Spacecraft Dynamics + Simulation:

* "Dynamics"

$$\dot{x} = f(\underbrace{t}_{\text{time}}, \overbrace{x}^{\text{state vector}}, \underbrace{u}_{\text{control input vector}})$$

- Can always put ODEs in this form (1^{st} order vector ODE)

$$F = ma = m\dot{v} \Rightarrow x = \begin{bmatrix} r \\ v \end{bmatrix}$$

position

velocity

$$\dot{x} = \begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ \frac{1}{m} F \end{bmatrix}$$

* "Simulation"

- Usually we can't solve these ODEs for $x(t)$ analytically:

$$x(t) = x(0) + \underbrace{\int_0^t f(t, x, u)\, dt}_{\text{this is a nasty integral}}$$

- We use numerical methods to approximate the solution
- Many options but most common are Runge-Kutta
- These fit a polynomial to $x(t)$ over a (short) time step by evaluating f
- "Order" ≈ order of polynomial
- "Stages" = number of f evals per time step

- Classic 4^{th}-order Runge-Kutta Method (RK4)

$$k_1 = f(t_n, x_n)$$
$$k_2 = f(t_n + \tfrac{1}{2}\Delta t, x_n + \tfrac{1}{2}\Delta t k_1)$$
$$k_3 = f(t_n + \tfrac{1}{2}\Delta t, x_n + \tfrac{1}{2}\Delta t k_2)$$
$$k_4 = f(t_n + \Delta t, x_n + \Delta t k_3)$$

$$x_{n+1} = x_n + \tfrac{1}{6}\Delta t(k_1 + 2k_2 + 2k_3 + k_4)$$

- Lots of extensions (e.g. adaptive step size) that you can find in libraries.

* Spacecraft Dynamics

- Spacecraft can mostly be modeled as rigid body

- Rigid body $\Rightarrow$ ignore flexible dynamics

$\Rightarrow \quad \omega_{flexible} \gg \omega_{RB/controller}$

- For a cubesat : Orbit dynamics $\approx$ 1/hour
  Attitude dynamics $\approx$ 1 Hz
  Flexible dynamics $\approx$ 1 KHz

decoupled ignore

* Orbit Dynamics

- What forces act on a satellite in LEO?

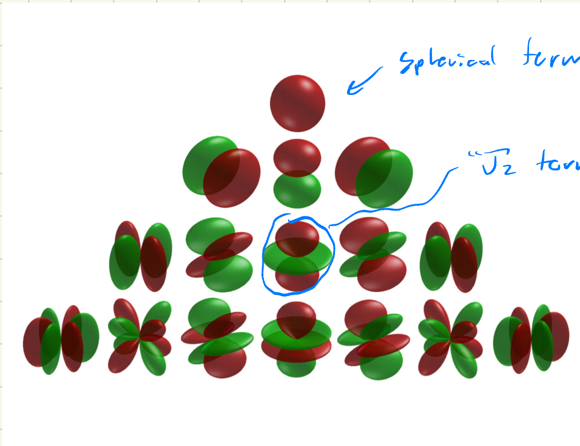
1) Gravity (not just spherical)
2) Atmospheric Drag
3) Solar radiation pressure

- Gravity

"Standard Gravitational Parameter" = GM

position $\in \mathbb{R}^3$

$$a_g = \underbrace{\frac{-\mu r}{\|r\|^3}}_{\text{"Spherical term"}} + \underbrace{J_2(r)}_{J_2 \text{ "doughnut" term}}$$



- In LEO $\|J_2\| \approx \|F_{drag}\|$. Next term is $\sim 1000\times$ smaller

"J_2 coefficient" for Earth $\approx 1.756 \times 10^{10}\ \mathrm{km^5/s^2}$

$$J_2(r) = \frac{J_2}{r^7}\begin{bmatrix} x\left(6z^2 - \frac{3}{2}(x^2+y^2)\right) \\ y\left(6z^2 - \frac{3}{2}(x^2+y^2)\right) \\ z\left(3z^2 - \frac{9}{2}(x^2+y^2)\right) \end{bmatrix}$$

- Drag:

Air density

velocity vector

$$F_{drag} = -\frac{1}{2} C_D \rho A v\|v\|$$

Drag Coefficient

Surface Area

$C_D \approx 2.2$ for spacecraft

$\rho \approx 10^{-15} - 10^{-11}\ \mathrm{kg/m^3}$ (use a model like Harris-Priestler)

