Carnegie Mellon
School of Computer Science

Deep Reinforcement Learning and Control

Pathwise derivatives, DDPG

CMU 10-703

Katerina Fragkiadaki

Computing Gradients of Expectations

Likelihood ratio gradient estimators:

$$\max_{\theta} . \quad \mathbb{E}_{x\sim P_\theta(x)} f(x)$$

$$\mathbb{E}_{x\sim P_\theta(x)} \nabla_\theta \log P_\theta(x) f(x)$$

$$\max_{\theta} . \ \mathbb{E}_{\tau\sim P_\theta(\tau)} \left[R(\tau) \right]$$

$$\mathbb{E}_{\tau\sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau) \right]$$

$$\mathbb{E}_{s\sim d^0(s),\ a\sim\pi_\theta(a|s)} \nabla_\theta \log \pi_\theta(a \mid s) \left[Q(s,a,\mathbf{w}) - V(s,\mathbf{w}) \right]$$

Qs:

- For continuous actions a, do we have access to the analytic gradients of rewards $R(\tau)$ w.r.t. actions a?
- For continuous actions a, do we have access to the analytic gradients of Q(s,a) or A(s,a)=Q(s,a)-V(s) w.r.t. actions a?
- Have we used the later anywhere?

What if we have a deterministic policy?

Here P does not anymore represent a distribution, just a deterministic function

$$y = P_\theta(x)$$

$$\max_\theta . \; f(P_\theta(x))$$

$$a = \pi_\theta(s)$$

$$\max_\theta . \; \mathbb{E} \sum_t R(S_t, \pi_\theta(S_t))$$

$$\max_\theta . \; \mathbb{E} \sum_t Q(S_t, \pi_\theta(S_t))$$

Qs:

- Can we backpropagate through R?
- Can we backpropagate through Q?

$$\frac{df(P_\theta(x))}{d\theta} = \frac{df(y)}{dy}\frac{dy}{d\theta}$$

$$\frac{d\mathbb{E}\sum_t Q(S_t, \pi_\theta(S_t))}{d\theta} = \mathbb{E}\sum_t \frac{dQ(S_t, \pi_\theta(S_t))}{da}\frac{da}{d\theta}$$

What if we have a deterministic policy?

Pathwise derivatives

$$y = P_\theta(x)$$

$$\max_\theta . \; f(P_\theta(x))$$

$$\frac{df(P_\theta(x))}{d\theta} = \frac{df(y)}{dy}\frac{dy}{d\theta}$$

Q: does this expectation depend on theta?

$$a = \pi_\theta(s)$$

$$\max_\theta . \; \mathbb{E} \sum_t R(S_t, \pi_\theta(S_t))$$

$$\max_\theta . \; \mathbb{E} \sum_t Q(S_t, \pi_\theta(S_t))$$

$$\frac{d\mathbb{E}\sum_t Q(S_t, \pi_\theta(S_t))}{d\theta} = \mathbb{E}\sum_t \frac{dQ(S_t, \pi_\theta(S_t))}{da}\frac{da}{d\theta}$$

Likelihood ratio gradient estimator

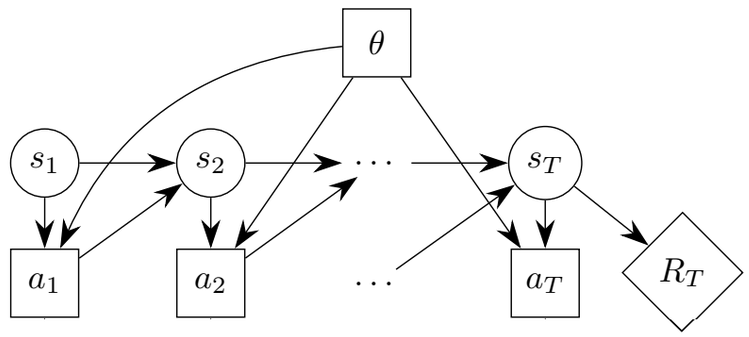
$$\max_\theta . \; \mathbb{E}_{x\sim P_\theta(x)} f(x)$$

$$\mathbb{E}_{x\sim P_\theta(x)} \nabla_\theta \log P_\theta(x) f(x)$$

$$\max_\theta . \; \mathbb{E}_{\tau\sim P_\theta(\tau)} \left[R(\tau)\right]$$

$$\mathbb{E}_{\tau\sim P_\theta(\tau)} \left[\nabla_\theta \log P_\theta(\tau) R(\tau)\right]$$

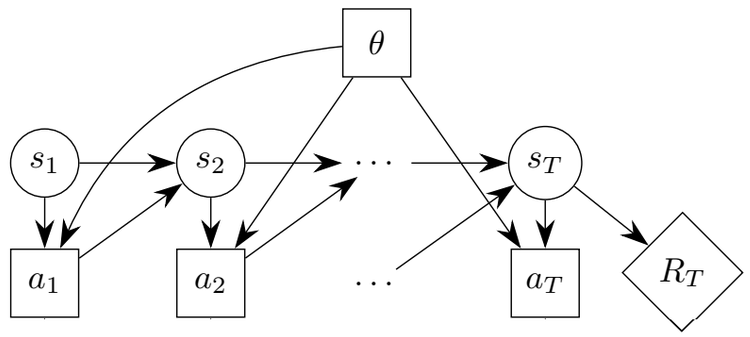
Deep Deterministic Policy Gradients



$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T} \frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$

