Carnegie Mellon

School of Computer Science

Deep Reinforcement Learning and Control

Monte Carlo Tree Search

Spring 2020, CMU 10-403

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Part of slides inspired by Sebag, Gaudel

Definitions

Learning: the acquisition of knowledge or skills through experience, study, or by being taught.

Planning: any computational process that uses a model to create or improve a policy

What is Online Planning?

Unroll the model of the environment forward in time to select the right action sequences to achieve your goal.

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current state

goal state

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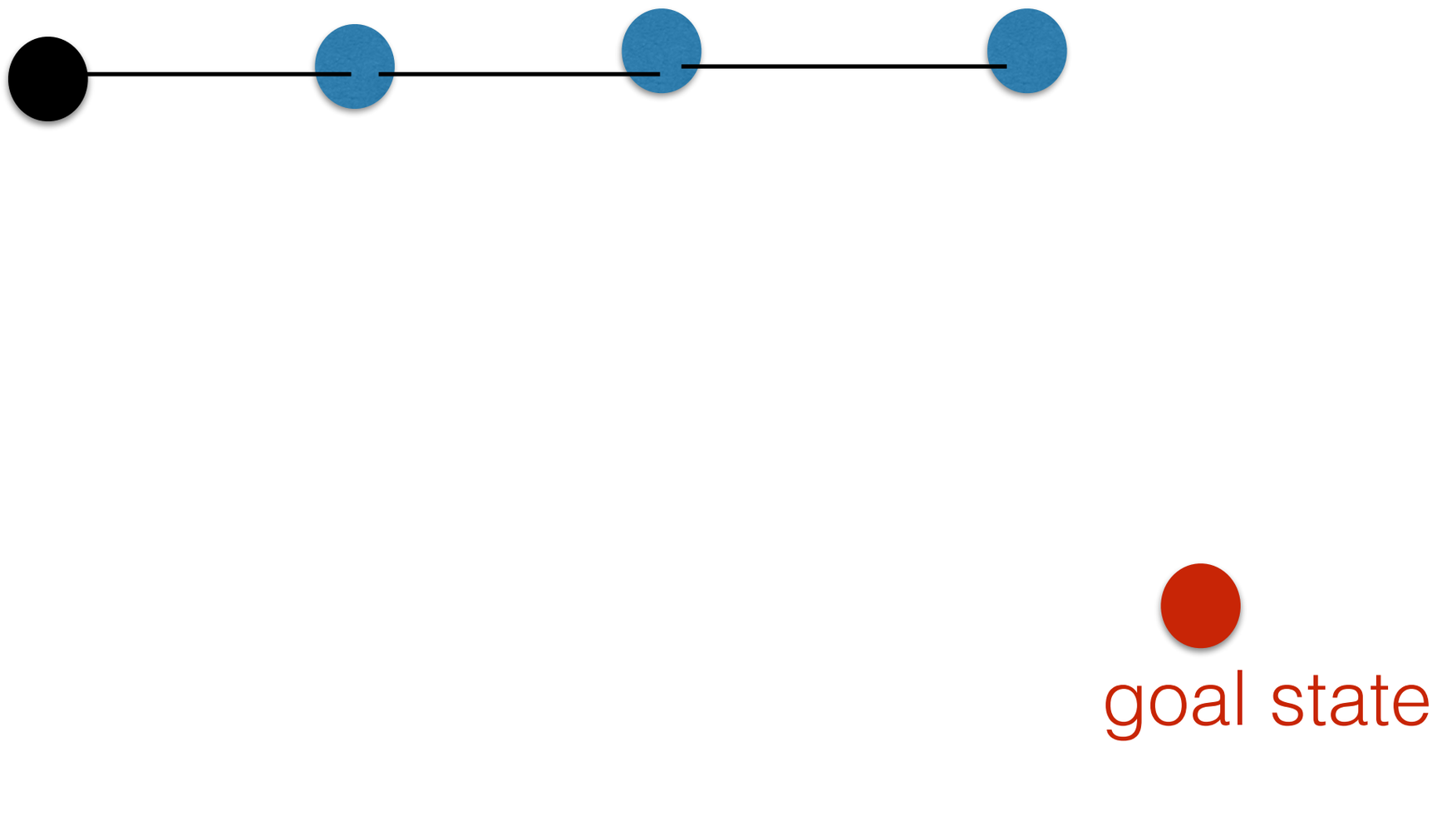

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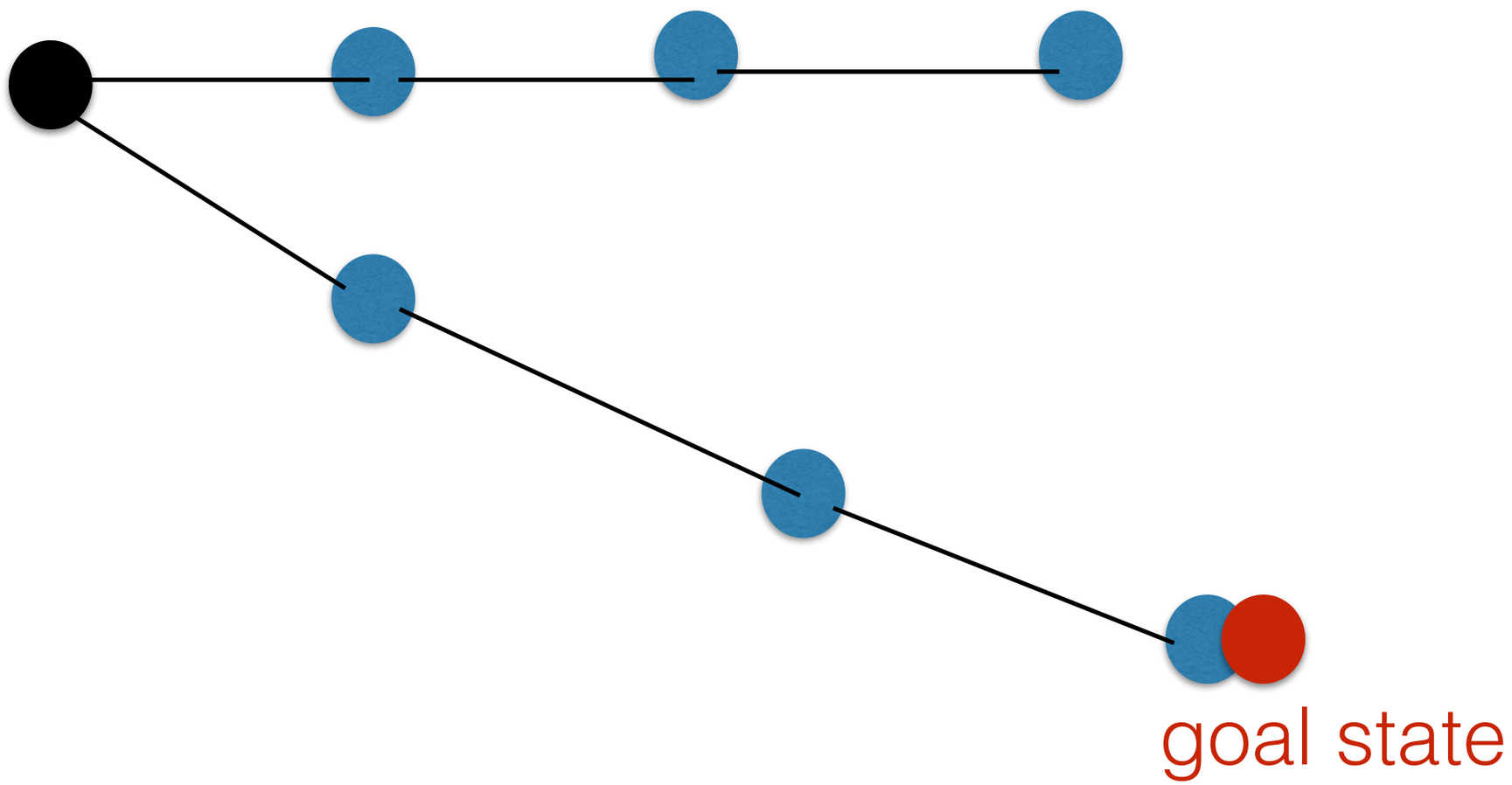


