Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Temporal Difference Learning

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Used Materials

- **Disclaimer**: Much of the material and slides for this lecture were borrowed from Russ Salakhutdinov who in turn borrowed some material from Rich Sutton's class and David Silver's class on Reinforcement Learning.

MC and TD Learning

- Goal: learn $v_\pi(s)$ from episodes of experience under policy π
- Incremental every-visit Monte-Carlo:
  - Update value $V(S_t)$ toward actual return G_t:

$$V(S_t) \leftarrow V(S_t) + \alpha\left(G_t - V(S_t)\right)$$

- Simplest Temporal-Difference learning algorithm: TD(0)
  - Update value V(St) toward estimated returns $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha\left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right)$$

- $R_{t+1} + \gamma V(S_{t+1})$ led the TD target
- $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ the TD error.

DP vs. MC vs. TD Learning

- Remember:

MC: sample average return approximates expectation

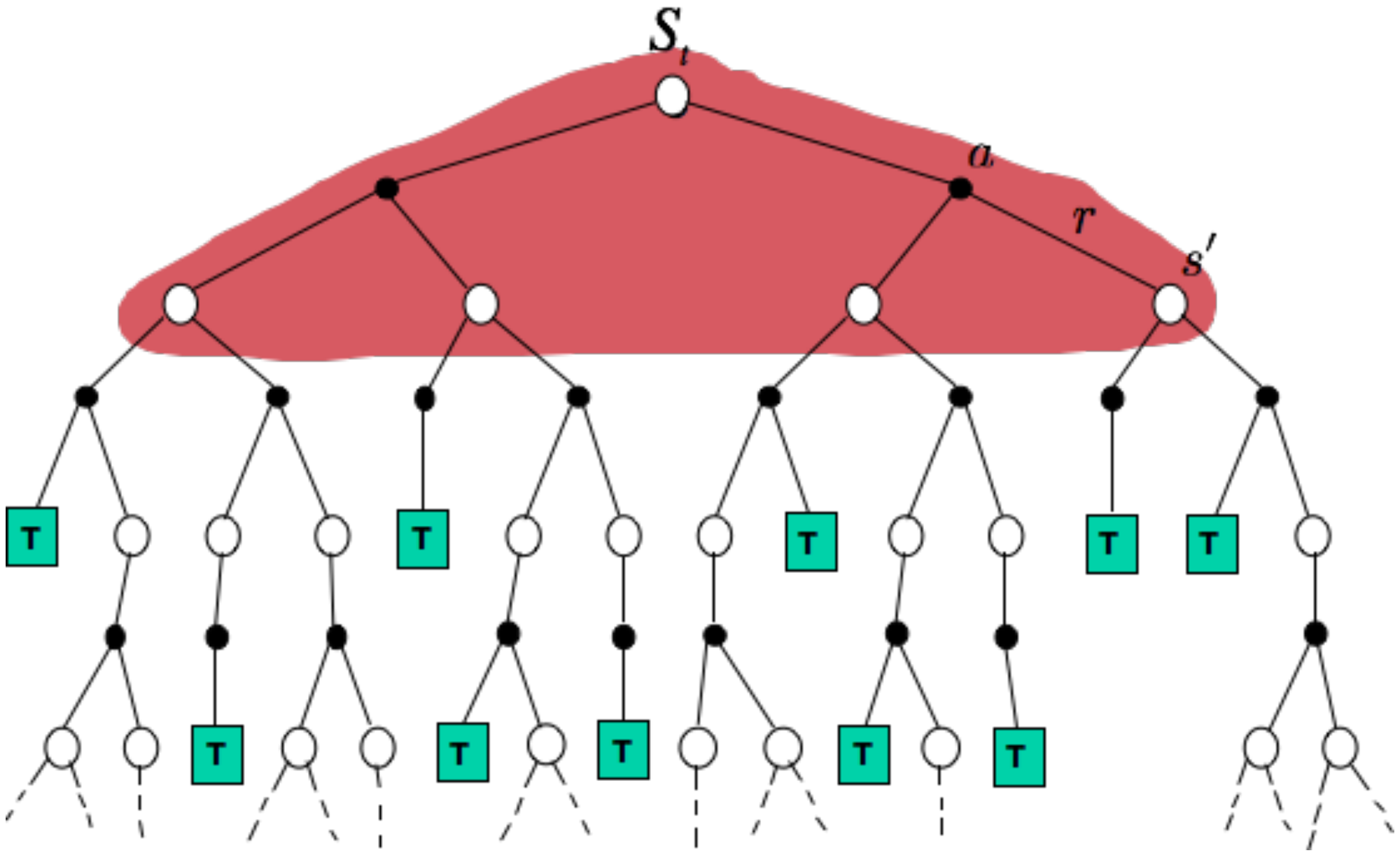
$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_{t+1} + \gamma \sum_{k=0}^{\infty} \gamma^k R_{t+k+2} \,\middle|\, S_t = s\right] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] \,.
\end{aligned}
$$

TD: combine both: Sample expected values and use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$

DP: the expected values are provided by a model. But we use a current estimate $V(S_{t+1})$ of the true $v_\pi(S_{t+1})$.