Continuous control with deep reinforcement learning, Lilicrap et al. 2016

Deep Deterministic Policy Gradients

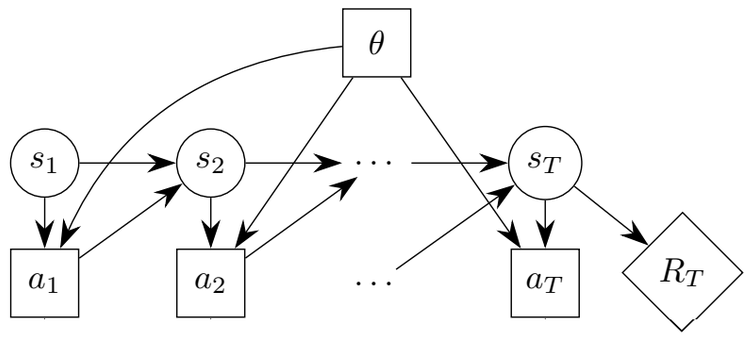


This expectation refers to the dynamics after time t

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$

Continuous control with deep reinforcement learning, Lilicrap et al. 2016

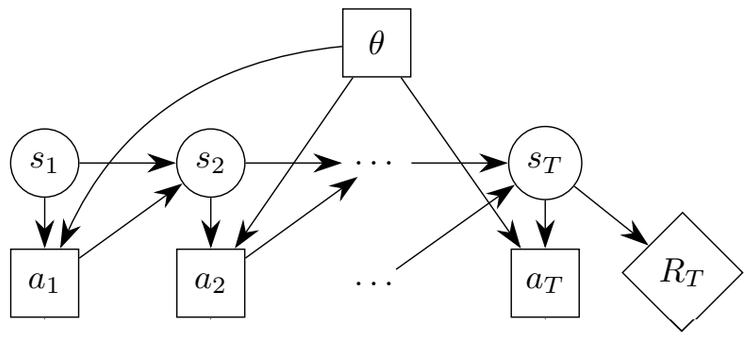
Deep Deterministic Policy Gradients



$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$
$$= \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}Q(s_t, a_t)}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$

Continuous control with deep reinforcement learning, Lilicrap et al. 2016

Deep Deterministic Policy Gradients



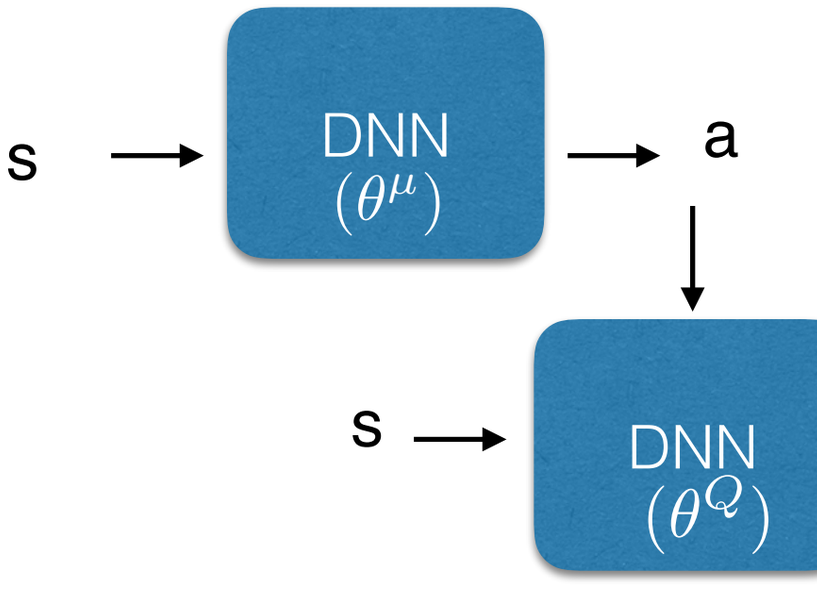
$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$

$$= \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}Q(s_t, a_t)}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}\theta}Q(s_t, \pi(s_t;\theta))\right]$$

Continuous control with deep reinforcement learning, Lilicrap et al. 2016

Deep Deterministic Policy Gradients

$$a = \mu(\theta)$$



We are following a stochastic behavior policy to collect data.
DDPG :Deep Q learning for continuous actions

Deep Deterministic Policy Gradients

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
  Execute action a_t and observe reward r_t and observe new state s_{t+1}
  Store transition (s_t, a_t, r_t, s_{t+1}) in R
  Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
  Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
  Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
  Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_i}$$

  Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

Deep Deterministic Policy Gradients

https://www.youtube.com/watch?v=tJBIqkC1wWM&feature=youtu.be

Model Free Methods - Comparison

Task	Random	REINFORCE	TNPG	RWR	REPS	TRPO	CEM	CMA-ES	DDPG
Cart-Pole Balancing	77.1 ± 0.0	4693.7 ± 14.0	**3986.4 ± 748.9**	**4861.5 ± 12.3**	565.6 ± 137.6	**4869.8 ± 37.6**	4815.4 ± 4.8	2440.4 ± 568.3	4634.4 ± 87.8
Inverted Pendulum*	−153.4 ± 0.2	13.4 ± 18.0	**209.7 ± 55.5**	84.7 ± 13.8	−113.3 ± 4.6	**247.2 ± 76.1**	38.2 ± 25.7	−40.1 ± 5.7	40.0 ± 244.6
Mountain Car	−415.4 ± 0.0	−67.1 ± 1.0	**-66.5 ± 4.5**	−79.4 ± 1.1	−275.6 ± 166.3	**-61.7 ± 0.9**	−66.0 ± 2.4	−85.0 ± 7.7	−288.4 ± 170.3
Acrobot	−1904.5 ± 1.0	−508.1 ± 91.0	−395.8 ± 121.2	−352.7 ± 35.9	−1001.5 ± 10.8	−326.0 ± 24.4	−436.8 ± 14.7	−785.6 ± 13.1	**-223.6 ± 5.8**
Double Inverted Pendulum*	149.7 ± 0.1	4116.5 ± 65.2	**4455.4 ± 37.6**	3614.8 ± 368.1	446.7 ± 114.8	**4412.4 ± 50.4**	2566.2 ± 178.9	1576.1 ± 51.3	2863.4 ± 154.0
Swimmer*	−1.7 ± 0.1	92.3 ± 0.1	**96.0 ± 0.2**	60.7 ± 5.5	3.8 ± 3.3	**96.0 ± 0.2**	68.8 ± 2.4	64.9 ± 1.4	85.8 ± 1.8
Hopper	8.4 ± 0.0	714.0 ± 29.3	**1155.1 ± 57.9**	553.2 ± 71.0	86.7 ± 17.6	**1183.3 ± 150.0**	63.1 ± 7.8	20.3 ± 14.3	267.1 ± 43.5
2D Walker	−1.7 ± 0.0	506.5 ± 78.8	**1382.6 ± 108.2**	136.0 ± 15.9	−37.0 ± 38.1	**1353.8 ± 85.0**	84.5 ± 19.2	77.1 ± 24.3	318.4 ± 181.6
Half-Cheetah	−90.8 ± 0.3	1183.1 ± 69.2	**1729.5 ± 184.6**	376.1 ± 28.2	34.5 ± 38.0	**1914.0 ± 120.1**	330.4 ± 274.8	441.3 ± 107.6	**2148.6 ± 702.7**
Ant*	13.4 ± 0.7	548.3 ± 55.5	**706.0 ± 127.7**	37.6 ± 3.1	39.0 ± 9.8	**730.2 ± 61.3**	49.2 ± 5.9	17.8 ± 15.5	326.2 ± 20.8
Simple Humanoid	41.5 ± 0.2	128.1 ± 34.0	**255.0 ± 24.5**	93.3 ± 17.4	28.3 ± 4.7	**269.7 ± 40.3**	60.6 ± 12.9	28.7 ± 3.9	99.4 ± 28.1
Full Humanoid	13.2 ± 0.1	262.2 ± 10.5	**288.4 ± 25.2**	46.7 ± 5.6	41.7 ± 6.1	**287.0 ± 23.4**	36.9 ± 2.9	N/A ± N/A	119.0 ± 31.2
Cart-Pole Balancing (LS)*	77.1 ± 0.0	420.9 ± 265.5	**945.1 ± 27.8**	68.9 ± 1.5	898.1 ± 22.1	**960.2 ± 46.0**	227.0 ± 223.0	68.0 ± 1.6	
Inverted Pendulum (LS)	−122.1 ± 0.1	−13.4 ± 3.2	**0.7 ± 6.1**	−107.4 ± 0.2	−87.2 ± 8.0	**4.5 ± 4.1**	−81.2 ± 33.2	−62.4 ± 3.4	
Mountain Car (LS)	−83.0 ± 0.0	−81.2 ± 0.6	**-65.7 ± 9.0**	−81.7 ± 0.1	−82.6 ± 0.4	**-64.2 ± 9.5**	**-68.9 ± 1.3**	**-73.2 ± 0.6**	
Acrobot (LS)*	−393.2 ± 0.0	−128.9 ± 11.6	**-84.6 ± 2.9**	−235.9 ± 5.3	−379.5 ± 1.4	**-83.3 ± 9.9**	−149.5 ± 15.3	−159.9 ± 7.5	
Cart-Pole Balancing (NO)*	101.4 ± 0.1	616.0 ± 210.8	**916.3 ± 23.0**	93.8 ± 1.2	99.6 ± 7.2	606.2 ± 122.2	181.4 ± 32.1	104.4 ± 16.0	
Inverted Pendulum (NO)	−122.2 ± 0.1	6.5 ± 1.1	**11.5 ± 0.5**	−110.0 ± 1.4	−119.3 ± 4.2	**10.4 ± 2.2**	−55.6 ± 16.7	−80.3 ± 2.8	
Mountain Car (NO)	−83.0 ± 0.0	−74.7 ± 7.8	**-64.5 ± 8.6**	−81.7 ± 0.1	−82.9 ± 0.1	**-60.2 ± 2.0**	−67.4 ± 1.4	−73.5 ± 0.5	
Acrobot (NO)*	−393.5 ± 0.0	**-186.7 ± 31.3**	**-164.5 ± 13.4**	−233.1 ± 0.4	−258.5 ± 14.0	**-149.6 ± 8.6**	−213.4 ± 6.3	−236.6 ± 6.2	
Cart-Pole Balancing (SI)*	76.3 ± 0.1	431.7 ± 274.1	**980.5 ± 7.3**	69.0 ± 2.8	702.4 ± 196.4	**980.3 ± 5.1**	746.6 ± 93.2	71.6 ± 2.9	
Inverted Pendulum (SI)	−121.8 ± 0.2	−5.3 ± 5.6	**14.8 ± 1.7**	−108.7 ± 4.7	−92.8 ± 23.9	**14.1 ± 0.9**	−51.8 ± 10.6	−63.1 ± 4.8	
Mountain Car (SI)	−82.7 ± 0.0	−63.9 ± 0.2	**-61.8 ± 0.4**	−81.4 ± 0.1	−80.7 ± 2.3	**-61.6 ± 0.4**	−63.9 ± 1.0	−66.9 ± 0.6	
Acrobot (SI)*	−387.8 ± 1.0	**-169.1 ± 32.3**	**-156.6 ± 38.9**	−233.2 ± 2.6	−216.1 ± 7.7	**-170.9 ± 40.3**	−250.2 ± 13.7	−245.0 ± 5.5	
Swimmer + Gathering	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Gathering	−5.8 ± 5.0	−0.1 ± 0.1	−0.4 ± 0.1	−5.5 ± 0.5	−6.7 ± 0.7	−0.4 ± 0.0	−4.7 ± 0.7	N/A ± N/A	−0.3 ± 0.3
Swimmer + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	N/A ± N/A	0.0 ± 0.0

Y. Duan, X. Chen, R. Houthooft, et al. "Benchmarking Deep Reinforcement Learning for Continuous Control"

Computing Gradients of Expectations

When the variable w.r.t. which we are differentiating appears in the distribution:

$$\nabla_\theta \mathbb{E}_{x\sim P_\theta(x)} f(x) = \mathbb{E}_{x\sim P_\theta(x)} \nabla_\theta \log P_\theta(x) f(x)$$

likelihood ratio gradient estimator

When the variable w.r.t. which we are differentiating appears inside the expectation:

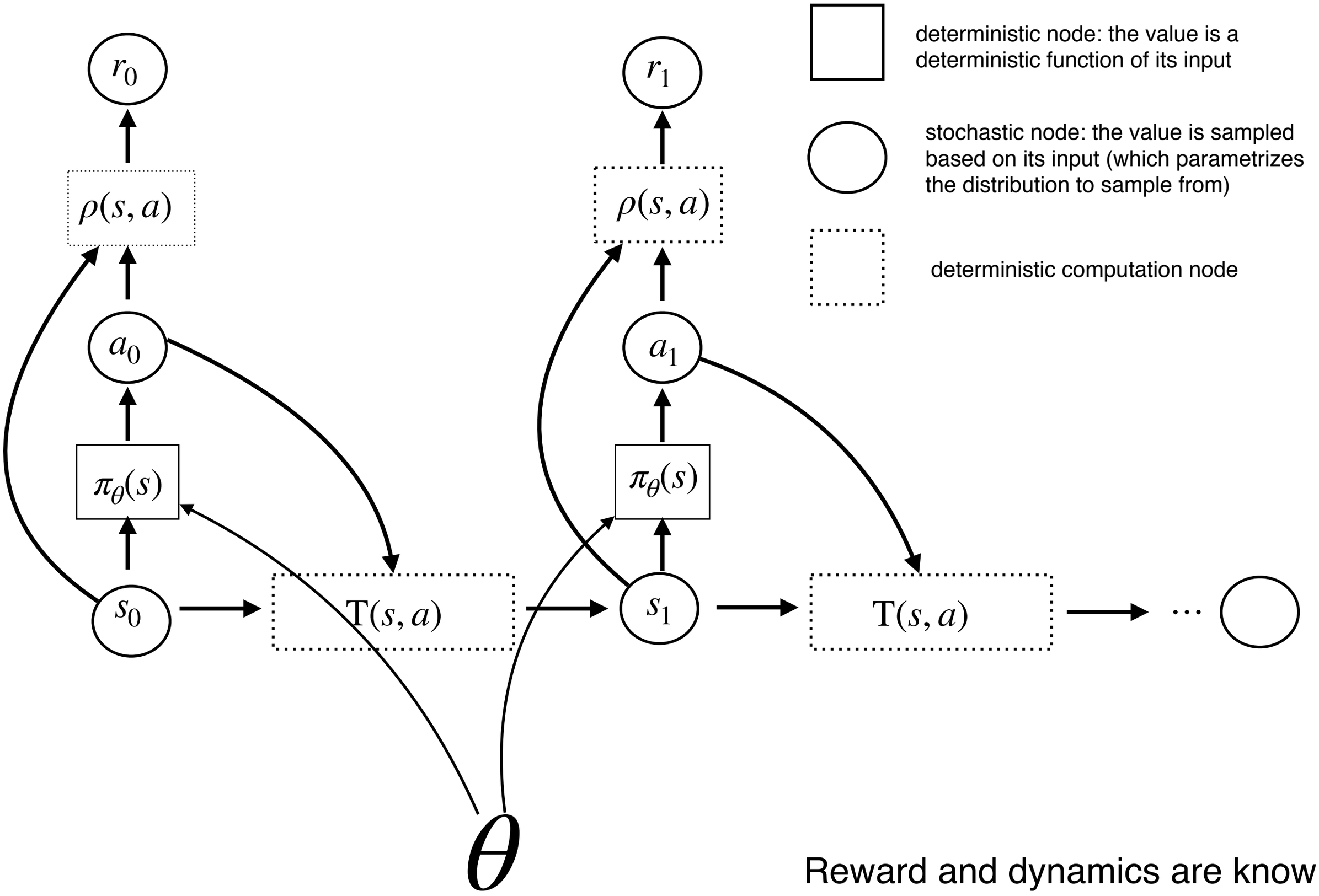
$$\nabla_\theta \mathbb{E}_{z\sim \mathcal{N}(0,1)} f(x(\theta), z) = \mathbb{E}_{z\sim \mathcal{N}(0,1)} \nabla_\theta f(x(\theta), z) = \mathbb{E}_{z\sim \mathcal{N}(0,1)} \frac{df(x(\theta), z)}{dx} \frac{dx}{d\theta}$$

pathwise derivative

Re-parametrization trick: For some distributions $P_\theta(x)$ we can switch from one gradient estimator to the other.

Q: From which to which? Why would we want to do so?

Known MDP (what if that was possible)



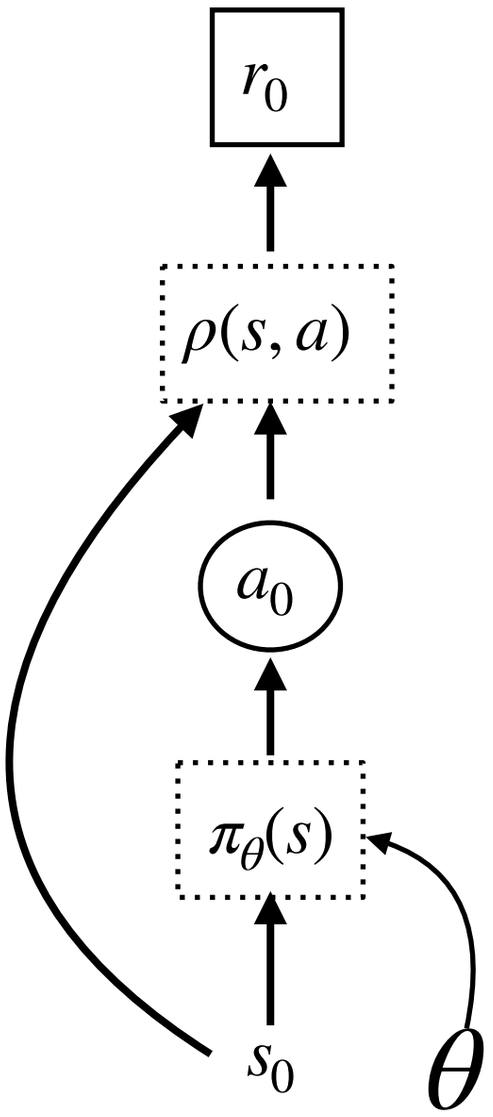
deterministic node: the value is a deterministic function of its input

stochastic node: the value is sampled based on its input (which parametrizes the distribution to sample from)

