CHAPTER

14

Biomedical Transport Processes

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OUTLINE

AT THE CONCLUSION OF THIS CHAPTER, STUDENTS WILL BE ABLE TO:

- Understand the function of the human lung in terms of mass balance and mass transfer.
- Understand the fundamental equations for mass transfer, including the concepts of diffusion and osmosis.
- Understand the fundamental concepts of mass transfer across pores in systemic capillaries.
- Understand the function of the human kidney and its most basic mass transfer element: the nephron.
- Understand how dialysis systems function and are controlled.
- Understand the role of blood cells in whole blood.
- Understand how the fundamental equations describing the conservation of mass and momentum equations are derived and the underlying physics.
- Understand the three basic elements of heat transfer, including conduction, convection, and radiation.
- Understand the process of heat exchange using a double pipe heat exchanger for heating and cooling of blood.
- Understand how heat is transferred from the core of the body to the periphery and then to the environment.
- Understand how heat transfer occurs in extreme heat environments.

14.1 BIOMEDICAL MASS TRANSPORT

Mass transport in the human body is a vital process that affects how the lungs function in transferring air and its components to the bloodstream, how the capillaries function in transferring nutrients and gases to surrounding body tissues, and how the kidneys function in transferring metabolic waste products and excess water from the blood into the urine. Mass transfer processes also occur in artificial devices such as artificial kidneys (dialysis) and artificial ventilators and respirators. Mass transfer in the body also affects transport across cell membranes, which controls processes in millions of cells affecting every area of the body. The majority of mass transfer occurs across small membranes of thin surfaces in order to shorten the distance over which substances must travel from point A to point B. This is true of cells in the body, which are quite thin, as well as artificial devices whose components are manufactured to be very thin.

14.1.1 Analysis of Respiration and Gas Transport

The human lungs control gas exchange from our environment into the bloodstream by means of pressure and concentration gradients. When we breath, air enters the lungs through a large entrance, the trachea, and eventually branches into smaller and smaller segments until reaching the smallest elements of the lungs, the alveoli. Each of these thin elements is in close proximity to blood in pulmonary capillaries, which are the smallest and thinnest of the blood vessels. With each of the alveoli in close proximity to a pulmonary capillary, the distance for gas exchange is very short, which thus shortens the time by which complete gas exchange occurs. A diagram of the lungs and its branching system is shown in Figure 14.1.

The two human lungs contain approximately 300 to 500 million alveoli, having a total surface area of about 75 m^2 in adults, the size of a tennis court. The branching of the airways into the alveoli represents millions of tiny sacs, which not only represents thinner membranes to speed gas exchange but also more surface area to speed gas exchange. The alveolar network is shown in Figure 14.2.

The lungs may be separated (for the purposes of mass transfer/gas exchange) as a dead space and an alveolar space. The dead space consists of the trachea, the bronchioles, and the bronchi, which are all large segments of the airway where there is air flow but no gas exchange with the bloodstream. The alveolar space is where the actual gas exchange occurs. However, both zones heat the inspired air to body temperature (37°C), as well as humidifying the air. Thus, expired air is heated to body temperature and is usually fully saturated with water vapor. In fact, it is possible to lose up to a half pound per day merely from losing water from the body via respiration. Figure 14.3 shows the relationship between the dead space and the alveolar space.

The amount of air that one inhales and exhales (without exertion) is called the *tidal volume*, which is 500 ml (about a fluid pint) per breath. The typical breathing rate is 12 breaths per minute at rest. The tidal volume represents only that portion of the lung volume where relatively easy breathing (in and out) occurs. Figure 14.4 shows all of the lung volumes and how the tidal volume compares to other (forced) breathing volumes.

Ambient air is approximately 79 percent nitrogen and 21 percent oxygen on a dry basis (not including any water vapor/humidity in the air). With a single breath, oxygen is transferred

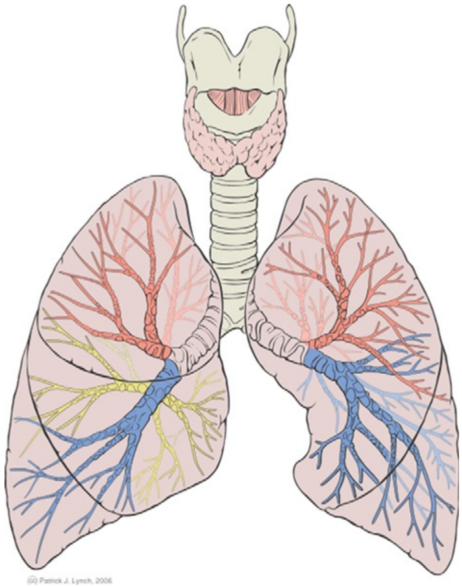

FIGURE 14.1 Branching of the airway network in the human lungs from the trachea ending in alveoli. *Patrick J. Lynch, illustrator; C. Carl Jaffe, MD, cardiologist; Yale University Center for Advanced Instructional Media Medical, Illustrations by Patrick Lynch, generated for multimedia teaching projects by the Yale University School of Medicine, Center for Advanced Instructional Media, 1987–2000.*

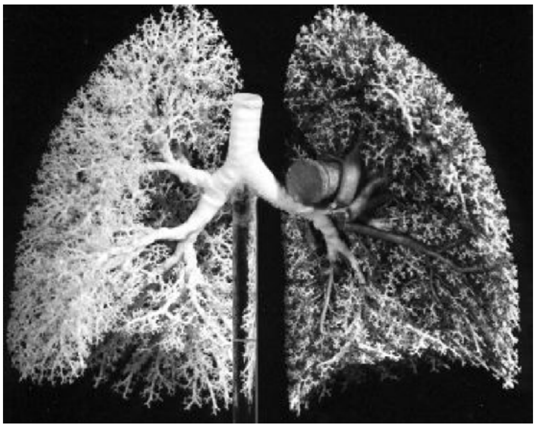

FIGURE 14.2 A cast of the human lungs showing the alveolar sacs. *From the Anatomical Institute, Bern.*

from the air to the pulmonary blood, and carbon dioxide is transferred from the blood to the air, across the alveolar membranes. Nitrogen is not transferred under normal conditions. Figure 14.5 shows the gas exchange between the alveoli and the pulmonary capillaries.

The close spatial relationship between the pulmonary capillaries is necessary to shorten the diffusion distance between the alveoli and the capillaries to ensure adequate gas transfer between the lungs and the capillaries. Figure 14.6 depicts this close spatial relationship.

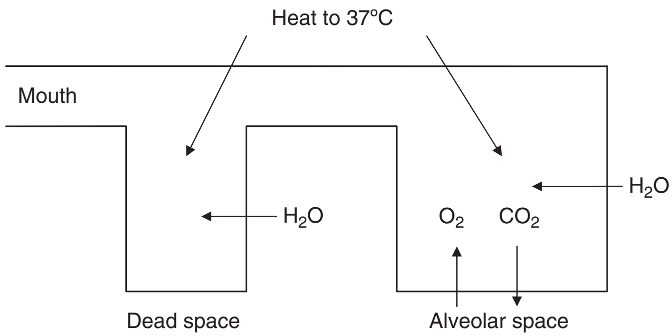

FIGURE 14.3 The dead space and alveolar space in the human lungs. Gas exchange occurs only in the alveolar space, while both zones heat the air and saturate with water vapor.

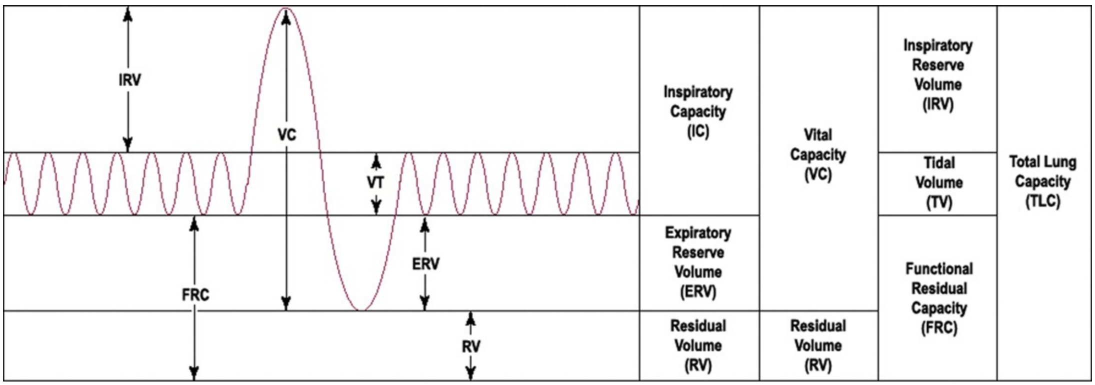

FIGURE 14.4 Lung capacity with various breathing volumes.

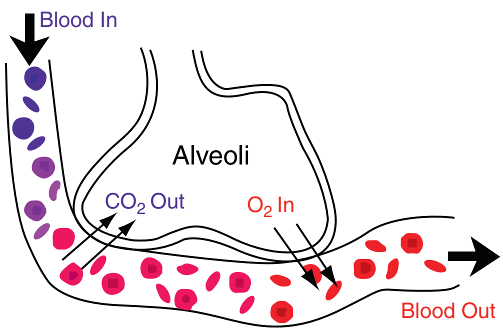

FIGURE 14.5 Gas exchange between the alveoli and the pulmonary capillaries. This event occurs across all of the millions of alveoli and associated capillaries.

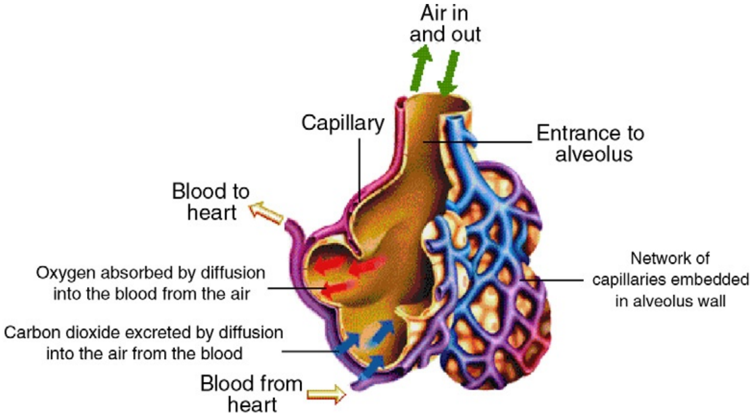

FIGURE 14.6 The close spatial relationship between alveoli and pulmonary capillaries.

The resulting single breath gas exchange between the alveoli and the pulmonary capillaries can be simplified via a mass balance. This is summarized in Table 14.1. The gases (nitrogen, oxygen, and carbon dioxide) are shown with "wet" percentages, which incorporate the water vapor in the air. Note that not all of the oxygen in the air is taken up by the lungs and that the expired gases from the lungs are heated to body temperature and saturated with water vapor—thus the higher water vapor content of the expired air. There is a trace amount of carbon dioxide in the ambient air but a higher percentage in the expired air. Thus, oxygen replaces carbon dioxide in the pulmonary bloodstream.

For the purposes of gas exchange, the components of the inspired air are described by means of their partial pressures. This is the fractional amount of total gas pressure due to the substance being measured. For example, at sea level, the total atmospheric pressure is 760 mm Hg. The amount of this pressure that is due to oxygen is approximately $0.21 \times 760 = 160$ mm Hg. We would say that the partial pressure of oxygen at sea level in dry air (no vapor in the air) is 160 mm Hg. The partial pressure of carbon dioxide in dry air at sea level is $0.03 \times 760 = 22.8$ mm Hg. The maximum amount of water vapor in the air varies with temperature and relative humidity. At body temperature (37°C) air can be saturated up to 47 mm Hg of water vapor pressure. Therefore, in the lung, where air is totally water-saturated, the partial pressure of water vapor would be 47 mm Hg.

TABLE 14.1 Single Breath Analysis for the Components of Air

Air Component	Atmospheric Air %	Expired Air %
Nitrogen and Inert Gases	78.62	74.9
Oxygen	20.85	15.3
Carbon Dioxide	0.03	3.6
Water Vapor	0.5	6.2
Total	100	100

Thus, in terms of partial pressures, and with the ambient humidity equal to a partial pressure of 10 mm Hg (as an example), Table 14.1 can be rewritten as follows: The partial pressure of water vapor is 10 mm Hg, and the partial volume would be equal to (10/760) × 500 ml tidal volume = 6.5 ml. The dry gases (nitrogen and oxygen) represent the remainder of the tidal volume, or 500 − 6.5 = 493.5. Thus, the nitrogen partial volume would be 493.5 × 79% = 389.9 ml, and the partial volume of oxygen would be 493.5 × 21% = 103.6 ml.

The nitrogen is not exchanged in the lungs, and thus its partial volume does not change. However, the oxygen is taken up by the bloodstream, and the carbon dioxide is released by the bloodstream to be exhaled from the lungs. The partial volumes of the exhaled gases are based on the metabolic load of these gases in the body. At rest, these are 284 ml/min of oxygen (at body temperature) and 226 ml/min for carbon dioxide. These are values at body temperature and pressure (BTP). The STP values are 250 ml/min for oxygen and 200 ml/min for carbon dioxide.

The BTP values are calculated by multiplying the STP values by the ration of the temperatures in absolute scale in °K. The STP temperature is 273°K, while the BTP temperature is 273 + 37 = 310°K. Thus, the BTP value for oxygen is 250 × (310/273) = 284, and the BTP value for carbon dioxide is 200 × (310/273) = 226.

The BTP values equal 23.67 ml/breath for oxygen (284/12 breaths per minute) and 18.83 ml/breath for carbon dioxide (226/12). The water vapor during exhalation equals the partial pressure of water vapor in the body (47 mm Hg) compared to the dry gas pressure (760 − 47 = 713 mm Hg) times the dry gas expired volume. Thus, the water vapor attaches to the dry gas during expiration.

Note that in the calculations shown in Table 14.2, the expired dry gas volume is the sum of nitrogen, oxygen, and carbon dioxide. The water vapor attaches to the dry gas. Also note that the dry gas pressure is only 760 – 47 = 713 mm Hg, since the water vapor accounts for the remaining 47 mm Hg. Thus, the ratio of the partial pressures of water vapor to dry gas is 47/713.

The preceding analysis does not consider expansion and contraction due to temperature differences, which do, in fact, exist. This is beyond the scope of this discussion but is available from several other sources.

In summary, gas exchange between the lungs and the pulmonary circulation occurs at the smallest elements of each, which are also in extremely close proximity to each other. With so many alveoli, the surface area for gas exchange is very large, which promotes faster

TABLE 14.2 Single Breath Analysis Based on Partial Volumes

Air Component	Inspired Gases Partial Volume	Expired Gases Partial Volume
Water Vapor	6.5	6.5 + (47/713) × 488.63 = 38.7 ml
Dry Gas	493.5	389.9 + 79.9 + 18.83 = 488.63
Nitrogen	389.9	389.9
Oxygen	103.6	103.6 – 23.67 = 79.9
Carbon Dioxide	0.0	0.0 + 18.83 = 18.83

mass transfer. The mass transfer of each of the components of air depends on the partial pressure of each, as well as the metabolic load of the body, along with the diffusion distance between alveoli and pulmonary capillaries. Oxygen is taken up by the bloodstream, and carbon dioxide is given off to the lungs from the bloodstream. The exhaled gases are heated to body temperature and are fully saturated with water vapor. Thus, when you place your hand over your mouth as you exhale, the air feels warm and moist.

14.1.2 Membranes, Pores, and Diffusion

The gas exchange between the alveoli and the pulmonary capillaries is affected by the partial pressures of each gas, the distance over which mass transfer occurs, and the surface area for mass transfer. We can think of mass transfer as occurring across a membrane in a steady-state fashion. This can be described by the one-dimensional, steady-state form of Fick's Law:

$$\text{Mass exchange rate} = D\ A\ dC/dx$$

where D is the *diffusivity*, A is the surface area for mass transfer, and dC/dx is the concentration gradient for mass transfer.

For gases, the concentration gradient is the difference in the partial pressure. For a liquid or for substances dissolved in a liquid (such as in blood or extracellular fluid), the concentration gradient can be approximated by dC/dx, the difference in concentration over a specified distance. The diffusivity (D) is a parameter that describes the relative ease by which a substance moves throughout the "medium"—the fluid through which the substance moves. An example of this is how sodium ions move through extracellular fluid. The diffusivity is thus affected by both the medium and the substance to be transferred; it is a material property of both.

For gases, mass transfer occurs across the entire membrane, since gases are lipid soluble, with membranes consisting of a layer of lipids and proteins. However, when the exchange is of a liquid or a substance dissolved in a liquid (such as an ion), then the mass transfer must occur through pores in the membrane. That is because most liquids are not lipid soluble. When substances do indeed travel across pores, then there is selective mass transfer due to the size of the pore as compared to the size of the substance that might transfer. This selective mass transfer by means of the relative sizes is called *filtration*. Fick's Law is modified to account for such selective mass transfer by adding a permeability term (P), which relates the area of the pores as compared to the total area of the membrane. At times, the area for mass transfer (A) is instead described as A_P rather than as the product of $A \times P$. An example of filtration is shown in Figure 14.7.

In general, the process by which mass transfer occurs via a concentration gradient is called *diffusion*. Diffusion is the macroscopic result of random thermal motion on a microscopic scale. For example, in Figure 14.8, oxygen and nitrogen molecules move in random directions, with kinetic energy on the order of kT, where k is the thermal conductivity and T is the absolute temperature. If there are more oxygen molecules on the left side of the plane A-A than on the right, more molecules will cross to the right than to the left; there will be a net movement even though the motion of each individual molecule is completely random. Diffusion in an open environment such as within extracellular fluid, and not constrained by a membrane for mass transfer, is shown in Figure 14.8.

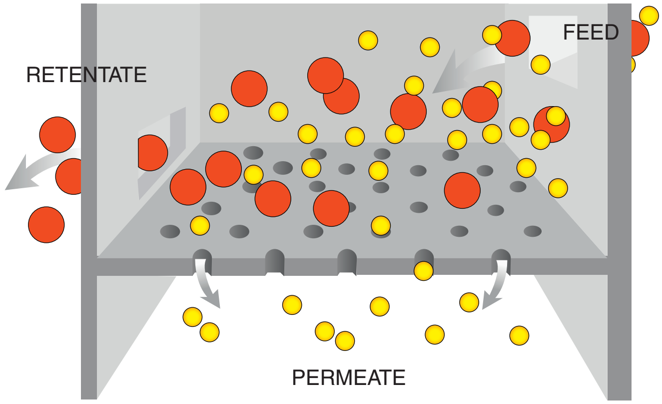

FIGURE 14.7 Filtration—selective mass transfer by means of pore size and size of substances compared to pore size.

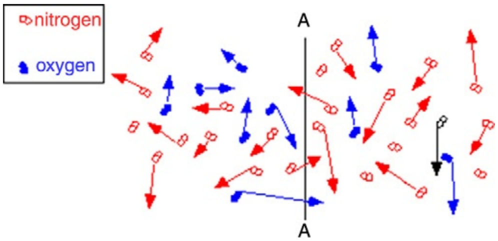

FIGURE 14.8 Random movement of particles resulting in a net movement across a dividing line based on relative concentrations on both sides of the line. *From Time Domain CVD.*

Again, the net movement of any substance (within a mixture of substances as just shown) across line A-A is a function of the concentration of that substance on each side of the line. It is certainly possible that the net movement of one substance is in the opposite direction of another, each based on its own concentration gradient. This is also true for mass transfer across pores. The concentration gradients control the amount and the direction of mass transfer for each substance. When a substance, such as an ion, is in a solution (such as within extracellular fluid), then the substance is known as a *solute* and the fluid as a *solvent*. Mass transfer of water is known as *osmosis*.

