CHAPTER

9

Bioinstrumentation

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OUTLINE

AT THE CONCLUSION OF THIS CHAPTER, STUDENTS WILL BE ABLE TO:

- Describe the components of a basic instrumentation system.
- Analyze linear circuits using the node-voltage method.
- Simplify complex circuits using Thévenin's equivalent circuits.
- Solve circuits involving resistors, capacitors, and inductors of any order.
- Analyze circuits that use operational amplifiers.
- Determine the steady-state response to sinusoidal inputs and work in the phasor domain.

- Understand the basic concepts of analog filter design and design basic filters.
- Design low-pass, high-pass, and band-pass filters.
- Explain the different types of noise in a biomedical instrument system.

9.1 INTRODUCTION

This chapter provides basic information about bioinstrumentation and electric circuit theory used in other chapters. Many biomedical instruments use a transducer or sensor to convert a signal created by the body into an electric signal. Our goal in this chapter is to develop expertise in electric circuit theory applied to bioinstrumentation. We begin with a description of variables used in circuit theory, charge, current, voltage, power, and energy. Next, Kirchhoff's current and voltage laws are introduced, followed by resistance, simplifications of resistive circuits, and voltage and current calculations. Circuit analysis techniques are then presented, followed by inductance and capacitance, and solutions of circuits using the differential equation method. Finally, the operational amplifier and time varying signals are introduced.

Before 1900, medicine had little to offer the typical citizen because its resources were mainly the education and little black bag of the physician. The origins of the changes that occurred within medical science are found in several developments that took place in the applied sciences. During the early nineteenth century, diagnosis was based on physical examination, and treatment was designed to heal the structural abnormality. By the late nineteenth century, diagnosis was based on laboratory tests, and treatment was designed to remove the cause of the disorder. The trend toward the use of technology accelerated throughout the twentieth century. During this period, hospitals became institutions of research and technology. Professionals in the areas of chemistry, physics, mechanical engineering, and electrical engineering began to work in conjunction with the medical field, and biomedical engineering became a recognized profession. As a result, medical technology advanced more in the twentieth century than it had in the rest of history combined (Figure 9.1).

During this period, the area of electronics had a significant impact on the development of new medical technology. Men such as Richard Caton and Augustus Desire proved that the human brain and heart depend on bioelectric events. In 1903, William Einthoven expanded on these ideas after he created the first string galvanometer. Einthoven placed two skin sensors on a man and attached them to the ends of a silvered wire that was suspended through holes drilled in both ends of a large permanent magnet. The suspended silvered wire moved rhythmically as the subject's heart beat. By projecting a tiny light beam across the silvered wire, Einthoven was able to record the movement of the wire as waves on a scroll of moving photographic paper. Thus, the invention of the string galvanometer led to the creation of the electrocardiogram (ECG), which is routinely used today to measure and record the electrical activity of abnormal hearts and to compare those signals to normal ones.

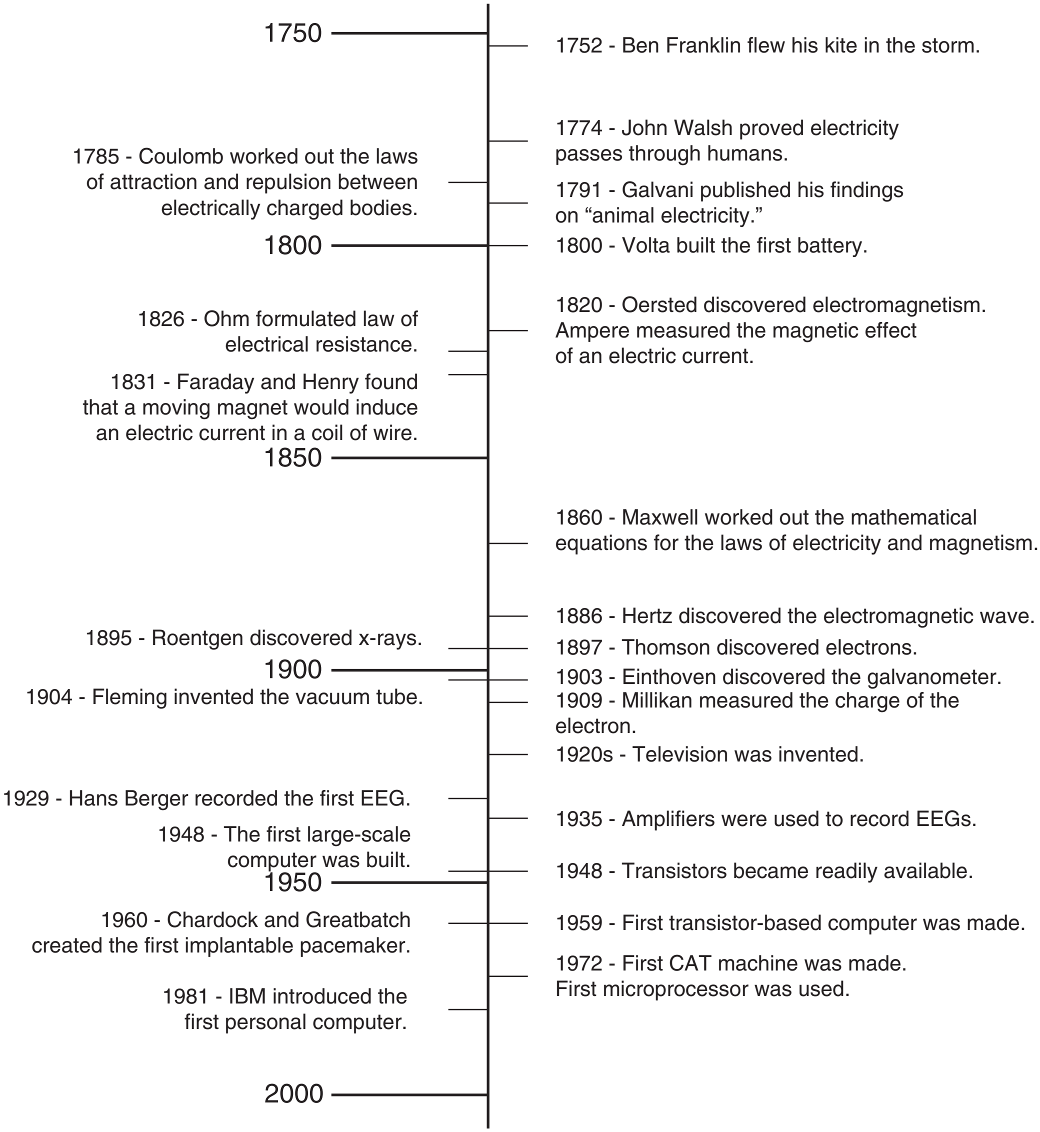


FIGURE 9.1 Timeline for major inventions and discoveries that led to modern medical instrumentation.

In 1929, Hans Berger created the first electroencephalogram (EEG), which is used to measure and record electrical activity of the brain. In 1935, electrical amplifiers were used to prove that the electrical activity of the cortex had a specific rhythm, and in 1960, electrical amplifiers were used in devices such as the first implantable pacemaker that was created by William Chardack and Wilson Greatbatch. These are just a small sample of the many examples in which the field of electronics has been used to significantly advance medical technology.

Many other advancements that were made in medical technology originated from research in basic and applied physics. In 1895, the x-ray machine, one of the most important technological inventions in the medical field, was created when W. K. Roentgen found that x-rays could be used to give pictures of the internal structures of the body. Thus, the x-ray machine was the first imaging device to be created. (Radiation imaging is discussed in detail in Chapter 15.)

Another important addition to medical technology was provided by the invention of the computer, which allowed much faster and more complicated analyses and functions to be performed. One of the first computer-based instruments in the field of medicine, the sequential multiple analyzer plus computer, was used to store a vast amount of data pertaining to clinical laboratory information. The invention of the computer made it possible for laboratory tests to be performed and analyzed faster and more accurately.

The first large-scale computer-based medical instrument was created in 1972 when the computerized axial tomography (CAT) machine was invented. The CAT machine created an image that showed all of the internal structures that lie in a single plane of the body. This new type of image made it possible to have more accurate and easier diagnosis of tumors, hemorrhages, and other internal damage from information that was obtained noninvasively (for details, see Chapter 15).

Telemedicine, which uses computer technology to transmit information from one medical site to another, is being explored to permit access to health care for patients in remote locations. Telemedicine can be used to let a specialist in a major hospital receive information on a patient in a rural area and send back a plan of treatment specific to that patient.

Today, a wide variety of medical devices and instrumentation systems are available. Some are used to monitor patient conditions or acquire information for diagnostic purposes—for example, ECG and EEG machines—while others are used to control physiological functions—for example, pacemakers and ventilators. Some devices, like pacemakers, are implantable, while many others are used noninvasively. This chapter focuses on those features that are common to devices that are used to acquire and process physiological data.

9.2 BASIC BIOINSTRUMENTATION SYSTEM

The quantity, property, or condition that is measured by an instrumentation system is called the measurand (Figure 9.2). This can be a bioelectric signal, such as those generated by muscles or the brain, or a chemical or mechanical signal that is converted to an electrical signal. As explained in Chapter 10, sensors are used to convert physical measurands into electric outputs. The outputs from these biosensors are analog signals—that is, continuous signals—that are sent to the analog processing and digital conversion block. There, the signals are amplified, filtered, conditioned, and converted to digital form. Methods for modifying analog signals, such as amplifying and filtering an ECG signal, are discussed

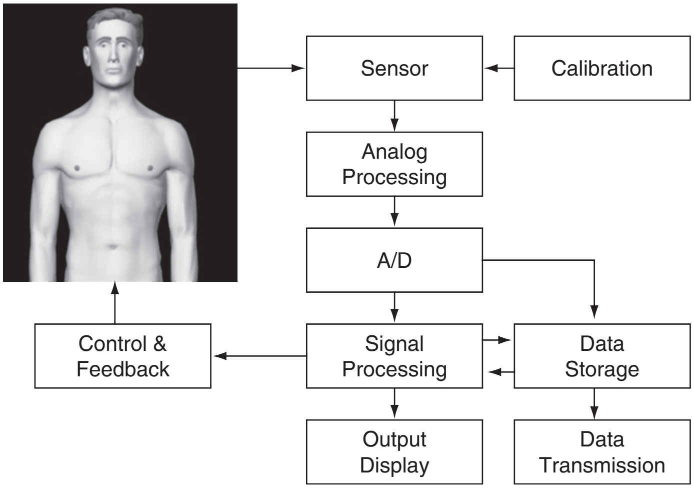


FIGURE 9.2 Basic instrumentation systems using sensors to measure a signal with data acquisition, storage, and display capabilities, along with control and feedback.

later in this chapter. Once the analog signals have been digitized and converted to a form that can be stored and processed by digital computers, many more methods of signal conditioning can be applied (for details, see Chapter 11).

Basic instrumentation systems also include output display devices that enable human operators to view the signal in a format that is easy to understand. These displays may be numerical or graphical, discrete or continuous, and permanent or temporary. Most output display devices are intended to be observed visually, but some also provide audible output—for example, a beeping sound with each heartbeat.

In addition to displaying data, many instrumentation systems have the capability of storing data. In some devices, the signal is stored briefly so further processing can take place or so an operator can examine the data. In other cases, the signals are stored permanently so different signal processing schemes can be applied at a later time. Holter monitors, for example, acquire 24 hours of ECG data that is later processed to determine arrhythmic activity and other important diagnostic characteristics.

With the invention of the telephone and now with the Internet, signals can be acquired with a device in one location, perhaps in a patient's home, and transmitted to another device for processing and/or storage. This has made it possible, for example, to provide quick diagnostic feedback if a patient has an unusual heart rhythm while at home. It has also allowed medical facilities in rural areas to transmit diagnostic images to tertiary care hospitals so that specialized physicians can help general practitioners arrive at more accurate diagnoses.

Two other components play important roles in instrumentation systems. The first is the calibration signal. A signal with known amplitude and frequency content is applied to the instrumentation system at the sensor's input. The calibration signal allows the components of the system to be adjusted so that the output and input have a known, measured relationship. Without this information, it is impossible to convert the output of an instrument system into a meaningful representation of the measurand.

Another important component, a feedback element, is not a part of all instrumentation systems. These devices include pacemakers and ventilators that stimulate the heart or the lungs. Some feedback devices collect physiological data and stimulate a response—a heartbeat or breath—when needed or are part of biofeedback systems in which the patient is made aware of a physiological measurement, such as blood pressure, and uses conscious control to change the physiological response.

9.3 CHARGE, CURRENT, VOLTAGE, POWER, AND ENERGY

9.3.1 Charge

Two kinds of charge, positive and negative, are carried by protons and electrons, respectively. The negative charge carried by an electron, q_e, is the smallest amount of charge that exists and is measured in units called coulombs (C).

$$q_e = -1.602 \cdot 10^{-19}\ \mathrm{C}$$

The symbol $q(t)$ is used to represent charge that changes with time, and Q is used for constant charge. The charge carried by a proton is the opposite of the electron.

9.3.2 Current

Electric current, $i(t)$, is defined as the change in the amount of charge that passes through a given point or area in a specified time period. Current is measured in amperes (A). By definition, one ampere equals one coulomb/second (C/s).

$$i(t) = \frac{dq}{dt} \tag{9.1}$$

and

$$q(t) = \int_{t_0}^{t} i(\lambda)\,d\lambda + q(t_0) \tag{9.2}$$

Current, defined by Eq. (9.1), also depends on the direction of flow, as illustrated in the circuit in Figure 9.3. Current is defined as positive if

a. A positive charge is moving in the direction of the arrow.
b. A negative charge is moving in the opposite direction of the arrow.

Since these two possibilities produce the same outcome, there is no need to be concerned as to which is responsible for the current. In electric circuits, current is carried by electrons in metallic conductors.

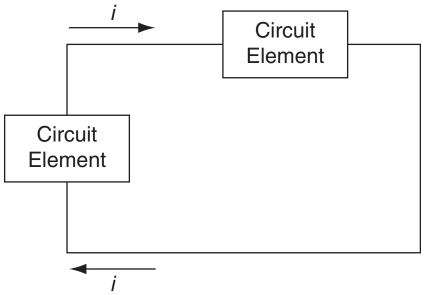


FIGURE 9.3 A simple electric circuit illustrating current flowing around a closed loop.

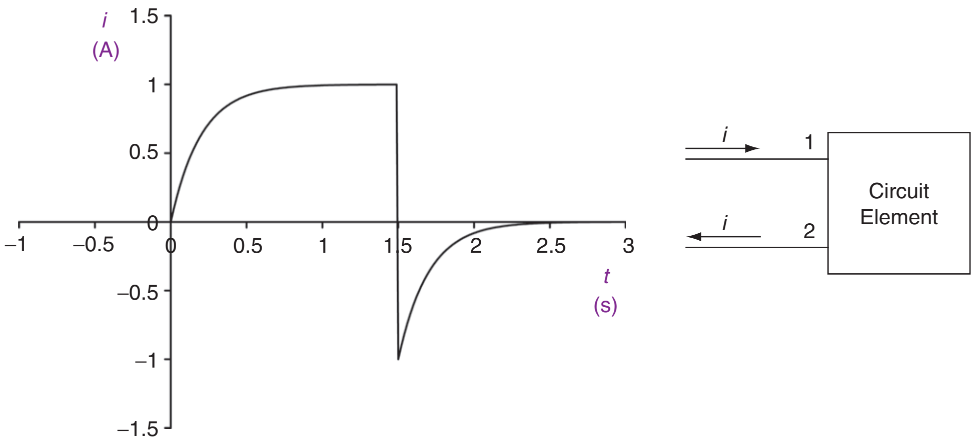


FIGURE 9.4 (Left) A sample current waveform. (Right) A circuit element with current entering terminal 1 and leaving terminal 2. Passive circuit elements have two terminals with a known voltage-current relationship. Examples of passive circuit elements include resistors, capacitors, and inductors.

Current is typically a function of time, as given by Eq. (9.1). Consider Figure 9.4, with the current entering terminal 1 in the circuit on the right. In the time interval 0 to 1.5 s, the current is positive and enters terminal 1. In the time interval 1.5 to 3 s, the current is negative and enters terminal 2 with a positive value. We typically refer to a constant current as a DC current and denote it with a capital letter such as I, indicating it doesn't change with time. We denote a time-varying current with a lowercase letter, such as $i(t)$, or just i.

Kirchhoff's Current Law

Current can flow only in a closed circuit, as shown in Figure 9.3. No current is lost as it flows around the circuit because net charge cannot accumulate within a circuit element and charge must be conserved. Whatever current enters one terminal must leave at the other terminal. Since charge cannot be created and must be conserved, the sum of the currents at any node—that is, a point at which two or more circuit elements have a common

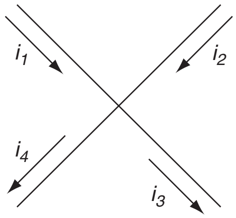


FIGURE 9.5 A node with four currents.

connection—must equal zero so no net charge accumulates. This principle is known as Kirchhoff's current law (KCL), given as

$$\sum_{n=1}^{N} i_n(t) = 0 \tag{9.3}$$

where there are N currents leaving the node. Consider the circuit in Figure 9.5. Using Eq. (9.3) and applying KCL for the currents *leaving* the node gives

$$-i_1 - i_2 + i_4 + i_3 = 0$$

The previous equation is equivalently written for the currents *entering* the node, since

$$i_1 + i_2 - i_4 - i_3 = 0$$

It should be clear that the application of KCL is for *all* currents whether they are all leaving or all entering the node.

In describing a circuit, we define its characteristics with the terms *node, branch, path, closed path,* and *mesh* as follows:

- **Node:** A point at which two or more circuit elements have a common connection.
- **Branch:** A circuit element or connected group of circuit elements. A connected group of circuit elements usually connect nodes together.
- **Path:** A connected group of circuit elements in which none is repeated.
- **Closed Path:** A path that starts and ends at the same node.
- **Mesh:** A closed path that does not contain any other closed paths within it.
- **Essential Node:** A point at which three or more circuit elements have a common connection.
- **Essential Branch:** A branch that connects two essential nodes.

Figure 9.6 shows five nodes—A, B, C, D, and E—that are all essential nodes. Kirchhoff's current law is applied to each of the nodes as follows:

$$\text{Node A: } -i_1 + i_2 - i_3 = 0$$
$$\text{Node B: } i_3 + i_4 + i_5 - i_6 = 0$$
$$\text{Node C: } i_1 - i_4 - i_8 = 0$$
$$\text{Node D: } -i_7 - i_5 + i_8 = 0$$
$$\text{Node E: } -i_2 + i_6 + i_7 = 0$$

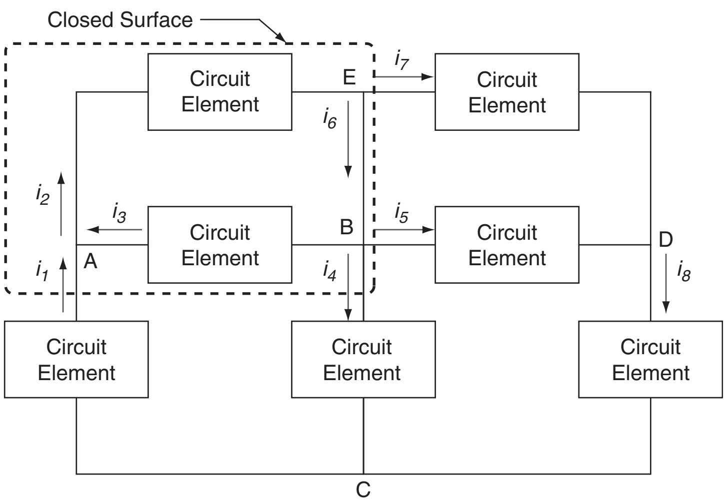


FIGURE 9.6 A circuit with a closed surface.

Kirchhoff's current law is also applicable to any closed surface surrounding a part of the circuit. It is understood that the closed surface does not intersect any of the circuit elements. Consider the closed surface drawn with dashed lines in Figure 9.6. Kirchhoff's current law applied to the closed surface gives

$$-i_1 + i_4 + i_5 + i_7 = 0$$

9.3.3 Voltage

Voltage represents the work per unit charge associated with moving a charge between two points (A and B in Figure 9.7) and is given as

$$v = \frac{dw}{dq} \tag{9.4}$$

The unit of measurement for voltage is the volt (V). A constant (DC) voltage source is denoted by V, while a time-varying voltage is denoted by $v(t)$, or just v. In Figure 9.7, the voltage, v, between two points (A and B) is the amount of energy required to move a charge from point A to point B.

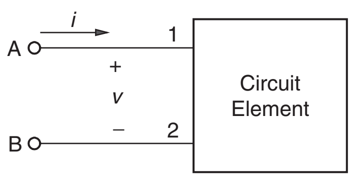


FIGURE 9.7 Voltage and current convention.

Kirchhoff's Voltage Law

Kirchhoff's voltage law (KVL) states that the sum of all voltages in a closed path is zero, or

$$\sum_{n=1}^{N} v_n(t) = 0 \tag{9.5}$$

where there are N voltage drops assigned around the closed path, with $v_n(t)$ denoting the individual voltage drops. The sign for each voltage drop in Eq. (9.5) is the first sign encountered while moving around the closed path.

Consider the circuit in Figure 9.8, with each circuit element assigned a voltage, v_n, with a given polarity and three closed paths, CP1, CP2, and CP3. Kirchhoff's voltage law for each closed path is given as

$$\begin{aligned}
&\text{CP1: } -v_3 + v_1 + v_4 = 0\\
&\text{CP2: } -v_4 + v_2 + v_5 = 0\\
&\text{CP3: } -v_3 + v_1 + v_2 + v_5 = 0
\end{aligned}$$

Kirchhoff's laws are applied in electric circuit analysis to determine unknown voltages and currents. Each unknown variable has its distinct equation. To solve for the unknowns using MATLAB, we create a matrix representation of the set of equations and solve using the techniques described in the appendix. This method is demonstrated in many examples in this chapter.

9.3.4 Power and Energy

Power is the rate of energy expenditure given as

$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = vi \tag{9.6}$$

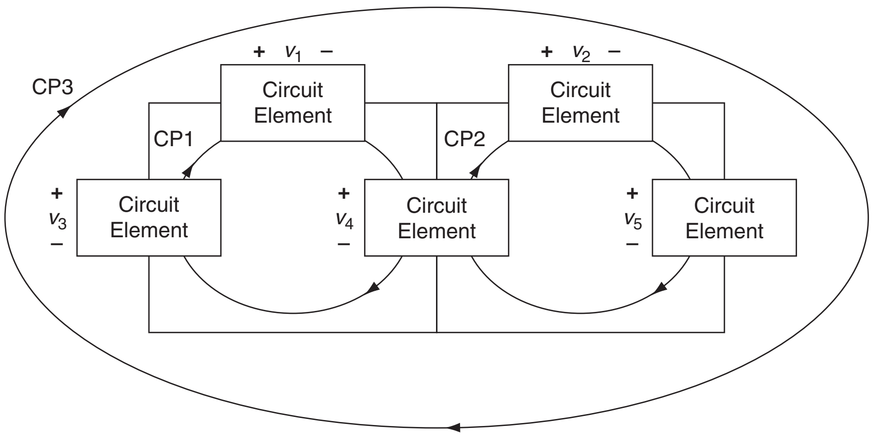


FIGURE 9.8 Circuit illustrating Kirchhoff's voltage law. Closed paths are identified as CP1, CP2, and CP3.

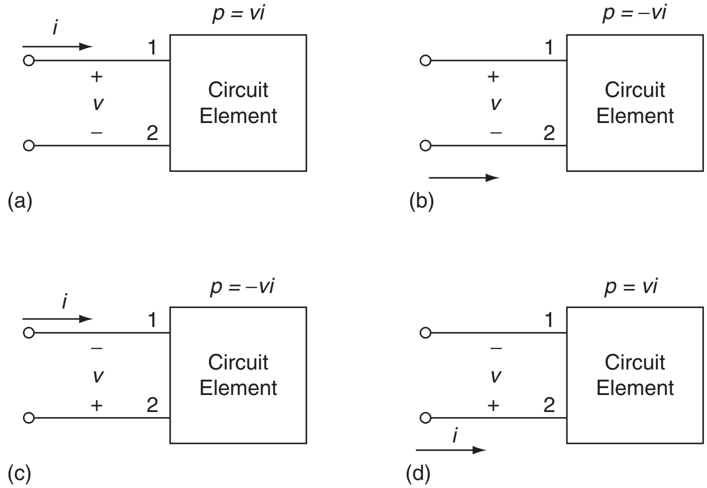


FIGURE 9.9 Polarity references for four cases of current and voltage. Cases (a) and (d) result in positive power being consumed by the circuit element. Cases (b) and (c) result in negative power being extracted from the circuit element.

where p is power measured in watts (W), and w is energy measured in joules (J). Power is usually determined by the product of voltage across a circuit element and the current through it. By convention, we assume that a positive value for power indicates that power is being delivered (or absorbed or consumed) by the circuit element. A negative value for power indicates that power is being extracted or generated by the circuit element—that is, a battery.

Figure 9.9 illustrates the four possible cases for a circuit element's current and voltage configuration. According to convention, if both i and v are positive, with the arrow and polarity shown in Figure 9.9(a), energy is absorbed (either lost by heat or stored). If either the current arrow or voltage polarity is reversed, as in (b) and (c), energy is supplied to the circuit. Note that if both the current direction and voltage polarity are reversed together, as in (d), energy is absorbed.

A passive circuit element is defined as an element whose power is always positive or zero, which may be dissipated as heat (resistance), stored in an electric field (capacitor), or stored in a magnetic field (inductor). We define an active circuit element as one whose power is negative and capable of generating energy. Energy is given by

$$w(t) = \int_{-\infty}^{t} p\,dt \tag{9.7}$$

9.3.5 Sources

Sources are two terminal devices that provide energy to a circuit. There is no direct voltage-current relationship for a source; when one of the two variables is given, the other

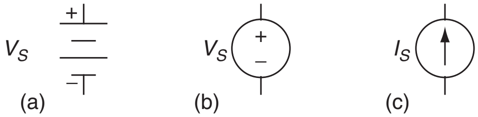


FIGURE 9.10 Basic symbols used for independent sources: (a) battery and (b) ideal voltage source. V_s can be a constant DC source (battery) or a time-varying source. (c) Ideal current source I_s.

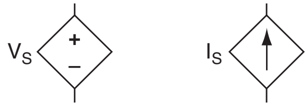


FIGURE 9.11 Basic symbols used for dependent or controlled sources. (Left) Controlled voltage source. The voltage v_s is a known function of some other voltage or current in the circuit. (Right) Controlled current source. The current i_s is a known function of some other voltage or current in the circuit.

cannot be determined without knowledge of the rest of the circuit. Independent sources are devices for which the voltage or current is given and the device maintains its value regardless of the rest of the circuit. A device that generates a prescribed voltage at its terminals, regardless of the current flow, is called an ideal voltage source. Figures 9.10a and b show the general symbols for an ideal voltage source. Figure 9.10c shows an ideal current source that delivers a prescribed current to the attached circuit. The voltage generated by an ideal current source depends on the elements in the rest of the circuit.

