Technical Proposal
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Title: Provably Near-Optimal Distributed Online Network Optimization

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1 Introduction

As communication devices and sensor infrastructures scale at a rapid rate, it has now become possible to deploy large ad-hoc networks that have the potential to carry out important tasks such as surveillance, environment monitoring, and threat detection. In order to realize this potential, however, we need to design ad hoc network protocols in which nodes communicate without a centralized control and implement complex functions with limited coordination and control. This naturally leads us to the consideration of *distributed* algorithms that use little or no central control.

The lack of control is further exacerbated by the lack of complete information about the environment in which the agents are located, adding an extra layer of uncertainty to the problem. The paucity of information can be modeled either by providing a probabilistic model of the information which leads to *stochastic optimization* models [8, 51], or by an adversarial model of information revelation that leads to competitive analysis in the sense of *online optimization* [11, 30]. When the scenarios of uncertainty are neither quantifiable using randomness, nor adversarial, the framework of *robust optimization* models [1, 19] is also useful.

In this proposal, we develop a variety of models for studying the fundamental tasks of establishing and maintaining connectivity and control in complex networked systems, which capture three important features.

1. Limited communication between agents (necessitating distributed algorithms),

2. Online revelation of information over time (including robust and competitive frameworks), with the input selection ranging from the worst-case adversarial to stochastic choice (leading respectively to online and stochastic optimization algorithms), and

3. Permit the development of rigorous approximation algorithms with a polynomial running time that provide provably good solutions with worst-case performance guarantees.

These three aspects of our model directly address in order the three main characteristics in this BRC topic detailed as follows.

1. Problems with "a high degree of decentralization" and "limited communication between system components",

2. "Not all relevant information of the environment is known a priori, and is revealed incrementally to individual system components", and

3. "The techniques are capable of producing high-quality solutions in a reasonable amount of time" where "solution quality is measured against optimality", and "analysis of these measures is expected to be mathematically rigorous."

1.1 Summary of technical goals

The main goal of this project is to develop a comprehensive theory of ***distributed online approximation*** algorithms for hard network optimization problems. Our framework encompasses three dimensions - locality, uncertainty, and temporality, with two complementary aspects to each dimension, see Figure 1. This gives rise to eight different categories of problems. The hardest problems are those that require *decentralized* solutions when faced with an *adversary* that *adaptively reveals the input over time.*

Different problems have different characteristics when investigated in the context of these different categories, and we hope to develop techniques that apply to several variants of fundamental network optimization problems. One of the performance measures we will adopt for analyzing our algorithms is the standard *competitive ratio*, which is the ratio of the cost of the online solution to that of an optimal offline solution for the worst-case instance [11, 58]. In addition, for certain network optimization problems, we will also quantify the *price of decentralization*, which we define as the worst-case ratio of the cost of the solution achieved by any distributed algorithm that does not perform global communication to that of an optimal centralized solution. In some fortunate circumstances, we can find solution concepts that

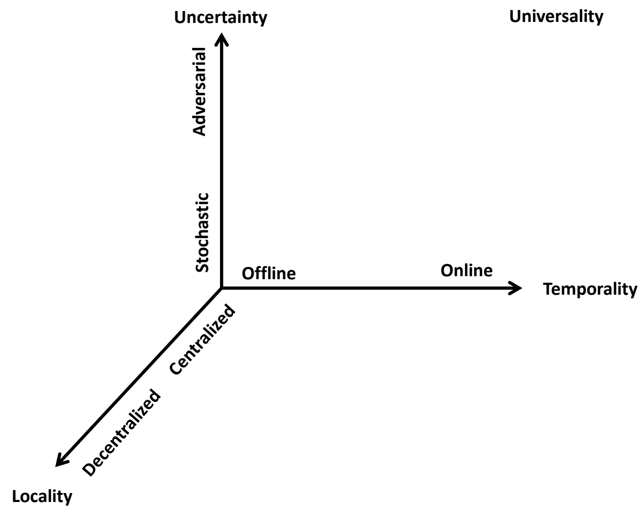

Figure 1: Our framework for Distributed Online Optimization

can handle multiple dimensions simultaneously as with the (meta-)phenomenon of *universality* [37] that, for certain problems of central significance such as TSP, Steiner Tree and Set Cover, produces guaranteed solutions (albeit, centralized) in the face of adversarial revelation of the input. Indeed, universal algorithms do not even exploit the adaptivity available to them and operate obliviously. One of the hopes of this proposal is to extend the phenomenon of universality to the distributed setting in addition to uncovering other similar (meta-)phenomena.

At a high level, the problems we plan to study in this project can be divided into two categories: ***network design*** and ***information flow***.

Network Design. Network design concerns the construction of overlay network structures that form the foundation for aggregation, point-to-point routing, broadcast, multicast and other critical network functions. The deployment of mission-critical military systems requires the ability to construct and maintain such large-scale network structures that will enable secure and reliable communication and operation in a highly dynamic and distributed environment. Research in network design has been at the forefront of major advances in approximation algorithms [56]. We believe that theoretical foundations of distributed online network design are essential to achieve major advances in this area. Within network design, we will study the general problem of *survivable network design*, with a focus on the important special case of *aggregation trees*.