Further, the relative size of the solute as compared to the pore size may affect the potential mass transfer by either (1) completely restricting the movement (if the species is larger than the pore size) or (2) merely restricting it because it may be nearly equal to the pore size. In such cases, Fick's Law can be modified to account not only for the limited pore area (via the permeability) but also via the pore to species size, which is given by the *restrictive diffusivity* (D_R). Thus, Fick's Law could be rewritten as

$$\text{Mass exchange rate} = D_R\, A_P\, dC/dx$$

A typical cell membrane is thin (80–100 Å thick) and elastic. Membranes typically consist of 60 to 65 percent proteins, 30 to 35 percent lipids, and 5 to 10 percent polysaccharides and

TABLE 14.3 Relative Diffusion Rates as a Function of Size of Species Compared to the Size of a Pore

Substance	Diameter (Å)	Ratio to Pore D	Relative Diffusion Rate
Water	3	0.38	50,000,000
Urea	3.6	0.45	40,000,000
Chloride ion	3.86	0.48	36,000,000
Potassium ion	3.96	0.49	200
Sodium ion	5.12	0.64	100
Glycerol	6.2	0.77	1
Galactose	8.4	1.03	0
Glucose	8.6	1.04	0
Lactose	10.8	1.35	0

From Cooney (1976).

other elements. The lipids include phospholipids (65 percent), cholesterol (25 percent), and other lipids (10 percent). As was previously mentioned, substances that are not lipid soluble must travel through pores within the cell membrane wall. Such transport of these substances depends on their relative size with respect to the pore size. Table 14.3 lists various substances that can only travel through pores and their relative diffusion rates, which are related to their permeability. An 8 Å pore diameter is chosen for the purposes of comparison of diffusion rates.

Note that potassium and sodium have smaller diffusion rates despite their size, due to a positively charged electrical potential. The pores also have a positive charge that affects the transport of positively charged ions. The transport of substances is primarily due to the substance's individual concentration gradients across the cell membrane. However, substances such as ions, which are suspended in water, are affected not only by their concentrations and charge but also by their diffusivities. The *Nernst equation* can be used to compute the relative electrical potential across a cell as a function of the concentrations of the ions as well as their diffusivities. The Nernst equation for the three primary ions (Na, K, Cl) is as follows:

$$-E = \frac{RT}{F} \ln \frac{D_{Cl}Cl_1 + D_K K_2 + D_{Na}Na_2}{D_{Cl}Cl_2 + D_K K_1 + D_{Na}Na_1}$$

where R is the gas constant, T is the absolute temperature, and F is Faraday's constant.

Using the resting concentrations of the ions inside and outside a cell membrane, it is possible to compute the resting electrical potential across a cell. Note that the inside and outside concentrations of the positive ions (Na, K) are opposite to those of the negatively charged ion (Cl). With RT/F = 26.5 mv (at body temperature), the resting transmembrane potential is

$$E = -26.5 \ln \frac{(1/60)(103) + (1)(141) + (1/75)(10)}{(1/60)(4) + (1)(5) + (1/75)(142)} = -74 \text{ mv}$$

As can be seen, the transport of ions across a cell membrane is affected by their charges and relative diffusivities as much as by their concentration gradients. Note that the diffusivities of all ions are normalized to that of potassium (1).

14.1.3 Mass Transport in Systemic Capillaries

Capillaries, the smallest and most numerous of the blood vessels, form the connection between the vessels that carry blood away from the heart (arteries) and the vessels that return blood to the heart (veins). The primary function of capillaries is the exchange of materials between the blood and tissue cells. Gas exchange occurs across capillary walls in a manner similar to that which occurs across alveoli and pulmonary capillaries. A partial pressure gradient exists for each gas (primarily oxygen and carbon dioxide) that controls the gas exchange process. As with pulmonary capillaries, the gas transfer across systemic capillaries occurs across the entire membrane wall, since the gases are lipid soluble.

However, unlike the pulmonary capillaries, mass transfer across systemic capillaries also includes liquids and ions (dissolved or free floating within the liquid) that travel across pores in the capillary walls. In addition to forming the connection between the arteries and veins, systemic capillaries have a vital role in the exchange of gases, nutrients, and metabolic waste products between the blood and the tissue cells. Substances pass through the capillary wall by diffusion, filtration, and osmosis. Oxygen and carbon dioxide move across the capillary wall by diffusion regulated by the partial pressure differences. Fluid movement across a capillary wall via the pores is determined by a combination of hydrostatic and osmotic pressure. The net result of the capillary microcirculation created by hydrostatic and osmotic pressure is that substances leave the blood at one end of the capillary and return at the other end, as the hydrostatic pressure drops along the length of the capillary, and thus the pressure difference (hydrostatic–osmotic) is different from the beginning of the capillary (arteriole side) to the end (venule side).

Thus, the driving mechanism for this mass transfer is twofold. The pressure (called the *hydrostatic pressure*) is a pushing pressure. Opposite in direction is the *osmotic pressure*, which is a pulling pressure that results from the concentration difference of substances that cannot fit through the pores. These are the nondiffusible components. These two pressure differences acting across the tube-like pores of the capillary are shown in Figure 14.9.

The capillaries form a network of minute vessels between arterioles and venules so as to maximize mass transfer by (a) shortening the distance for mass transfer and (b) maximizing the overall surface area for mass transfer (Figure 14.10). The anatomy of capillaries is well suited to the task of efficient exchange. Capillary walls are composed of a single layer of endothelial cells. The thin nature of the capillary wall facilitates efficient diffusion of oxygen and carbon dioxide, as well as containing short pore lengths to facilitate the bulk motion of liquids and dissolved ions (Figure 14.11).

As was just stated, the hydrostatic pressure in a capillary is higher on the arteriole end of the capillary and lower on the venule end. This is due to the normal pressure gradient along a pipe or tube that produces the axial (down the vessel) flow rate. In fact, a pressure drop is required in order to propel the blood downstream. As a result, the higher hydrostatic pressure near the arteriole end is greater than the osmostic pressure, and the net pressure is outward—pushing fluid out of the capillary. As the hydrostatic pressure drops

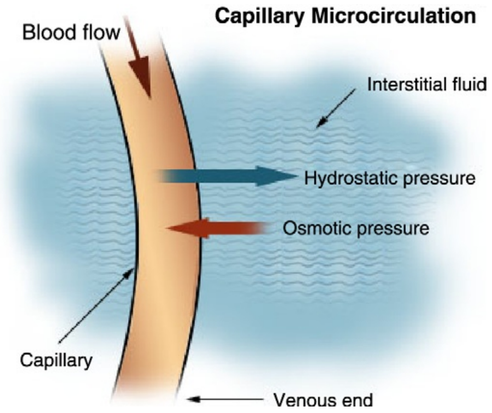

FIGURE 14.9 Mass transfer across systemic capillaries.

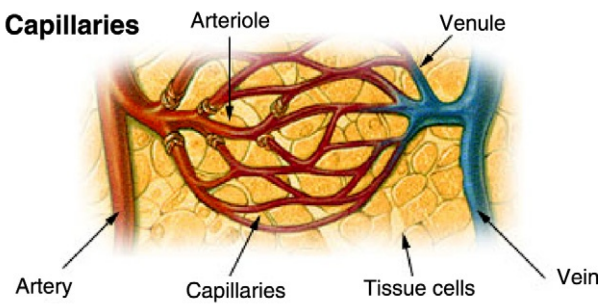

FIGURE 14.10 Capillary network allowing close proximity to tissues, which shortens the distance for mass transfer.

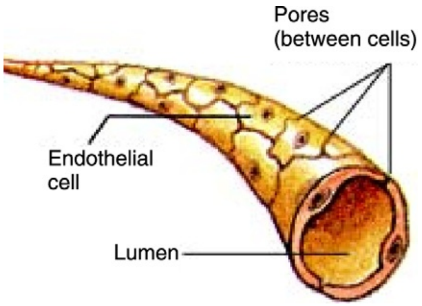

FIGURE 14.11 Thin wall of a systemic capillary that facilitates mass transfer by creating a short distance for transport.

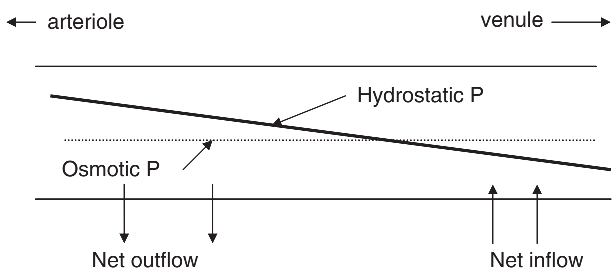

FIGURE 14.12 Relative inflow and outflow of liquids across capillary pores as a function of the relationship between the decreasing hydrostatic pressure and the constant osmostic pressure.

along the length of the capillary, the pressure difference (hydrostatic–osmotic) is reduced and eventually reverses as blood reaches the venule end. Thus, there is a net inflow of fluid at the venule end (into the capillary) (Figure 14.12).

In addition, arterioles are known as the resistance vessels in the circulatory system in that they can constrict or dilate in response to neural and/or hormonal feedback, given their large smooth muscle composition. Arterioles constrict in response to a decrease in arterial pressure and dilate in response to a rise in arterial pressure. When arterioles constrict, they produce an added resistance to blood flow, which drops the hydrostatic pressure downstream. This is because the pressure gradient is proportional to the blood vessel resistance. When the downstream pressure at the end of arterioles is reduced, this corresponds to the inlet pressure to capillaries. Therefore, with arteriolar constriction, the hydrostatic pressure is reduced all along the capillary, which results in less pushing pressure out of the capillary. This produces a net increase in fluid entering the capillary and eventually leaving the capillary via blood flow into the venules. With the increase in fluid level, the blood volume increases, which results in greater venous return of blood into the heart and a resulting increase in cardiac output and arterial pressure (Figure 14.13).

With dilation of the arterioles, the opposite effect is created. The resistance of the arteriole is reduced, which produces a smaller pressure gradient across the arteriole. This then

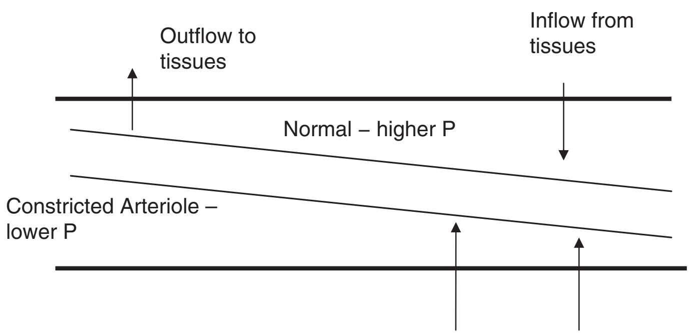

FIGURE 14.13 Hydrostatic pressure gradient inside a capillary for a normal condition and for a constricted arteriole. The hydrostatic pressure is reduced all along the capillary from the resulting reduction at the inlet. This produces a greater inflow of fluid into the capillary.

TABLE 14.4 Relationship between Arteriole Diameter, Capillary Hydrostatic Pressure, Transcapillary Fluid Flow, and Arterial Pressure

Initial Arterial Pressure	Arteriole Diameter Change	Capillary Hydrostatic Effect	Net Fluid	Venous Return	Cardiac/Arterial Output/ Pressure
Low	constrict	reduced	into capillary	increase	increase
High	dilate	increased	out of capillary	decrease	decrease

produces a rise in the inlet hydrostatic pressure to capillaries, which results in a greater pushing pressure for fluid moving out of the capillary and into the tissues and extracellular fluid. With a net increase in fluid leaving the capillaries, there is a reduction in blood volume and in venous return. This results in a reduction in cardiac output and in arterial pressure. Table 14.4 summarizes the cause and effect of changes in arteriole diameter.

When there is an increase in fluid leaving the capillary as a result of arteriole dilation, then the extracellular fluid pressure rises. The increase in fluid volume will eventually cause edema, a buildup of fluid that produces swelling and potential damage to cells and tissues. To alleviate this condition, there is a network of lymphatic vessels that parallels the circulatory system. The peripheral lymphatic vessels are similar in size to capillaries and are networked alongside the capillaries. Their function is to collect excess fluid from extracellular spaces and transport it along the lymphatic vessel network of larger and larger vessels until the lymph reaches the vena cava, where it is returned to the bloodstream. The lymph flow is normally quite small compared with the blood flow. Even the terminal lymph flow is quite small (1 ml/hr) compared with the capillary axial blood flow (1 ml/sec). The terminal lymphatic vessels located in close proximity to systemic capillaries are shown in Figure 14.14.

Propulsion of lymph through the larger vessels is produced by contraction of the walls of the vessels in a fashion similar to that provided for flow through the human intestines and somewhat similar to the action of muscles on veins in the cardiovascular system. Lymph vessels, like the venous system, have valves within the vessels to limit backflow and to promote forward flow along the lymphatic circulation. This is in contrast with flow through the bloodstream, which is produced by a driving hydrostatic pressure gradient from the aorta through arteries, arterioles, capillaries, venules, and veins. The lymph flow originates in the terminal vessels and therefore has no embedded driving pressure gradient. The contraction of the lymph vessels in concert with the added fluid volume (from all terminal lymph vessels joining together within the network) produces the required lymph flow headed to the vena cava.

14.1.4 Mass Transport in the Kidneys and Dialysis

As with the alveoli in the lungs, mass transfer in the kidneys is also processed via a large array of extremely small elements. Each of these elements is a *nephron*. Although nephrons and alveoli represent the essence of mass transfer in the body—small size and large overall surface area—the nephron is quite different from an alveolus. The alveolus is solely

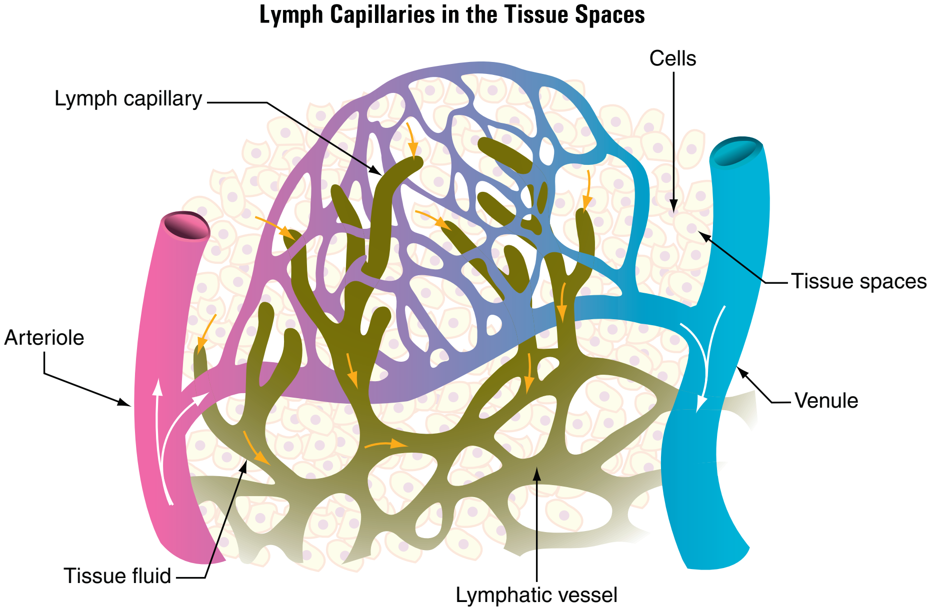

FIGURE 14.14 Lymphatic vessels embedded within the systemic capillary bed.

responsible for mass transfer of gases in one dimension and one direction. Mass transfer in the nephron includes the movement of ions, water, and metabolic waste products, such as urea, uric acid, and creatinine. Furthermore, the nephron initially removes more fluid and ions than the body can safely lose. It is the mechanism by which much of this fluid and ions are reabsorbed into the bloodstream that makes the nephron and the kidneys a unique mass transfer system. There are millions of nephrons in both kidneys (Figure 14.15).

The nephron consists of several segments. The initial filter is the glomerulus, which is embedded inside Bowman's capsule (Figure 14.16).

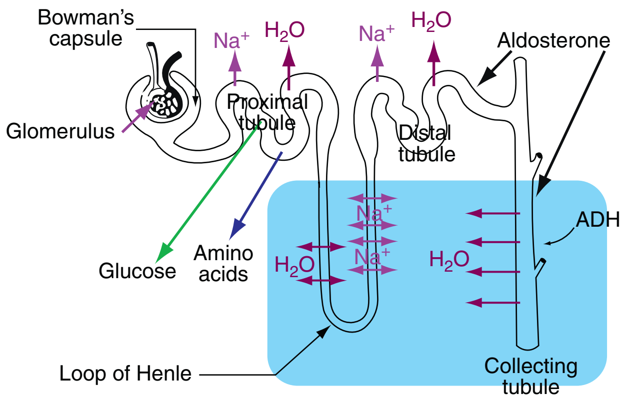

FIGURE 14.15 A nephron in the human kidneys.

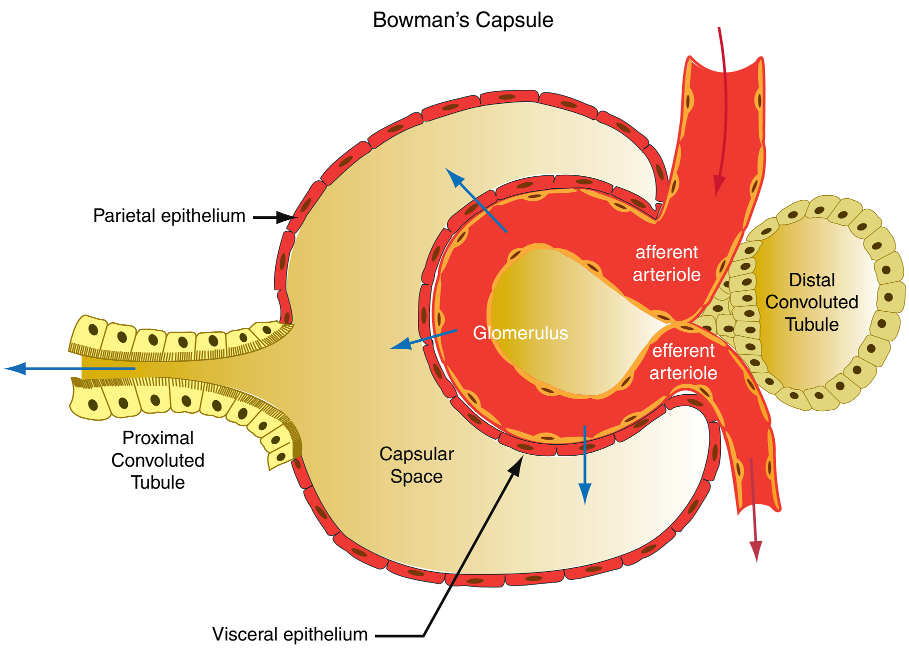

FIGURE 14.16 The glomerulus inside Bowman's capsule.