Figure 9.11 shows a dependent voltage and current source. A dependent source takes on a value equaling a known function of some other voltage or current value in the circuit. We use a diamond-shaped symbol to represent a dependent source. Often, a dependent source is called a controlled source. The current generated for a dependent voltage source and the voltage for a dependent current source depend on circuit elements in the rest of the circuit. Dependent sources are very important in electronics. Later in this chapter, we will see that the operational amplifier uses a controlled voltage source for its operation.

9.4 RESISTANCE

9.4.1 Resistors

A resistor is a circuit element that limits the flow of current through it and is denoted with the symbol ⩘⩘. Resistors are made of different materials, and their ability to impede current is given with a value of resistance, denoted R. Resistance is measured in ohms (Ω), where $1\ \Omega = 1\ \mathrm{V/A}$. A theoretical bare wire that connects circuit elements together has a resistance of zero. A gap between circuit elements has a resistance of infinity. An ideal resistor follows Ohm's law, which describes a linear relationship between voltage and current, with a slope equal to the resistance.

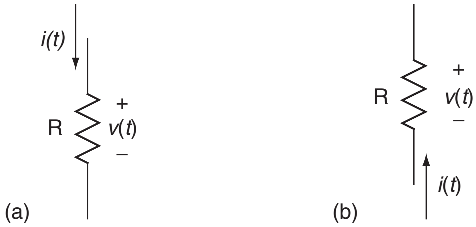


FIGURE 9.12 An ideal resister with resistance R in ohms (Ω).

There are two ways to write Ohm's law, depending on the current direction and voltage polarity. Ohm's law is written for Figure 9.12a as

$$v = iR \tag{9.8}$$

and for Figure 9.12b as

$$v = -iR \tag{9.9}$$

In this book, we use the convention shown in Figure 9.12a to write the voltage drop across a resistor. As described, the voltage across a resistor is equal to the product of the current flowing through the element and its resistance, R. This linear relationship does not apply at very high voltages and currents. Some electrically conducting materials have a very small range of currents and voltages in which they exhibit linear behavior. This is true of many physiological models as well: linearity is observed only within a range of values. Outside this range, the model is nonlinear. We define a short circuit as shown in Figure 9.13a, with $R = 0$ and having a 0 V voltage drop. We define an open circuit as shown in Figure 9.13b, with $R = \infty$ and having 0 A current pass through it.

Each material has a property called resistivity (ρ) that indicates the resistance of the material. Conductivity (σ) is the inverse of resistivity, and conductance (G) is the inverse of resistance. Conductance is measured in units called siemens (S) and has units of A/V. In terms of conductance, Ohm's law is written as

$$i = Gv \tag{9.10}$$

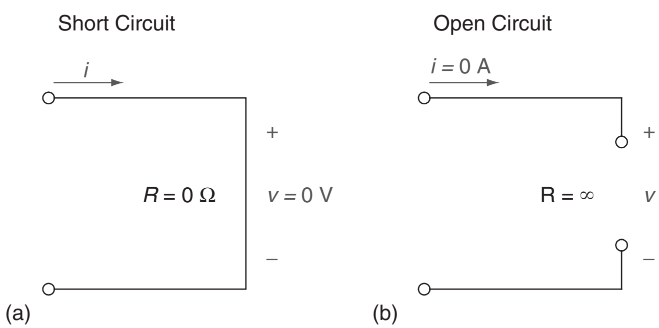
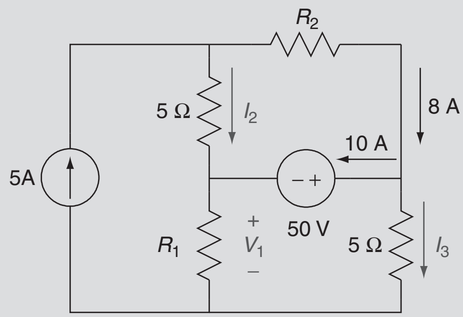


FIGURE 9.13 Short and open circuits.

EXAMPLE PROBLEM 9.1

From the following circuit, find I_2, I_3, and V_1.



Solution

Find I_2 first by applying KCL at the node in the upper left of the circuit.

$$-5 + I_2 + 8 = 0$$

and

$$I_2 = -3 \text{ A}$$

Current I_3 is determined by applying KCL at the node on the right of the circuit.

$$10 + I_3 - 8 = 0$$

and

$$I_3 = -2 \text{ A}$$

Voltage V_1 is determined by applying KVL around the lower right closed path and using Ohm's law.

$$-V_1 - 50 + 5I_3 = 0$$
$$V_1 = -50 + 5 \times (-2) = -60 \text{ V}$$

9.4.2 Power

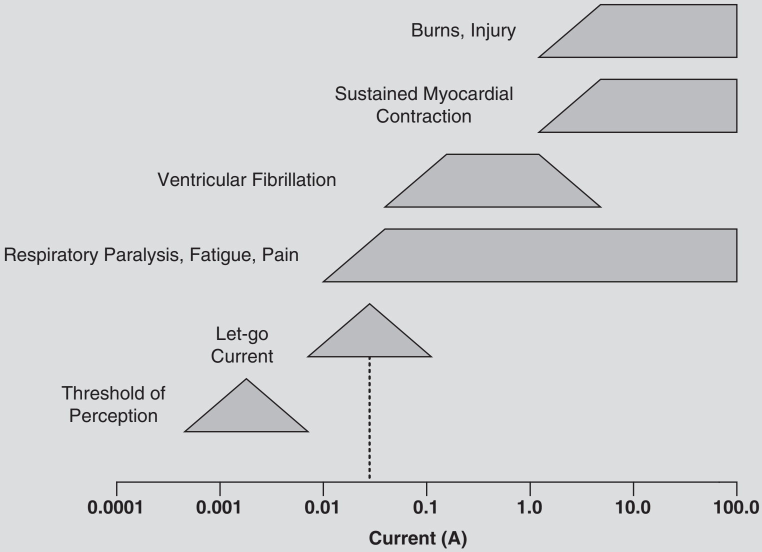
The power consumed by a resistor is given by the combination of Eq. (9.6) and either Eq. (9.8) or (9.9) as

$$p = vi = \frac{v^2}{R} = i^2 R \tag{9.11}$$

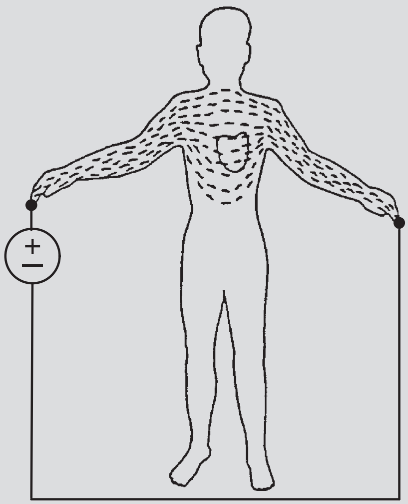
and given off as heat. Equation (9.11) demonstrates that regardless of the voltage polarity and current direction, power is consumed by a resistor. Power is always positive for a resistor, which is true for any passive element.

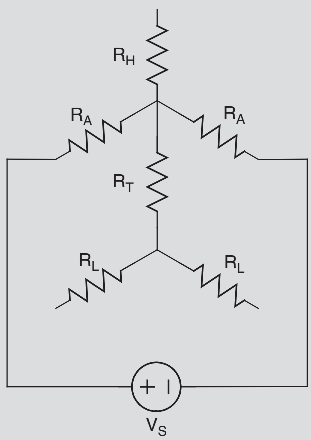
EXAMPLE PROBLEM 9.2

Electric safety is of paramount importance in a hospital or clinical environment. If sufficient current is allowed to flow through the body, significant damage can occur, as illustrated in the following figure.



For instance, a current of magnitude 50 mA (dashed line) is enough to cause ventricular fibrillation, as well as other conditions. The figure on the left shows the current distribution from a macroshock from one arm to another. A crude electric circuit model of the body consisting of two arms (each with resistance $R_{A)}$, two legs (each with resistance R_L), body trunk (with resistance R_T), and head (with resistance R_H) is shown in the following figure on the right.





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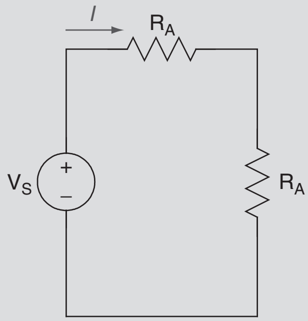
Since the only elements that form a closed path through which the current can flow are given by the source in series with the two arms, we reduce the body electric circuits to



If $R_A = 400\ \Omega$ and $V_s = 120$ V, then find I.

Solution

Using Ohm's law, we get

$$I = \frac{V_s}{R_A + R_A} = \frac{120}{800} = 0.15\ \text{A}$$

The current I is the current passing through the heart, and at this level it would cause ventricular fibrillation.

9.4.3 Equivalent Resistance

It is sometimes possible to reduce complex circuits into simpler, equivalent circuits. Two circuits are considered equivalent if they cannot be distinguished from each other by voltage and current measurements—that is, the two circuits behave identically. Consider the two circuits A and B in Figure 9.14, consisting of combinations of resistors, each stimulated

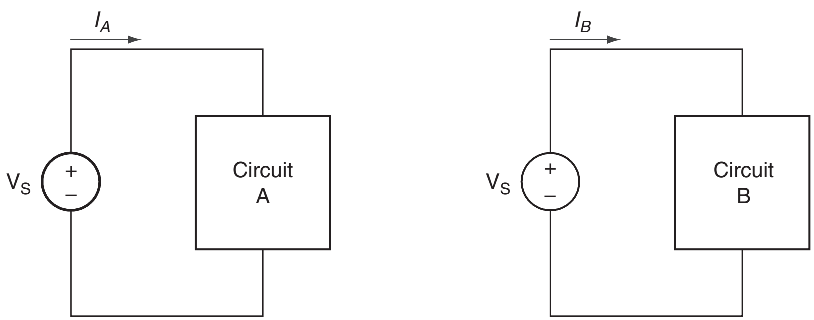


FIGURE 9.14 Two circuits.

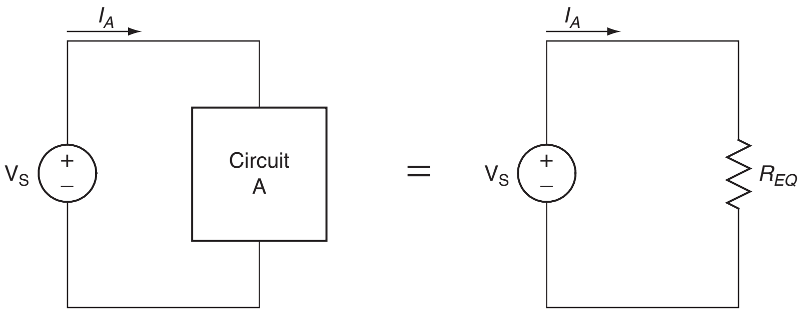


FIGURE 9.15 Equivalent circuits.

by a DC voltage V_s. These two circuits are equivalent if $I_A = I_B$. We represent the resistance of either circuit using Ohm's law as

$$R_{EQ} = \frac{V_s}{I_A} = \frac{V_s}{I_B} \tag{9.12}$$

Thus, it follows that any circuit consisting of resistances can be replaced by an equivalent circuit, as shown in Figure 9.15. When we discuss a Thévenin equivalent circuit later in this chapter, we will expand this remark to include any combination of sources and resistances.

9.4.4 Series and Parallel Combinations of Resistance

Resistors in Series

If the same current flows from one resistor to another, the two are said to be in series. If these two resistors are connected to a third and the same current flows through all of them, then the three resistors are in series. In general, if the same current flows through N resistors, then the N resistors are in series. Consider Figure 9.16 with three resistors in series. An equivalent circuit can be derived through KVL as

$$-V_s + IR_1 + IR_2 + IR_3 = 0$$

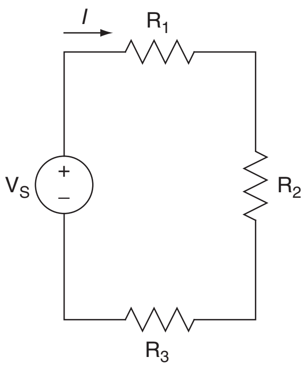


FIGURE 9.16 A series circuit.

or rewritten in terms of an equivalent resistance R_{EQ} as

$$R_{EQ} = \frac{V_s}{I} = R_1 + R_2 + R_3$$

In general, if we have N resistors in series,

$$R_{EQ} = \sum_{i=1}^{N} R_i \tag{9.13}$$

Resistors in Parallel

Two or more elements are said to be in parallel if the same voltage is across each of the resistors. Consider the three parallel resistors shown in Figure 9.17. We use a shorthand notation to represent resistors in parallel using the $\|$ symbol. Thus, in Figure 9.17, $R_{EQ} = R_1 \| R_2 \| R_3$. An equivalent circuit for Figure 9.17 is derived through KCL as

$$-I + \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} = 0$$

or rewritten in terms of an equivalent resistance R_{EQ} as

$$R_{EQ} = \frac{V_s}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

In general, if we have N resistors in parallel,

$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \tag{9.14}$$

For just two resistors in parallel, Eq. (9.14) is written as

$$R_{EQ} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} \tag{9.15}$$

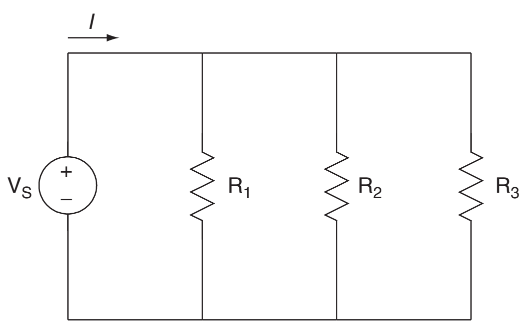
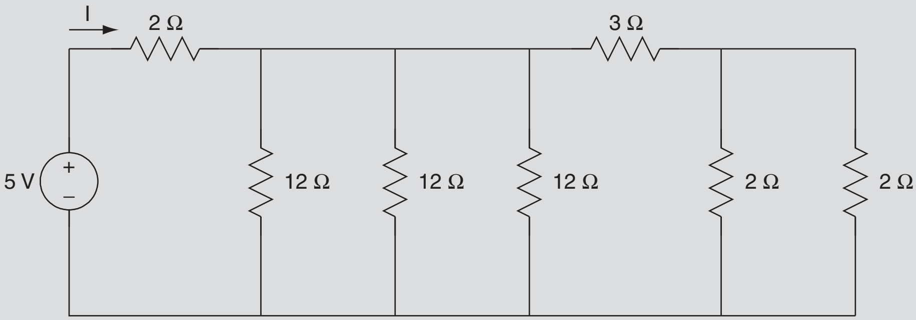


FIGURE 9.17 A parallel circuit.

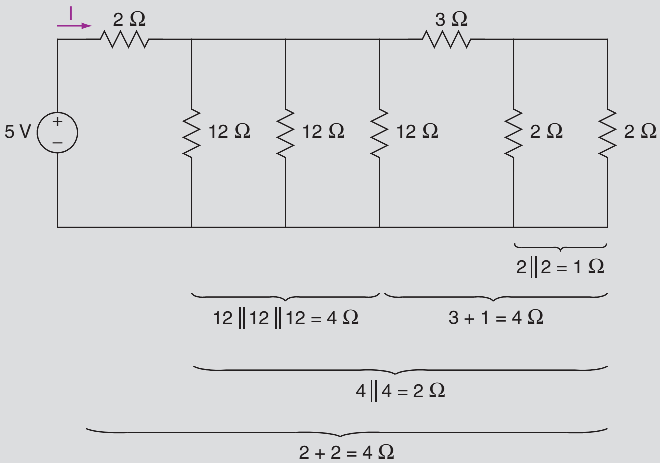
EXAMPLE PROBLEM 9.3

Find R_{EQ} and the power supplied by the source for the following circuit.



Solution

To solve for R_{EQ}, apply from right to left the parallel and series combinations. First, we have two 2 Ω resistors in parallel that are in series with the 3 Ω resistor. Next, this group is in parallel with the three 12 Ω resistors. Finally, this group is in series with the 2 Ω resistor. These combinations are shown in the following figure and calculation:



$$
\begin{aligned}
R_{EQ} &= 2\,\Omega + ((12\,\Omega \parallel 12\,\Omega \parallel 12\,\Omega) \parallel (3\,\Omega + (2\,\Omega \parallel 2\,\Omega))) \\
&= 2 + \left(\left(\frac{1}{\frac{1}{12}+\frac{1}{12}+\frac{1}{12}}\right) \parallel \left(3 + \frac{1}{\frac{1}{2}+\frac{1}{2}}\right)\right) \\
&= 2 + ((4) \parallel (3+1)) = 2 + 2 = 4\,\Omega
\end{aligned}
$$

Continued

Accordingly,

$$I = \frac{5}{R_{EQ}} = \frac{5}{4} = 1.25 \text{ A}$$

and

$$p = 5 \times I = 6.25 \text{ W}$$

9.4.5 Voltage and Current Divider Rules

Let us now extend the concept of equivalent resistance, $R_{EQ} = \frac{V}{I}$, to allow us to quickly calculate voltages in series resistor circuits and currents in parallel resistor circuits without digressing to the fundamentals.

Voltage Divider Rule

The voltage divider rule allows us to easily calculate the voltage across a given resistor in a series circuit. Consider finding V_2 in the series circuit shown in Figure 9.18, where $R_{EQ} = R_1 + R_2$. Accordingly,

$$I = \frac{V_s}{R_{EQ}} = \frac{V_s}{R_1 + R_2}$$

and therefore

$$V_2 = IR_2 = V_s \frac{R_2}{R_1 + R_2}$$

This same analysis can be used to find V_1 as

$$V_1 = V_s \frac{R_1}{R_1 + R_2}$$

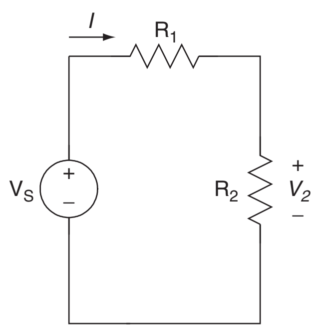


FIGURE 9.18 Voltage divider rule circuit.

In general, if a circuit contains N resistors in series, the voltage divider rule gives the voltage across any one of the resistors, R_i, as

$$V_i = V_s \frac{R_i}{R_1 + R_2 + \cdots R_N} \tag{9.16}$$

Current Divider Rule

The current divider rule allows us to easily calculate the current through any resistor in parallel resistor circuits. Consider finding I_2 in the parallel circuit shown in Figure 9.19, where $R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$. Accordingly,

$$I_2 = \frac{V_s}{R_2}$$

and

$$V_s = I \frac{R_1 R_2}{R_1 + R_2}$$

yielding after substituting V_s

$$I_2 = I \frac{\frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

In general, if a circuit contains N resistors in parallel, the current divider rule gives the current through any one of the resistors, R_i, as

$$I_i = I \frac{\frac{1}{R_i}}{\frac{1}{R_1} + \frac{1}{R_2} \cdots + \frac{1}{R_N}} \tag{9.17}$$

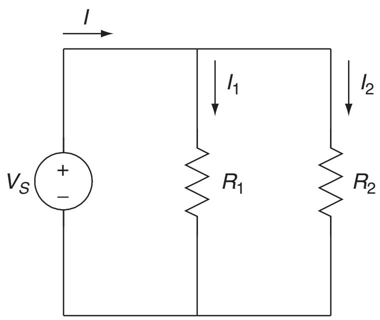
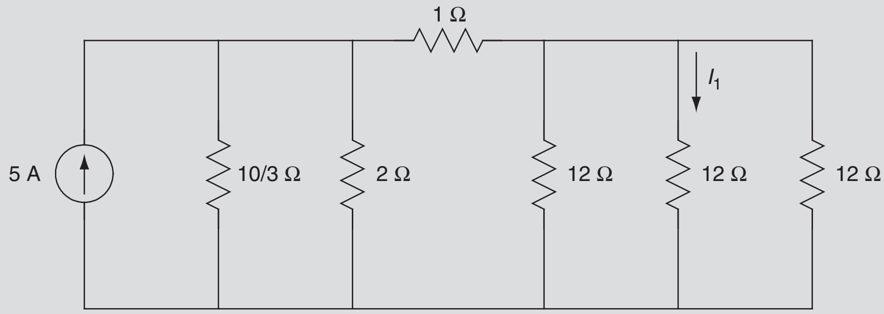


FIGURE 9.19 Current divider rule circuit.

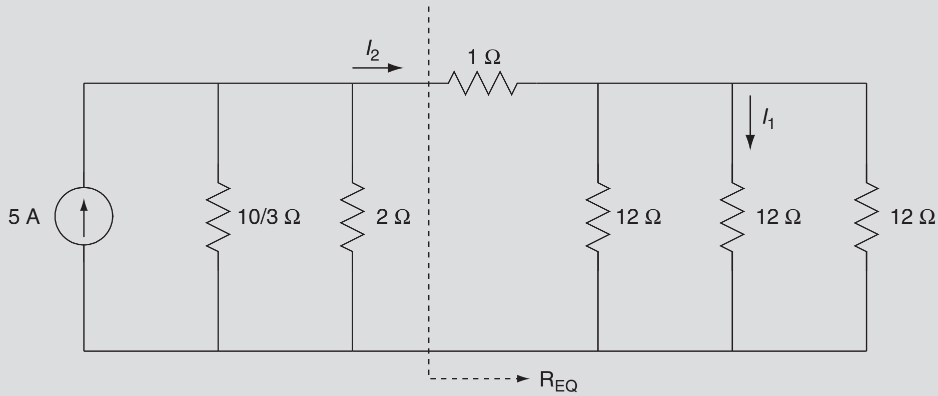
EXAMPLE PROBLEM 9.4

For the following circuit, find I_1.



Solution

This circuit problem is solved in two parts, as is evident from the redrawn circuit that follows, by first finding I_2 and then I_1.



To begin, first find R_{EQ}, which, when placed into the circuit, reduces to three parallel resistors from which I_2 is calculated. The equivalent resistance is found as

$$R_{EQ} = 1 + (12 \parallel 12 \parallel 12) = 1 + \frac{1}{\frac{1}{12} + \frac{1}{12} + \frac{1}{12}} = 5\ \Omega$$

Applying the current divider rule on the three parallel resistors, $\frac{10}{3} \parallel 2 \parallel R_{EQ}$, we have

$$I_2 = 5\left(\frac{\frac{1}{5}}{\frac{3}{10} + \frac{1}{2} + \frac{1}{5}}\right) = 1\ \text{A}$$

I_2 flows through the 1 Ω resistor and then divides into three equal currents of $\frac{1}{3}$ A through each 12 Ω resistor. The current I_1 can also be found by applying the current divider rule as

$$I_1 = I_2\left(\frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{12}+\frac{1}{12}}\right) = \frac{\frac{1}{12}}{\frac{1}{12}+\frac{1}{12}+\frac{1}{12}} = \frac{1}{3}\text{ A}$$

9.5 LINEAR NETWORK ANALYSIS

Our methods for solving circuit problems up to this point have included applying Ohm's law and Kirchhoff's laws, resistive circuit simplification, and the voltage and current divider rules. This approach works for all circuit problems, but as the circuit complexity increases, it becomes more difficult to solve problems. In this section, we introduce the node-voltage method to provide a systematic and easy solution of circuit problems. The application of the node-voltage method involves expressing the branch currents in terms of one or more node voltages and applying KCL at each of the nodes. This method provides a systematic approach that leads to a solution that is efficient and robust, resulting in a minimum number of simultaneous equations that save time and effort.

The use of node equations provides a systematic method for solving circuit analysis problems by the application of KCL at each essential node. The node-voltage method involves the following two steps:

1. Assign each node a voltage with respect to a reference node (ground). The reference node is usually the one with the most branches connected to it and is denoted with the symbol ⏚. All voltages are written with respect to the reference node.
2. Except for the reference node, we write KCL at each of the *N-1* nodes.

The current through a resistor is written using Ohm's law, with the voltage expressed as the difference between the potential on either end of the resistor with respect to the reference node, as shown in Figure 9.20. We express node-voltage equations as the currents leaving the node. Two adjacent nodes give rise to the current moving to the right (like Figure 9.20a) for one node and the current moving to the left (like Figure 9.20b) for the other node. The current is written for (a) as $I_A = \frac{V}{R} = \frac{V_1 - V_2}{R}$ and for (b) as $I_B = \frac{V}{R} = \frac{V_2 - V_1}{R}$. It is easy to verify in (a) that $V = V_1 - V_2$ by applying KVL.

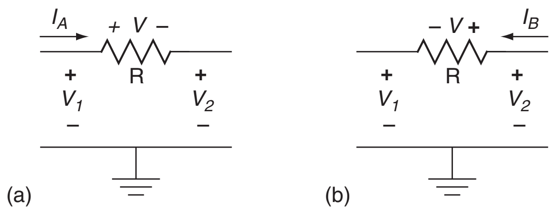
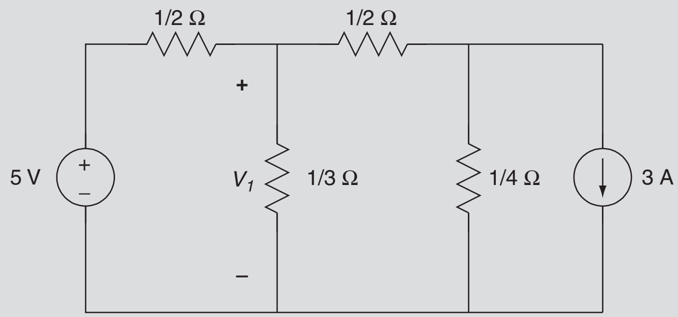


FIGURE 9.20 Ohm's law written in terms of node voltages.

If one of the branches located between an essential node and the reference node contains an independent or dependent voltage source, we do not write a node equation for this node because the node voltage is known. This reduces the number of independent node equations by one and the amount of work in solving for the node voltages. In writing the node equations for the other nodes, we write the value of the independent voltage source in those equations. Consider Figure 9.20a and assume the voltage V_2 results from an independent voltage source of 5 V. Since the node voltage is known, we do not write a node-voltage equation for node 2 in this case. When writing the node-voltage equation for node 1, the current I_A is written as $I_A = \frac{V_1 - 5}{R}$. Example Problem 9.5 further illustrates this case.