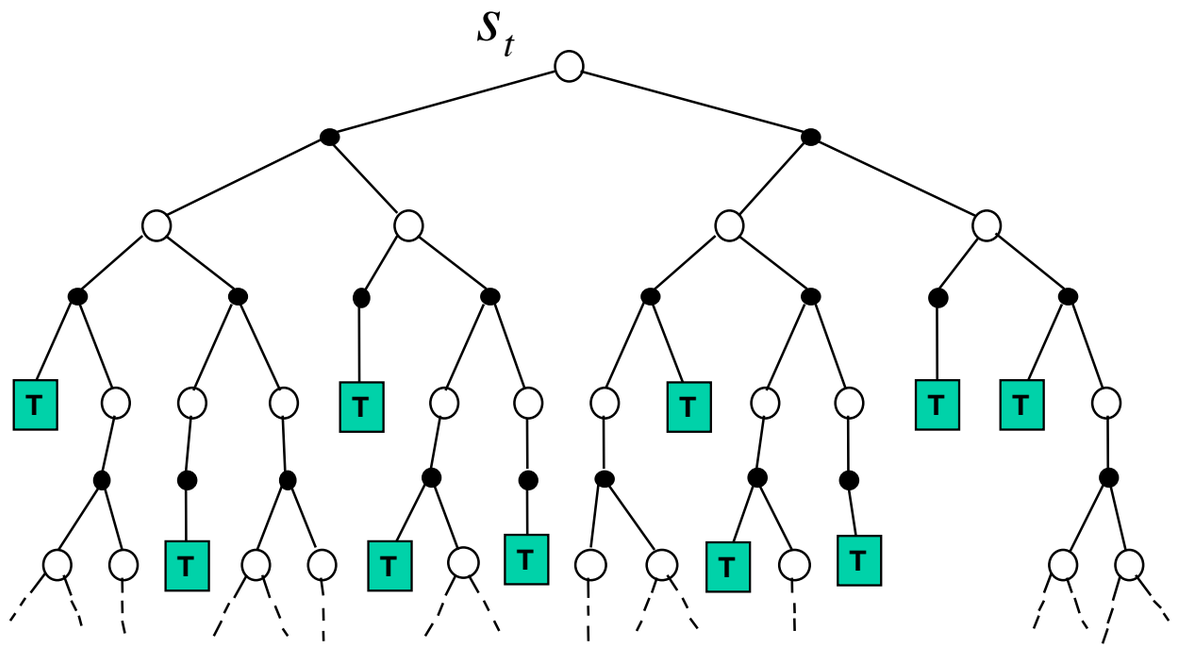
Why online planning?

Why don't we *just* learn a value function directly for every state offline, so that we do not waste time online?

- Because the environment has many many states (consider Go 10^170, Chess 10^48, real world ….)
- Very hard to compute a good value function for each one of them, most of them you will never visit at training time.
- Thus, condition on the current state you are in, try to estimate the value function of the relevant part of the state space online.
- Focus your resources on sub-MDP starting from now, often dramatically easier than solving the whole MDP.

Online Planning with Search

1. Build the full search tree **with the current state of the agent** at the root
2. Select the next move to execute using heuristics
3. Execute it
4. GOTO 1



Curse of dimensionality

The sub-MDP rooted at the current state the agent is in may still be very large (too many states are reachable), despite much smaller than the original one.

Too many actions possible: large tree branching factor

Too many steps: large tree depth

I cannot exhaustively search the full tree

Curse of dimensionality

Consider hex on an NxN board.

branching factor ≤ N^2

$2N \leq \text{depth} \leq N^2$

board size	max branching factor	min depth	tree size	depth of 10^{10} nodes
6x6	36	12	$>10^{17}$	7
8x8	64	16	$>10^{28}$	6
11x11	121	22	$>10^{44}$	5
19x19	361	38	$>10^{96}$	4

Goal of HEX: to make a connected line that links two antipodal points of the grid

How to handle the curse of dimensionality?

Intelligent instead of exhaustive search

The depth of the search may be reduced by position evaluation: truncating the search tree at state s and replacing the subtree below s by an approximate value function $v(s) = v*(s)$that predicts the outcome from state s.

The breadth of the search may be reduced by sampling actions from a policy $p(a \mid s)$, that is, a probability distribution over plausible moves a in position s, instead of trying every action.

Position evaluation

We can estimate values for states in two ways:

- Engineering them using human experts (DeepBlue)
- Learning them from self-play (TD-gammon)

Problems with human engineering:

- tiring
- non transferrable to other domains.

YET: that's how Kasparov was first beaten.