The glomerulus contains very large pores, each 50 angstroms in diameter and 500 angstroms long. These large pores allow a cumulative 125–150 ml/minute of filtered blood through to the remainder of the nephron. These pores allow water, ions, and metabolic waste products through, but they are too small to allow blood cells, proteins, and large sugars to pass. Once the fluid passes through the glomerulus, it is then known as *filtrate*. Thus, the term *glomerular filtration rate* (*GFR*) corresponds to the 125–150 ml/min traveling through glomerular pores into the tubules of the nephron. The pressure gradient across the glomerulus that causes the filtrate to pass through the glomerular pores is 50 mmHg with a back pressure (osmotic pressure) of 25 mm Hg for a net 25 mm Hg. One can estimate the number of glomeruli in both kidneys by using a model for steady flow through a tube: Poiseuille flow. The Poiseuille flow equation, which will be described later, is

$$Q\,(\mathrm{GFR}) = -n\pi r^4 \Delta P/8\mu L$$

where r is the pore radius, μ is the fluid viscosity, L is the pore length, and n is the number of total pores.

The pore radius is 25 angstroms (25×10^{-8} cm), the pressure gradient is 25 mm Hg (3.33×10^4 g/cm-sec^2), the pore length is 500 angstroms (500×10^{-8} cm), and the filtrate viscosity is 0.012 g/cm-sec.

Obviously, one cannot survive if 125 ml/minute of fluid were to leave the body. Thus, most of this fluid is reabsorbed back into the bloodstream, with the concentrated remainder resulting in urine. Normally, up to 124 ml are reabsorbed, resulting in a urine output of 1 ml/min, which is stored in the bladder.

The remainder of the nephron includes the proximal tubule, the loop of Henle, the distal tubule, and finally the collecting duct, where urine is formed and collected. At each segment of the nephron, water and ions are removed from the tubules to be reabsorbed into the bloodstream. The end result is more concentrated filtrate culminating in urine. Blood hormones control the amount of ions and water that are reabsorbed into the bloodstream, which results in more or less dilute urine. A more detailed description of mass transfer of ions and water from the tubules of the nephron is shown in Figure 14.17.

Parallel to the loop of Henle in the nephron is a loop of capillaries, called the *peritubular capillaries* or *vasa recta*. It is here where the water and ions in the filtrate are reabsorbed into the bloodstream. This parallel network is shown in Figure 14.18.

Mass transfer within the tubules of the nephron is a combination of concentration-driven diffusion as well as active transport. The loop of Henle is configured in a countercurrent approach that allows for some substances that leave the descending loop of Henle to be reabsorbed into the ascending loop in order to maximize the concentration of waste products by controlling water transport. The water transport follows ion transport so as to equalize the concentrations of both ions and water on either side of the tubule. Thus, the concentration at the bottom of the loop of Henle is very different from either top ends (both descending—near the proximal tubule and ascending—near the distal tubule). This is shown in Figures 14.19a and b.

The process by which waste products, water, and excess ions are removed from a failing kidney is *dialysis*. Dialysis is a means of artificially cleaning the blood by accessing the

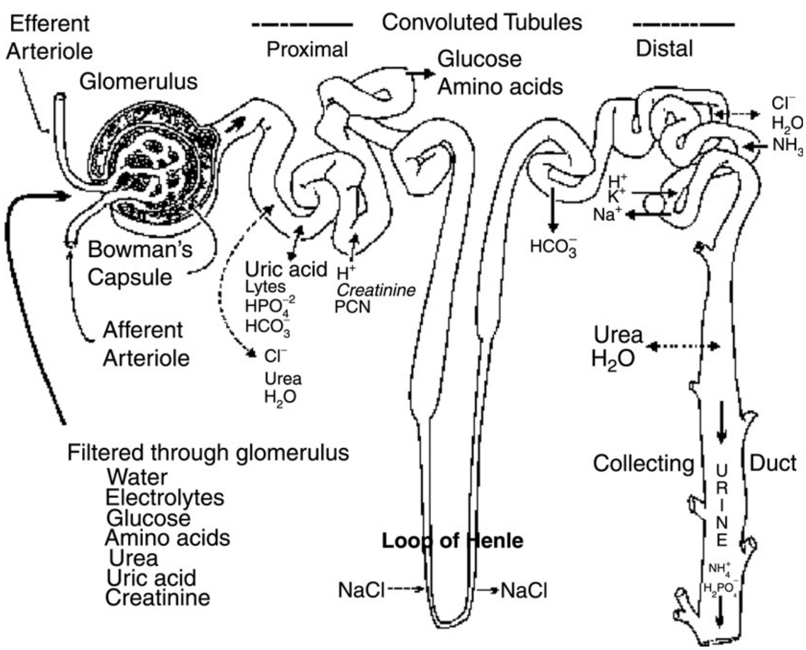

FIGURE 14.17 Mass transfer of ions, water, and waste products along the length of the nephron and its segments.

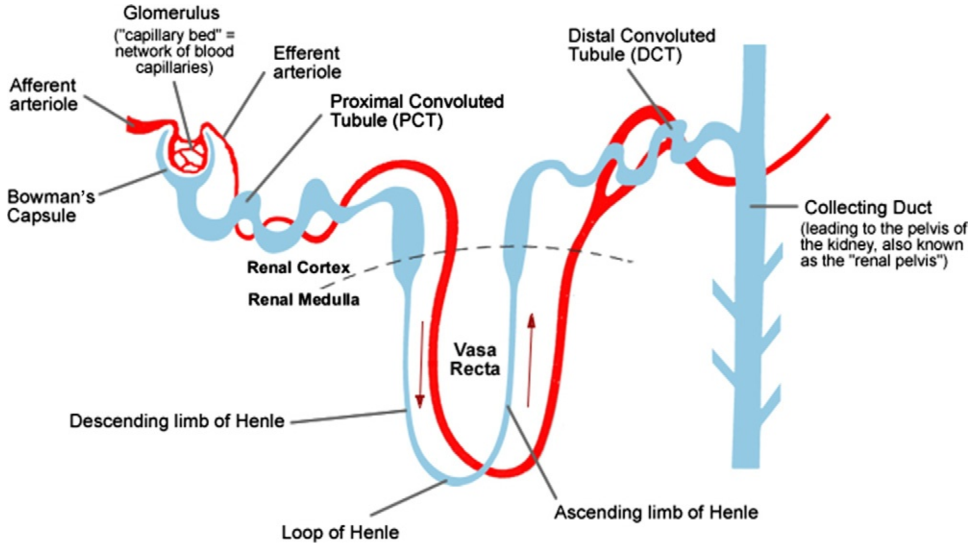

FIGURE 14.18 Vasa recta parallels the loop of Henle for reabsorption into the bloodstream.

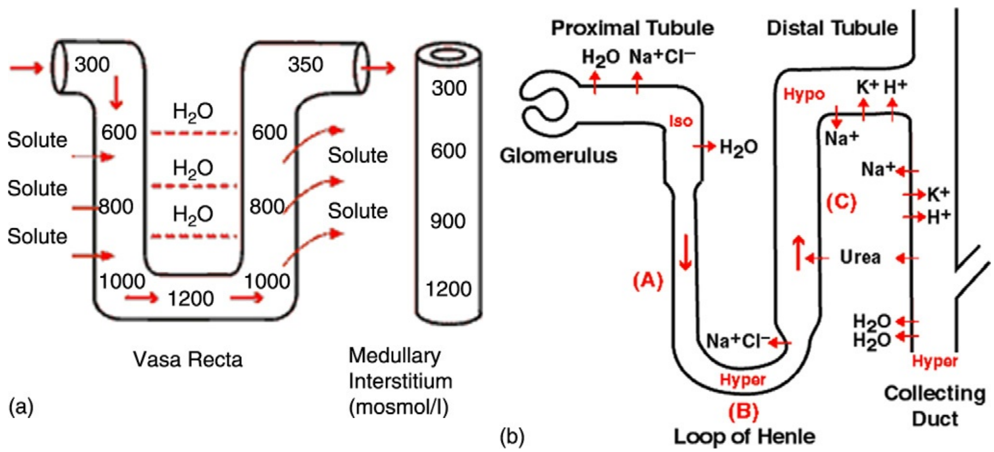

FIGURE 14.19 (a) Changes in osmolarity. (b) Countercurrent system.

radial artery for blood flow into a dialyzer cartridge embedded in a dialysis machine. The resulting clean blood is then returned through the cephalic vein into the body. This is shown Figure 14.20a, with a plastic shunt between the artery and the vein to protect the vessels from continual puncture during chronic dialysis shown in Figure 14.20b.

Within the dialysis cartridge are thousands of cellulose capillary tubes, each with pores. Blood flows inside these capillary tubes, and a concentration-driven diffusion occurs across the pores, which are large enough for transport of water, ions, and the metabolic wastes but too small for blood cells, proteins, or other vital blood components. Inside the cartridge is a

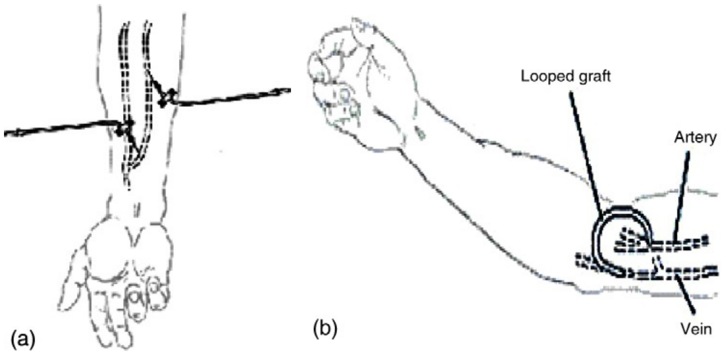

FIGURE 14.20 (a) Blood access for dialysis through the radial artery and the cephalic vein. (b) Plastic shunt connecting the radial artery to the cephalic vein.

fluid that accepts the transported substances. This fluid is called *dialysate*. The process of dialysis is shown in Figure 14.21. The dialysis cartridge is shown in Figure 14.22, which depicts both the blood flow and the dialysate flow pathways.

The mass transport in the radial direction across the dialyzer cartridge capillary tube pores is a concentration-driven diffusion process from blood to dialysate. Blood has a high

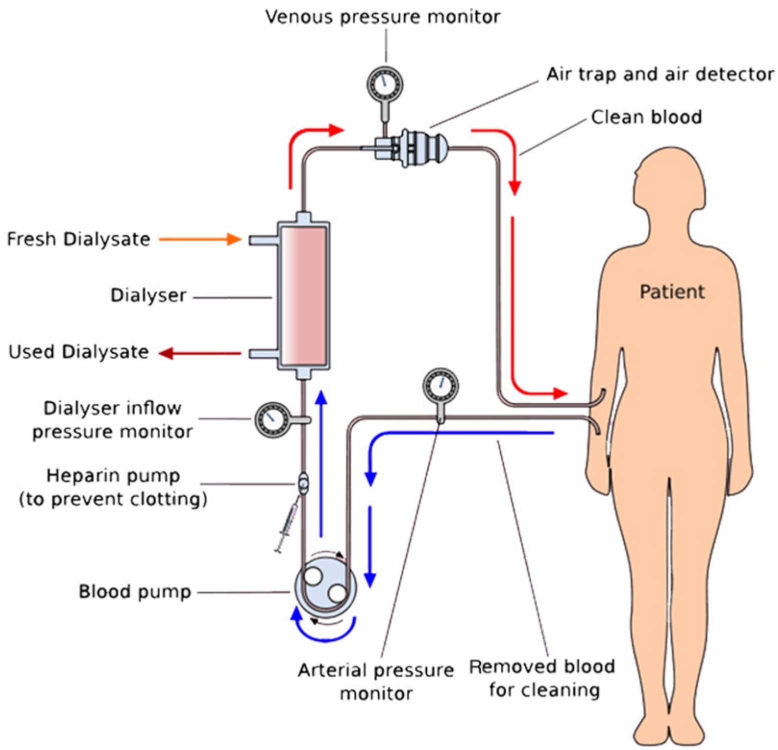

FIGURE 14.21 Dialysis system.

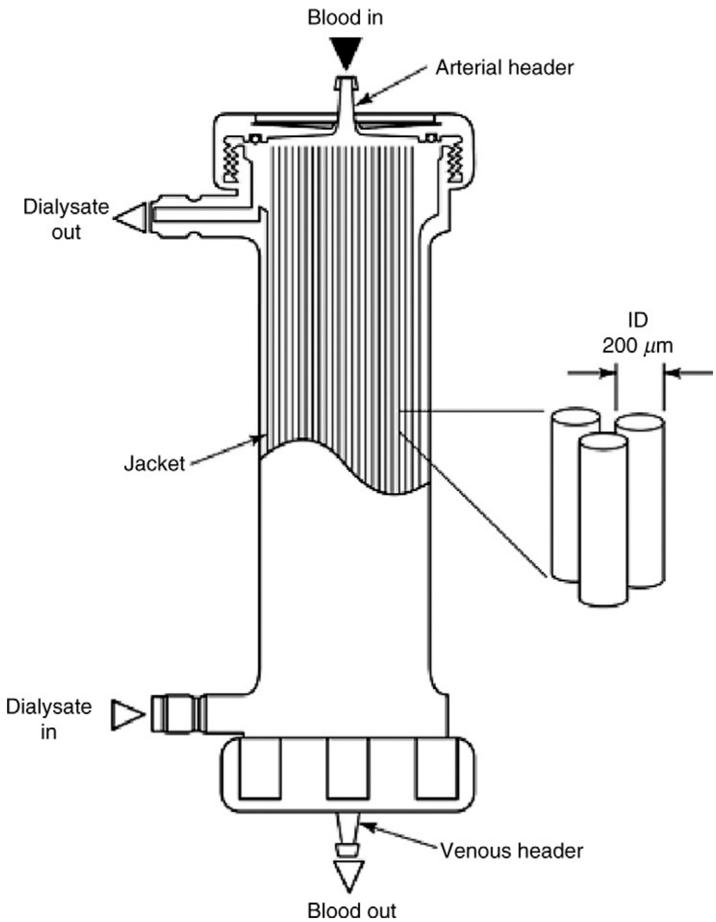

FIGURE 14.22 Dialyzer cartridge depicting blood and dialysate pathways.

concentration of wastes, and dialysate is waste free. Blood has a high concentration of ions, and dialysate has a normal concentration (Figure 14.23).

The solubility of diffused substances within the capillary tube pore (labeled membrane in Figure 14.23) is different from that of the freely moving substances within blood and/or dialysate. Thus, there is a discontinuity in the concentration of substances within the membrane, which is noted by a membrane distribution coefficient. Note that the diffusion is from high concentration within the blood toward the lower concentration within the dialysate. There is also a free flow zone for both blood and dialysate (where the concentration is flat, indicating no mass transfer), as well as a boundary layer near the wall of the capillary tube (where the concentrations are now altered due to mass transfer). The boundary layer produces slower axial flow rates and allows for the radial mass transfer to proceed. The slower flow rates near the wall of the capillary tube within the boundary layer are due to shearing forces within flowing blood. This will be described in the subsequent

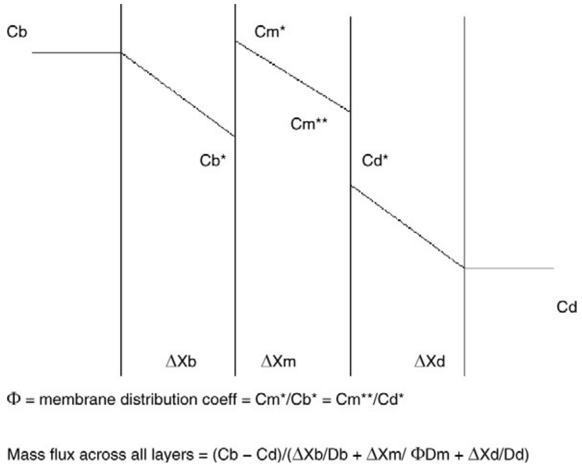

FIGURE 14.23 Diffusion of wastes, ions, and water in the radial direction across a dialysis capillary tube pore.

section on fluid flow (momentum transport). The dialysate flow rate is typically much higher than the blood flow rate in order to move the "dirty" dialysate (now having received the wastes and excess ions from the blood) out of the dialyzer cartridge and bring in clean dialysate. This keeps the concentration gradient for mass transfer large. A typical blood flow rate through the cartridge is 200 ml/min, while the dialysate flow rate is 500–800 ml/min.

Blood entering the top of the dialyzer cartridge is cleared of wastes and excess ions by the time blood leaves through the bottom of the cartridge. However, this blood is then returned to the bloodstream to be mixed with the remainder of the blood. Therefore, in order to completely clean the entire blood supply within the body, dialysis requires four hours to complete.

Fick's Law describes the concentration-driven diffusion between blood and dialysate across the pores of the capillary tubes. As such, each layer of Figure 14.23 indicates a diffusivity (D), a thickness of the diffusion zone (∇X), and a concentration gradient ($C_b - C_b{}^*$), which are all factors associated with one-dimensional Fick's Law. Each layer has its own diffusivity, since D is a function of both the substance to be transported along with the material through which it moves. Each layer has its own diffusion thickness and concentration gradient. It is true that the mass transfer of a given substance is the same for all three layers, since there is no accumulation of any substance in an individual layer. Therefore, an overall mass transfer rate across all three layers can be derived as shown on the bottom of Figure 14.23 that uses the overall concentration gradient but incorporates the individual diffusivities and individual thicknesses.

Water transport via dialysis cannot be conducted by means of concentration-driven diffusion, since the water concentration gradient is in the wrong direction (from dialysate toward blood). Therefore, water transport from blood to dialysate is controlled by means of a hydrostatic pressure gradient, as shown in Figure 14.24.

The water transport is actually controlled via the dialysate flow rate, per the Bernoulli effect, similar to the way a vacuum cleaner works. By having a large flow rate near an

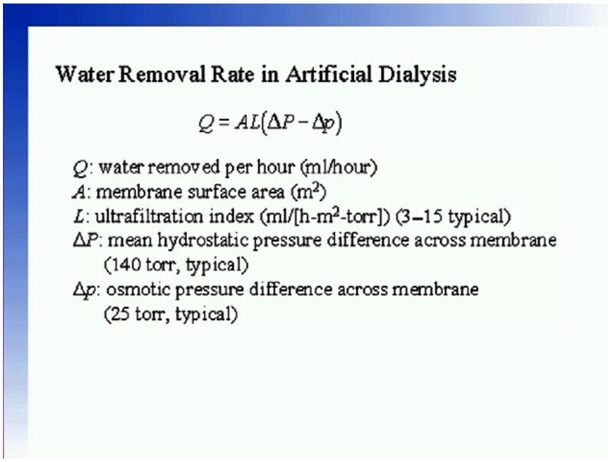

FIGURE 14.24 Water removal during dialysis via a hydrostatic pressure gradient and an osmotic pressure gradient.

opening at right angles to that flow, there is a vacuum created at the opening, thus pulling the water out of the blood and into the dialysate.

As can be seen from this section, mass transfer across membranes, either within the body or within artificial organs, is vital to keeping us alive. Mass transfer allows cells to control nervous impulses, blood chemistry, kidney and lung function, and hemostasis of bodily functions. Mass transport occurs in virtually every cell in the body and is thus evaluated on a smaller scale than that of fluid flow and fluid momentum, which is the subject of the next section.