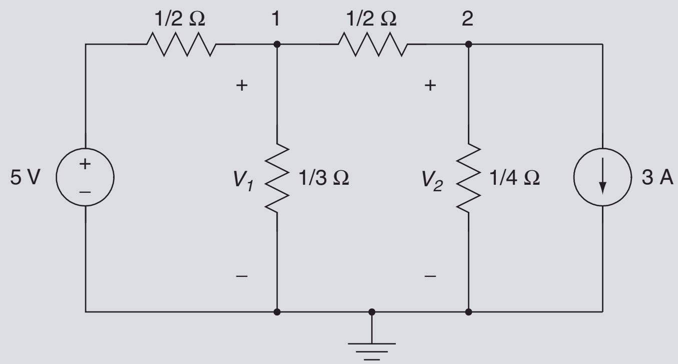
EXAMPLE PROBLEM 9.5

Find V_1 using the node-voltage method.



Solution

This circuit has two essential nodes, labeled 1 and 2 in the redrawn circuit that follows, with the reference node and two node voltages, V_1 and V_2, indicated. The node involving the 5 V voltage source has a known node voltage, and therefore we do not write a node equation for it.



Summing the currents leaving node 1 gives

$$2(V_1 - 5) + 3V_1 + 2(V_1 - V_2) = 0$$

which simplifies to

$$7V_1 - 2V_2 = 10$$

Summing the currents leaving node 2 gives

$$2(V_2 - V_1) + 4V_2 + 3 = 0$$

which simplifies to

$$-2V_1 + 6V_2 = -3$$

The two node equations are written in matrix format as

$$\begin{bmatrix} 7 & -2 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -3 \end{bmatrix}$$

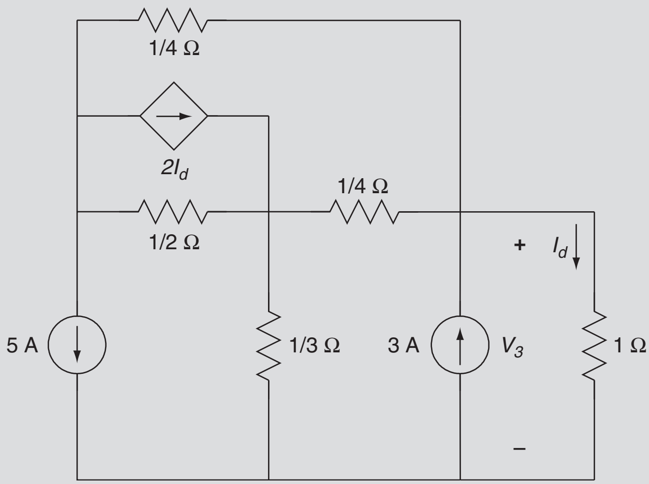
and solved with MATLAB as follows:

```
>> A = [7 -2;-2 6];
>> F = [10;-3];
>> V = A\F

V =

   1.4211
   -0.0263
```

Thus, $V_1 = 1.4211$ V.

EXAMPLE PROBLEM 9.6

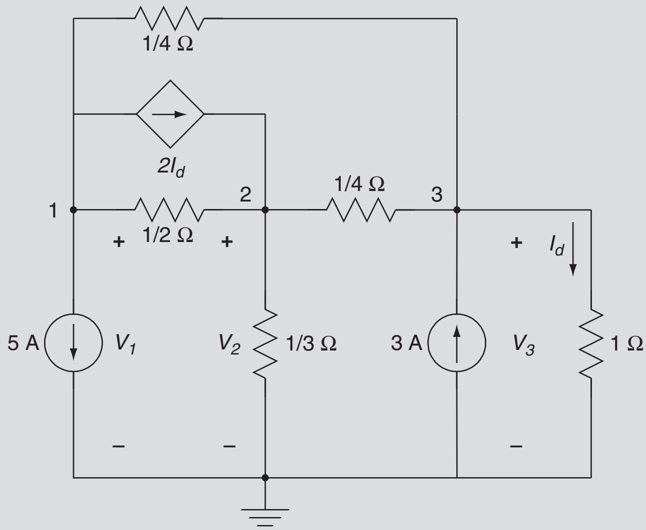
For the following circuit, find V_3 using the node-voltage method.



Continued

Solution

Notice that this circuit has three essential nodes and a dependent current source. We label the essential nodes 1, 2, and 3 in the redrawn circuit, with the reference node at the bottom of the circuit and three node voltages V_1, V_2, and V_3, as indicated.



Note that $I_d = V_3$ according to Ohm's law. Summing the currents leaving node 1 gives

$$5 + 2(V_1 - V_2) + 2I_d + 4(V_1 - V_3) = 0$$

which reduces to

$$6V_1 - 2V_2 - 2V_3 = -5$$

Summing the currents leaving node 2 gives

$$-2I_d + 2(V_2 - V_1) + 3V_2 + 4(V_2 - V_3) = 0$$

which simplifies to

$$-2V_1 + 9V_2 - 6V_3 = 0$$

Summing the currents leaving node 3 gives

$$4(V_3 - V_2) - 3 + V_3 + 4(V_3 - V_1) = 0$$

reducing to

$$-4V_1 - 4V_2 + 9V_3 = 3$$

The three node equations are written in matrix format as

$$\begin{bmatrix} 6 & -2 & -2 \\ -2 & 9 & -6 \\ -4 & -4 & 9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3 \end{bmatrix}$$

Notice that the system matrix is no longer symmetrical because of the dependent current source, and two of the three nodes have a current source, giving rise to a nonzero term on the right-hand side of the matrix equation.

Solving with MATLAB gives

```
>> A = [6 -2 -2;-2 9 -6;-4 -4 9];
>> F = [-5; 0; 3];
>> V = A\F

V =

   -1.1471
   -0.5294
   -0.4118
```

Thus, $V_3 = -0.4118$ V.

If one of the branches has an independent or controlled voltage source located between two essential nodes, as shown in Figure 9.21, the current through the source is not easily expressed in terms of node voltages. In this situation, we form a supernode by combining the two nodes. The supernode technique requires only one node equation in which the current, I_A, is passed through the source and written in terms of currents leaving node 2. Specifically, we replace I_A with $I_B + I_C + I_D$ in terms of node voltages. Because we have two unknowns and one supernode equation, we write a second equation by applying KVL for the two node voltages 1 and 2 and the source as

$$-V_1 - V_\Delta + V_2 = 0$$

or

$$V_\Delta = V_1 - V_2$$

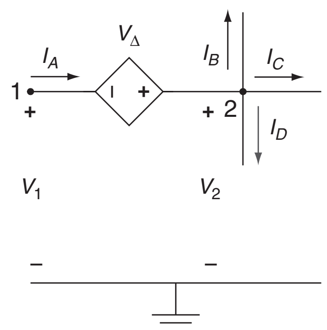
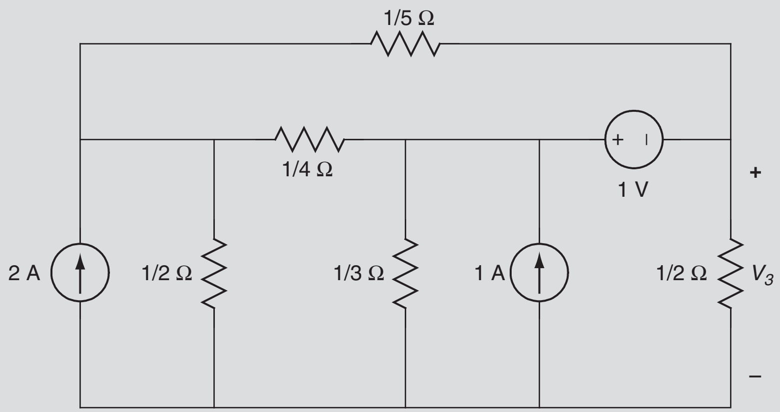


FIGURE 9.21 A dependent voltage source is located between nodes 1 and 2.

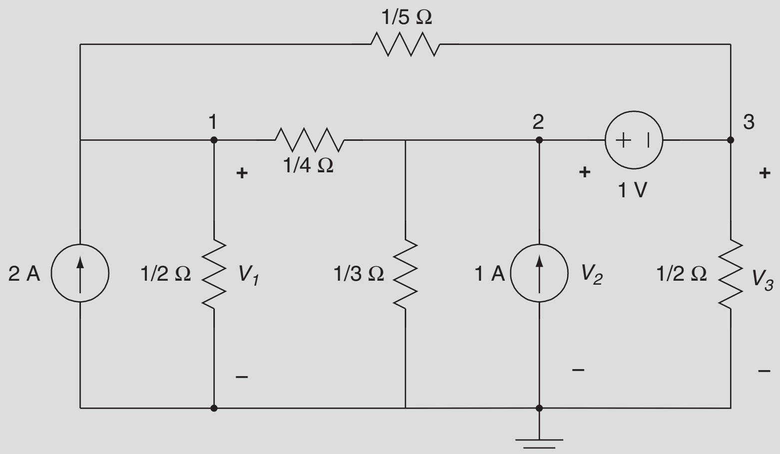
EXAMPLE PROBLEM 9.7

For the following circuit, find V_3.



Solution

The circuit has three essential nodes, two of which are connected to an independent voltage source and form a supernode. We label the essential nodes as 1, 2, and 3 in the redrawn circuit, with the reference node at the bottom of the circuit and three node voltages, V_1, V_2, and V_3 as indicated.



Summing the currents leaving node 1 gives

$$-2 + 2V_1 + 5(V_1 - V_3) + 4(V_1 - V_2) = 0$$

Simplifying gives

$$11V_1 - 4V_2 - 5V_3 = 2$$

Nodes 2 and 3 are connected by an independent voltage source, so we form a supernode 2+3. Summing the currents leaving the supernode 2+3 gives

$$4(V_2 - V_1) + 3V_2 - 1 + 2V_3 + 5(V_3 - V_1) = 0$$

Simplifying yields

$$-9V_1 + 7V_2 + 7V_3 = 1$$

The second supernode equation is KVL through the node voltages and the independent source, giving

$$-V_2 + 1 + V_3 = 0$$

or

$$-V_2 + V_3 = -1$$

The two node and KVL equations are written in matrix format as

$$\begin{bmatrix} 11 & -4 & -5 \\ -9 & 7 & 7 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

Solving with MATLAB gives

```
≫ A = [11 −4 −5;−9 7 7; 0 −1 1];
≫ F = [2;1;−1];
≫ V = A\F

V =

  0.4110
  0.8356
  −0.1644
```

Thus, $V_3 = -0.1644$.

9.6 LINEARITY AND SUPERPOSITION

If a linear system is excited by two or more independent sources, then the total response is the sum of the separate individual responses to each input. This property is called the principle of superposition. Specifically for circuits, the response to several independent sources is the sum of responses to each independent source with the other independent sources dead, where

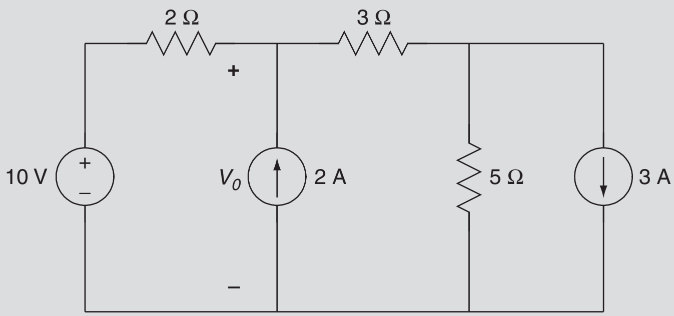
- A dead voltage source is a short circuit.
- A dead current source is an open circuit.

In linear circuits with multiple independent sources, the total response is the sum of each independent source taken one at a time. This analysis is carried out by removing all of the sources except one and assuming the other sources are dead. After the circuit is analyzed with the first source, it is set equal to a dead source, and the next source is applied with the remaining sources dead. When each of the sources has been analyzed, the total response is obtained by summing the individual responses. Note carefully that this principle holds true solely for independent sources. Dependent sources must remain in the circuit when applying this technique, and they must be analyzed based on the current or voltage for which it is defined. It should be apparent that voltages and currents in one circuit differ among circuits and that we cannot mix and match voltages and currents from one circuit with another.

Generally, superposition provides a simpler solution than is obtained by evaluating the total response with all of the applied sources. This property is especially valuable when dealing with an input consisting of a pulse or delays. These are considered in future sections.

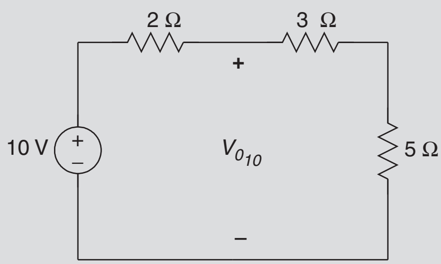
EXAMPLE PROBLEM 9.8

Using superposition, find V_0 as shown in the following figure.



Solution

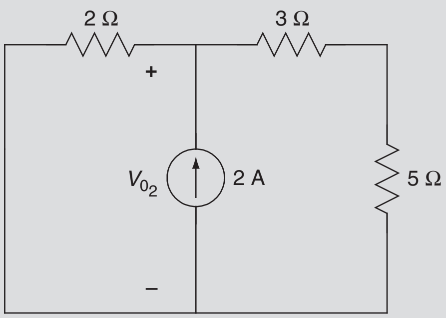
Start by analyzing the circuit with just the 10 V source active and the two current sources dead, as shown in the following figure.



The voltage divider rule easily gives the response, $V_{0_{10}}$, due to the 10 V source

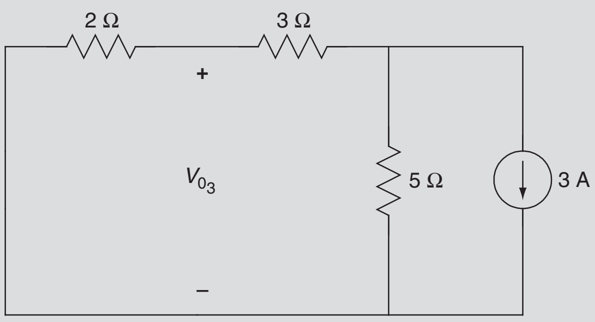
$$V_{0_{10}} = 10\left(\frac{8}{2+8}\right) = 8\ \text{V}$$

Next consider the 2 A source active and the other two sources dead, as shown in the following circuit.



Combining the resistors in an equivalent resistance, $R_{EQ} = 2 \parallel (3+5) = \frac{2 \times 8}{2+8} = 1.6\ \Omega$, and then applying Ohm's law gives $V_{0_2} = 2 \times 1.6 = 3.2$ V.

Finally, consider the response,V_{0_3}, to the 3 A source as shown in the following figure.



To find V_{0_3}, note that the 3 A current splits into 1.5 A through each branch ($2+3\ \Omega$ and $5\ \Omega$), and $V_{0_3} = -1.5 \times 2 = -3$ V.

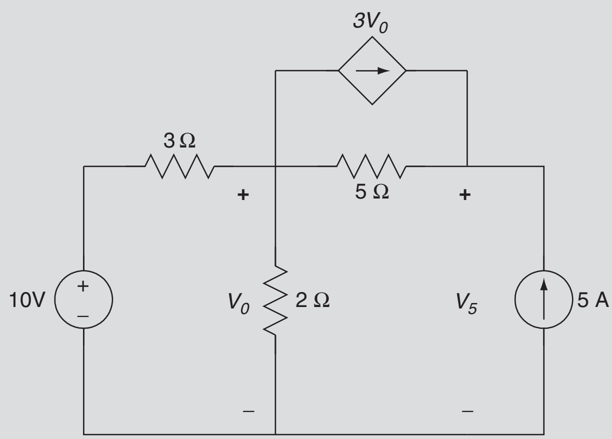
The total response is given by the sum of the individual responses as

$$V_0 = V_{0_{10}} + V_{0_2} + V_{0_3} = 8 + 3.2 - 3 = 8.2\ \text{V}$$

This is the same result we would have found if we analyzed the original circuit directly using the node-voltage method.

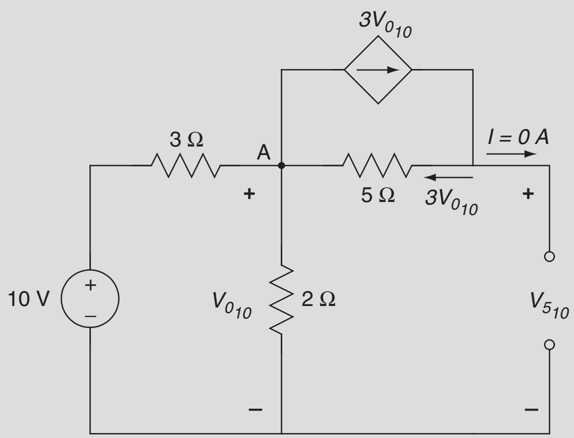
EXAMPLE PROBLEM 9.9

Find the voltage across the 5 A current source, V_5, in the following figure using superposition.



Solution

First consider finding the response, $V_{0_{10}}$, due to the 10 V source only, with the 5 A source dead, as shown in the following figure. As required during the analysis, the dependent current source is kept in the modified circuit and should not be set dead.

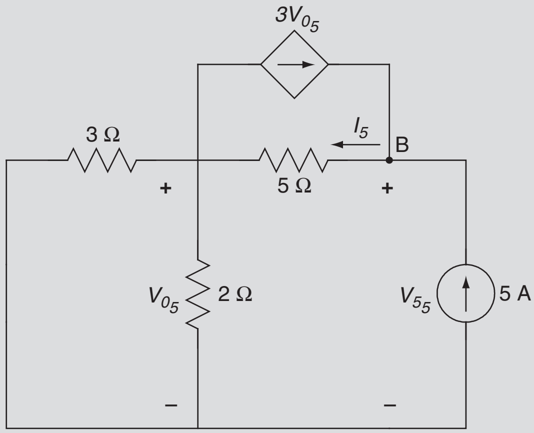


Notice that no current flows through the open circuit created by the dead current source and that the current flowing through the 5 Ω resistor is $3V_0$. Therefore, applying KCL at node A gives

$$\frac{V_{0_{10}} - 10}{3} + \frac{V_{0_{10}}}{2} + 3V_{0_{10}} - 3V_{0_{10}} = 0$$

which gives $V_{0_{10}} = 4\text{V}$. KVL gives $-V_{0_{10}} - 5 \cdot 3V_{0_{10}} + V_{5_{10}} = 0$, and therefore $V_{5_{10}} = 64$ V.

Next, consider finding the response, V_{0_5}, due to the 5 A source, with the 10 V source dead.



First combine the two resistors in parallel (3Ω || 2Ω), giving 1.2 Ω. V_{0_5} is easily calculated by Ohm's law as $V_{0_5} = 5 \cdot 1.2 = 6$V. KCL is then applied at node B to find I_5, giving

$$-3V_{0_5} + I_5 - 5 = 0$$

With $V_{0_5} = 6$ V, $I_5 = 3 \cdot 6 + 5 = 23$ A. Finally, apply KVL around the closed path

$$-V_{0_5} - 5I_5 + V_{5_5} = 0$$

or

$$V_{5_5} = V_{0_5} + 5I_5 = 6 + 5 \cdot 23 = 121 \text{ V}.$$

The total response is given by the sum of the individual responses as

$$V_5 = V_{5_{10}} + V_{5_5} = 64 + 121 = 185 \text{ V}$$

9.7 THÉVENIN'S THEOREM

Any combination of resistances, controlled sources, and independent sources with two external terminals (A and B, denoted A,B) can be replaced by a single resistance and an independent source, as shown in Figure 9.22. A Thévenin equivalent circuit reduces the original circuit into a voltage source in series with a resistor. This theorem helps reduce complex circuits into simpler circuits. We refer to the circuit elements connected across the terminals A,B (that are not shown) as the *load*. The Thévenin equivalent circuit is

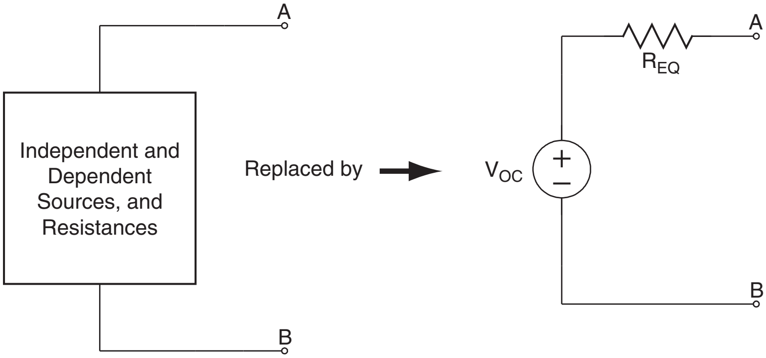


FIGURE 9.22 A general circuit consisting of independent and dependent sources can be replaced by a voltage source (V_{oc}) in series with a resistor (R_{EQ}).

equivalent to the original circuit in that the same voltage and current are observed across any load. Usually the load is not included in the simplification because it is important for other analysis, such as maximum power expended by the load. Although we focus here on sources and resistors, this theorem can be extended to any circuit composed of linear elements with two terminals.

Thévenin's Theorem states that an equivalent circuit consisting of an ideal voltage source, V_{OC}, in series with an equivalent resistance, R_{EQ}, can be used to replace any circuit that consists of independent and dependent voltage and current sources and resistors. V_{OC} is equal to the open circuit voltage across terminals A,B, as shown in Figure 9.23, and calculated using standard techniques such as the node-voltage method.

The resistor R_{EQ} is the resistance seen across the terminals A,B when all sources are *dead*. Recall that a dead voltage source is a short circuit, and a dead current source is an open circuit.

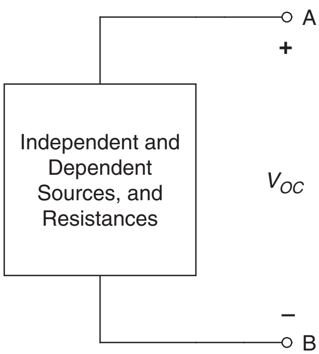
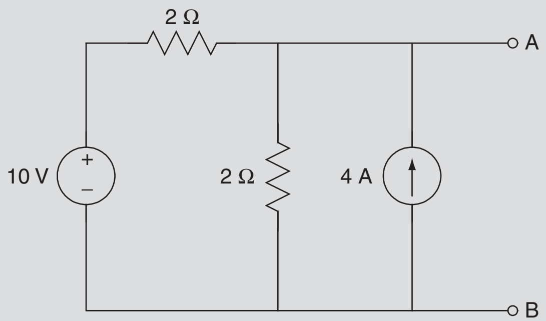


FIGURE 9.23 The open circuit voltage, V_{oc}, is calculated across the terminals A,B using standard techniques such as the node-voltage method.

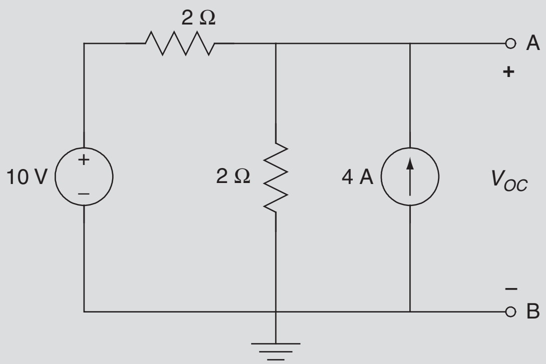
EXAMPLE PROBLEM 9.10

Find the Thévenin equivalent circuit with respect to terminals A,B for the following circuit.



Solution

The solution to finding the Thévenin equivalent circuit is done in two parts: first finding V_{OC} and then solving for R_{EQ}. The open circuit voltage, V_{OC}, is easily found using the node-voltage method, as shown in the following circuit.



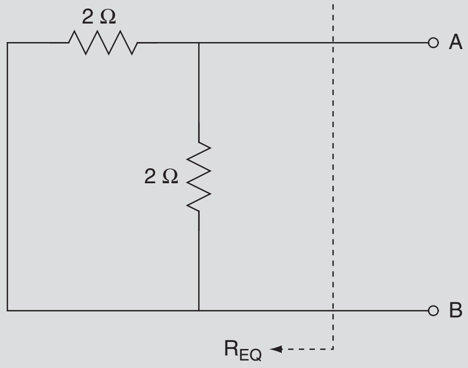
The sum of currents leaving the node is

$$\frac{V_{OC} - 10}{2} + \frac{V_{OC}}{2} - 4 = 0$$

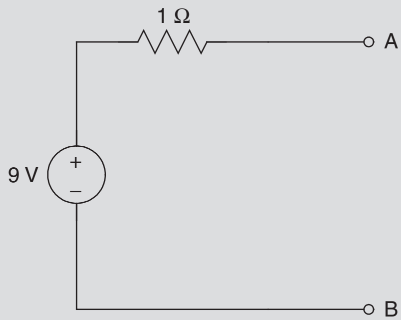
and $V_{OC} = 9$ V.

Next, R_{EQ} is found by first setting all sources dead (the current source is an open circuit and the voltage source is a short circuit) and then finding the resistance seen from the terminals A,B, as shown in the following figure.

Continued



From the previous circuit, it is clear that R_{EQ} is equal to 1Ω (that is, 2Ω || 2Ω). Thus, the Thévenin equivalent circuit is



It is important to note that the circuit used in finding V_{OC} is not to be used in finding R_{EQ} as not all voltages and currents are relevant in the other circuit and one cannot simply mix and match.

If the terminals A,B are shorted as shown in Figure 9.24, the current that flows is denoted I_{SC}, and the following relationship holds:

$$R_{EQ} = \frac{V_{OC}}{I_{SC}} \tag{9.18}$$

9.8 INDUCTORS

In the previous sections of this chapter, we considered circuits involving sources and resistors that are described with algebraic equations. Any changes in the source are instantaneously observed in the response. In this section we examine the inductor, a passive element that relates the voltage-current relationship with a differential equation. Circuits that

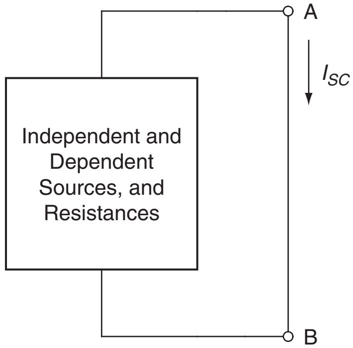


FIGURE 9.24 The short circuit current, I_{sc}, is calculated by placing a short across the terminals A,B and finding the current through the short using standard techniques such as the node-voltage method.

contain inductors are written in terms of derivatives and integrals. Any changes in the source with circuits that contain inductors—that is, a step input—have a response that is not instantaneous but have a natural response that changes exponentially and a forced response that is the same form as the source.

An inductor is a passive element that is able to store energy in a magnetic field and is made by winding a coil of wire around a core that is an insulator or a ferromagnetic material. A magnetic field is established when current flows through the coil. We use the symbol  to represent the inductor in a circuit; the unit of measure for inductance is the henry or henries (H), where 1 H = 1 V – s/A. The relationship between voltage and current for an inductor is given by

$$v = L\frac{di}{dt} \tag{9.19}$$

The convention for writing the voltage drop across an inductor is similar to that of a resistor, as shown in Figure 9.25.

