

Monte Carlo Learning

CMU 10-03

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Used Materials

- **Disclaimer:** Much of the material and slides for this lecture were borrowed from Russ who in turn borrowed some materials from Rich Sutton's class and David Silver's class on Reinforcement Learning.

Summary so far

- So far, to estimate value functions we have been using dynamic programming with *known rewards and dynamics* functions:

$$v_{[k+1]}(s) = \sum_a \pi(a|s) \left(r(s, a) + \gamma \sum_{s'} p(s'|s, a) v_{[k]}(s') \right), \forall s$$

$$v_{[k+1]}(s) = \max_{a \in \mathcal{A}} \left(r(s, a) + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a) v_{[k]}(s') \right), \forall s$$

Q: Was our agent interacting with the world? Was our agent *learning* something?

Coming up

- So far, to estimate value functions we have been using dynamic programming with *known rewards and dynamics* functions:

$$v_{[k+1]}(s) = \sum_a \pi(a|s) \left(r(s, a) + \gamma \sum_{s'} p(s'|s, a) v_{[k]}(s') \right), \forall s$$

$$v_{[k+1]}(s) = \max_{a \in \mathcal{A}} \left(r(s, a) + \gamma \sum_{s' \in \mathcal{S}} p(s'|s, a) v_{[k]}(s') \right), \forall s$$

- Next: estimate value functions and policies from *interaction experience, without known rewards or dynamics*.
- How? By sampling all the way. Instead of probabilities distributions to compute expectations, we will use empirical expectations by averaging sampled returns!

Monte Carlo (MC) Methods

- ▶ Monte Carlo methods are learning methods
 - Experience → values, policy
- ▶ Monte Carlo methods learn from complete sampled trajectories and their returns
 - Only defined for episodic tasks
 - All episodes must terminate
- ▶ Monte Carlo uses the simplest possible idea: value = mean return

Monte-Carlo Policy Evaluation

- ▶ Goal: learn $v_\pi(s)$ from episodes of experience under policy π

$$S_1, A_1, R_2, \dots, S_k \sim \pi$$

- ▶ Remember that the return is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T$$

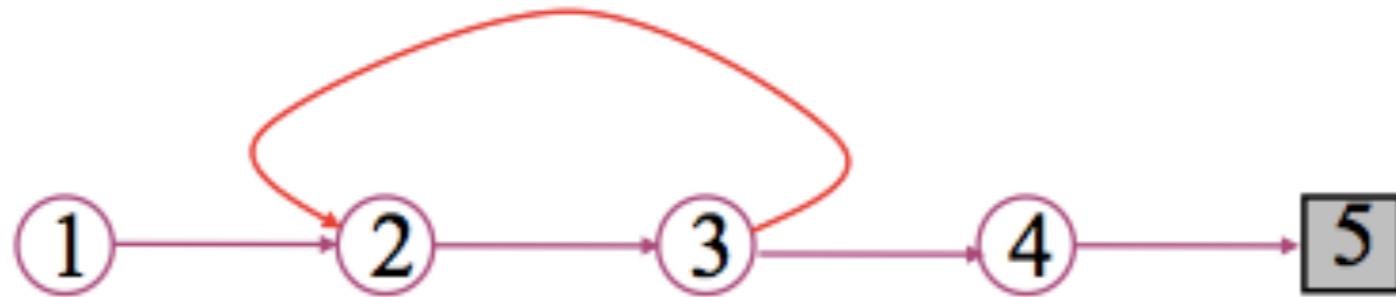
- ▶ Remember that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi [G_t \mid S_t = s]$$

- ▶ Monte-Carlo policy evaluation uses empirical mean return instead of expected return

Monte-Carlo Policy Evaluation

- ▶ **Goal:** learn $v_\pi(s)$ from episodes of experience under policy π
- ▶ **Idea:** Average returns observed after visits to s:



- ▶ **Every-Visit MC:** average returns for every time s is visited in an episode
- ▶ **First-visit MC:** average returns only for first time s is visited in an episode
- ▶ Both converge asymptotically

First-Visit MC Policy Evaluation

- ▶ To evaluate state s
- ▶ The **first** time-step t that state s is visited in an episode,
 - ▶ Increment counter: $N(s) \leftarrow N(s) + 1$
 - ▶ Increment total return: $S(s) \leftarrow S(s) + G_t$
- ▶ Value is estimated by mean return $V(s) = S(s)/N(s)$
- ▶ By law of large numbers $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$

Every-Visit MC Policy Evaluation

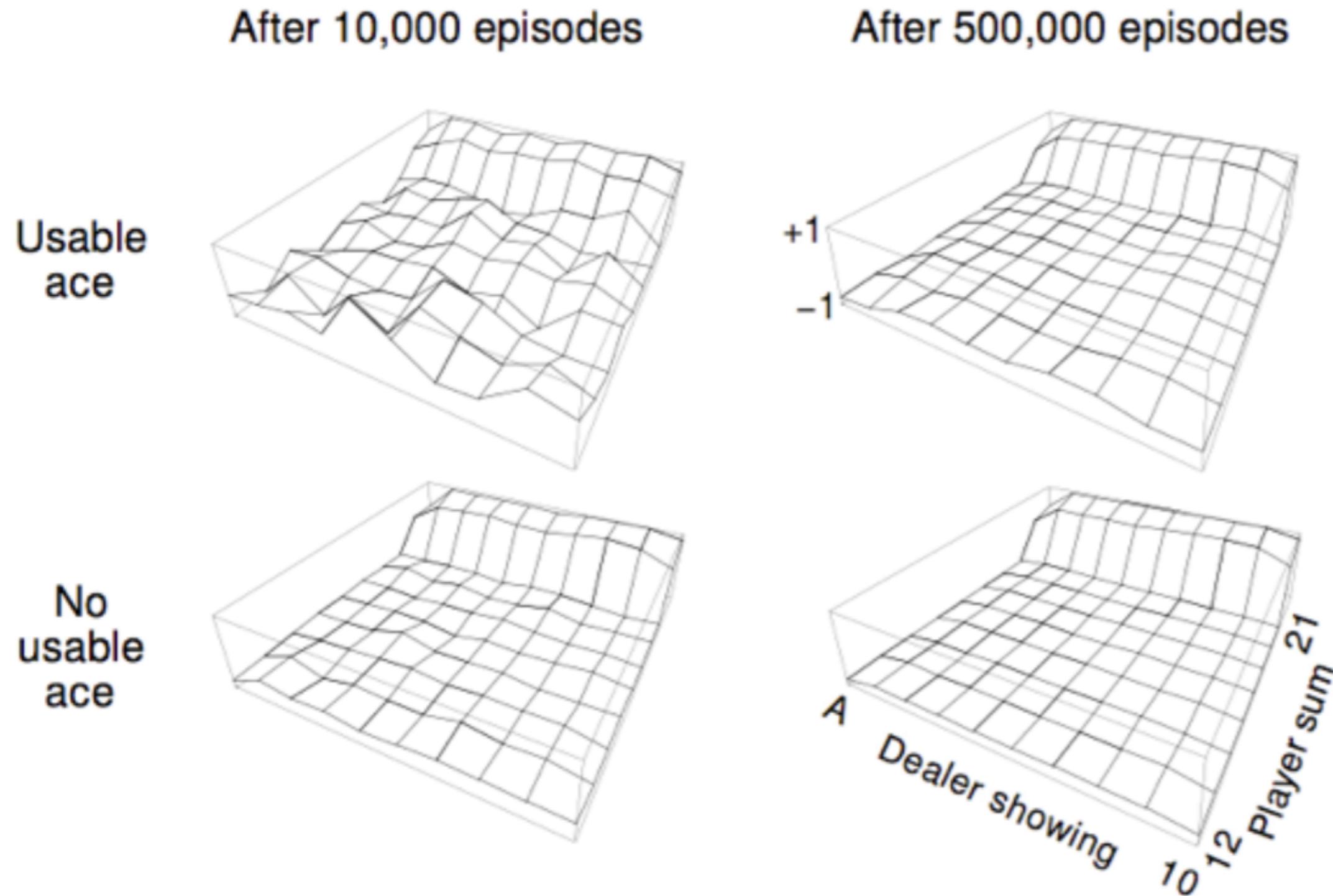
- ▶ To evaluate state s
- ▶ **Every** time-step t that state s is visited in an episode,
 - ▶ Increment counter: $N(s) \leftarrow N(s) + 1$
 - ▶ Increment total return: $S(s) \leftarrow S(s) + G_t$
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Blackjack Example

- ▶ **Objective:** Have your card sum be greater than the dealer's without exceeding 21.
- ▶ **States** (200 of them):
 - current sum (12-21)
 - dealer's showing card (ace-10)
 - do I have a useable ace?
- ▶ **Reward:** +1 for winning, 0 for a draw, -1 for losing
- ▶ **Actions:** stick (stop receiving cards), hit (receive another card)
- ▶ **Policy:** Stick if my sum is 20 or 21, else hit
- ▶ No discounting ($\gamma=1$)

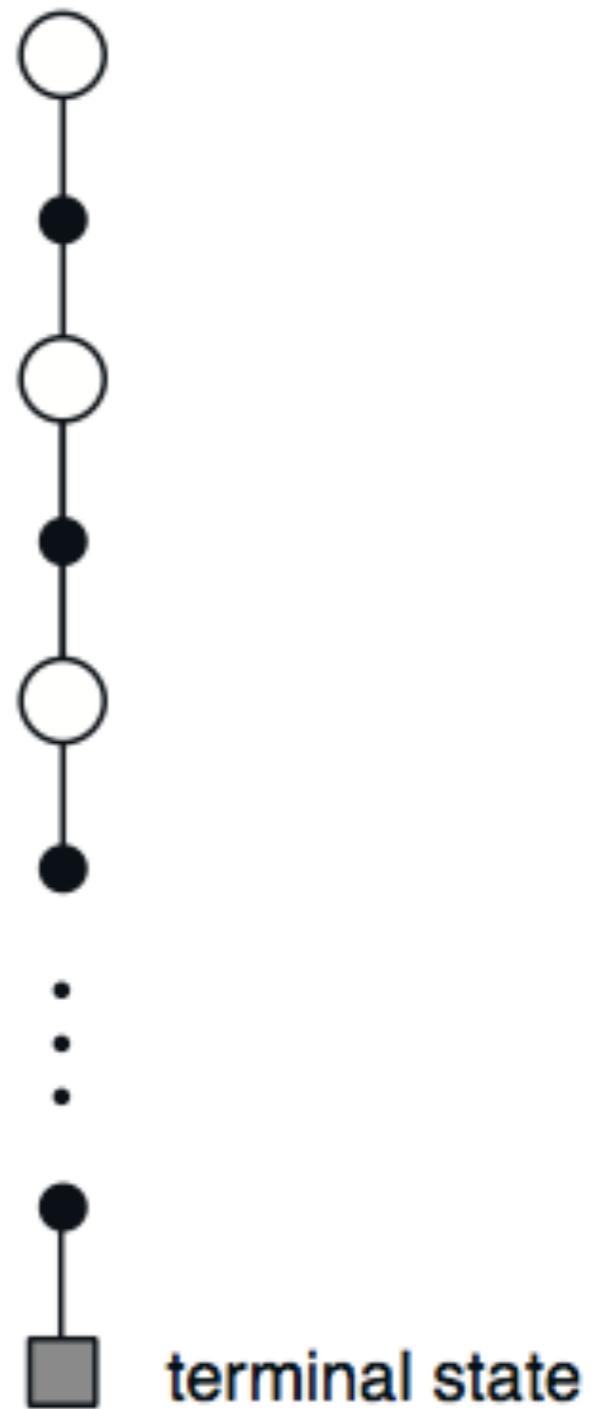


Learned Blackjack State-Value Functions



Backup Diagram for Monte Carlo

- ▶ Entire rest of episode included
- ▶ **Only one choice considered at each state (unlike DP)**
 - thus, there will be an explore/exploit dilemma
- ▶ Does **not bootstrap** from successor state's values (unlike DP)
- ▶ Value is estimated by mean return



Incremental Mean

- The mean μ_1, μ_2, \dots of a sequence x_1, x_2, \dots can be computed incrementally:

$$\begin{aligned}\mu_k &= \frac{1}{k} \sum_{j=1}^k x_j \\ &= \frac{1}{k} \left(x_k + \sum_{j=1}^{k-1} x_j \right) \\ &= \frac{1}{k} (x_k + (k-1)\mu_{k-1}) \\ &= \mu_{k-1} + \frac{1}{k} (x_k - \mu_{k-1})\end{aligned}$$

Incremental Monte Carlo Updates

- ▶ Update $V(s)$ incrementally after episode $S_1, A_1, R_2, \dots, S_T$
- ▶ For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)} (G_t - V(S_t))$$

- ▶ In non-stationary problems, it can be useful to track a running mean, i.e. forget old episodes.