14.2 BIOFLUID MECHANICS AND MOMENTUM TRANSPORT

We have already seen examples of mass transport of substances across membranes within the human body and through those of artificial organs, such as dialyzers. In addition to mass transport, there are numerous examples of *momentum transport*, with momentum being the product of mass times velocity. This momentum transport is manifested in fluid, such as air flow in the lungs and blood flow through the human circulatory system. Much as there were guiding equations, such as Fick's Law, to govern mass transfer, there are also guiding equations to govern fluid flow. In addition, much as there were parameters that affected mass transfer and were properties of a given material (diffusivity is an example), such is also the case for momentum transfer/fluid mechanics. Like diffusivity is a proportionality constant for mass transfer, so is *viscosity* a proportionality constant for fluid mechanics. In mass transfer, the driving force was either a concentration gradient (for liquids or solutes embedded in solvents) or a partial pressure gradient (for gases). For fluid flow and momentum transport, the driving force is a pressure gradient. This is not a partial pressure gradient but rather a standard pressure gradient, often called the hydrostatic pressure.

However, the total pressure gradient for a flowing fluid is the sum of the hydrostatic pressure, the dynamic pressure (related to the fluid velocity), and the pressure associated with gravity and a height differential. The gravity component is easily described by the height difference in a fluid manometer. An example of a manometer, which can be used to measure a pressure difference as a result of the density and gravitational difference between two fluids, is shown in Figure 14.25.

The total pressure of a flowing fluid is given by

$$P_T = P\,(\text{hydrostatic}) + \tfrac{1}{2}\rho V^2 + \rho g h$$

Where the hydrostatic pressure is what is normally thought of as "pressure," the dynamic pressure is given by ½ ρV^2, and the last term is the pressure as a result of a height differential and is measured by a manometer. A pressure transducer can be used to measure the hydrostatic pressure, a flowmeter is used to measure the fluid velocity (flow rate divided by vessel cross-sectional area) resulting in the dynamic pressure, and a manometer for the gravity term. Depending on the density of the fluid and the value for fluid velocity, it is possible that one of the three pressure components can far outweigh the others. The Bernoulli equation is a modification of the total pressure equation. It states that the total pressure at two points is the same or that the sum of the three pressures is equal, as shown following.

$$P_1 = \tfrac{1}{2}\rho {V_1}^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho {V_2}^2 + \rho g h_2$$

14.2.1 Blood and Fluid Viscosity

Viscosity is a measure of the resistance of a flowing fluid that is being deformed by either shear stress or normal stress. In general terms, it is the resistance of a liquid to flow or its "thickness." Viscosity describes a fluid's internal resistance to flow and may be thought

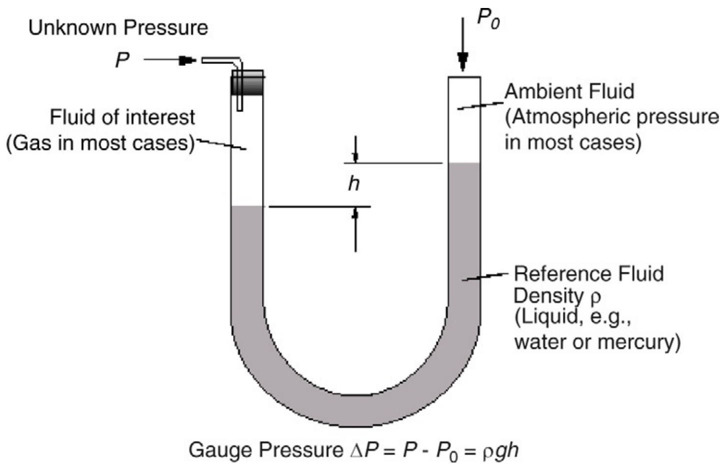

FIGURE 14.25 Fluid manometer used to measure the density/gravitational pressure difference between two fluids with varying heights within the manometer. The pressure gradient is a function of the density multiplied by the height difference in the two fluids.

of as a measure of fluid friction. Thus, water is "thin," having a lower viscosity, while vegetable oil is "thick," having a higher viscosity. All real fluids (except superfluids, such as liquid nitrogen) have some resistance to stress and fluid flow, but a fluid that has no resistance to shear stress is known as an ideal fluid or inviscid fluid. A real fluid with a non-zero viscosity is called a viscous fluid. The study of viscosity of a real fluid is known as rheology.

Isaac Newton postulated that for straight, parallel, and uniform flow, the shear stress, τ, between fluid layers is proportional to the velocity gradient, $\partial u/\partial y$, in the direction perpendicular to the layers.

$$\tau = \mu \frac{\partial u}{\partial y}$$

Here, the constant μ is known as the *coefficient of viscosity*, the *viscosity*, the *dynamic viscosity*, or the Newtonian viscosity. Many fluids, such as water and most gases, satisfy Newton's criterion and are known as Newtonian fluids. Non-Newtonian fluids exhibit a more complicated relationship between shear stress and velocity gradient than simple linearity. Many such non-Newtonian fluids have a high concentration of suspended particles. Blood may be one such fluid, since blood contains almost 50 percent of its volume as blood cells. In the metric system, also known as the cgs (centimeter-gram-second) system, the unit of viscosity is the *poise*, named after Jean Louis Marie Poiseuille, who formulated Poiseuille's law of viscous flow. The value of viscosity is so small for many fluids (liquids and gases) that their viscosity is often listed in cP, centipoise, or one-hundredth of a poise. A poise is equal to one dyne/cm-sec.

As an example, the viscosity of water is approximately 1 cP with a density of 1 g/cc. Blood has a viscosity of 3–4 cP, with a density of 1.05 g/cc. As can be seen, although the densities of water and blood are fairly similar, their viscosities are not. This is due to the large suspension of cells within blood, which adds to the resistance to blood flow. The more solid/liquid boundaries in a fluid, where the shearing stresses tend to be higher, the higher the viscosity. The cells within blood add numerous "solid"/liquid boundaries. Denser pure fluids, such as honey and molasses, are thicker fluids with a much higher viscosity than water or blood. When a pure liquid (without a suspension of particles) has a higher density, its viscosity also tends to be proportionally higher.

Fluids with large suspensions, such as blood, have viscosities that are proportional to the concentrations of particles. For blood, this is the hematocrit, which is the percentage of red cells by volume in whole blood. Normally, the hematocrit is 45 percent. Figure 14.26 depicts the relationship between blood hematocrit and viscosity.

The blood consists of a suspension of cells in a liquid called *plasma.* In an adult man, the blood is about $\frac{1}{12}$ the body weight, and this corresponds to 5–6 liters. Blood consists of 55 percent plasma and 45 percent cells, sometimes called *formed elements*. The blood performs many important functions. By means of the hemoglobin contained in the erythrocytes (red cells), it carries oxygen to the tissues and collects the carbon dioxide. It also conveys nutritive substances (e.g., amino acids, sugars, mineral salts) and gathers the excreted material that will be eliminated through the kidney. The blood also carries hormones, enzymes, and vitamins. It performs the defense of the body by means of the phagocytic activity of the leukocytes (white cells).

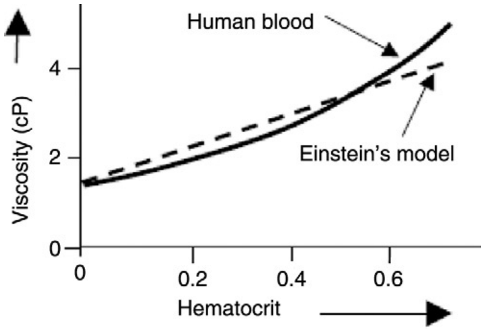

FIGURE 14.26 Blood viscosity as a function of blood hematocrit. Albert Einstein developed a model of viscosity of a fluid with a large concentration of substances where $c > 10\%$.

The erythrocytes are the most numerous blood cells, about 4–6 millions/mm^3. They are also called red cells. In humans and all mammals, erythrocytes are devoid of a nucleus and have the shape of a biconcave lens. The red cells are rich in hemoglobin, a protein able to bind to oxygen. Hence, these cells are responsible for providing oxygen to tissues and partly for recovering carbon dioxide produced as waste. However, most CO_2 is carried by plasma in the form of soluble carbonates. Red cells, which are the most prolific cells in the bloodstream, form the basis of the hematocrit, which is a factor in the determination of blood viscosity. A typical red cell is shown in Figure 14.27. The other blood cells are the leucocytes (white cells), which are designed to combat infections, and the thrombocytes (platelets), which are designed to assist in the clotting process.

White blood cells are colorless without hemoglobin. They contain a nucleus and have an irregular shape. Though there are fewer white blood cells than red blood cells, they are much bigger in size. They can change their shape easily, and this allows them to squeeze through the walls of the blood vessels into the intercellular spaces. Unlike red blood cells or platelets, there are five different types of white blood cells, each serving a different purpose in our body's immune system. Neutrophils make up 55 to 70 percent of the total white

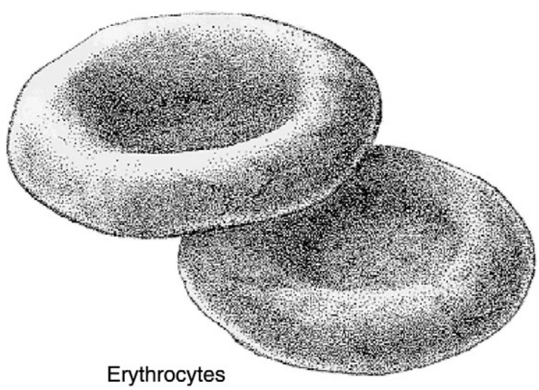

FIGURE 14.27 Red cells consisting of biconcave disks, each 8 microns in diameter and 2 microns thick.

blood count in the bloodstream. They have segmented nuclei and are "C" shaped. Neutrophils can be most commonly found near sites of infection or injury, where they will stick to the walls of the blood vessels and engulf any foreign particles that try to enter the bloodstream. They can also be found in the pus of wounds.

Eosinophils make up 2 to 5 percent of the total blood count and mainly attack parasites and any antigen complexes. These cells are also responsible for allergic response within the blood. Basophils make up less than 1 percent of the total white blood count. They secrete anticoagulant and antibodies, which mediate hypersensitivity reactions within the blood. They are known to have phagocytory features, although they are more often related to immediate immune reaction against external germs and diseases. Monocytes, having only 5 to 8 percent of the total white blood count, are the largest of the five types of white blood cells. They act as tissue macrophages and remove foreign particles and prevent the invasion of germs that cannot be effectively dealt with by the neutrophils. They have been known to have phagocytic functions.

Lymphocytes produce antibodies against toxins secreted by bacteria and infecting germs. These antibodies will be excreted into the plasma to kill bacteria in the blood as well as act as antitoxins. These antibodies will cause the foreign particles to cluster together, rendering them easily engulfed by the phagocytes. However, the nature of lymphocytes is highly specific, and they can recognize only certain antigens. Figure 14.28 shows several white cell subtypes.

Platelets are only about 20 percent of the diameter of red blood cells, the most numerous cell of the blood. The normal platelet count is 150,000–350,000 per microliter of blood, but since platelets are so small, they make up just a tiny fraction of the blood volume. The principal function of platelets is to prevent bleeding. Platelets are produced in the bone marrow, the same as the red cells and most of the white blood cells. Platelets are actually not true cells but merely circulating fragments of cells. But even though platelets are merely cell

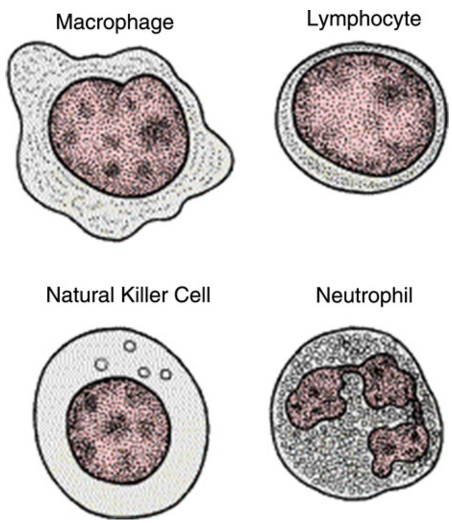

FIGURE 14.28 Sizes and shapes of various white cell types.

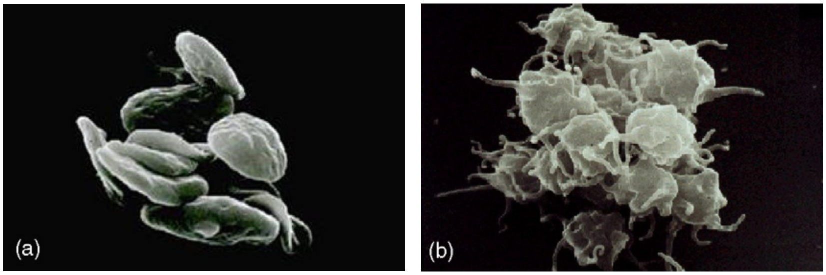

FIGURE 14.29 (a) Normal platelet. (b) Activated platelet.

fragments, they contain many structures that are critical to stop bleeding. They contain proteins on their surface that allow them to stick to breaks in the blood vessel wall and also to stick to one another. They contain granules that can secrete other proteins required for creating a firm plug to seal blood vessel breaks. Also, platelets contain proteins similar to muscle proteins that allow them to change shape when they become sticky. Figures 14.29a and b depict a normal platelet and an activated platelet. The normal platelet is shaped like a plate, thus lending itself to the name. An activated platelet extends tentacles to allow it to adhere to surfaces where bleeding exists.

The measurement of fluid viscosity is conducted with a device known as a viscometer. For liquids with viscosities that vary with flow conditions, an instrument called a rheometer is used. Viscometers only measure viscosity under one flow condition. In general, either the fluid remains stationary and an object moves through it, or the object is stationary and the fluid moves past it. The drag caused by relative motion of the fluid and a surface is a measure of the viscosity.

A falling sphere viscometer is one of the simpler types of viscometers. It is based on Stokes's law, which describes the flow of a fluid past a sphere in a vertical tube, where the flow is very slow. A sphere of known size and density is allowed to descend through the liquid. If correctly selected, it reaches terminal velocity, which can be measured by the time it takes to pass two marks on the tube. Knowing the terminal velocity, the size and density of the sphere, and the density of the liquid, Stokes's law can be used to calculate the viscosity of the fluid. A series of steel ball bearings of different diameters are normally used in the classic experiment to improve the accuracy of the calculation. Stokes's law is given as

$$F = 6\pi r \eta v$$

where

F = the frictional force
r = the radius of the spherical object
η = the fluid viscosity
v = the particle's velocity

An example of a functional relationship for a falling sphere viscometer is shown in Figure 14.30.

Since blood is opaque, sensors within the tube indicate when the sphere passes points along the tube. Another type of viscometer is the *Couette* or concentric cylinder viscometer.

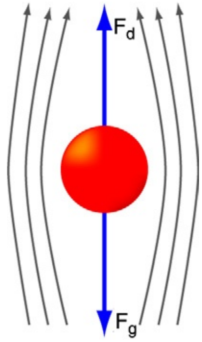

FIGURE 14.30 Slow flow past a falling sphere, which forms the basis of a falling sphere viscometer.

The fluid is placed between two cylinders with large radii and with a gap width between the cylinders that is very small compared to the radii. One of the cylinders rotates while the other remains stationary. With a small gap width, the velocity between the cylinders approaches a linear profile. The torque required to spin the cylinder as a function of the cylinder rotation rate provides the fluid viscosity. Different fluids would have different linear flow profiles between the cylinders, indicating different values of viscosities. A typical concentric cylinder viscometer is shown in Figure 14.31.

14.2.2 Conservation of Mass

The equation that describes fluid flow as a function of the pressure gradient in a tube is used to describe blood flow in a blood vessel or air flow in the trachea and bronchioles. As was the case with diffusion occurring in the direction of decreasing concentration gradient, so is the case of fluid flow in the direction of decreasing pressure in a blood vessel. In the human systemic circulation, the aortic pressure is 120 systolic pressure over 80 diastolic pressure (in mm Hg). This is because blood flow is pulsatile when it comes from the aortic valve as the ventricles undergo systolic contraction followed by diastolic relaxation. The blood pressure is 100 mm Hg mean and eventually decreases to about 5–10 mm Hg at the vena cava. Thus, there is a 90–95 mm Hg pressure gradient that pushes blood around the systemic circulation. Blood flow in the aorta is highly accelerated and time varying and continues around the sharp bend of the aortic arch. The equations that describe blood flow and fluid mechanics are time varying in nature, as well as affected by spatial variation due to the changing geometry/anatomy of the circulatory system. These equations include the equation relating to the conservation of mass (sometimes called the continuity equation) and the equations of fluid momentum (sometimes called the equations of fluid motion).

As was just noted, the type of fluid (Newtonian or non-Newtonian) can affect the equations related to fluid mass and fluid momentum. For simplicity, we can assume that blood acts Newtonian and can then alter that assumption after deriving the preceding equations. The continuity equation for a Newtonian fluid is shown in the following equation. We are using the version of this equation in cylindrical coordinates because blood vessels are cylinders. Thus, the velocities associated with this coordinate system are in the z direction (axially downstream),

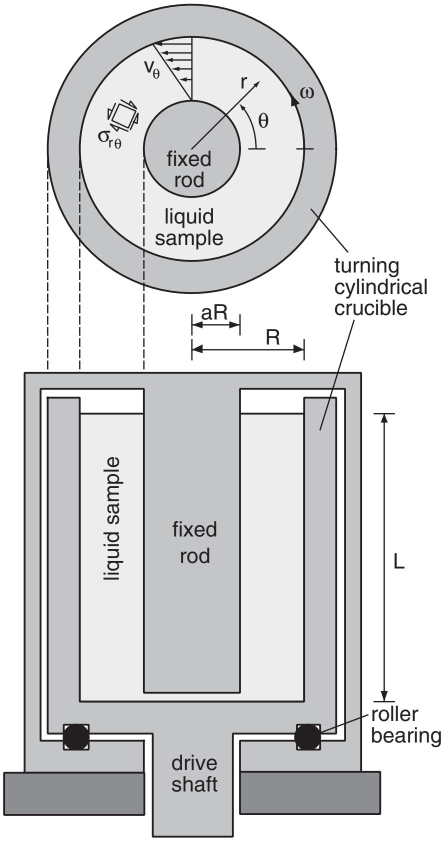

FIGURE 14.31 Concentric cylinder viscometer.

the r direction (radially from the center to the edge of the vessel), and the theta direction (circumferentially). This equation assumes that the density is a constant. A constant density assumes the fluid to be incompressible, which is a realistic assumption for most liquids (such as blood) and is unrealistic for most gases, which can compress at higher pressures.

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0.$$

Because mass is a scalar quantity, there is only one conservation of mass equation, although there are three terms in that equation, relating the derivatives of the three orthogonal velocities.

14.2.3 Fluid Momentum, the Equations of Motion, and the Navier Stokes Equations

The Navier-Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of viscous fluids such as liquids and gases and therefore the conservation of fluid momentum. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress (τ in the preceding section describing viscosity) is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term. As was the case for the continuity equation, the Navier-Stokes equations may be written in various coordinate systems, but the cylindrical coordinate system is most appropriate for blood flow, since blood vessels are cylinders.