Physically, current cannot change instantaneously through an inductor, since an infinite voltage is required, according to Eq. (9.19) (i.e., the derivative of current at the time of the instantaneous change is infinity). Mathematically, a step change in current through an inductor is possible by applying a voltage that is a Dirac delta function. For convenience, when a circuit has just DC currents (or voltages), the inductors can be replaced by short circuits, since the voltage drops across the inductors are zero.

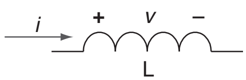
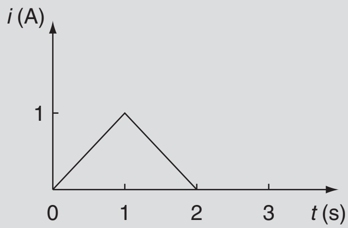
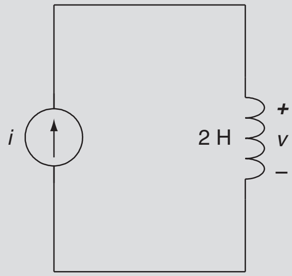


FIGURE 9.25 An inductor.

EXAMPLE PROBLEM 9.11

Find v in the following circuit.





Solution

The solution to this problem is best approached by breaking it up into time intervals consistent with the changes in input current. Clearly, for $t < 0$ and $t > 2$, the current is zero and therefore $v = 0$. We use Eq. (9.19) to determine the voltage in the other two intervals as follows.

For $0 < t < 1$

In this interval, the input is $i = t$, and

$$v = L\frac{di}{dt} = 2\frac{d(t)}{dt} = 2\text{ V}$$

For $1 \le t \le 2$

In this interval, the input is $i = -(t - 2)$, and

$$v = L\frac{di}{dt} = 2\frac{d(-(t-2))}{dt} = -2\text{ V}$$

Equation (9.19) defines the voltage across an inductor for a given current. Suppose one is given a voltage across an inductor and asked to find the current. We start from Eq. (9.19) by multiplying both sides by dt, giving

$$v(t)dt = Ldi$$

Integrating both sides yields

$$\int_{t_0}^{t} v(\lambda)d\lambda = L\int_{i(t_0)}^{i(t)} d\alpha \tag{9.20}$$

or

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\lambda)d\lambda + i(t_0)$$

For $t_0 = 0$, as is often the case in solving circuit problems, Eq. (9.20) reduces to

$$i(t) = \frac{1}{L}\int_0^t v(\lambda)d\lambda + i(0) \tag{9.21}$$

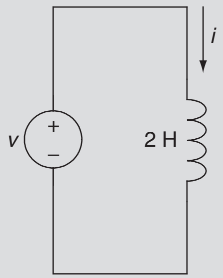
and for $t_0 = -\infty$, the initial current is by definition equal to zero, and therefore Eq. (9.20) reduces to

$$i(t) = \frac{1}{L}\int_{-\infty}^t v(\lambda)d\lambda \tag{9.22}$$

The initial current in Eq. (9.20), $i(t_0)$, is usually defined in the same direction as i, which means $i(t_0)$ is a positive quantity. If the direction of $i(t_0)$ is in the opposite direction of i (as will happen when we write node equations), then $i(t_0)$ is negative.

EXAMPLE PROBLEM 9.12

Find i for $t \geq 0$ if $i(0) = 2$ A and $v(t) = 4e^{-3t}u(t)$ in the following circuit.



Solution

From Eq. (9.20), we have

$$i(t) = \frac{1}{L}\int_{t_0}^t v d\lambda + i(t_0) = \frac{1}{2}\int_0^t 4e^{-3\lambda}d\lambda + 2$$

$$= 2\frac{e^{-3\lambda}}{-3}\bigg|_{\lambda=0}^t + 2$$

$$= \frac{2}{3}\left(4 - e^{-3t}\right)u(t) \text{ V}$$

9.9 CAPACITORS

A capacitor is a device that stores energy in an electric field by charge separation when appropriately polarized by a voltage. Simple capacitors consist of parallel plates of conducting material that are separated by a gap filled with a dielectric material. Dielectric materials—that is, air, mica, or Teflon—contain a large number of electric dipoles that become polarized in the presence of an electric field. The charge separation caused by the polarization of the dielectric is proportional to the external voltage and given by

$$q(t) = C\,v(t) \tag{9.23}$$

where C represents the capacitance of the element. The unit of measure for capacitance is the farad or farads (F), where 1 F = 1 C/V. We use the symbol C to denote a capacitor; most capacitors are measured in terms of microfarads (1 μF = 10^{-6} F) or picofarads (1 pF = 10^{-12} F). Figure 9.26 illustrates a capacitor in a circuit.

Using the relationship between current and charge, Eq. (9.23) is written in a more useful form for circuit analysis problems as

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \tag{9.24}$$

The capacitance of a capacitor is determined by the permittivity of the dielectric ($\varepsilon = 8.854 \times 10^{-12}\,\frac{\mathrm{F}}{\mathrm{m}}$ for air) that fills the gap between the parallel plates, the size of the gap between the plates, d, and the cross-sectional area of the plates, A, as

$$C = \frac{\varepsilon A}{d} \tag{9.25}$$

As described, the capacitor physically consists of two conducting surfaces that store charge, separated by a thin insulating material that has a very large resistance. In actuality, current does not flow through the capacitor plates. Rather, as James Clerk Maxwell hypothesized when he described the unified electromagnetic theory, a displacement current flows internally between capacitor plates, and this current equals the current flowing into the capacitor and out of the capacitor. Thus, KCL is maintained. It should be clear from Eq. (9.29) that dielectric materials do not conduct DC currents; capacitors act as open circuits when DC currents are present.

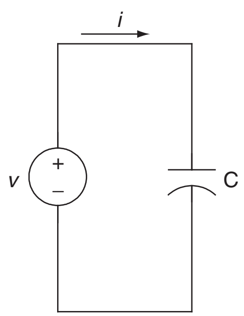
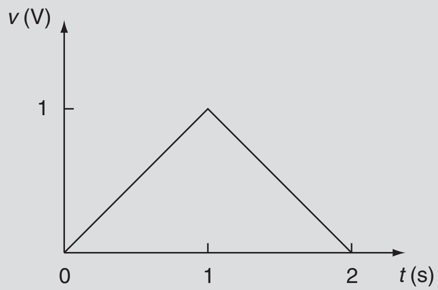
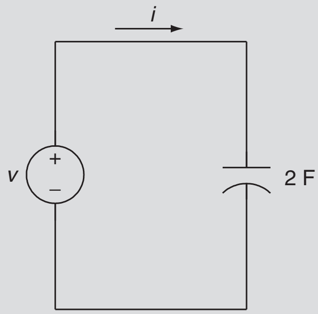


FIGURE 9.26 Circuit with a capacitor.

EXAMPLE PROBLEM 9.13

Find i for the following circuit.





Solution

For $t < 0$ and $t > 2$, $v = 0$ V, and therefore $i = 0$ in this interval. For nonzero values, the voltage waveform is described with two different functions: $v = t$ V for $0 \leq t \leq 1$, and $v = -(t-2)$V for $1 < t \leq 2$. Equation (9.24) is used to determine the current for each interval as follows.

For 0 < t < 1

$$i = C\frac{dv}{dt} = 2 \times \frac{d}{dt}(t) = 2 \text{ A}$$

For 1 ≤ t ≤ 2

$$i = C\frac{dv}{dt} = 2 \times \frac{d}{dt}(-(t-2)) = -2 \text{ A}$$

Voltage cannot change instantaneously across a capacitor. To have a step change in voltage across a capacitor, an infinite current must flow through the capacitor, and that is not physically possible. Of course, this is mathematically possible using a Dirac delta function.

Equation (9.24) defines the current through a capacitor for a given voltage. Suppose one is given a current through a capacitor and asked to find the voltage. To find the voltage, we start from Eq. (9.24) by multiplying both sides by dt, giving

$$i(t)dt = C\, dv$$

Integrating both sides yields

$$\int_{t_o}^{t} i(\lambda)d\lambda = C \int_{v(t_o)}^{v(t)} dv$$

or

$$v(t) = \frac{1}{C}\int_{t_o}^{t} i dt + v(t_o) \tag{9.26}$$

For $t_0 = 0$, Eq. (9.26) reduces to

$$v(t) = \frac{1}{C}\int_{0}^{t} i dt + v(0) \tag{9.27}$$

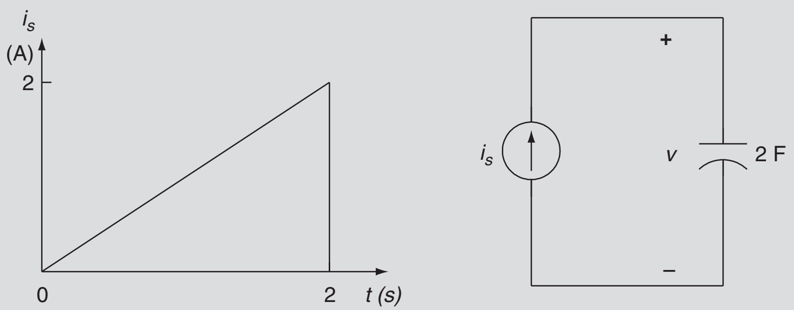
and for $t_0 = -\infty$, Eq. (9.27) reduces to

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\lambda) d\lambda \tag{9.28}$$

The initial voltage in Eq. (9.26), $v(t_0)$, is usually defined with the same polarity as v, which means $v(t_0)$ is a positive quantity. If the polarity of $v(t_0)$ is in the opposite direction of v, then $v(t_0)$ is negative.

EXAMPLE PROBLEM 9.14

Find v for the circuit that follows.



Solution

The current waveform is described with three different functions: for the interval $t \le 0$, for the interval $0 < t \le 2$, and for $t > 2$. To find the voltage, we apply Eq. (9.28) for each interval as follows.

For t < 0

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i dt = \frac{1}{2}\int_{-\infty}^{0} 0 dt = 0 \text{ V}$$

For $\mathbf{0 \le t \le 2}$

$$v(t) = \frac{1}{C}\int_0^t i dt + v(0)$$

and with $v(0) = 0$, we have

$$v(t) = \frac{1}{2}\int_0^t \lambda\, d\lambda = \frac{1}{2}\left(\frac{\lambda^2}{2}\right)\Bigg|_0^t = \frac{t^2}{4}\text{ V}$$

The voltage at $t = 2$ needed for the initial condition in the next part is

$$v(2) = \frac{t^2}{4}\Bigg|_{t=2} = 1\text{ V}$$

For $\mathbf{t > 2}$

$$v(t) = \frac{1}{C}\int_2^t i dt + v(2) = \frac{1}{2}\int_2^t 0 dt + v(2) = 1\text{ V}$$

9.10 A GENERAL APPROACH TO SOLVING CIRCUITS INVOLVING RESISTORS, CAPACITORS, AND INDUCTORS

Sometimes a circuit consisting of resistors, inductors, and capacitors cannot be simplified by bringing together like elements in series and parallel combinations. Consider the circuit shown in Figure 9.27. In this case, the absence of parallel or series combinations of resistors, inductors, or capacitors prevents us from simplifying the circuit for ease in solution. In this section, the node-voltage method is applied to write equations involving integrals and differentials using element relationships for resistors, inductors, and capacitors. From these equations, any unknown currents and voltages of interest can be solved using the standard differential equation approach.

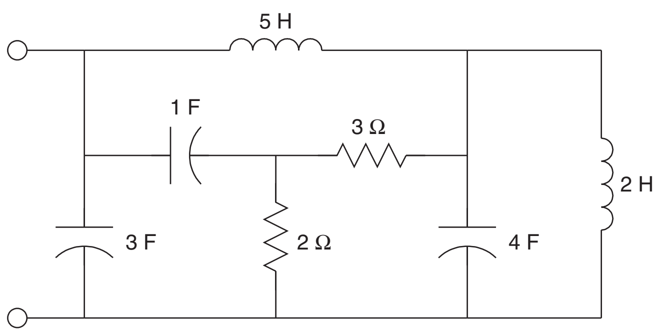
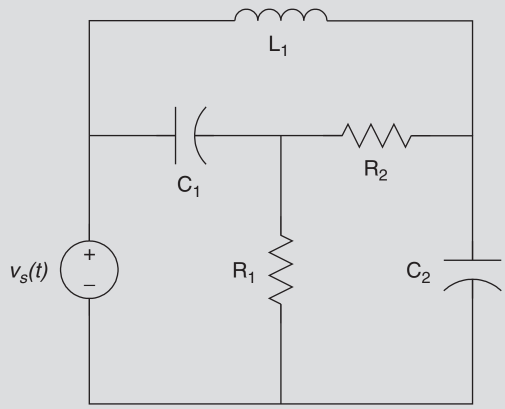


FIGURE 9.27 A circuit that cannot be simplified.

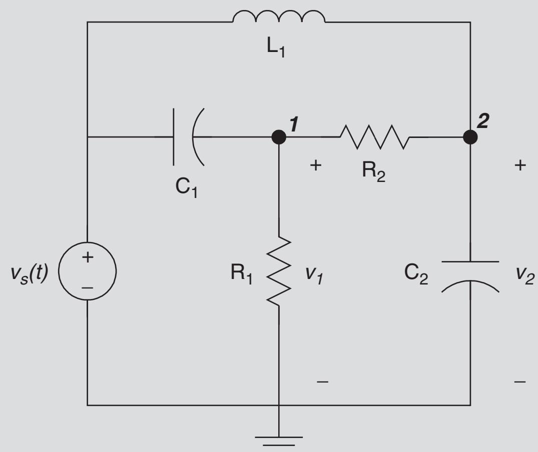
EXAMPLE PROBLEM 9.15

Write the node equations for the following circuit for $t \geq 0$ if the initial conditions are zero.



Solution

With the reference node at the bottom of the circuit, we have two essential nodes, as shown in the following redrawn circuit. Recall that the node involving the voltage source is a known voltage and that we do not write a node equation for it. When writing the node-voltage equations, the current through a capacitor is $i_c = C\,\Delta\dot{v}$, where $\Delta\dot{v}$ is the derivative of the voltage across the capacitor, and the current through an inductor is $i_L = \frac{1}{L}\int_0^t \Delta v d\lambda + i_L(0)$, where Δv is the voltage across the inductor. Since the initial conditions are zero, the term $i_L(0) = 0$.



Summing the currents leaving node 1 gives

$$C_1(\dot{v}_1 - \dot{v}_s) + \frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} = 0$$

which simplifies to

$$C_1\dot{v}_1 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v_1 - \frac{1}{R_2}v_2 = C_1\dot{v}_s$$

Summing the currents leaving node 2 gives

$$\frac{v_2 - v_1}{R_2} + C_2\dot{v}_2 + \frac{1}{L_1}\int_0^t (v_2 - v_s)\, d\lambda = 0$$

Typically we eliminate integrals in the node equations by differentiating. When applied to the previous expression, this gives

$$\frac{1}{R_2}\dot{v}_2 - \frac{1}{R_2}\dot{v}_1 + C_2\ddot{v}_2 + \frac{1}{L_1}v_2 - \frac{1}{L_1}v_s = 0$$

and after rearranging yields

$$\ddot{v}_2 + \frac{1}{C_2R_2}\dot{v}_2 + \frac{1}{C_2L_1}v_2 - \frac{1}{C_2R_2}\dot{v}_1 = \frac{1}{C_2L_1}v_s$$

When applying the node-voltage method, we generate one equation for each essential node. To write a single differential equation involving just one node voltage and the inputs, we use the other node equations and substitute into the node equation of the desired node voltage. Sometimes this involves differentiation as well as substitution. The easiest case involves a node equation containing an undesired node voltage without its derivatives. Another method for creating a single differential equation is to use the D operator or the Laplace transform.

Consider the node equations for Example Problem 9.15, and assume that we are interested in obtaining a single differential equation involving node voltage v_1 and its derivatives, and the input. For ease in analysis, let us assume that the values for the circuit elements are $R_1 = R_2 = 1\ \Omega, C_1 = C_2 = 1$ F, and $L_1 = 1$ H, giving us

$$\dot{v}_1 + 2v_1 - v_2 = \dot{v}_s$$

and

$$\ddot{v}_2 + \dot{v}_2 + v_2 - \dot{v}_1 = v_s$$

Using the first equation, we solve for v_2, calculate $\dot{v}_2$ and $\ddot{v}_2$, and then substitute into the second equation as follows.

$$\begin{aligned} v_2 &= \dot{v}_1 + 2v_1 - \dot{v}_s \\ \dot{v}_2 &= \ddot{v}_1 + 2\dot{v}_1 - \ddot{v}_s \\ \ddot{v}_2 &= \dddot{v}_1 + 2\ddot{v}_1 - \dddot{v}_s \end{aligned}$$

After substituting into the second node equation, we have

$$\dddot{v}_1 + 2\ddot{v}_1 - \dddot{v}_s + \ddot{v}_1 + 2\dot{v}_1 - \ddot{v}_s + \dot{v}_1 + 2v_1 - \dot{v}_s - \dot{v}_1 = v_s$$

and after simplifying

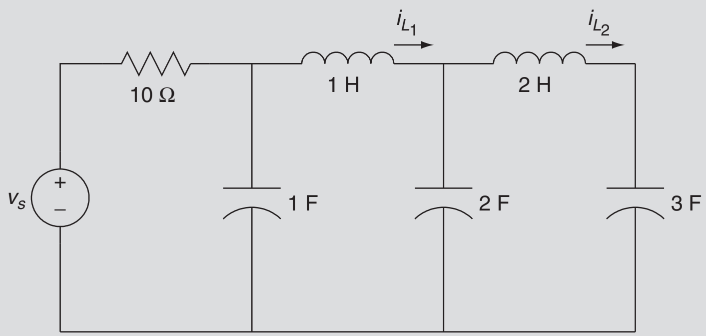
$$\dddot{v}_1 + 3\ddot{v}_1 + 2\dot{v}_1 + 2v_1 = \dddot{v}_s + \ddot{v}_s + \dot{v}_s - v_s$$

In general, the order of the differential equation relating a single output variable and the inputs is equal to the number of energy storing elements in the circuit (capacitors and inductors). In some circuits, the order of the differential equation is less than the number of capacitors and inductors in the circuit. This occurs when capacitor voltages and inductor currents are not independent; that is, there is an algebraic relationship between the capacitor—specifically, voltages and the inputs, or the inductor currents and the inputs. This occurs when capacitors are connected directly to a voltage source or when inductors are connected directly to a current source.

Example Problem 9.15 involved a circuit with zero initial conditions. When circuits involve nonzero initial conditions, our approach remains the same as before except that the initial inductor currents are included when writing the node-voltage equations.

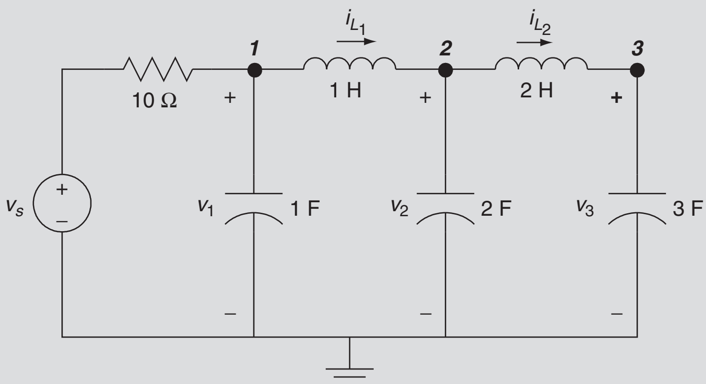
EXAMPLE PROBLEM 9.16

Write the node equations for the following circuit for $t \geq 0$ assuming the initial conditions are $i_{L_1}(0) = 8$ A and $i_{L_2}(0) = -4$ A.



Solution

With the reference node at the bottom of the circuit, there are three essential nodes, as shown in the redrawn circuit that follows.



Summing the currents leaving node 1 gives

$$\frac{(v_1 - v_s)}{10} + \dot{v}_1 + \int_0^t (v_1 - v_2)\, d\lambda + 8 = 0$$

where $i_{L_1}(0) = 8$ A.

Summing the currents leaving node 2 gives

$$\int_0^t (v_2 - v_1)\, d\lambda - 8 + 2\dot{v}_2 + \frac{1}{2}\int_0^t (v_2 - v_3)\, d\lambda - 4 = 0$$

where $i_{L_2}(0) = -4$ A. Notice that the sign for the initial inductor current is negative because the direction is from right to left and the current is defined on the circuit diagram in the opposite direction for the node 2 equation.

Summing the currents leaving node 3 gives

$$\frac{1}{2}\int_0^t (v_3 - v_2)\, d\lambda + 4 + 3\dot{v}_3 = 0$$

In this example, the node equations were not simplified by differentiating to remove the integral, which would have eliminated the initial inductor currents from the node equations. If we were to write a single differential equation involving just one node voltage and the input, a fifth-order differential equation would result because there are five energy storing elements in the circuit. To solve the differential equation, we would need five initial conditions, the initial node voltage for the variable selected, and the first through fourth derivatives at time zero.

9.10.1 Discontinuities and Initial Conditions in a Circuit

Discontinuities in voltage and current occur when an input such as a unit step is applied or a switch is thrown in a circuit. As we have seen, when solving an nth order differential equation, one must know n initial conditions, typically the output variable and its $(n-1)$ derivatives at the time the input is applied or the switch thrown. As we will see, if the inputs to a circuit are known for all time, we can solve for initial conditions directly based on energy considerations and not have to depend on being provided with them in the problem statement. Almost all of our problems involve the input applied at time zero, so our discussion here is focused on time zero, but it may be easily extended to any time an input is applied.

Energy cannot change instantaneously for elements that store energy. Thus, there are no discontinuities allowed in current through an inductor or voltage across a capacitor at any time—specifically, the value of the variable remains the same at $t = 0^-$ and $t = 0^+$. In the previous problem when we were given initial conditions for the inductors and capacitors, this implied, $i_{L_1}(0^-) = i_{L_1}(0^+)$ and $i_{L_2}(0^-) = i_{L_2}(0^+)$, and $v_1(0^-) = v_1(0^+), v_2(0^-) = v_2(0^+)$, and $v_3(0^-) = v_3(0^+)$. With the exception of variables associated with current through an inductor and voltage across a capacitor, other variables can have discontinuities, especially at a time when a unit step is applied or when a switch is thrown; however, these variables must obey KVL and KCL.

While it may not seem obvious at first, a discontinuity is allowed for the derivative of the current through an inductor and voltage across a capacitor at $t = 0^-$ and $t = 0^+$, since

$$\frac{di_L(0+)}{dt} = \frac{v_L(0+)}{L} \text{ and } \frac{dv_C(0+)}{dt} = \frac{i_C(0+)}{L}$$

as discontinuities are allowed in $v_L(0+)$ and $i_C(0+)$. Keep in mind that the derivatives in the previous expression are evaluated at zero after differentiation—that is,

$$\frac{di_L(0+)}{dt} = \left.\frac{di_L(t)}{dt}\right|_{t=0^+} \text{ and } \frac{dv_C(0+)}{dt} = \left.\frac{dv_C(t)}{dt}\right|_{t=0^+}$$

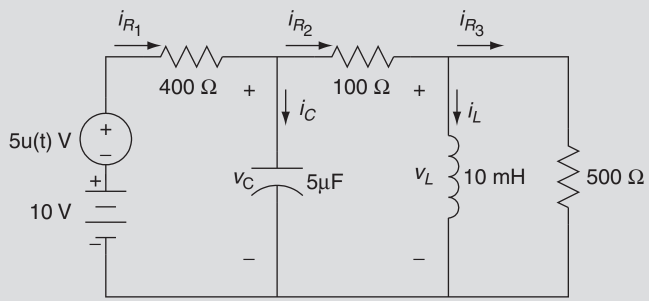
In calculations to determine the derivatives of variables not associated with current through an inductor and voltage across a capacitor, the derivative of a unit step input may be needed. Here we assume the derivative of a unit step input is zero at $t = 0^+$.

The initial conditions for variables not associated with current through an inductor and voltage across a capacitor at times of a discontinuity are determined only from the initial conditions from variables associated with current through an inductor and voltage across a capacitor and any applicable sources. The analysis is done in two steps involving KCL and KVL or using the node-voltage method.

1. First, we analyze the circuit at $t = 0^-$. Recall that when a circuit is at steady state, an inductor acts as a short circuit and a capacitor acts as an open circuit. Thus, at steady-state at $t = 0^-$, we replace all inductors by short circuits and capacitors by open circuits in the circuit. We then solve for the appropriate currents and voltages in the circuit to find the currents through the inductors (actually the shorts connecting the sources and resistors) and voltages across the capacitors (actually the open circuits among the sources and resistors).
2. Second, we analyze the circuit at $t = 0^+$. Since the inductor current cannot change in going from $t = 0^-$ to $t = 0^+$, we replace the inductors with current sources whose values are the currents at $t = 0^-$. Moreover, since the capacitor voltage cannot change in going from $t = 0^-$ to $t = 0^+$, we replace the capacitors with voltage sources whose values are the voltages at $t = 0^-$. From this circuit we solve for all desired initial conditions necessary to solve the differential equation.

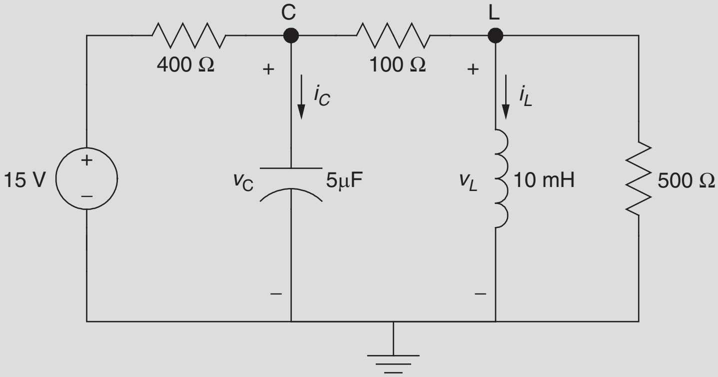
EXAMPLE PROBLEM 9.17

Use the node-voltage method to find v_c for the following circuit for $t \geq 0$.



Solution

For $t \geq 0$, the circuit is redrawn for analysis in the following figure.



