

# Were We Already There? Applying Minimal Generalization to the SIGMORPHON-UniMorph Shared Task on Cognitively Plausible Morphological Inflection

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## Abstract

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## 1 Introduction

In a landmark paper, Albright and Hayes (2003) proposed a model that learns morphological rules by recursive **minimal generalization** from lexeme-specific examples (e.g.,  $\text{ɪ} \rightarrow \text{ʌ} / \text{st} \_ \eta$  for *sting*  $\sim$  *stung* and  $\text{ɪ} \rightarrow \text{ʌ} / \text{fl} \_ \eta$  for *fling*  $\sim$  *flung* generalized to  $\text{ɪ} \rightarrow \text{ʌ} / \text{X} [-\text{syllabic}, +\text{coronal}, +\text{anterior}, \dots] \_ \eta$ ).<sup>1</sup> The model was presented more formally in Albright and Hayes (2002), along with evidence that the rules it learns for the English past tense give a good account of native speakers' productions and ratings in wug-test experiments (e.g. judgments that *splung* is quite acceptable as the past tense of the novel verb *spling*). In addition to providing further analysis of the experimental data, Albright and Hayes (2003) compared their proposal with early connectionist models of morphology (e.g., Plunkett and Juola, 1999) and an analogical or 'family resemblance' model inspired by research on psychological categories (Nakisa et al., 2001).

Along with Albright (2002), which presents a parallel treatment of Italian inflection, Albright & Hayes's study of the English past tense is a paradigm example of theory-driven, multiple-methodology, open and reproducible research in cognitive science.<sup>2</sup> Their model has enduring significance for the study of morphological learning

and productivity in English (e.g., Rácz et al., 2014, 2020; Corkery et al., 2019) and many other languages (e.g., Hijazi Arabic: Ahyad 2019; Japanese: Oseki et al. 2019; Korean: Albright and Kang 2009; Navajo: Albright and Hayes 2006; Portuguese: Veríssimo and Clahsen 2014; Russian: Kapatsinski 2010; Tgdaya Seediq: Kuo 2020; Spanish: Albright and Hayes 2003; Swedish: Strik 2014).

In this paper, we apply a partial reimplementa-tion of the Albright & Hayes model to wug-test rating data from three languages (Dutch, English, and German) collected for the SIGMORPHON-UniMorph 2021 Shared Task. Our version of the model is based purely on minimal generalization of morphological rules, as described in §3.1 of Albright and Hayes (2002) and reviewed below. It does not include additional mechanisms for learning phonological rules, and expanding or reigning in morphological rules, that were part of the original model (see Albright and Hayes, 2002, §3.3 - §3.7). We think there is much to be gained from considering minimal generalization on its own, with the other mechanisms ablated away, as borne out by competitive results on the shared task. For convenience, we refer to the original model as **min-gen** and our bare-bones version as **mingen0**.

### 1.1 Outline

## 2 Model Description

**Inputs** The model takes as input a set of word-form pairs, one per lexeme, that instantiate the same morphological relationship. In simulations of English past tense formation, these are pairs of bare verb stems and past tense forms such as  $\langle \text{ʌwɔk}, \text{ʌwɔkt} \rangle$ ,  $\langle \text{ʌtɔk}, \text{ʌtɔkt} \rangle$ ,  $\langle \text{ʌstɪŋ}, \text{ʌstɪŋ} \rangle$ ,  $\langle \text{ʌflɪŋ}, \text{ʌflɪŋ} \rangle$ , and  $\langle \text{ʌkæt}, \text{ʌkæt} \rangle$ . Word-forms consist of phonological segments (here, in broad transcription) delimited by special beginning and end of string symbols. The set  $\Sigma$  of phonological segments, and the set  $\Sigma_{\#} = \Sigma \cup \{ \text{ʌ}, \text{ʌ} \}$ , are assumed to be given to the model.

<sup>1</sup>The square brackets contain the shared phonological feature specifications of /t/ and /l/, which in the feature system used here are xxx.

<sup>2</sup>Albright & Hayes released both the results of their wug-test experiments and an implementation of their model (visit <http://www.mit.edu/~albright/mgl/> and <https://linguistics.ucla.edu/people/hayes/RulesVsAnalogy/index.html>). An impediment to large-scale simulation with the model is that it runs from a GUI interface only. As part of the present project, we have added a command line interface to the original source code (available on request).

The model also requires a phonological feature specification for each of the symbols that appears in wordforms. We used a well-known feature set, augmenting it with orthogonal feature specifications for the delimiters  $\bowtie$  and  $\ltimes$ .<sup>3</sup>  $\Phi$  is the set of all possible (partial) feature specifications over the chosen set and  $\phi(x)$  gives the features of  $x \in \Sigma_{\#}$ .

