Introductory Electricity, Magnetism, and Optics Practice Problems

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Contents

Useful Constants

Electron Mass = 9.11×10^{-31} kg
Proton Mass = 1.67×10^{-27} kg
Elementary Charge = 1.602×10^{-19} C
Coulomb's Constant = 8.99×10^{9} Nm^2/C^2
Gravitational Constant = 6.67×10^{-11} m^3/kg$\cdot s^2$
Avogadro's Number = 6.02×10^{23} atoms/mole

Atomic Number of Uranium = 92
Atomic Number of Copper = 29
Molar Mass of Copper = 63.5 g/mole
Mass of the Earth = 5.97×10^{24} kg
Mass of the Moon = 7.35×10^{22} kg

1 Quantized Electric Charge and Charging Objects

1.1 Electric Charge Units

What is the basic unit of electric charge?

$\Rightarrow$ Coulomb

1.2 Charge on an Electron

True or False? The charge on an electron is constant?

$\Rightarrow$ True, $e = 1.6 \times 10^{-19}C$

1.3 Transfer of Electrons

I rub a piece of silk against a glass rod in order to charge up the rod. If the final charge on the rod is +2.3 μC, how many electrons were transferred from the rod to the silk?

$\Rightarrow 1.4 \times 10^{13}$ electrons

1.4 Charge Within a Uranium Nucleus

What is the net charge within the nucleus of an atom of U-238?

$\Rightarrow 1.5 \times 10^{-17}$ C

1.5 Number of Electrons in Copper

Estimate the number of electrons in a 1.0 kg block of copper that has been charged to +10 μC.

$\Rightarrow 2.7 \times 10^{26}$ electrons

1.6 Charging an Object #1

I bring a charged insulator close to an uncharged conductor (not touching). I then ground the conductor. This method of charging the conductor is called charging by _____.

$\Rightarrow$ Induction

1.7 Charging an Object #2

I touch an uncharged conductor to a second negatively charged conductor. This method of charging an object is called charging by _____.

$\Rightarrow$ Contact

1.8 Charging an Object #3

I rub a glass rod against a silk cloth in order to charge up the glass rod. This is an example of charging by _____.

$\Rightarrow$ Friction

2 Coulomb's Force Law

2.1 Charge at a Distance

True or False? Two charged spheres must be touching in order for them to feel an electric force.

$\Rightarrow$ False

2.2 Hydrogen Atom

In a hydrogen atom, a proton is separated from an electron by an average distance of about 5.3×10^{-11} meters. Calculate the electrostatic force of attraction by the proton on the electron. Do the same for the electron on the proton.

$\Rightarrow 8.2 \times 10^{-8}$ N, Same For Both

2.3 Force at the Center of a Square

Suppose I place four charges (each +Q) at the four vertices of a square with side length L. What is the magnitude of the net force on a positive point charge (+q) located at the center of the square?

$\Rightarrow$ 0 N

2.4 Equilibrium Point

Suppose I place a charge of Q_1 = +1.0 nC at the point (1 m, 0 m) and a charge of Q_2 = -2.5 nC at the point (0 m, 0 m). At what point in the xy-plane could I put a negative charge of Q_3 = -5.0 μC such that Q_3 would feel no net electrostatic force?

$\Rightarrow$ (2.7 m, 0 m)

2.5 Electrical vs. Gravitational Force

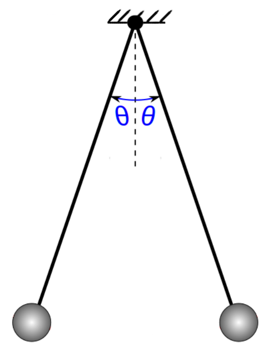
Suppose the moon were held in orbit due to an electrical force instead of a gravitational force. Assume that the Earth and the moon have equal charge magnitudes with opposite sign (i.e. Q_{Earth} = +q and Q_{Moon} = -q). How large would these charges have to be in order to maintain the current stable orbit?

$\Rightarrow |Q_{Earth}| = |Q_{Moon}| = 5.71 \times 10^{13}$ C

2.6 Charged Pendulum

Consider the double pendulum setup shown below. Two small spherical bobs, each with mass M hang from strings of length L. After a short time, the bobs reach the equilibrium position as shown where the angle θ is measured from the vertical. Derive an expression for the charge on each bob.