Summing the currents leaving node C gives

$$\frac{v_C - 15}{400} + 5 \times 10^{-6}\dot{v}_C + \frac{v_C - v_L}{100} = 0$$

which simplifies to

$$\dot{v}_C + 2500v_C - 2000v_L = 7500$$

Summing the currents leaving node L gives

$$\frac{v_L - v_C}{100} + \frac{1}{10 \times 10^{-3}}\int_0^t v_L d\lambda + i_L(0^+) + \frac{v_L}{500} = 0$$

which, after multiplying by 500 and differentiating, simplifies to

$$6\dot{v}_L + 50 \times 10^3 v_L - 5\dot{v}_C = 0$$

Using the D operator method, the two differential equations are written as

$$Dv_C + 2500v_C - 2000v_L = 7500 \text{ or } (D + 2500)v_C - 2000v_L = 7500$$

$$6Dv_L + 50 \times 10^3 v_L - 5Dv_C = 0 \text{ or } \left(6D + 50 \times 10^3\right)v_L - 5Dv_C = 0$$

We then solve for v_L from the first equation,

$$v_L = \left(0.5 \times 10^{-3}D + 1.25\right)v_C - 3.75$$

and then substitute v_L into the second equation, giving

$$\left(6D + 50 \times 10^3\right)v_L - 5Dv_C = \left(6D + 50 \times 10^3\right)\left(\left(0.5 \times 10^{-3}D + 1.25\right)v_C - 3.750\right) - 5Dv_C = 0$$

Reducing this expression yields

$$D^2 v_C + 10.417 \times 10^3 Dv_C + 20.83 \times 10^6 v_C = 62.5 \times 10^6$$

Continued

Returning to the time domain gives

$$\ddot{v}_C + 10.417 \times 10^3 \dot{v}_C + 20.83 \times 10^6 v_C = 62.5 \times 10^6$$

The characteristic equation for the previous differential equation is

$$s^2 + 10.417 \times 10^3 s + 20.833 \times 10^6 = 0$$

with roots -7.718×10^3 and -2.7×10^3 and the natural solution

$$v_{C_n}(t) = K_1 e^{-7.718 \times 10^3 t} + K_2 e^{-2.7 \times 10^3 t} \text{ V}$$

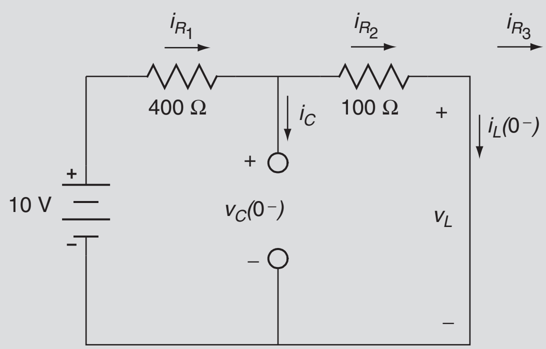
Next, we solve for the forced response, assuming that $v_{C_f}(t) = K_3$. After substituting into the differential equation, this gives

$$20.833 \times 10^6 K_3 = 62.5 \times 10^6$$

or $K_3 = 3$. Thus, our solution is now

$$v_C(t) = v_{C_n}(t) + v_{C_f}(t) = K_1 e^{-7.718 \times 10^3 t} + K_2 e^{-2.7 \times 10^3 t} + 3 \text{ V}$$

Initial conditions for $v_C(0^+)$ and $\dot{v}_C(0^+)$ are necessary to solve for K_1 and K_2. For $t = 0^-$, the capacitor is replaced by an open circuit and the inductor by a short circuit as shown in the following circuit.



Notice $v_L(0^-) = 0$ V because the inductor is a short circuit. Also note that the 500 Ω resistor is not shown in the circuit, since it is shorted out by the inductor, and so $i_{R_3}(0^-) = 0$A. Using the voltage divider rule, we have

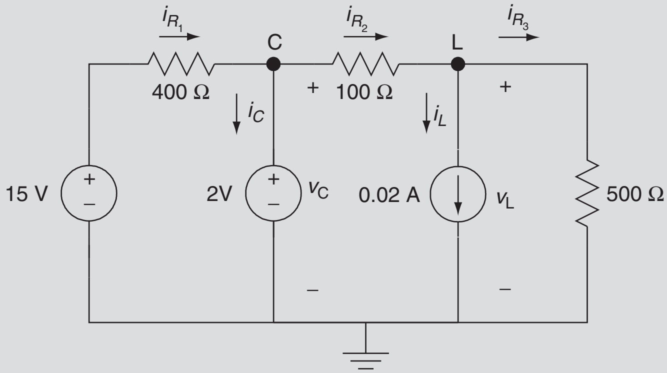
$$v_C(0^-) = 10 \times \frac{100}{400 + 100} = 2 \text{ V}$$

and by Ohm's law

$$i_L(0^-) = \frac{10}{100 + 400} = 0.02 \text{ A}$$

It follows that $i_{R_1}(0^-) = i_{R_2}(0^-) = i_L(0^-) = 0.02$ A. Because voltage across a capacitor and current through an inductor are not allowed to change from $t = 0^-$ to $t = 0^+$ we have $v_C(0^+) = v_C(0^-) = 2$V and $i_L(0^+) = i_L(0^-) = 0.02$ A.

The circuit for $t = 0^+$ is drawn by replacing the inductors in the original circuit with current sources whose values equal the inductor currents at $t = 0^-$ and the capacitors with voltage sources whose values equal the capacitor voltages at $t = 0^-$, as shown in the following figure with nodes C and L and reference. Note also that the input is now $10 + 5u(t) = 15$ V.



To find $v_L(0^+)$, we sum the currents leaving node L, yielding

$$\frac{v_L - 2}{100} + 0.02 + \frac{v_L}{500} = 0$$

which gives $v_L(0^+) = 0$ V. Now $i_{R_3}(0^+) = \dfrac{v_L(0^+)}{500} = 0$ A, $i_{R_2}(0^+) = 0.02 + i_{R_3}(0^+) = 0.02$ A, and $i_{R_1}(0^+) = \dfrac{15 - 2}{400} = 0.0325$ A.

To find $i_C(0^+)$, we write KCL at node C, giving

$$-i_{R_1}(0^+) + i_C(0^+) + i_{R_2}(0^+) = 0$$

or

$$i_C(0^+) = i_{R_1}(0^+) - i_{R_2}(0^+) = 0.0325 - 0.02 = 0.125 \text{ A}$$

To find $\dot{v}_C(0^+)$, note that $i_C(0^+) = C\dot{v}_C(0^+)$ or

$$\dot{v}_C(0^+) = \frac{i_C(0^+)}{C} = \frac{0.0125}{5 \times 10^{-6}} = 2.5 \times 10^3 \frac{\text{V}}{\text{s}}.$$

With the initial conditions, the constants K_1 and K_2 are solved as

$$v_C(0) = 2 = K_1 + K_2 + 3$$

Next,

$$\dot{v}_C(t) = -7.718 \times 10^3 K_1 e^{-7.718 \times 10^3 t} - 2.7 \times 10^3 K_2 e^{-2.7 \times 10^3 t}$$

Continued

and at $t = 0$,

$$\dot{v}_C(0) = 2.5 \times 10^3 = -7.718 \times 10^3 K_1 - 2.7 \times 10^3 K_2$$

Solving gives $K_1 = 0.04$ and $K_2 = -1.04$. Substituting these values into the solution gives

$$v_C(t) = 0.04e^{-7.718\times10^3 t} - 1.04e^{-2.7\times10^3 t} + 3 \text{ V}$$

for $t \geq 0$.

9.11 OPERATIONAL AMPLIFIERS

Section 9.3 introduced controlled voltage and current sources that are dependent on a voltage or current elsewhere in a circuit. These devices were modeled as a two-terminal device. In this section, we look at the operational amplifier, also known as an op amp, which is a multiterminal device. An operational amplifier is an electronic device that consists of large numbers of transistors, resistors, and capacitors. Fully understanding its operation requires knowledge of diodes and transistors—topics that are not covered in this book. However, fully understanding how an operational amplifier operates in a circuit involves a topic already covered: the controlled voltage source. Circuits involving operational amplifiers form the cornerstone for any bioinstrumentation, from amplifiers to filters. Amplifiers used in biomedical applications have very high-input impedance to keep the current drawn from the system being measured low. Most body signals have very small magnitudes. For example, an ECG has a magnitude in the millivolts, and the EEG has a magnitude in the microvolts. Analog filters are often used to remove noise from a signal, typically through frequency domain analysis to design the filter.

As the name implies, the operation amplifier is an amplifier, but as we will see, when it is combined with other circuit elements, it integrates, differentiates, sums, and subtracts. One of the first operational amplifiers appeared as an eight-lead dual-in-line package (DIP), shown in Figure 9.28.

Differing from previous circuit elements, this device has two input and one output terminals. Rather than draw the operational amplifier using Figure 9.28, the operational amplifier is drawn with the symbol in Figure 9.29. The input terminals are labeled the noninverting input (+) and the inverting input (–). The power supply terminals are labeled V+ and V–, which are frequently omitted, since they do not affect the circuit behavior except in saturation conditions, as will be described. Most people shorten the name of the operational amplifier to the "op amp."

Figure 9.30 shows a model of the op amp, focusing on the internal behavior of the input and output terminals. The input-output relationship is

$$v_o = A\left(v_p - v_n\right) \tag{9.29}$$

Since the internal resistance is very large, we will replace it with an open circuit to simplify analysis, leaving us with the op amp model show in Figure 9.31.

With the replacement of the internal resistance with an open circuit, the currents $i_n = i_p = 0$ A. In addition, current i_A, the current flowing out of the op amp, is not zero. Because i_A is unknown, seldom is KCL applied at the output junction. In solving op amp problems, KCL is almost always applied at input terminals.

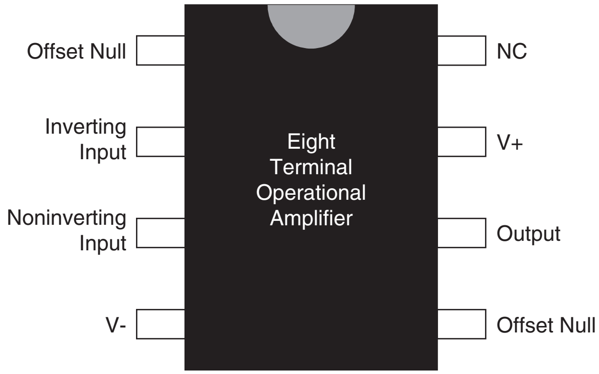


FIGURE 9.28 An eight-terminal operational amplifier. The terminal NC is not connected, and the two terminal offset nulls are used to correct imperfections (typically not connected). V+ and V– are terminal power to provide energy to the circuit. Keep in mind that a ground exists for both V+ and V–, a ground that is shared by other elements in the circuit. Modern operational amplifiers have ten or more terminals.

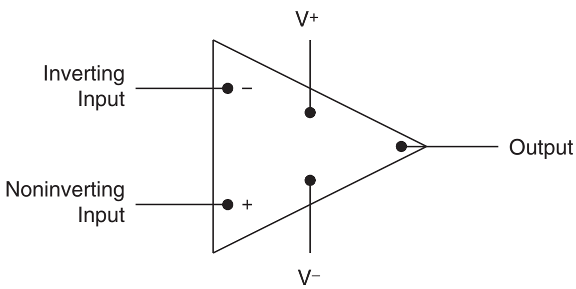


FIGURE 9.29 Circuit element symbol for the operational amplifier.

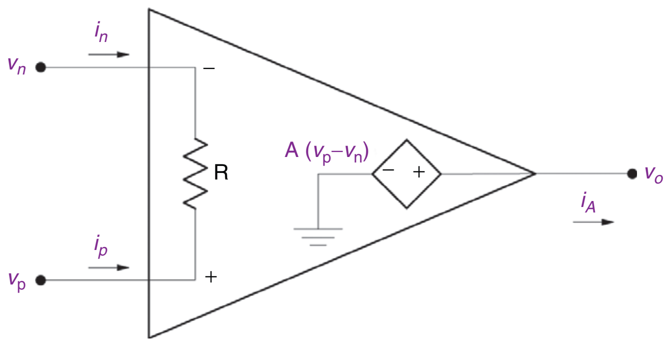


FIGURE 9.30 An internal model of the op amp. The internal resistance between the input terminals, R, is very large, exceeding 1 MΩ. The gain of the amplifier, A, is also large, exceeding 10^4. Power supply terminals are omitted for simplicity.

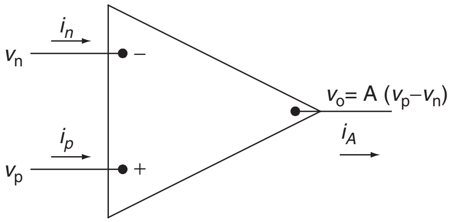
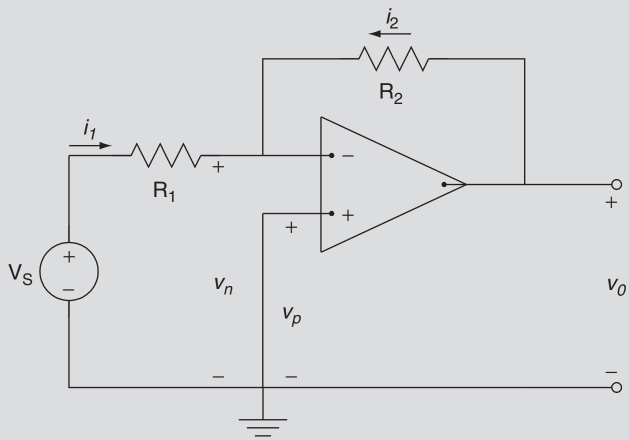


FIGURE 9.31 Idealized model of the op amp with the internal resistance, R, replaced by an open circuit.

EXAMPLE PROBLEM 9.18

Find v_0 for the following circuit.



Solution

Using the op amp model of Figure 9.31, we apply KCL at the inverting terminal giving

$$-i_1 - i_2 = 0$$

since no current flows into the op amp's input terminals. Replacing the current using Ohm's law gives

$$\frac{v_s - v_n}{R_1} + \frac{v_o - v_n}{R_2} = 0$$

Multiplying by R_1R_2 and collecting like terms, we have

$$R_2v_s = (R_1 + R_2)v_n - R_1v_o$$

Now $v_o = A(v_p - v_n)$, and since the noninverting terminal is connected to ground, $v_p = 0$,

$$v_o = -Av_n$$

or

$$v_n = -\frac{v_o}{A}$$

Substituting v_n into the KCL inverting input equation gives

$$R_s v_s = (R_1 + R_2)\left(-\frac{v_o}{A}\right) - R_1 v_o$$

$$= \left(\frac{R_1 + R_2}{A} + R_1\right) v_o$$

or

$$v_o = \frac{-R_2 v_s}{\left(R_1 + \dfrac{R_1 + R_2}{A}\right)}$$

As A goes to infinity, the previous equation goes to

$$v_o = -\frac{R_2}{R_1} v_s$$

Interestingly, with A going to infinity, v_0 remains finite due to the resistor R_2. This happens because a negative feedback path exists between the output and the inverting input terminal through R_2. This circuit is called an inverting amplifier with an overall gain of $-\frac{R_2}{R_1}$.

An operational amplifier with a gain of infinity is known as an ideal op amp. Because of the infinite gain, there must be a feedback path between the output and input, and we cannot connect a voltage source directly between the inverting and noninverting input terminals. When analyzing an ideal op amp circuit, we simplify the analysis by letting

$$v_n = v_p$$

Consider the previous example. Because $v_p = 0$, $v_n = 0$. Applying KCL at the inverting input gives

$$-\frac{v_s}{R_1} + \frac{-v_o}{R_2} = 0$$

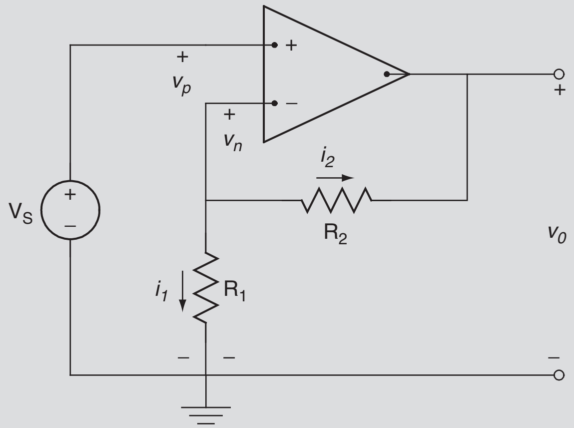
or

$$v_o = -\frac{R_2}{R_1} v_s$$

Notice how simple the analysis becomes when we assume $v_n = v_p$. Keep in mind that this approximation is valid as long as A is very large (infinity) and a feedback is included.

EXAMPLE PROBLEM 9.19

Find the overall gain for the following circuit.



Solution

Assuming the op amp is ideal, we start with $v_n = v_p$. Then, since the op amp's noninverting terminal is connected to the source, $v_n = v_p = v_s$. Because no current flows into the op amp, by KCL we have

$$i_1 + i_2 = 0$$

and

$$\frac{v_s}{R_1} + \frac{v_s - v_o}{R_2} = 0$$

or

$$v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s$$

The overall gain is

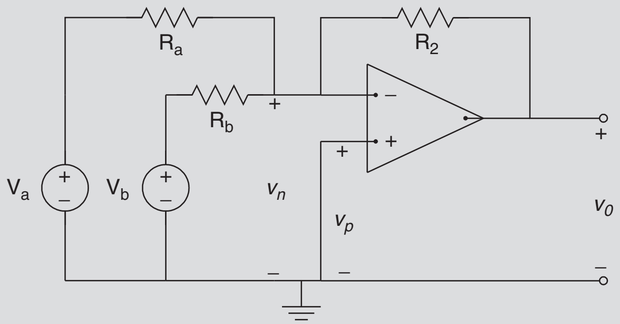
$$\frac{v_o}{v_s} = \frac{R_1 + R_2}{R_1}$$

This circuit is a noninverting op amp circuit used to amplify the source input. Amplifiers are used in almost all clinical instrumentation for ECG, EEG, EOG, and so on.

Example Problem 9.20 describes a summing op amp circuit.

EXAMPLE PROBLEM 9.20

Find the overall gain for the following circuit.



Solution

As before, we start the solution with $v_n = v_p$ and note that the noninverting input is connected to ground, yielding $v_n = v_p = 0$ V. Applying KCL at the inverting input node gives

$$-\frac{V_a}{R_a} - \frac{V_b}{R_b} - \frac{v_o}{R_2} = 0$$

or

$$v_o = -\left(\frac{R_2}{R_a} V_a + \frac{R_2}{R_b} V_b\right)$$

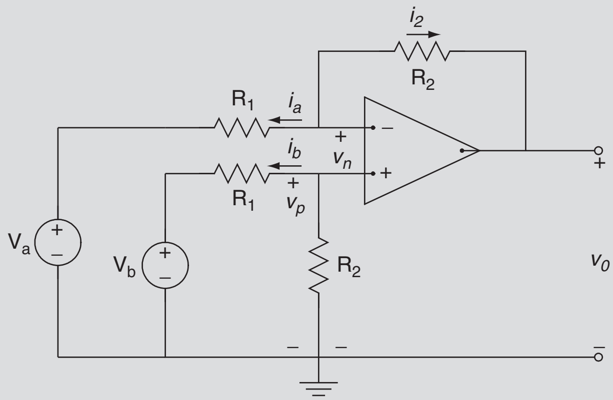
This circuit is a weighted summation of the input voltages. We can add additional source resistor inputs so that in general

$$v_o = -\left(\frac{R_2}{R_a} V_a + \frac{R_2}{R_b} V_b + \ldots + \frac{R_2}{R_m} V_m\right)$$

The op amp circuit in Example Problem 9.21 provides an output proportional to the difference of two input voltages. This op amp is often referred to as a differential amplifier.

EXAMPLE PROBLEM 9.21

Find the overall gain for the following circuit.



Solution

Assuming an ideal op amp, we note no current flows into the input terminals and that $v_n = v_p$. Apply KCL at the inverting input terminal gives

$$i_a = -i_2$$

or

$$\frac{v_n - V_a}{R_1} + \frac{v_n - v_o}{R_2} = 0$$

and

$$(R_1 + R_2)v_n - R_2 V_a = R_1 v_o$$

The previous equation involves two unknowns, so we need another equation easily found by applying the voltage divider at the noninverting input.

$$v_p = \frac{R_2}{R_1 + R_2} v_b = v_n$$

Substituting this result for v_n into the KCL equation at the inverting terminal gives

$$R_2 V_b - R_2 V_a = R_1 v_o$$

or

$$v_o = \frac{R_2}{R_1}(V_b - V_a)$$

As shown, this op amp circuit, also known as the differential amplifier, subtracts the weighted input signals. This amplifier is used for bipolar measurements involving ECG and EEG, since the typical recording is obtained between two bipolar input terminals. Ideally, the measurement contains only the signal of interest uncontaminated by noise from the environment. The noise is typically called a *common-mode signal*. A common-mode signal comes from lighting, 60-Hz power line signals, inadequate grounding, and power supply leakage. A differential amplifier with appropriate filtering can reduce the impact of a common-mode signal.

The response of a differential amplifier can be decomposed into differential-mode and common-mode components:

$$v_{dm} = v_b - v_a$$

and

$$v_{cm} = \frac{(v_a + v_b)}{2}$$

As described, the common-mode signal is the average of the input voltages. Using the two previous equations, one can solve v_a and v_b in terms of v_{dm} and v_{cm} as

$$v_a = v_{cm} - \frac{v_{dm}}{2}$$

and

$$v_b = v_{cm} + \frac{v_{dm}}{2}$$

When substituted into the response in Example Problem 9.21, we get

$$v_0 = \left(\frac{R_1R_2 - R_1R_2}{R_1(R_1 + R_2)}\right)v_{cm} + \left(\frac{R_2(R_1 + R_2) + R_2(R_1 + R_2)}{2R_1(R_1 + R_2)}\right)v_{dm} = A_{cm}v_{cm} + A_{dm}v_{dm}$$

Notice the term multiplying v_{cm}, A_{cm}, is zero, characteristic of the ideal op amp that amplifies only the differential-mode of the signal. Since real amplifiers are not ideal and resistors are not truly exact, the common-mode gain is not zero. So when one designs a differential amplifier, the goal is to keep A_{cm} as small as possible and A_{dm} as large as possible.

The rejection of the common-mode signal is called *common-mode rejection*, and the measure of how ideal the differential amplifier is called the *common-mode rejection ratio*, given as

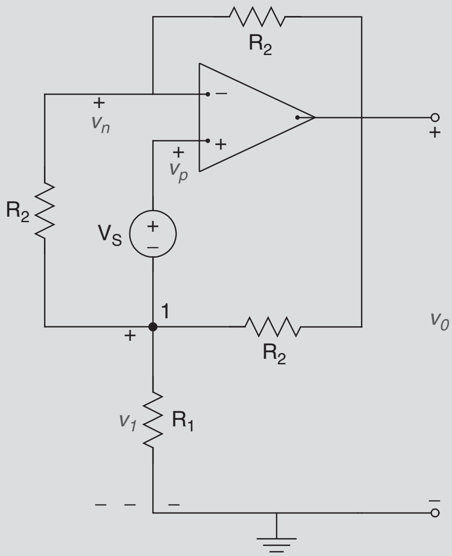
$$CMRR = 20\log_{10}\left|\frac{A_{dm}}{A_{cm}}\right|$$

where the larger the value of *CMRR*, the better. Values of *CMRR* for a differential amplifier for EEG, ECG, and EMG are 100 to 120 db.

The general approach to solving op amp circuits is to first assume that the op amp is ideal and $v_p = v_n$. Next, we apply KCL or KVL at the two input terminals. In more complex circuits, we continue to apply our circuit analysis tools to solve the problem, as Example Problem 9.22 illustrates.

EXAMPLE PROBLEM 9.22

Find v_0 for the following circuit.



Solution

With $v_n = v_p$, we apply KCL at the inverting input

$$\frac{v_n - v_1}{R_2} + \frac{v_n - v_o}{R_2} = 0$$

and

$$2v_n - v_1 - v_o = 0$$

Next, we apply KVL from ground to node 1 to the noninverting input and back to ground, giving

$$-v_1 - V_s + v_p = 0$$

and with $v_n = v_p$, we have $v_n - v_1 = V_s$.

Now we apply KCL at node 1, noting no current flows into the noninverting input terminal:

$$\frac{v_1}{R_1} + \frac{v_1 - v_o}{R_2} + \frac{v_1 - v_n}{R_2} = 0$$

Combining like terms in the previous equation gives

$$-R_1 v_n + (2R_1 + R_2)v_1 - R_1 v_o = 0$$

With three equations and three unknowns, we first eliminate v_1 by subtracting the inverting input KCL equation by the KVL equation, giving

$$v_1 = v_o - 2V_s$$

Next, we eliminate v_n by substituting v_1 into the inverting input KCL equation, as follows:

$$v_n = \frac{1}{2}(v_1 + v_o)$$

$$= \frac{1}{2}(v_o - 2V_s + v_o)$$

$$= v_o - V_s$$

Finally, we substitute the solutions for v_1 and v_n into the node 1 KCL equation, giving

$$-R_1 v_n + (2R_1 + R_2)v_1 - R_1 v_o = 0$$
$$-R_1(v_o - V_s) + (2R_1 + R_2)(v_o - 2V_s) - R_1 v_o = 0$$

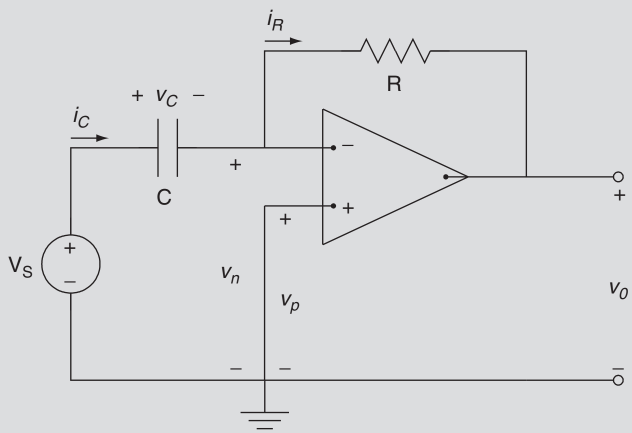
After simplification, we have

$$v_o = \frac{(3R_1 + 2R_2)}{R_2} V_s$$

Example Problems 9.23 and 9.24 illustrate an op amp circuit that differentiates and integrates by using a capacitor.

EXAMPLE PROBLEM 9.23

Find v_0 for the following circuit.



Solution

With the noninverting input connected to ground, we have $v_p = 0 = v_n$. From KVL

$$v_C = V_s$$

Continued

and it follows that

$$i_C = C\frac{dv_C}{dt} = C\frac{dV_s}{dt}$$

Since no current flows into the op amp, $i_C = i_R$. With

$$i_R = \frac{v_n - v_o}{R} = -\frac{v_o}{R}$$

and

$$i_C = C\frac{dV_s}{dt} = i_R = -\frac{v_o}{R}$$

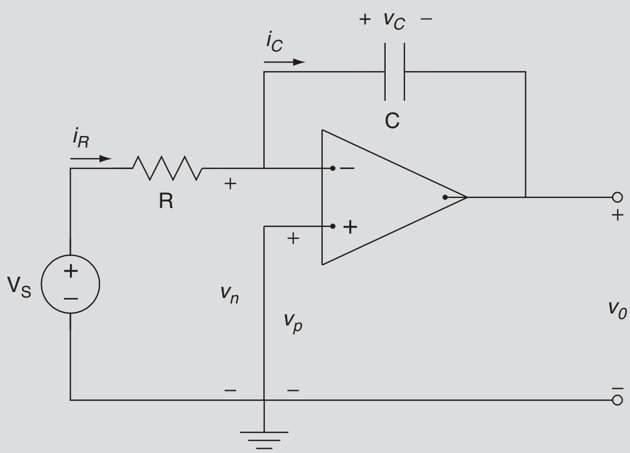
we have

$$v_o = -RC\frac{dV_s}{dt}$$

If $R = \frac{1}{C}$, the circuit in this example differentiates the input, $v_o = -\frac{dV_s}{dt}$.