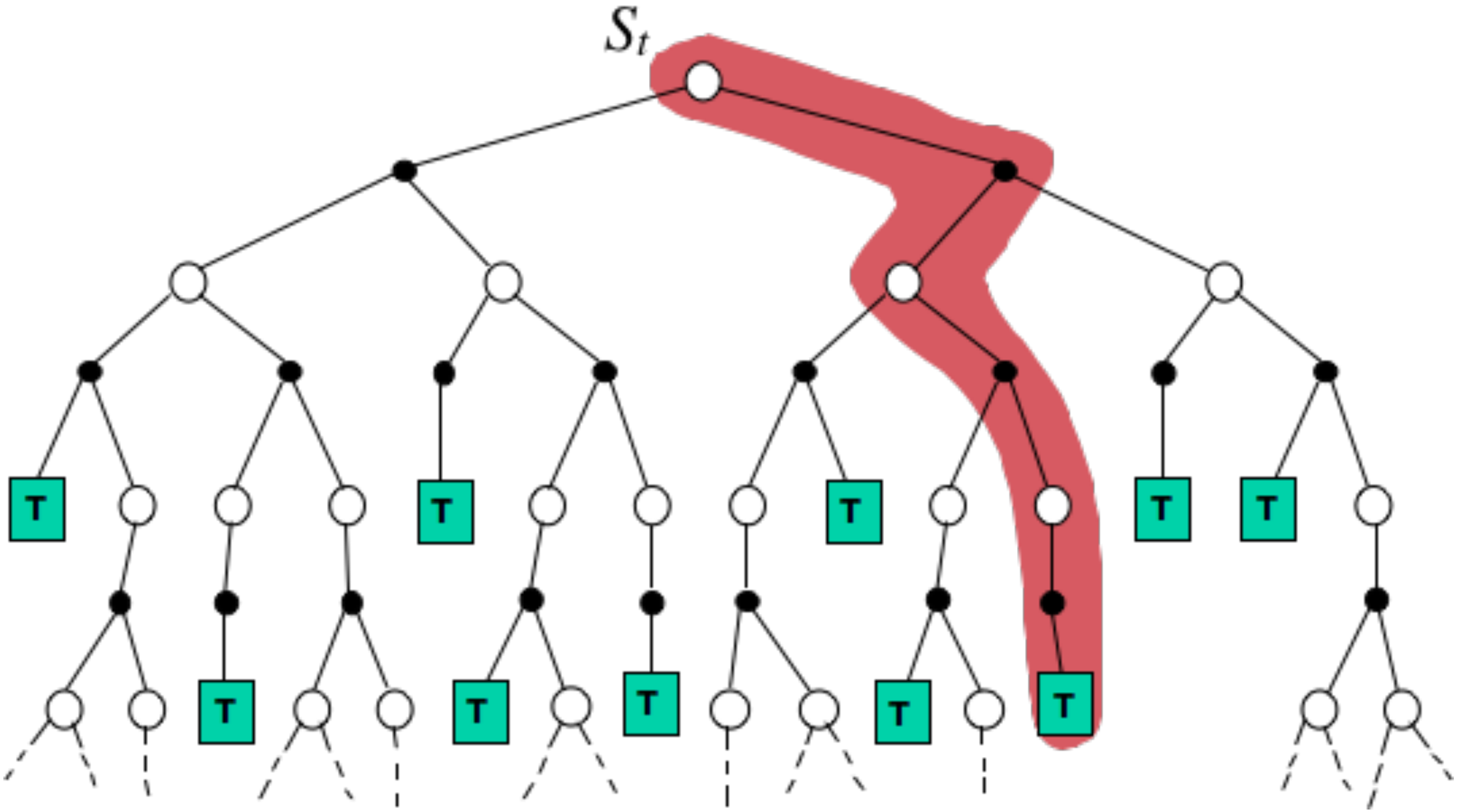
Dynamic Programming

$$V(S_t) \leftarrow E_\pi\left[R_{t+1} + \gamma V(S_{t+1})\right] = \sum_a \pi(a|S_t) \sum_{s',r} p(s', r|S_t, a)[r + \gamma V(s')]$$



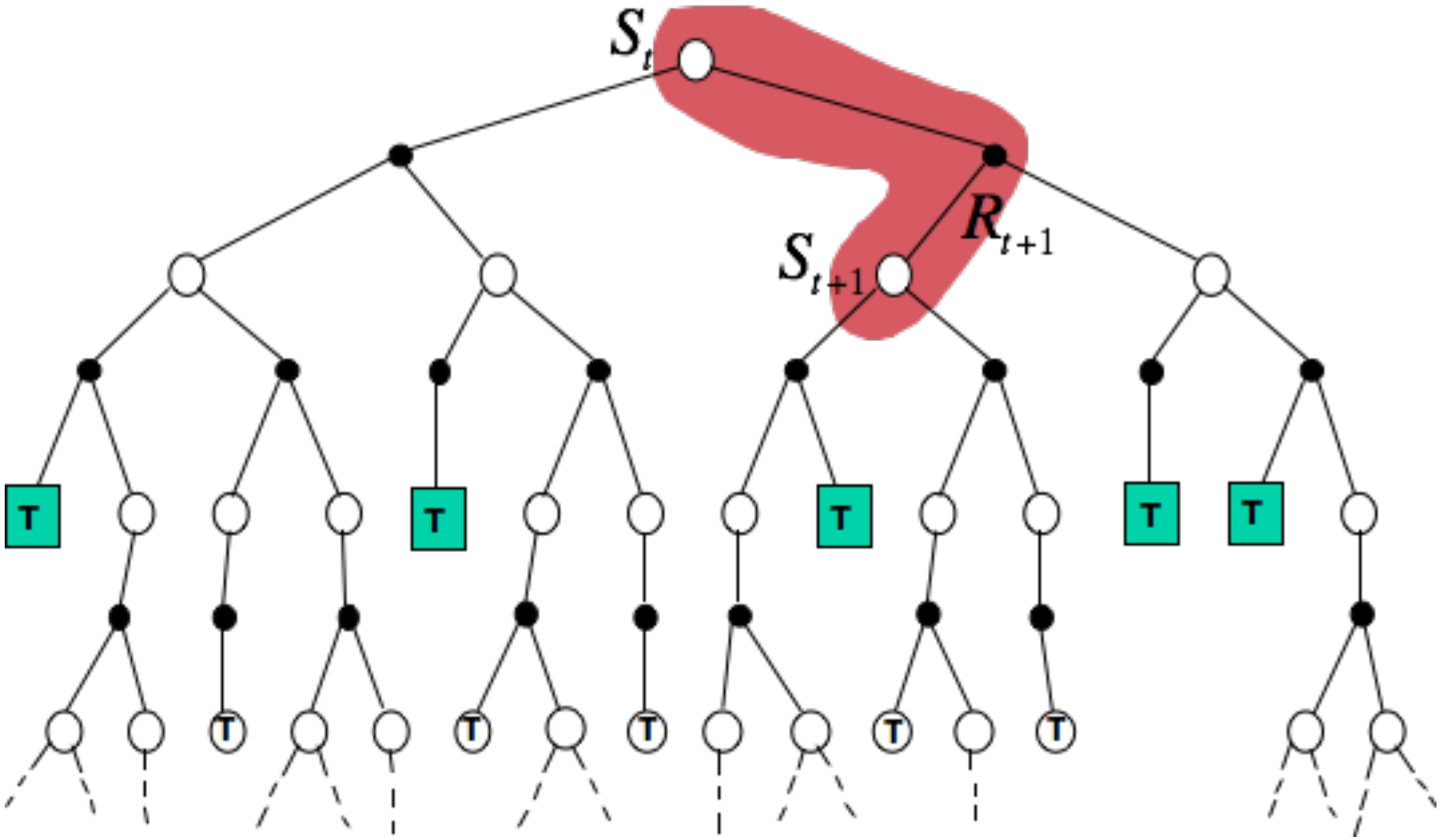
Monte Carlo

$$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$$



Simplest TD(0) Method

$$V(S_t) \leftarrow V(S_t) + \alpha \left(R_{t+1} + \gamma V(S_{t+1}) - V(S_t) \right)$$



TD Methods Bootstrap and Sample

- Bootstrapping: update involves an estimate
  - MC does not bootstrap
  - DP bootstraps
  - TD bootstraps
- Sampling: update does not involve an expected value
  - MC samples
  - DP does not sample
  - TD samples

TD Prediction

- Policy Evaluation (the prediction problem):
  - for a given policy π, compute the state-value function v_π.
- Remember: Simple every-visit Monte Carlo method:

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big]$$

target: the actual return after time t

- The simplest Temporal-Difference method TD(0):