http://stanford.edu/~cpiech/cs221/apps/deepBlue.html

Monte-Carlo position evaluation

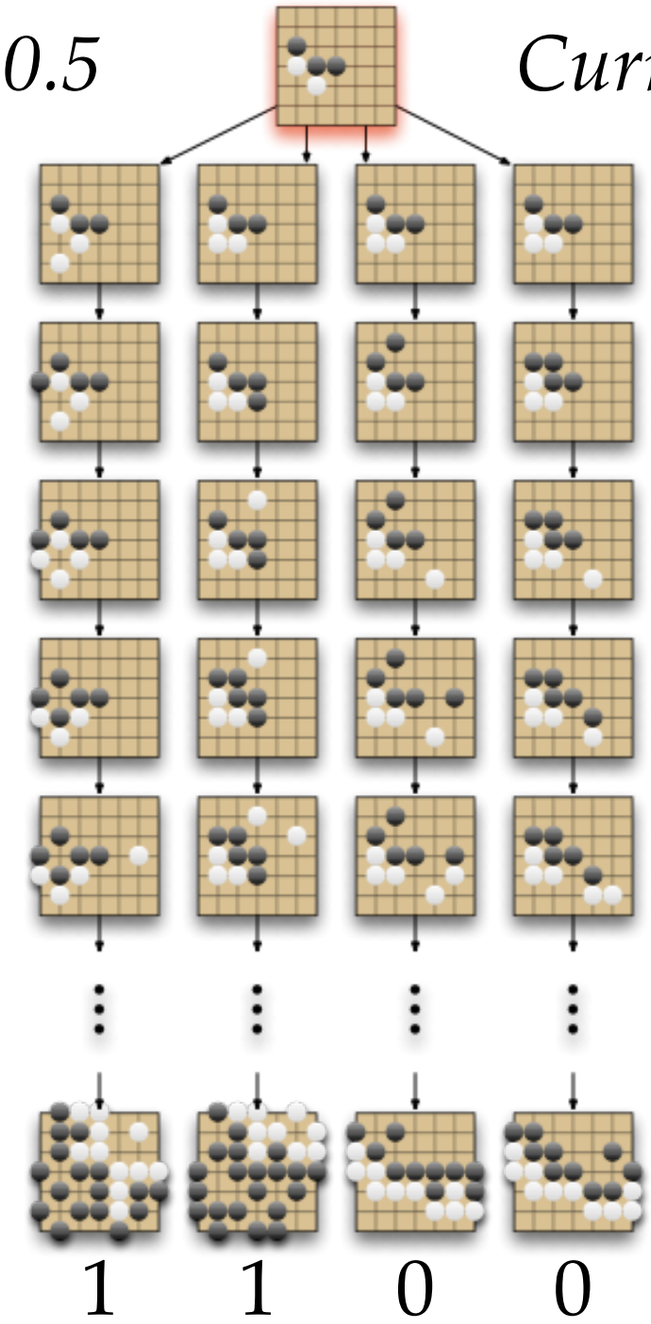
```
function MC_BoardEval(state):
    wins = 0
    losses = 0
    for i=1:NUM_SAMPLES
        next_state = state
        while non_terminal(next_state):
            next_state = random_legal_move(next_state)
        if next_state.winner == state.turn: wins++
        else: losses++ #needs slight modification if draws possible
return (wins - losses) / (wins + losses)
```

What policy shall we use to draw our simulations?

The cheapest one is random..

Monte-Carlo position evaluation in Go

$V(s) = 2/4 = 0.5$



Current position s

Simulation

1 1 0 0

Outcomes

Averaging sampled returns..

Simplest Monte-Carlo Search

- For action selection, I need to be estimating not state but rather state-action values.
- But! Since we assume dynamics given, we can simply use one step look-ahead!

Simplest Monte-Carlo Search

Given a deterministic transition function T, a root state s and a simulation policy π (potentially random)

Simulate K episodes from current (real) state:

$$\{s, a, R_1^k, S_1^k, A_1^k, R_2^k, S_2^k, A_2^k, \ldots, S_T^k\}_{k=1}^K \sim \mathrm{T}, \pi$$

Evaluate action value function of the root by mean return:

$$Q(s,a) = \frac{1}{K} \sum_{k=1}^{K} G_k \to q_\pi(s,a)$$

Select root action: $a = \mathrm{argmax}_{a \in \mathcal{A}} Q(s,a)$

Simplest Monte-Carlo Search

Given a deterministic transition function T, a root state s and a simulation policy π (potentially random)

For each action $a \in \mathscr{A}$

$$Q(s,a) = \text{MC-boardEval}(s'), \quad s' = \mathrm{T}(s,a)$$

Select root action: $a = \mathrm{argmax}_{a \in \mathscr{A}} Q(s,a)$

Can we do better?

- Could we be improving our simulation policy the more simulations we obtain?
- Yes we can! We can have two policies:
  - Internal to the tree: keep track of action values Q not only for the root but also for nodes internal to a tree we are expanding, and use to improve the simulation policy over time
  - External to the tree: we do not have Q estimates and thus we use a random policy

In MCTS, the simulation policy improves

- Can we think anything better than ϵ — greedy?

Upper Confidence Bound (UCB)

$$A_t \sim \left[Q_t(a) + c\sqrt{\frac{\log t}{N_t(a)}} \right]$$

- t : parent node visits
- $N_t(a)$: times the action has been tried out

- Probability of choosing an action:
  - decreases with the number of visits (explore)
  - increases with a node's value (exploit)
- Always tries every option once.

- A better exploration-exploitation than ϵ − greedy

Finite-time Analysis of the Multiarmed Bandit Problem, *Auer, Cesa-Bianchi, Fischer, 2002*

Monte-Carlo Tree Search

1. **Selection**
    - Used for nodes we have seen before
    - Pick according to UCB
2. **Expansion**
    - Used when we reach the frontier
    - Add one node per playout
3. **Simulation**
    - Used beyond the search frontier
    - Don’t bother with UCB, just play randomly
4. **Back-propagation**
    - After reaching a terminal node
    - Update value and visits for states expanded in selection and expansion

Bandit based Monte-Carlo Planning, *Kocsis and Szepesvari, 2006*

Monte-Carlo Tree Search

Basic MCTS pseudocode

```
function MCTS_sample(state)
    state.visits++
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    else:
        next_state = state
    winner = random_playout(next_state)
update_value(state, winner)
```

For every state within the search tree we bookkeep # of visits and # of wins

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Monte-Carlo Tree Search

MCTS helper functions

Sample actions based on UCB score

```
function UCB_sample(state):
    weights = []
    for child of state:
        w = child.value + C * sqrt(ln(state.visits) / child.visits)
        weights.append(w)
    distribution = [w / sum(weights) for w in weights]
    return child sampled according to distribution

function random_playout(state): (unrolling)
    if is_terminal(state):
        return winner
    else: return random_playout(random_move(state))
```