EXAMPLE PROBLEM 9.24

Find v_0 for the following circuit.



Solution

It follows that

$$v_n = v_p = 0$$

and

$$i_C = i_R = \frac{V_s}{R}$$

Therefore,

$$v_C = \frac{1}{C}\int_{-\infty}^{t} i_C d\lambda = \frac{1}{C}\int_{-\infty}^{t} \frac{V_s}{R} d\lambda$$

From KVL, we have

$$v_C + v_o = 0$$

and

$$v_o = -\frac{1}{RC}\int_{-\infty}^{t} V_s d\lambda$$

With $R = \frac{1}{C}$, the circuit operates as an integrator

$$v_o = -\int_{-\infty}^{t} V_s d\lambda$$

9.11.1 Voltage Characteristics of the Op Amp

In the preceding examples involving the op amp, we did not consider the supply voltage (shown in Figure 9.29) and that the output voltage of an ideal op amp is constrained to operate between the supply voltages V^+ and V^-. If analysis determines v_0 is greater than V^+, v_0 saturates at V^+. If analysis determines v_0 is less than V^-, v_0 saturates at V^-. The output voltage characteristics are shown in Figure 9.32.

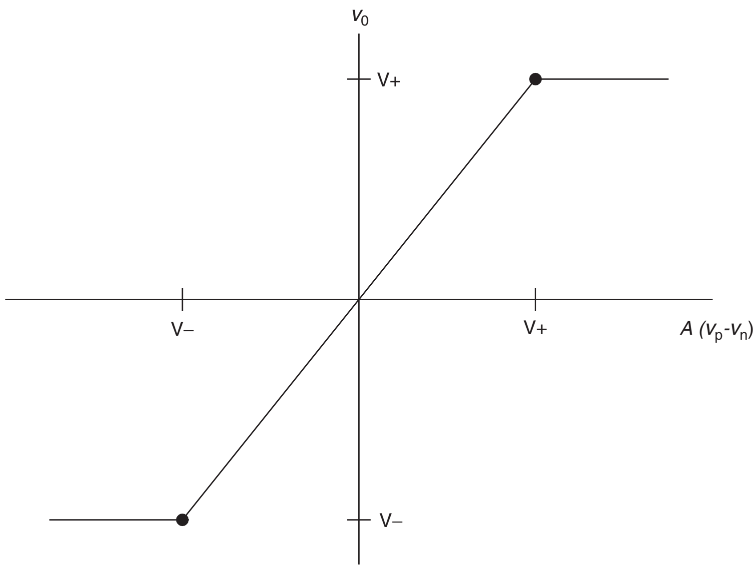


FIGURE 9.32 Voltage characteristics of an op amp.

EXAMPLE PROBLEM 9.25

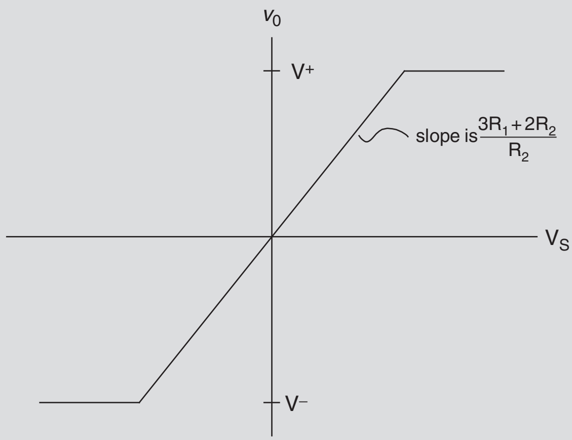
For the circuit shown in Example Problem 9.22, let $V^+ = +10$ V and $V^- = -10$ V. Graph the output voltage characteristics of the circuit.

Solution

The solution for Example Problem 9.22 is

$$v_o = \left(\frac{3R_1 + 2R_2}{R_2}\right) V_s$$

which saturates whenever v_0 is less than V^- and greater than V^+, as shown in the following graph.



9.12 TIME-VARYING SIGNALS

An alternating current (*a-c*) or sinusoidal source of 50 or 60 Hz is common throughout the world as a power source supplying energy for most equipment and other devices. While most of this chapter has focused on the transient response, when dealing with sinusoidal sources, attention is now focused on the steady-state or forced response. In bioinstrumentation, analysis in the steady-state simplifies the design by focusing only on the steady-state response, which is where the device actually operates. A sinusoidal voltage source is a time-varying signal given by

$$v_s = V_m \cos(\omega t + \phi) \tag{9.30}$$

where the voltage is defined by angular frequency (ω in radians/s), phase angle (ϕ in radians or degrees), and peak magnitude (V_m). The period of the sinusoid T is related to frequency f (Hz or cycles/s) and angular frequency by

$$\omega = 2\Pi f = \frac{2\Pi}{T} \tag{9.31}$$

An important metric of a sinusoid is its *rms value* (square root of the mean value of the squared function), given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t + \phi)dt} \tag{9.32}$$

which reduces to $V_{rms} = \frac{V_m}{\sqrt{2}}$.

To appreciate the response to a time-varying input, $v_s = V_m \cos(\omega t + \phi)$, consider the circuit shown in Figure 9.33, in which the switch is closed at $t = 0$ and there is no initial energy stored in the inductor. Applying KVL to the circuit gives

$$L\frac{di}{dt} + iR = V_m \cos(\omega t + \phi)$$

and after some work, the solution is

$$\begin{aligned} i &= i_n + i_f \\ &= \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\phi - \frac{\omega L}{R}\right) e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t + \phi - \frac{\omega L}{R}\right) \end{aligned}$$

The first term is the natural response that goes to zero as t goes to infinity. The second term is the forced response that has the same form as the input (i.e., a sinusoid with the same frequency ω, but a different phase angle and maximum amplitude). If all you are interested in is the steady-state response, as in most bioinstrumentation applications, then the only unknowns are the response amplitude and phase angle. The remainder of this section deals with techniques involving the *phasor* to efficiently find these unknowns.

9.12.1 Phasors

The phasor is a complex number that contains amplitude and phase angle information of a sinusoid and for the signal in Eq. (9.30) is expressed as

$$\mathbf{V} = V_m e^{j\phi} = V_m \underline{|\phi} \tag{9.33}$$

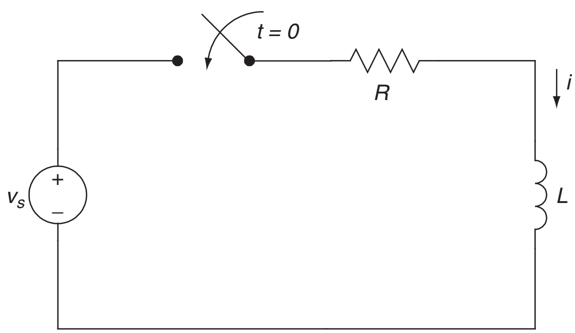


FIGURE 9.33 An RL circuit with sinusoidal input.

In Eq. (9.33), by practice, the angle in the exponential is written in radians, and in the $\underline{|\phi}$ notation is written in degrees. Work in the phasor domain involves the use of complex algebra in moving between the time and phasor domain, so the rectangular form of the phasor is also used, given as

$$\mathbf{V} = V_m(\cos\phi + j\sin\phi) \tag{9.34}$$

9.12.2 Passive Circuit Elements in the Phasor Domain

To use phasors with passive circuit elements for steady-state solutions, the relationship between voltage and current is needed for the resistor, inductor, and capacitor. Assume that

$$i = I_m \cos(\omega t + \theta)$$

$$I = I_m \underline{|\theta} = I_m e^{j\theta}$$

For a resistor,

$$v = IR = RI_m \cos(\omega t + \theta)$$

and the phasor of v is

$$\mathbf{V} = RI_m \underline{|\theta} = R\mathbf{I} \tag{9.35}$$

where $\mathbf{I} = I_m \underline{|\theta}$. Note that there is no phase shift for the relationship between the phasor current and voltage for a resistor.

For an inductor,

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta) = -\omega L I_m \cos(\omega t + \theta - 90^\circ)$$

and the phasor of v is

$$\begin{aligned}\mathbf{V} &= -\omega L I_m \underline{|\theta - 90^\circ} = -\omega L I_m e^{j(\theta - 90^\circ)} \\ &= -\omega L I_m e^{j\theta} e^{-j90^\circ} = -\omega L I_m e^{j\theta}(-j) \\ &= j\omega L I_m e^{j\theta} \\ &= j\omega L\mathbf{I}\end{aligned} \tag{9.36}$$

Note that inductor current and voltage are out of phase by 90°—that is, current lags behind voltage by 90°.

For a capacitor, define $v = V_m \cos(\omega t + \theta)$ and $\mathbf{V} = V_m \underline{|\theta}$. Now

$$\begin{aligned}i &= C\frac{dv}{dt} = C\frac{d}{dt}(V_m \cos(\omega t + \theta)) \\ &= -CV_m\omega \sin(\omega t + \theta) = -CV_m\omega\cos(\omega t + \theta - 90^\circ)\end{aligned}$$

and the phasor for i is

$$\begin{aligned}\mathbf{I} &= -\omega C V_m \underline{|\theta - 90°} = -\omega C V_m e^{j\theta} e^{-j90°} \\ &= -\omega C V_m e^{j\theta}(\cos(90°) - j\sin(90°)) \\ &= j\omega C V_m e^{j\theta} \\ &= j\omega C\mathbf{V}\end{aligned}$$

or

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I} = \frac{-j}{\omega C}\mathbf{I} \tag{9.37}$$

Note that capacitor current and voltage are out of phase by 90°—that is, voltage lags behind current by 90°.

Equations (9.35)–(9.37) all have the form of $\mathbf{V}=Z\mathbf{I}$, where Z represents the impedance of the circuit element and is, in general, a complex number, with units of ohms. The impedance for the resistor is R, the inductor, $j\omega L$, and the capacitor, $\frac{-j}{\omega C}$. The impedance is a complex number and not a phasor even though it may look like one. The imaginary part of the impendence is called reactance.

The final part to working in the phasor domain is to transform a circuit diagram from the time to phasor domain. For example, the circuit shown in Figure 9.34 is transformed into the phasor domain shown in Figure 9.35 by replacing each circuit element with their impedance equivalent and sources by their phasor. For the voltage source, we have

$$v_s = 100\sin 500t = 100\cos(500t - 90°)\ \text{mV} \quad \leftrightarrow \quad 500\underline{|-90°}\ \text{mV}$$

For the capacitor, we have

$$0.5\ \mu\text{F} \quad \leftrightarrow \quad \frac{-j}{\omega C} = -j4000\ \Omega$$

For the resistor, we have

$$1000\ \Omega \quad \leftrightarrow \quad 1000\ \Omega$$

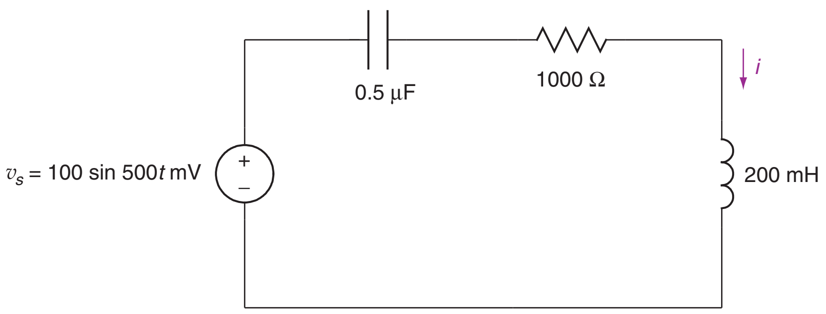


FIGURE 9.34 A circuit diagram.

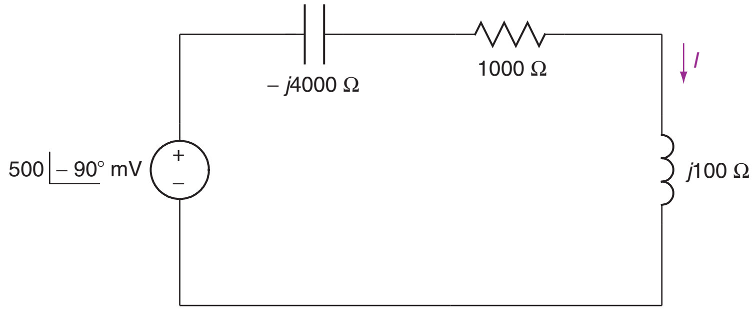


FIGURE 9.35 Phasor and impedance equivalent circuit for Figure 9.34.

For the inductor, we have

$$200\text{ mH} \quad \leftrightarrow \quad j\omega L = j100\ \Omega$$

Each of the elements is replaced by its phasor and impedance equivalents, as shown in Figure 9.35.

9.12.3 Kirchhoff's Laws and Other Techniques in the Phasor Domain

It is fortunate that all of the material presented before in this chapter involves Kirchhoff's current and voltage laws, and all of the other techniques apply to phasors. That is, for KVL, the sum of phasor voltages around any closed path is zero

$$\sum \mathbf{V}_\mathrm{i} = 0 \tag{9.38}$$

and for KCL, the sum of phasor currents leaving any node is zero

$$\sum \mathbf{I}_\mathrm{i} = 0 \tag{9.39}$$

Impedances in series are given by

$$Z = Z_1 + \cdots + Z_n \tag{9.40}$$

Impedances in parallel are given by

$$Z = \frac{1}{\frac{1}{Z_1} + \cdots + \frac{1}{Z_n}} \tag{9.41}$$

The node-voltage method, superposition and Thévenin equivalent circuits are also applicable in the phasor domain. Example Problems 9.26 and 9.27 illustrate the process, with the most difficult aspect involving complex algebra.

EXAMPLE PROBLEM 9.26

For the circuit shown in Figure 9.35, find the steady-state response *i*.

Solution

The impedance for the circuit is

$$Z = -j4000 + 1000 + j100 = 1000 - j3900\ \Omega$$

Using Ohm's law,

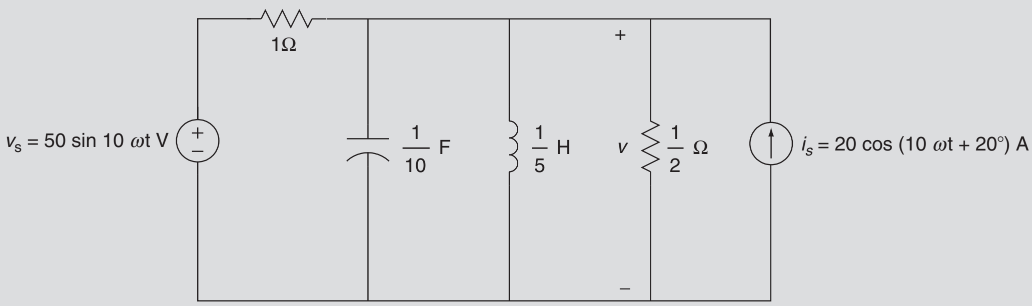
$$\mathbf{I} = \frac{\mathbf{V}}{Z} = \frac{0.5\angle -90^\circ}{1000 - j3900} = \frac{0.5\angle -90^\circ}{4026\angle -76^\circ} = 124\angle -14^\circ\ \mu\text{A}$$

Returning to the time domain, the steady-state current is

$$i = 124\cos(500t - 14^\circ)\ \mu\text{A}$$

EXAMPLE PROBLEM 9.27

Find the steady-state response v using the node-voltage method for the following circuit.



Solution

The first step is to transform the circuit elements into their impedances, which for the capacitor and inductor are

$$\frac{1}{10}\ \text{F} \leftrightarrow \frac{-j}{\omega C} = -j\ \Omega$$

$$\frac{1}{5}\ \text{H} \leftrightarrow j\omega L = j2\ \Omega$$

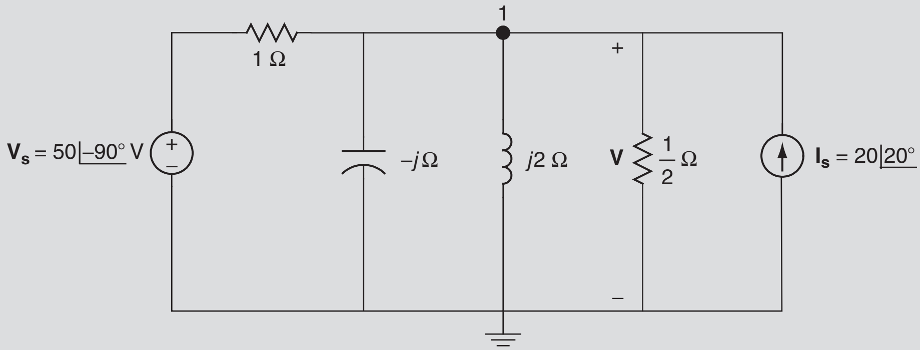
The phasors for the two sources are

$$v_s = 50\sin\omega t\ \text{V} \quad \leftrightarrow \quad \mathbf{V}_s = 50\angle -90^\circ\ \text{V}$$

$$i_s = 20\cos(\omega t + 20^\circ)\ \text{A} \quad \leftrightarrow \quad \mathbf{I}_s = 20\angle 20^\circ$$

Continued

Since the two resistors retain their values, the phasor drawing of the circuit is shown in the following figure with the ground at the lower node.



Writing the node-voltage equation for node 1 gives

$$\mathbf{V} - 50\angle{-90^\circ} + \frac{\mathbf{V}}{-j} + \frac{\mathbf{V}}{j2} + 2\mathbf{V} - 20\angle{20^\circ} = 0$$

Collecting like terms, converting to rectangular form, and converting to polar form gives

$$\mathbf{V}\left(3 + \frac{j}{2}\right) = 50\angle{-90^\circ} + 20\angle{20^\circ}$$

$$\mathbf{V}\left(3 + \frac{j}{2}\right) = -50j + 18.8 + j6.8 = 18.8 - j43.2$$

$$\mathbf{V} \times 3.04\angle{9.5^\circ} = 47.1\angle{-66.5^\circ}$$

$$\mathbf{V} = \frac{47.1\angle{-66.5^\circ}}{3.04\angle{9.5^\circ}} = 15.5\angle{-76^\circ}$$

The steady-state solution is

$$v = 15.6\cos\,(10t - 76^\circ)\ \text{V}$$

9.13 ACTIVE ANALOG FILTERS

This section presents several active analog filters involving the op amp. Passive analog filters use passive circuit elements: resistors, capacitors, and inductors. To improve performance in a passive analog filter, the resistive load at the output of the filter is usually increased. By using the op amp, fine control of the performance is achieved without increasing the load at the output of the filter. Filters are used to modify the measured signal by removing noise. A filter is designed in the frequency domain so the measured signal to be retained is passed through and noise is rejected.

Figure 9.36 shows the frequency characteristics of four filters: low-pass, high-pass, band-pass, and notch filters. The signal that is passed through the filter is indicated by the frequency interval called the passband. The signal that is removed by the filter is indicated by the frequency interval called the stopband. The magnitude of the filter, $|H(j\omega)|$, is one in the passband and zero in the stopband. The low-pass filter allows slowly changing signals with frequency less than ω_1 to pass through the filter and eliminates any signal or noise above ω_1. The high-pass filter allows quickly changing signals with frequency greater than ω_2 to pass through the filter and eliminates any signal or noise with frequency less than ω_2. The band-pass filter allows signals in the frequency band greater than ω_1 and less than ω_2 to pass through the filter and eliminates any signal or noise outside this interval. The notch filter allows signals in the frequency band less than ω_1 and greater than ω_2 to pass through

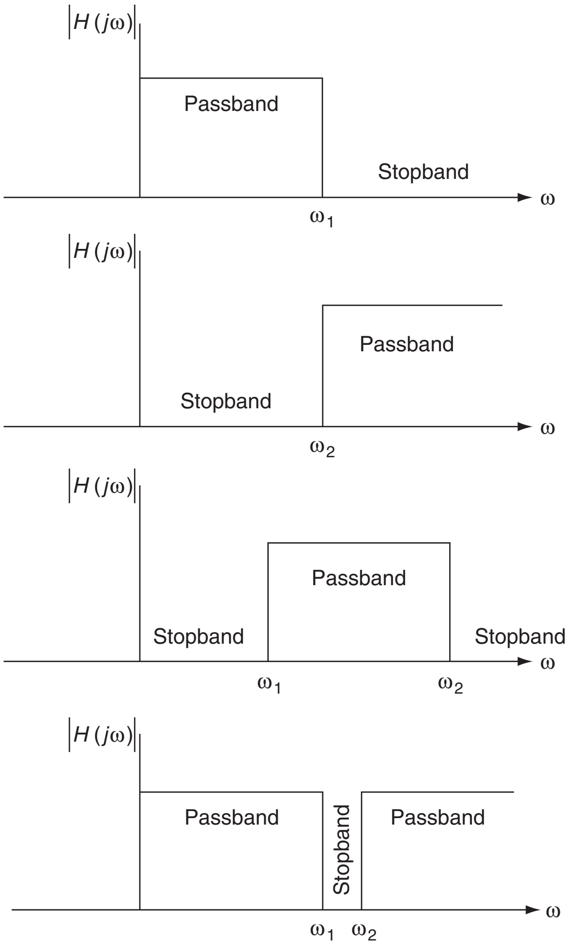


FIGURE 9.36 Ideal magnitude-frequency response for four filters, from top to bottom: low-pass, high-pass, band-pass, and notch.

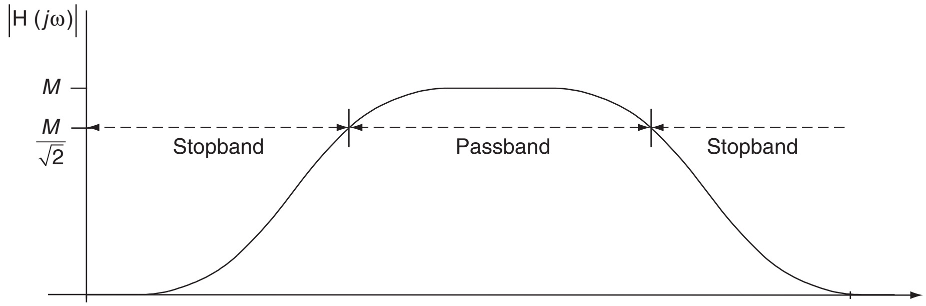


FIGURE 9.37 A realistic magnitude-frequency response for a band-pass filter. Note that the magnitude M does not necessarily need to be one. The passband is defined as the frequency interval when the magnitude is greater than $\frac{M}{\sqrt{2}}$.

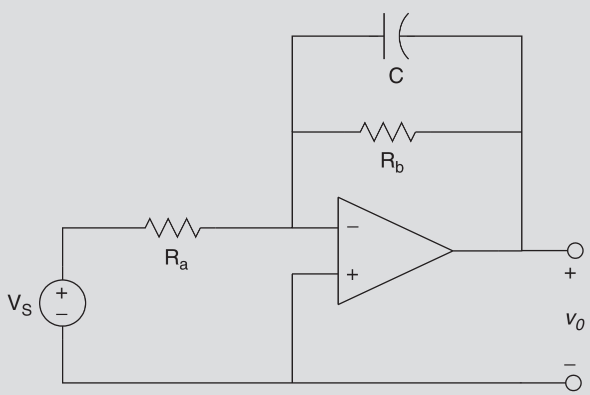
the filter and eliminates any signal or noise outside this interval. The frequencies ω_1 and ω_2 are typically called cutoff frequencies for the low-pass and high-pass filters.

In reality, any real filter cannot possibly have these ideal characteristics but instead has a smooth transition from the passband to the stopband, as shown, for example, in Figure 9.37 (the reason for this behavior is discussed in Chapter 11). Further, it is sometimes convenient to include both amplification and filtering in the same circuit, so the maximum of the magnitude does not need to be one, but it can be a value of M specified by the needs of the application.

To determine the filter's performance, the filter is driven by a sinusoidal input. One varies the input over the entire spectrum of interest (at discrete frequencies) and records the output magnitude. The critical frequencies are when $|H(j\omega)| = \frac{M}{\sqrt{2}}$.

EXAMPLE PROBLEM 9.28

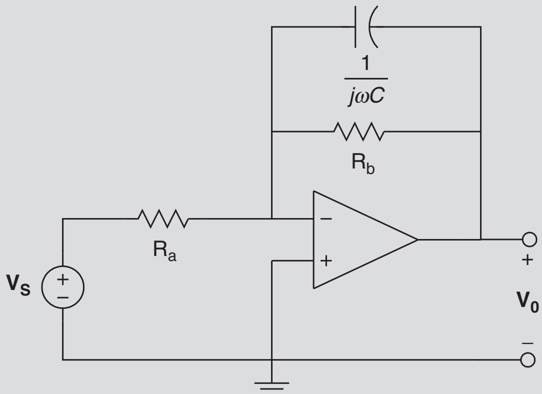
Using the low-pass filter in the following circuit, design the filter to have a gain of 5 and a cutoff frequency of 500 $\frac{\text{rad}}{\text{s}}$.



Solution

By treating the op amp as ideal, note that the noninverting input is connected to ground and, therefore, the inverting input is also connected to ground. The operation of this filter is readily apparent because at low frequencies, the capacitor acts like an open circuit, reducing the circuit to an inverting amplifier that passes low-frequency signals. At high frequencies, the capacitor acts like a short circuit, which connects the output terminal to the inverting input and ground.

The phasor method will be used to solve this problem by first transforming the circuit into the phasor domain, as shown in the following figure.