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

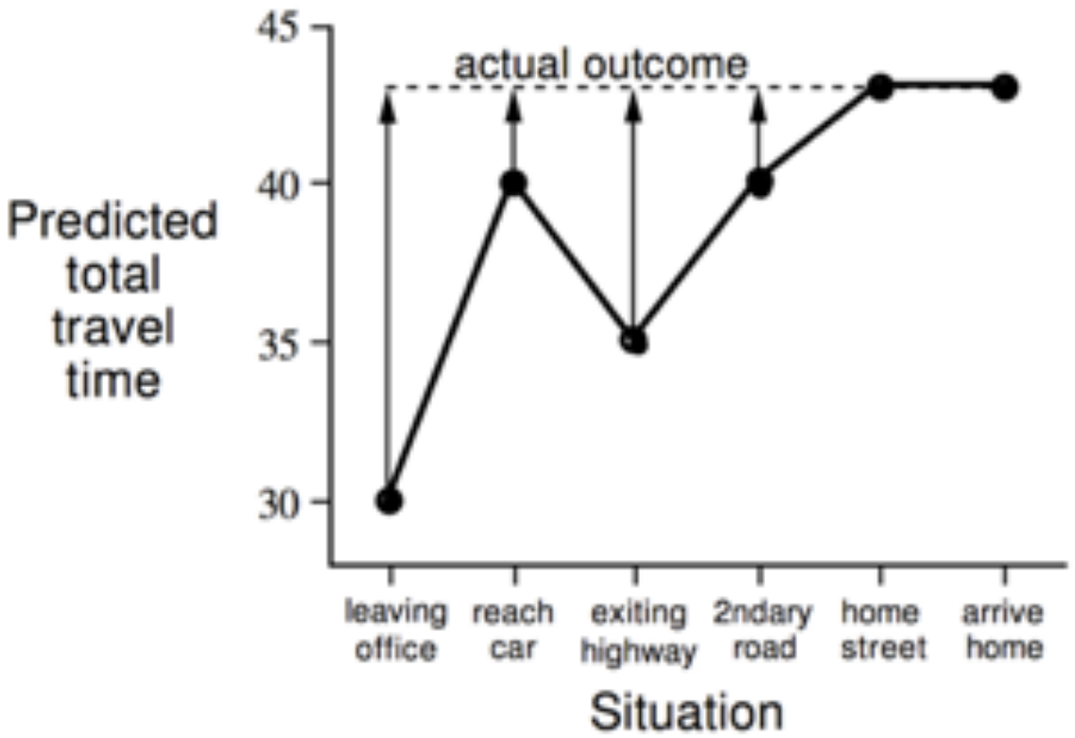
target: an estimate of the return

Example: Driving Home

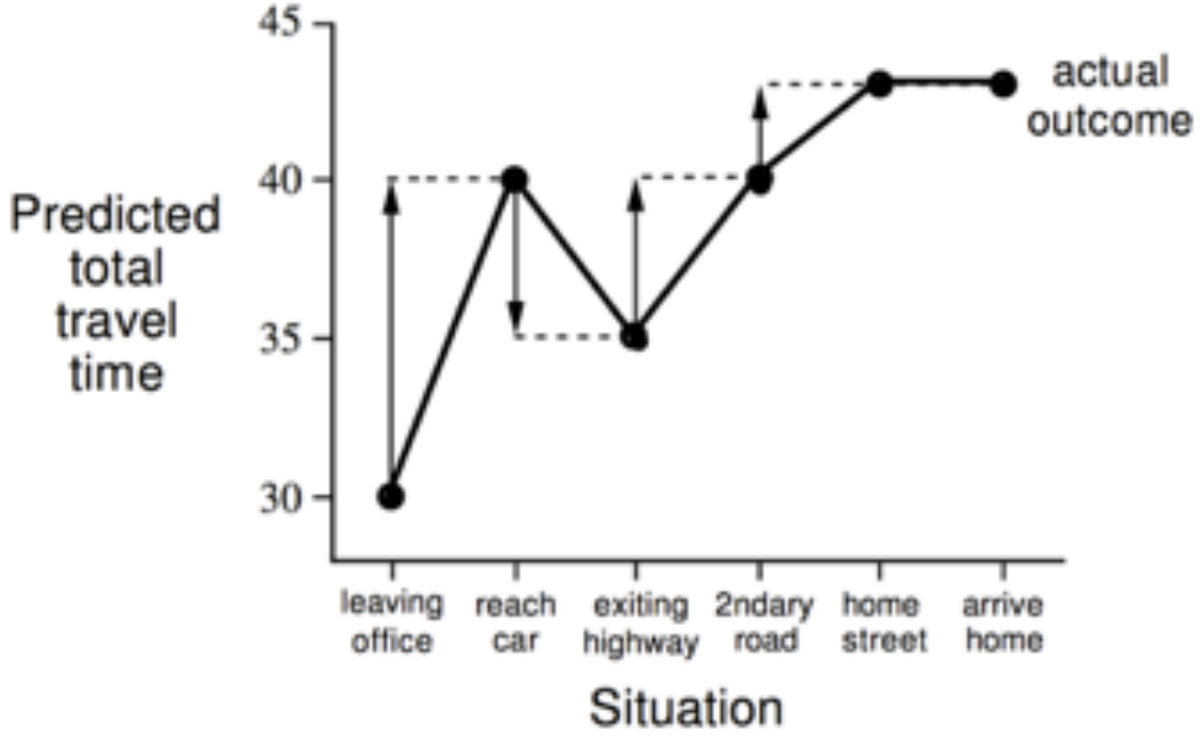
State	*Elapsed Time (minutes)*	*Predicted Time to Go*	*Predicted Total Time*
leaving office, friday at 6	0	30	30
reach car, raining	5	35	40
exiting highway	20	15	35
2ndary road, behind truck	30	10	40
entering home street	40	3	43
arrive home	43	0	43

Example: Driving Home

Changes recommended by Monte Carlo methods (α=1)



Changes recommended by TD methods (α=1)



Advantages of TD Learning

- TD methods do not require a model of the environment, only experience

- TD, but not MC, methods can be fully incremental

- You can learn before knowing the final outcome
    - Less memory
    - Less computation
- You can learn without the final outcome
    - From incomplete sequences

- Both MC and TD converge (under certain assumptions to be detailed later), but which is faster?

Bias-Variance Trade-Off

- Monte-Carlo: Update value $V(S_t)$ toward actual return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha\,(G_t - V(S_t))$$

- Return $G_t = R_{t+1} + \gamma R_{t+2} + \ldots + \gamma^{T-1} R_T$ is unbiased estimate of $v_\pi(S_t)$

- TD: Update value V(St) toward estimated returns $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha\,(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

- True TD target: $R_{t+1} + \gamma v_\pi(S_{t+1})$ is unbiased estimate of $v_\pi(S_t)$

- TD target: $R_{t+1} + \gamma V(S_{t+1})$ is biased estimate of $v_\pi(S_t)$.

- TD target is much lower variance than the return:
  - Return depends on many random actions, transitions, rewards
  - TD target depends on one random action, transition, reward

Bias-Variance Trade-Off

- MC has high variance, zero bias
  - Good convergence properties
  - Even with function approximation
  - Not very sensitive to initial value
  - Very simple to understand and use
- TD has low variance, some bias
  - Good Usually more efficient than MC
  - TD(0) converges to $v_\pi(s)$
  - More sensitive to initial value

Batch Updating in TD and MC methods

- Batch Updating: train completely on a finite amount of data,
  - e.g., train repeatedly on 10 episodes until convergence.
- Compute updates according to TD or MC, but only update estimates after each complete pass through the data.
- For any finite Markov prediction task, under batch updating, TD converges for sufficiently small α.
- Constant-α MC also converges under these conditions, but may converge to a different answer.