Monte-Carlo Tree Search

MCTS helper functions

```
function expand(state):
    state.visits = 1
    state.value = 0
```

```
function update value(state, winner):

    if winner == state.turn:
        state.value += 1
    else:
        state.value -= 1
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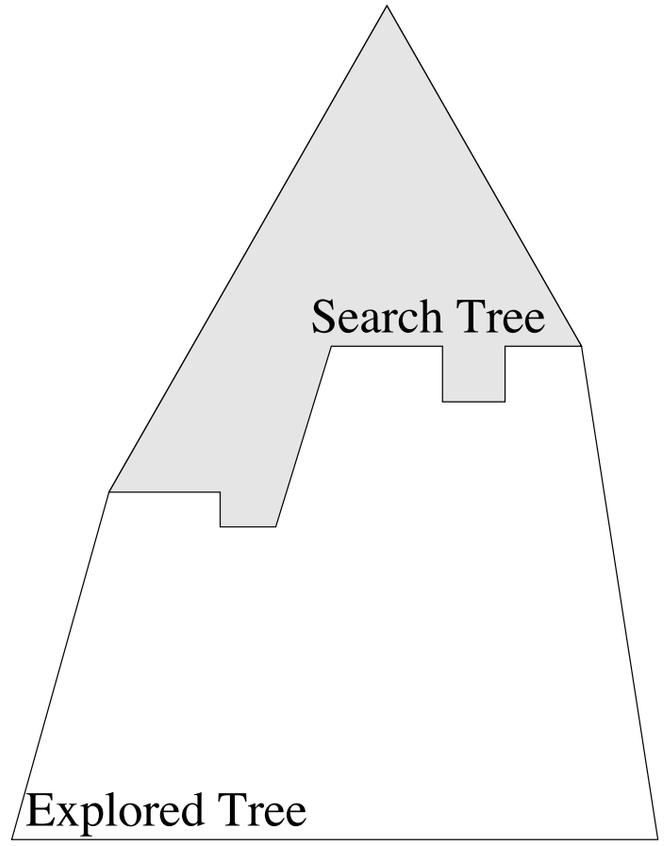

Search tree contains states whose all children have been tried at least once

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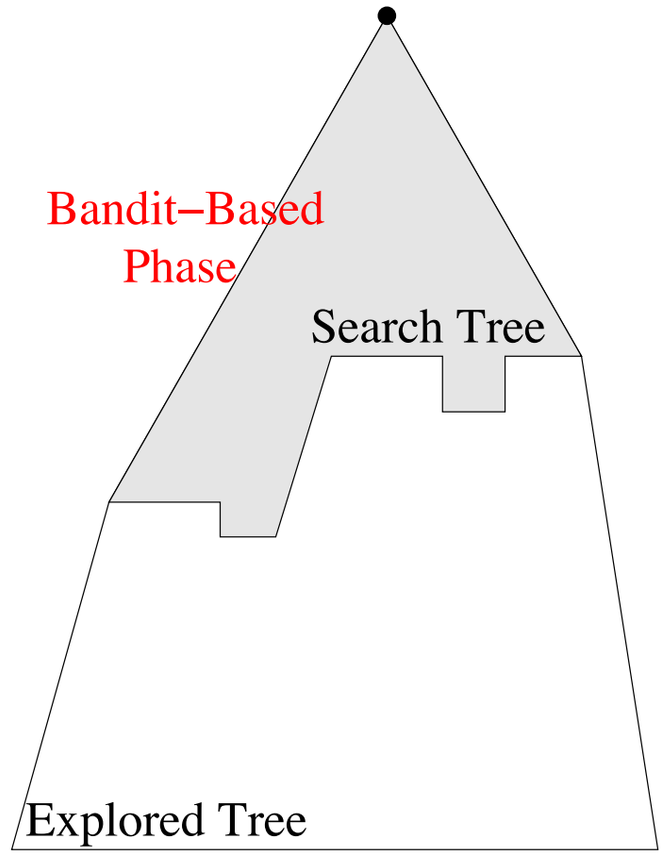

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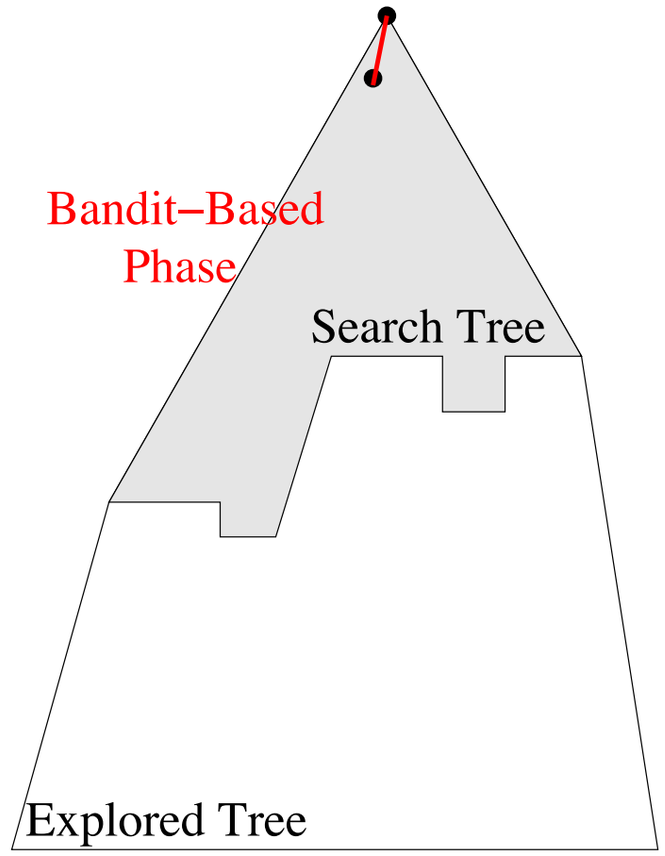

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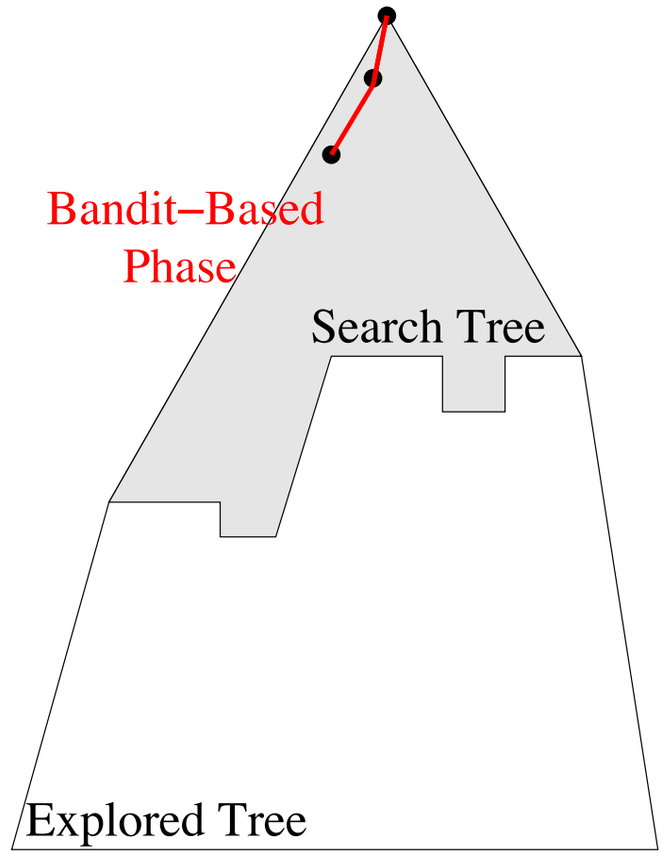

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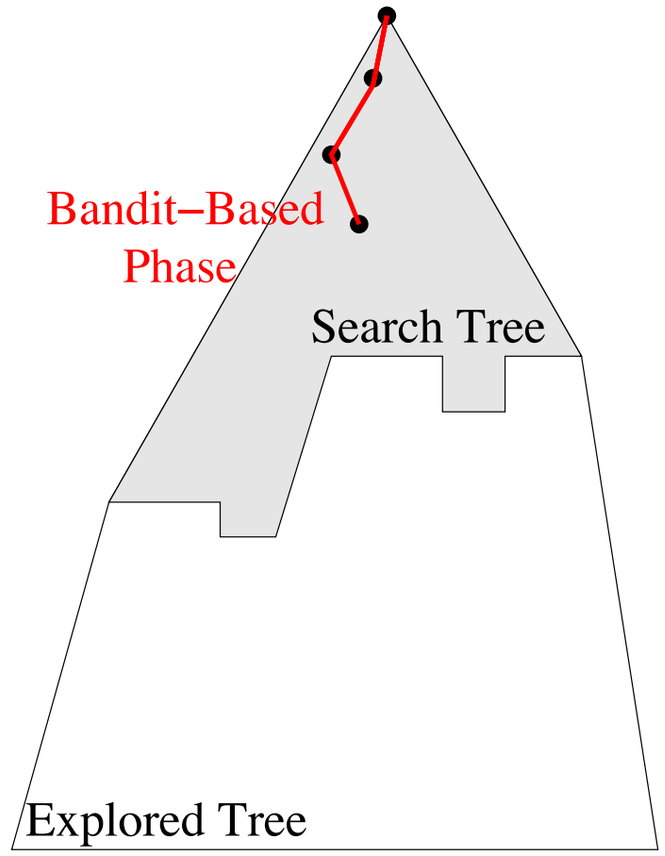

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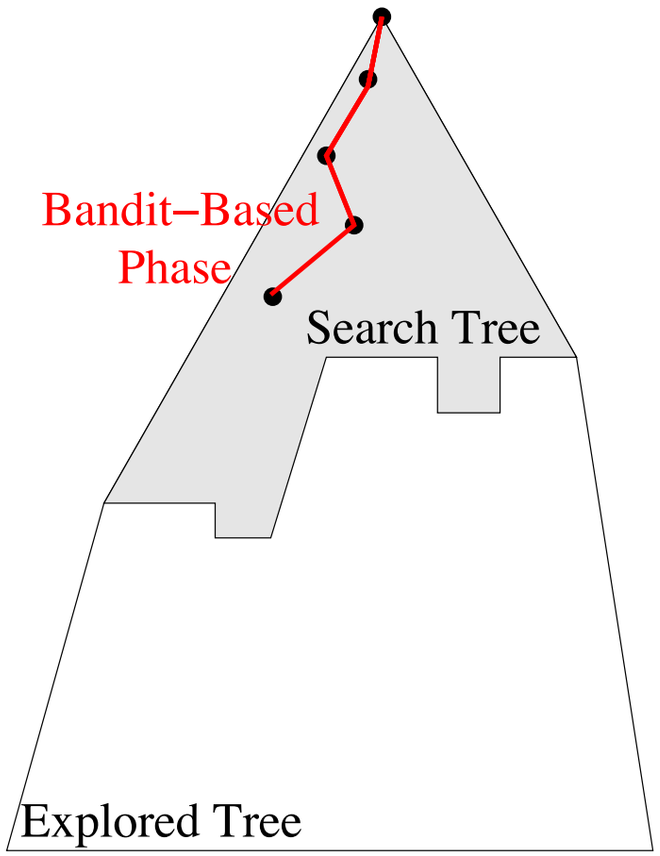

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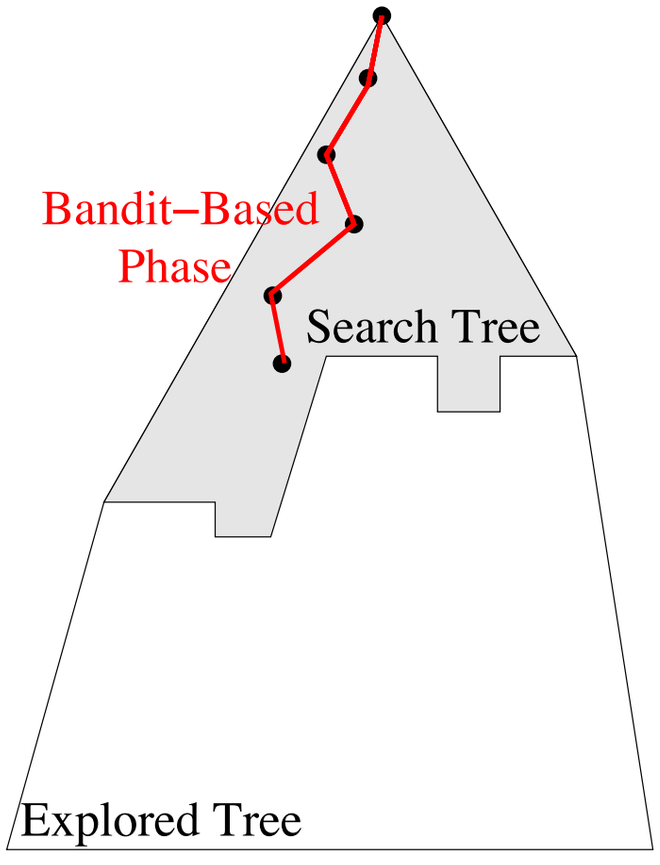

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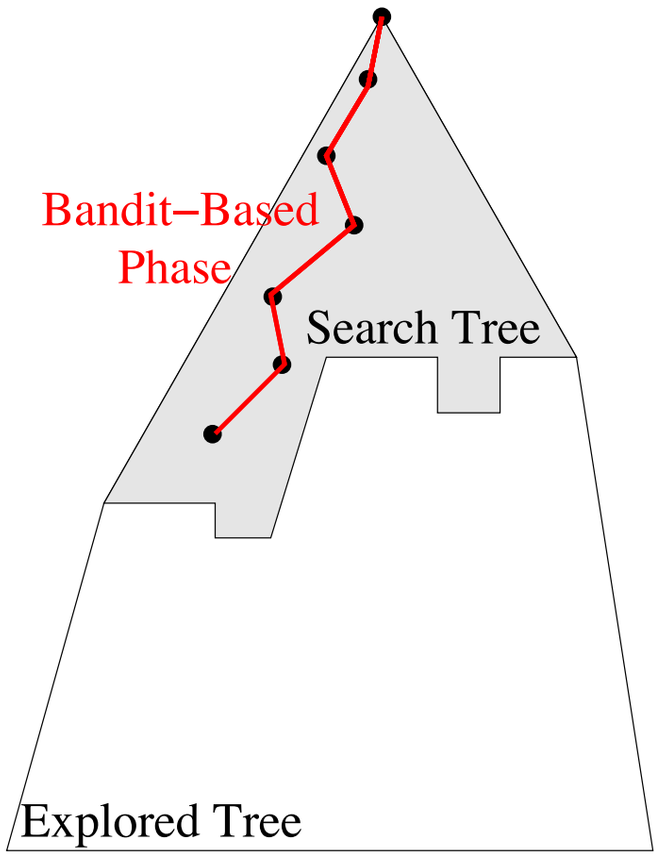