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

MC Estimation of Action Values (Q)

- ▶ Monte Carlo (MC) is most useful when a model is not available
 - We want to learn $q^*(s,a)$
- ▶ $q_\pi(s,a)$ - average return starting from state s and action a following π

$$\begin{aligned} q_\pi(s, a) &= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s, A_t = a] \\ &= \sum_{s', r} p(s', r | s, a) [r + \gamma v_\pi(s')]. \end{aligned}$$

- ▶ Converges asymptotically if every state-action pair is visited

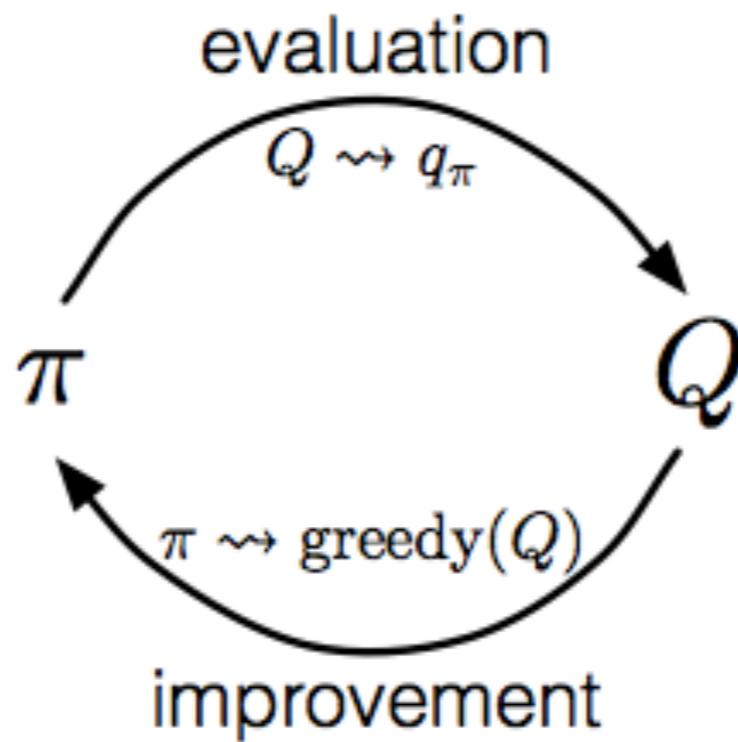
Q: Is this possible if we are using a deterministic policy?

The Exploration problem

- If we always follow the deterministic policy to collect experience, we will never have the opportunity to see and evaluate (estimate q) of alternative actions...
- ALL learning methods face a dilemma: they seek to learn action values conditioned on subsequent optimal behaviour but they need to act suboptimally in order to explore all actions (to discover the optimal actions). The exploration-exploitation dilemma.
- Solutions:
 1. **exploring starts**: Every state-action pair has a non-zero probability of being the starting pair
 2. Give up on deterministic policies and only search over **ϵ -soft policies**
 3. **Off policy**: use a different policy to collect experience than the one you care to evaluate

Monte-Carlo Control

$$\pi_0 \xrightarrow{E} q_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} q_{\pi_1} \xrightarrow{I} \pi_2 \xrightarrow{E} \dots \xrightarrow{I} \pi_* \xrightarrow{E} q_*$$



- ▶ **MC policy iteration step:** Policy evaluation using MC methods followed by policy improvement
- ▶ **Policy improvement step:** greedify with respect to value (or action-value) function

Greedy Policy

- ▶ For any action-value function q , the corresponding **greedy policy** is the one that:
 - For each s , deterministically chooses an action with maximal action-value:

$$\pi(s) \doteq \arg \max_a q(s, a).$$

- ▶ **Policy improvement** then can be done by constructing each π_{k+1} as the greedy policy with respect to q_{π^k} .

Convergence of MC Control

- ▶ Greedified policy meets the conditions for policy improvement:

$$\begin{aligned} q_{\pi_k}(s, \pi_{k+1}(s)) &= q_{\pi_k}\left(s, \operatorname{argmax}_a q_{\pi_k}(s, a)\right) \\ &= \max_a q_{\pi_k}(s, a) \\ &\geq q_{\pi_k}(s, \pi_k(s)) \\ &\geq v_{\pi_k}(s). \end{aligned}$$

- ▶ And thus must be $\geq \pi_k$.
- ▶ This assumes exploring starts and infinite number of episodes for MC policy evaluation

Monte Carlo Exploring Starts

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:

$Q(s, a) \leftarrow$ arbitrary

$\pi(s) \leftarrow$ arbitrary

$Returns(s, a) \leftarrow$ empty list

Fixed point is optimal policy π^*

Repeat forever:

Choose $S_0 \in \mathcal{S}$ and $A_0 \in \mathcal{A}(S_0)$ s.t. all pairs have probability > 0

Generate an episode starting from S_0, A_0 , following π

For each pair s, a appearing in the episode:

$G \leftarrow$ return following the first occurrence of s, a

Append G to $Returns(s, a)$

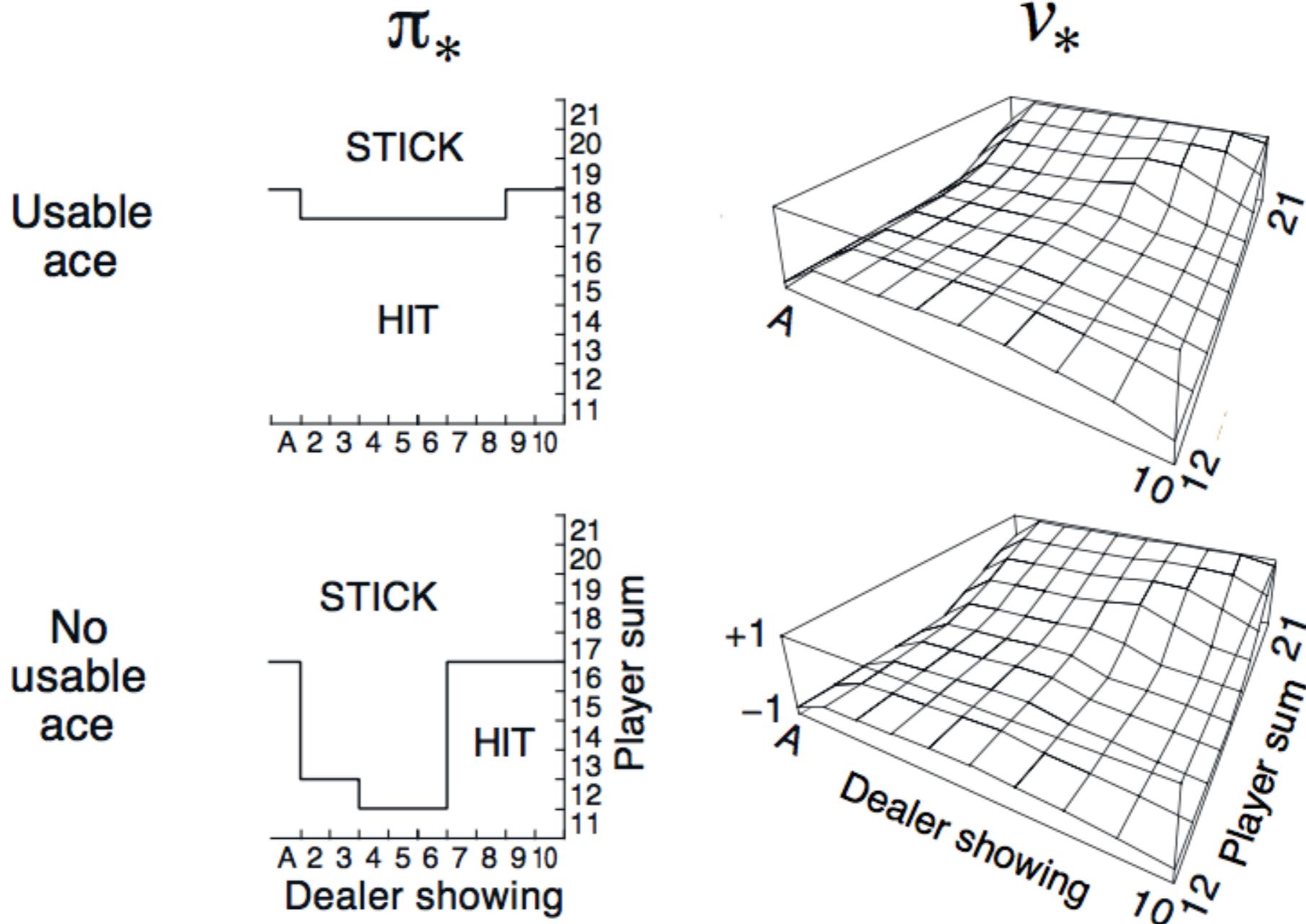
$Q(s, a) \leftarrow$ average($Returns(s, a)$)

For each s in the episode:

$\pi(s) \leftarrow \text{argmax}_a Q(s, a)$

Blackjack example continued

- ## ► With exploring starts



On-policy Monte Carlo Control

- ▶ **On-policy**: learn about policy currently executing
- ▶ How do we get rid of exploring starts?
 - The policy must be **eternally soft**: $\pi(a|s) > 0$ for all s and a .

- ▶ For example, for **ϵ -soft policy**, probability of an action, $\pi(a|s)$,

$$= \frac{\epsilon}{|\mathcal{A}(s)|} \quad \text{or} \quad 1 - \epsilon + \frac{\epsilon}{|\mathcal{A}(s)|}$$

non-max max (greedy)

- ▶ Similar to GPI: move policy towards **greedy policy**
- ▶ **Converges to the best ϵ -soft policy.**

On-policy Monte Carlo Control

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:

$$Q(s, a) \leftarrow \text{arbitrary}$$

$$Returns(s, a) \leftarrow \text{empty list}$$

$$\pi(a|s) \leftarrow \text{an arbitrary } \varepsilon\text{-soft policy}$$

Repeat forever:

(a) Generate an episode using π

(b) For each pair s, a appearing in the episode:

$$G \leftarrow \text{return following the first occurrence of } s, a$$

Append G to $Returns(s, a)$

$$Q(s, a) \leftarrow \text{average}(Returns(s, a))$$

(c) For each s in the episode:

$$A^* \leftarrow \arg \max_a Q(s, a)$$

For all $a \in \mathcal{A}(s)$:

$$\pi(a|s) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(s)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(s)| & \text{if } a \neq A^* \end{cases}$$

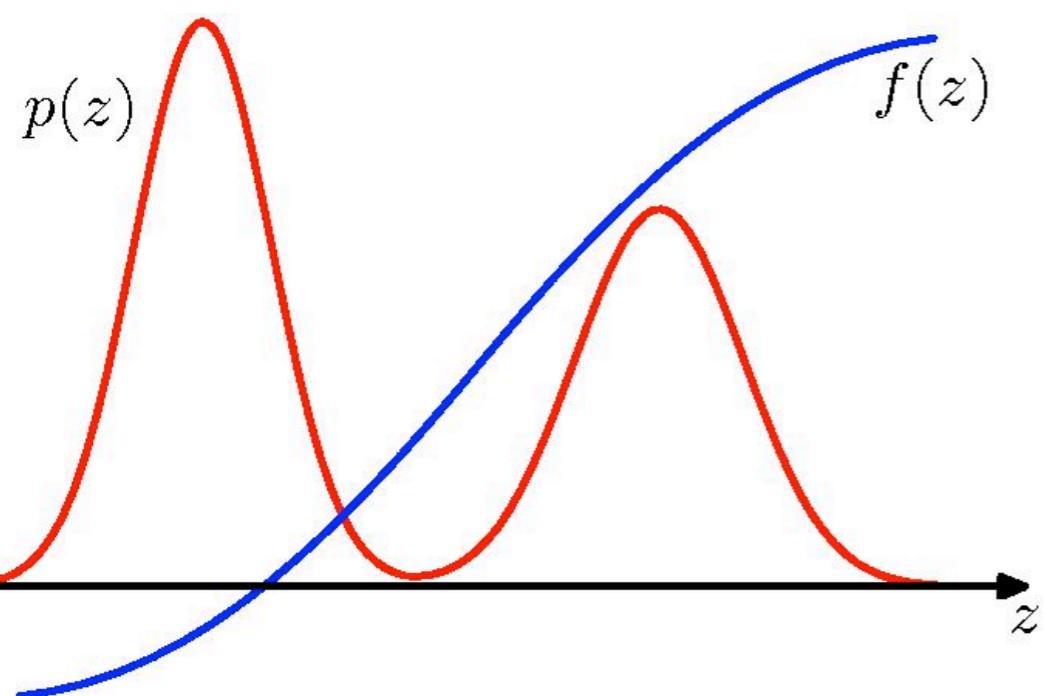
Off-policy methods

- ▶ Learn the value of the **target policy** π from experience due to **behavior policy** μ .
- ▶ For example, π is the greedy policy (and ultimately the optimal policy) while μ is exploratory (e.g., ϵ -soft) policy
- ▶ In general, we only require coverage, i.e., that μ generates behavior that **covers**, or includes, π

$$\mu(a|s) > 0 \quad \text{for every } s, a \text{ at which} \quad \pi(a|s) > 0$$

- ▶ Idea: **Importance Sampling**:
 - Weight each return by the ratio of the probabilities of the trajectory under the two policies.