- **Survivable network design:** In the *survivable network design problem (SNDP)*, we are given a graph $G = (V, E)$ with edge-costs, and edge-connectivity requirements $r_{ij} \in Z_{\geq 0}$ for every pair of vertices $i, j \in V$, and need to find an (approximately) minimum-cost network that provides the required connectivity. This is one of the most fundamental problems in network design that generalizes several graph-theoretic optimization problems including shortest paths, spanning and Steiner trees. The edge-connectivity requirements capture the need for increased reliability in mission-critical systems; furthermore, the general statement of the problem allows one to develop algorithmic paradigms that may have broad applicability. *We propose the development of distributed online algorithms for SNDP against strong adaptive adversaries.*

- **Aggregation trees:** A special case of survivable network design is the fundamental problem of constructing a tree that aggregates information from important agents in a distributed network to a central point (HQ). The agents are modeled as nodes in a graph, whose edges capture the communication links between the agents. Suppose the primary task of the network is to detect and monitor specific targets whose location the agents are unaware of a priori. As the agents explore their local terrain in detail they may become aware of the presence of valuable targets. This converts such a node into a *terminal* node that must now communicate with the HQ node henceforth called the *root*. This problem is a variant of the classical minimum Steiner tree problem. Though Steiner tree and their variants have been extensively studied, there are no distributed algorithms for computing near-optimal Steiner trees in dynamic or uncertain environments. *We propose to build new distributed methods for constructing near-optimal aggregation trees in stochastic and online environments, and design even more robust solutions in the form of universal approximations.*

Information Flow. Our proposed work on decentralized network design will guarantee that the nodes can maintain a well-connected overlay structure useful for exchanging critical information among the nodes. Information flow concerns the development of decentralized algorithms that route higher-level application information over networks. Within information flow, we will study distributed algorithms for constructing and maintaining routing tables, and gossip-based information spreading in highly dynamic networks.

- **Minimizing congestion in dynamic networks:** The capacity of links in wireless (ad hoc and sensor) networks often changes due to fading and multi-path effects. Further, the mobility of the nodes themselves leads to changes in the network topology. A major challenge in such a dynamic environment is to compute and maintain bottleneck-free routes for information flow. There are two main sources of difficulty: first, the routes must satisfy certain degree constraints for keeping routing information within manageable limits [17]; second, each node only has knowledge of the state of links to which they are connected, while link capacities can be dynamically varying. Given a traffic demand matrix over the set of nodes, our goal is to devise a distributed algorithm that computes and maintains *congestion minimizing degree-bounded flows* between all source-destination pairs. The algorithm must be rapidly convergent and must utilize minimal control information to avoid congesting the links that must be conserved for data transport. *We propose to build distributed algorithms for congestion-minimizing multcommodity flows that also obey degree bounds.*

- **Information spreading in adversarially dynamic networks** The dynamics of networks arising in military and critical infrastructure settings are not only extensive, but can also be influenced by external adversaries. A natural model is that of an adversarial *time-evolving network*: in step t, the network $G_t = (V_t, E_t)$ is an arbitrary graph over the set V_t of n nodes, where the edge sets E_t are determined by an adversary. We propose to develop purely local lightweight algorithms for fundamental information dissemination tasks in highly dynamic networks. Our starting point will be the basic *k-broadcast problem* in which k of the n nodes each have a message and would like to disseminate them to every node in the network. A fundamental open problem is the following: Can the k-broadcast problem be solved on an dynamic, always-connected, n-node network in $O(n + k)$ steps? *We propose to design an efficient and effective distributed algorithm for this basic k-broadcast problem and develop distributed gossip-based algorithms for the more general information dissemination problems of consensus, aggregation, and computing separable functions.*

1.2 Future Naval Relevance

The problems presented in this proposal are of direct relevance to a number of Naval (and other military) initiatives. The most notable example is JTRS [21] - Joint Tactical Radio System. JTRS is a Department of Defense initiative to develop a family of revolutionary software-defined tactical radios that will provide networked voice, data and video communications, as well as interoperability across the joint battlespace. Originally conceived in 1997 it has expanded into an ongoing family of programs for the development and delivery of tactical communications and networking solutions for the Air Force, Army, Navy and Marines. Though portions of the JTRS program have been scrapped or undergone transformation, nevertheless the research issues raised by some of its subinitiatives continue to be of significance today. The Airborne, Maritime, Fixed Station (AMF) subinitiative of the JTRS project is of particular relevance. It is concerned with the design, development, integration, testing and delivery of advanced communication systems to modernize current radio systems with next-generation Software Defined Radio technology. The properties of reliable network-centric capability and legacy compatibility are to provide joint interoperability and secure information flow for military platforms such as aircraft, submarines, surface ships and DoD installations. The problems of *network design* and *information flow* are directly motivated by applications to the AMF JTRS program.

There is immense ongoing interest in the development of an IP-based Air-and-Surface borne Network based on mobile ad hoc networking concepts. Such a network may consist of disparate mobile airborne and sea-based platforms; will provide reliable ad hoc interconnectivity to terrestrial and airborne nodes; and will be applicable to net-centric military communications. A wireless network that includes an airborne component is architecturally reminiscent of conventional wireless telecommunications networks. Relatively quasi-stable airborne platforms, equipped with directional data links, could form a high capacity backbone core akin to the back-haul in telecommunications networks. This directional backbone core would serve as a reliable data transport infrastructure for highly dynamic terrestrial and airborne platforms at the network edge, which are typically equipped with lower capacity omni-directional radios. The latter are akin to end users in conventional telecommunications networks. As in any mobile network, the performance of such a network depends to a great extent on the prevailing network topology. The network topology determines the availability and the characteristics of the data path between any two nodes of the network. These characteristics directly dictate the performance of data transmission, such as throughput and delay, between the nodes. For a network of fixed nodes the topology is essentially static. It is determined by deployed physical connections between nodes. For a network with mobile nodes the topology is dynamic with several topology choices possible. The design problems of *survivable networks* and *aggregation trees* are motivated by the need for active topology management and optimization in these Air-and-Surface borne networks with the aim to improve overall communication and decision-making performance.