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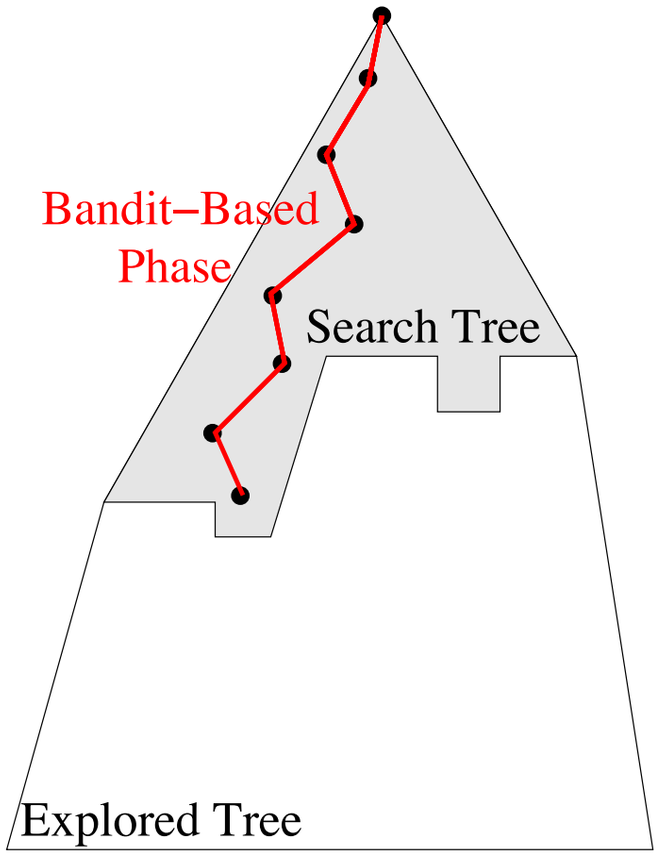

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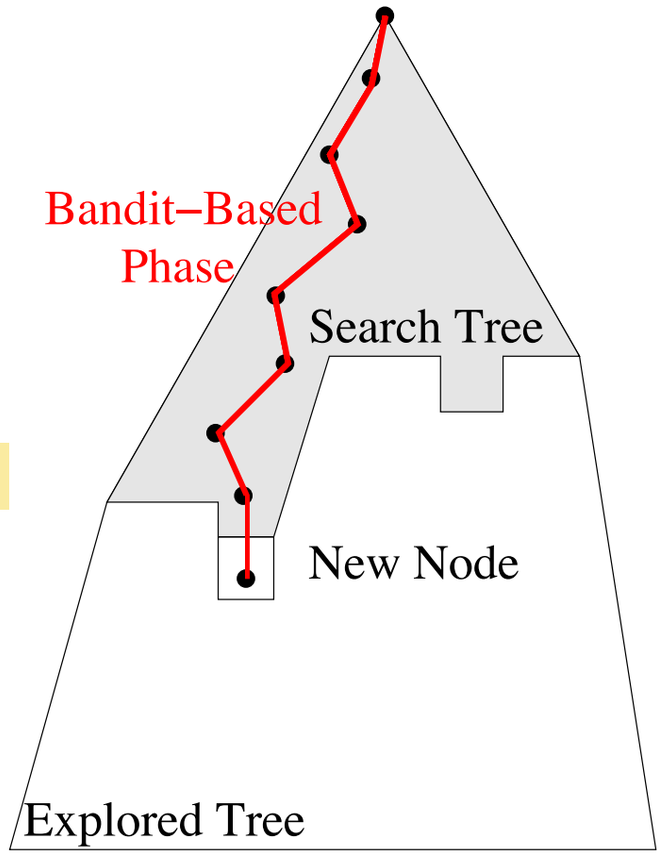

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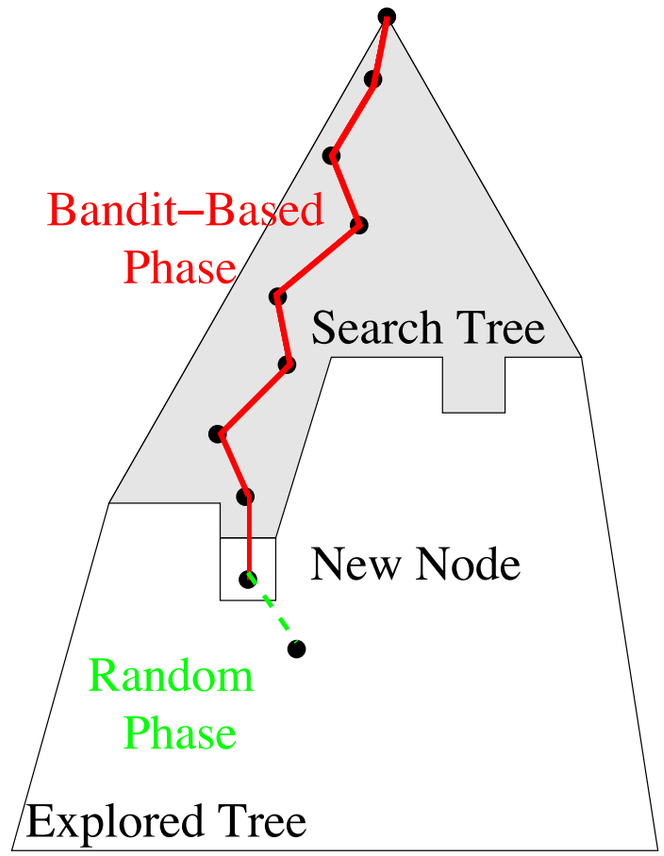

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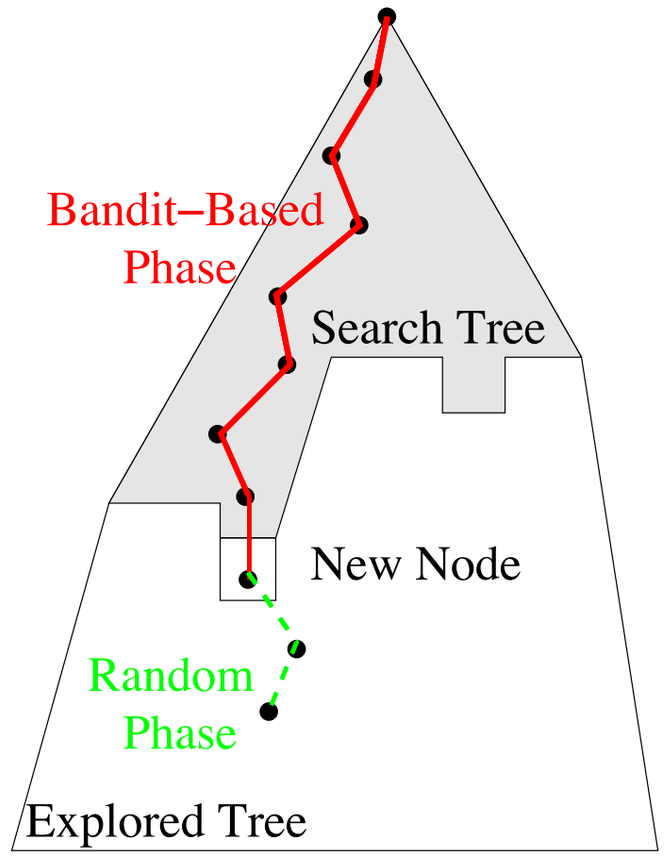

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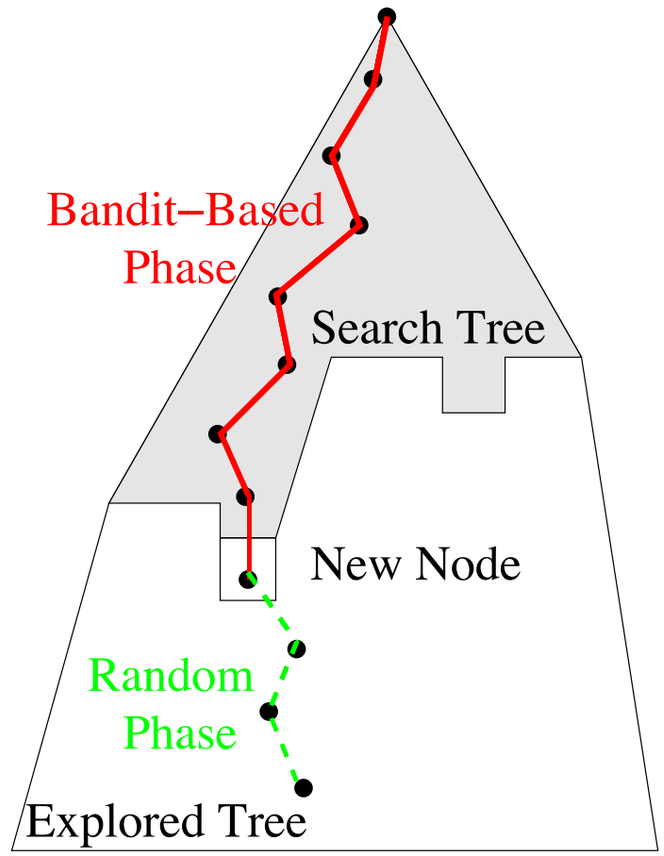


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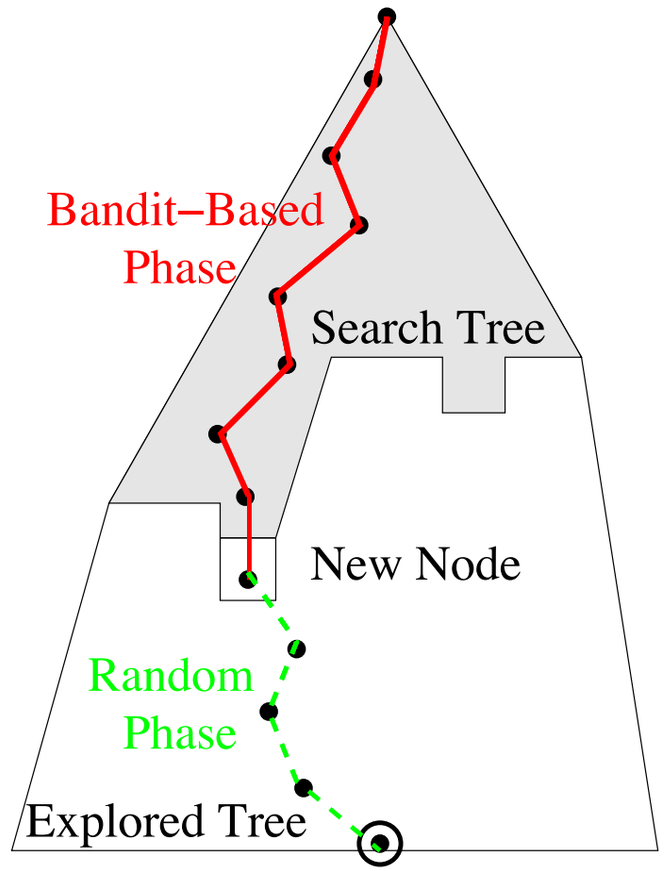


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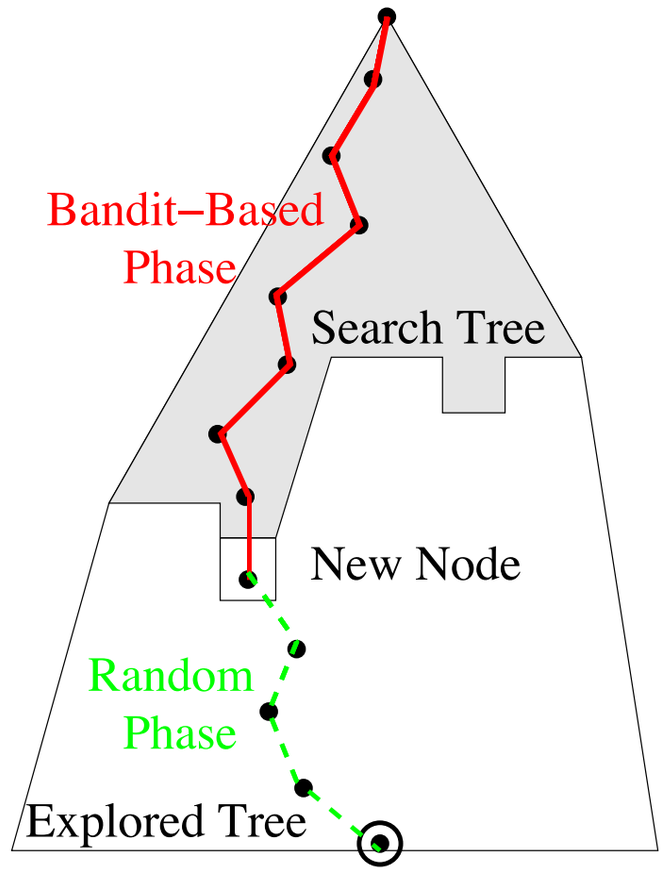
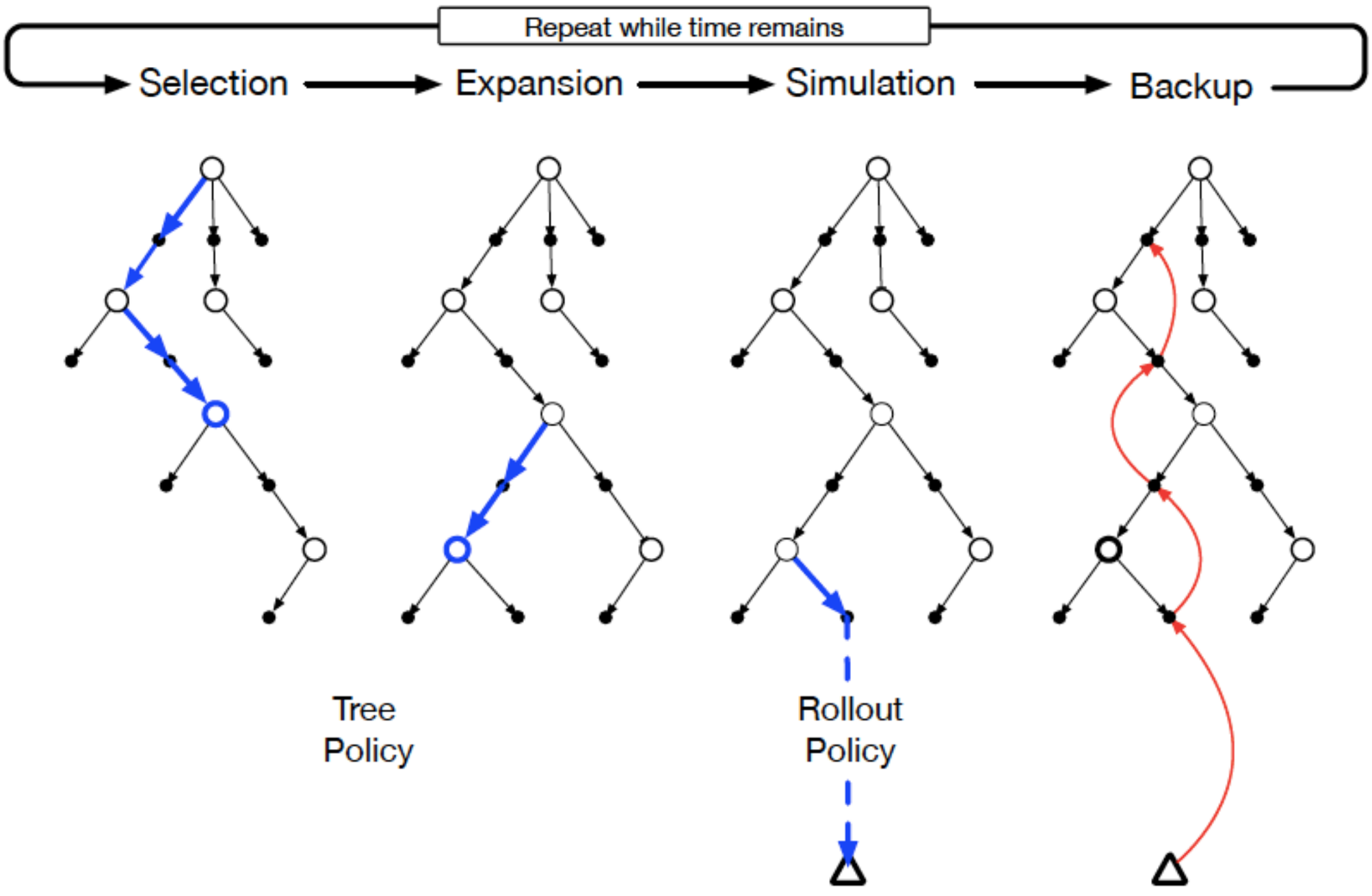


Monte-Carlo Tree Search



Monte-Carlo Tree Search planner

- Estimates action-state values $Q(s, a)$ by look-ahead planning.
- Questions:
  - Are those estimates more or less accurate than those discovered with model-free RL methods, e.g., DQN?
  - Why don’t we simply use MCTS to select actions during playing of Atari games (no prior knowledge)?
