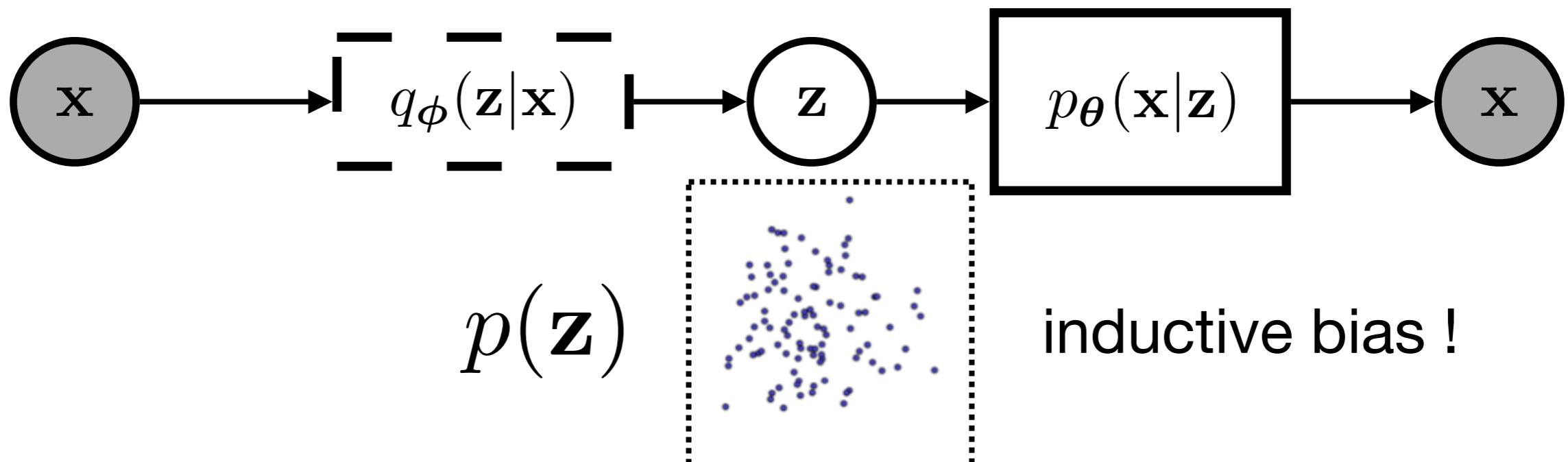
inductive bias !