AB Example

- Suppose you observe the following 8 episodes:

A, 0, B, 0
B, 1
B, 1
B, 1
B, 1
B, 1
B, 1
B, 0

$V(\mathrm{B})?$ 0.75

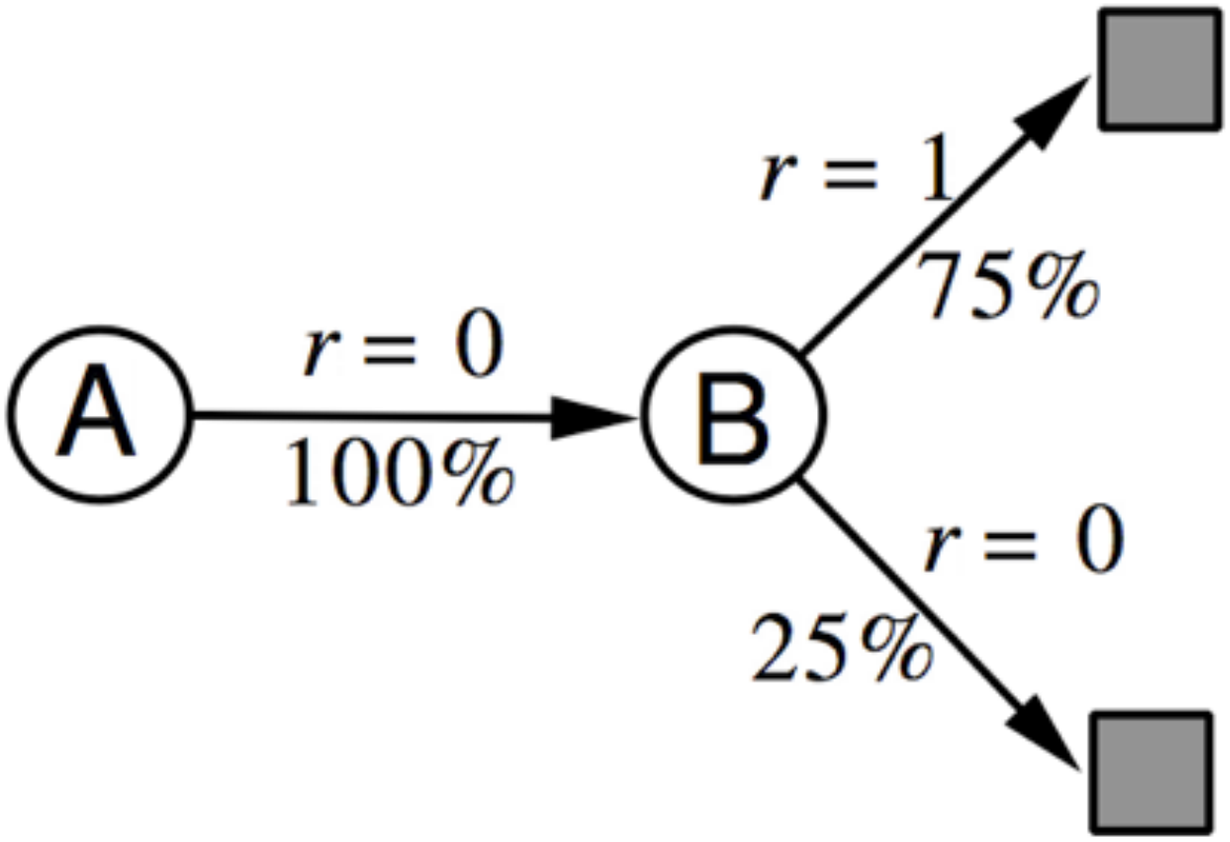
$V(\mathrm{A})?$ 0

- Assume Markov states, no discounting ($\gamma = 1$)

AB Example

- The prediction that best matches the training data is V(A)=0
  - This minimizes the mean-square-error on the training set
  - This is what a batch Monte Carlo method gets

AB Example



$V(\mathrm{A})?$ 0.75

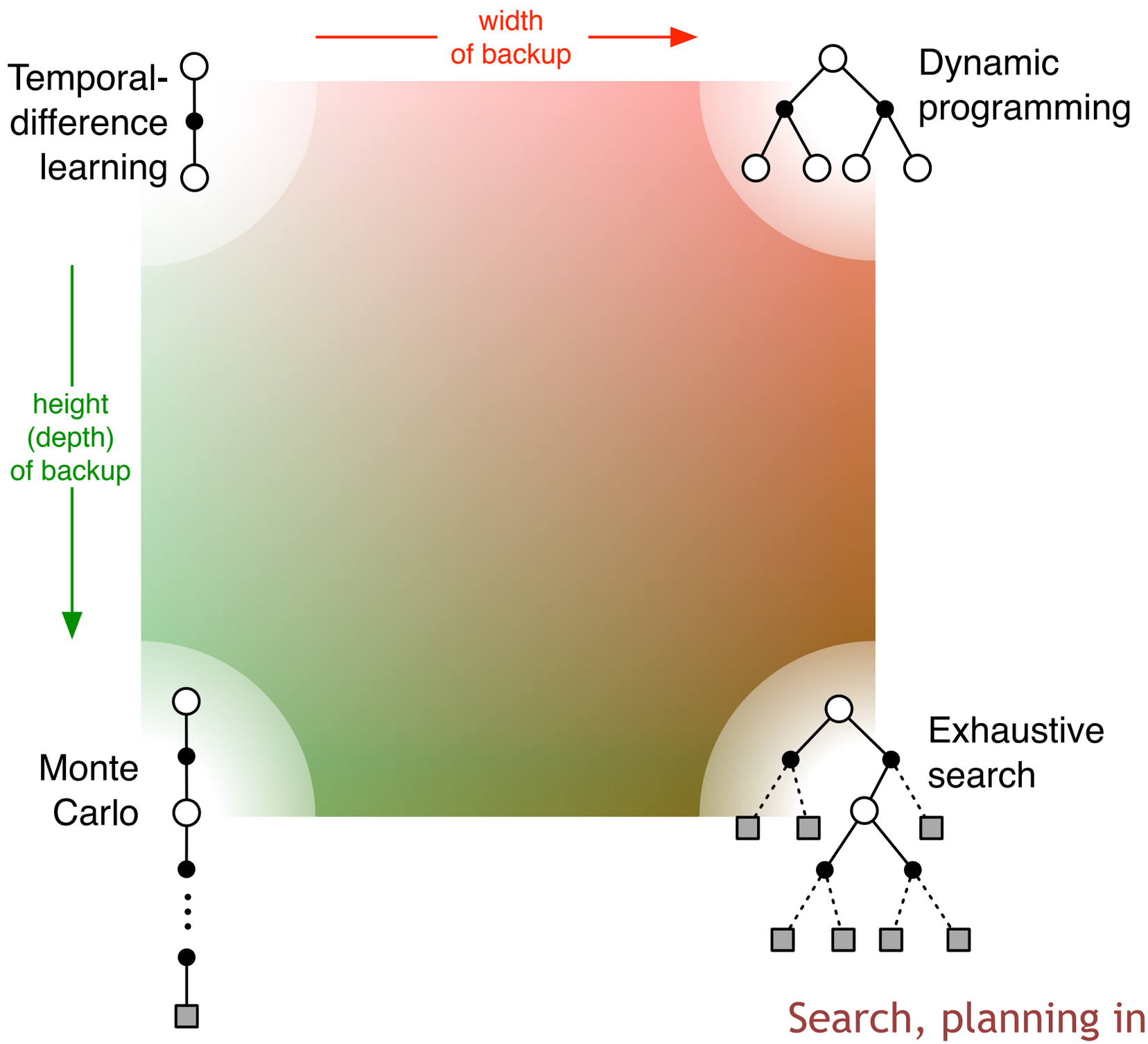
AB Example

- The prediction that best matches the training data is V(A)=0
  - This minimizes the mean-square-error on the training set
  - This is what a batch Monte Carlo method gets
- If we consider the sequentiality of the problem, then we would set V(A)=.75
  - This is correct for the maximum likelihood estimate of a Markov model generating the data
  - i.e, if we do a best fit Markov model, and assume it is exactly correct, and then compute what it predicts.
  - This is called the certainty-equivalence estimate
  - This is what TD gets

