Communication Link Budgets

* Noise limits our ability to detect/decode signals
* The metric that matters is signal-to-noise ratio (SNR or S/N) or the closely related E_b/N_0

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* How do we calculate the power recieved by a radio?

P_T = Transmitter power
G_T = Transmitter antenna gain
L_{FS} = Free-Space Loss
G_R = Receiver antenna gain
P_R = Received power

- In a real system there are additional effects: cable losses, multi-path, atmospheric attenuation, rain fade, polarization losses.
- Can be time varying and frequency dependent
- Add everything up:

$$P_R = \frac{P_T \, G_T \, G_R}{L_{FS}}$$

- Typically done in decibels with units dB-milliwatts (dBm)

$$\left.\underset{(dBm)}{P_R} = \underset{(dBm)}{P_T} + \underset{(dB)}{G_T} - \underset{(dB)}{L_{FS}} + \underset{(dB)}{G_R}\right\} \text{ "Friis Equation"}$$

* Free-Space Loss

distance

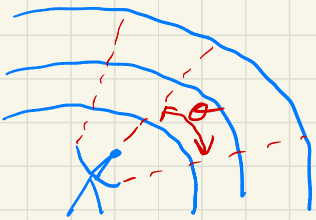
$$L_{FS} = \left(\frac{4\pi r}{\lambda}\right)^2 = 20\log_{10}(r) - 20\log_{10}(\lambda) + 20\log_{10}(4\pi)$$
$$= 20\log_{10}(r) + 20\log_{10}(f) + 20\log_{10}\left(\frac{4\pi}{c}\right)$$

wavelength

Frequency

speed of light

- Inverse-square law plus a frequency-dependent term



$$E(r) = \frac{P_T}{A(r)}$$

Irradiance (W/m^2)

- Diffraction limit $\Rightarrow \theta \approx \frac{\lambda}{D} \approx \frac{\lambda}{\sqrt{4\pi A_T}}$

diameter of dish

Area of dish

$$\Rightarrow A(r) \approx \pi(\theta r)^2 = \frac{\pi \lambda^2 r^2}{4\pi A_T} \approx \frac{\lambda^2 r^2}{A_T}$$

$$P_R = E(r) A_R = \underbrace{P_T\left(\frac{A_T A_R}{\lambda^2 r^2}\right)}$$

Another version of the Friis equation in terms of dish area instead of gain (actually the original)

* Antenna Gain:

  - A unitless quantity that measures how directed an antenna is.
  - Directly proportional to collecting area or "aperture"
  - In general, larger antenna ⇒ higher gain
  - Higher gain ⇒ tighter pointing requirements.

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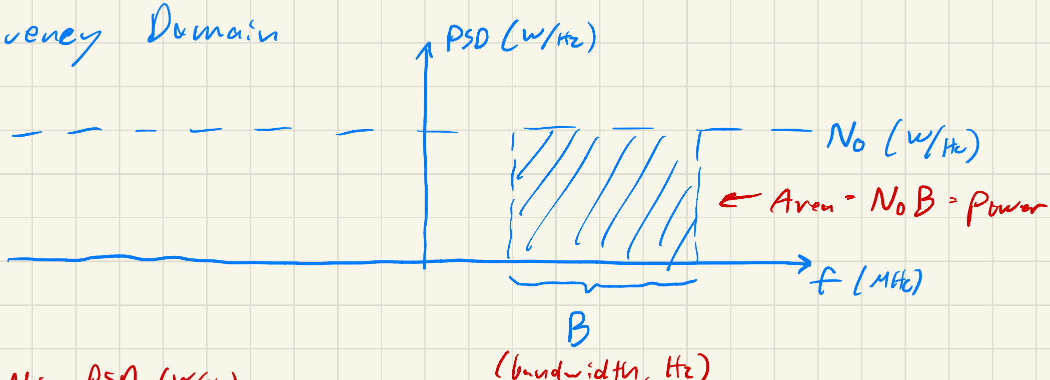
* How do we calculate noise power received by a radio?

  - Noise is everywhere due to thermal excitation of (mostly) electrons.
  - This ambient thermal noise is "white" (equal power at all frequencies) and it scales with temperature.
  - There is more noise if the background is hot vs. cold (e.g. deep space vs. warm planet)
  - Frequency Domain



Noise PSD (W/Hz)

$$N_0 = K_B T$$

← Temperature in Kelvin

Boltzmann's Constant (J/K)

$$N = N_0 B = K_B T B$$

Noise power (W)   Bandwidth (Hz)

- For analog signals $P_R/N = S/N$ is used to analyze links

- For digital signals, we use something like S/N integrated over a bit:

$$\frac{E_B}{N_0} = \frac{S}{N} \cdot \frac{B}{R_B}$$

Bandwidth (Hz)

Bit rate (Bits/sec)

Shannon-Hartley Theorem:

- The maximum bit rate achievable over a communication channel with white noise is given by:

$$C = B \log_2\left(1 + \frac{S}{N}\right)$$

"Channel capacity" = max R_B (bits/sec)

- With a little math, you can show:

$$\frac{E_B}{N_0} > \ln(2) \simeq -1.59 \text{ dB}$$

* It is actually possible to decode signals below the noise floor!

* Modern systems can get very close to the Shannon limit in practice.

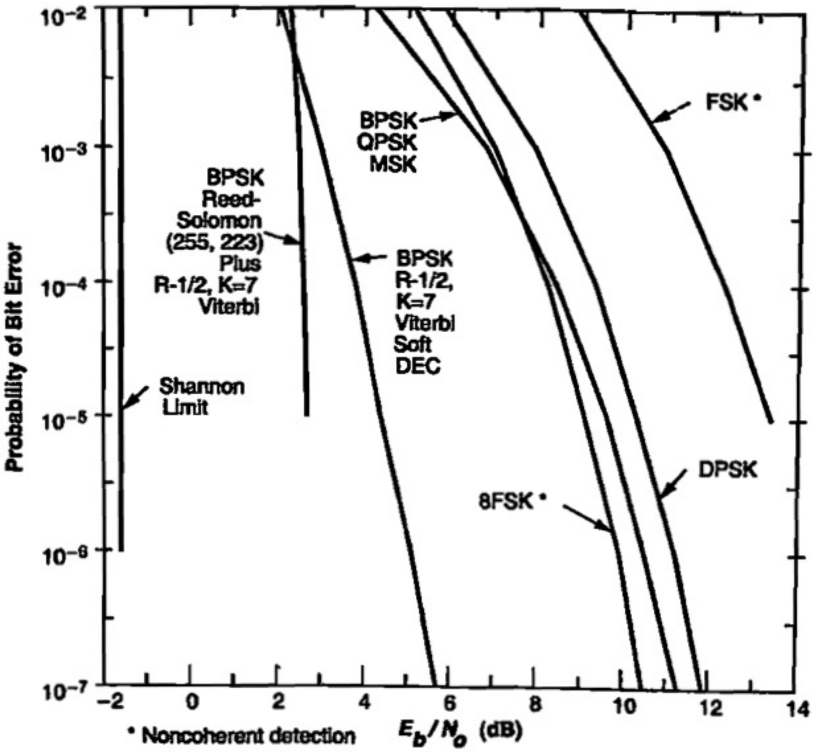


Fig. 13-9. Bit Error Probability as a Function of E_b/N_o. The theoretical performance limit can be approached by use of error correction coding.

* Typically add several dB of margin in a practical system.