AE



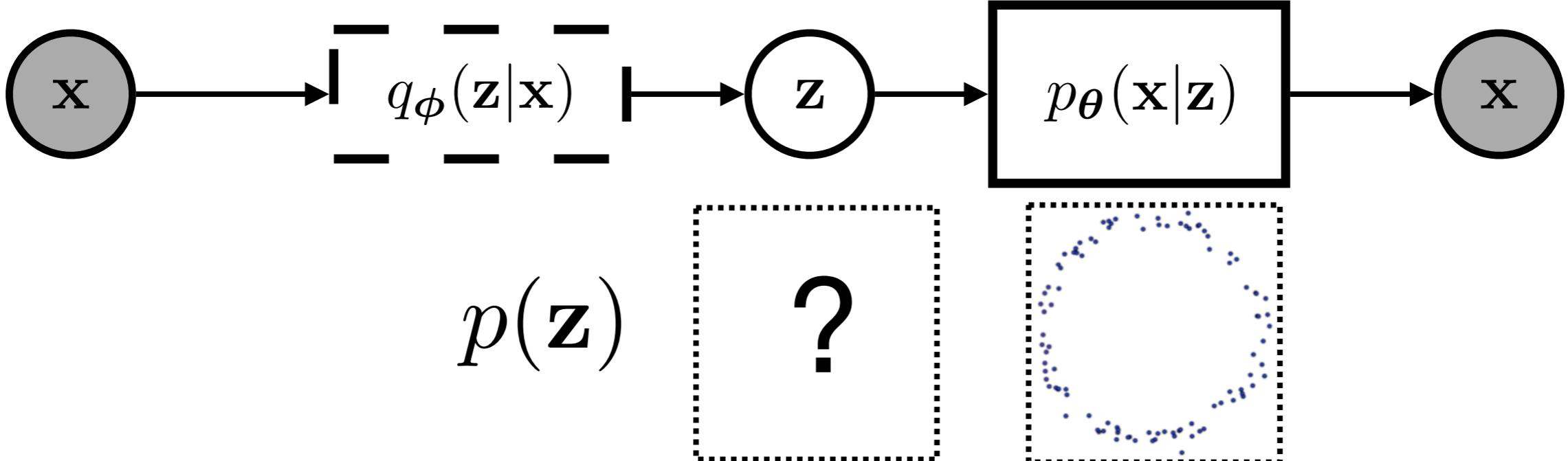
VAE vs. AE

VAE



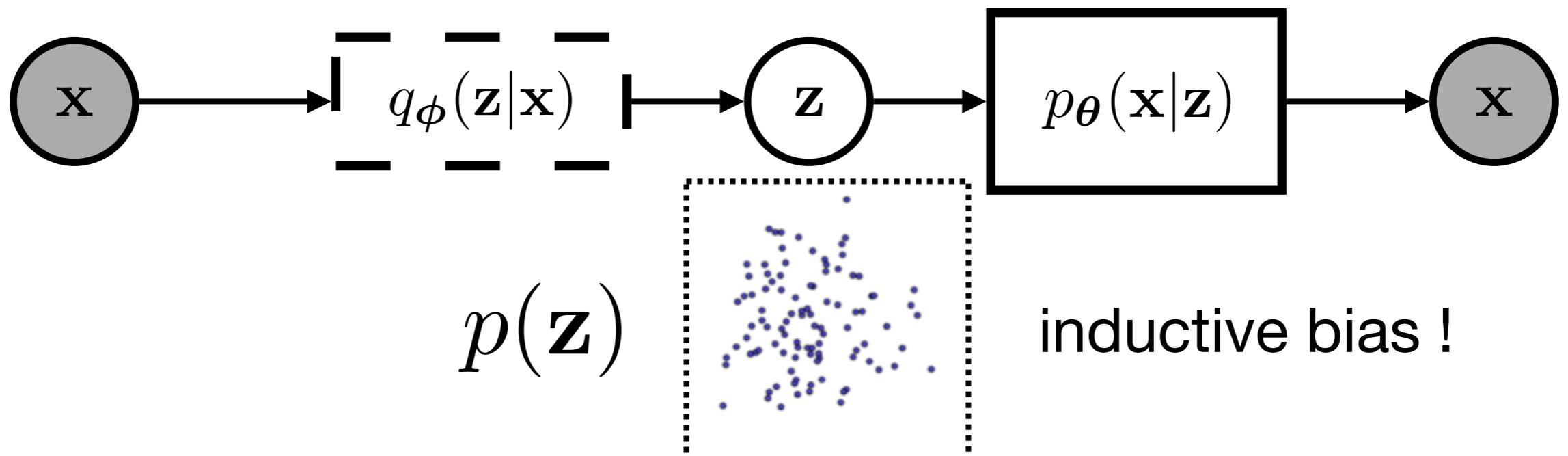
inductive bias !

AE



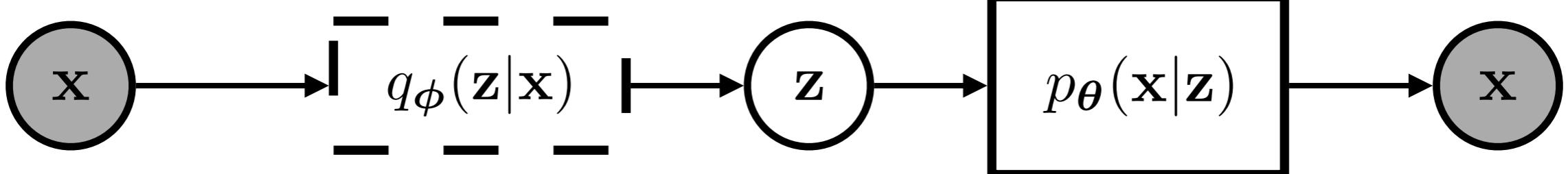
VAE vs. AE

VAE



inductive bias !

AE

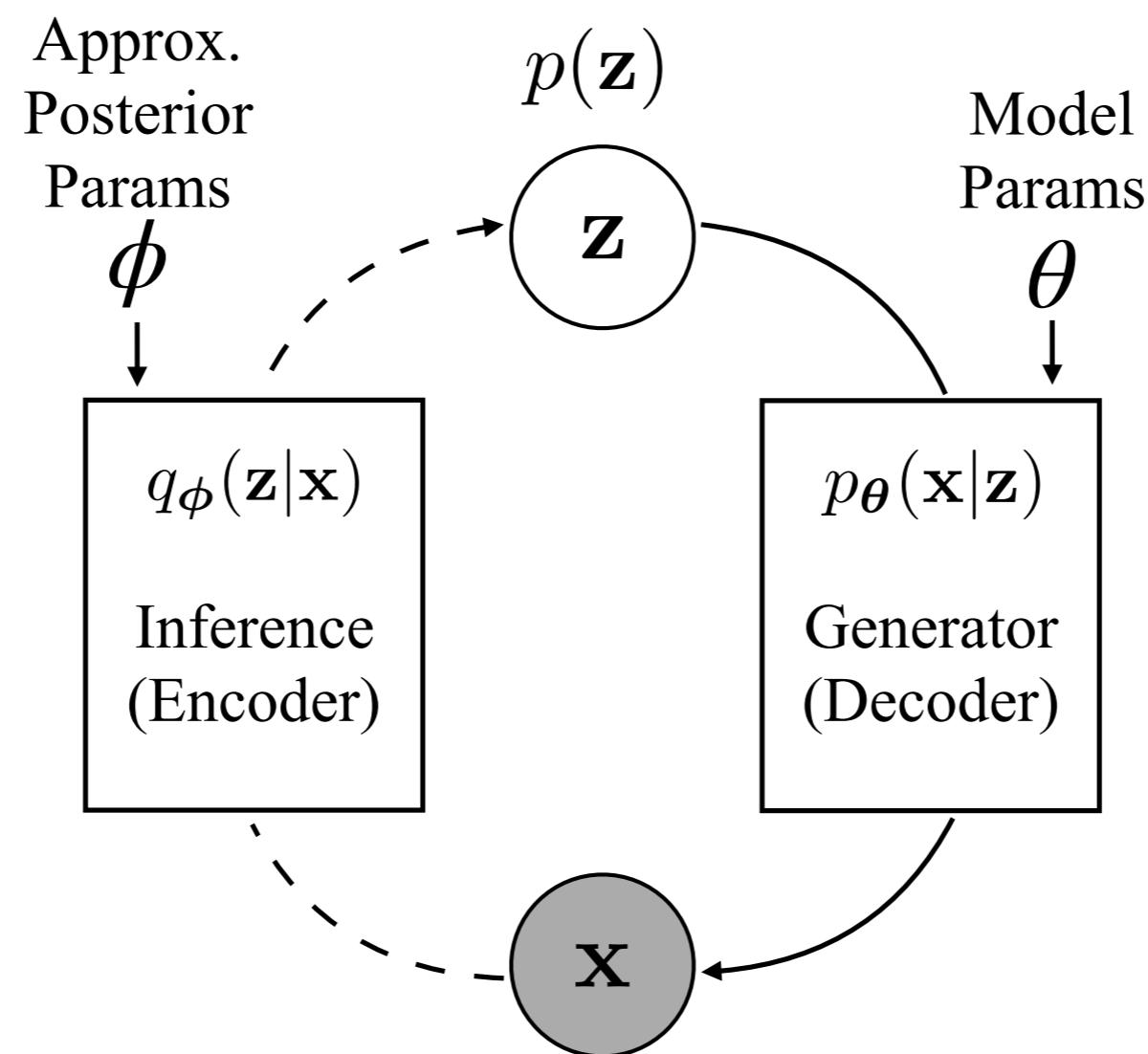


AE is not a generative model:

- (1) Can't sample new data from AE
- (2) Can't compute or bound the log likelihood of data x

Learning VAE

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\text{KL Regularizer}}$$



Problem!