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+u_z\frac{\partial u_r}{\partial z}-\frac{u_\theta^2}{r}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{\partial^2 u_r}{\partial z^2}-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right]+\rho g_r$$

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+u_z\frac{\partial u_\theta}{\partial z}-\frac{u_r u_\theta}{r}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{\partial^2 u_\theta}{\partial z^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}-\frac{\partial^2 u_z}{\partial z^2}\right]+\rho g_z$$

The Navier-Stokes equations also assume a constant density as well as a constant viscosity, indicating not only an incompressible fluid but a Newtonian one as well. Note that there are three fluid flow equations, since fluid momentum is a vector quantity, the product of mass and velocity, with velocity being a vector quantity with three components. Also note that each of the three fluid flow equations also contain multiple elements owing to the three derivatives of the fluid velocity.

14.2.4 Poiseuille Flow

As we just saw, even with the assumption of a Newtonian fluid, these equations are time varying with partial derivatives and are difficult to solve. As such, the Poiseuille assumptions, named after Jean Louis Marie Poiseuille, who formulated Poiseuille's law of viscous flow (as mentioned previously), are as follows:

1. The flow is steady ($\partial(\ldots)/\partial t=0$).
2. The radial and swirl components of the fluid velocity are zero ($u_r=u_\theta=0$).
3. The flow is axisymmetric ($\partial(\ldots)/\partial\theta=0$) and fully developed ($\partial u_z/\partial x=0$).

As a result, the first two of the three Navier-Stokes momentum equations and the continuity equation are identically satisfied. The third (z) momentum equation reduces to

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)=\frac{1}{\mu}\frac{\partial p}{\partial z}$$

The two boundary conditions for the flow of a viscous fluid (such as blood) are $(dV_z/dr)=0$, when $r=0$ (center of blood vessel), and $V_z=O$ at $r=R$ (edge of tube), which are the result of axisymmetric flow and viscous "no slip at the wall," respectively.

The solution is

$$u_z=\frac{1}{4\mu}\frac{\partial p}{\partial z}r^2+c_1\ln r+c_2$$

Since u_z needs to be finite at $r = 0$, $c_1 = 0$. The no slip boundary condition at the pipe wall requires that $u_z = 0$ at $r = R$ (radius of the pipe), which yields

$$c_2 = -\frac{1}{4\mu}\frac{\partial p}{\partial z}R^2.$$

Thus, we have finally the following parabolic velocity profile:

$$u_z = -\frac{1}{4\mu}\frac{\partial p}{\partial z}(R^2 - r^2).$$

The maximum velocity occurs at the pipe centerline ($r = 0$):

$$u_{z\,max} = -\frac{R^2}{4\mu}\left(-\frac{\partial p}{\partial z}\right).$$

The blood flow is thus a paraboloid with a maximum velocity at the center of the blood vessel with a zero velocity at the wall, as shown in Figure 14.32. The velocity, varying in the radial direction, can be thought of as flowing within concentric layers, each with a different velocity at a different radial value. The zero value at the wall corresponds to the no slip condition associated with Poiseuille flow. The fluid velocity is symmetric about the center of the tube, corresponding to the axisymmetric assumption associated with Poiseuille flow. The no slip condition is a result of the flow of a viscous fluid against a nonmoving boundary.

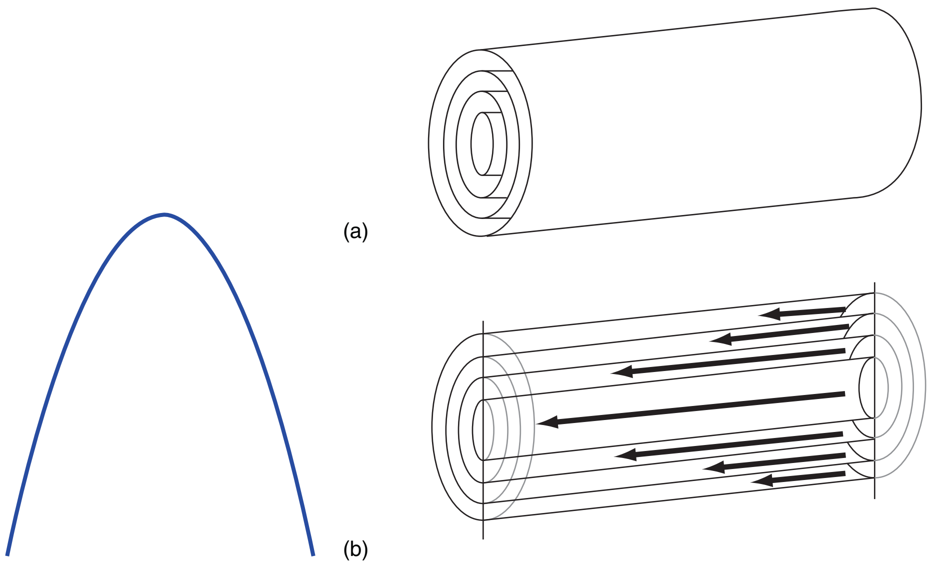

FIGURE 14.32 The parabolic velocity profile associated with the solution of the Navier-Stokes equations for Poiseuille flow. The velocity is maximal at the center of the vessel and zero at the wall. The velocity is axisymmetric, corresponding to the associated Poiseuille assumption.

If one integrates the velocity over the cross-sectional area of the blood vessel, this results in the well-known Poiseuille law for steady, incompressible, axisymmetric flow in a cylindrical vessel

$$Q = \pi R^4 \nabla P / 8\,\mu l$$

where Q is the volume flow rate (cardiac output for blood), R is the vessel radius, ∇P is the pressure gradient, μ is the blood viscosity, and l is the length of the blood vessel over which the pressure gradient is acting.

Poiseuille's law (sometimes called the Hagen-Poiseuille law) states that assuming laminar flow in a tube, flow (Q) is proportional to its radius to the fourth power (R^4) and inversely proportional to its length (l) and viscosity of the fluid (μ). The general principle of this law is that small changes in the internal diameter of a blood vessel lumen can make a big difference to the rate of flow. The flow in the aorta is large—in part because the radius of the aorta is large. The flow in smaller vessels is less, not only because the branching blood vessels segment the available flow but also because the vessel radius is smaller. As already stated, when the hematocrit is smaller, the blood viscosity is as well. As a result, an anemic patient has a smaller blood viscosity, and therefore, according to the Poiseuille law, the flow rate is larger. The resistance to blood flow (Poiseuille) corresponds to the relationship between volume flow rate (Q) and the pressure gradient (∇P) by the equation

$$Q = \nabla P / R_s$$

where R_s is the resistance to flow. If one uses the Poiseuille flow equation, then the flow resistance takes the form of

$$R_s = 8\mu l / \pi R^4$$

Thus, the resistance to flow inside a given blood vessel is proportional to the fluid viscosity and inversely related to the vessel radius to the fourth power.

Throughout the analysis of fluid flow in a vessel, the driving mechanism has been the pressure gradient (∇P). An example of the relationship between pressure and flow is the relationship between left ventricular pressure in the heart, left atrial pressure, and aortic pressure. The heart valves associated with the left heart (mitral valve between the left atrium and left ventricle, and aortic valve between the left ventricle and the aorta) open and close via the difference in pressure across each valve. This is evident in Figure 14.33, which shows the pressure and timing relationship across the two left heart valves.

Another manifestation of pressure drop producing a fluid flow is the relationship between pressure across the systemic circulation, resulting from flow across the aortic valve, through the aorta, arteries, arterioles, capillaries, venules, veins, and the vena cava. The pressure across the systemic circulation begins as a pulsatile pressure in the aorta (at a mean pressure of about 100 mm Hg) and ends at the vena cava with a mean pressure of about 5 mm Hg. This is shown in Figure 14.34, along with the smaller pressures associated with the right heart and the pulmonary circulation.

14.2.5 Boundary Layers

The Poiseuille assumption of fully developed flow ($\partial u_z / \partial z = 0$) refers to the fluid boundary layer, the region of the flow field where viscous forces predominate. What is "fully

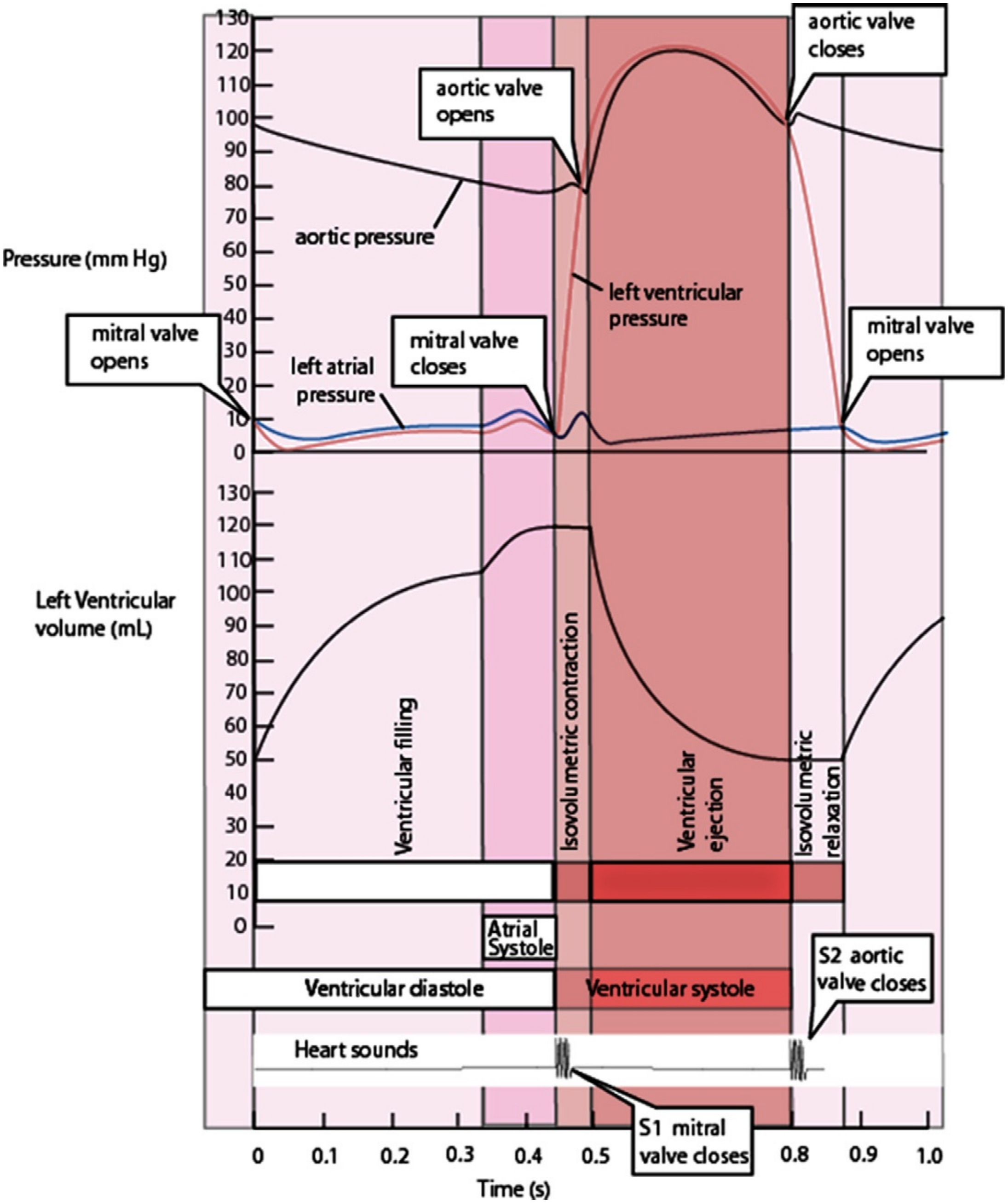

FIGURE 14.33 Pressure relationships across the left heart, which produce flow through the left atrium and left ventricle (across the mitral valve), or through the left ventricle and aorta (across the aortic valve).

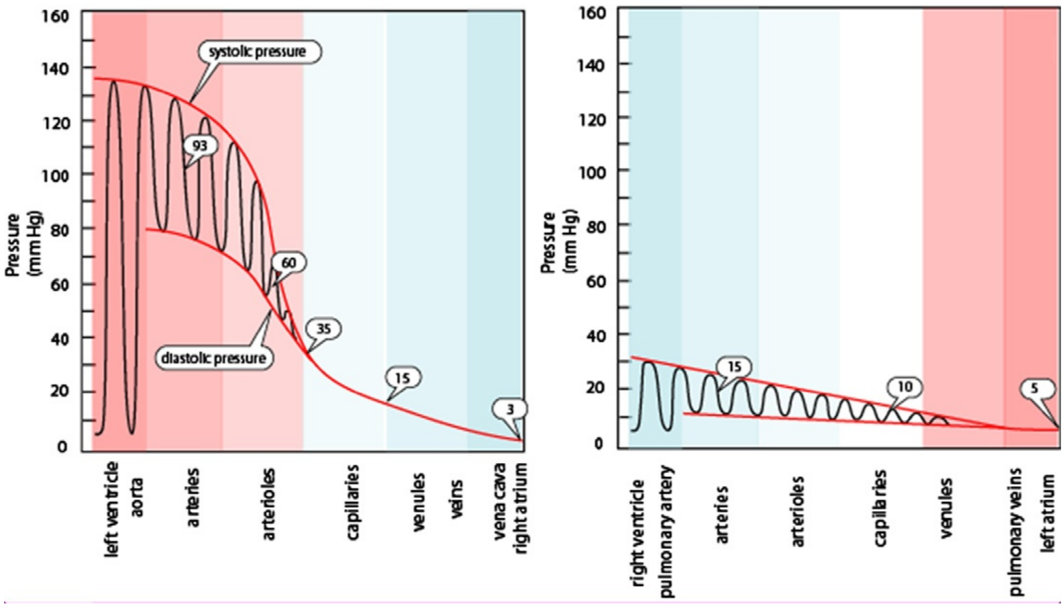

FIGURE 14.34 Pressure reduction across the systemic circulation (left side) and the pulmonary circulation (right side) indicating the continual pressure gradient that drives blood flow along the circulatory systems.

developed" is the boundary layer. Thus, for Poiseuille flow, the entire flow field is viscous dominated. This is manifested by the parabolic flow profile just mentioned. However, in reality, the fluid flow in the aorta is not truly parabolic but is more of a blunt profile. The flow profile becomes more rounded as the boundary layer grows from the outside of the vessel toward the center.

From a momentum transport perspective, the outside of the blood vessel, where the boundary layer is growing, is said to undergo convective deceleration, while in the center of the vessel, the flow is undergoing convective acceleration. Thus, there is a transfer of fluid momentum from the outside of the vessel toward the inside until finally the entire flow pattern is completely rounded and parabolic. The growth of the boundary layer is shown in Figure 14.35 for laminar flow, which is flow of a fluid that is smooth and orderly.

The boundary layer thickness is given by the symbol δ, whose value is quite small during "plug" flow at the entrance to a blood vessel but that slowly grows during viscous flow until it equals the vessel radius. At that point, the velocity profile is fully parabolic and the flow is said to be "fully developed." This is the point where the Poiseuille assumptions are now valid. Note that the pulsatile nature of blood flow in an artery has been neglected in this analysis and in Figure 14.35.

When the fluid flow is turbulent, the flow profile is more chaotic. This results in more fluid mixing from the areas of higher-velocity flow at the center with lower-velocity flow at the periphery of the vessel (toward the blood vessel wall). Thus, turbulent flow has more mixing and therefore more momentum transport between the center of the vessel and the wall. In laminar flow, the fluid shear stress is highest at the wall and zero at the center. Even so,

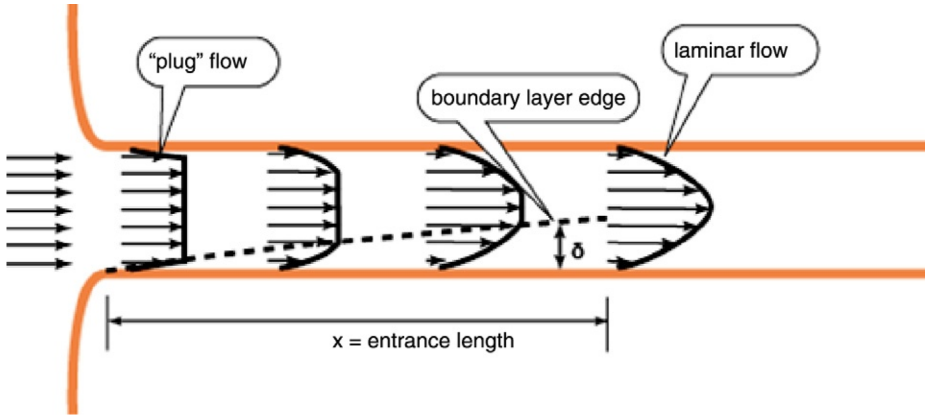

FIGURE 14.35 Growth of the laminar boundary layer from a blunt profile on the left toward a more rounded profile on the right, where the boundary layer becomes fully developed.

the laminar wall shear stress is not huge. However, in turbulent flow, the higher velocities are convected toward the wall, and the wall shear stress is higher than that of laminar flow. The turbulent boundary layer grows at a different rate than that for laminar flow, and the fully developed flow profile is not parabolic but more blunt in shape (Figure 14.36).

Turbulent flow has more energy and therefore a more chaotic flow as compared to the relatively smooth profile of laminar flow. With the added energy of turbulent flow, there are fluctuating components of the flow that can be periodically large. Turbulent flow is normally three-dimensional and time varying, while laminar flow is normally two-dimensional and time varying only during boundary layer development in the cardiovascular system. As such, it is possible to hear turbulent flow, and this approach often forms the basis to determine the prevalence of fluid turbulence in the cardiovascular system.

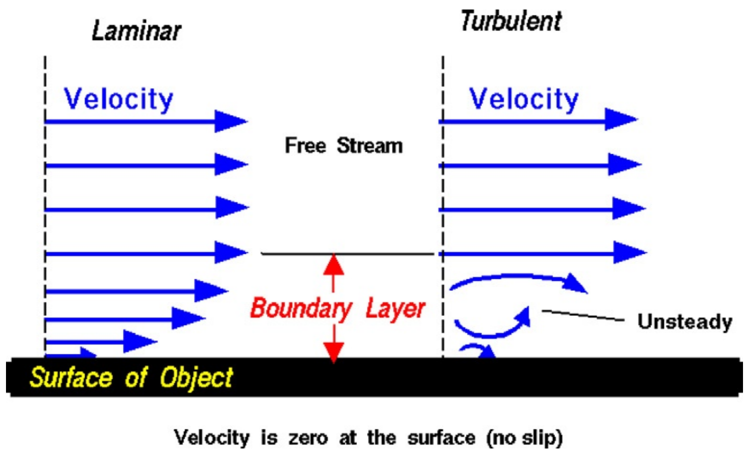

FIGURE 14.36 Laminar and turbulent boundary layers with laminar version more rounded than turbulent version. Turbulent version has greater mixing and greater momentum transport.

If the boundary layer of either type of flow separates from the wall, there is a zone of stagnant fluid. For blood, this would be a zone where thrombosis (clotting) would occur. Such a clot might become dislodged, where it would be transported downstream to lodge in a smaller blood vessel. This might result in a stroke if the dislodged clot (called an embolism) were to become trapped in the cerebral (brain) circulation. Such boundary layer separation might occur at sharp bends where the fluid flow cannot adequately follow the curvature (Figure 14.37). Boundary layer separation might also occur at branches of blood vessels as shown in Figure 14.38.