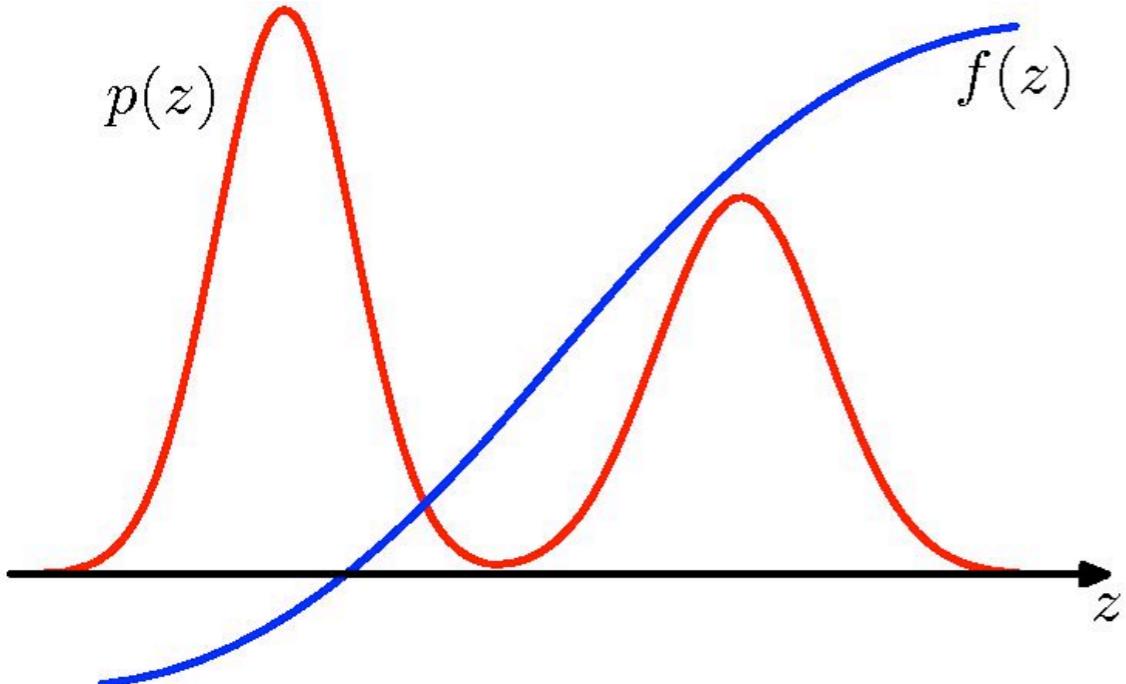
Expectations



$$\mathbb{E}[f] = \int f(z)p(z)dz$$

Estimating Expectations

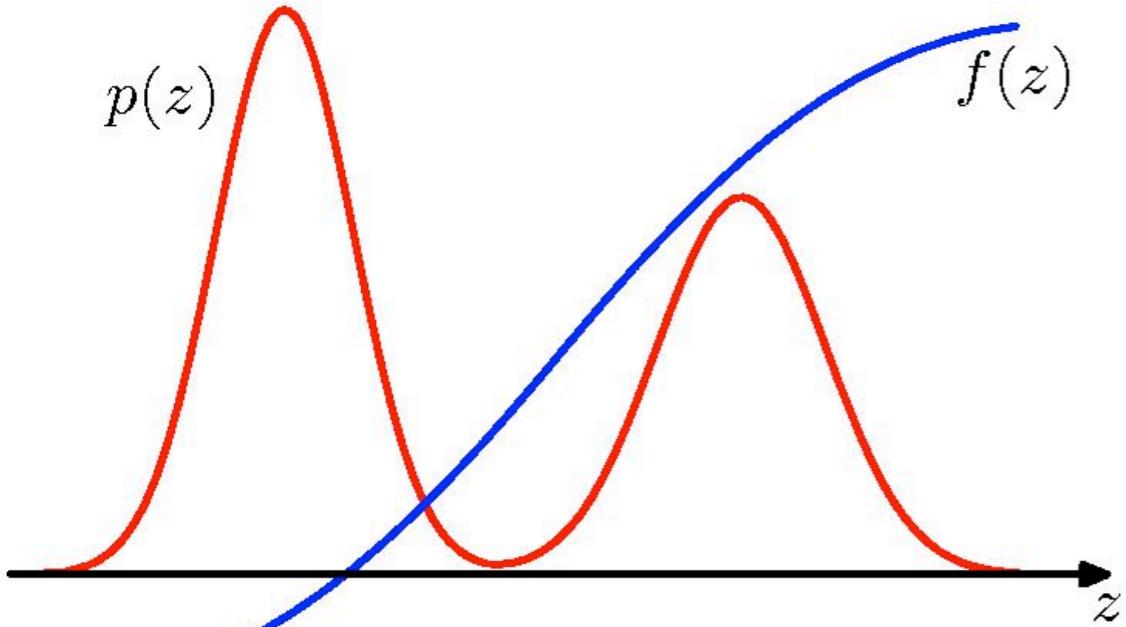
- General Idea: Draw independent samples $\{z^1, \dots, z^n\}$ from distribution $p(z)$ to approximate expectation:



$$\mathbb{E}[f] = \int f(z)p(z)dz \approx \frac{1}{N} \sum_{n=1}^N f(z^n) = \hat{f}.$$

Estimating Expectations

- General Idea: Draw independent samples $\{z^1, \dots, z^n\}$ from distribution $p(z)$ to approximate expectation:



so the estimator has correct mean (unbiased).

- The variance:

$$\text{var}[\hat{f}] = \frac{1}{N} \mathbb{E}[(f - \mathbb{E}[f])^2].$$

- Variance decreases as $1/N$.

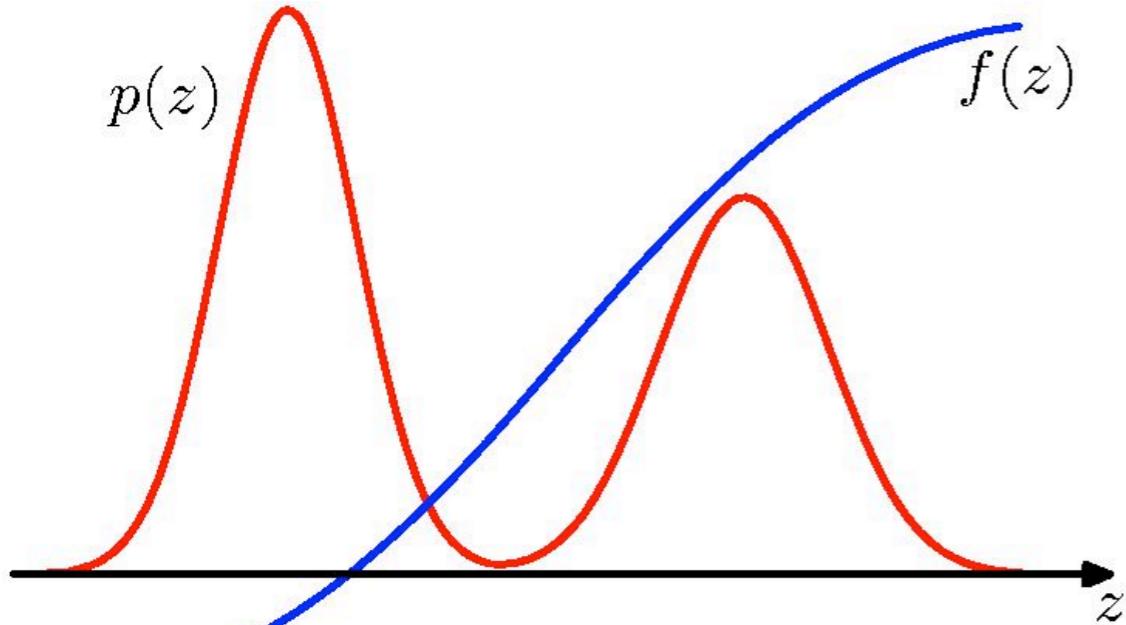
$$\begin{aligned}\mathbb{E}[f] &= \int f(z)p(z)dz \approx \\ &\frac{1}{N} \sum_{n=1}^N f(z^n) = \hat{f}.\end{aligned}$$

Note that:

$$\mathbb{E}[f] = \mathbb{E}[\hat{f}].$$

Estimating Expectations

- General Idea: Draw independent samples $\{z^1, \dots, z^n\}$ from distribution $p(z)$ to approximate expectation:



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$$\text{var}[\hat{f}] = \frac{1}{N} \mathbb{E}[(f - \mathbb{E}[f])^2].$$

- Variance decreases as $1/N$.

- Remark:** The accuracy of the estimator **does not depend on dimensionality** of z .

$$\begin{aligned}\mathbb{E}[f] &= \int f(z)p(z)dz \approx \\ &\frac{1}{N} \sum_{n=1}^N f(z^n) = \hat{f}.\end{aligned}$$

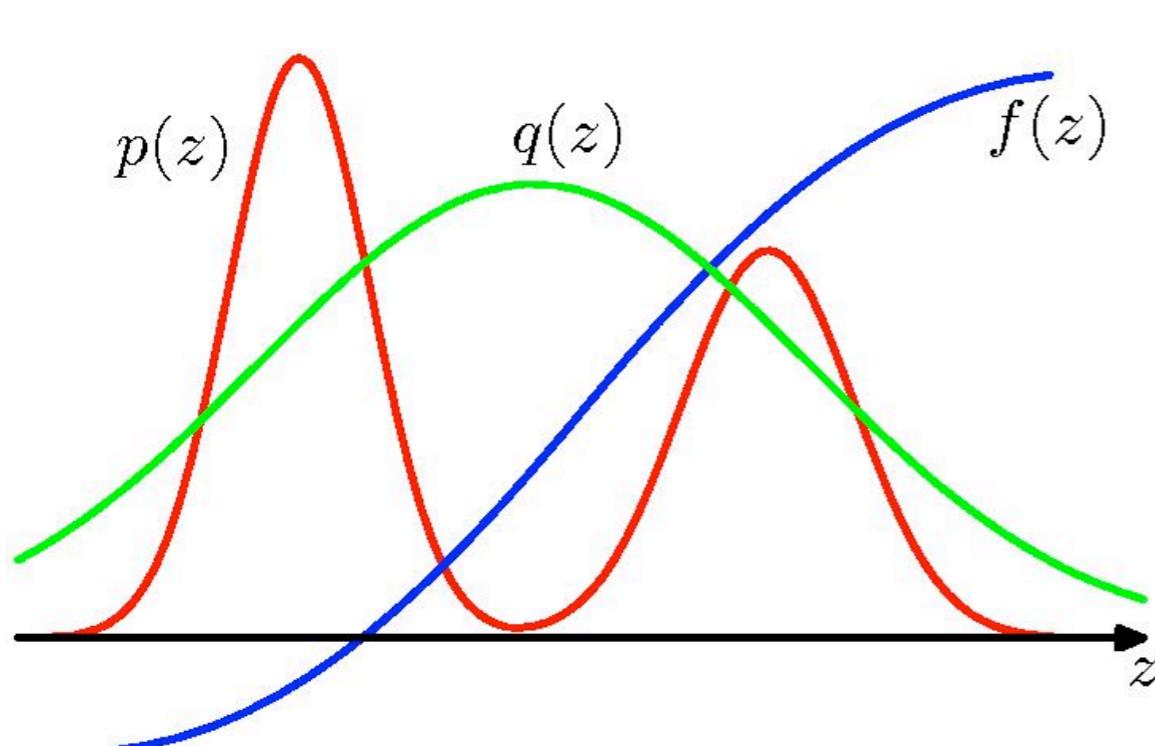
Note that:

$$\mathbb{E}[f] = \mathbb{E}[\hat{f}].$$

Importance Sampling

- Suppose we have an **easy-to-sample** proposal distribution $q(z)$, such that

$$q(z) > 0 \text{ if } p(z) > 0. \quad \mathbb{E}[f] = \int f(z)p(z)dz$$



$$\begin{aligned} &= \int f(z) \frac{p(z)}{q(z)} q(z) dz \\ &\approx \frac{1}{N} \sum_n \frac{p(z^n)}{q(z^n)} f(z^n), \quad z^n \sim q(z). \end{aligned}$$

- The quantities

$$w^n = p(z^n)/q(z^n)$$

are known as **importance weights**.