Sampling Breaks Backprop

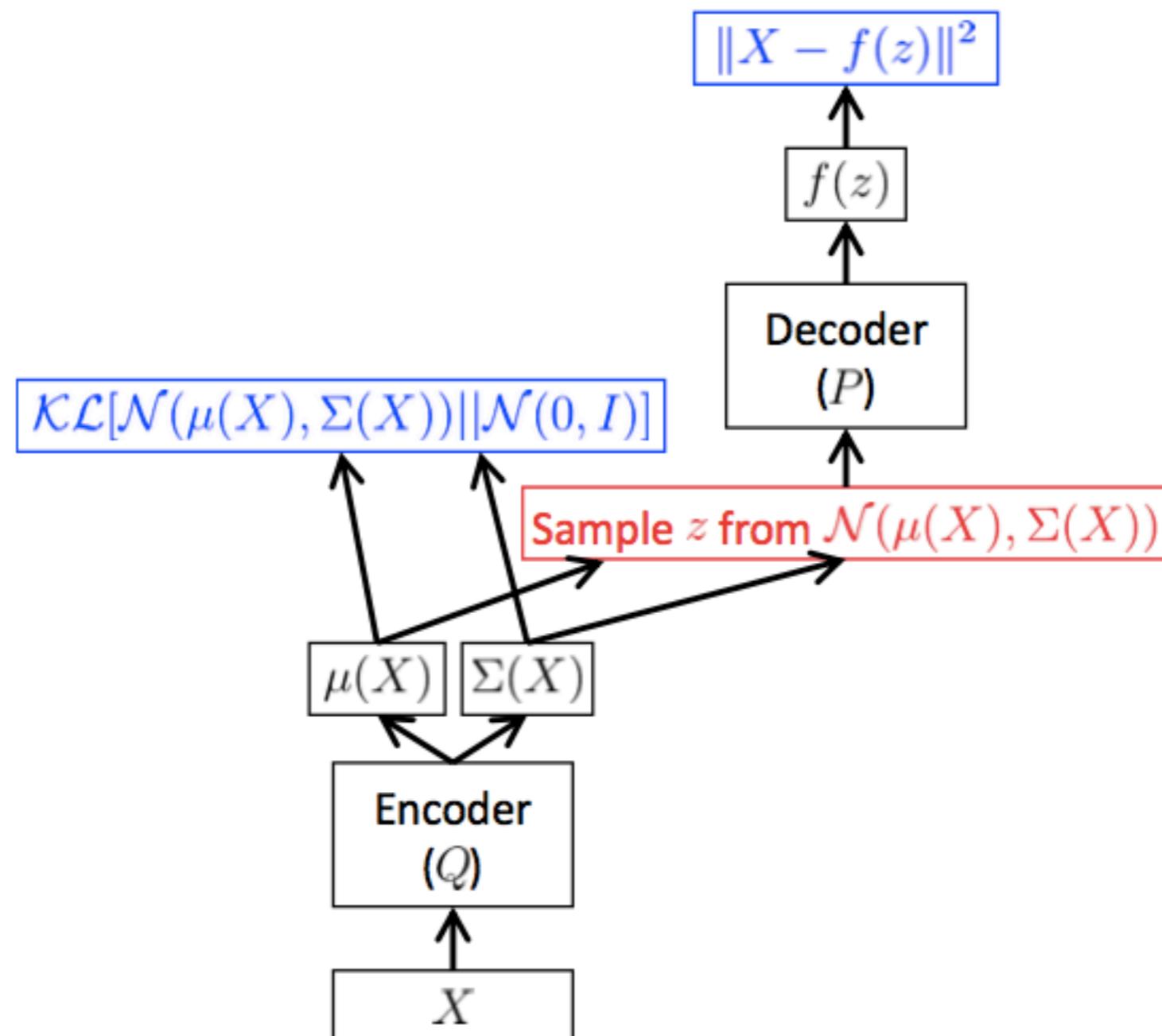


Figure Credit: Doersch (2016)