  - How can we use the estimates discovered with MCTS but at the same time play fast at test time?

Deep Learning for Real-Time Atari Game Play Using Offline Monte-Carlo Tree Search Planning

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Idea: For state-action value estimation at training time. At test time just use the reactive policy network, without any look-ahead planning. In other words, imitate the MCTS planner.

Learning from offline MCTS

- The MCTS agent plays against himself and generates $(s, a, Q(s, a))$ tuples. Use this data to train:
  - **UCTtoRegression**: A regression network, that given 4 frames regresses to $Q(s, a, w)$for all actions. Select actions using argmax Q.
  - **UCTtoClassification**: A classification network, that given 4 frames predicts the best action.
- Q: Could we use the learned policies to play the game?

Learning from offline MCTS

- The state distribution visited using actions of the MCTS planner will not match the state distribution obtained from the learned policy.
  - **UCTtoClassification-Interleaved**: Interleave UCTtoClassification with data collection:

    1. Start from 200 runs with MCTS.
    2. Train the policy **UCTtoClassification**.
    3. Deploy the policy for 200 runs allowing 5% of the time a random action to be sampled.
    4. Use MCTS to decide best action for those states,
    5. GOTO 2

- At test time, just deploy the learnt policy.

Results

Agent	*B.Rider*	*Breakout*	*Enduro*	*Pong*	*Q*bert*	*Seaquest*	*S.Invaders*
DQN	4092	168	470	20	1952	1705	581
-best	5184	225	661	21	4500	1740	1075
UCC	5342 (20)	175(5.63)	558(14)	19(0.3)	11574(44)	2273(23)	672(5.3)
-best	10514	351	942	21	29725	5100	1200
-greedy	5676	269	692	21	19890	2760	680
UCC-I	5388(4.6)	215(6.69)	601(11)	19(0.14)	13189(35.3)	2701(6.09)	670(4.24)
-best	10732	413	1026	21	29900	6100	910
-greedy	5702	380	741	21	20025	2995	692
UCR	2405(12)	143(6.7)	566(10.2)	19(0.3)	12755(40.7)	1024 (13.8)	441(8.1)

Table 2: Performance (game scores) of the off-line UCT game playing agent.

Agent	*B.Rider*	*Breakout*	*Enduro*	*Pong*	*Q*bert*	*Seaquest*	*S.Invaders*
UCT	7233	406	788	21	18850	3257	2354