$\Rightarrow Q = \sqrt{\frac{mgL^2 sin^3\theta}{kcos\theta}}$



3 Continuous Electric Charge

3.1 Direction

True or False? When solving for the electric field due to a continuous charge distribution using Coulomb's Law, we must take direction into account since electric field is a vector.

$\Rightarrow$ True

3.2 Charge at the Center of a Ring

Consider a charged ring in the xy-plane, centered at the origin, with charge density $\rho = \cos(\theta)$ where θ is the angle about the ring in standard orientation. If I place a positive charge at the center of the ring, which way will it move?

$\Rightarrow -\hat{x}$ direction

3.3 Uniformly Charged Ring

Calculate the electric field along the axis of a uniformly charged ring (radius R, net charge Q).

$\Rightarrow \vec{E} = \frac{kQz}{(z^2+R^2)^{3/2}}\hat{z}$

3.4 Uniformly Charged Ring (Difficult - Extra Credit)

Show that for small distances, the electric field around the center of the uniformly charged ring is linear with distance. Then, show that a particle released near the center of the ring experiences simple harmonic motion.

$$\vec{E} = \frac{kQz}{(z^2+R^2)^{3/2}}\hat{z} \approx \frac{kQz}{R^3}\hat{z} \tag{1}$$

$$\vec{F} = m\vec{a} \tag{2}$$

$$-\frac{kQz}{R^3} = m\frac{d^2z}{dt^2} \tag{3}$$

$$\frac{d^2z}{dt^2} + \frac{kQz}{mR^3} = 0 \tag{4}$$

$$z(t) = Asin(\omega t) \tag{5}$$

where

$$\omega = \sqrt{\frac{kQ}{mR^3}} \tag{6}$$

3.5 Charged Line #1

A uniform line of charge extends from the point (0,0) to (1,0). It has a net charge of +Q. What is the electric field at the point (2,0)?

$\Rightarrow \frac{1}{2}kQ$

3.6 Charged Line #2 (Difficult)

A uniform line of charge extends from the point (0,0) to (1,0). It has a net charge of +Q. What is the electric field at the point (2,1)? Hint: You might need to consult a table of integrals for this problem.

$\Rightarrow \vec{E} = \frac{kQ}{\sqrt{10}}(\sqrt{5} - \sqrt{2})\hat{x} + \frac{kQ}{\sqrt{10}}(2\sqrt{2} - \sqrt{5})\hat{y}$

4 Electric Field

4.1 Field Due to Proton and Electron

A proton is placed at x = +2 m and an electron is placed at x = -2 m. What is the magnitude of the electric field at the point x = 0 m?

$\Rightarrow 7.2 \times 10^{-10}$ N/C

4.2 Electron in an Electric Field #1

An electron is fired into a uniform electric field. The initial velocity of the electron is given by $\vec{v} = 500\hat{x} + 100\hat{y} - 300\hat{z}$, and the electric field is given by $\vec{E} = 100\hat{x} + 200\hat{y} - 150\hat{z}$. Calculate the magnitude of the acceleration of the electron at time t = 5 s.

$\Rightarrow |\vec{a}| = 4.735 \times 10^{13}$ m/s^2

4.3 Electron in an Electric Field #2

An electron is fired into a region of uniform electric field given by $\vec{E} = 0.10\hat{y}$ N/C. At time t = 0 s, it is located at the origin with an initial velocity of $\vec{v_o} = 5.0 \times 10^5\hat{x} + 3 \times 10^5\hat{y}$ m/s. What is the velocity of the particle at time t = 1.0 μs?

$\Rightarrow \vec{v} = (3.0 \times 10^5)\hat{x} + (2.8 \times 10^5)\hat{y}$ m/s

4.4 Electron in an Electric Field #3

An electron is fired into a region of uniform electric field given by $\vec{E} = 10\hat{y}$. At time t = 0 s, it is located at the origin with an initial velocity of $\vec{v_o} = 5.0 \times 10^5\hat{x} + 3 \times 10^5\hat{y}$ m/s. At what time during its trajectory does it intersect the x-axis again?

$\Rightarrow$ t = 341.6 ns

4.5 Multiple Choice

Which of the following statements is NOT true?

A) Electric field lines emanate from positive charges and terminate on negative charges.

B) Electric field lines that are closely spaced imply a strong electric field.

C) Positive point charges feel an electric force parallel to electric field lines.

D) A positive point charge does not produce an electric field.