In this proposal, we are interested not just in the management and optimization of the topology but also the sharing and routing of relevant information in a timely manner so as to satisfy mission-critical needs. The movement of information on the network core is a challenging proposition [32] due, in particular, to the following factors: (1) The nodes have a limited number of directional links (and this number may be different for different nodes), (2) The links may have different and time-varying data rates (for instance, due to differences in received Signal to Interference and Noise Ratio (SINR)), (3) The nodes may be moving in space causing changes in underlying topology; with incoming and outgoing links of different radio types, such incompatibilities introducing communication constraints over and above range restrictions, and (4) There may be important information that needs to get from one part of the network to the other at the same time as jamming or eavesdropping adversaries are rendering certain links inoperational. Motivated by these issues we study the problems of *congestion minimization* and *information dissemination* in dynamic networks.

1.3 PI Information

Prof. R. Ravi works in the intersection of Operations Research and Computer Science and has pioneered work in various network optimization problems such as the Buy-at-Bulk Network Design problem with economies of scale, the Group Steiner Tree problem that combines the Set Covering and Steiner Tree problems, and in designing provably near-optimal approximation algorithms with performance guarantees for problems in stochastic and robust combinatorial optimization [19, 29, 31, 25, 51, 54]. He has also advanced the development of tools for the design of approximation algorithms for network optimization such as the primal-dual method, dependent randomized rounding, Lagrangean relaxations and iterative methods [3, 43, 50].

Co-PI Rajaraman's research expertise covers distributed computing theory, approximation algorithms, network optimization, and algorithmic game theory [15, 39, 40]. His work on distributed hash tables is widely-cited and has been implemented in several peer-to-peer network systems [48]. Co-PI Sundaram's research expertise covers networks, algorithms, complexity theory and combinatorics [15, 40, 42, 53]. Before joining academics, he was Director of Engineering at Akamai Technologies, where he established the mapping group for the world's leading content delivery network, which is responsible for directing browser requests (over 10 billion a day) to the optimal Akamai server.

The PIs have a strong history of collaboration. For instance, PI Ravi and co-PI Sundaram were co-authors on a widely cited foundational paper on bicriteria approximations in network design [44]. Co-PIs Rajaraman and Sundaram were co-authors on a ICDCS 2006 paper on Internet capacity that won the best paper award [16]. With deep synergistic expertise in optimization theory, online and approximation algorithms, and distributed computing, the team is uniquely positioned to address the basic research challenges of this call.

2 Technical Approaches and Recent Related Work

The optimization problems raised in this proposal pose two major challenges. First, how do we deal with uncertainty, whether it is in the node pairs being connected in the survivable network design problem, the location of targets in the aggregation tree problem, or the dynamic network topology in the information flow problems? Second, how do we ensure that our algorithms can be implemented with little coordination among the network nodes?

We plan to attack the issue of uncertainty by developing algorithms for a spectrum of models spanning from *stochastic*, where the unknown online inputs are assumed to be drawn from some known probability distribution, to *adversarial*, where the inputs or network dynamics are completely under adversarial control. For the design of fully-distributed algorithms, we propose to study new *distributed methods* for constructing network decompositions that are at the heart of many effective network design algorithms. We will also focus on gossip-style algorithms for information flow, which are inherently decentralized, and have the promise of achieving near-optimality in highly dynamic environments. We now elaborate on the specific technical approaches for the problems listed in Section 1.1.

2.1 Network Design

Our research in network design will concern distributed online algorithms for the general survivable network design problem under certain stochastic and adversarial scenarios (Section 2.1.1), and explore more robust solutions for several variants of aggregation tree problems that often arise in network applications (Sections 2.1.2 and 2.1.3).

2.1.1 Survivable Network Design

Recall that in the *survivable network design problem (SNDP)*, we are given a graph $G = (V, E)$ with edge-costs, and edge-connectivity requirements $r_{ij} \in Z_{\geq 0}$ for every pair of vertices $i, j \in V$, and need to find an (approximately) minimum-cost network that provides the required connectivity. Since SNDP is NP-hard (it contains the Steiner tree problem as a special case), it has been widely studied from the viewpoint of approximation algorithms (See e.g., [28] for a survey of results). These problems were one of the earliest applications of the primal-dual method in this area which led, over a sequence of papers, to the development of an $O(\log r_{\max})$-approximation algorithm [26]. Subsequently, one of the first uses of iterative rounding in approximation algorithms led to a 2-approximation for this problem (and for the general problem of network design with weakly-supermodular functions) [36].

In [30], we studied these problems in the *online* setting: we are given a graph with edge-costs, and an upper bound $r_{\max}$ on the connectivity demand. A sequence of vertex pairs $\{i, j\} \in V \times V$ is presented to us over time, each with some edge-connectivity demand r_{ij}—at this point we may need to buy some edges to ensure that all the edges bought by the algorithm provide an edge-connectivity of r_{ij} between vertices i and j. The goal is to remain competitive with the optimal offline solution of the current demand set. Work in this online setting has mainly focused only on one-connected problems such as Steiner and generalized Steiner trees [49]. No online algorithms were previously known for this problem even for the online rooted 2-connectivity problem (i.e., for the case where all the vertex pairs share a root vertex r and the connectivity requirement is 2 for all pairs) while we noted a lower bound of $\Omega(\min\{|D|, \log n\})$ on the competitive ratio for this special case, where D is the set of terminal pairs given to the algorithm. This is in contrast to the case of online 1-connectivity (i.e., online Steiner forest) where the best online algorithm is $\Theta(\log |D|)$-competitive [7]. Our main result for the edge-connected survivable network design problem is an $O(r_{\max} \log^3 n)$-competitive randomized online algorithm against oblivious adversaries.