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Online planning (without aided by any neural net!) outperforms DQN policy. It takes though "a few days on a recent multicore computer to play for each game".

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Classification is doing much better than regression! indeed, we are training for exactly what we care about.

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Interleaving is important to prevent mismatch between the training data and the data that the trained policy will see at test time.

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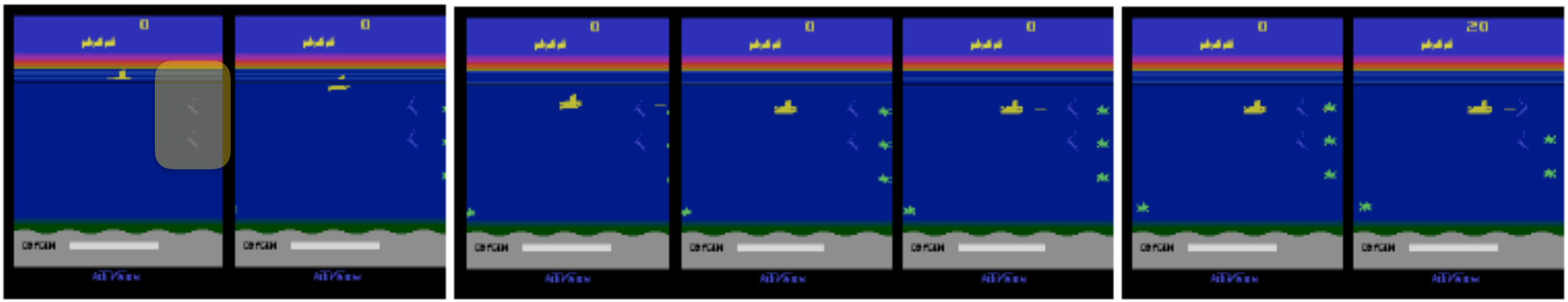
Results improve further if you allow MCTS planner to have more simulations and build more reliable Q estimates.

Problem



Step 69: FIRE | Step 70: DOWN+FIRE | Step 74:DOWN+FIRE | Step 75:RIGHT+FIRE | Step 76:RIGHT+FIRE | Step 78: RIGHT+FIRE | Step 79:DOWN+FIRE

We do not learn to save the divers. Saving 6 divers brings very high reward, but exceeds the depth of our MCTS planner, thus it is ignored.