Solution: Re-parameterization Trick

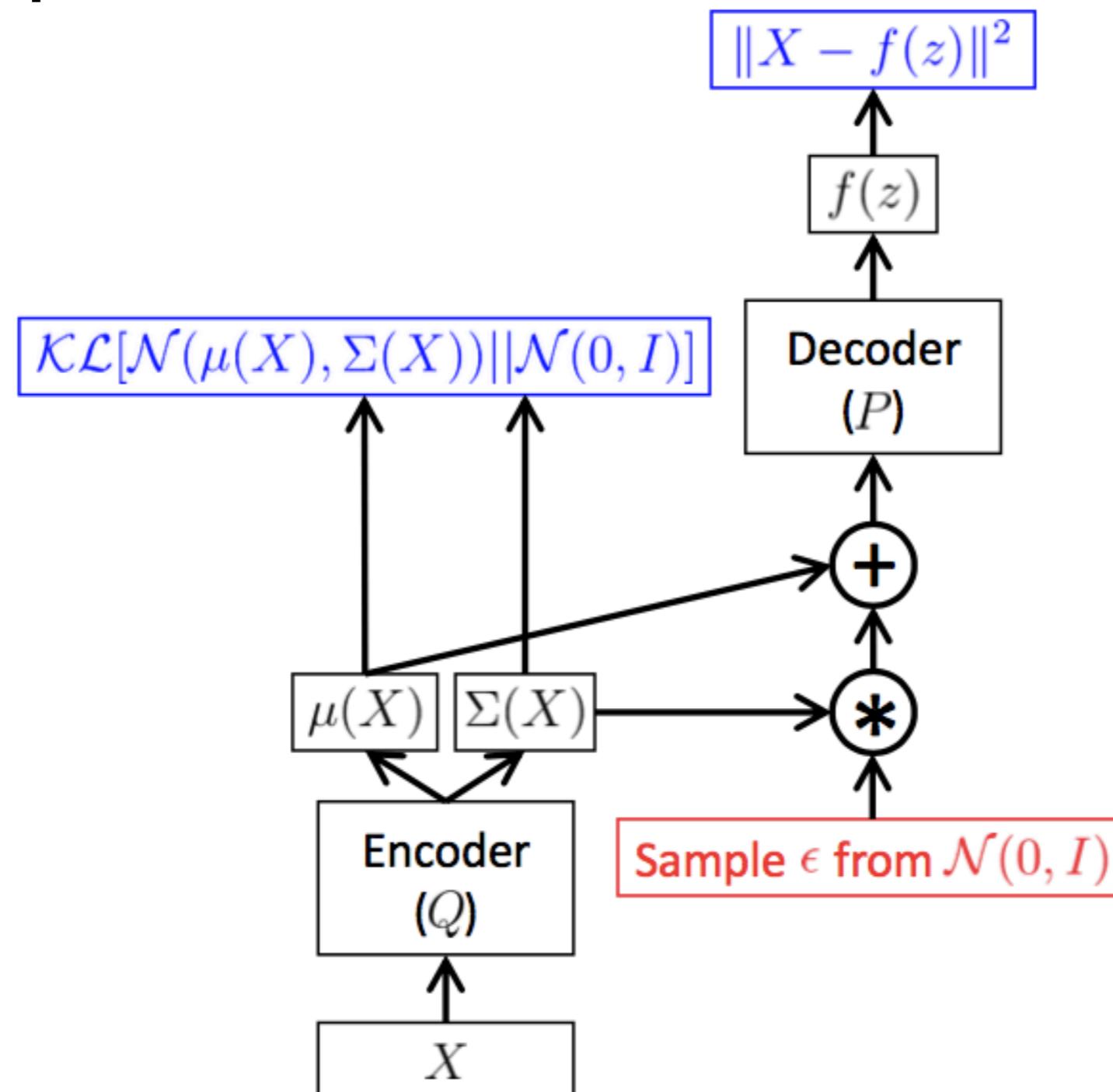


Figure Credit: Doersch (2016)

An Example: Generating Sentences
w/ Variational Autoencoders

Generating from Language Models

- **Remember:** using autoregressive sampling, we can generate from a normal language model

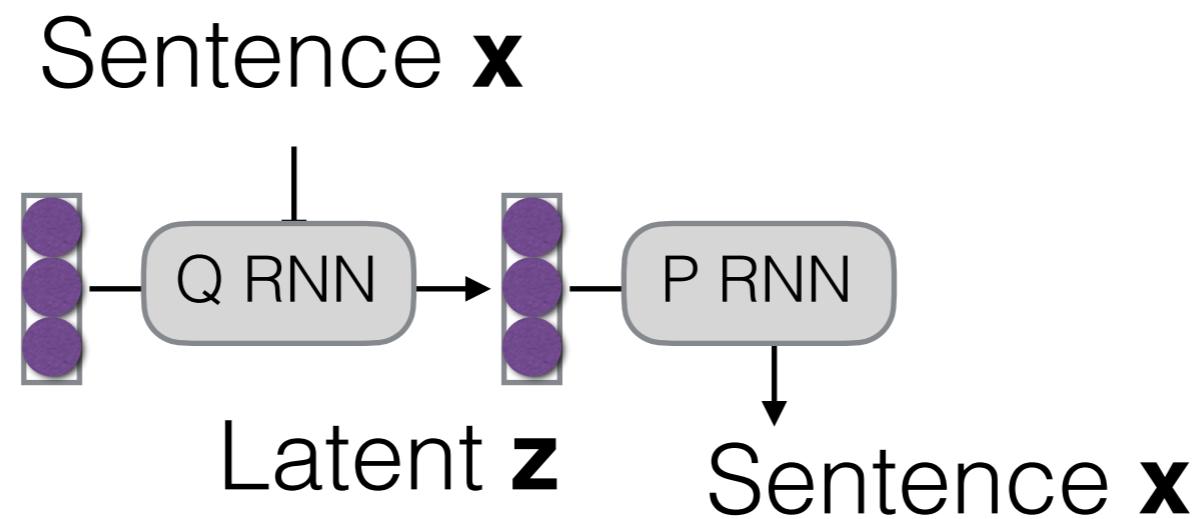
```
while  $x_{j-1} \neq "$ </s>" $":$   
 $x_j \sim P(x_j | x_1, \dots, x_{j-1})$ 
```

- We can also generate conditioned on something $P(\mathbf{y}|\mathbf{x})$ (e.g. translation, image captioning)

```
while  $y_{j-1} \neq "$ </s>" $":$   
 $y_j \sim P(y_j | X, y_1, \dots, y_{j-1})$ 
```

Generating Sentences from a Continuous Space (Bowman et al. 2015)

- The VAE-based approach is conditional language model that conditions on a latent variable \mathbf{z}
- Like an encoder-decoder, but latent representation is latent variable, input and output are identical



Motivation for Latent Variables

- Allows for a **consistent latent space** of sentences?
 - e.g. interpolation between two sentences

Standard encoder-decoder

i went to the store to buy some groceries .
i store to buy some groceries .
i were to buy any groceries .
horses are to buy any groceries .
horses are to buy any animal .
horses the favorite any animal .
horses the favorite favorite animal .
horses are my favorite animal .