Our algorithms use the standard embeddings of graphs into random subtrees (i.e., into *singly connected* subgraphs) as an intermediate step to get algorithms for higher connectivity. As a consequence of using these random embeddings, our algorithms are competitive only against oblivious adversaries. A natural extension that is still open is whether our methods can be extended to work against adaptive adversaries. Furthermore, our algorithms relying on tree embeddings and inherently centralized, and it is unclear how to extend even the one-connected online algorithms for Steiner trees to the distributed setting.

Problem 1 *Devise distributed online algorithms for the Survivable Network Design Problem with good competitive ratios against adaptive adversaries.*

We propose to study decentralized algorithms for survivable network design in an online setting where the underlying graph and costs are static while the connectivity requirements change online. To design decentralized algorithms, we will consider two sets of techniques, one used for global structures such as minimum spanning trees [23, 24], and the other for local structures such as minimum dominating sets [39]. We believe that a combination of these techniques will be helpful in tackling several special cases of SNDP where the connectivity requirements satisfy some locality constraints. We also plan to quantify the price of decentralization of SNDP, which will help determine what network design problems can be effectively solved using decentralizated algorithms, and which others may even be infeasible to solve in a decentralized manner.

Another direction to extend the SNDP is the stochastic case when the instance is drawn according to a probability distribution. In [30], we considered the case when we have a product distribution: for each pair i, j of vertices we are given a probability p_{ij}, and are guaranteed that *tomorrow* each pair will flip their coins independently, and if the coin turns up heads, they would demand k-connectivity. We can buy some edges today at cost $c(\cdot)$, but if we wait for the actual set D, the edges will cost $\lambda c(\cdot)$ tomorrow, for a pre-specified inflation parameter $\lambda \geq 1$; the goal is to minimize the sum of the cost of edges bought today and the expected cost of augmentation edges bought tomorrow (at the inflated price). In that work, we assumed for simplicity that all pairs have the the same connectivity requirement of k. A simple generalization would allow each arriving pair to specify even the connectivity to be any value in $\{0, 1 \ldots, k\}$ according to a pre-defined probability distribution – we'll now have values $p_{ij}(\kappa)$ for $\kappa \in \{0, 1 \ldots, k\}$, summing to one. We propose to extend our previous work to this more interesting setting that allows for more general stochastic specifications of connectivity requirements.

2.1.2 Aggregation trees

Consider the problem of building an aggregation tree for tracking targets. In a stochastic model of uncertainty, we may view each target as an object that is detected by one of the various agents according to a previously determined probability distribution. In order to build a backbone network of the graph for coordination and control, we need to determine a subtree of the graph containing the root such that given the probability distribution of target materialization, the subtree maximizes the expected number of targets that can be covered by the agent nodes in the tree. If this backbone tree is a spanning tree, then all targets (each of which materializes at any one of the nodes) will be covered. At the other end, if no backbone or tree edges can be built a priori, the root can only detect targets that materialize in its vicinity. We thus have a trade-off between the allowed size of the Steiner tree and its expected target coverage and this is what we propose to study in detail.

We call this problem the ***Target Maximizing Rooted Steiner Tree*** problem. We are given an undirected graph $G = (V, E)$, a root node r, and a set of targets $T_i, i = 1 \ldots t$. Each target T_i is a probability distribution p_i over V where $p_i(v)$ is the probability that the target i is at node v (Note that $p_i(v) \geq 0 \ \forall v$ and $\sum_{v \in V} p_i(v) = 1 \ \forall i$). Assume that the distributions p_i and p_j for $i \neq j$ are independent of each other. Given a budget S on the size, our goal is to find a tree F containing the root of at most S edges such that the expected size of the targets that materialize in the node set of the tree is maximized.

This problem can be generalized in various ways. First, the size bound S can be extended to take into account distances between nodes or the relative costs of establishing connection between these pairs of agents. Second, the objective function can be changed to reflect other notions of reward such as the probability of detecting at least N targets for a given N. Third, the independence assumption on target materialization can be relaxed; this would not make any difference for the expected target size function but is interesting for other objectives. Fourth, the tree construction process can be modeled as the vehicle tour of a control packet in the ad hoc network, leading to vehicle routing problems that try to minimize expected size to cover all targets say. Fifth, the notion of stochastic uncertainty can be replaced by the requirement that the method is robust towards the worst of many possible scenarios of target materialization. We investigate one particularly strong notion of robustness for this problem in Section 2.1.3.

> **Problem 2** *Devise offline approximation algorithms for the Target Maximizing Rooted Steiner Tree problem with good constant performance ratio. Extend the techniques to the various generalizations and to the robust and distributed settings.*

As stated, the problem is closely related to the k-MST problem that PIs Ravi and Sundaram introduced in an early paper [52], and for which we eventually designed the first constant-factor approximation algorithm [10]. In this problem we are given an undirected edge-weighted graph with a root and an integer k, and the goal is to find a minimum-weight spanning tree connecting the root with at least k other nodes. Our target maximizing version is close to the complementary version of the problem where we are given a bound on the cost of the tree and the goal is to maximize the number of nodes covered. This is also related to the *Orienteering Problem* where the object collecting the nodes is an orienteering path rather than a tree, and for which constant factor approximations are known [9]. We have recently also devised approximation algorithms for stochastic versions of the orienteering problem [29] but the stochasticity is related to delays at the nodes to fetch the rewards rather than the location of targets on the visited nodes. These are all interesting approaches that we propose to investigate to solve the target maximizing problems we propose.

Two other challenging directions that we will pursue are to extend these centralized methods to work in a distributed setting, and to bound the price of decentralization for the general problem. For the former, we expect that it will be important to have the nodes exchange appropriate information about the target probability distributions to one another; in this regard, our proposed local methods for disseminating information (Section 2.2.2) may be helpful.