Summing the currents leaving the inverting input gives

$$-\frac{\mathbf{V}_s}{R_a} - \frac{\mathbf{V}_0}{\frac{1}{j\omega C}} - \frac{\mathbf{V}_0}{R_b} = 0$$

Collecting like terms and rearranging yields

$$-\mathbf{V}_0\left(\frac{1}{\frac{1}{j\omega C}} + \frac{1}{R_b}\right) = \frac{\mathbf{V}_s}{R_a}$$

After further manipulation,

$$\frac{\mathbf{V}_0}{\mathbf{V}_s} = -\frac{1}{R_a}\left(\frac{1}{\frac{1}{\frac{1}{j\omega C}} + \frac{1}{R_b}}\right) = -\frac{1}{R_a}\left(\frac{1}{j\omega C + \frac{1}{R_b}}\right)$$

$$\frac{\mathbf{V}_0}{\mathbf{V}_s} = -\frac{1}{R_aC}\left(\frac{1}{j\omega + \frac{1}{R_bC}}\right)$$

Continued

Similar to the reasoning for the characteristic equation for a differential equation, the cutoff frequency is defined as $\omega_c = \frac{1}{R_b C}$, (i.e., the denominator term, $j\omega + \frac{1}{R_b C}$ is set equal to zero). Thus, with the cutoff frequency set at $\omega_c = 500\frac{\text{rad}}{\text{s}}$, then $\frac{1}{R_b C} = 500$. The cutoff frequency is also defined as when $|H(j\omega)| = \frac{M}{\sqrt{2}}$, where $M = 5$. The magnitude of $\frac{\mathbf{V}_0}{\mathbf{V}_s}$ is given by

$$\left|\frac{\mathbf{V}_0}{\mathbf{V}_s}\right| = \frac{\frac{1}{R_a C}}{\sqrt{\omega^2 + \left(\frac{1}{R_b C}\right)^2}}$$

and at the cutoff frequency, $\omega_c = 500\frac{\text{rad}}{\text{s}}$,

$$\frac{5}{\sqrt{2}} = \frac{\frac{1}{R_a C}}{\sqrt{\omega_c^2 + \left(\frac{1}{R_b C}\right)^2}}$$

With $\frac{1}{R_b C} = 500$, the magnitude is

$$\frac{5}{\sqrt{2}} = \frac{\frac{1}{R_a C}}{\sqrt{\omega_c^2 + \left(\frac{1}{R_b C}\right)^2}} = \frac{\frac{1}{R_a C}}{\sqrt{500^2 + 500^2}} = \frac{\frac{1}{R_a C}}{500\sqrt{2}}$$

which gives

$$R_a C = \frac{1}{2500}$$

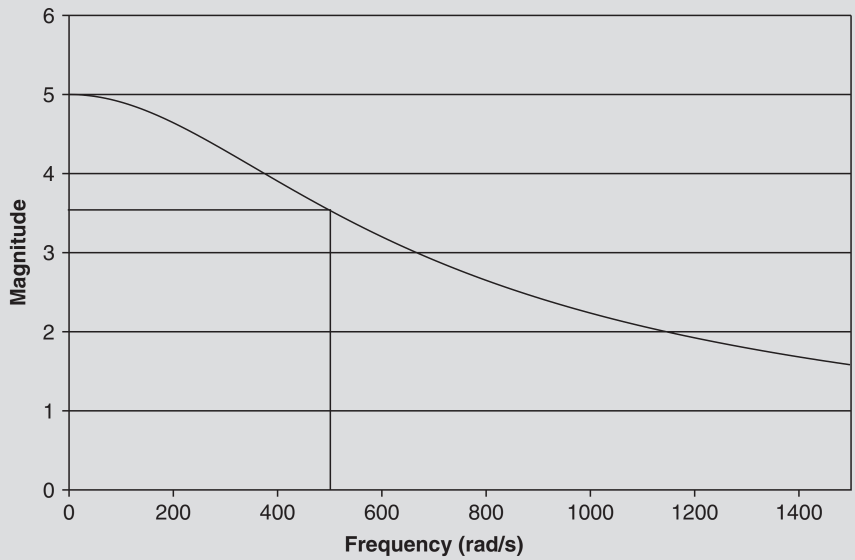
Since we have three unknowns and two equations ($R_a C = \frac{1}{2500}$ and $\frac{1}{R_b C} = 500$), there are an infinite number of solutions. Therefore, one can select a convenient value for one of the elements—say, $R_a = 20\ \text{k}\Omega$—and the other two elements are determined as

$$C = \frac{1}{2500 \times R_a} = \frac{1}{2500 \times 20000} = 20\ \text{nF}$$

and

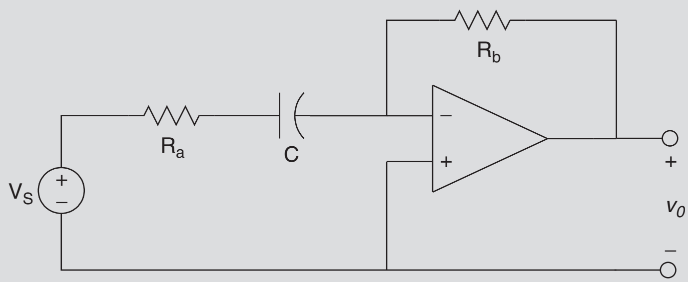
$$R_b = \frac{1}{500 \times C} = \frac{1}{500 \times 20 \times 10^{-9}} = 100\ \text{k}\Omega$$

A plot of the magnitude versus frequency is shown in the following figure. As can be seen, the cutoff frequency gives a value of magnitude equal to 3.53 at 100 Hz, which is the design goal.



EXAMPLE PROBLEM 9.29

Using the high-pass filter in the following circuit, design the filter to have a gain of 5 and a cutoff frequency of 100 $\frac{\text{rad}}{\text{s}}$.

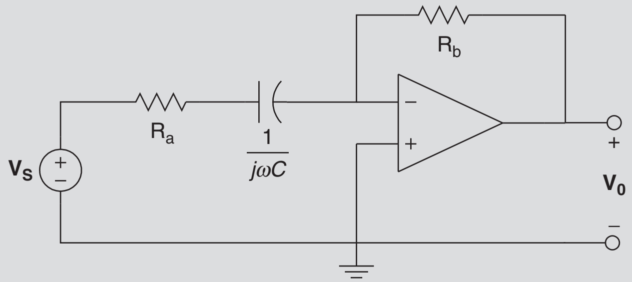


Solution

Since the op amp is assumed ideal and the noninverting input is connected to ground, the inverting input is also connected to ground. The operation of this filter is readily apparent because at low frequencies, the capacitor acts like an open circuit, so no input voltage is seen at the non-inverting input. Since there is no input, then the output is zero. At high frequencies, the capacitor acts like a short circuit, which reduces the circuit to an inverting amplifier that passes through high-frequency signals.

Continued

As before, the phasor method will be used to solve this problem by first transforming the circuit into the phasor domain, as shown in the following figure.



Summing the currents leaving the inverting input gives

$$-\frac{\mathbf{V}_s}{R_a + \frac{1}{j\omega C}} - \frac{\mathbf{V}_0}{R_b} = 0$$

Rearranging yields

$$\frac{\mathbf{V}_0}{\mathbf{V}_s} = -\frac{R_b}{R_a + \frac{1}{j\omega C}} = -\frac{R_b}{R_a}\frac{j\omega}{j\omega + \frac{1}{R_a C}}$$

At cutoff frequency $\omega_c = 100\frac{\text{rad}}{\text{s}} = \frac{1}{R_a C}$. The magnitude of $\frac{\mathbf{V}_0}{\mathbf{V}_s}$ is given by

$$\left|\frac{\mathbf{V}_0}{\mathbf{V}_s}\right| = \frac{R_b}{R_a}\frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{R_a C}\right)^2}}$$

and at the cutoff frequency,

$$\frac{5}{\sqrt{2}} = \frac{R_b}{R_a}\frac{\omega_c}{\sqrt{\omega_c^2 + \left(\frac{1}{R_a C}\right)^2}}$$

With $\frac{1}{R_a C} = 100$ and $\omega_c = 100\frac{\text{rad}}{\text{s}}$, gives

$$\frac{5}{\sqrt{2}} = \frac{R_b}{R_a}\frac{\omega_c}{\sqrt{\omega_c^2 + \left(\frac{1}{R_a C}\right)^2}} = \frac{R_b}{R_a}\frac{\frac{1}{R_a C}}{\sqrt{100^2 + 100^2}} = \frac{R_b}{R_a}\frac{100}{100} = \frac{R_b}{\sqrt{2}R_a}$$

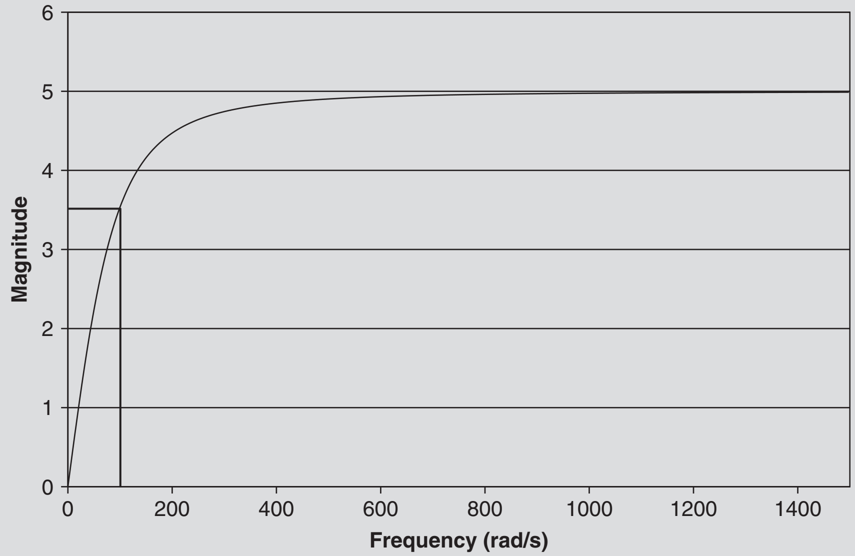
Thus, $\frac{R_b}{R_a} = 5$. Since we have three unknowns and two equations, one can select a convenient value for one of the elements—say, $R_b = 20\ \text{k}\Omega$—and the other two elements are determined as

$$R_a = \frac{R_b}{5} = \frac{20000}{5} = 4\ \text{k}\Omega$$

and

$$C = \frac{1}{100R_a} = \frac{1}{100 \times 4000} = 2.5\ \mu\text{F}$$

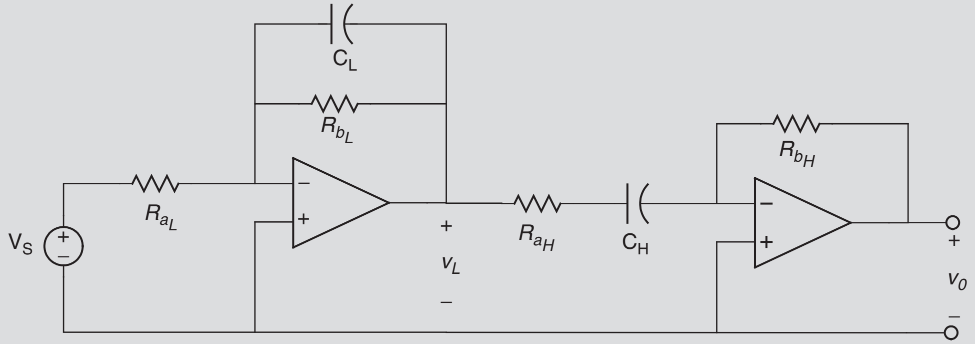
A plot of the magnitude versus frequency is shown in the following figure. As can be seen, the cutoff frequency gives a value of magnitude equal to 3.53 at 100 Hz, which is the design goal.



Example Problem 9.30 demonstrates the technique to create band-pass filters (which require two cutoff frequencies).

EXAMPLE PROBLEM 9.30

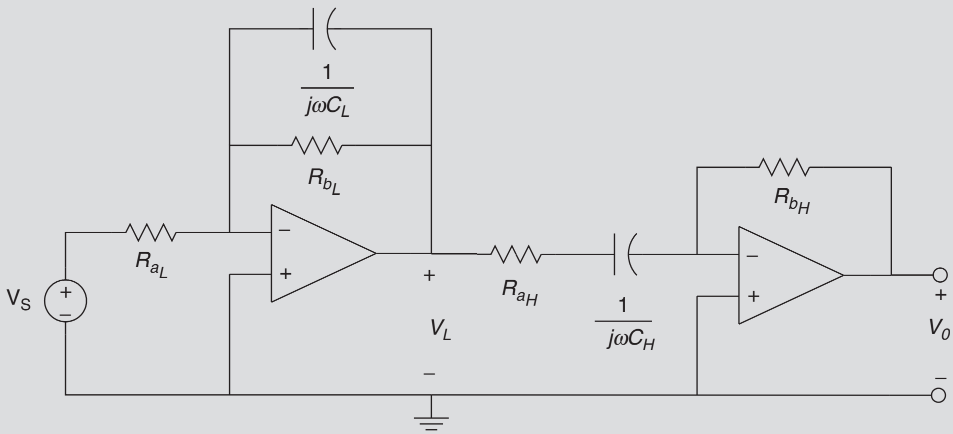
Using the band-pass filter in the following circuit, design the filter to have a gain of 5 and pass through frequencies from 100 to 500 $\frac{\text{rad}}{\text{s}}$.



Continued

Solution

As usual, the design of the filter is done in the phasor domain and uses the work done in the previous two examples. Note that the elements around the op amp on the left are the low-pass filter circuit elements, and those on the right are the high-pass filter circuit elements. In fact, when working with op amps, filters can be cascaded together to form other filters, so a low-pass and high-pass filter cascaded together will form a band-pass. The phasor domain circuit is given in the next figure.



As before the noninverting input to the op amps is connected to ground, which means that the inverting input is also connected to ground. Summing the currents leaving the inverting input for each op amp gives

$$-\frac{\mathbf{V}_{\mathrm{s}}}{R_{a_L}} - \frac{\mathbf{V}_{\mathrm{L}}}{\frac{1}{j\omega C_L}} - \frac{\mathbf{V}_{\mathrm{L}}}{R_{b_L}} = 0$$

$$-\frac{\mathbf{V}_{\mathrm{L}}}{R_{a_H} + \frac{1}{j\omega C_H}} - \frac{\mathbf{V}_0}{R_{b_H}} = 0$$

Solving the first equation for $\mathbf{V}_{\mathrm{L}}$ gives

$$\mathbf{V}_{\mathrm{L}} = -\frac{1}{R_{a_L} C_L}\left(\frac{1}{j\omega + \frac{1}{R_{b_L} C_L}}\right)\mathbf{V}_{\mathrm{s}}$$

Solving the second equation for $\mathbf{V}_0$ gives

$$\mathbf{V}_0 = -\frac{R_{b_H}}{R_{a_H}} \frac{j\omega}{j\omega + \frac{1}{R_{a_H} C_H}} \mathbf{V}_{\mathrm{L}}$$

Substituting $\mathbf{V}_L$ into the previous equation yields

$$\mathbf{V}_0 = \frac{R_{b_H}}{R_{a_H}} \frac{j\omega}{j\omega + \frac{1}{R_{a_H} C_H}} \times \frac{1}{R_{a_L} C_L} \left(\frac{1}{j\omega + \frac{1}{R_{b_L} C_L}} \right) \mathbf{V}_s$$

The form of the solution is simply the product of each filter. The magnitude of the filter is

$$\left| \frac{\mathbf{V}_0}{\mathbf{V}_s} \right| = \frac{R_{b_H}}{R_{a_H}} \frac{\omega}{\sqrt{\omega^2 + \left(\frac{1}{R_{a_H} C_H} \right)^2}} \frac{\frac{1}{R_{a_L} C_L}}{\sqrt{\omega^2 + \left(\frac{1}{R_{b_L} C_L} \right)^2}}$$

Since there are two cutoff frequencies, two equations evolve:

$$\omega_{c_H} = \frac{1}{R_{a_H} C_H} = 100 \frac{\text{rad}}{\text{s}}$$

and

$$\omega_{c_L} = \frac{1}{R_{b_L} C_L} = 500 \frac{\text{rad}}{\text{s}}$$

At either cutoff frequency, the magnitude is $\frac{5}{\sqrt{2}}$, such that at $\omega_{c_H} = 100 \frac{\text{rad}}{\text{s}}$

$$\frac{5}{\sqrt{2}} = \frac{R_{b_H}}{R_{a_H}} \frac{\omega_{c_H}}{\sqrt{\omega_{c_H}^2 + \left(\frac{1}{R_{a_H} C_H} \right)^2}} \frac{\frac{1}{R_{a_L} C_L}}{\sqrt{\omega_{C_H}^2 + \left(\frac{1}{R_{b_L} C_L} \right)^2}}$$

$$= \frac{R_{b_H}}{R_{a_H}} \frac{100}{\sqrt{100^2 + 100^2}} \frac{\frac{1}{R_{a_L} C_L}}{\sqrt{100^2 + 500^2}}$$

Therefore,

$$500\sqrt{26} = \frac{R_{b_H}}{R_{a_H} R_{a_L} C_L}$$

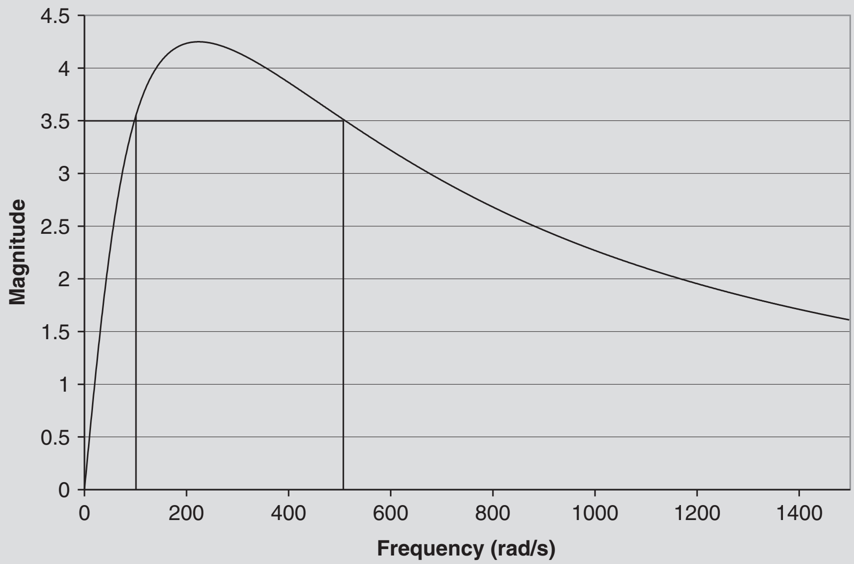
The other cutoff frequency gives the same result as the previous equation. There are now three equations $\left(\frac{1}{R_{a_H} C_H} = 100, \frac{1}{R_{b_L} C_L} = 500, \text{ and } 500\sqrt{26} = \frac{R_{b_H}}{R_{a_H} R_{a_L} C_L} \right)$, and six unknowns. For convenience, set $R_{b_L} = 100\ \text{k}\Omega$ and $R_{a_H} = 100\ \text{k}\Omega$, which gives $C_L = \frac{1}{500 R_{b_L}} = 20\ \text{nF}$ and $C_H = \frac{1}{100 R_{a_H}} = 0.1\ \mu\text{F}$. Now from $500\sqrt{26} = \frac{R_{b_H}}{R_{a_H}} \frac{1}{R_{a_L} C_L}$,

$$\frac{R_{b_H}}{R_{a_L}} = 500\sqrt{26} C_L R_{a_H} = 5.099$$

Once again, one can specify one of the resistors—say, $R_{a_L} = 10\ \text{k}\Omega$—giving $R_{b_H} = 50.099\ \text{k}\Omega$.

Continued

A plot of the magnitude versus frequency is shown in the following figure. As can be seen, the cutoff frequency gives a value of magnitude equal to 3.53 at 500 Hz, which is the design goal.



None of the filters in Example Problems 9.28–9.30 have the ideal characteristics of Figure 9.36. To improve the performance from the pass-band to stopband in a low-pass filter with a sharper transition, one can cascade identical filters together—that is, connect the output of the first filter to the input of the next filter and so on. The more cascaded filters, the better the performance. The magnitude of the overall filter is the product of the individual filter magnitudes.

While this approach is appealing for improving the performance of the filter, the overall magnitude of the filter does not remain a constant in the pass-band. Better filters are available with superior performance, such as a Butterworth filter. Two Butterworth filters are shown in Figure 9.38. Analysis of these filters is carried out in Exercises 53 and 55.

9.14 BIOINSTRUMENTATION DESIGN

Figure 9.2 described the various elements needed in a biomedical instrumentation system. The purpose of this type of instrument is to monitor the output of a sensor or sensors and to extract information from the signals that are produced by the sensors.

Acquiring a discrete-time signal and storing this signal in computer memory from a continuous-time signal is accomplished with an analog-to-digital (A/D) converter. The A/D converter uniformly samples the continuous-time waveform and transforms it into a sequence of numbers, one every t_k *seconds.* The A/D converter also transforms the

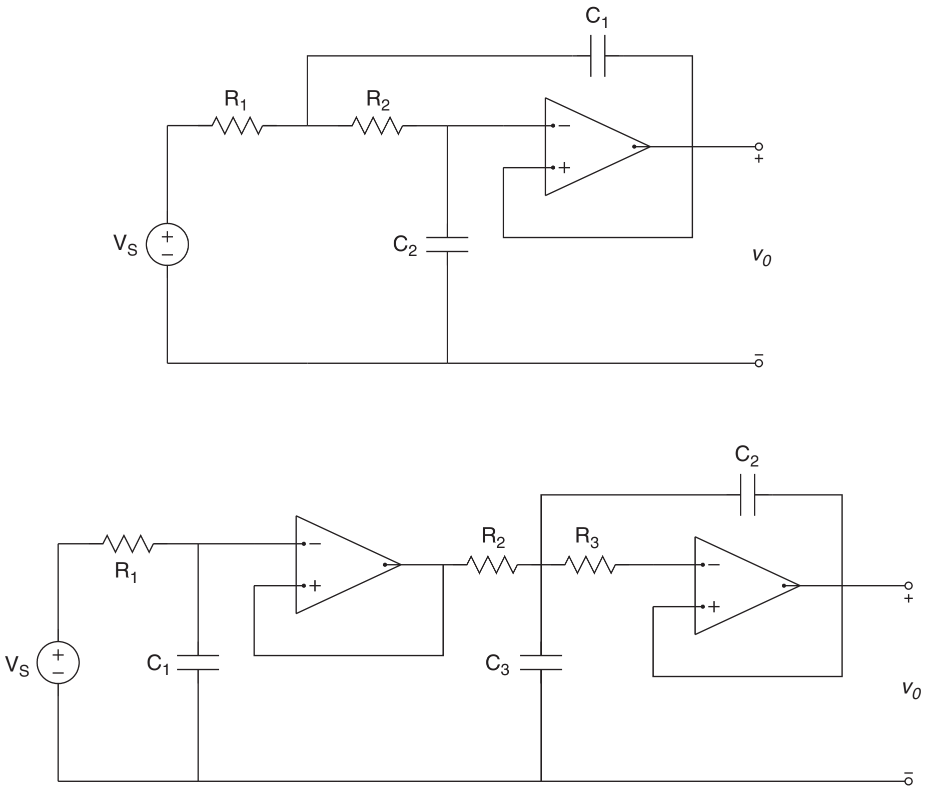


FIGURE 9.38 (Top) Second-order Butterworth low-pass filter. (Bottom) Third-order Butterworth low-pass filter.

continuous-time waveform into a digital signal (i.e., the amplitude takes one of 2^n discrete values), which is converted into computer words and stored in computer memory. To adequately capture the continuous-time signal, the sampling instants t_k must be selected carefully so information is not lost. The minimum sampling rate is twice the highest frequency content of the signal (based on the sampling theorem from communication theory). Realistically, we often sample at five to ten times the highest frequency content of the signal so as to achieve better accuracy by reducing aliasing error.

9.14.1 Noise

Measurement signals are always corrupted by noise in a biomedical instrumentation system. Interference noise occurs when unwanted signals are introduced into the system by outside sources, such as power lines and transmitted radio and television electromagnetic waves. This kind of noise is effectively reduced by careful attention to the circuit's wiring configuration to minimize coupling effects.

Interference noise is introduced by power lines (50 or 60 Hz), fluorescent lights, AM/FM radio broadcasts, computer clock oscillators, laboratory equipment, and cellular phones.

Electromagnetic energy radiating from noise sources is injected into the amplifier circuit or into the patient by capacitive and/or inductive coupling. Even the action potentials from nerve conduction in the patient generate noise at the sensor/amplifier interface. Filters are used to reduce the noise and to maximize the signal-to-noise (S/N) ratio at the input of the A/D converter.

Low-frequency noise (amplifier d.c. offsets, sensor drift, temperature fluctuations, etc.) is eliminated by a high-pass filter with the cutoff frequency set above the noise frequencies and below the biological signal frequencies. High-frequency noise (nerve conduction, radio broadcasts, computers, cellular phones, etc.) is reduced by a low-pass filter with the cutoff set below the noise frequencies and above the frequencies of the biological signal that is being monitored. Power line noise is a very difficult problem in biological monitoring, since the 50- or 60-Hz frequency is usually within the frequency range of the biological signal that is being measured. Band-stop filters are commonly used to reduce power line noise. The notch frequency in these band-stop filters is set to the power line frequency of 50 or 60 Hz with the cutoff frequencies located a few Hertz to either side.

The second type of corrupting signal is called inherent noise. Inherent noise arises from random processes that are fundamental to the operation of the circuit's elements and thus is reduced by good circuit design practice. While inherent noise can be reduced, it can never be eliminated. Low-pass filters can be used to reduce high-frequency components. However, noise signals within the frequency range of the biosignal being amplified cannot be eliminated by this filtering approach.

9.14.2 Computers

Computers consist of three basic units: the central processing unit (CPU), the arithmetic and logic unit (ALU), and memory. The CPU directs the functioning of all other units and controls the flow of information among the units during processing procedures. It is controlled by program instructions. The ALU performs all arithmetic calculations (add, subtract, multiply, and divide) as well as logical operations (AND, OR, NOT) that compare one set of information to another.

Computer memory consists of read only memory (ROM) and random access memory (RAM). ROM is permanently programmed into the integrated circuit that forms the basis of the CPU and cannot be changed by the user. RAM stores information temporarily and can be changed by the user. RAM is where user-generated programs, input data, and processed data are stored.

Computers are binary devices that use the presence of an electrical signal to represent 1 and the absence of an electrical pulse to represent 0. The signals are combined in groups of 8 bits, a byte, to code information. A word is made up of 2 bytes. Most desktop computers that are used today are 32-bit systems, which means they can address 4.295×10^9 locations in memory. Most new computers today are 64-bit systems that can address 1.8447×10^{19} locations in memory. The first microcomputers were 8-bit devices that could interact with only 256 memory locations.

Programming languages relate instructions and data to a fixed array of binary bits so the specific arrangement has only one meaning. Letters of the alphabet and other symbols such as punctuation marks are represented by special codes. ASCII stands for American Standard Code for Information Exchange. ASCII provides a common standard that allows different types of computers to exchange information. When word processing files are saved as text files, they are saved in ASCII format. Ordinarily, word processing files are saved in special program-specific binary formats, but almost all data analysis programs can import and export data in ASCII files.

The lowest level of computer languages is machine language and consists of the 0s and 1s that the computer interprets. Machine language represents the natural language of a particular computer. At the next level, assembly languages use English-like abbreviations for binary equivalents. Programs written in assembly language can manipulate memory locations directly. These programs run very quickly and are often used in data acquisition systems that must rapidly acquire a large number of samples, perhaps from an array of sensors, at a very high sampling rate.