deterministic computation node

Reward and dynamics are known

Known MDP-let's make it simpler



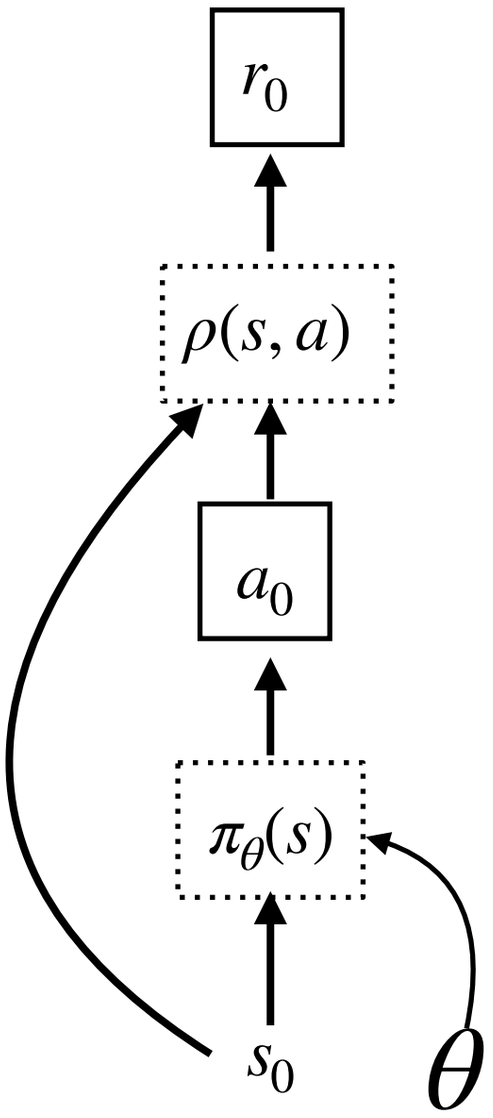
deterministic node: the value is a deterministic function of its input

stochastic node: the value is sampled based on its input (which parametrizes the distribution to sample from)

deterministic computation node

I want to learn θ to maximize the average reward obtained.

Deterministic policy



$a = \pi_\theta(s)$

deterministic node: the value is a deterministic function of its input

stochastic node: the value is sampled based on its input (which parametrizes the distribution to sample from)

deterministic computation node

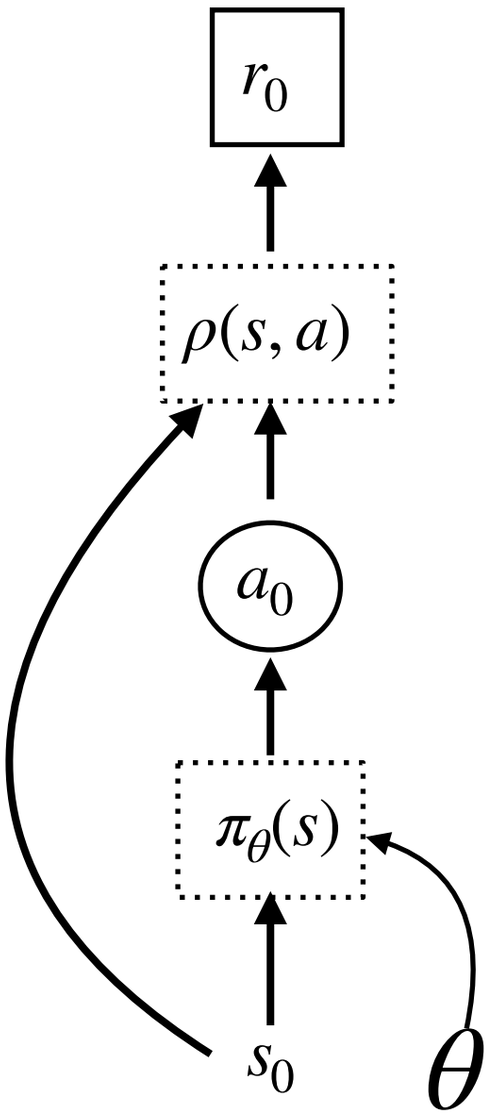
I want to learn θ to maximize the average reward obtained.

I can compute the gradient with backpropagation.

$$\nabla_\theta \rho(s, a) = \rho_a \pi_{\theta\theta}$$

Derivative of the *known* reward function w.r.t. the action

Stochastic policy



deterministic node: the value is a deterministic function of its input

stochastic node: the value is sampled based on its input (which parametrizes the distribution to sample from)

deterministic computation node

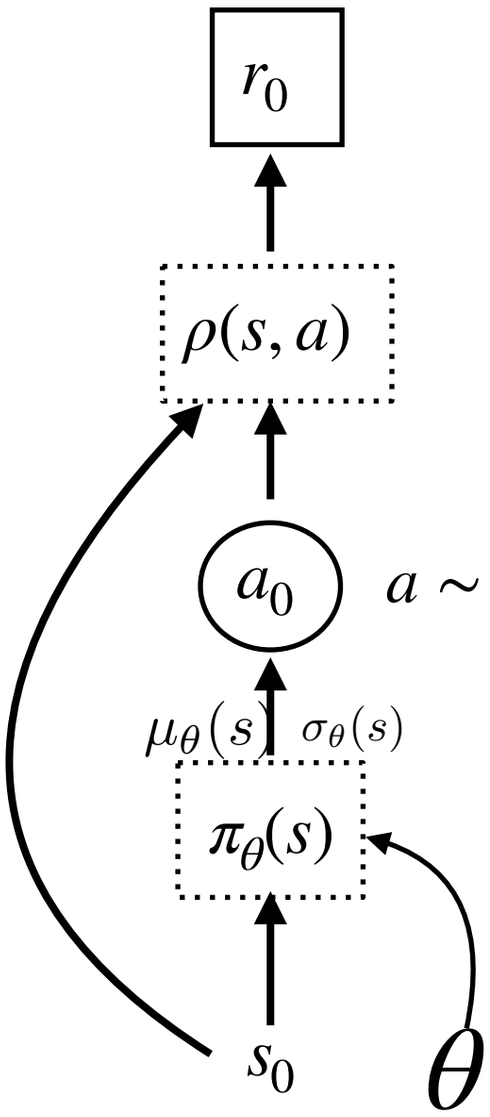
I want to learn θ to maximize the average reward obtained.

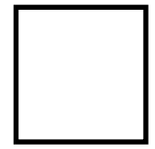
Likelihood ratio estimator, works for both continuous and discrete actions

$$\mathbb{E}_a \nabla_\theta \log \pi_\theta(s) \rho(s, a)$$

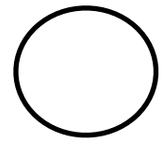
It does not use the derivative of the reward w.r.t. the action.