2.1.3 Universal approximations for Steiner tree and related problems

We also plan to study aggregation trees under an adversarial model of uncertainty. We have introduced the framework of *universal approximations* that provides a robust notion of quality with respect to *any* online sequence of arrivals [37]. Universality is a framework for dealing with uncertainty by guaranteeing a certain quality of goodness for all possible completions of the partial information set. Formally, an instance of the Universal Steiner Tree (UST) problem is a pair $\langle G, r\rangle$ where $G = (V, E)$ is a weighted undirected graph, and r is a distinguished vertex in V that we refer to as the *root*. For any spanning tree T of G, define the *stretch* of T as $\max_{S\subseteq V} \mathrm{cost}(T_{S\cup r})/\mathrm{cost}(\mathrm{OptSt}_{S\cup r})$, where OptSt_X is an optimal Steiner tree connecting the vertices in X. The goal of the UST problem is to determine a spanning tree with minimum stretch.

Similar to the UST problem, we can define universal versions of Traveling Salesperson Problem (UTSP), group Steiner tree, group TSP, and generalized Steiner network problems. An instance of UTSP is a metric space (V, d). For any cycle (tour) C containing all the vertices in V and a subset S of V, let C_S denote the unique cycle over S in which the ordering of vertices in S is consistent with their ordering in C. The stretch of C is defined as $\max_{S\subseteq V} \mathrm{cost}(C_S)/\mathrm{cost}(\mathrm{OptTr}_S)$, where OptTr_S denotes the minimum cost tour on set S. The UTSP is to find a tour on V with minimum stretch.

> **Problem 3** *What are the best stretch achievable for* UST*,* UTSP*, and universal variants of the group Steiner tree, and special cases of SNDP?*

The notion of universality is captured by the complexity class Σ_2^P, and we conjecture that determining the best universal algorithms for the above problems is Σ_2^P-hard. In previous work, however, we have been able to achieve near-optimal universal solutions in some cases. For any metric space, there exist Steiner trees that have universal poylogarithmic approximation ratios: our prior results [37], and the improved results of $O(\log^2 n)$ [27] that introduce the notion of oblivious network design. Though these results reveal new insights to the structural properties of Steiner trees and metrics, a major drawback is that *they do not apply to arbitrary graphs.*

In very recent work, we have made some progress on this front [12], and have identified a promising approach to attack the problem. One of the major challenges in constructing a universal Steiner tree is that any tree will be forced to place some vertices far apart (in the tree), even though they may be "nearby" according to the underlying graph distances. As a result, the resulting spanning tree may perform poorly on a subset that includes this set and some other carefully chosen vertices. To address this challenge, we introduce the notion of a *hierarchy of graph partitions*, each of which guarantees small strong cluster diameter and bounded local neighbourhood intersections. We have shown that the such a suitable hierarchy of graph partitions is essentially both sufficient and necessary for constructing low-stretch universal Steiner trees. For metric spaces – i.e., weighted complete graphs satisfying triangle inequality – such a hierarchy can be constructed using the seminal work of Awerbuch and Peleg on sparse partitions [6, 47].

It is a major open problem, however, to build such hierarchies for arbitrary graphs. We have made preliminary progress by presenting partition hierarchies for general graphs that yield a $2^{O(\sqrt{\log n})}$-stretch UST for general graphs, and partition hierarchies for minor-free graphs that yield a polylogarithmic-stretch UST for minor-free graphs. Furthermore, all of the solutions proposed thus far are centralized. We propose to develop distributed algorithms for constructing sparse partition hierarchies. In an earlier work on aggregation trees for sensor network applications, we showed that this is achievable for the highly specialized case of grid graphs [38], thus ensuring a small price of decentralization for the universal Steiner tree problem on grid-like graphs. For general graphs, we plan to build on these ideas and the techniques developed in [47] for dstributed construction of related network decompositions.

2.2 Information Flow

Our proposed work on decentralized network design will ensure that the underlying communication network connecting the agent nodes satisfies mission-critical connectivity properties. The network structures thus designed are very likely to change with time due to the mobility of the nodes, failures in the communication links, as well as adversarial attacks on the network. The second major component of this proposal to develop distributed algorithms for efficient and effective flow of information over such dynamic networks. Our proposed work on network design allows us to make assumptions about minimal connectivity of the underlying network; however, to guarantee high-throughput information flow for a dynamic distributed environment, we need to control congestion, and focus on local lightweight algorithms that maintain limited state. In this project, we will study two important information flow problems: congestion-minimizing degree-bounded flows (Section 2.2.1), and information spreading under adversarial dynamics (Section 2.2.2).

2.2.1 Congestion-minimizing degree-bounded flows in dynamic networks

A key requirement of scalable routing algorithms is to severely limit the amount of information that each network node maintains; such lightweight protocols are much more effective in a large-scale dynamic setting. Providing good guarantees on performance measures such as congestion, however, then poses a major challenge. In earlier work [5], distributed algorithms for finding congestion minimizing flows in certain dynamic networks were presented. However the routes undertaken by these flows were unconstrained and expensive to maintain. We plan to build on our past work on efficient algorithms for converting optimal flows into near-optimal *degree-bounded* flows [15, 16]; we call a traffic flow d-bounded if all flow through a node v destined for a sink t leaves v through at most d neighbors. A small bound for d ensures that each node maintains very limited state per destination. Routing on the Internet is, in fact, 1-bounded or confluent [16]. For reducing congestion in an ad-hoc network environment, however, d-bounded flows for a small constant d will achieve significantly better performance. Our current algorithms for obtaining degree-bounded flows involve some refinement techniques that require global knowledge. We believe that the local balancing approach of [5], in conjunction with our flow refinement techniques, will enable us to maintain near-optimal degree-bounded flows in dynamic networks.