There is certainly a danger in boundary layer separation owing to the potential for blood clotting. Separation also occurs downstream of an atherosclerotic plaque that adheres to the vessel wall and narrows the vessel diameter. As the diameter becomes smaller with growth of the plaque, there is separation downstream of the stenosis (narrowing of the vessel), which creates clotting and further promotes the stenosis downstream. Thus, at some point in the progression of arterial disease, there is a tipping point where the stenosis becomes

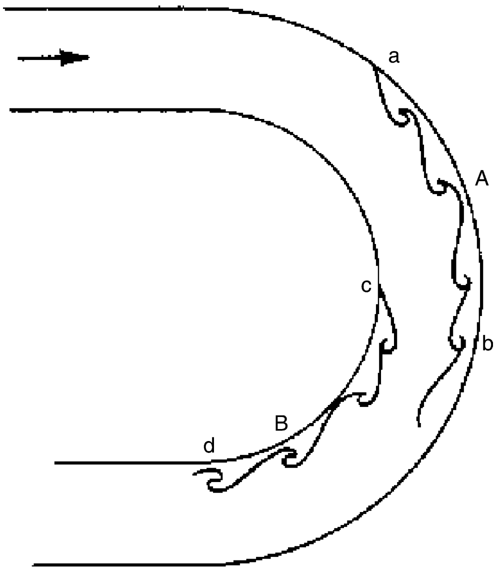

FIGURE 14.37 Boundary layer separation at a tight curve, such as the aortic arch.

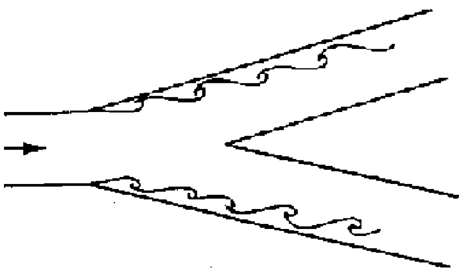

FIGURE 14.38 Boundary layer separation in branching blood vessels.

large enough to extend and grow via fluid separation. At that stage, the disease grows rapidly. However, since turbulence may exist downstream of the separation region, it is possible to diagnose the disease by listening for turbulence with a stethoscope or other diagnostic tool, such as ultrasound-based echocardiography.

14.2.6 Reynolds Number and Types of Fluid Flow

The differences between laminar and turbulent flow are considerable. Laminar flow, sometimes known as streamline flow, occurs when a fluid flows in parallel layers, with no disruption between the layers. It is the opposite of turbulent flow. In nonscientific terms, laminar flow is "smooth," while turbulent flow is "rough." The dimensionless Reynolds number is an important parameter in the equations that describe whether flow conditions lead to laminar or turbulent flow. It indicates the relative significance of the viscous effect compared to the inertia effect, with laminar flow being slower and more viscous in nature, while turbulent flow can be faster and more inertial (accelerating) in nature. The Reynolds number is proportional to the ratio of the inertial force (acceleration) to the viscous force (fluid deceleration). The values of the Reynolds number for various types of flow are as follows:

- **laminar** if $\mathrm{Re} < 2{,}000$
- **transient** if $2{,}000 < \mathrm{Re} < 3{,}000$
- **turbulent** if $3{,}000 < \mathrm{Re}$

These are approximate values. The Reynolds number can be affected by the anatomy/geometry of the fluid flow field, the roughness of the vessel wall, and irregularities in pressure or external forces acting on the fluid. When the Reynolds number is much less than 1, creeping motion or Stokes flow occurs. This is an extreme case of laminar flow where viscous (friction) effects are much greater than the virtually nonexistent inertial forces. Stokes flow was previously described regarding the falling sphere viscometer and is also typical of blood flow in capillaries.

As already noted, the Reynolds number is dimensionless and gives a measure of the ratio of inertial forces ($V\rho$) to viscous forces (μ/L), and, consequently, it quantifies the relative importance of these two types of forces for given flow conditions. The equation for the Reynolds number is

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu} = \frac{QD}{\nu A}$$

where

V= the mean fluid velocity in (cm/s)
D = the diameter (cm)
μ = the dynamic viscosity of the fluid (g/cm-sec)
ν = the kinematic viscosity ($\nu = \mu/\rho$) (cm^2/s)
r = the density of the fluid (g/cm^3)
Q = the flow rate (cm^3/s)
A= the pipe cross-sectional area (cm^2)

The first form of the equation is the most common form. In the circulatory system, the Reynolds number is 3,000 (mean value) and 7,500 (peak value) for the aorta, 500 for a typical artery, 0.001 in a capillary, and 400 for a typical vein. This corresponds to turbulent flow in the aorta (where the aortic wall is strengthened to overcome the turbulent forces), laminar flow in arteries and veins, and creeping flow in capillaries. Thus, there are added forces, mixing, and momentum transport in the aorta and virtually no momentum transport in capillaries. However, as was noted in the discussion on mass transfer in systemic capillaries, it is the low axial blood flow that allows for the radial mass transfer to occur. This could not happen in arteries, since the fluid velocity is too large to allow any significant radial mass transfer to occur. However, with turbulent flow in the aorta, there is considerable mixing of fluid layers and added wall shear stresses. As a result, there is added mass transfer across the artery walls, which is why arterial disease is prominent in the aorta. Such a disease is caused by mass transfer of lipids across the arterial wall and by distortion of the endothelial cells lining the vessel wall, allowing for such mass transfer to occur in the gaps.

A depiction of the systemic capillaries is shown in Figure 14.39. The Reynolds number is less than 1 and the flow is creeping flow. Such flow produces entirely viscous flow with no inertial effects, no turbulence, and no flow separation. As such, there are no negative effects due to the circuitous nature of the capillary bed nor to sudden sharp turns or bifurcations. In larger vessels, such vessel geometry would produce turbulence and/or flow separation.

14.2.7 Blood Pressure Measurement

The measurement of blood pressure noninvasively is conducted by the well-known blood pressure cuff assembly: the sphygmomanometer. Most individuals have had their blood pressure taken with this mechanism. It involves inflating a pressure cuff just above the elbow to a pressure significantly exceeding systolic blood pressure (120 mm Hg). At such a high pressure, the artery just below the surface (brachial artery) collapses as the externally applied pressure from the cuff exceeds the internal blood pressure inside the artery. A stethoscope is placed just below the cuff. As the pressure is reduced, the external pressure will eventually fall slightly below the systolic blood pressure, and the brachial artery will open slightly. The blood at that high pressure will jet through the small opening, producing turbulent flow. Such jetting turbulent flow can be heard via the stethoscope. This is similar to a heart murmur heard through a stethoscope, which is indicative of a leaky heart valve, also producing turbulence, as well as separated flow downstream of a stenosis.

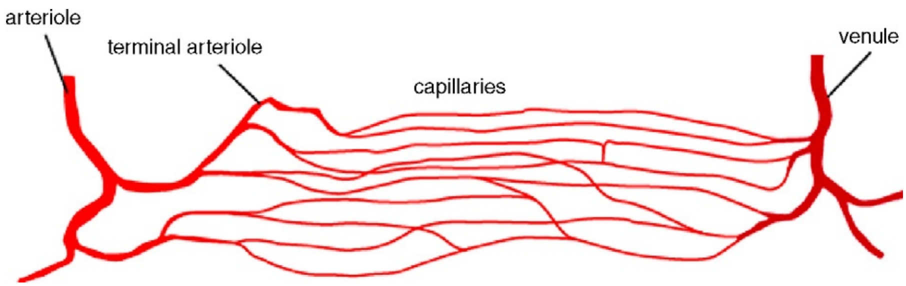

FIGURE 14.39 The network of the systemic capillaries leading from arterioles and ending in venules, with bending and bifurcating elements in the capillary bed that are possible with creeping flow.

As the cuff pressure continues to fall, the duration that the brachial artery pressure exceeds the cuff pressure will increase. As such, the (small) opening will increase in size and duration, until finally the vessel remains open as the cuff pressure falls below diastolic pressure in the artery. At that point, the turbulent sounds, called Korotkoff sounds, fade away.

A manometer is connected to the pressure cuff so the physician can see what the pressures are when the sound begins (systolic pressure) and when the sound finally fades away (diastolic pressure). Thus, the sphygmomanometer can be used as a noninvasive means of determining the two extremes of blood pressure, normally at 120/80 mm Hg. The relationships between cuff pressure, vessel pressure, and sounds via the stethoscope are shown in Figure 14.40.

The measurement of blood pressure depends greatly on the location of the measurement. As shown in Figure 14.34, the blood pressure falls as blood circulates through the systemic

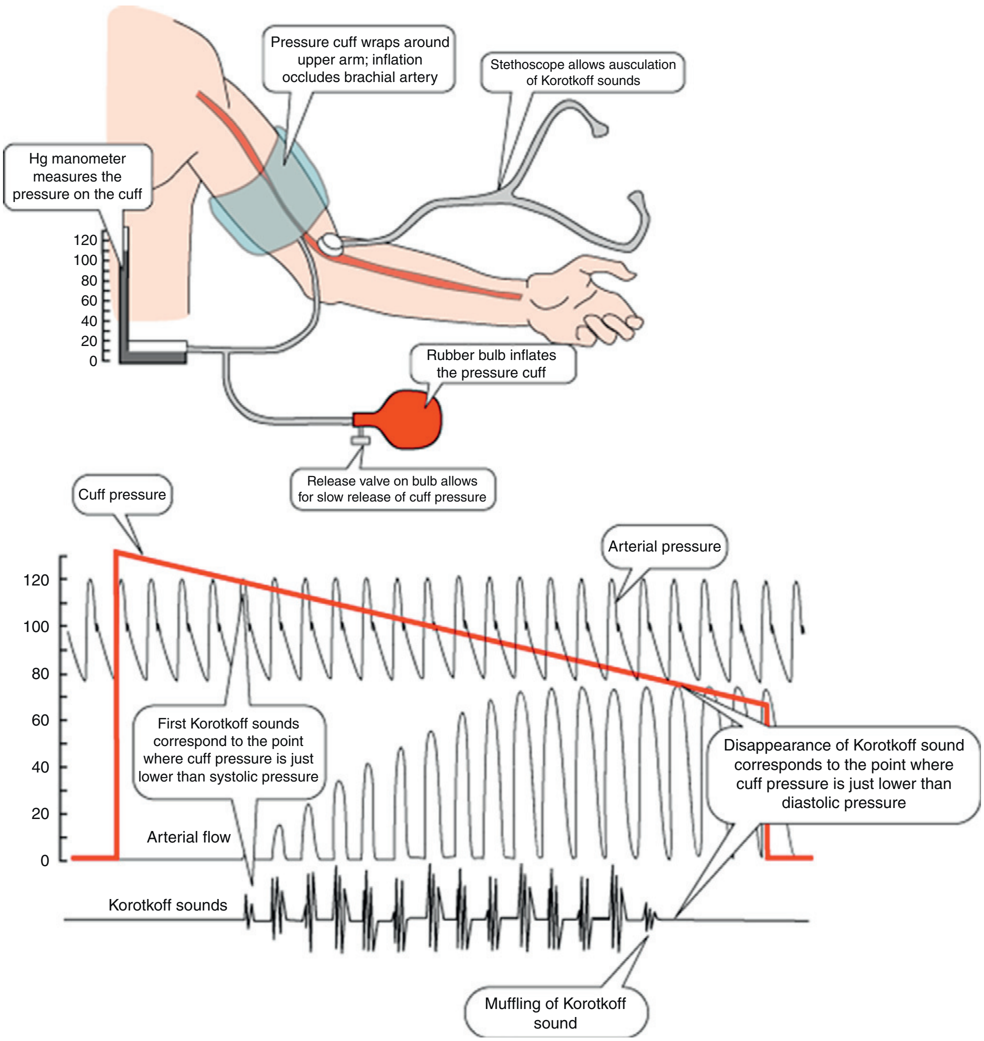

FIGURE 14.40 The sphygmomanometer and the generation of Korotkoff sounds indicating systolic and diastolic arterial pressure.

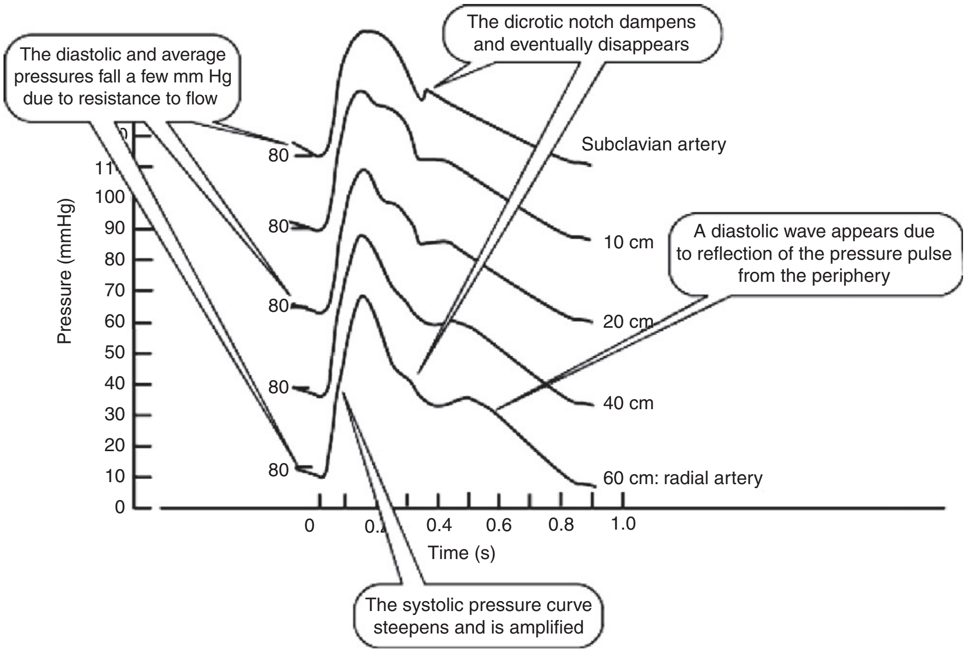

FIGURE 14.41 The blood pressure waveform and the resulting pulse pressure (difference between systolic and diastolic) can vary in differing midlevel arteries. This would result in different measured systolic and diastolic values of blood pressure via a sphygmomanometer.

circulatory system. However, even though blood pressure is normally taken in the arterial network, the location of that measurement can affect the systolic and diastolic values. As shown in Figure 14.41, the blood pressure waveform can be steeper with a corresponding larger difference between systolic and diastolic pressure, depending on the location within one of the midsized arteries. The increase in the pulse pressure (difference between systolic and diastolic) is sometimes due to a reflection of the pressure wave, which is the propagation of the pulse wave (moving far faster than the fluid time-varying pressure).

As can be seen, blood flow and velocity patterns can be quite complex and varying in direction, time, and dimensions. Simplifying assumptions such as Poiseuille flow, a Newtonian fluid, and an incompressible fluid aid in our ability to develop first-order solutions to rather complex waveforms and flow systems. Fluid mechanics is a more macroscopic description of body functions, where mass transfer was a more cellular description of body functions. Both mass transfer and fluid flow (momentum transfer) can affect heat transfer within and outside the body. This is the topic of the next section.

14.3 BIOMEDICAL HEAT TRANSPORT

Heat transport within the human body provides the mechanism for the body's core temperature to remain constant by shunting heat from the core to the periphery, where heat is lost through the skin and through respiration. Heat is generated inside the body as it metabolizes food, which produces heat as a by-product. Heat transfer from the body to the environment is also affected by outside temperature and wind speed. The latter affects windchill, which can produce a substantial heat loss from the body. Heat transport also occurs in blood heating and cooling devices, which are often used during surgical procedures and as part of heart-lung machines. Finally, heat transport is vital for extreme conditions such as a firefighter's suit or a spacesuit, both used to protect the body from extreme temperatures in the environment.

The heat generated by the body at rest is called the *basal metabolic rate*, equal to 72 kcal/hr. Obviously, as metabolic activity increases, such as during exercise, the level of heat production rises.

The equations describing heat transport are similar in form to the equations relating transport of mass (Fick's Law) and momentum. Such transport is always in the direction of a decreasing gradient. For mass, the gradient is the concentration difference between two points or across membranes. For momentum transport of fluids, it is the pressure differential causing fluid flow. For heat transfer, the driving gradient is the temperature difference between two points.

14.3.1 Conduction, Convection, and Radiation

Heat transport can occur by three basic mechanisms. Thermal *conduction* is the process by which heat transfer occurs by molecular interaction. It can occur in gases, liquids, and solids. An example is heat transfer across a closed window or across a wall. In the human body, the example of conductive heat transfer is across tissue from the core toward the periphery (skin). Thermal *convection* is the process by which heat transfer occurs via bulk motion of a fluid. An example might be forced air flow from an air conditioner vent or windchill on a windy day. In the body, examples of convective heat transfer include blood flow from the core toward the periphery or air flow in the lungs from the alveoli through the trachea and out the mouth and nose. Both conduction and convection require that there be a material involved, although convection does not occur in solids, whereas conduction can. The third method of heat transfer is thermal *radiation*, which occurs as a result of electromagnetic transport processes. An example is heat gain from the sun to the earth (and your own skin). Radiation is a surface-to-surface phenomenon and does not require a material interaction. The greenhouse effect is an example of thermal radiation. As an example, Figure 14.42 depicts the three types of heat transfer from a campfire.

14.3.2 Thermal Conduction

Conduction (or heat conduction) is the transfer of thermal energy between neighboring molecules in a substance due to a temperature gradient. It always takes place from a region of higher temperature to a region of lower temperature and acts to equalize temperature differences. Conduction takes place in all forms of matter, including solids, liquids, and gases, but does not require any bulk motion of matter. In solids, it is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion. Conduction is the movement of heat from a warmer object to a cooler one when they are in direct contact with one another. This serves to even the temperature

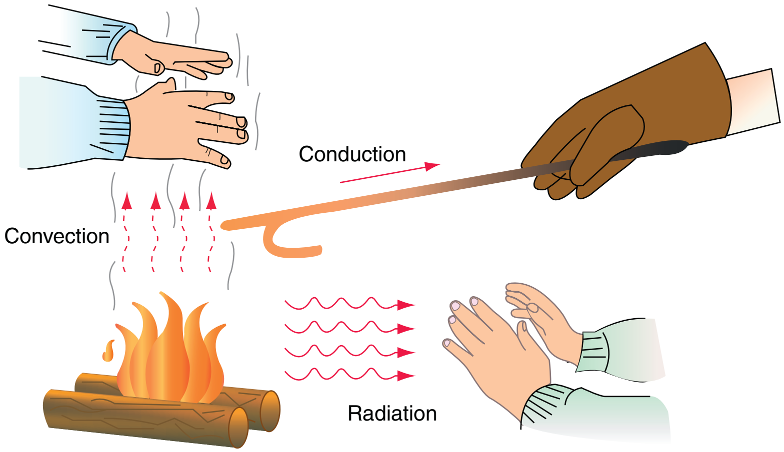

FIGURE 14.42 The three types of heat transfer: conduction, convection, and radiation.

difference between them over time. The rate of heat transfer between two objects of different temperatures depends on several factors:

- The temperature difference between the two objects
- The total surface area where the two objects are in contact
- The efficiency of the insulation that is between the objects

The greater the temperature difference between two objects in contact, the more heat that is transferred between them in a given time. For example, when you place your hand on a very hot stove top, you will quickly receive a great heat input from the stove to your hand. If the stove top is only warm, it will take much longer to receive the same amount of heat into your hand.

The more surface area in contact between two objects, the more quickly heat is transferred between them. Stick your finger on an icicle for a minute and it feels cold, but you will probably not feel too uncomfortable. Strip naked and lay on a block of ice for a minute and you will most likely be very uncomfortable indeed as the ice absorbs heat from your body at a very fast rate.