Importance Sampling Ratio

- ▶ Probability of the rest of the trajectory, after S_t , under policy π

$$\begin{aligned} & \Pr\{A_t, S_{t+1}, A_{t+1}, \dots, S_T \mid S_t, A_{t:T-1} \sim \pi\} \\ &= \pi(A_t|S_t)p(S_{t+1}|S_t, A_t)\pi(A_{t+1}|S_{t+1}) \cdots p(S_T|S_{T-1}, A_{T-1}) \\ &= \prod_{k=t}^{T-1} \pi(A_k|S_k)p(S_{k+1}|S_k, A_k), \end{aligned}$$

- ▶ **Importance Sampling:** Each return is weighted by the relative probability of the trajectory under the target and behavior policies

$$\rho_t^T = \frac{\prod_{k=t}^{T-1} \pi(A_k|S_k)p(S_{k+1}|S_k, A_k)}{\prod_{k=t}^{T-1} \mu(A_k|S_k)p(S_{k+1}|S_k, A_k)} = \prod_{k=t}^{T-1} \frac{\pi(A_k|S_k)}{\mu(A_k|S_k)}$$

- ▶ This is called the **Importance Sampling Ratio**

Importance Sampling

- ▶ Ordinary importance sampling forms estimate

$$V(s) \doteq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{|\mathcal{T}(s)|}.$$

First time of termination
following time t

return after t up through
 $T(t)$

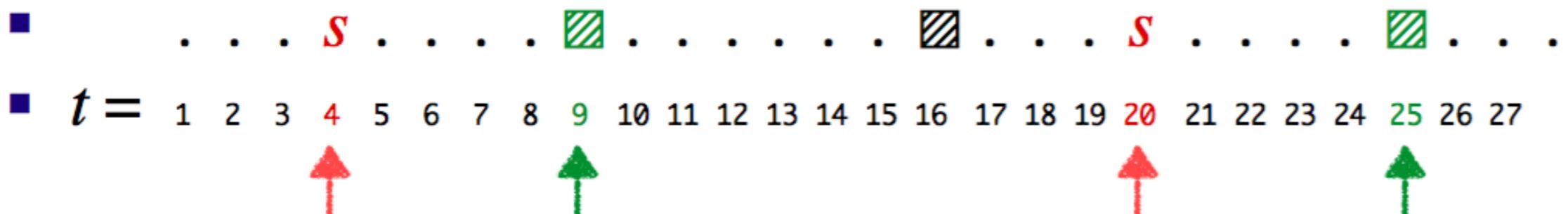
Every time: the set of all time
steps in which state s is visited

Importance Sampling

- ▶ Ordinary importance sampling forms estimate

$$V(s) \doteq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{|\mathcal{T}(s)|}.$$

- ▶ New notation: time steps increase across episode boundaries:



$\mathcal{T}(s) = \{4, 20\}$
set of start times

$T(4) = 9 \quad T(20) = 25$
next termination times

Importance Sampling

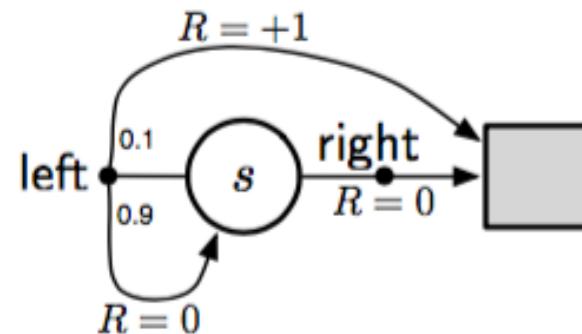
- ▶ Ordinary importance sampling forms estimate:

$$V(s) \doteq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{|\mathcal{T}(s)|}.$$

- ▶ Weighted importance sampling forms estimate:

$$V(s) \doteq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)}}$$

Example of Infinite Variance under Ordinary Importance Sampling



$$\pi(\text{left}|s) = 1$$

$$\mu(\text{left}|s) = \frac{1}{2}$$

$$\gamma = 1$$

$$v_\pi(s) = 1$$

$$\frac{\pi(\text{right}|s)}{\mu(\text{right}|s)} = 0$$

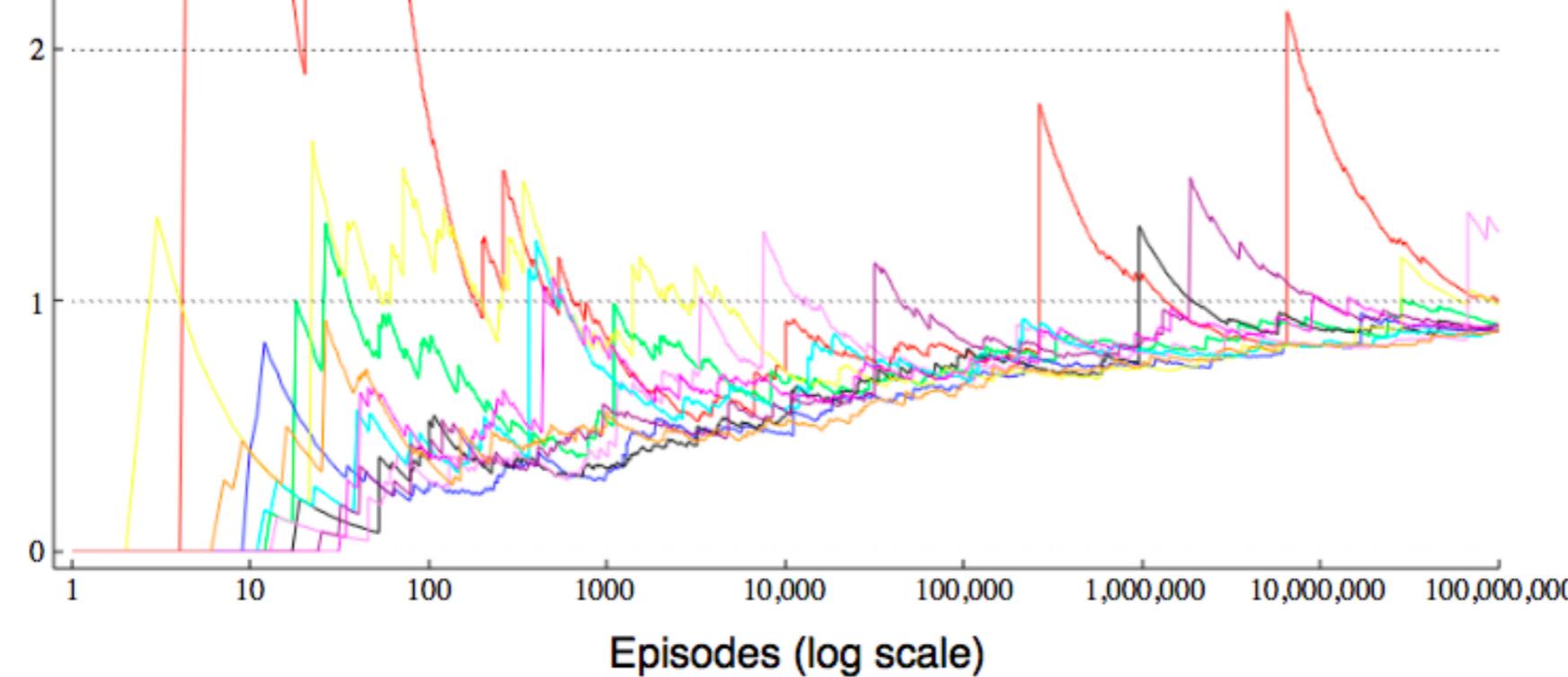
$$\frac{\pi(\text{left}|s)}{\mu(\text{left}|s)} = 2$$

Trajectory	G_0	ρ_0^T
$s, \text{left}, 0, s, \text{left}, 0, s, \text{left}, 0, s, \text{right}, 0, \blacksquare$	0	0
$s, \text{left}, 0, s, \text{left}, 0, s, \text{left}, 0, s, \text{left}, +1, \blacksquare$	1	16

OIS:

$$V(s) \triangleq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{|\mathcal{T}(s)|}$$

Monte-Carlo estimate of $v_\pi(s)$ with ordinary importance sampling (ten runs)

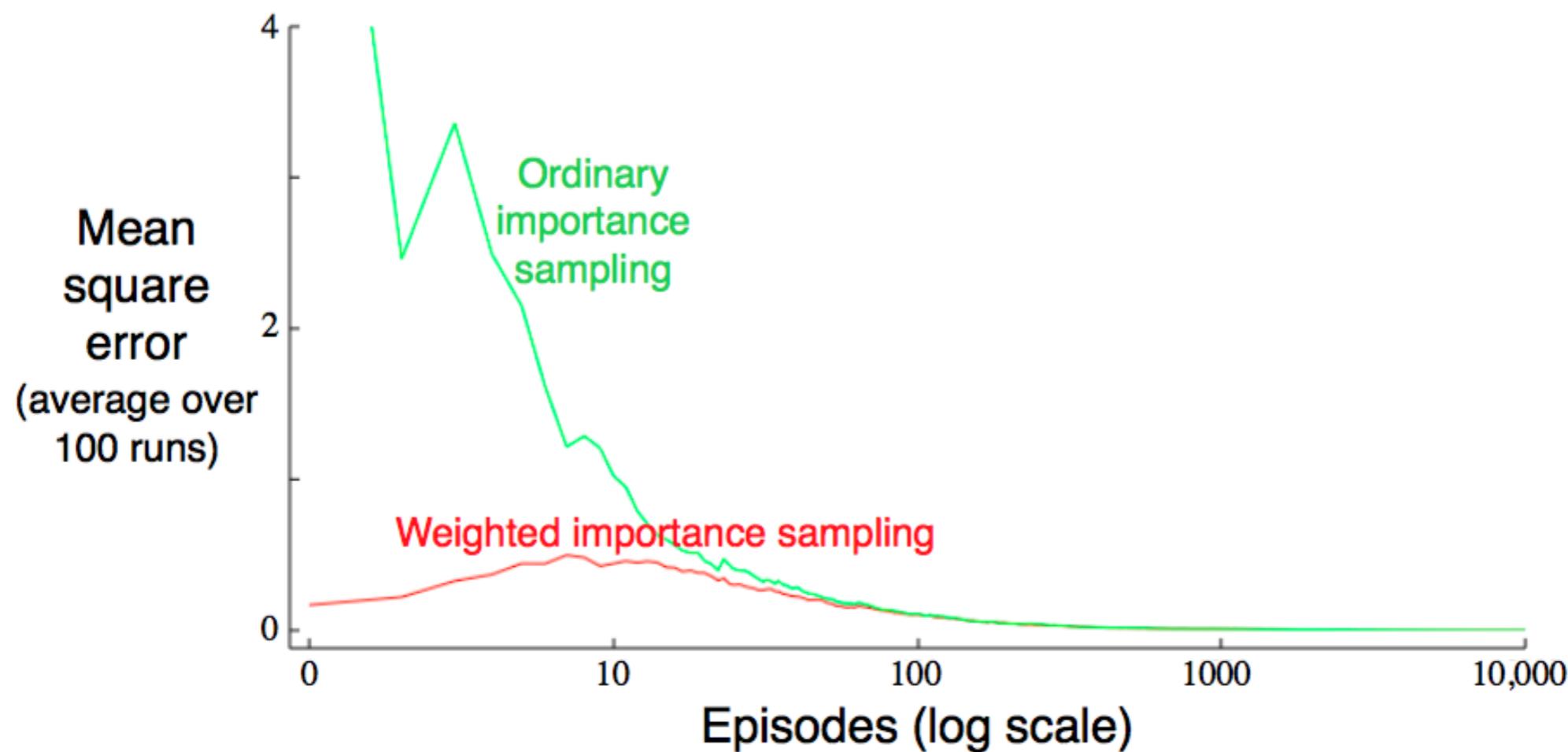


WIS:

$$V(s) \triangleq \frac{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)} G_t}{\sum_{t \in \mathcal{T}(s)} \rho_t^{T(t)}}$$