**Base rules** For each wordform pair, the model constructs a lexeme-specific morphological rule by first identifying the longest common prefix excluding  $\ltimes$  (C), then the longest common suffix from the remainder (D), and finally identifying the remaining symbols in the first (A) and second (B) members of the pair. The symbol  $\emptyset \notin \Sigma_{\#}$  denotes the empty string.<sup>4</sup> The rule formed from  $\langle \bowtie \text{w} \text{o} \text{k} \ltimes, \bowtie \text{w} \text{o} \text{k} \text{t} \ltimes \rangle$  has the components  $C = \bowtie \text{w} \text{o} \text{k}$ ,  $D = \ltimes$ ,  $A = \emptyset$  and  $B = \text{t}$  (i.e.,  $\emptyset \rightarrow \text{t} / \bowtie \text{w} \text{o} \text{k} \_ \ltimes$ ). The rule for  $\langle \bowtie \text{k} \text{a} \text{t} \ltimes, \bowtie \text{k} \text{a} \text{t} \ltimes \rangle$  is  $\emptyset \rightarrow \emptyset \bowtie \text{k} \text{a} \text{t} \_ \ltimes$ .

**Minimal Generalization** Given any two base rules  $R_1$  and  $R_2$  that make the same change ( $A \rightarrow B$ ), the model forms a possibly more general rule by aligning and comparing their contexts. The minimal generalization operation,  $R = R_1 \sqcap R_2$ , carries over the common change of the two base rules and applies independently to their left-hand ( $C_1, C_2$ ) and right-hand ( $D_1, D_2$ ) contexts. For convenience, we define minimal generalization of the right-hand contexts. Minimal generalization of the left-hand contexts can be performed by reversing  $C_1$  and  $C_2$ , applying the definition for right-hand contexts, and reversing the result.

The minimal generalization  $D = D_1 \sqcap D_2$  is defined procedurally by first extracting the longest common prefix ( $\sigma_{1 \wedge 2}$ ) of the two contexts and then operating on the remainders ( $D'_1, D'_2$ ). If both  $D'_1$  and  $D'_2$  are empty then  $D = \sigma_{1 \wedge 2}$ . If one but not both of them are empty then  $D = \sigma_{1 \wedge 2} X$ , where  $X \notin \Sigma_{\#}$  is a variable over symbol sequences (i.e.,  $X$  stands for  $\Sigma_{\#}^*$ ). If neither is empty, then the operation determines whether their first two symbols have any shared features; for this purpose it is useful to consider  $\phi(x)$  as a function from symbols to sets of feature-value pairs, in which case the common features are found by set intersection.

If there are no common features,  $\phi_{1 \cap 2} = \emptyset$ , then as before  $D = \sigma_{1 \wedge 2} X$ . Otherwise, the set of common features  $\phi_{1 \cap 2} \neq \emptyset$  is appended to  $\sigma_{1 \wedge 2}$ , the first symbol is removed from  $D'_1$  and  $D'_2$ , and the operation applies to the remainders. If both remainders are empty then  $D = \sigma_{1 \wedge 2} \phi_{1 \cap 2}$ , otherwise  $D = \sigma_{1 \wedge 2} \phi_{1 \cap 2} X$ .

In summary, the generalized right-hand context  $D$  consists of the longest common prefix shared by  $D_1$  and  $D_2$ , followed by a single set of shared features (if any), followed by  $X$  in case there are no shared features or one context is longer than the other. With the change and generalized left-hand context  $C$  determined as already described, the result of applying minimal generalization to the two base rules is  $R = A \rightarrow B/C \_ D$ .<sup>5</sup>

**Recursive Minimal Generalization** Applying Minimal Generalization to all base rule pairs

### 3 Results

### 4 Conclusions and Future Directions

<sup>3</sup>The phonological feature specifications are available from Bruce Hayes's website, <https://linguistics.ucla.edu/people/hayes/120a/Index.htm#features>. xxx binary with 0s xxx original features distributed with the model included scalar features xxx for example. Alternative binary (with underspecification) feature sets are xxx phoible xxx panphon.

<sup>4</sup>In other common notations, the empty string is denoted by  $\lambda$ . xxx by construction can occur only in A or B

<sup>5</sup>There could be a small difference between our definition of context generalization and that in Albright and Hayes (2002), hinging on whether the empty feature set is allowed in rules. In our definition,  $\phi_{1 \cap 2} = \emptyset$  is replaced by the variable  $X$ . It is possible that the original proposal intended for empty and non-empty feature sets to be treated alike. The definitions can diverge when applied to contexts that are of identical length and share all but the last (resp. first) segments, in which case our version would result in a broader rule.

These instructions are for authors submitting papers to \*ACL conferences using L<sup>A</sup>T<sub>E</sub>X. They are not self-contained. All authors must follow the general instructions for \*ACL proceedings,<sup>6</sup> and this document contains additional instructions for the L<sup>A</sup>T<sub>E</sub>X style files.

The templates include the L<sup>A</sup>T<sub>E</sub>X source of this document (`acl.tex`), the L<sup>A</sup>T<sub>E</sub>X style file used to format it (`acl.sty`), an ACL bibliography style (`acl_natbib.bst`), an example bibliography (`custom.bib`), and the bibliography for the ACL Anthology (`anthology.bib`).