Example: Gaussian policy





deterministic node: the value is a deterministic function of its input



stochastic node: the value is sampled based on its input (which parametrizes the distribution to sample from)

deterministic computation node

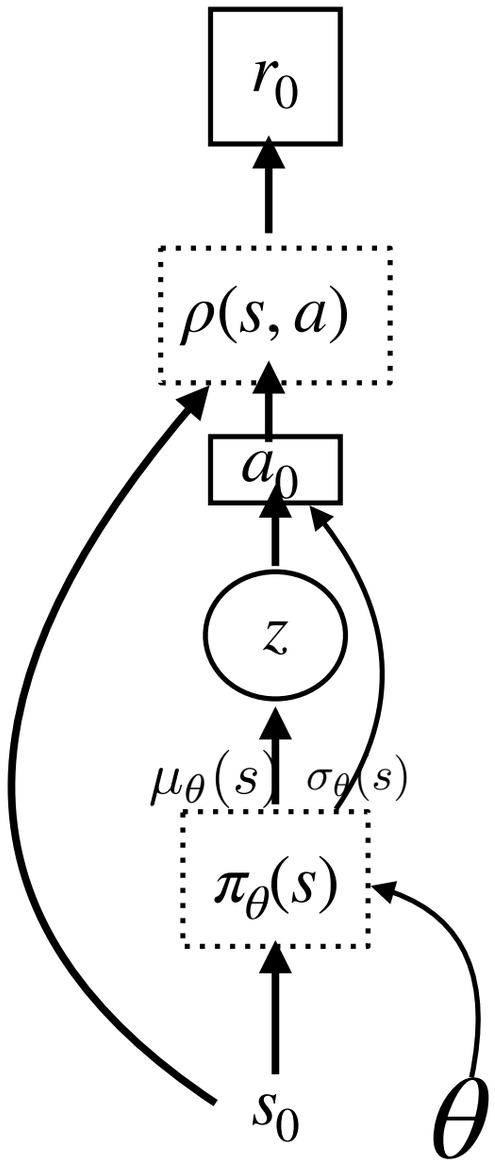
I want to learn θ to maximize the average reward obtained.

$$\mathbb{E}_a \nabla_\theta \log \pi_\theta(s) \rho(s, a)$$

If σ^2 is constant:

$$\nabla_\theta \log \pi_\theta(s, a) = \frac{(a - \mu(s; \theta)) \frac{\partial \mu(s;\theta)}{\partial \theta}}{\sigma^2}$$

Re-parametrization for Gaussian

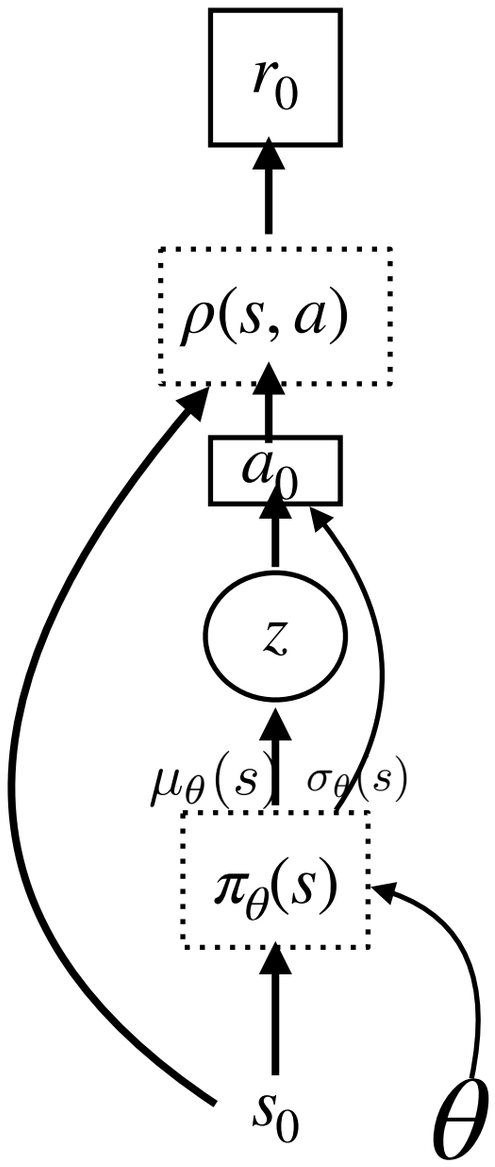


$$a \sim \mathcal{N}(\mu(s,\theta), \Sigma(s,\theta))$$

We can either:

- Assume σ fixed
- Learn $\sigma(s,\theta)$ one value for all action coordinates
- Learn $\sigma^i(s,\theta), i = 1 \cdots n$, (sperical gaussian,diagonal covariance)
- Learn a full covariance matrix $\Sigma(s,\theta)$

Re-parametrization for Gaussian



Instead of: $a \sim \mathcal{N}(\mu(s,\theta), \Sigma(s,\theta))$

We can write: $a = \mu(s,\theta) + z \odot \sigma(s,\theta)$ $\quad z \sim \mathcal{N}(0,I)$

Why?

Because: $\mathbb{E}_z(\mu(s,\theta) + z\sigma(s,\theta)) = \mu(s,\theta)$

$$\mathrm{Var}_z(\mu(s,\theta) + z\sigma(s,\theta)) = \sigma(s,\theta)^2$$

What do we gain?

$$\nabla_\theta \mathbb{E}_z\left[\rho\left(a(\theta,z),s\right)\right] = \mathbb{E}_z \frac{d\rho\left(a(\theta,z),s\right)}{da}\frac{da(\theta,z)}{d\theta}$$

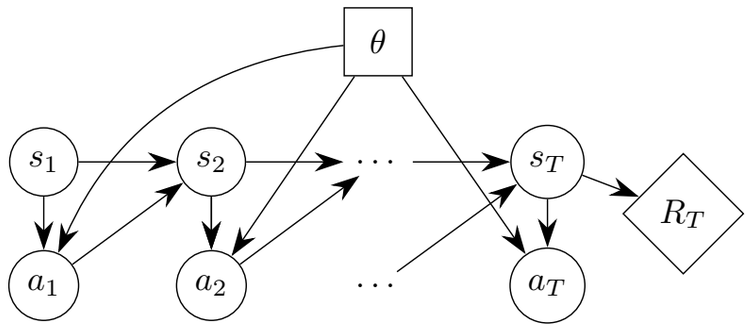
$$\frac{da(\theta,z)}{d\theta} = \frac{d\mu(s,\theta)}{d\theta} + z \odot \frac{d\sigma(s,\theta)}{d\theta}$$