The amount of heat being transferred between two objects of different temperatures can be slowed by the use of effective insulation. Insulation retards the movement of heat between them by creating pockets of dead air space that trap the flow of heat or by otherwise slowing the overall heat transfer rate by adding a low-conductance/high-resistance layer.

The law of heat conduction, also known as Fourier's law, states that the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area at right angles to that gradient through which the heat is flowing. We can state this law in two equivalent forms: the integral form, in which we look at the amount of energy flowing into or out of a body as a whole, and the differential form, in which we look at the flows or fluxes of energy locally. The differential form is

$$\vec{q} = -k\nabla T$$

where $\vec{q}$ is the local heat flux, $[W \cdot m^{-2}]$, k is the material's conductivity, $[W \cdot m - 1 \cdot K^{-1}]$, and ∇T is the temperature gradient, $[K \cdot m^{-1}]$. "K" refers to °Kelvin, an absolute scale for temperature. For materials with a high thermal conductivity, such as metals, there is little if any insulating capacity. For materials with a low thermal conductivity, the material has more insulating characteristics. Air and many gases have low thermal conductivities. Human tissue also has a relatively low thermal conductivity and is thus relatively insulating, which aids in reducing heat transfer to the core of your body on extremely hot or cold days. Clothing helps to further insulate the body by providing extra layers for heat conduction and by trapping air, which is itself a thermal insulator.

For many simple applications, a one-dimensional form of Fourier's law is written as

$$q_x = -k\frac{dT}{dx}$$

The thermal conductivity, k, is often treated as a constant, though this is not always true. While the thermal conductivity of a material generally varies with temperature, the variation can be small over a significant range of temperatures for some common materials. In anisotropic materials, the thermal conductivity typically varies with orientation. For the human body, trapped air in the lungs has a different conductivity as opposed to human tissue, but both have smaller values than denser materials, such as human bone. Water (and in many ways human blood) has a larger conductivity value, but it is also a thermal capacitor, able to retain heat.

Since the surface area can also affect heat conduction occurring orthogonal to a plane, Fourier's law is also written as

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$$

where A is the cross-sectional surface area, ΔT is the temperature difference between the ends of a thickness, and Δx is the distance between the ends.

The thermal conductivities for various materials, including human tissue, are as follows:

Material	k (W/mK)
Human tissue (organs/muscle)	0.5
Human tissue (fat)	0.2
Human tissue (skin)	0.3
Blood	0.5
Bone	0.5–0.6
Water	0.62
Air	0.03
Glass	1.1
Aluminum	200
Copper	400

As can be seen from the preceding list, metals have high thermal conductivities, which is to be expected. Most human tissue has relatively low thermal conductivities, suggesting that conduction is not rapid within the human body. The skin has a lower conductivity than human tissue, suggesting that conduction into the body (as on a hot day) is also limited. It is interesting to note that air, like most gases, has a very low thermal conductivity. Thus, trapped air within clothing provides excellent resistance to thermal conduction in either direction—into or out of the body. On an extremely cold day, a parka with trapped air within a nylon covering provides excellent thermal resistance. A three-layer model can be utilized to examine heat transfer via thermal conduction, as shown in Figure 14.43.

By applying Fourier's law to each layer and assuming that there is no accumulation of heat within any layer, the equations become

$$Q1 = K1(T1\text{-}T2)/\Delta X1 \qquad Q2 = K2(T2\text{-}T3)/\Delta X2 \qquad Q3 = K3(T3\text{-}T4)/\Delta X3$$

Since there is no accumulation of heat, then Q1 = Q2 = Q3 = Q. Rearranging the preceding equations gives

$$T1\text{-}T2 = Q\Delta X1/K1 \qquad T2\text{-}T3 = Q\,\Delta X2/K2 \qquad T3\text{-}T4 = Q\,\Delta X3/K3$$

Summing these equations results in

$$T1\text{-}T4 = Q/(\Delta X1/K1 + \Delta X2/K2 + \Delta X3/K3) \text{ or}$$
$$Q = (T1\text{-}T4)(\Delta X1/K1 + \Delta X2/K2 + \Delta X3/K3)$$

Since the thermal conductivities are in the denominator, any one term can affect the overall heat transfer rate. Thus, if air is one of the layers, its thermal conductivity is so low that with 1/K as a factor, it would dominate the other layers. As a result, it is usually the layer with the lowest thermal conductivity that affects the overall heat transfer rate via conduction. This is why trapped air is such a good thermal insulator. Note that T1-T4 will eventually equal out if there is no continual heat source, but the rate of transfer will be slower with air as one of the layers.

If the body maintains a certain level of heat production, then a coat with trapped air will help to maintain skin and body temperature without a danger of hypothermia. Figure 14.44 shows the temperature distribution inside and at the body surface.

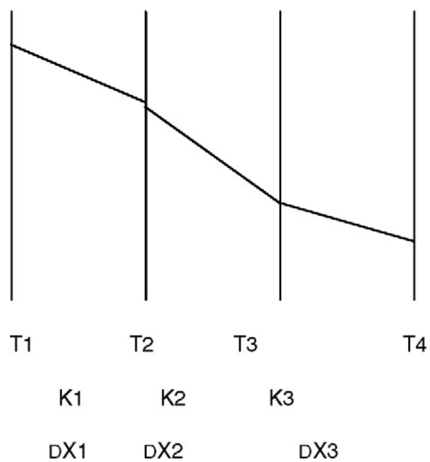

FIGURE 14.43 A three-layer model for steady-state heat conduction.

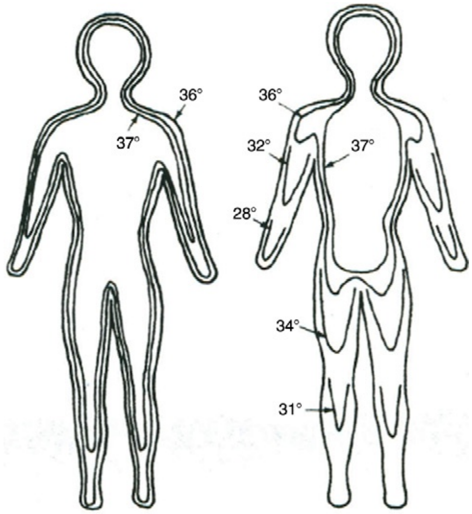

FIGURE 14.44 Temperature distribution in the human body, with the left figure depicting the temperature distribution in a warm environment and the right figure for a cold environment. *From Ruch and Patton (1965).*

Although the temperature nearest the skin is lower in a cold environment, the use of clothing with trapped air prevents the skin temperature from becoming dangerously cold. For someone in an extremely cold environment, such as outer space, the use of an encapsulated suit is needed because the exterior temperature is extremely low. In an earth-bound environment, the issues associated with insulating clothing do not address the added issue of heat loss (or gain) via respiration. We will soon see how respiration can be an important element in heat loss from the body.

14.3.3 Thermal Convection

Convection is heat transfer via bulk motion of a fluid, which can be a liquid or a gas. Examples of convective heat transfer include air conditioning coming from a compressor, blower, and ductwork; windchill; heat exchangers; and blood warming/cooling as part of heart/lung machines (also called cardiopulmonary bypass).

Convective heat loss can be forced or via natural (unforced) mechanisms. Forced convection is commonly due to a pressure gradient or other forcing function that would move a fluid (with its associated heat) to another position or location. Thus, a forced air system consisting of an air conditioner, blower, and vents is an example of a forced convection system. Another is windchill, which results from a pressure gradient moving air across a surface and removing heat from that surface. If an individual experiences windchill, he or she would feel colder because the windchill would remove heat from the skin.

Natural or unforced convection is due to thermal gradients in a fluid that result in fluid motion due to buoyancy differences. Thus, hot air rises because it is lighter than cold air.

The movement of the hot air by differences in fluid density is a prime example of unforced convection. Normally, free convection is much smaller in magnitude than forced convection, since the movement of fluid via density differences is often far smaller than the movement of fluid via pressure gradients. Hot air rises but at a rate far smaller than forced hot air from a heat pump through vents or via windchill from a strong wind.

Convective heat loss can generally be approximated by a convective heat loss coefficient used in the generalized equation

$$Q_c = h_c \, A \, \Delta T$$

where

Q_c = convection heat transfer rate
h_c = convective heat transfer coefficient
A = area over which heat transfer occurs
ΔT = temperature gradient between the surface and the environment

Typically, the convective heat transfer coefficient is a function of fluid velocity for forced convective heat transfer. There are several versions of the equation relating the convective heat transfer coefficient to fluid velocity. One such example is

$$h_c = 5.6 \, v^{0.67}$$

where v is in m/sec and h_c is in kcal/m^2 – hr – °C.

As the fluid velocity increases, so does the convection heat transfer, which increases the convective heat transfer rate. Thus, as the wind velocity increases, so does the windchill, which is an example of convection cooling. The convection heat transfer rate for free (natural) convection is normally a constant at about 2–2.3. If one equates the free convective coefficient with the forced coefficient equation, then the air velocity required for the forced term to exceed the free term is

$$2.3 = 5.6 \, v^{0.67} \text{with } v^{0.67} = 2.3/5.6 = 0.4107$$

and therefore

$$v = 0.263 \text{ m/sec or } v = 0.588 \text{ mph}$$

Thus, any wind speed over about a half mile per hour would exceed the free convective heat loss.

However, the more accurate method of computing the forced convective heat loss is to use the Reynolds number (indicative of laminar vs. turbulent flow), the Prandtl number (relating viscous effects to thermal effects/conduction), and the Nusselt number (convective heat transfer to thermal effects/conduction). This allows for the specific calculation of the convection heat transfer rate for laminar or turbulent flow. Because these two types of fluid flow are very different in terms of velocities and velocity profile, it is not surprising that the heat transfers associated with these two types of fluid flow are quite different.

In general, the Nusselt number (Nu) = h_c D/k, where D is the tube diameter or representative length, k is the thermal conductivity, and h_c is the convective heat transfer coefficient.

However, the Nusselt number is related to the Reynolds number (Re) and the Prandtl number (Pr) by the following representative equations:

$$\begin{aligned} Nu &= 0.322\ Re^{0.5}\ Pr^{0.33} && \text{for laminar flow} \\ Nu &= 0.023\ Re^{0.8}\ Pr^{0.33} && \text{for turbulent flow} \end{aligned}$$

where

$$\begin{aligned} Re &= \rho\ V\ D/\upsilon \\ Pr &= c_p\ \upsilon/k \\ \rho &= \text{density} \\ \upsilon &= \text{viscosity} \\ c_p &= \text{specific heat} \end{aligned}$$

Since the Nusselt number is related to both the Reynolds number and the Prandtl number, as well as to the convective (forced) heat transfer coefficient, by computing these three parameters, one can determine the convective heat transfer for the appropriate type of flow. Obviously, this is a more complex set of calculations than the simple form of $h_c = 5.6\ v^{0.67}$, but it is more accurate, since the simplistic form does not designate the type of fluid flow.

14.3.4 Heat Exchangers

Heat exchangers are commonly used in medical settings as blood heaters/coolers for open heart-lung machines or for patient heating/cooling for standard surgeries. The purpose of cooling the patient during surgery is to reduce the metabolic load, which lowers the need for larger blood flow rates or ventilation rates. The patient is then heated (or the blood heated) toward the end of surgery. A typical type of heat exchanger for blood is the double pipe heat exchanger, which consists of two concentric pipes with blood in one pipe and water in the other. By using cold water, the blood is cooled, while using warm water heats the blood. A double pipe heat exchanger combines convective heat transfer, as the fluids are moving, along with heat conduction through the pipe walls. A typical double pipe heat exchanger is shown in Figure 14.45. Because the blood and the water can either be moving in the same direction (cocurrent flow) or in opposite directions (countercurrent flow), there are two versions of the heat exchanger. The physical device is shown in Figure 14.46.

The design equations for a double pipe heat exchanger combine Fourier's law for heat conduction through the wall of the inner pipe, together with the complex form of the convective heat transfer coefficients with the Reynolds, Prandtl, and Nusselt numbers.

This equation is $Q = U\ A\ \Delta T_{lm}$, where U is the overall heat transfer coefficient and is a function of the thermal conductivity and the convective heat transfer coefficient, A is the surface area of the pipe, and ΔT_{lm} is the effective temperature difference between the two fluids from one end of the exchanger to the other. This term is known as the log mean of the temperature gradient. Since the surface area of the pipe could be the inner or outer surface, then the overall heat transfer coefficient is synched with the surface area as U_o and A_o, or U_i and A_i. The temperature gradient is unaffected by inner or outer diameter but is affected by whether the flow is cocurrent or countercurrent. In general, countercurrent flow is more efficient for overall heat transfer.

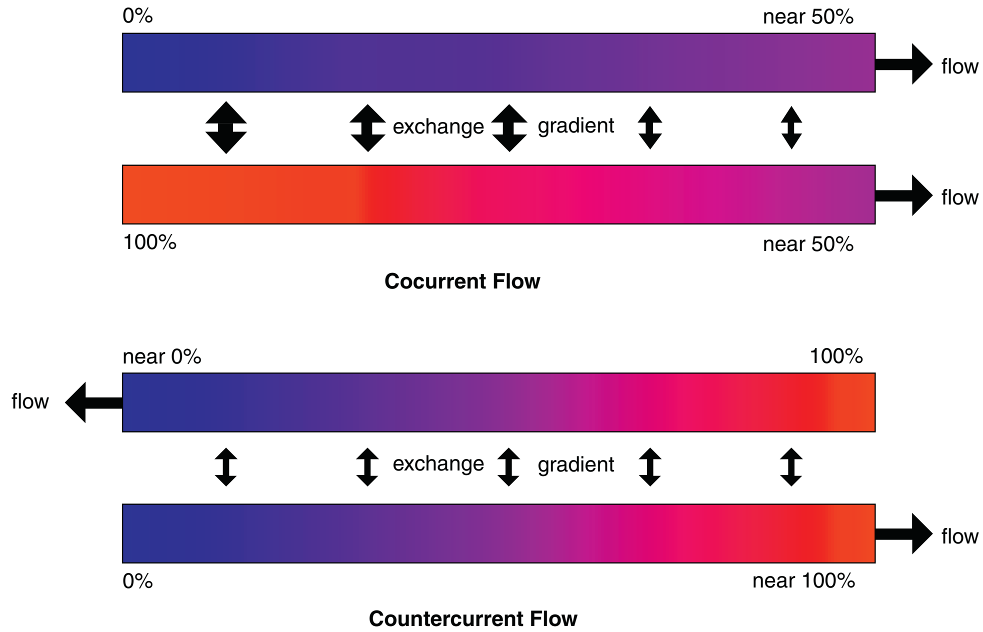

FIGURE 14.45 Double pipe heat exchangers for cocurrent and countercurrent flows.

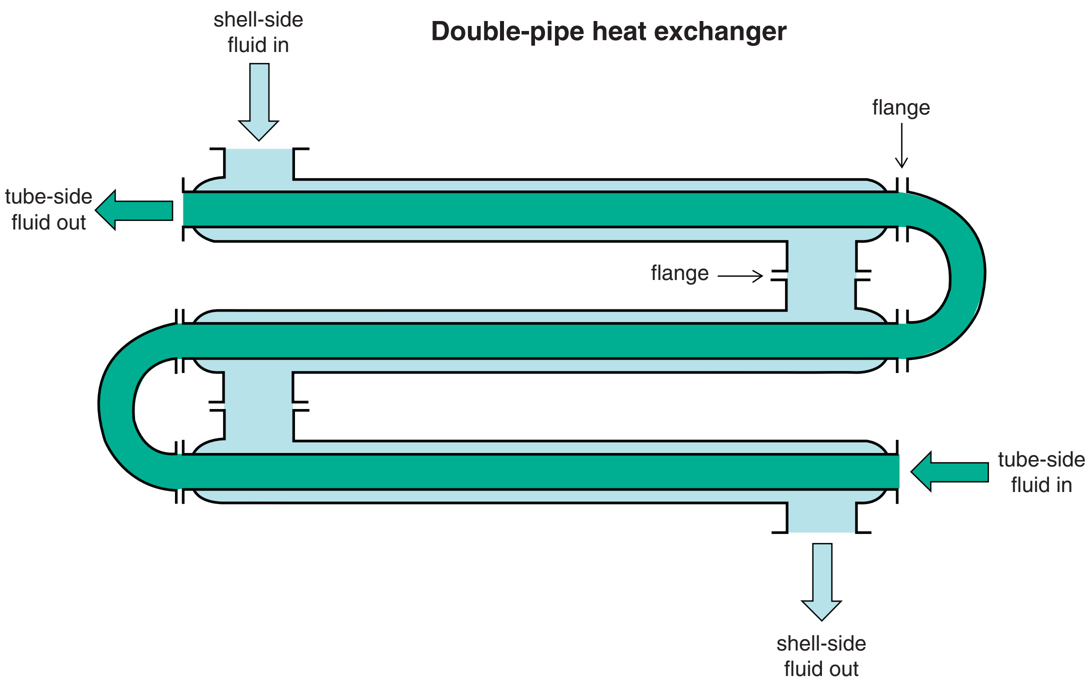

FIGURE 14.46 Double pipe heat exchanger design.

14.3.5 Thermal Radiation

Thermal radiation is electromagnetic radiation emitted from a material that is due to the heat of the material, the characteristics of which depend on its temperature. An example of thermal radiation is the infrared radiation emitted by a common household radiator or electric heater. A person near a raging bonfire will feel the radiated heat of the fire, even if the surrounding air is very cold. Thermal radiation is generated when heat from the movement of charges in the material (electrons and protons in common forms of matter) is converted to electromagnetic radiation. Sunshine, or solar radiation, is thermal radiation from the extremely hot gasses of the sun, and this radiation heats the earth. The earth also emits thermal radiation but at a much lower intensity because it is cooler. The balance between heating by incoming solar thermal radiation and cooling by the earth's outgoing thermal radiation is the primary process that determines the earth's overall temperature. As such, radiation is the only form of heat transfer that does not require a material to transmit the heat. Radiative heat is transferred from surface to surface, with little heat absorbed between surfaces. However, the surfaces, once heated, can release the heat via conduction or convection to the surroundings.

Thermal radiation is conducted via electromagnetic waves. As such, this form of heat transfer is not only a function of the temperature difference between the two surfaces but also the frequency range of the emitted and received energy. As an example, sunlight is composed of the visible light spectrum as well as infrared energy and ultraviolet energy. Figure 14.47 depicts the effects of temperature and wavelength of the thermal energy on the heat transfer rate.