Problem 4 *Design distributed algorithms for congestion-minimizing degree-bounded multicommodity flows in dynamic networks.*

We will begin with a stochastic model of traffic demand that assumes a fixed rate of communication among the various source-destination pairs, and a dynamic network model that bounds the rate of change of the network. We will then consider more adversarial models of changing traffic demand and network dynamics assuming some minimum network capacity, while ensuring that we maintain the degree-bound constraint on the traffic flows. We also plan to consider generalizations of the problem incorporating link errors and coding techniques.

2.2.2 Information spreading in adversarial networks

Problem 4 considers the task of routing point-to-point multicommodity flows in networks with some restricted dynamics. We also plan to conduct a comprehensive study of fully distributed algorithms for the more specialized task of information spreading in highly dynamic networks controlled by adversaries. In such scenarios, the algorithms need to be lightweight, with each node continually making decisions without any knowledge of the future, and possibly even no information about the current network neighborhood. Many basic coordination tasks are very difficult or even impossible to achieve in dynamic networks [22]. We will begin by studying the basic k-broadcast problem. Recall that in the k-broadcast problem, k of the n nodes each has a message that needs to be disseminated to every node in the network. Suppose that the nodes are synchronized and in each step, each node can broadcast the equivalent of a bounded number of tokens to its neighbors [41]. What is the minimum number of steps needed to complete the dissemination? If the network is completely static and connected, then a local token-forwarding process on a spanning tree of the network can accomplish the task in $O(n + k)$ steps, independent of the structure of the network. In a dynamic network as in the above model however, the problem is much more challenging.

Problem 5 *Is there a distributed algorithm that completes any k-broadcast instance in $O(n + k)$ steps on any adversarially dynamic n-node network?*

In recent work [20], we studied the class of *forwarding* algorithms that do not manipulate tokens in any way other than copying, storing, and forwarding them. We show that *any* forwarding algorithm will take $\Omega(nk/\log(n))$ steps to complete k-broadcast, thus resolving an open problem of [41]. Given that almost any local greedy forwarding procedure completes k-broadcast in $O(nk)$ steps in any dynamic network, our lower bound essentially captures the severe limitations imposed by highly adversarial network dynamics.

A natural and attractive alternative to forwarding algorithms is to use coding (either end-to-end [13, 57] or network [4]). Recent work [33, 34] has shown that information spreading based on network coding can solve k-broadcast in $O(n+k)$ steps, assuming the sizes of the messages are $\Omega(n \log n)$ bits. While this message size lower bound is prohibitively large and impractical (since it scales with the size of the network), our lower bound [20] together with this upper bound establish, in theory, a fundamental gap between flow-based and coding-based dissemination procedures.

One approach we plan to pursue is to consider a hybrid forwarding-coding algorithm in which nodes exchange information in the symmetric difference of what they currently hold, which can be done in $O(\log n)$ rounds of communication using fingerprinting [45]. We have shown that if the entropy of the initial distribution of information is high, then convergence to full dissemination is rapid. We will also consider weaker notions of the adversary (e.g., offline or oblivious), which model real-world settings where the adversary has significant control but is not cognizant of all the network actions. The offline dissemination problem can be reduced to the problem of packing Steiner trees in directed graphs [18, 20], and thus has deep connections with the long-standing open problems of approximating directed Steiner trees [14, 35, 59] and bounding the network coding advantage in multicast over directed networks [2, 55].

Following the k-broadcast problem, we will study more general information spreading problems including consensus, computing aggregates (that often arise in sensor network environments), and larger classes of computations such as separable functions [46].

Problem 6 *Design and analyze fully distributed gossip-based algorithms for consensus and aggregation, and computing separable functions.*

We expect to quantify all our results in terms of relevant parameters including locality of the dynamics, conductance/expansion of the evolving network, initial entropy of the information distribution, and knowledge available to the dissemination algorithm.

3 Project Schedule and Milestones

The schedule for this project is given in the Table 1. We expect each topic to span approximately 4 years of the project. The total effort is evenly distributed over the 5 year span of the project. The three PIs plan to work closely on all of the research proposed in this project; the table lists the lead PI for each of the topics.

Topic	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	**Lead PI**
T1: Survivable network design						Ravi
T2: Stochastic aggregation tree problems						Ravi
T3: Universal aggregation trees						Sundaram
T4: Congestion-minimizing degree-bounded flows						Sundaram
T5: Information dissemination in adversarial networks						Rajaraman

Table 1: Project timeline

Milestones. We will produce annual reports at the end of each of the project years. During the course of the project, we will document all of the problem formulations and results in papers, following ONR guidelines for publication. At a high level, we will have the following milestones for each of the topics. For each milestone, we include in parenthesis its deadline during the 4-year period devoted to the topic:

- *Problem formulation:* Precise formulations for the core problem and its variants. (end of Y1 for T1, T3, and T5; end of Y2 for T4 and T5)
- *Preliminary bounds:* Preliminary bounds on the core problem. (end of Y1 for T1, T3, and T5; end of Y2 for T4 and T5)
- *Results for online models:* Results for suitable online stochastic and online adversarial models. (end of Y2 for T1, T3, and T5; end of Y3 for T4 and T5)
- *Decentralization:* Quantify the price of decentralization. (end of Y3 for T1, T3, and T5; end of Y4 for T4 and T5)
- *Extensions and integration:* Adapt results to model extensions and integrate the technical results across topics. (end of Y5)

4 Management Approach

The whole research team, consisting of the three PIs and all graduate students working on the project will coordinate via bi-weekly meetings arranged online (through a service such as Skype). Research progress in the form of meeting reports, working papers, and any experimental code generated will be shared through a project repository online. A face-to-face annual meeting of the project members will be held every year just before the annual report is due, to gather all the information for the report and re-evaluate the project goals and schedule. If appropriate, we will plan this meeting as part of a visit to a professional meeting that we all attend.