Example: Off-policy Estimation of the Value of a Single Blackjack State

- ▶ State is player-sum 13, dealer-showing 2, useable ace
- ▶ Target policy is stick only on 20 or 21
- ▶ Behavior policy is equiprobable
- ▶ True value ≈ -0.27726



So far

- ▶ MC has several advantages over DP:
 - Can learn directly from **interaction with environment**
 - No need for full models
- ▶ MC methods provide an alternate policy evaluation process
- ▶ One issue to watch for: maintaining **sufficient exploration**
- ▶ Looked at distinction between **on-policy** and **off-policy** methods
- ▶ Looked at **importance sampling** for off-policy learning
- ▶ Looked at distinction between **ordinary** and **weighted IS**

Coming up next

- MC methods are different than Dynamic Programming in that they:
 1. use experience in place of known dynamics and reward functions
 2. do not bootstrap
- Next lecture we will see temporal difference learning which
 3. use experience in place of known dynamics and reward functions
 4. bootstrap!

Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Temporal Difference Learning

CMU 10-703

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Used Materials

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MC and TD Learning

- ▶ Goal: learn $v_\pi(s)$ from episodes of experience under policy π

- ▶ Incremental every-visit Monte-Carlo:

- Update value $V(S_t)$ toward actual return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

- ▶ Simplest Temporal-Difference learning algorithm: TD(0)

- Update value $V(S_t)$ toward estimated returns $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha (R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

- ▶ $R_{t+1} + \gamma V(S_{t+1})$ is called the TD target

- ▶ $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ called the TD error.

DP vs. MC vs. TD Learning

- Remember:

$$\begin{aligned} v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\ &= \mathbb{E}_\pi \left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \mid S_t = s \right] \\ &= \mathbb{E}_\pi \left[R_{t+1} + \gamma \sum_{k=0}^{\infty} \gamma^k R_{t+k+2} \mid S_t = s \right] \\ &= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s]. \end{aligned}$$

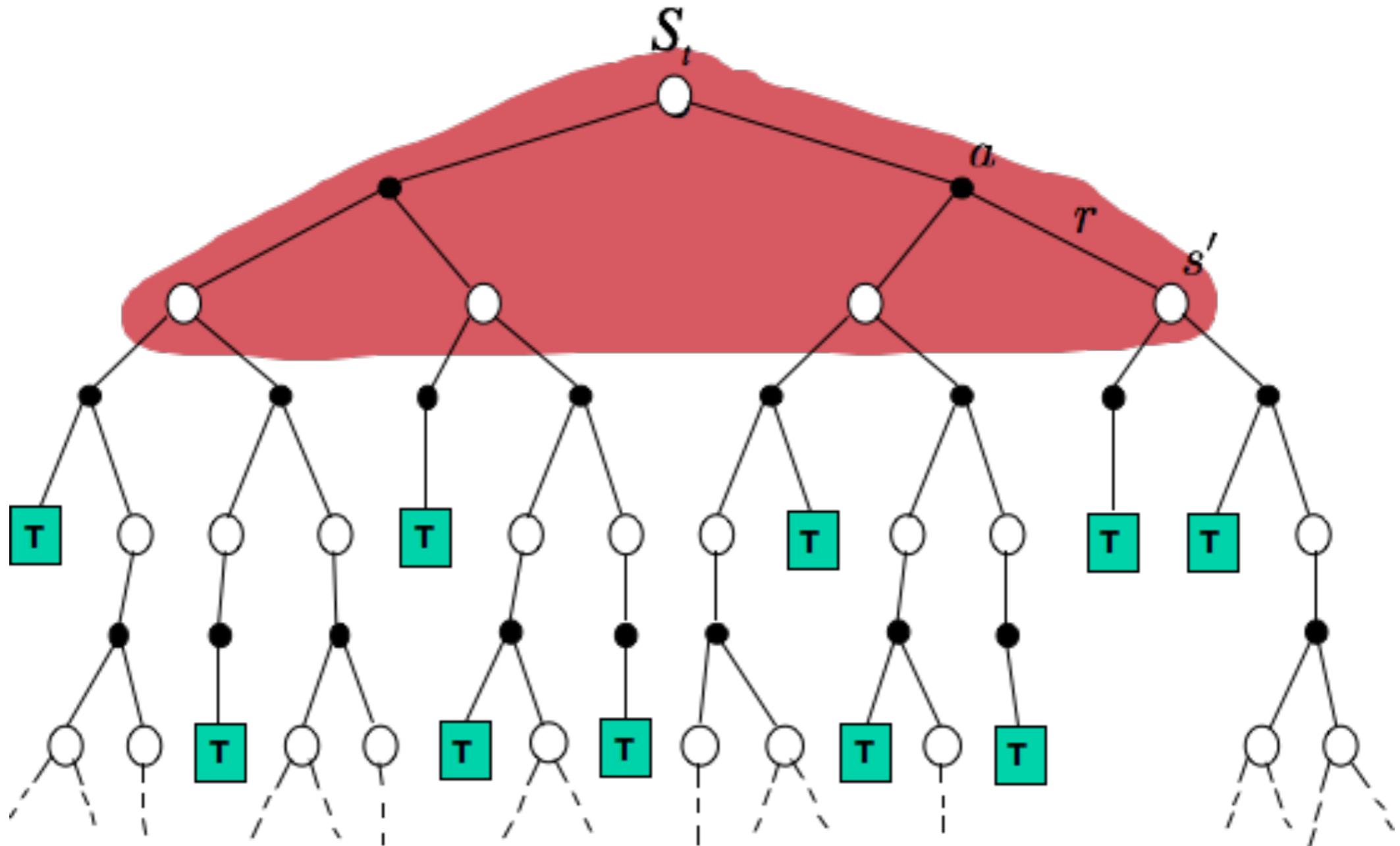
TD: combine both: Sample expected values *and* use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$

MC: sample average return approximates expectation

DP: the expected values are provided by a model. But we use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$

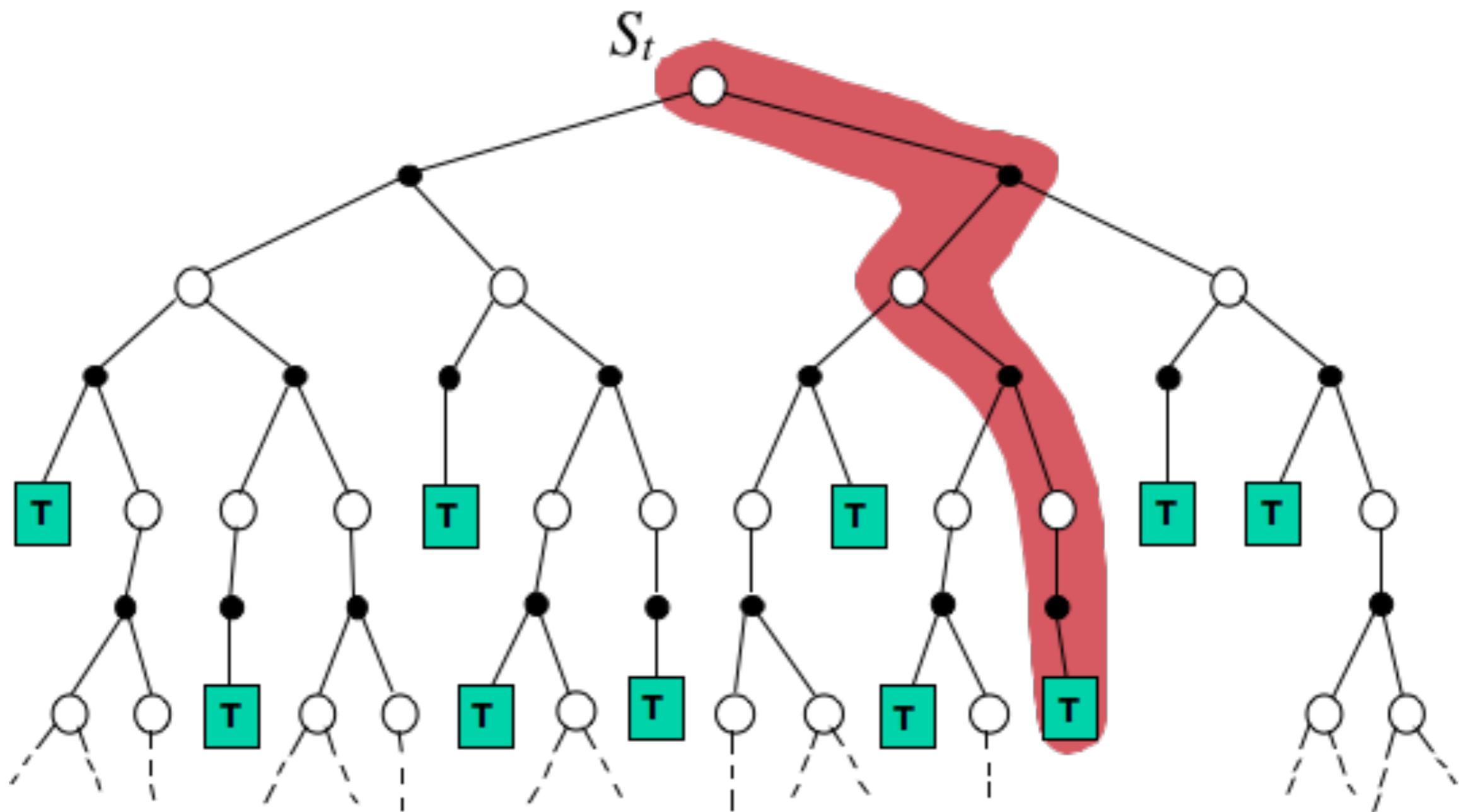
Dynamic Programming

$$V(S_t) \leftarrow E_{\pi} \left[R_{t+1} + \gamma V(S_{t+1}) \right] = \sum_a \pi(a|S_t) \sum_{s',r} p(s',r|S_t,a) [r + \gamma V(s')]$$