When radiant energy reaches a surface, the energy can be absorbed, transmitted (through), or reflected (or any combination). The sum of these three effects equals the total energy transmitted, and the parameters that describe these three phenomena are given by

$$\alpha + \rho + \tau = 1,$$

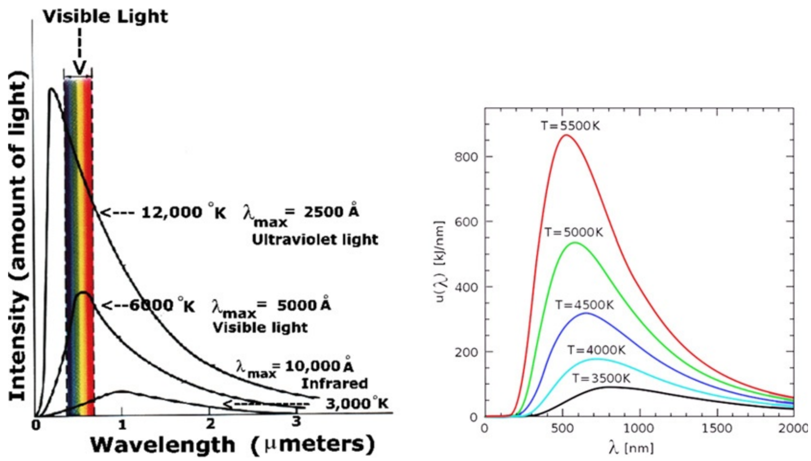

FIGURE 14.47 Peak wavelength and total radiated amount vary with temperature. Although the plots show relatively high temperatures, the same relationships hold true for any temperature down to absolute zero. Visible light is between 380 and 750 nm.

where α represents spectral absorption factor, ρ represents the spectral reflection factor, and τ represents the spectral transmission factor.

The radiative heat transfer rate is given by the Stefan-Boltzmann law

$$Q_r = \sigma A\, T^4$$

where σ is the Boltzmann constant and A is the surface area of the radiating source. The temperature is in an absolute scale (°Kelvin, corresponding to °C, or °Rankin, corresponding to °F).

To predict the exact amount of radiative heat transfer between two surfaces, the preceding equation is expanded as

$$Q_r = \sigma F_1 A_1 ({T_1}^4 - {T_2}^4)$$

where F is the facing factor that represents the amount of the emitting surface (1) facing the receiving surface (2) with the surface area A representing surface 1. Correspondingly, this equation can use F_2 and A_2 to represent the facing factor for the receiving surface toward the emitting surface with the surface area of 2. Boltzmann's constant and the temperature gradient are unchanged for either form of the equation. The facing factor can be approximated as a disk of radius R if the distance between the two surfaces is large, such as the earth to the sun. For shorter distances, the facing factor is a complex interaction between the angles of the two surfaces that face each other.

As can be seen, thermal radiation is affected by the frequency of the emitted energy. This is why sunscreen ointments have ultraviolet protection, since this type of energy can be damaging to skin. In addition, it is common to feel warmer on the sunny side of the street as opposed to the shady side, given the radiative heat transfer. Radiation can be a significant source of heat as compared to the other forms (conduction and convection) because radiation is composed primarily of sunlight and the respective heating of the earth.

14.3.6 The Greenhouse Effect

Emitted radiation from the sun consists of a spectrum of electromagnetic radiation of varying frequencies and wavelengths. The sun's surface temperature is approximately 10,000°C. When this energy hits the earth's surface, it can be absorbed, reflected, or transmitted through. The amount of each of these factors depends on the surface temperature of the earth and the exact material (water, soil, man-made materials). However, the reflectivity, absorptivity, or transmissivity is frequency dependent. In addition, the frequency spectrum of the incoming thermal energy is shifted due to the different surface temperature of the earth as compared to the sun.

As the energy reached the earth through its atmosphere, the transmissivity was high (allowing it to pass through the atmosphere) and the reflectivity and absorptivity were low. However, as the energy hit the earth's surface, the new temperature shifted the spectrum such that the new reflectivity was high and the transmissivity was low. Thus, when some of the thermal energy was reflected back from the earth's surface toward the atmosphere, it did not transmit through toward outer space but instead was reflected back toward the earth's surface. At each reflection back, more energy was absorbed into the earth's surface, heating the earth. Normal conduction and convection would heat the

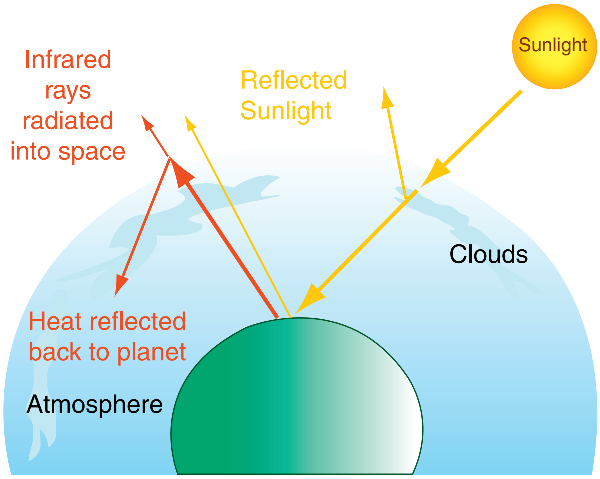

FIGURE 14.48 Depiction of the greenhouse effect resulting in thermal radiation being reflected back toward the earth's surface.

surrounding air and buildings, resulting in the warm temperatures that allow us to live on this planet. This phenomenon is known as the *greenhouse effect* (Figure 14.48).

You have probably experienced the greenhouse effect firsthand if you ever left your car unattended on a sunny day. The thermal radiation is transmitted through the windshield and hits the seats and dashboard. The frequency shift occurs, and the reflected energy is then reflected back to the windshield, only now at a high reflectivity and low transmissivity. As such, the inside of the car heats up, often up to 20–30 degrees higher than the ambient air. A sunscreen on the window will reduce the amount of radiation entering the car and keep it cooler.

Commercial greenhouses purposefully trap the heat via the greenhouse effect and use some limited screening to control the amount of trapped heat. The increased heat allows the plants to remain at a relatively warm temperature and avoid the changes in temperature with changing air currents and air temperature. By keeping the doors to the greenhouse closed, the temperature can be somewhat maintained overnight. A commercial greenhouse is shown in Figure 14.49.

Heat loss at night via radiation occurs as reemitted energy from the earth's surface to the dark night sky. The amount of radiation energy released at night depends on whether the sky is clear or cloudy.

14.3.7 Heat Loss via Respiration

When air is inspired into the lungs, heat and water vapor are transferred to the air by convection and evaporation from the surface lining the respiratory tract. By the time the air has reached the deepest parts of the lungs, the air is at deep body temperature (37°C) and saturated with water vapor (47 mm Hg partial pressure). As the air moves outward through the respiratory tract during expiration, some heat is transferred back to the body

FIGURE 14.49 Commercial greenhouses.

and some water is condensed. However, the inspired air still contains significantly more heat and water than the inspired air.

Respiration results in a latent heat loss and a sensible heat loss. The latent heat loss is based on the latent heat of vaporization of water by the equation

$$Q_{el} = (dm_a/dt)(Y_o - Y_i)\lambda$$

where

dm_a/dt = the kilograms of air breathed in and out per hour
$Y_o - Y_i$ = the difference in the expired and inspired air water content
λ = the latent heat of vaporization of water at the expired air temperature

The pulmonary ventilation rate (dm_a/dt) is primarily a function of the body metabolic rate via the following relationship:

$$(dm_a/dt) = 0.006\ M$$

where M is the metabolic rate in Kcal per hour.

It is somewhat rare to breath in bone-dry air, so the inspired air temperature and water content can affect the expired air from the lungs. Fanger and colleagues (1968) found that there is a relationship between the inspired water content of air and the expired water content

$$Y_o = 0.029 + 0.20\ Y_i$$

for normal conditions. Thus, if the entering air is very dry ($Y_i = 0$), the expired air would be less humid than if the inspired air were humid. McCutchan and Taylor (1951) determined that the temperature of the expired air is dependent on the inspired air in the following fashion:

$$T_o = 32.6 + 0.006\ T_i + 32\ Y_i$$

Although the relationship between T_o and T_i is expected, the inspired air humidity also plays a role. The air, upon being expired, is somewhat cooler than it would have been if the inspired air had been humid.

Besides the evaporative latent heat loss associated with respiration, there is also a sensible heat loss (when $T_o > T_i$) or heat gain (when $T_o < T_i$) that is described by

$$Q_{sl} = (dm_a/dt)\, C_p\, (T_o - T_i)$$

where C_p is the specific heat. This is the heat loss due to heating up the dry air component of the inspired air. An example of calculating heat loss follows.

Assume that 6 liters/min of bone-dry air at 20°C is inspired (12 breaths per minute at 500 ml tidal volume per breath = 6 liters per minute). Also assume that the expired air is fully saturated with water vapor (47 mm Hg) and is at 37°C. The physical properties are

$$C_{p,\ air} \text{ at } 20^\circ\text{C} = 0.25 \text{ cal/g}^\circ\text{C}$$
$$\lambda \text{ at } 37^\circ\text{C} = 577 \text{ cal/g}$$
$$\text{Vapor pressure of water at } 37^\circ\text{C} = 47 \text{ mm Hg}$$

The dry air mass flow rate in grams per minute can be calculated via the ideal gas law:

$$\begin{aligned}(dm_s/dt) &= (6 \text{ liters/min})(1 \text{ mol}/22.4 \text{ liters})(273^\circ\text{K}/293^\circ\text{K})(28.9 \text{ g/mol})\\ &= 7.2 \text{ g/mol of dry air}\end{aligned}$$

The amount of water vapor in the expired air is

$$(dm_w/dt) = (7.2/28.9)(47/760 - 47)(18) = 0.295 \text{ g/min}$$

Note that the true ratio of the partial pressure of water vapor in the lungs is not 47/760 but rather 47/760 – 47, since the water vapor is attached to the dry gas, which is the remainder of the partial pressure (713). The molecular weight of water is 18. It is necessary to compute the moles of water vapor when multiplying by the partial pressure ratio, since the partial pressures are related to the moles rather than the mass. Multiplying by the molecular weight returns the calculation back to units of mass.

The latent heat loss is

$$Q_{el} = 0.295\ (577) = 170 \text{ cal/min}$$

The sensible heat loss is

$$Q_{sl} = 7.2\ (0.25)(37 - 20) = 30.4 \text{ cal/min}$$

Note that the latent heat loss is far more significant than the sensible heat loss. The total heat loss of approximately 200 cal/min equals 12.0 kcal/hr, which is 17 percent of the basal metabolic rate of the body—an appreciable quantity. During exertion, one's respiration increases, which provides even greater heat loss by this mode. The overall rise in the metabolic heat production rate is also greater, such that an additional heat loss mode is necessary, such as sweating. Evaporation of sweat from the body produces the same latent heat loss as was evident via respiration. However, if one sweats profusely, such that sweat rolls off the body, then dehydration may occur, which can be quite dangerous.

An interesting note is that the panting of dogs is the primary mechanism of heat rejection, since dogs cannot sweat. By interchanging air with the dead space on a rapid basis, rather than via deep breathing, the dogs avoid hyperventilation yet produce heating of inspired air as well as humidification. Thus, there is latent and sensible heat loss. Many dogs have thick coats that provide insulation against heat gain or loss via conduction. Thus, if the dog were to get overheated, the solution is to hose down the dog, which would reduce the trapped air insulation via the fur. The same effect occurs when skiing in a cold environment. A ski parka has trapped air that serves as an insulator. However, if the parka were to become wet, the trapped air would be mixed with water, thus reducing its effectiveness as an insulator.

14.3.8 Heat Loss inside the Body

Many of the mechanisms for heat transfer that have been discussed refer to heat loss from the body to the environment. However, heat must be transferred from the body core to the skin or lungs. The two mechanisms available for such transfer are via conduction and via convection. Conduction is a slow process. Convection is accomplished via blood flow. As blood reaches the body core, it absorbs heat. Blood then is channeled toward the periphery, where heat is released. Ruch and Patton (1965) discuss the mechanisms by which the blood circulation affects internal heat distribution in three ways:

1. It minimizes temperature differences within the body. Tissues having high metabolic rates (e.g., the liver) are more highly perfused and are thus kept at nearly the same temperature as less active tissues. Cooler tissues are warmed by blood coming from the active organs.
2. It controls the effective body insulation in the skin region. Warm blood is increased to the skin via vasodilation, when the body wishes to reject heat. Blood is bypassed from arteries to veins via deeper channels through vasoconstriction when conservation of body heat is vital.
3. Countercurrent heat exchange between major arteries and veins often occurs to a significant extent. If heat conservation is necessary, arterial blood flowing along the body's extremities is precooled by loss of heat to adjacent venous streams.

A model of heat transfer from the core toward the skin can be split into two sections: from the core to a muscle region and from the muscle region to the skin (Cooney, 1976). The heat transfer includes both a conductive term using Fourier's law and a convective term using a forced convective form as follows:

$$Q = \underbrace{k_{cm}\, A\, \frac{(T_c - T_m)}{\Delta Z_{cm}}}_{\text{conduction}} + \underbrace{(dm_b/dt)\, C_{pB}\, (T_c - T_m)}_{\text{convection}}$$

$$Q = k_{ms}\, A\, \frac{(T_m - T_s)}{\Delta Z_{ms}} + (dm_b/dt)\, C_{pB}\, (T_m - T_s)$$

Since the heat transfer rate is assumed to be the same for both zones and the convection term is likely far greater than the conduction term, then an overall heat transfer rate from core to skin is of the form

$$Q = (dm_b/dt)\, C_{pB}\, (T_c - T_s)$$

This form does not account for any shunting of blood to allow for heat gain or loss via vasoconstriction or vasodilation.

14.3.9 Heat Loss in Extreme Environments

One example of heat loss (or gain) in an extreme environment is in a fire, which firefighters must deal with. As such, fire protection clothing and gear must protect against excessive heat conduction and therefore must have considerable insulating effect. It must also protect against radiation, since an open flame produces significant radiant energy, as was shown in Figure 14.42. As water is used at high flow rates, the firefighter's suit must also be water resistant. A typical firefighter's clothing, called turnout gear, is shown in Figure 14.50.

FIGURE 14.50 Typical firefighter's turnout gear. (1) Nomex hood. (2) Cotton T-shirt with department logo, small on chest and large on back. (3) Suspenders with retroflective striping, connecting to the pants at eight points. (4) Insulated pants with retroflective striping. They are held closed with Velcro and spring hooks. They are reinforced with leather at the knees and bottoms and have two large side pockets and one smaller rear pocket.

Continued

FIGURE 14.50, cont'd (5) Steel-toed insulated rubber boots with Vibram soles. These boots have handles at their tops to help pull them on and come up to just below the knees. (6) Helmet, with goggles and department logo. These helmets have a Velcro/buckle chinstrap, adjustable headband, and a protective cloth flap that hangs over the collar, further protecting the neck and preventing embers from falling down the collar. (7) Goggles, attached to helmet. Used for eye protection when not wearing breathing apparatus. (8) Radio with clip-on microphone. These radios are waterproof and inherently safe (nonsparking) for use in explosive atmospheres if necessary. (9) D-ring carabiner, used to clip additional equipment to the coat (not standard issue). (10) Flashlight. Department jackets have custom loops to hold the flashlights at center chest. (11) Insulated leather gloves. (12) Insulated jacket with retroflective striping. Oversized pockets hold radio, gloves, a hose strap, and other equipment. Like the pants, it is held closed with Velcro and spring hooks. (13) Air-line and pressure gauge. On this particular brand of SCBA, there are two air gauges: one at the bottom of the tank in back (for checking the pressure when the tank isn't being worn) and one in front. (14) SCBA harness, comprised of shoulder and waist straps. (15) PASS device. Current issue is an integrated PASS/SCBA, which activates automatically when the air supply is turned on. (16) Name label on back of helmet. (17) SCBA shoulder straps. (18) Department identification. (19) Air tank bottle and backpack frame. The bottle is quick-swappable because at a working fire a firefighter often goes through several bottles. (20) Regulator and main supply valve. (21) Name label (under tank). *Courtesy of the Santa Clara, California, Fire Department.*

The coat and pants are insulated and reinforced. They are made out of a fire-resistant fabric called PBI (other materials like Nomex are also used). They have reflective stripes to make them reflect when a light is pointed at them so they can be better seen in the dark, as well as glow-in-the-dark patches. They also have the firefighter's name and department printed on the back, like a football player, to help identify the individual, since when everyone is suited up and wearing masks, it is difficult to tell who is who. They are also equipped with several large pockets for holding gloves, tools, radios, and so forth. Rubber or leather waterproof steel-toed boots protect the firefighter's feet. The rubber boots are usually stored in the "turned-out" pants so they can be quickly donned—hence the term *turnouts*. A fire-retardant hood covers the firefighter's head and neck, protecting the ears and other parts that would be exposed under a helmet. When properly worn, no part of the firefighter's skin is exposed or unprotected.

Helmets are color-coded so the wearer can be quickly identified at a fire scene. For many fire departments, the color codes shown in Figure 14.51 are used.

Heat transfer can affect the human body in terms of sustaining life, protecting human life in hazardous settings, and maintaining comfort. Without thermal convection via blood flow, the body core temperature would rise to dangerous levels, since conduction is too

Helmet Color Coding

Yellow:	Firefighter/Paramedic
Black:	Volunteer Firefighter
Red:	Captain
White:	Chief (Battalion or District)
Blue:	Contract EMT/Paramedic (AMR)

FIGURE 14.51 Firefighter protective helmet with color coding to indicate function. *From Santa Clara, California, Fire Department.*

slow a process to remove all heat from the body core. Without the greenhouse effect, the earth's temperature would fall, since most of the thermal radiation from the sun would be reflected back into space. Even heat exchangers for blood cooling and reheating make extended surgeries possible.

14.4 EXERCISES

1. What would be the new values in Table 14.2 if the outside water partial pressure were 25 mm Hg?
2. During cell depolarization, the relative diffusivity of sodium increases 1,000-fold. What would be the new electrical potential across the cell membrane in mv?
3. How long would it take for changes in the arteriolar diameter that affect transcapillary fluid exchange to affect the venous return? The cardiac output is 5,000 ml/min. Assume an average diameter of a blood vessel and the average time course of blood through the cardiovascular system.
4. If the glomerular filtration rate (GFR) is 125 ml/min (125 cc/min), then how many pores are there in all glomeruli?
5. Based on the relative densities, compute the ratio of pressure differences in the manometer (shown in Figure 14.25) as a function of the same height difference for the following fluids (use water as a reference):
   Mercury/water ratio:
   Glycerin/water ratio:
   Oil to water ratio:
6. Research and describe the nonlinear behavior (stress vs. strain rate) for several non-Newtonian fluids such as a dilatant fluid, a pseudoplastic fluid, and a Bingham plastic fluid. Use the form of the stress versus strain rate equation for a Newtonian fluid as a reference point.
7. In Figure 14.33, there is a notch in the aortic pressure waveform where the aortic valve closes. Explain why that might happen in terms of valve movement.
8. Using the Reynolds number values listed in Section 14.2.6 for various blood vessels, compute the ratios of $V \times D$ for these vessels, assuming that the blood density and viscosity do not change. Then, researching the average vessel diameters, compute their average velocities.
9. Using the combination equation relating Q to (T1-T4) in the three-layer model for heat conduction, compute the overall term consisting of the thermal conductivities for a thermal window with glass on the outer sides and air in the middle. Use the K values in the list of thermal conductivities and assume that all thicknesses (ΔX) are the same.
10. Over a ten-day period, note the nighttime low temperature on each day, as well as whether the sky was clear at night, partly cloudy, or cloudy. How does that relate to the greenhouse effect?
11. Using a breathing rate of 18 breaths per minute instead of 12 and an external temperature of 30°C (303°K) instead of the 20° used in the example in the text, compute the new latent and sensible heat losses from the body. Assume all other parameters are unchanged.
12. Research Nomex and PBI and determine their thermal conductivities. Is one a better insulator than the other for use in firefighter protective clothing? Gortex is also used as a material. Compare that as well.

Suggested Readings

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