VAE

“ i want to talk to you . ”
“i want to be with you . ”
“i do n’t want to be with you . ”
“i do n’t want to be with you . ”
she did n’t want to be with him .

he was silent for a long moment .
he was silent for a moment .
it was quiet for a moment .
it was dark and cold .
there was a pause .
it was my turn .

- **More robust to noise?** VAE can be viewed as standard model + regularization.

Difficulties in Training

- Of the two components in the VAE objective, the KL divergence term is much easier to learn!

$$\mathbb{E}_{z \sim Q(z|x)} [\log P(x | z)] - \mathcal{KL}[Q(z | x) || P(z)]$$

Requires good
generative model

Just need to
set the mean/variance
of Q to be same as P

- Results in the model learning to rely solely on decoder and ignore latent variable ($P(x|z) = P(x)$)
-> **Posterior Collapse**

Solution 1: KL Divergence Annealing

- Basic idea: Multiply KL term by a constant λ starting at zero, then gradually increase to 1
- Result: model can learn to use **z** before getting penalized

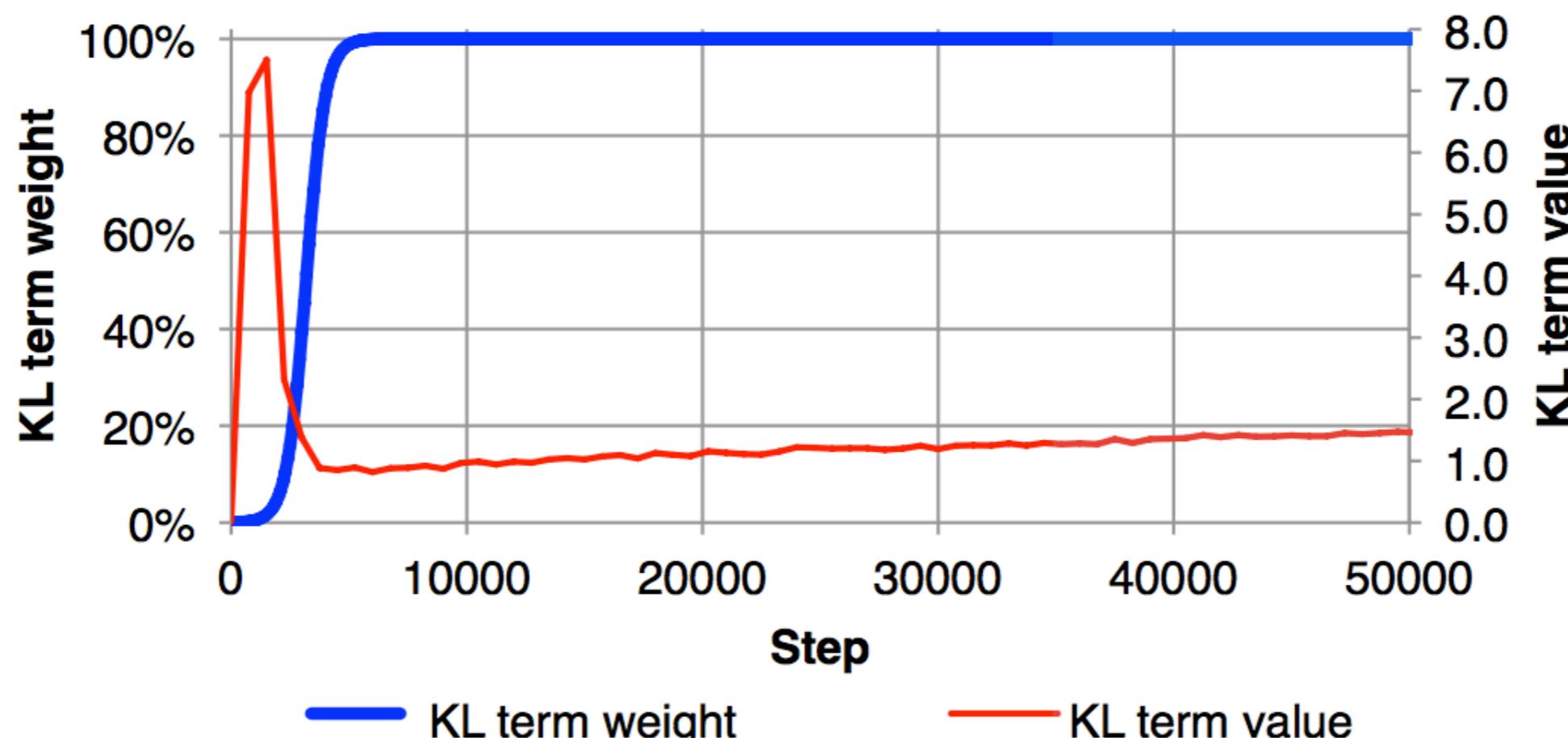


Figure Credit: Bowman et al. (2017)

Solution 2: Free bits / KL thresholding

- Free bits replaces the KL term in ELBO with a hinge loss that maxes each component of the original KL with a constant:

$$\sum_i \max[\lambda, D_{\text{KL}}(q_{\phi}(z_i|x) || p(z_i))]$$

- λ :Target rate

Solution 3: Weaken the Decoder

- But theoretically still problematic: it can be shown that the optimal strategy is to ignore \mathbf{z} when it is not necessary (Chen et al. 2017)
- Solution: weaken decoder $P(\mathbf{x}|\mathbf{z})$ so using \mathbf{z} is essential
 - Use word dropout to occasionally skip inputting previous word in \mathbf{x} (Bowman et al. 2015)
 - Use a convolutional decoder w/ limited context (Yang et al. 2017)