5 Current and Pending Support

5.1 Current awards

NSF EAGER grant for R. Ravi.

1. Investigator: R. Ravi
2. Title of Proposal and Summary: **EAGER: New Techniques for Graph-TSP**. The traveling salesperson problem (TSP) is a benchmark problem for combinatorial optimization that asks for a shortest tour that visits all the cities in a given network. The graph version where the distances arise from an underlying undirected graph captures a significant portion of the difficulty of designing good algorithms for solving this problem. This proposal will develop a consolidated understanding of the new techniques used in recent developments in the design of improved approximation algorithms for the graph-TSP problem and suggest new ones to move towards optimal performance guarantees.
3. Source and amount of funding: NSF CISE directorate CCF division award number 1143998, for $99,277.
4. Percentage effort devoted: 2 summer months in 2012
5. Identity of prime Offeror: Carnegie Mellon University, Pittsburgh.
6. Technical Contact: Prof. Ramamoorthi Ravi, Tepper School of Business at Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213-3890. Tel: +1 412 268 3694; Fax: +1 412 268 7345; Email: ravi@cmu.edu.
7. Administrative Contact: Richard Ling, Tepper School of Business at Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213-3890. Tel: +1 412 268 5915; Fax: +1 412 268 2810; Email: rjling@andrew.cmu.edu
8. Period of performance: September 1, 2011 - August 31, 2012.
9. Relation to current proposal: There is no overlap with the currently proposed effort.

BAE subcontract for R. Rajaraman and R. Sundaram.

1. Title of Proposal and Summary: **Communications in Extreme RF Spectrum Conditions (Commex)**. This subcontract consists of three tasks towards the design of a communication protocol for ad hoc networks under extreme adversarial (jamming) conditions: (i) Develop a non-cooperative game theory engine, conduct a theoretical analysis, and develop the associated algorithms and software; (ii) Develop algorithms and techniques that provide optimized protocol mechanism hopping at the physical and media access control layers; (iii) Perform theoretical analysis and trade studies to characterize the performance of jammer jujitsu interference deception algorithms against a range of jammer/interference models.
2. Source and amount of funding: BAE Systems subcontract on DARPA grant, for $650K.

3. Percentage effort devoted: 1 summer month in 2012, and 1 course buyout in 2012, for each PI
4. Identity of prime Offeror: Northeastern University
5. Technical Contact: Prof. Ravi Sundaram, 202 WVH, CCIS, Northeastern University Boston MA 02115. Tel: +1 617 373 5876; Fax: +1 617 373 5121; Email: koods@ccs.neu.edu
6. Administrative Contact: Deborah Grupp-Patrutz, Director, Research Administration and Finance, Northeastern University, 360 Huntington Avenue, 960 RP, Boston, MA 02115. Phone: (617) 373-5600, Fax: (617) 373-4595 Email: oraf@neu.edu
7. Period of performance: July 1, 2011 - February 28, 2013.
8. Relation to current proposal: There is no overlap with the currently proposed effort.

5.2 Pending proposals (under review)

NSF AF proposal for R. Ravi.

1. Title of Proposal and Summary: **AF: Small: Approximation Algorithms for Network Design**. Problems in network design have a two-fold importance in current practice and theory. The key societal advances of the last decade are enabled by the surge of interconnection technologies that have given rise to a variety of such problems. Examples include the design, analysis and operation of supply chain networks, social networks and telecommunication networks. On the other hand, theoretical problems in network design have served as model problems for developing new techniques in the evolution of combinatorial optimization in general and in the design of approximation algorithms in particular. Examples include the primal-dual and iterative methods. This proposal will study several fundamental connectivity problems in network design that have already spurred much algorithmic advance; The focus will be on the traveling salesperson problem and its closely related variants such as the bridge connectivity augmentation problem, two-edge-connected subgraph problem and vertex connectivity network design problems; The goal is to design improved approximation algorithms by advancing current techniques and developing new methods. As a second goal, we propose two new models for network design problems that take into account subadditive demands, and that integrate inventory storage and vehicle routing costs. Thus the two main thrusts of this proposal are to develop new techniques by studying fundamental connectivity problems in network design and new models that will further necessitate such new methods as well as extend and improve existing techniques.
2. Source and amount of funding requested: NSF CISE directorate CCF division proposal number 1218382, for $494,050.
3. Percentage effort devoted: 2 summer months in 2013, 2014 and 2015.
4. Identity of prime Offeror: Carnegie Mellon University, Pittsburgh.
5. Technical Contact: Prof. Ramamoorthi Ravi, Tepper School of Business at Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213-3890. Tel: +1 412 268 3694; Fax: +1 412 268 7345; Email: ravi@cmu.edu.
6. Administrative Contact: Richard Ling, Tepper School of Business at Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh PA 15213-3890. Tel: +1 412 268 5915; Fax: +1 412 268 2810; Email: rjling@andrew.cmu.edu
7. Period of performance: September 1, 2012 - August 31, 2015.
8. Relation to current proposal: There is no overlap with the currently proposed effort.

NSF ICES proposal for R. Rajaraman and R. Sundaram.