## 5 Engines

To produce a PDF file, pdfL<sup>A</sup>T<sub>E</sub>X is strongly recommended (over original L<sup>A</sup>T<sub>E</sub>X plus dvips+ps2pdf or dvi2pdf). XeL<sup>A</sup>T<sub>E</sub>X also produces PDF files, and is especially suitable for text in non-Latin scripts.

## 6 Preamble

The first line of the file must be

```
\documentclass[11pt]{article}
```

To load the style file in the review version:

```
\usepackage[review]{acl}
```

For the final version, omit the `review` option:

```
\usepackage{acl}
```

To use Times Roman, put the following in the preamble:

```
\usepackage{times}
```

(Alternatives like `txfonts` or `newtx` are also acceptable.)

Please see the L<sup>A</sup>T<sub>E</sub>X source of this document for comments on other packages that may be useful.

Set the title and author using `\title` and `\author`. Within the author list, format multiple authors using `\and` and `\And` and `\AND`; please see the L<sup>A</sup>T<sub>E</sub>X source for examples.

By default, the box containing the title and author names is set to the minimum of 5 cm. If you need more space, include the following in the preamble:

```
\setlength\titlebox{<dim>}
```

where `<dim>` is replaced with a length. Do not set this length smaller than 5 cm.

<sup>6</sup><http://acl-org.github.io/ACLPUB/formatting.html>

Command	Output	Command	Output
<code>\"a</code>	ä	<code>\c c</code>	ç
<code>\^e</code>	ê	<code>\u g</code>	ğ
<code>\`i</code>	ì	<code>\l</code>	ł
<code>\.I</code>	İ	<code>\~n</code>	ñ
<code>\o</code>	ø	<code>\H o</code>	ő
<code>\'u</code>	ú	<code>\v r</code>	ř
<code>\aa</code>	å	<code>\ss</code>	ß

Table 1: Example commands for accented characters, to be used in, *e.g.*, BibT<sub>E</sub>X entries.

## 7 Document Body

### 7.1 Footnotes

Footnotes are inserted with the `\footnote` command.<sup>7</sup>

### 7.2 Tables and figures

See Table 1 for an example of a table and its caption. **Do not override the default caption sizes.**

### 7.3 Hyperlinks

Users of older versions of L<sup>A</sup>T<sub>E</sub>X may encounter the following error during compilation:

```
\pdfendlink ended up in
different nesting level
than \pdfstartlink.
```

This happens when pdfL<sup>A</sup>T<sub>E</sub>X is used and a citation splits across a page boundary. The best way to fix this is to upgrade L<sup>A</sup>T<sub>E</sub>X to 2018-12-01 or later.

### 7.4 Citations

Table 2 shows the syntax supported by the style files. We encourage you to use the `natbib` styles. You can use the command `\citet` (cite in text) to get “author (year)” citations, like this citation to a paper by ?. You can use the command `\citep` (cite in parentheses) to get “(author, year)” citations (?). You can use the command `\citealp` (alternative cite without parentheses) to get “author, year” citations, which is useful for using citations within parentheses (*e.g.* ?).

### 7.5 References

The L<sup>A</sup>T<sub>E</sub>X and BibT<sub>E</sub>X style files provided roughly follow the American Psychological Association format. If your own bib file is named `custom.bib`, then placing the following before any appendices in

<sup>7</sup>This is a footnote.

Output	natbib command	Old ACL-style command
(?)	<code>\citep</code>	<code>\cite</code>
?	<code>\citealp</code>	no equivalent
?	<code>\citet</code>	<code>\newcite</code>
(?)	<code>\citeyearpar</code>	<code>\shortcite</code>

Table 2: Citation commands supported by the style file. The style is based on the natbib package and supports all natbib citation commands. It also supports commands defined in previous ACL style files for compatibility.

your L<sup>A</sup>T<sub>E</sub>X file will generate the references section for you:

```
\bibliographystyle{acl_natbib}
\bibliography{custom}
```

You can obtain the complete ACL Anthology as a BibT<sub>E</sub>X file from <https://aclweb.org/anthology/anthology.bib.gz>. To include both the Anthology and your own .bib file, use the following instead of the above.

```
\bibliographystyle{acl_natbib}
\bibliography{anthology, custom}
```

Please see Section 8 for information on preparing BibT<sub>E</sub>X files.

## 7.6 Appendices

Use `\appendix` before any appendix section to switch the section numbering over to letters. See Appendix A for an example.

## 8 BibT<sub>E</sub>X Files

Unicode cannot be used in BibT<sub>E</sub>X entries, and some ways of typing special characters can disrupt BibT<sub>E</sub>X’s alphabetization. The recommended way of typing special characters is shown in Table 1.

Please ensure that BibT<sub>E</sub>X records contain DOIs or URLs when possible, and for all the ACL materials that you reference. Use the `doi` field for DOIs and the `url` field for URLs. If a BibT<sub>E</sub>X entry has a URL or DOI field, the paper title in the references section will appear as a hyperlink to the paper, using the `hyperref` L<sup>A</sup>T<sub>E</sub>X package.

## Acknowledgements

This research was partially supported by NSF grant BCS-1844780.

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## A Example Appendix

This is an appendix.