Solution 4:

Aggressive Inference Network Learning

- Freeze the generation network p during some training of the inference network q

$$\max_{\theta, \phi} \underbrace{\log p_{\theta}(\mathbf{x})}_{\text{marginal log data likelihood}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})|p_{\theta}(\mathbf{z}|\mathbf{x}))}_{\text{agreement between approximate and model posteriors}}$$

↓

$$\max_{\theta} \max_{\phi} \underbrace{\log p_{\theta}(\mathbf{x})}_{\text{marginal log data likelihood}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})|p_{\theta}(\mathbf{z}|\mathbf{x}))}_{\text{agreement between approximate and model posteriors}}$$

(He et al. 2019)

Handling Discrete Latent Variables

Discrete Latent Variables?

- Many variables are better treated as discrete
 - Part-of-speech of a word
 - Class of a question
 - Writer traits (left-handed or right-handed, etc.)
- How do we handle these?

Method 1: Enumeration

- For discrete variables, our integral is a sum

$$P(\mathbf{x}; \theta) = \sum_{\mathbf{z}} P(\mathbf{x} \mid \mathbf{z}; \theta) P(\mathbf{z})$$

- If the number of possible configurations for \mathbf{z} is small, we can just sum over all of them

Method 2: Sampling

- Randomly sample a subset of configurations of \mathbf{z} and optimize with respect to this subset
- Various flavors:
 - Minimum risk training
 - Maximize ELBO loss
- Score function gradient estimator - Policy Gradient Method
 - Like RL, but with $p(x | z)$ as the reward!
 - Unbiased estimator but high variance - need to control variance

Method 3: Reparameterization

(Maddison et al. 2017, Jang et al. 2017)

- Reparameterization also possible for discrete variables!

Original Categorical Sampling Method:

$$\hat{z} = \text{cat-sample}(P(z | x))$$

Reparameterized Method

$$\hat{z} = \operatorname{argmax}(\log P(z | x) + \text{Gumbel}(0,1))$$

where the Gumbel distribution is

$$\text{Gumbel}(0, 1) = -\log(-\log(\text{Uniform}(0,1)))$$

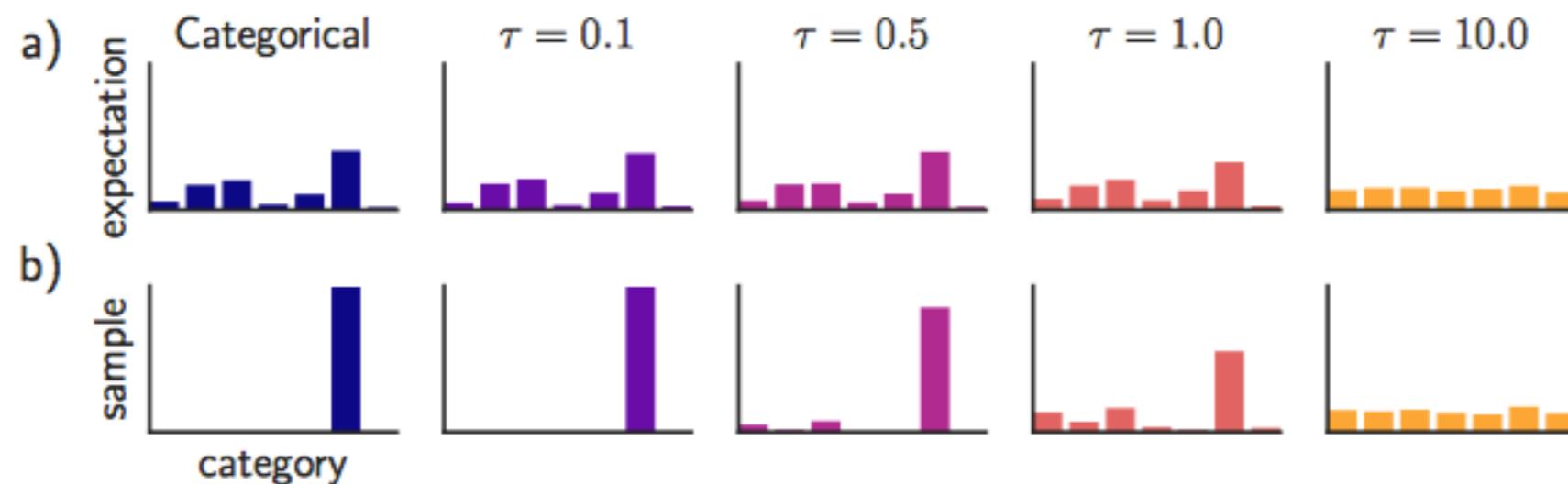
- Backprop is still not possible, due to argmax

Gumbel-Softmax

- A way to soften the decision and allow for continuous gradients
- Instead of argmax, take softmax with temperature τ

$$\hat{\mathbf{z}} = \text{softmax}((\log P(\mathbf{z} \mid \mathbf{x}) + \text{Gumbel}(0,1))^{1/\tau})$$