Summary so far

- Introduced one-step tabular model-free TD methods
- These methods bootstrap and sample, combining aspects of DP and MC methods

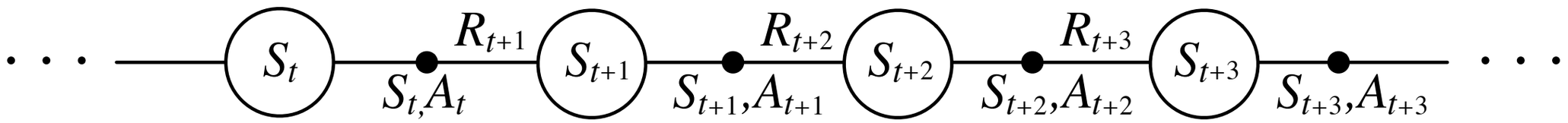
Unified View



Search, planning in a later lecture!

Learning An Action-Value Function

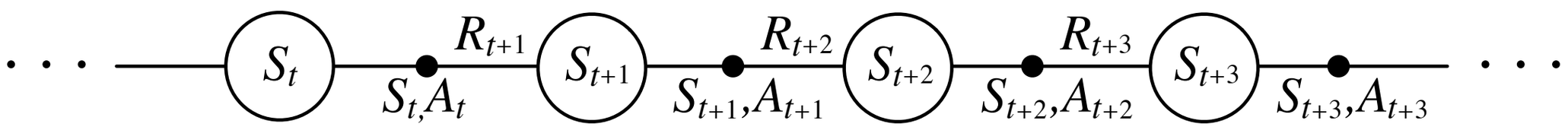
- Estimate q_π for the current policy π



- Can we come up with the TD update equation for Q values?

Learning An Action-Value Function

- Estimate q_π for the current policy π



After every transition from a nonterminal state, S_t, do this:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\left[R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t)\right]$$

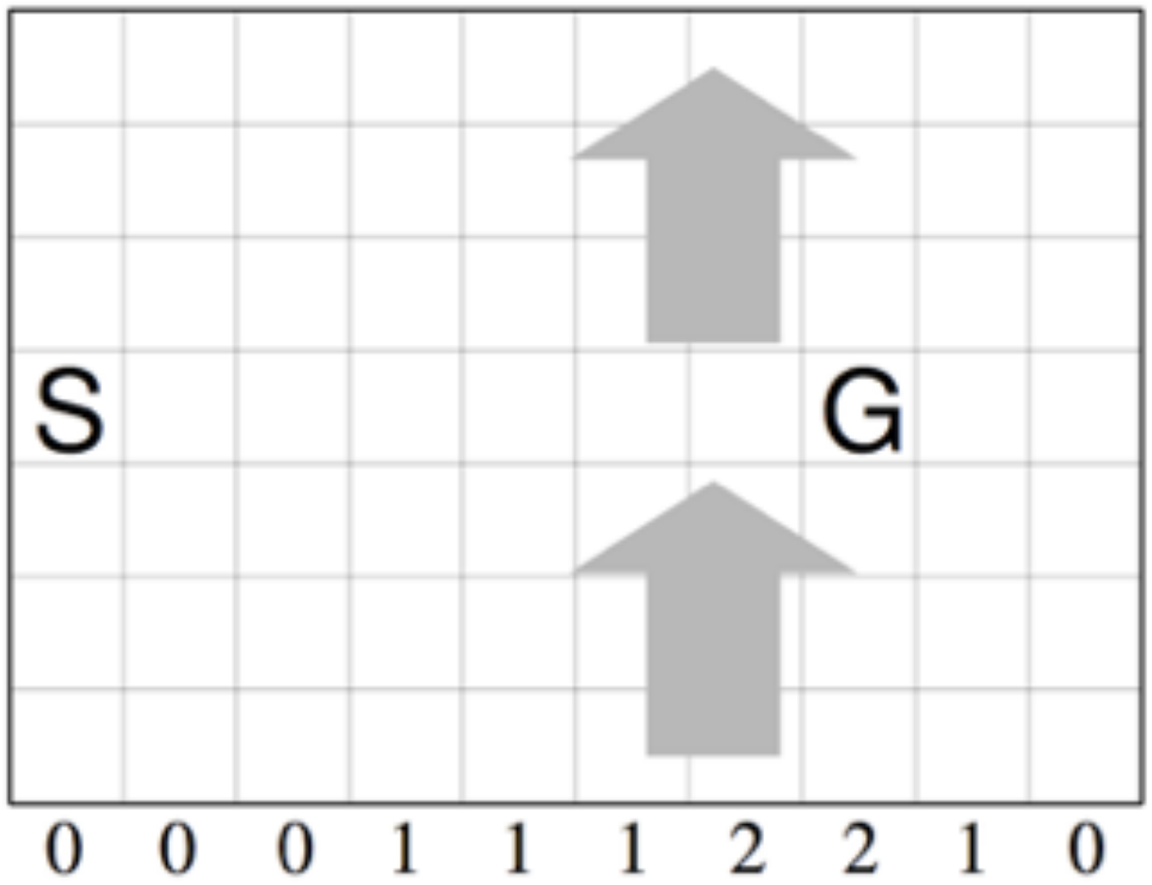
If S_{t+1} is terminal, then define $Q(S_{t+1}, A_{t+1}) = 0$

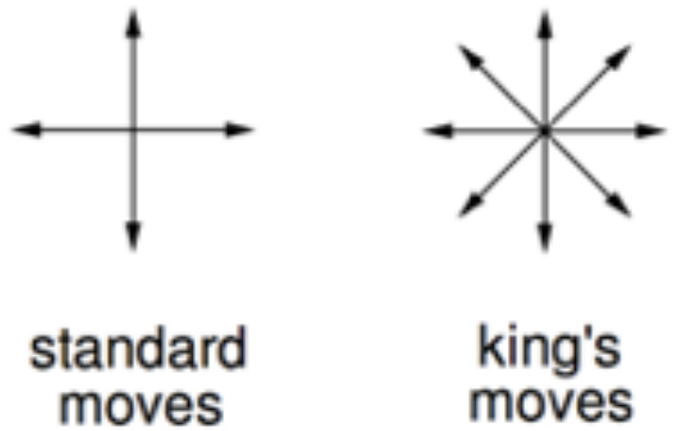
Sarsa: On-Policy TD Control

- Turn this into a control method by always updating the policy to be greedy with respect to the current estimate:

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

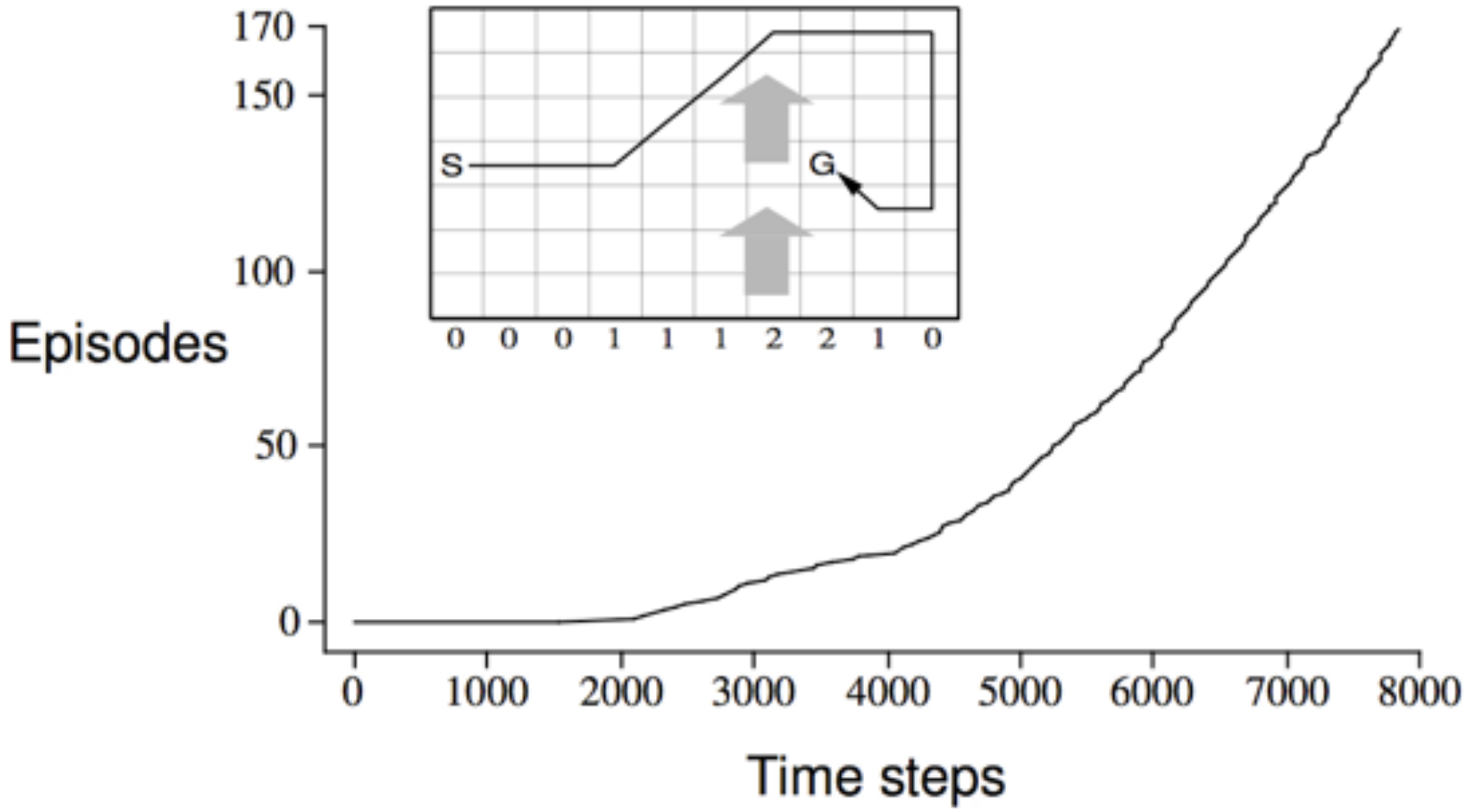
Windy Gridworld





- undiscounted, episodic, reward = −1 until goal

Results of Sarsa on the Windy Gridworld



Q: Can a policy result in infinite loops? What will MC policy iteration do then?

- If the policy leads to infinite loop states, MC control will get trapped as the episode will not terminate.
- Instead, TD control can update continually the state-action values and switch to a different policy.

Q-Learning: Off-Policy TD Control

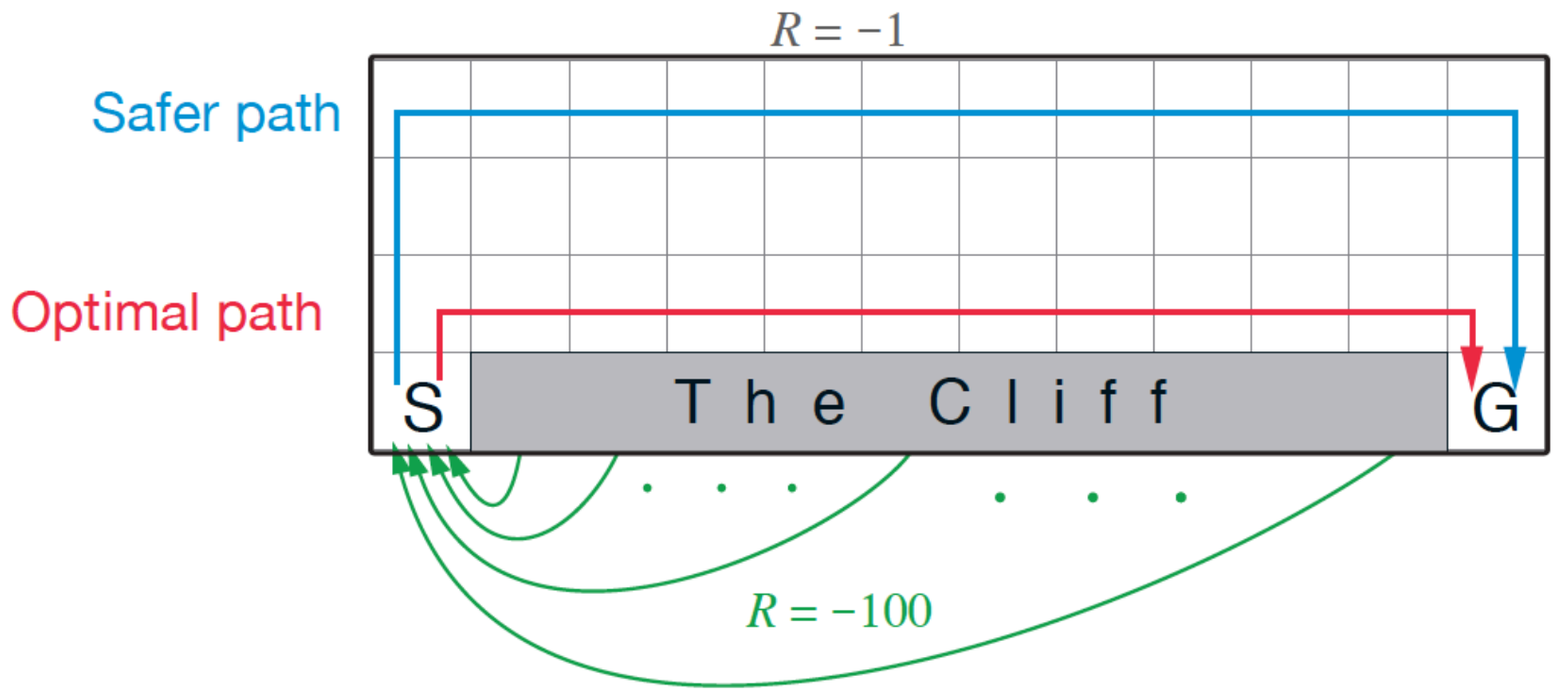
- One-step Q-learning:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha\Big[R_{t+1} + \gamma \max_a Q(S_{t+1}, a) - Q(S_t, A_t)\Big]$$

