Communication + Link Budgets

* Noise limits our ability to detect/decode signals

* The metric that matters is signal-to-noise ratio (SNR or S/N) or closely related E_B/N_0 for digital systems

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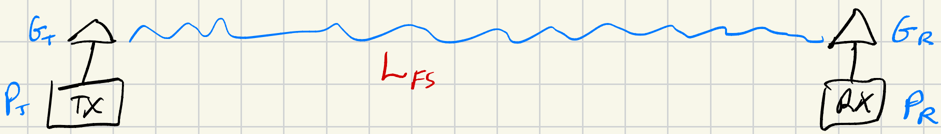
* How do we calculate power received by a radio?



P_T = Transmit Power
G_T = Transmitter antenna gain
L_{FS} = Free-Space Loss
G_R = Receiver Antenna gain
R_R = Received power

- In a real system there are additional effects: cable losses, multi-path, atmotspheric attenuation, rain fade, polarization losses.

- Can be time varying and frequency dependent

- Add everything up:

$$P_R = \frac{P_T\, G_T\, G_R}{L_{FS}}$$

- Typically done in decibels with units of dB-milliwatts (dBm)

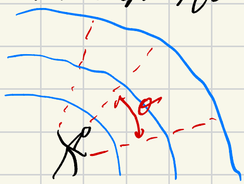
$$\left.\underset{(dBm)}{P_R} = \underset{(dBm)}{P_T} + \underset{(dB)}{G_T} - \underset{(dB)}{L_{FS}} + \underset{(dB)}{G_R}\right\}$$ "Friis Equation"

* Free-Space Loss

$$L_{FS} = \left(\frac{4\pi \overset{\text{distance}}{r}}{\underset{\text{wavelength}}{\lambda}}\right)^2 = 20\log_{10}(r) - 20\log_{10}(\lambda) + 20\log_{10}(4\pi)$$

$$= 20\log_{10}(r) + 20\log(\underset{\text{frequency}}{f}) + 20\log_{10}\left(\frac{4\pi}{c}\right)$$

- Inverse-square law plus a frequency-dependent term



$$\underset{\text{Irradiance } (W/m^2)}{E(r)} = \frac{P_T}{A(r)}$$

- Diffraction limit $\Rightarrow$ $\theta \approx \frac{\overset{\text{wavelength}}{\lambda}}{\underset{\text{dish diameter}}{D}} \approx \frac{\lambda}{\sqrt{4\pi \underset{\text{Area of dish}}{A_T}}}$

$$\Rightarrow A(r) \approx \pi(\theta r)^2 = \frac{\pi \lambda^2 r^2}{4\pi A_T} \approx \frac{\lambda^2 r^2}{A_T}$$

$$P_R = E(r)\, A_R = \underbrace{P_T\left(\frac{A_T A_R}{\lambda^2 r^2}\right)}$$

Another version of the Friis equation in terms of dish areas instead of Gain (actually original)

* Antenna Gain

- A unitless quantity that measures how directed an antenna is
- Directly proportional to collecting-area or "aperture"
- In general, larger antenna $\Rightarrow$ higher gain
- Higher gain $\Rightarrow$ tighter pointing requirements

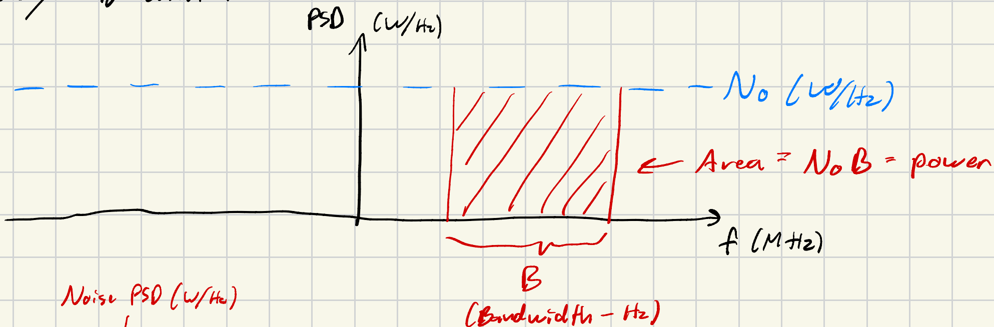
* How do we calculate noise power received by a radio?

- Noise is everywhere due to thermal excitation of (mostly) electrons

- This ambient thermal noise is "white" (equal power at all frequencies) and it scales with temperature

- There is more noise if the background is hot vs. cold (e.g. deep space vs. planet).

- Frequency Domain



Noise PSD (W/Hz)

$$N = N_0 B$$ ← Bandwidth (Hz)

Noise power (W)

$$N_0 = K_B T$$ ← temperature (Kelvin)

Botzmann's Constant (J/K)

$$\Rightarrow N = K_B T B$$

- For analog signals $P_R/N = S/N$ is used to analyze links

- For digital signals, we use something like S/N integrated over a bit:

$$\frac{E_B}{N_0} = \frac{S}{N} \cdot \frac{B}{R_B}$$

Bandwidth (Hz)

← Bit rate (bits/sec)

Shannon-Hartley Theorem

- The maximum bit rate achievable over a comms link with white noise is:

$$C = B \log_2 \left(1 + \frac{S}{N}\right)$$

"Channel capacity" = max R_B (bits/sec)

- With a little math, you can show:

$$\frac{E_B}{N_0} > \ln(2) \approx -1.59 \text{ dB}$$

* It is actually possible to decode signals below the noise floor

* Modern systems can get very close to the Shannon limit in practice.

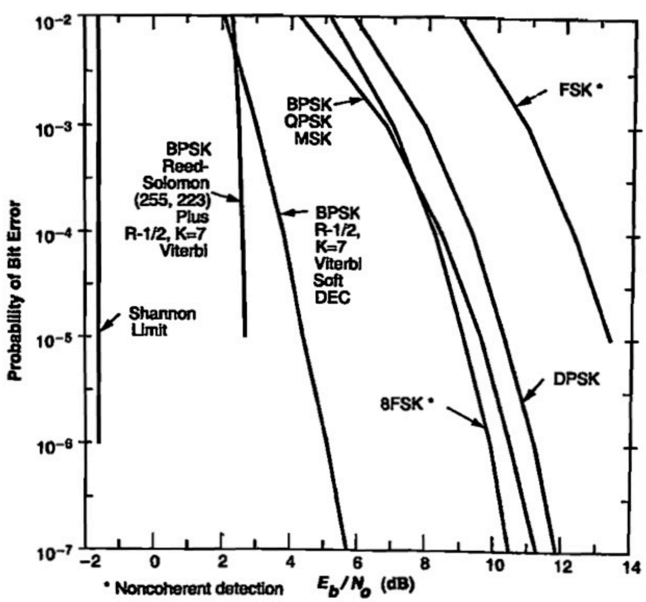


Fig. 13-9. Bit Error Probability as a Function of E_b/N_o. The theoretical performance limit can be approached by use of error correction coding.

* Typically add several dB of margin in a practical system