Filter Design

ECE3151 Linear Systems Lab

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ABSTRACT

The purpose of this lab was to facilitate understanding of practical applications for Fourier Transforms by use of filter design. Additionally, this lab demonstrates one of the potential applications for filters. Using a windowed impulse response for a low-pass filter, the relationship between tau and the rolloff. Once that calibration was done, a filter function was constructed and the efficacy of the function was tested. The final test for the filter function was taking a section of a song with a distortion at a specific frequency and filtering out the distortion.

INTRODUCTION

Ideal filters can be characterized by their impulse response. The impulse response for a low-pass filter (1-1) is dependent upon the time, t, and the cutoff frequency, fc [1].

$$h_I(t) = 2f_c sinc(2f_c t) \tag{1-1}$$

This creates a non-causal filter. A method of developing a more precise filter is known as the window method. The impulse response is multiplied by the window, w, to have a narrow time in which there is signal and can be shifted by τ, related to the rolloff, such that the impulse response is causal (1-2) [1].

$$h_{LPF}(t) = (t - \tau)w(t - \tau) \tag{1-2}$$

A band-pass filter can be constructed using a low-pass filter and a cosine to frequency shift the low-pass filter (1-3).

$$h_{BPF}(t) = h_{LPF}(t) 2cos[2\pi f_0(t - \tau)] \tag{1-3}$$

Where the fc for the low-pass filter is half the bandwidth and f0 is the center frequency.

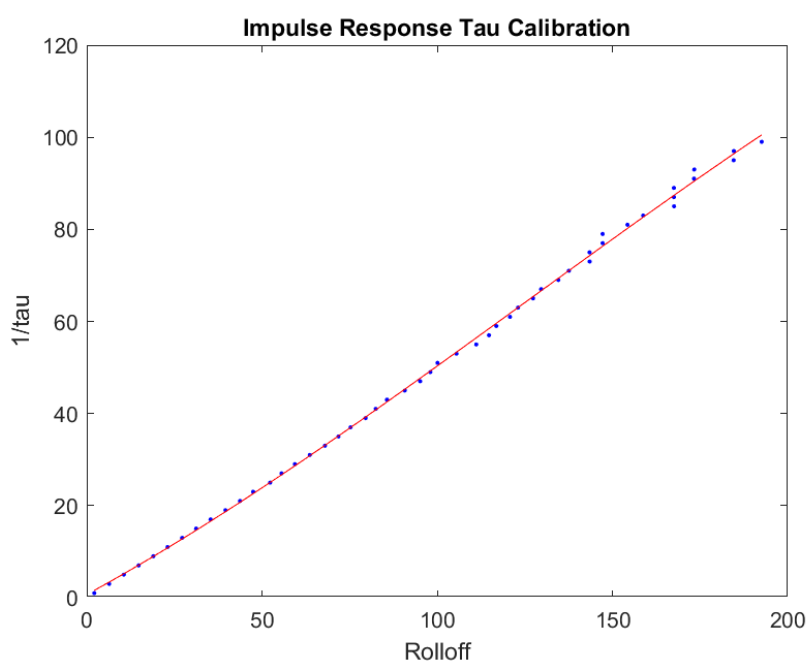
The first step in completing the lab was determining the relationship between tau and rolloff. A calibration curve relating tau and rolloff was found for values of tau ranging from 0.01

to 1. At each tau value, a windowed impulse response was calculated and the rolloff value for that response was found.

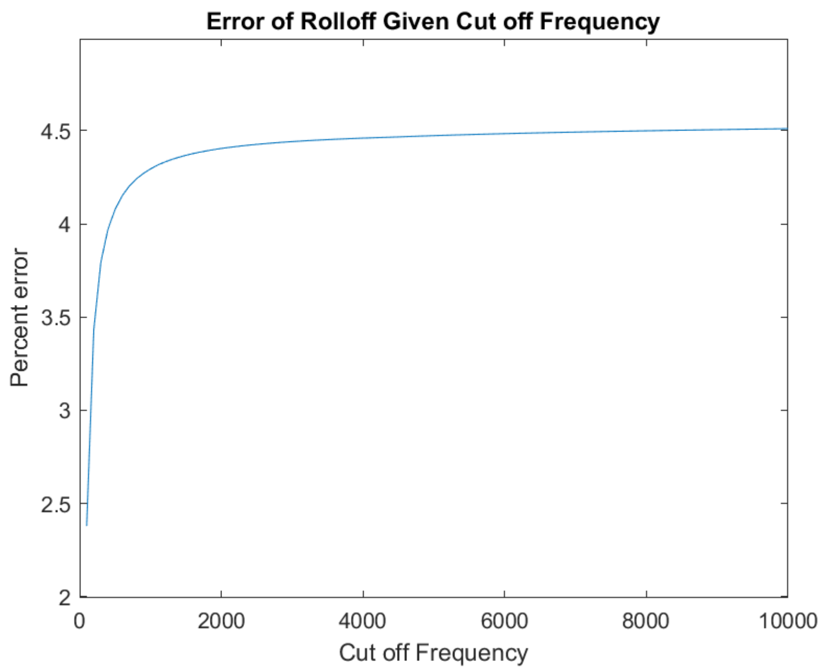
To determine the accuracy of the calibration curve, the tau value was calculated for a given rolloff value. The tau value was used to determine the window response. For a range of cutoff frequencies, the impulse response was calculated and windowed. At each frequency value, the actual rolloff was calculated. The percent error of the given rolloff value and the actual rolloff value was determined and plotted versus frequency.

RESULTS

A. The rolloff for various values of tau was calculated using the window method of designing a filter. The calculated rolloff values were graphed as compared to 1/tau and a line of best fit was determined (Figure1). After trying several different polynomial orders, it was decided that an equation with order 3 provided the best fit. This was determined by examining the percent error between the calculated tau value and the actual tau value.



B. To determine the efficacy of the line of best fit found in the previous section, a lowpass filter response was generated for various frequency values using a rolloff of 10 Hz. The actual rolloff was calculated and the percent error was plotted (Figure 2). The percent error ranged from just under 2.5% to almost 4.5%. The error curve is exponential and approaches 4.5% as the cutoff frequency approaches infinity.



C. After graphing the signal of the song, the frequency range chosen was [585,615] as the interference was from [595,605]. The rolloff chosen was 5 Hz as the interference was only within a 10 Hz range and the frequency range chosen for the calculations included a 10 Hz buffer on each side. After filtering, the song was much improved. Without headphones, it was almost impossible to hear any of the original interference.

CONCLUSIONS

In conclusion, this lab allowed students to see one of the many important applications of Fourier Transform - filter design. The lab allowed the students to observe how in frequency domain, signals which are additive to some of various sinusoidal signals can be separated based on their frequency. By designing the required filter, the noise which would lie in a certain frequency range could be removed. Additionally a major part of the lab focused on how to create real life filters, since the real life lowpass or bandpass filters would differ from the ideal filters since they would have a certain rolloff and would also need to be causal. Overall, this lab turned out to be crucial for solidifying the understanding of signals in frequency domain and digital filter design, it was also one of the best ways to develop an intuition of fourier transform in a real world scenario.

BIBLIOGRAPHY

[1] Rodger E. Ziemer, William H. Tranter, and D. Ronald Fannin, *Signals and Systems: Continuous and Discrete*, Fourth. Prentice Hall, 1998.