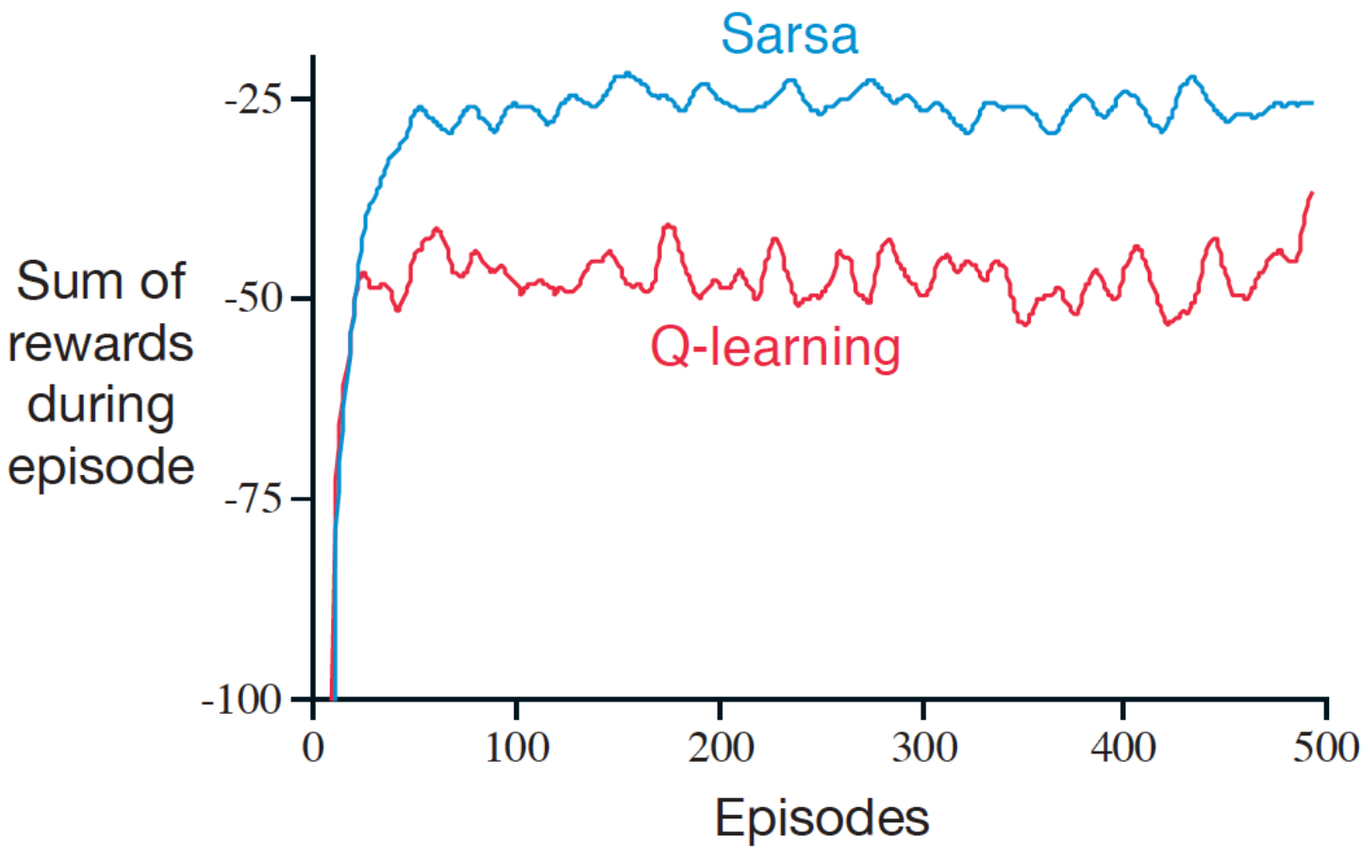
Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma \max_a Q(S',a) - Q(S,A)]$
    $S \leftarrow S'$;
  until S is terminal

Remember SARSA: $Q(S,A) \leftarrow Q(S,A) + \alpha[R + \gamma Q(S',A') - Q(S,A)]$

Cliffwalking

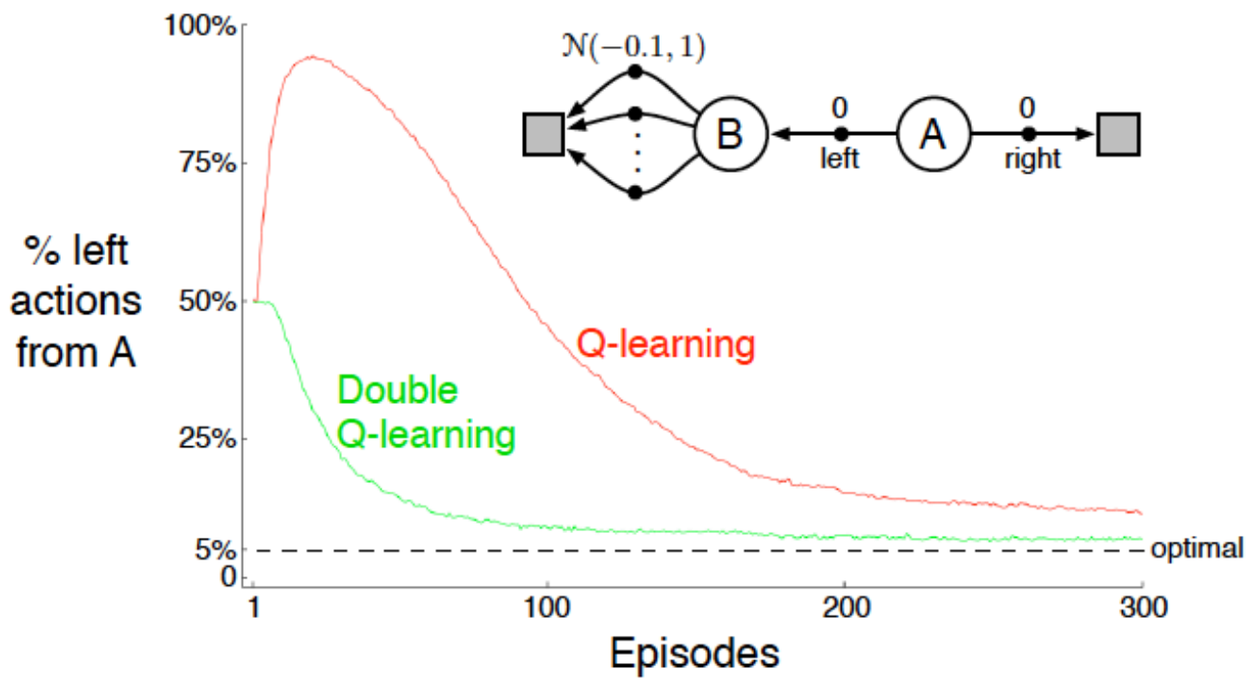


ϵ – greedy, $\epsilon = 0.1$



Maximization Bias

- We often need to maximize over our value estimates. The estimated maxima suffer from maximization bias
- Consider a state for which all ground-truth $q_*(s, a) = 0$. Our estimates $Q(s, a)$ are uncertain, some are positive and some negative.
- $Q(s, \text{argmax}_a Q(s, a)) > 0$ while $q_*(s, \text{argmax}_a q_*(s, a)) = 0$.