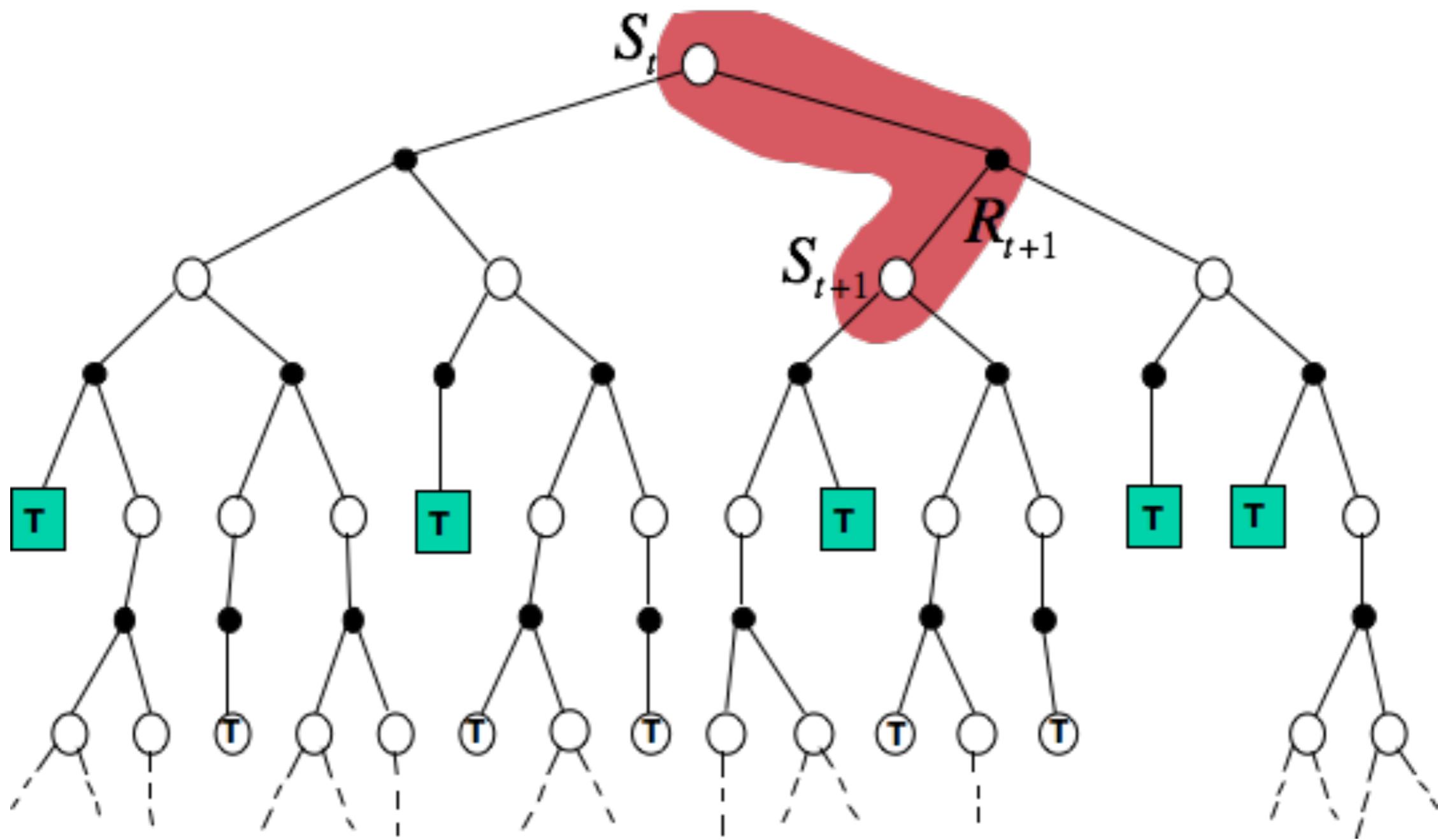
Monte Carlo

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$



Simplest TD(0) Method

$$V(S_t) \leftarrow V(S_t) + \alpha (R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$



TD Methods Bootstrap and Sample

- ▶ **Bootstrapping:** update involves an estimate
 - MC does not bootstrap
 - DP bootstraps
 - TD bootstraps
- ▶ **Sampling:** update does not involve an expected value
 - MC samples
 - DP does not sample
 - TD samples

TD Prediction

- ▶ Policy Evaluation (the prediction problem):
 - for a given policy π , compute the state-value function v_π

- ▶ Remember: Simple every-visit Monte Carlo method:

$$V(S_t) \leftarrow V(S_t) + \alpha [G_t - V(S_t)]$$



target: the actual return after time t

- ▶ The simplest Temporal-Difference method TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha [R_{t+1} + \gamma V(S_{t+1}) - V(S_t)]$$



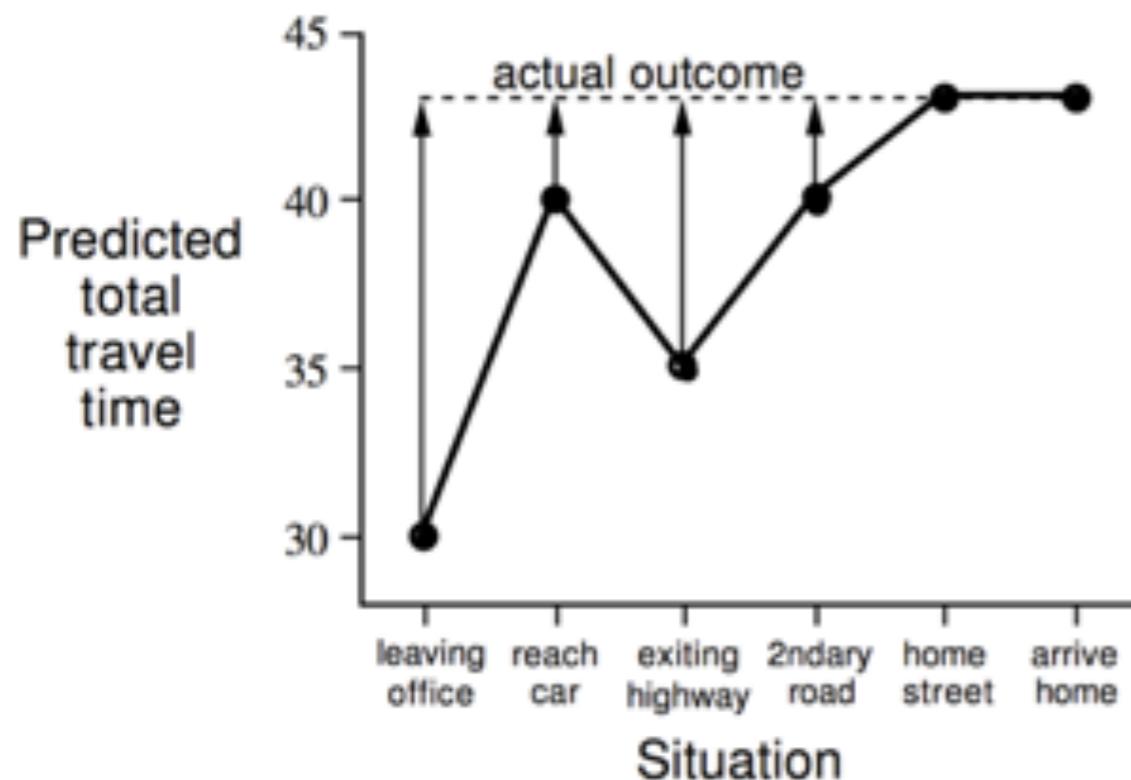
target: an estimate of the return

Example: Driving Home

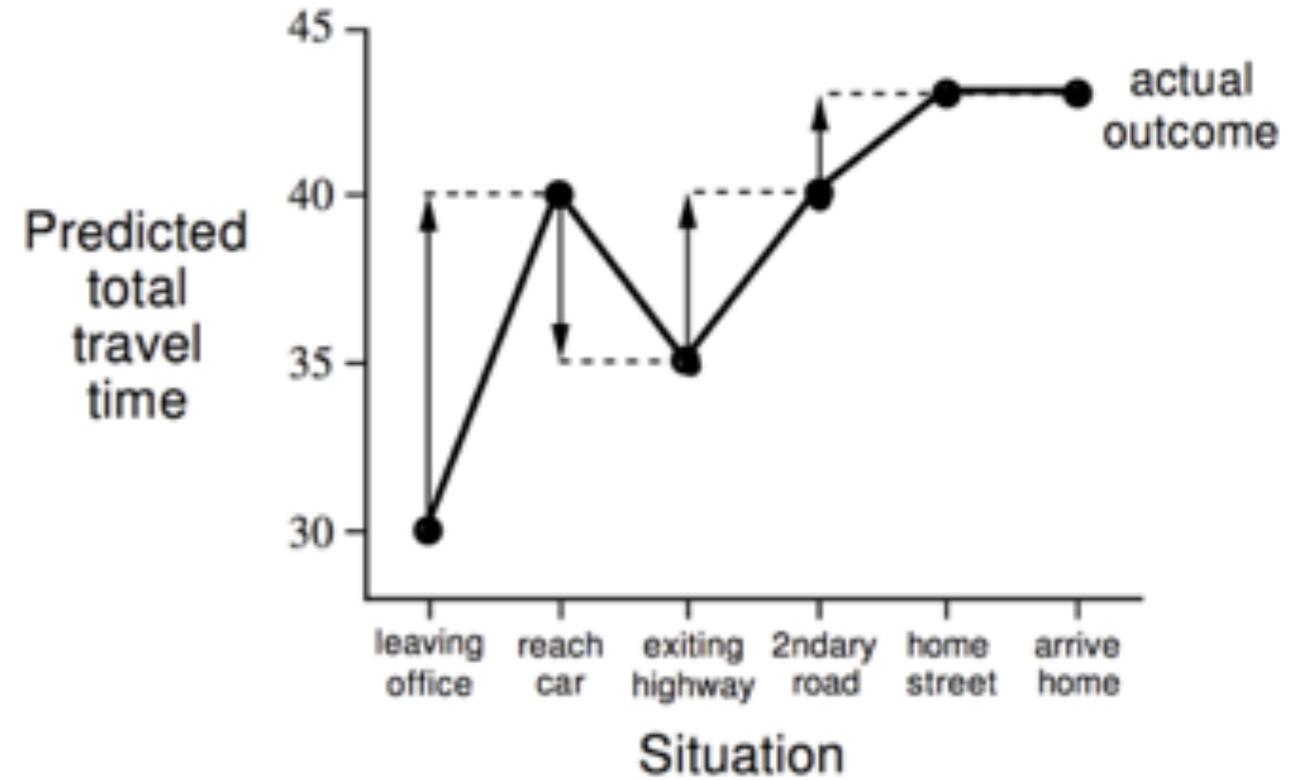
<i>State</i>	<i>Elapsed Time</i> (minutes)	<i>Predicted</i> <i>Time to Go</i>	<i>Predicted</i> <i>Total Time</i>
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Example: Driving Home

Changes recommended by Monte Carlo methods ($\alpha=1$)



Changes recommended by TD methods ($\alpha=1$)



Advantages of TD Learning

- ▶ TD methods **do not require a model of the environment**, only experience
- ▶ TD, but not MC, methods can be **fully incremental**
- ▶ You can learn **before** knowing the final outcome
 - Less memory
 - Less computation
- ▶ You can learn **without** the final outcome
 - From incomplete sequences
- ▶ Both MC and TD converge (under certain assumptions to be detailed later), but which is faster?

Batch Updating in TD and MC methods

- ▶ **Batch Updating:** train completely on a finite amount of data,
 - e.g., train repeatedly on 10 episodes until convergence.
- ▶ Compute updates according to TD or MC, but only update estimates after **each complete pass through the data**.
- ▶ For any **finite** Markov prediction task, under batch updating, TD converges for sufficiently small α .
- ▶ Constant- α MC also converges under these conditions, but may converge to a different answer.