- Add up all forces to get the dynamics:

$$\underbrace{\begin{bmatrix} \dot{r} \\ \dot{v} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} v \\ a_g(r) + \frac{1}{m} F_{drag}(r, v) \end{bmatrix}}_{f(x)}$$

- Initial Conditions:

    - To simulate, we need a starting position r_0 and velocity v_0

* Vis-Viva Equation:

$$v^2 = \mu\left(\frac{2}{\|r\|} - \frac{1}{a}\right)$$

"semimajor axis"

- circular orbit $\Rightarrow$ $a = \|r\|$ $\Rightarrow$ $v = \sqrt{\frac{\mu}{a}}$

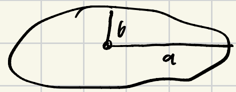
Orbital Elements:

* Assumes perfectly spherical planet with no perturbations

* Just a way of writing down shape + orientation of an elliptical orbit.

Semimajor Axis (a):

- Units of length (km)
- Defines size of orbit
- for a circular orbit $a = r$
- For ellipse:



(b = "semiminor axis")

Eccentricity (e)

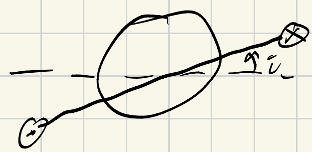
- Unitless
- Measures how "stretched" the ellipse is

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$e = 0$ for a circle
$0 < e < 1$ for ellipse

Inclination (i)

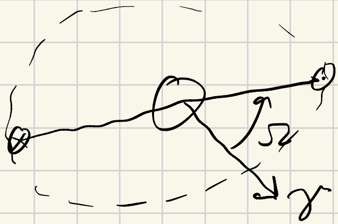
- Units of radians or degrees
- Angle between orbital plane and "reference plane" (equator)



← side view

Longitude of Ascending Node (RAAN) (Ω):

- Units of rad/deg
- Rotation of the orbital plane about $\hat{z}$ axis (north pole)
- Measured relative to a reference direction. "First point of Aries" or "Ram's Head" constellation γ

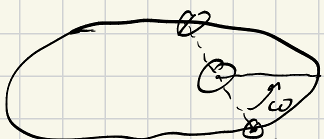


- Measured in "right hand" direction γ

Argument of Perigee/Periapsis (ω)

- Units of rad/deg
- Rotates orbit about $\underline{h}$ vector (orbit normal defined by satellite motion)
- Measured from the ascending node to periapsis

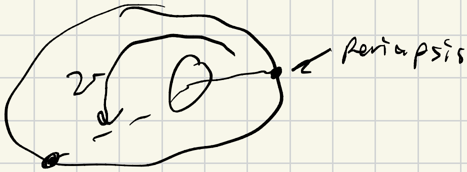
equator crossing
closest point to earth

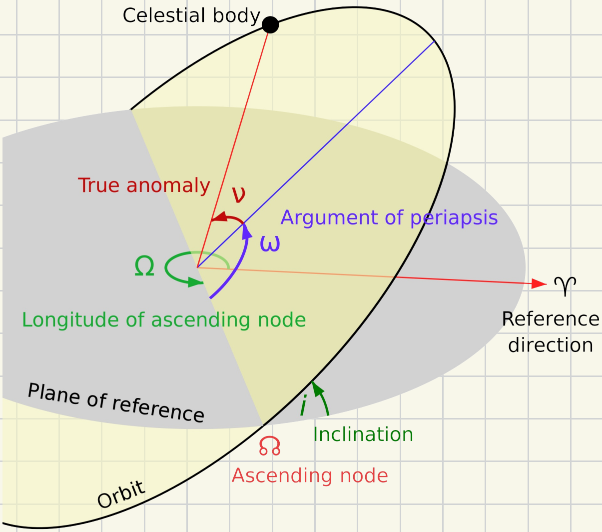


- Measured in the direction of spacecraft motion

True Anomaly (ν)

- Units of rad/deg
- Gives the position of the spacecraft along the orbit at a particular time
- Measured from periapsis


Periapsis


Celestial body
True anomaly
ν
Argument of periapsis
ω
Ω
♈
Longitude of ascending node
Reference
direction
Plane of reference
i
Inclination
☊
Ascending node
Orbit