Double Q-Learning

- Train 2 action-value functions, Q1 and Q2
- Do Q-learning on both, but
  - never on the same time steps (Q1 and Q2 are independent)
  - pick Q1 or Q2 at random to be updated on each step
- If updating Q1, use Q2 for the value of the next state:

$$Q_1(S_t, A_t) \leftarrow Q_1(S_t, A_t) + \\ + \alpha\Big(R_{t+1} + Q_2\big(S_{t+1}, \operatorname*{arg\,max}_a Q_1(S_{t+1}, a)\big) - Q_1(S_t, A_t)\Big)$$

- Action selections are ε-greedy with respect to the sum of Q1 and Q2

Double Tabular Q-Learning

Initialize $Q_1(s,a)$ and $Q_2(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily
Initialize $Q_1(\textit{terminal-state},\cdot) = Q_2(\textit{terminal-state},\cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q_1 and Q_2 (e.g., ε-greedy in $Q_1 + Q_2$)
    Take action A, observe R, S'
    With 0.5 probabilility:

$$Q_1(S,A) \leftarrow Q_1(S,A) + \alpha\Big(R + \gamma Q_2\big(S', \operatorname{argmax}_a Q_1(S',a)\big) - Q_1(S,A)\Big)$$

    else:

$$Q_2(S,A) \leftarrow Q_2(S,A) + \alpha\Big(R + \gamma Q_1\big(S', \operatorname{argmax}_a Q_2(S',a)\big) - Q_2(S,A)\Big)$$

    $S \leftarrow S'$;
  until S is terminal

Hado van Hasselt 2010

Expected Sarsa

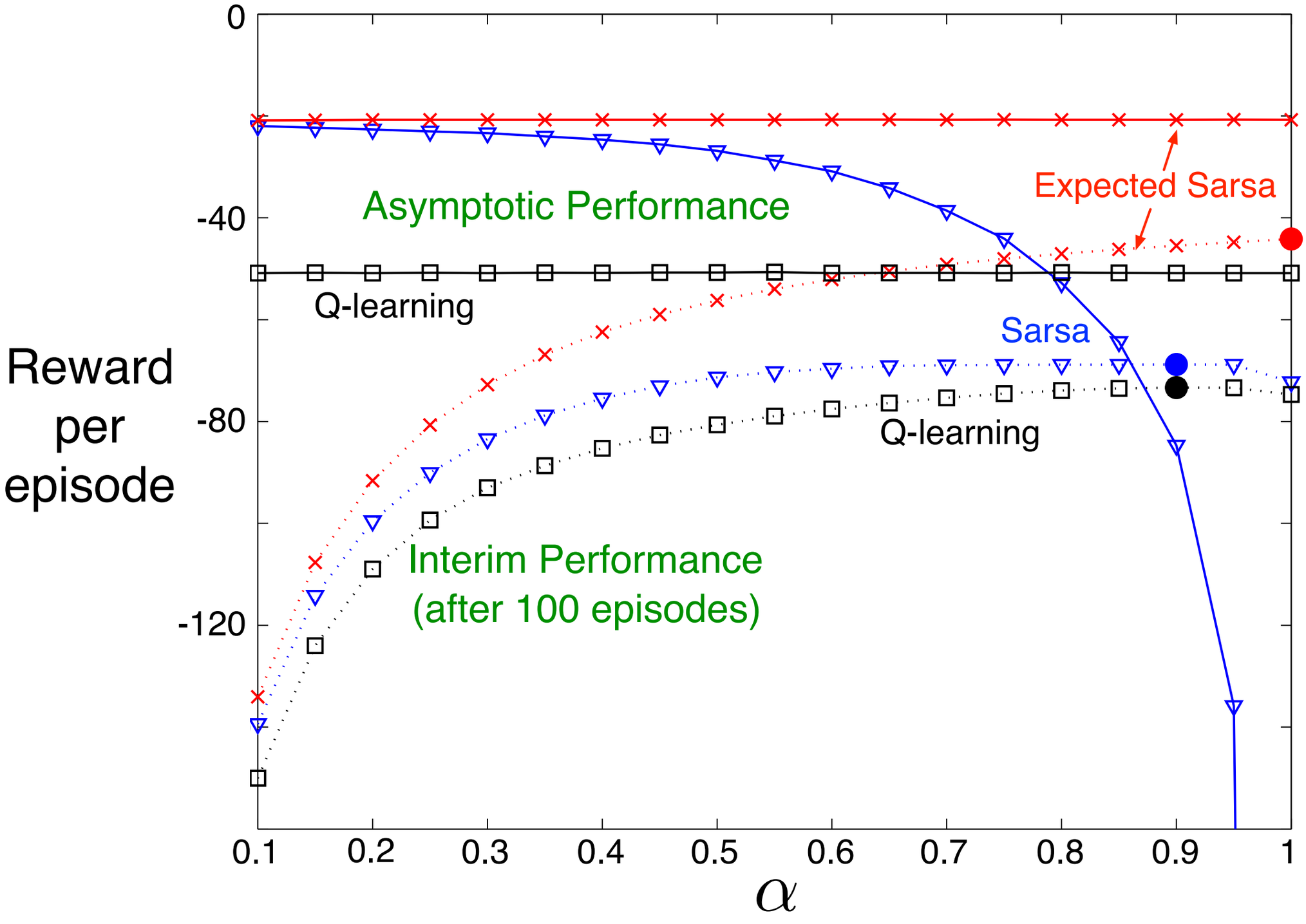
- Instead of the sample value-of-next-state, use the expectation!

$$
\begin{aligned}
Q(S_t, A_t) &\leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \mathbb{E}[Q(S_{t+1}, A_{t+1}) \mid S_{t+1}] - Q(S_t, A_t) \Big] \\
&\leftarrow Q(S_t, A_t) + \alpha \Big[R_{t+1} + \gamma \sum \pi(a|S_{t+1}) Q(S_{t+1}, a) - Q(S_t, A_t) \Big]
\end{aligned}
$$

- Expected Sarsa performs better than Sarsa (but costs more)
  - Q: why?

Q: Is expected SARSA on policy or off policy?
What if π is the greedy deterministic policy?

Performance on the Cliff-walking Task



Summary

- Introduced one-step tabular model-free TD methods
- These methods bootstrap and sample, combining aspects of DP and MC methods
- TD methods are computationally congenial

- Extend prediction to control by employing some form of GPI
  - On-policy control: Sarsa
  - Off-policy control: Q-learning