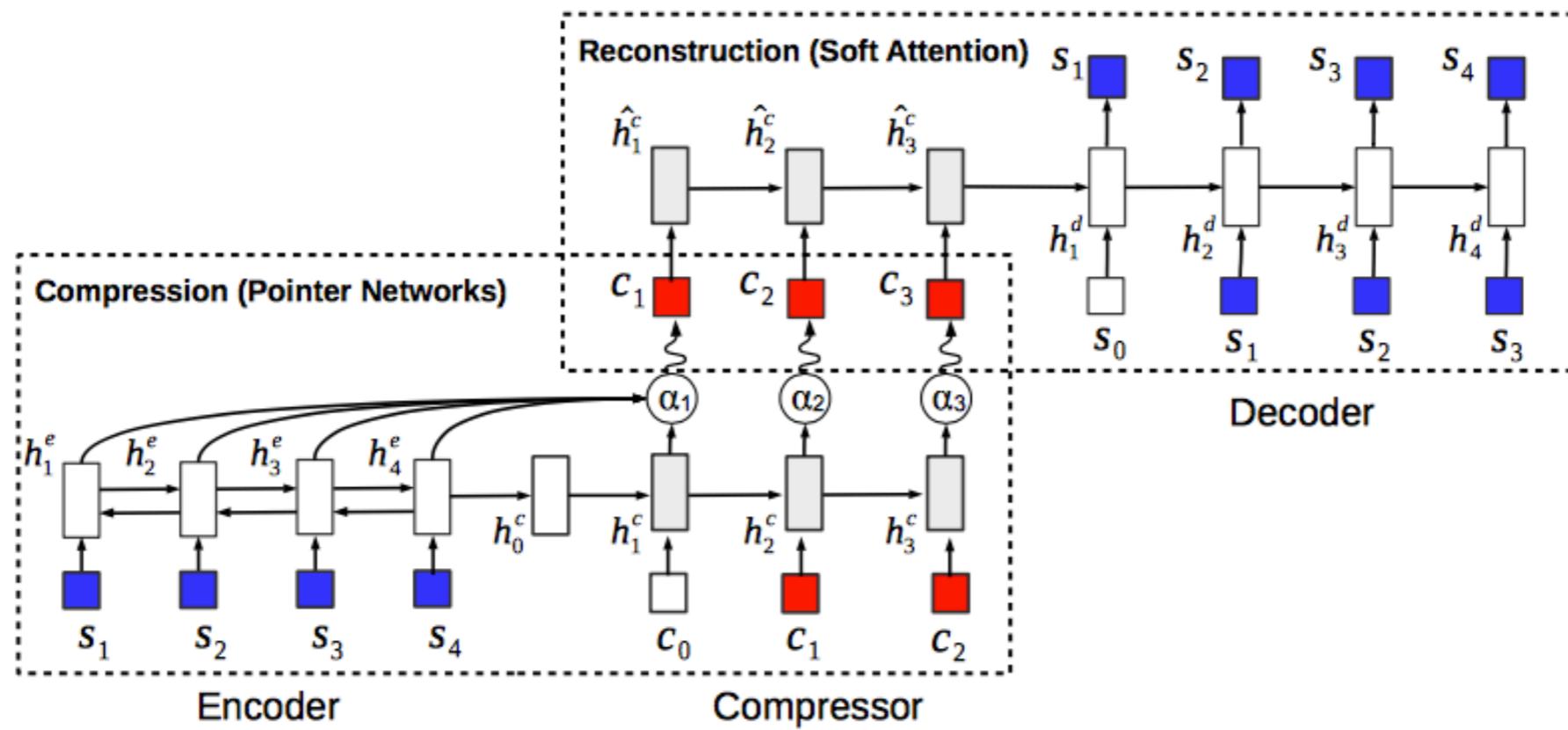
- As τ approaches 0, will approach max



Application Examples in NLP

Symbol Sequence Latent Variables (Miao and Blunsom 2016)

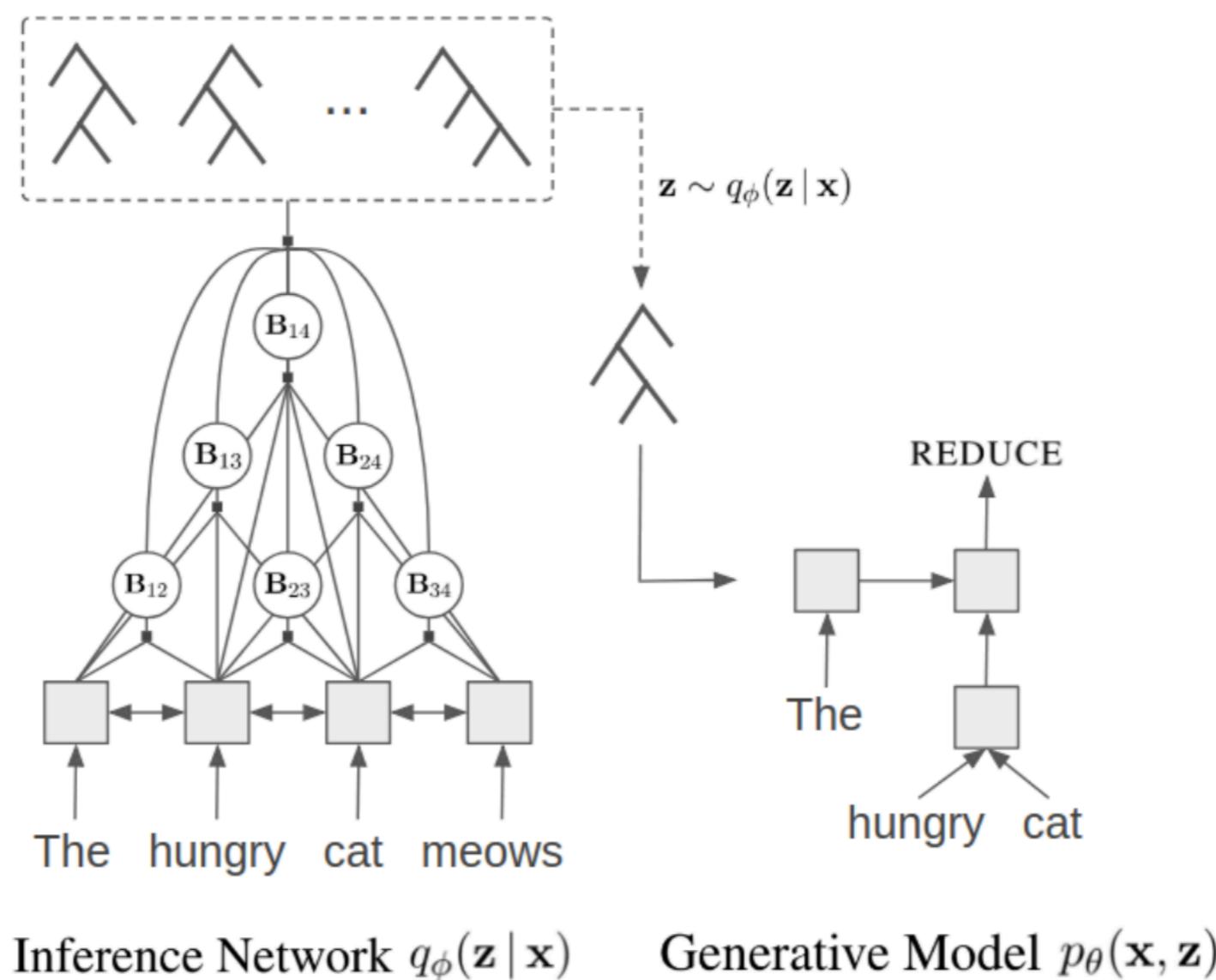
- Encoder-decoder with a sequence of latent symbols