Sample estimate:

$$\nabla_\theta \frac{1}{N}\sum_{i=1}^{N}\left[\rho\left(a(\theta,z_i),s\right)\right] = \frac{1}{N}\sum_{i=1}^{N}\frac{d\rho\left(a(\theta,z),s\right)}{da}\frac{da(\theta,z)}{d\theta}\Big|_{z=z_i}$$

Re-parametrized Policy Gradients

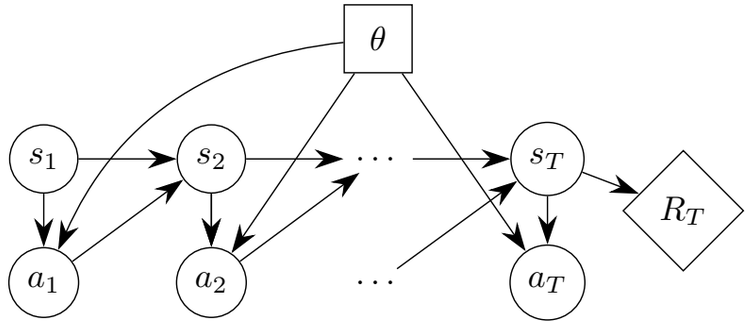
- Episodic MDP:



We want to compute: $\nabla_\theta \mathbb{E}[R_T]$

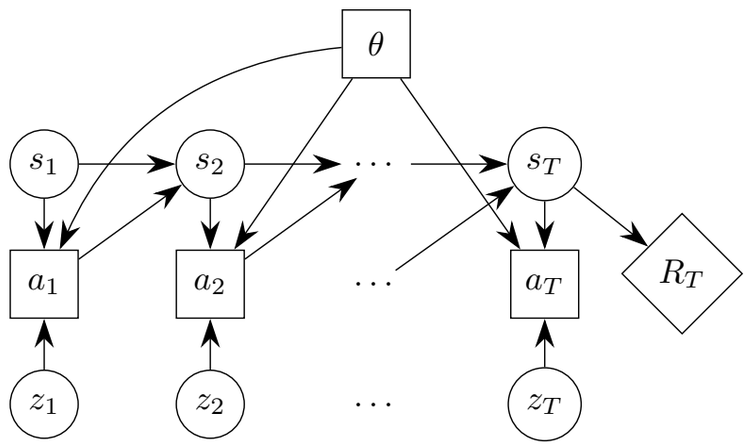
Re-parametrized Policy Gradients

- Episodic MDP:



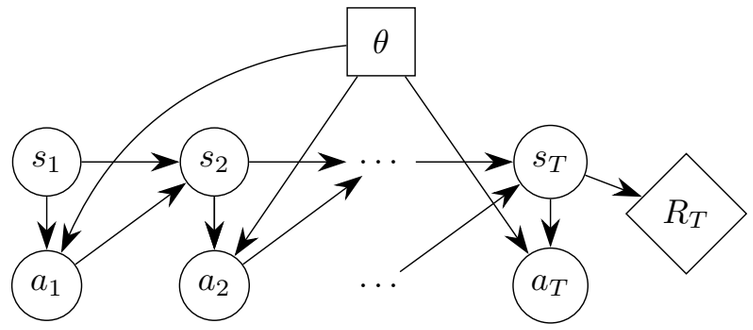
We want to compute: $\nabla_\theta \mathbb{E}[R_T]$

- Reparameterize: $a_t = \pi(s_t, z_t; \theta)$. z_t is noise from fixed distribution.



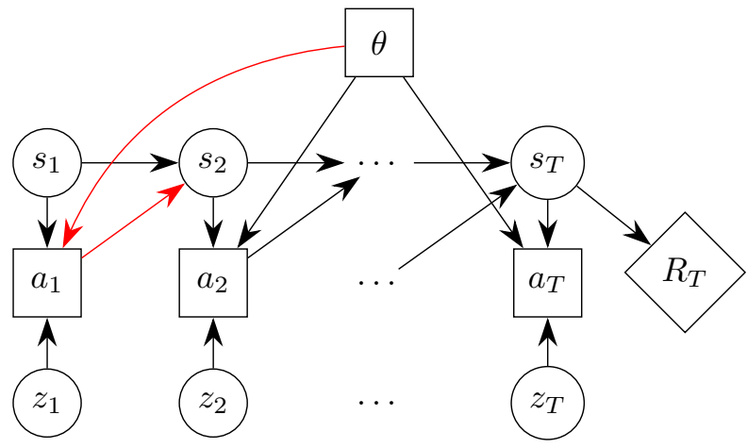
Re-parametrized Policy Gradients

- Episodic MDP:



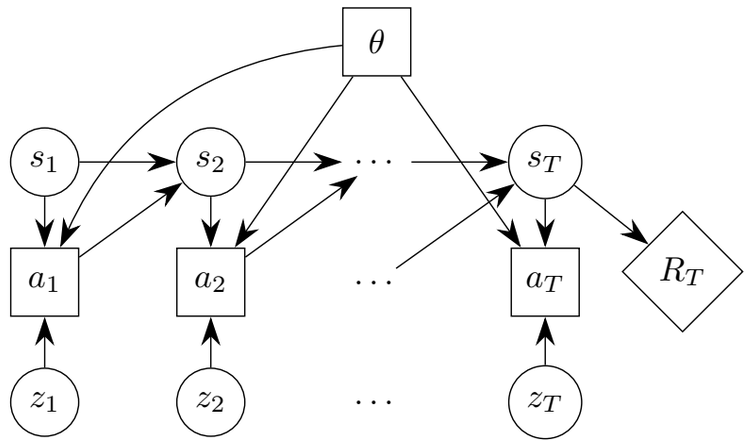
We want to compute: $\nabla_\theta \mathbb{E}[R_T]$

- Reparameterize: $a_t = \pi(s_t, z_t; \theta)$. z_t is noise from fixed distribution.