1. Title of Proposal and Summary: **NSF:ICES:CollaborativeResearch: The Role of Space, Time, and Information in Controlling Epidemics.** The control of epidemics, broadly defined to range from human diseases such as influenza and smallpox to malware in communication networks, relies crucially on interventions such as vaccinations and anti-virals (in human diseases) or software patches (for malware). These interventions are almost always voluntary directives from public agencies; however, people do not always adhere to such recommendations, and make individual decisions based on their specific self interest. Additionally, people alter their contacts dynamically, and these behavioral changes have a huge impact on the dynamics and the effectiveness of these interventions, so that good intervention strategies might, in fact, be ineffective, depending upon the individual response. The goal of this proposal is to study the foundations of policy design for controlling epidemics, using a broad class of epidemic games on complex networks involving uncertainty in network information, temporal evolution and learning. We propose models that capture the complexity of static and temporal interactions and patterns of information exchange, including the possibility of failed interventions and the potential for moral hazard. We also consider specific policies posed by public agencies and network security providers for controlling the spread of epidemics and malware, and study their efficacy and resource constrained mechanisms to implement them in this framework.

2. Source and amount of funding: NSF CISE directorate CCF division proposal number 1216038, for $275,000.

3. Percentage effort devoted: 0.25 summer months for each PI in 2013, 2014, and 2015.

4. Identity of prime Offeror: Northeastern University

5. Technical Contact: Prof. Rajmohan Rajaraman, 202 WVH, CCIS, Northeastern University Boston MA 02115. Tel: +1 617 373 2075; Fax: +1 617 373 5121; Email: rraj@ccs.neu.edu

6. Administrative Contact: Deborah Grupp-Patrutz, Director, Research Administration and Finance, Northeastern University, 360 Huntington Avenue, 960 RP, Boston, MA 02115. Phone: (617) 373-5600, Fax: (617) 373-4595 Email: oraf@neu.edu

7. Period of performance: September 1, 2012 - August 31, 2015.

8. Relation to current proposal: There is no overlap with the currently proposed effort.

NSF NeTS proposal for R. Rajaraman.

1. Title of Proposal and Summary: **NeTS: Small: Scaling Wireless and Data Access the Power of Community Networks**. PI Guevara Noubir. Wireless communication systems have been a key enabler of the smart phones revolution. However, such systems are under a continuously increasing demand for bandwidth and are reaching their limit forcing the operators to deploy, operate, and maintain base stations in the users homes (femto-BS) without added incentives to the users. At the same time, Wireless Access Points (APs) already have an extremely high density in urban areas, operate on unlicensed bands (greater than 200Mhz of ISM bands), provide high data rates, have high availability, are closer to the users than content delivery networks (CDN), consume little energy, and can easily be extended to provide storage and caching services. They therefore have several characteristics that make them a good candidate for a community infrastructure that provides truly ubiquitous wireless and data access. However, several fundamental and systems challenges have to be addressed to make such networks a reality. The goal of this research is to develop enabling mechanisms, protocols, and demonstrators for community networks that provide ubiquitous wireless and data access.

2. Source and amount of funding: NSF CISE directorate CNS division proposal number 1218146, amount $499,954.00.

3. Percentage effort devoted: 0.25 summer months in 2013, 2014, and 2015.
4. Identity of prime Offeror: Northeastern University
5. Technical Contact: Prof. Guevara Noubir, 202 WVH, CCIS, Northeastern University Boston MA 02115. Tel: +1 617 373 5205; Fax: +1 617 373 5121; Email: noubir@ccs.neu.edu
6. Administrative Contact: Deborah Grupp-Patrutz, Director, Research Administration and Finance, Northeastern University, 360 Huntington Avenue, 960 RP, Boston, MA 02115. Phone: (617) 373-5600, Fax: (617) 373-4595 Email: oraf@neu.edu
7. Period of performance: September 1, 2012 - August 31, 2015.
8. Relation to current proposal: There is no overlap with the currently proposed effort.

NSF CSR proposal for R. Rajaraman.

1. Title of Proposal and Summary: **CSR: Small: Collaborative Research: Pursuing High Performance on Clouds and Other Dynamically Heterogeneous Computing Platforms**. PI Arnold Rosenberg. The proposed research will develop a transformative computing paradigm that will enable high performance computing on modern platforms such as (computing) clouds and many genres of (computing) grids. Clouds and grids promise to make high-performance computing platforms accessible to the public; but realizing this promise requires one to cope with the platforms dynamic heterogeneity, i.e., the fact that their constituent computers relative powers and speeds can change at unpredicable times and in unpredictable ways. (For instance, a computer in a cloud may slow down because it is assigned more work from another user.) The new paradigm replaces traditional schedulers attempts to accommodate the particulars of a computing platforma goal that dynamic heterogeneity confutesby orchestrating a complex computation in a way that honors the relevant details of the computations inherent structure. Specifically, the paradigm aims to enhance the average rate at which the computations constituent tasks are rendered eligible for execution, by executing tasks in an appropriate order. (Because of this goal, the paradigms schedules are termed area-oriented for reasons detailed in the project description.) This strategy aims to: (a) increase opportunities for executing independent tasks in parallel and (b) minimize the chance of the computations stalling pending completion of already allocated tasks.
2. Source and amount of funding: NSF CISE directorate CNS division proposal number 1217981, amount $315,513.
3. Percentage effort devoted: 0.25 summer months in 2013, 2014, and 2015.
4. Identity of prime Offeror: Northeastern University
5. Technical Contact: Prof. Arnold Rosenberg, 202 WVH, CCIS, Northeastern University Boston MA 02115. Fax: +1 617 373 5121; Email: rsnbrg@ccs.neu.edu
6. Administrative Contact: Deborah Grupp-Patrutz, Director, Research Administration and Finance, Northeastern University, 360 Huntington Avenue, 960 RP, Boston, MA 02115. Phone: (617) 373-5600, Fax: (617) 373-4595 Email: oraf@neu.edu
7. Period of performance: September 1, 2012 - August 31, 2015.
8. Relation to current proposal: There is no overlap with the currently proposed effort.

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