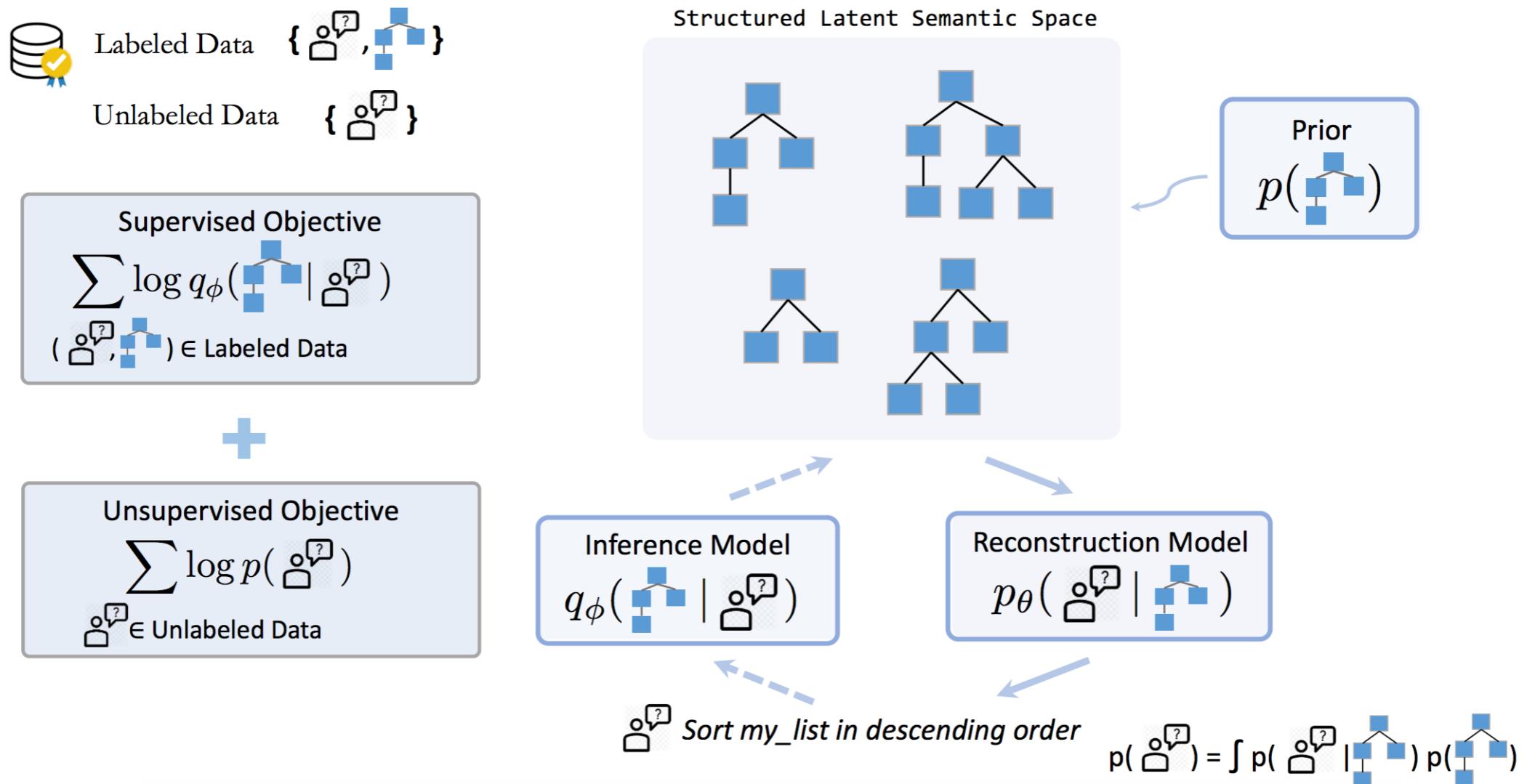
- Summarization in Miao and Blunsom (2016)
- Attempts to “discover” language (e.g. Havrylov and Titov 2017)
 - But things may not be so simple! (Kottur et al. 2017)

Unsupervised Recurrent Neural Network Grammars

(Kim et al., 2019)



STRUCTVAE: Tree-structured Latent Variable Models for Semi-supervised Semantic Parsing (Yin et al. 2018)



Questions?