For pathwise derivative to work, we need transition dynamics and reward function to be known.

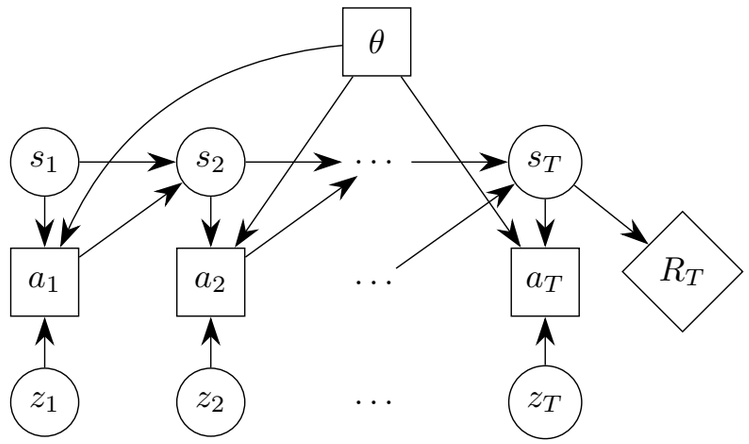
Re-parametrized Policy Gradients



$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$

For path wise derivative to work, we need transition dynamics and reward function to be known, or…

Re-parametrized Policy Gradients



$$\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right]$$
$$= \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}Q(s_t, a_t)}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}\theta}Q(s_t, \pi(s_t, z_t; \theta))\right]$$

- Learn Q_ϕ to approximate $Q^{\pi,\gamma}$, and use it to compute gradient estimates.

Learning continuous control by stochastic value gradients, Hees et al.

Stochastic Value Gradients V0

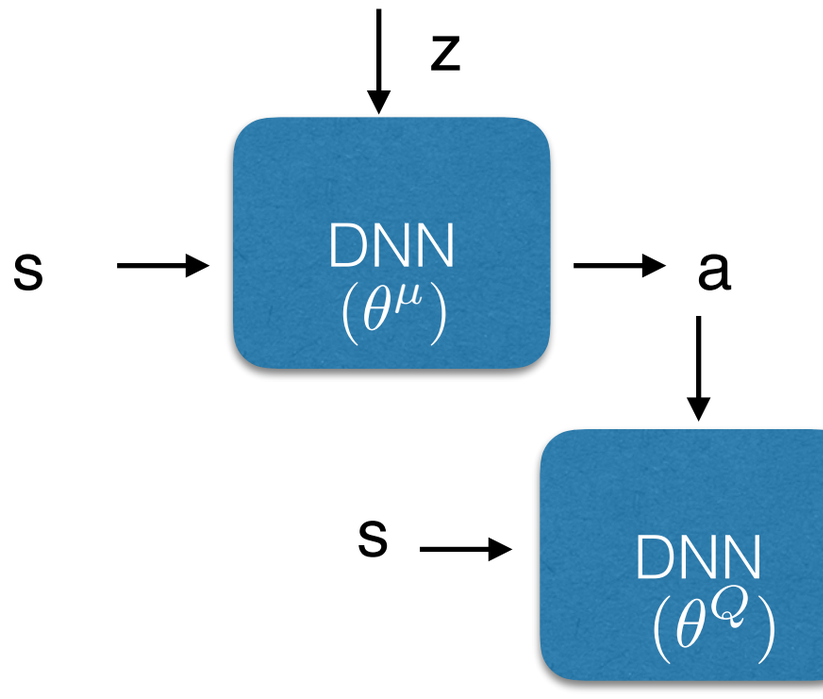
- Learn Q_ϕ to approximate $Q^{\pi,\gamma}$, and use it to compute gradient estimates.
- Pseudocode:

for iteration=$1, 2, \ldots$ **do**
  Execute policy π_θ to collect T timesteps of data
  Update π_θ using $g \propto \nabla_\theta \sum_{t=1}^{T} Q(s_t, \pi(s_t, z_t; \theta))$
  Update Q_ϕ using $g \propto \nabla_\phi \sum_{t=1}^{T} (Q_\phi(s_t, a_t) - \hat{Q}_t)^2$, e.g. with TD($\lambda$)
end for

Learning continuous control by stochastic value gradients, Hees et al.

Stochastic Value Gradients V0

$$z \sim \mathcal{N}(0,1)$$

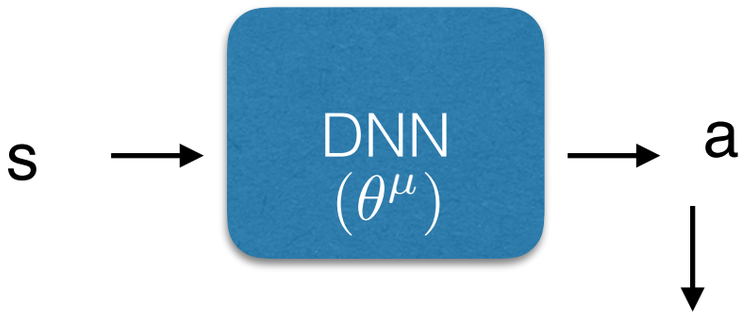


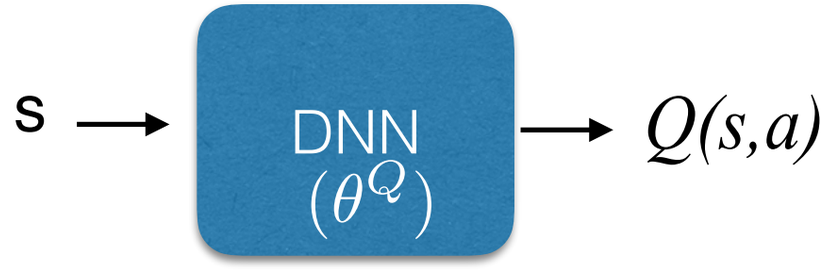
$$a = \mu(s;\theta) + z\sigma(s;\theta)$$

Learning continuous control by stochastic value gradients, Hees et al.

Compare with: Deep Deterministic Policy Gradients

$$a = \mu(\theta)$$





No z!