AB Example

- ▶ Suppose you observe the following 8 episodes:

A, 0, B, 0

B, 1

B, 1

$V(B)? \textcolor{blue}{0.75}$

B, 1

$V(A)? \textcolor{blue}{0?}$

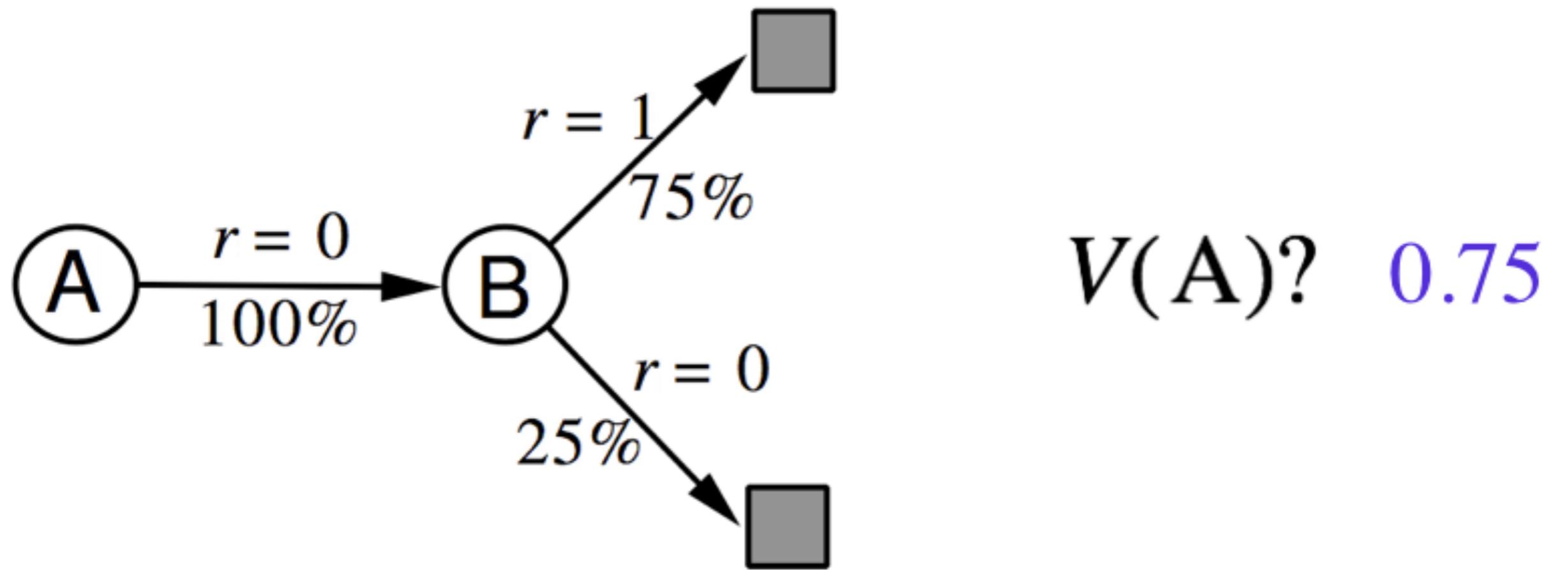
B, 1

B, 1

B, 0

- ▶ Assume Markov states, no discounting ($\gamma = 1$)

AB Example



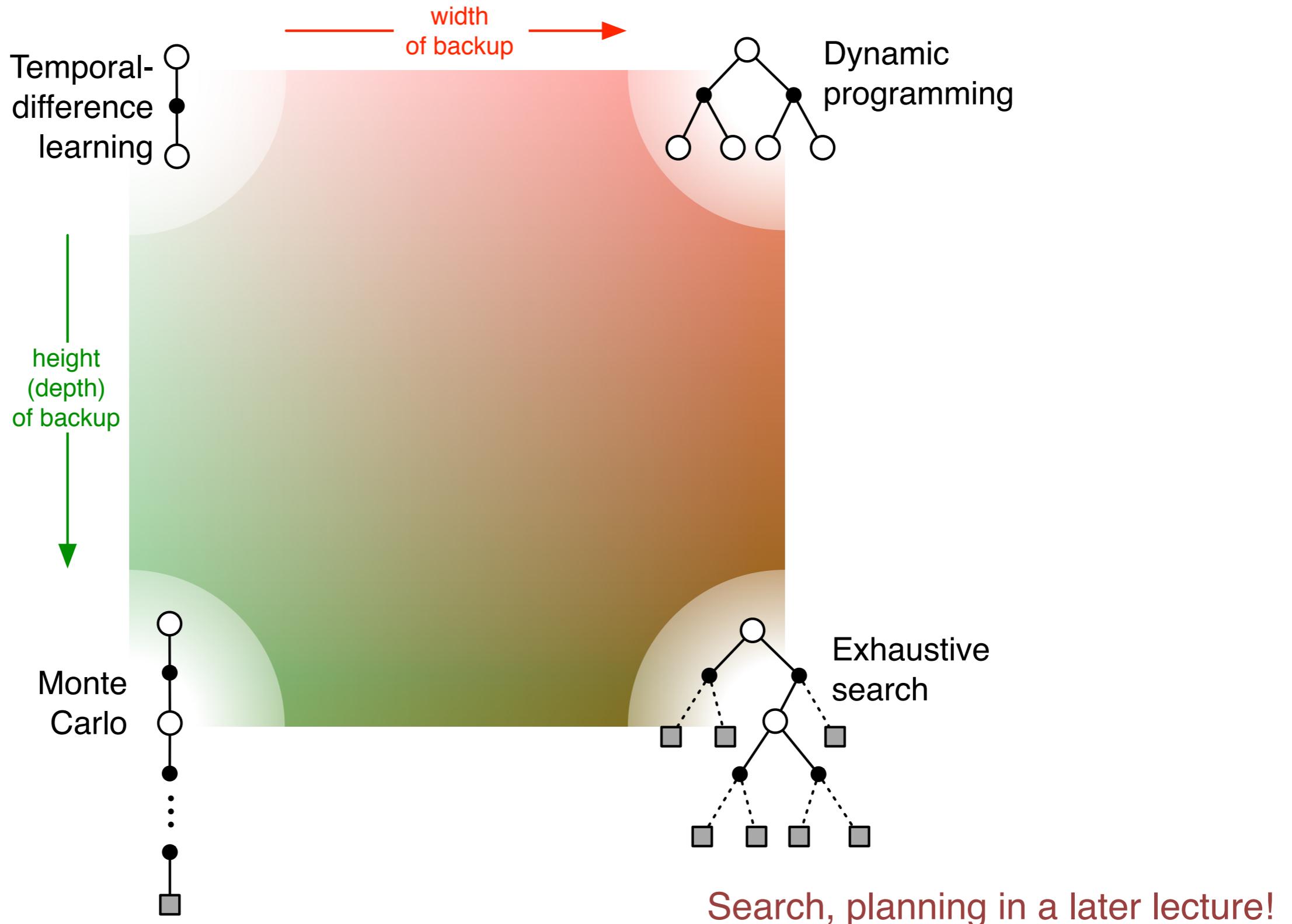
AB Example

- ▶ The prediction that best matches the training data is $V(A)=0$
 - This minimizes the mean-square-error on the training set
 - This is what a batch Monte Carlo method gets
- ▶ If we consider the sequentiality of the problem, then we would set $V(A)=.75$
 - This is correct for the maximum likelihood estimate of a Markov model generating the data
 - i.e, if we do a best fit Markov model, and assume it is exactly correct, and then compute what it predicts.
 - This is called the certainty-equivalence estimate
 - This is what TD gets

Summary so far

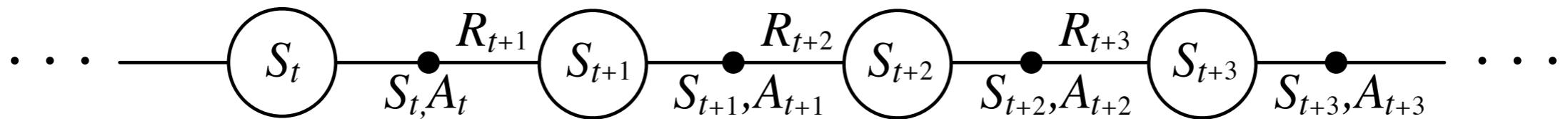
- ▶ Introduced one-step tabular model-free TD methods
- ▶ These methods bootstrap and sample, combining aspects of DP and MC methods
- ▶ If the world is truly Markov, then TD methods will learn faster than MC methods

Unified View



Learning An Action-Value Function

- ▶ Estimate q_{π} for the **current policy** π



After every transition from a nonterminal state, S_t , do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha [R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)]$$

If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

Sarsa: On-Policy TD Control

- Turn this into a control method by always updating the policy to be **greedy** with respect to the current estimate:

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

 Initialize S

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Repeat (for each step of episode):

 Take action A , observe R, S'

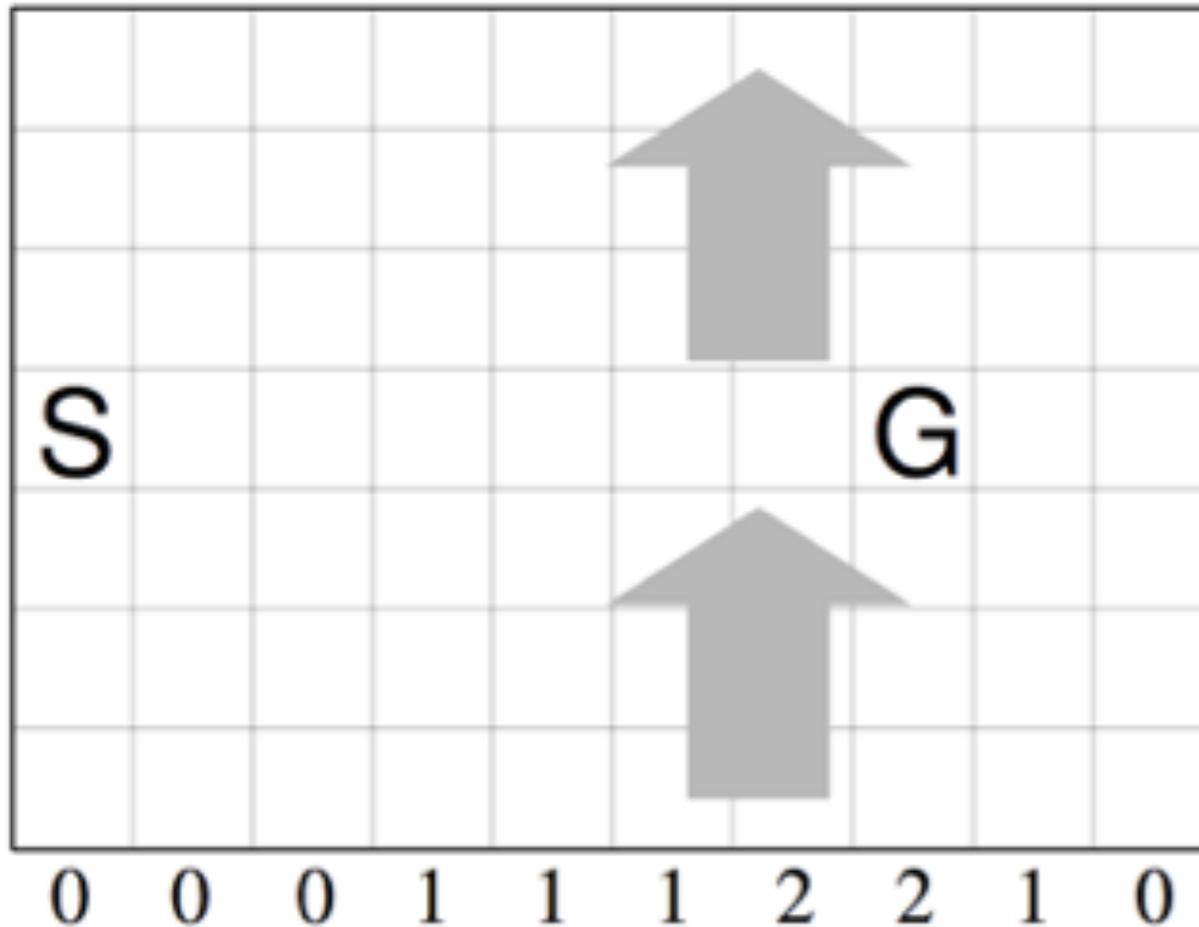
 Choose A' from S' using policy derived from Q (e.g., ε -greedy)

$$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$$

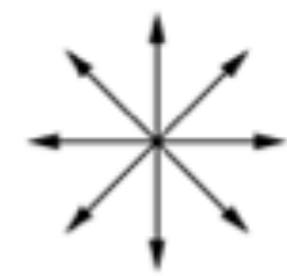
$S \leftarrow S'; A \leftarrow A'$;