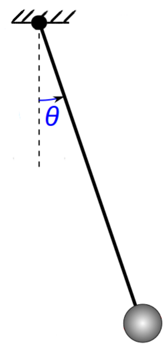
E) There is a non-zero electric field at every finite distance around an electric dipole.

$\Rightarrow$ D

4.6 Pendulum in an Electric Field

Consider a simple pendulum that has a bob with mass M and charge +Q as shown below. The string of the pendulum has a length L. A uniform electric field of magnitude E keeps the pendulum bob at equilibrium. Derive an expression for θ.

$\Rightarrow \theta = tan^{-1}(\frac{qE}{mg})$



5 Electric Dipoles

5.1 Electric Field Around a Perfect Electric Dipole

True or False? There is a point somewhere around a perfect electric dipole where the electric field is equal to zero.

$\Rightarrow$ False

5.2 Electric Field Due to a Dipole

Consider an electric dipole with charges +Q and -Q located at x = +1 m and x = -1 m respectively. Calculate the net electric field due to the dipole at x = 0 m.

$\Rightarrow -2kQ$ N/C $\hat{x}$

5.3 Potential Energy of an Electric Dipole

I place a dipole with dipole moment p = 10$\hat{z}$ C·m in an electric field E = -100$\hat{x}$ N/C. What is the potential energy of the dipole?

$\Rightarrow$ 0 J

5.4 Maximum Potential Energy of an Electric Dipole

Suppose I put an object with dipole moment 2.00 C·m in a uniform electric field of magnitude 300 N/C. What is the maximum possible value for the potential energy of the system?

$\Rightarrow$ 600 J

5.5 Multiple Choice

Which of the following statements is NOT true?

A) If a perfect electric dipole is placed into a uniform electric field, it can experience a net force.

B) A perfect electric dipole consists of two equal and opposite point charges separated by a small distance.

C) If a perfect electric dipole is placed into a non-uniform electric field, the dipole can experience a net force.

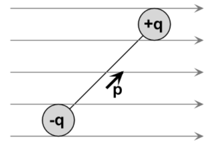
D) If a perfect electric dipole is placed into a uniform electric field, the dipole can experience a net torque.

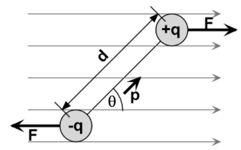
E) If a perfect electric dipole is placed into a non-uniform electric field, the dipole can experience a net force.

$\Rightarrow$ A

5.6 Electric Dipole in an Electric Field

I place an electric dipole in a uniform electric field as shown below. What is the net force on the dipole? What is the net torque on the dipole? What is the electric potential energy of the dipole?





$\Rightarrow \vec{F} = 0, \quad \tau = dFsin\theta = dqEsin\theta, \quad U_E = -pEcos\theta$

6 Electric Flux

6.1 Operators

When we describe electric flux, we say that a surface is oriented in a certain direction with respect to an electric field. When we try to calculate how much electric field passes through the surface, we make use of what operator?

$\Rightarrow$ Dot Product

6.2 Charge Density

Charge per unit volume is defined as _____.

$\Rightarrow$ Charge Density

6.3 Flux Through a Square Sheet

Suppose I put a sheet of paper (10 cm by 10 cm) into a uniform electric field of magnitude E = 2000 N/C. What is the largest possible value for the electric flux passing through the sheet?

$\Rightarrow 20Nm^2/C$

6.4 Flux Through a Sphere

Some region of space contains a uniform electric field given by $\vec{E} = 3\hat{\imath} + 4\hat{\jmath}$ N/C. I place a metal spherical shell of radius R = 1 m into the electric field. What is the net electric flux through the sphere?

$\Rightarrow$ 0 Nm$^2/C$

6.5 Flux Around an Electric Dipole #1

An electric dipole is formed by two point charges (+Q and -Q) located at (1 m, 0 m) and (-1 m, 0 m) respectively. I draw a closed elliptical surface such that the charges +Q and -Q are both located outside the ellipse. What is the net electric flux through the ellipse?

$\Rightarrow$ 0 Nm$^2/C$

6.6 Flux Around an Electric Dipole #2

An electric dipole is formed by two point charges (+Q and -Q) located at (1 m, 0 m) and (-1 m, 0 m) respectively. I draw a closed elliptical surface such that the charges +Q and -Q are both located inside the ellipse. What is the net electric flux through the ellipse?

$\Rightarrow$ 0 Nm$^2/C$