 until S is terminal

Windy Gridworld



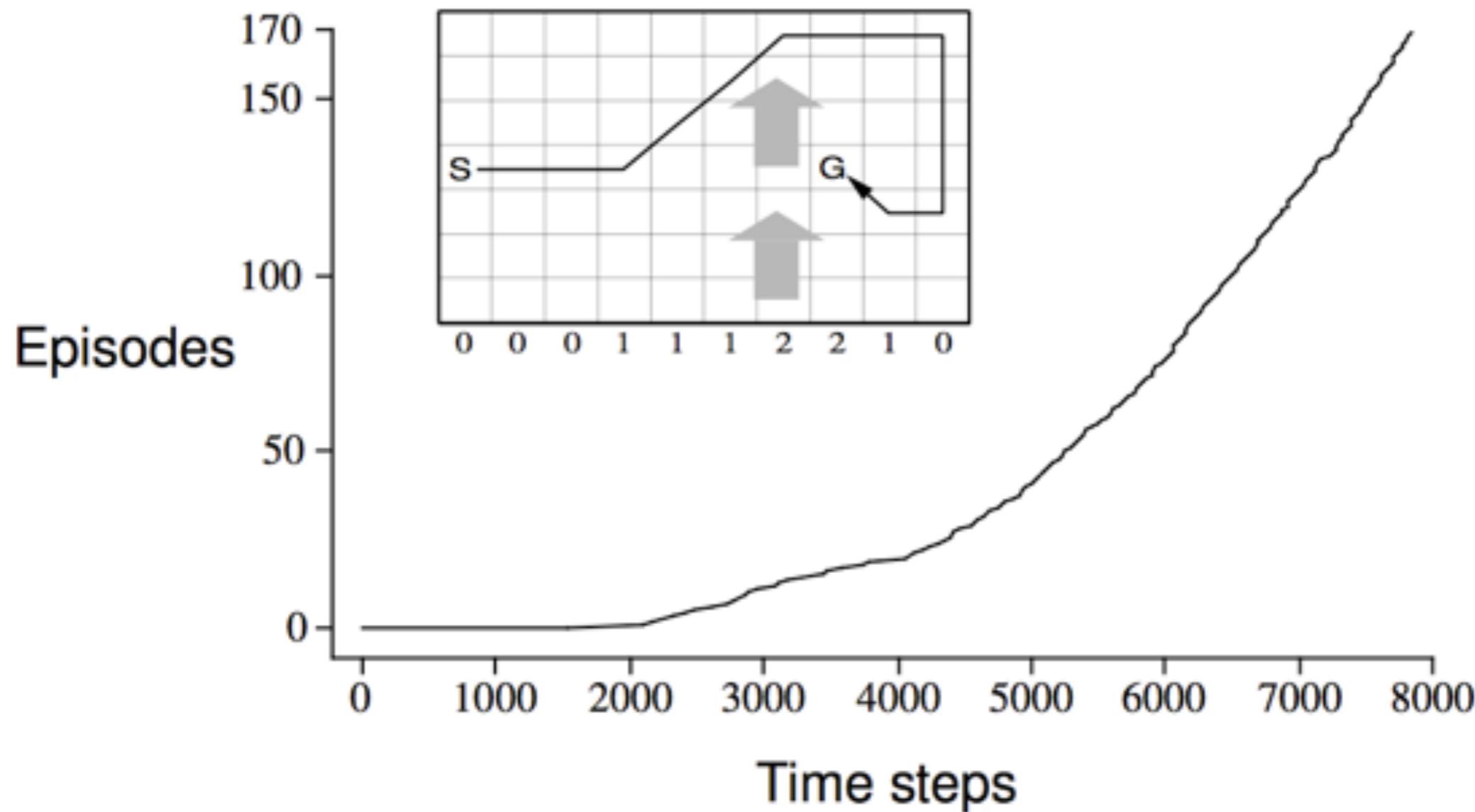
standard
moves



king's
moves

- undiscounted, episodic, reward = -1 until goal

Results of Sarsa on the Windy Gridworld



Q: Can a policy result in infinite loops? What will MC policy iteration do then?

- If the policy leads to infinite loop states, MC control will get trapped as the episode will not terminate.
- Instead, TD control can update continually the state-action values and switch to a different policy.

Q-Learning: Off-Policy TD Control

- ▶ One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t) \right]$$

Initialize $Q(s, a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\text{terminal-state}, \cdot) = 0$

Repeat (for each episode):

 Initialize S

 Repeat (for each step of episode):

 Choose A from S using policy derived from Q (e.g., ε -greedy)

 Take action A , observe R, S'

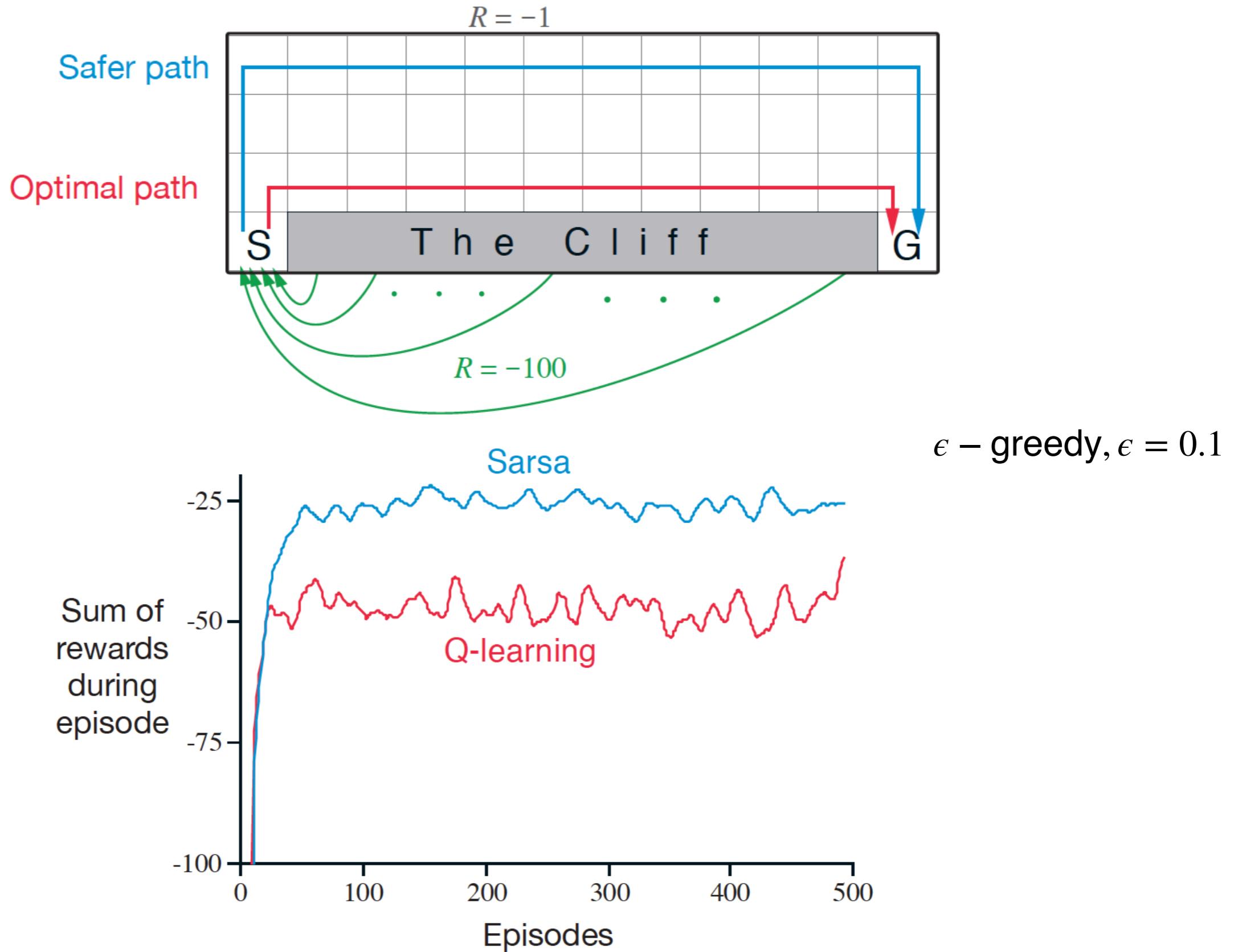
$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$;

 until S is terminal

$$Q(S, A) \leftarrow Q(S, A) + \alpha[R + \gamma Q(S', A') - Q(S, A)]$$

Cliffwalking



Expected Sarsa

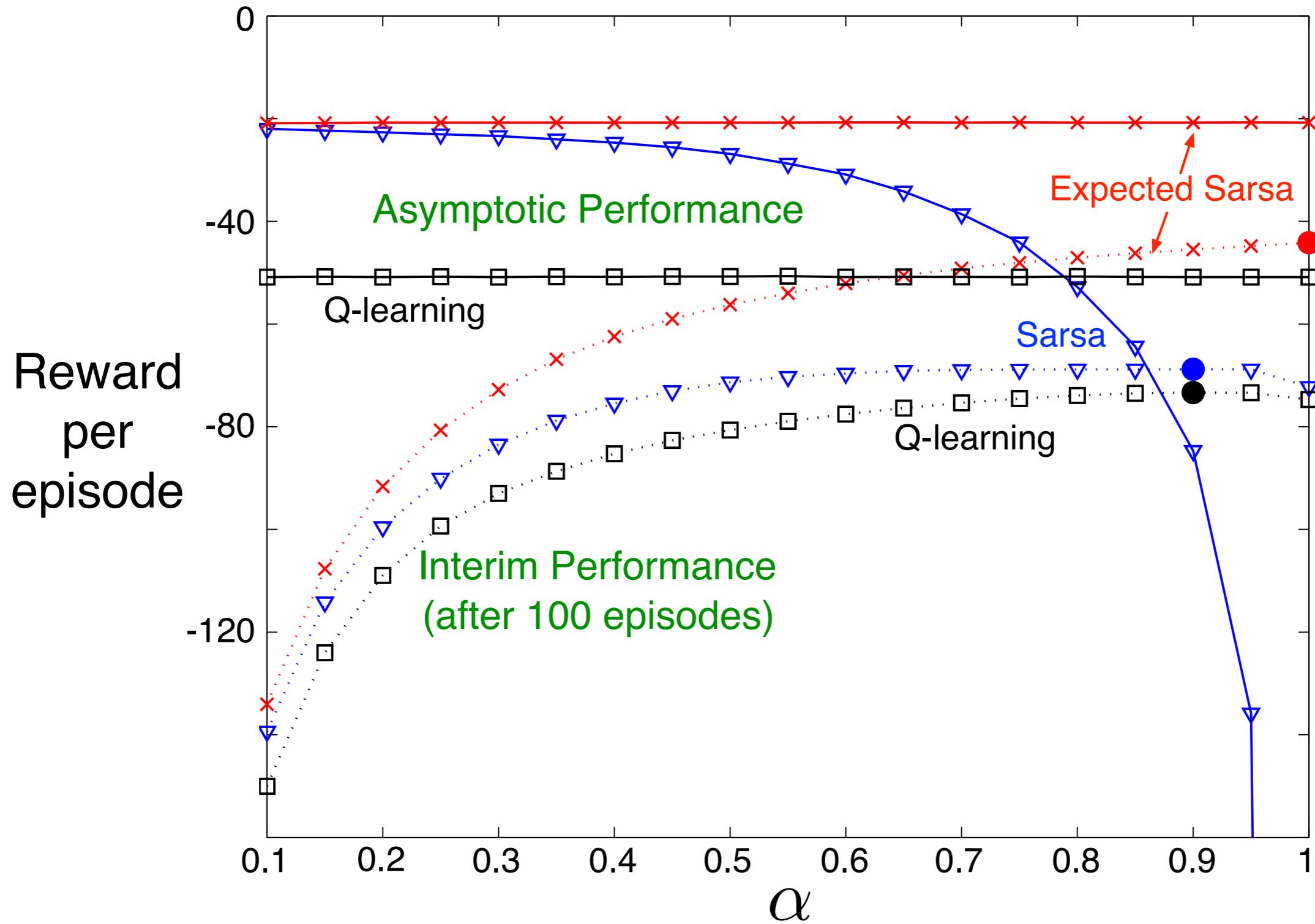
- ▶ Instead of the sample value-of-next-state, use the expectation!

$$\begin{aligned} Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t) \right] \\ &\leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \sum \pi(a|S_{t+1})Q(S_{t+1}, a) - Q(S_t, A_t) \right] \end{aligned}$$

- ▶ Expected Sarsa performs better than Sarsa (but costs more)
 - ▶ **Q:** why?

Q: Is expected SARSA on policy or off policy? What if π is the greedy deterministic policy?

Performance on the Cliff-walking Task



Summary

- ▶ Introduced one-step tabular model-free TD methods
- ▶ These methods **bootstrap and sample**, combining aspects of DP and MC methods
- ▶ TD methods are computationally congenial
- ▶ If the world is truly Markov, then TD methods will learn faster than MC methods
- ▶ Extend prediction to control by employing some form of GPI
 - **On-policy control**: Sarsa, Expected Sarsa
 - **Off-policy control**: Q-learning