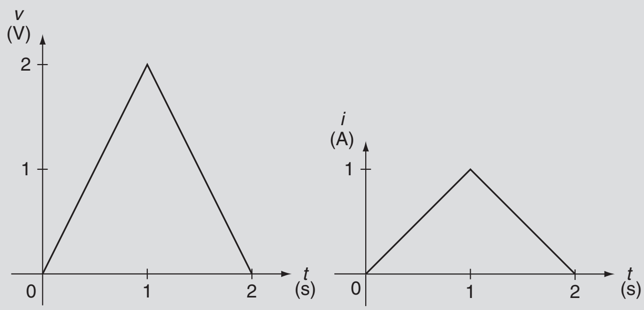
Higher-level languages such as FORTRAN, PERL, and C++ contain statements that accomplish tasks that require many machine or assembly language statements. Instructions in these languages often resemble English and contain commonly used mathematical notations. Higher-level languages are easier to learn than machine and assembly languages. Program instructions are designed to tell computers when and how to use various hardware components to solve specific problems. These instructions must be delivered to the CPU of a computer in the correct sequence in order to give the desired result. Newer programing languages such as MATLAB and LabView are easier to use and more user friendly.

When computers are used to acquire physiological data, programming instructions tell the computer when data acquisition should begin, how often samples should be taken from how many sensors, how long data acquisition should continue, and where the digitized data should be stored. The rate at which a system can acquire samples depends on the speed of the computer's clock—233 MHz—and the number of computer instructions that must be completed in order to take a sample. Some computers can also control the gain on the input amplifiers so signals can be adjusted during data acquisition. In other systems, the gain of the input amplifiers must be manually adjusted.

9.15 EXERCISES

1. Find the power absorbed for the circuit element in Figure 9.7 if

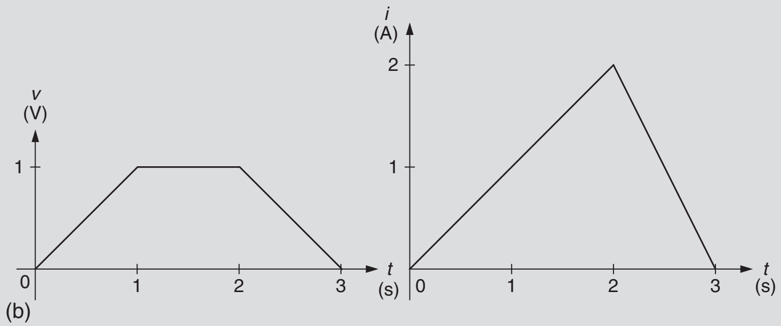
(a)



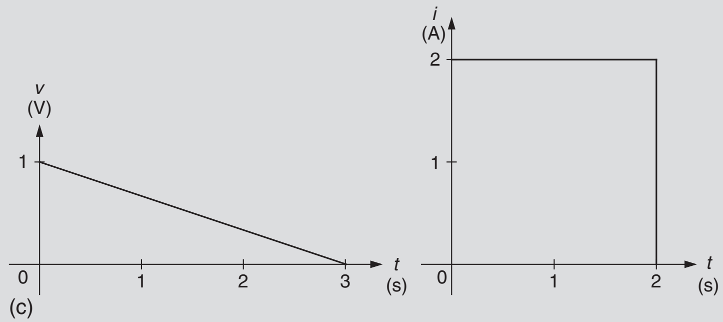
(a)

Continued

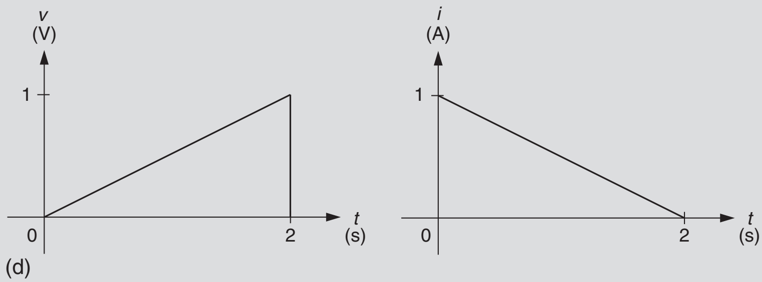
(b)


v
(V)
1
0
1
2
3
t
(s)
(b)
i
(A)
2
1
0
1
2
3
t
(s)

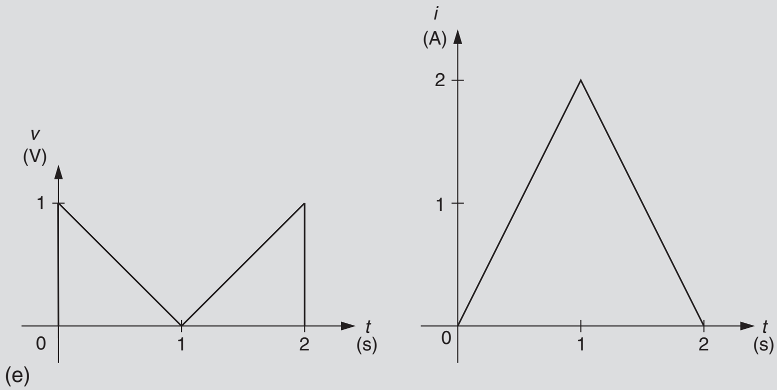
(c)


v
(V)
1
0
1
2
3
t
(s)
(c)
i
(A)
2
1
0
1
2
t
(s)

(d)


v
(V)
1
0
2
t
(s)
(d)
i
(A)
1
0
2
t
(s)

(e)

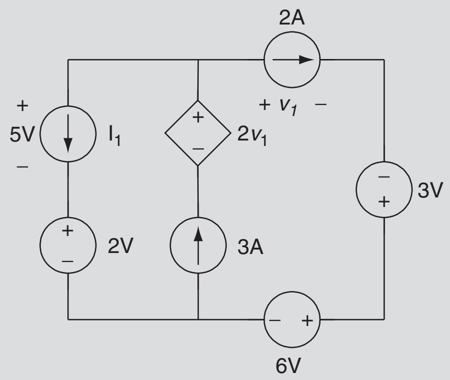

v
(V)
1
0
1
2
t
(s)
(e)
i
(A)
2
1
0
1
2
t
(s)

2. The voltage and current at the terminals in Figure 9.7 are

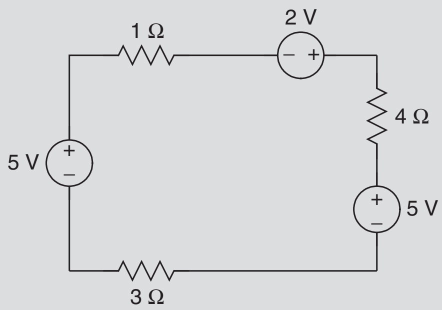
$$v = te^{-10,000t}u(t)\ \mathrm{V}$$
$$i = (t+10)e^{-10,000t}u(t)\ \mathrm{A}$$

(a) Find the time when the power is at its maximum.
(b) Find the maximum power.
(c) Find the energy delivered to the circuit at $t = 1 \times 10^{-4}$ s.
(d) Find the total energy delivered to the circuit element.

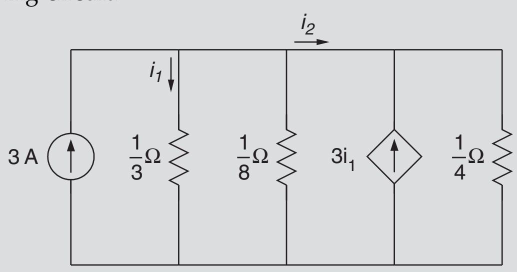
3. For the following circuit find (a) v_1, (b) the power absorbed and delivered.



4. For the following circuit, find the power in each circuit element.



5. Find i_2 in the following circuit.



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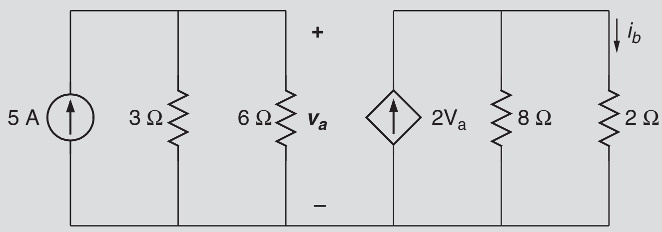
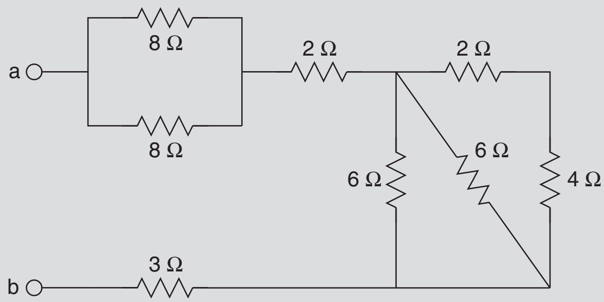
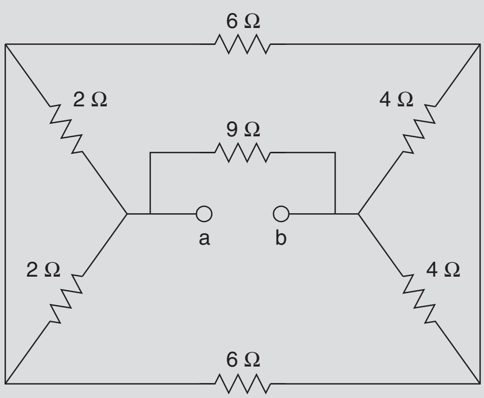
6. Find i_b for the following circuit.



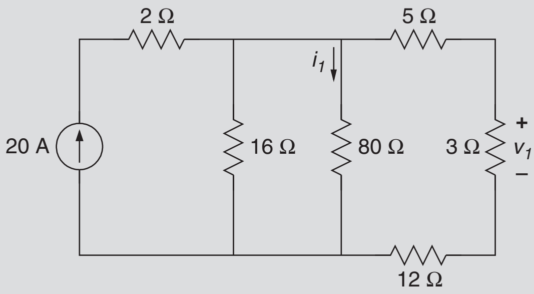
7. Find the equivalent resistance R_{ab} for the following circuit.



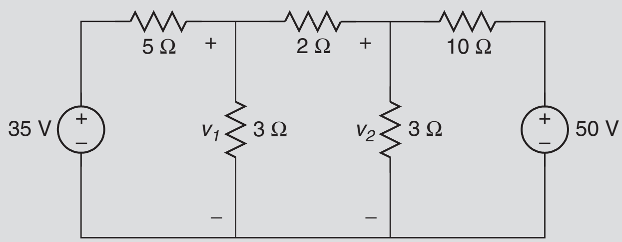
8. Find the equivalent resistance R_{ab} for the following circuit.



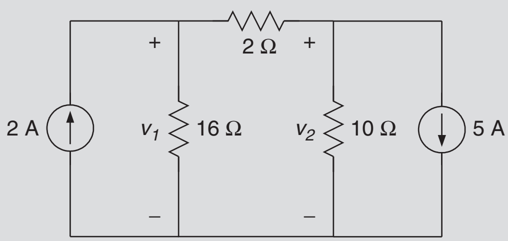
9. Find i_1 and v_1 for the following circuit.



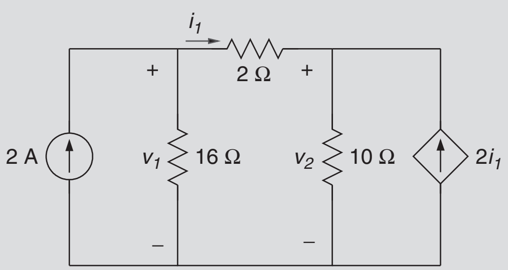
10. Use the node-voltage method to determine v_1 and v_2.



11. Use the node-voltage method to determine v_1 and v_2.



12. Use the node-voltage method to determine v_1 and v_2.



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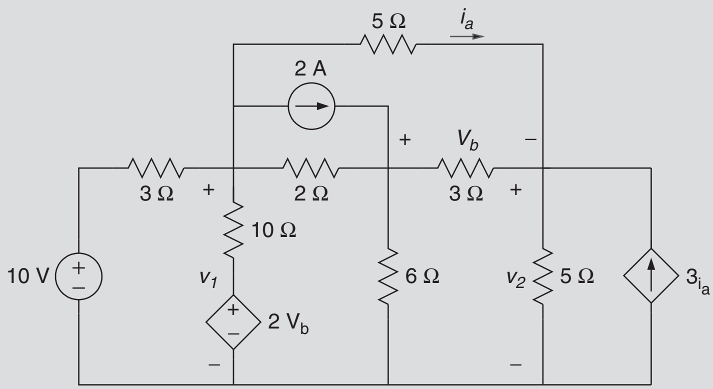
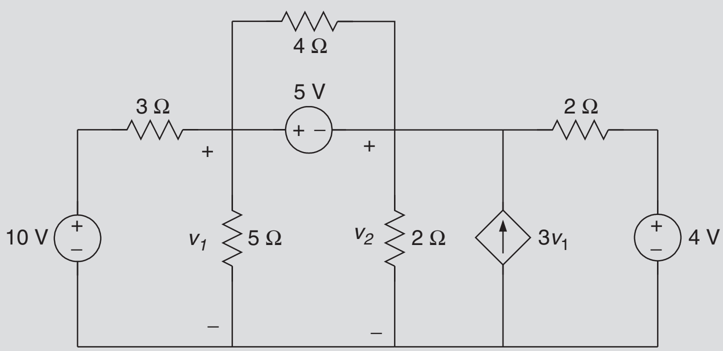
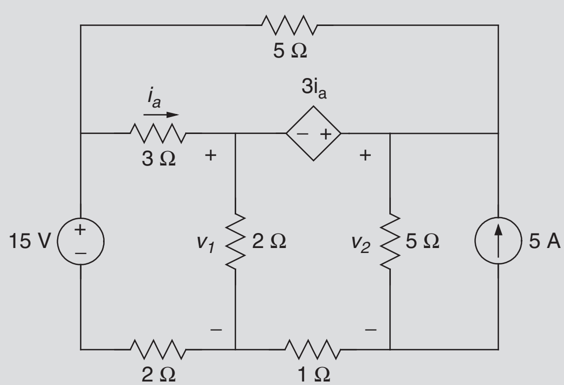
13. Use the node-voltage method to determine v_1 and v_2.



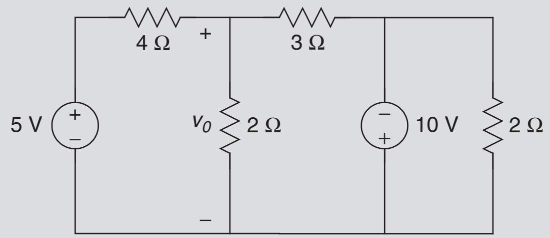
14. Use the node-voltage method to determine v_1 and v_2.



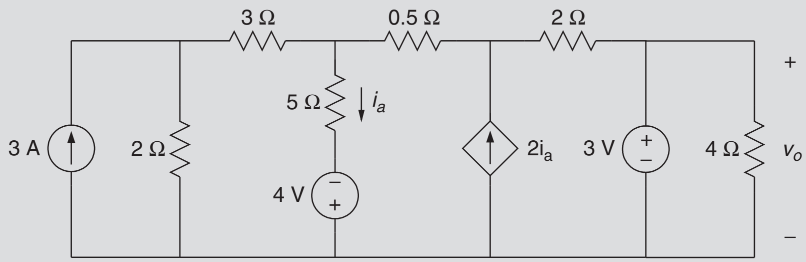
15. Use the node-voltage method to determine v_1 and v_2.



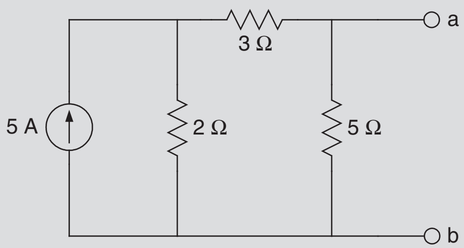
16. Use the superposition method to find v_o.



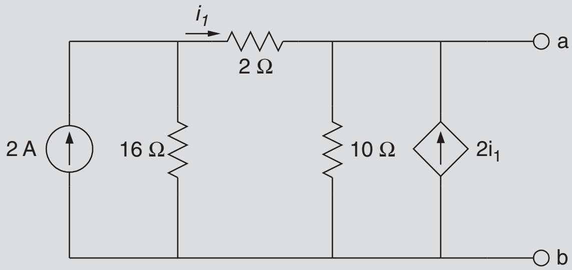
17. Use the superposition method to find v_o.



18. Find the Thévenin equivalent with respect to terminals a and b.



19. Find the Thévenin equivalent with respect to terminals a and b.



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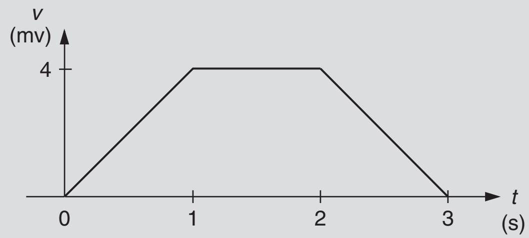
20. A current pulse given by $i(t) = \left(2 + 10e^{-2t}\right)u(t)$ is applied through a 10-mH inductor. (a) Find the voltage across the inductor. (b) Sketch the current and voltage. (c) Find the power as a function of time.

21. The voltage across an inductor is given by the following figure. If $L = 30$ mH and $i(0) = 0$ A, find $i(t)$ for $t \geq 0$.



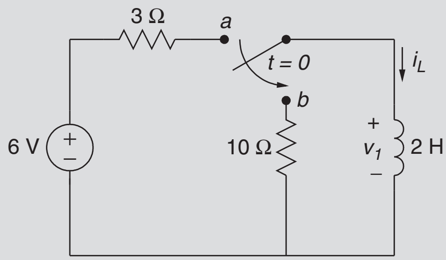
22. The voltage across a 4 μF capacitor is $v(t) = (200{,}000t - 50{,}000)e^{-2000t}u(t)$ V. Find (a) the current through the capacitor, (b) power as a function of time, and (c) energy.

23. The current through a 5 μF capacitor is

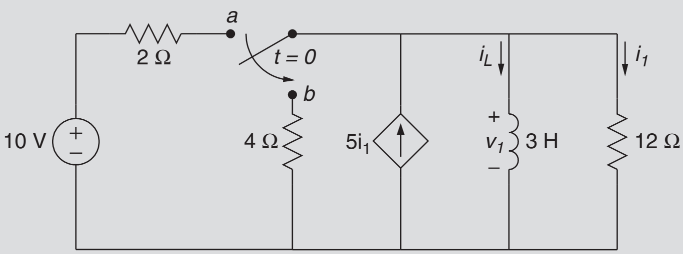
$$i(t) = \begin{cases} 0 \text{ mA} & t < 0 \text{ ms} \\ 5t^2 \text{ mA} & 0 \leq t < 1 \text{ ms} \\ 5\left(2\text{-}t^2\right) \text{ mA} & t \geq 1 \text{ ms} \end{cases}$$

Find the voltage across the capacitor.

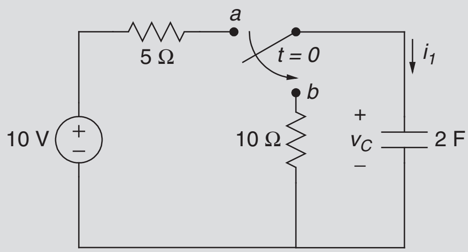
24. The switch has been in position a for a long time. At $t = 0$, the switch instantaneously moves to position b. Find i_L and v_1 for $t > 0$.



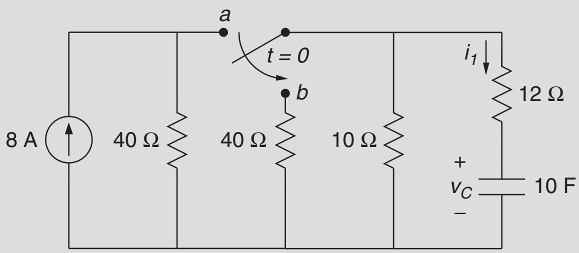
25. The switch has been in position a for a long time. At $t = 0$, the switch instantaneously moves to position b. Find i_L, i_1, and v_1 for $t > 0$.



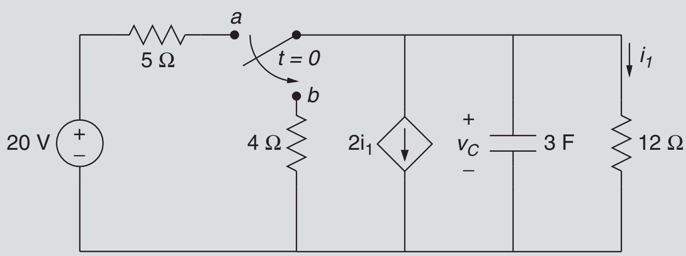
26. The switch has been in position a for a long time. At $t = 0$, the switch instantaneously moves to position b. Find v_c and i_1 for $t > 0$.



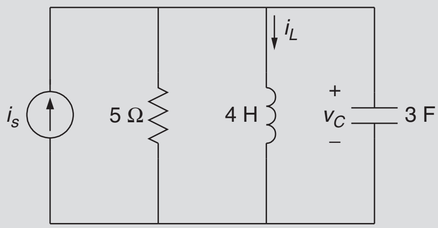
27. The switch has been in position a for a long time. At $t = 0$, the switch instantaneously moves to position b. Find v_c and i_1 for $t > 0$.



28. The switch has been in position a for a long time. At $t = 0$, the switch instantaneously moves to position b. Find v_c and i_1 for $t > 0$.



29. Find i_L and v_c for $t > 0$ for the following circuit if (a) $i_s = 3u(t)$ A; (b) $i_s = 1 + 3u(t)$ A.



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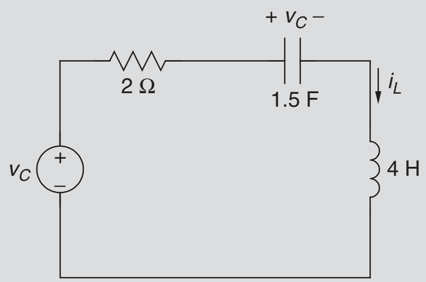
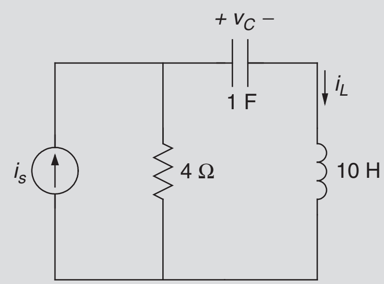
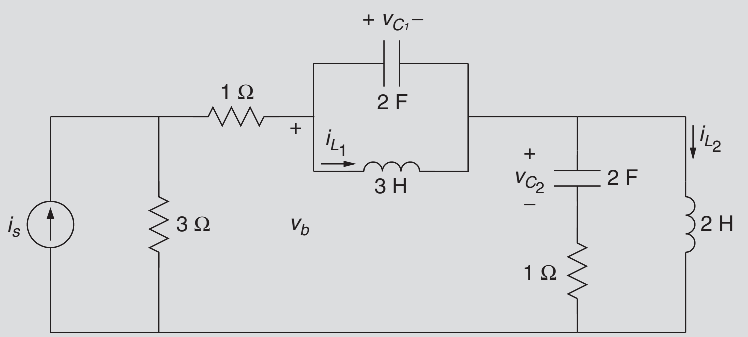
30. Find i_L and v_c for $t > 0$ for the following circuit if (a) $v_s = 5u(t)$ V; (b) $v_s = 5u(t) + 3$ V.



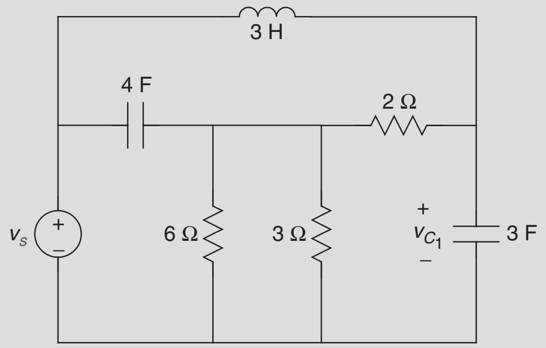
31. Find i_L and v_c for $t > 0$ for the following circuit if (a) $i_s = 3u(t)$ A; (b) $i_s = 3u(t) - 1$ A.



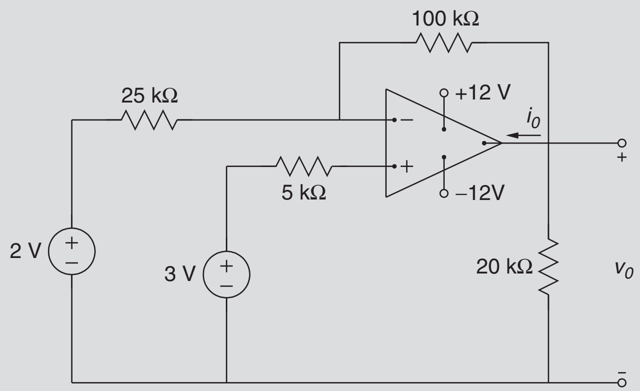
32. For the following circuit we are given that $i_{L_1}(0) = 2$ A, $i_{L_2}(0) = 5$ A, $v_{c_1}(0) = 2$ V, $v_{c_2}(0) = -3$ V, and $i_s = 2e^{-2t}u(t)$ A. Use the node-voltage method to find V_b for $t > 0$.



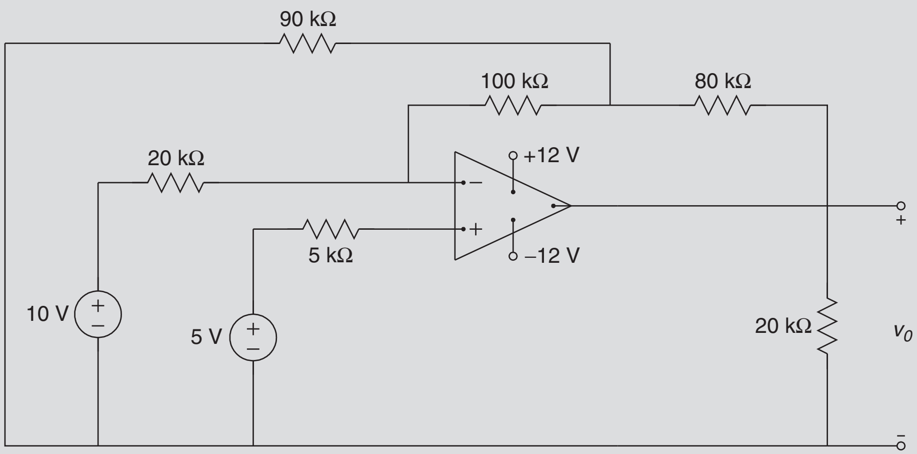
33. Use the node-voltage method to find v_{c_1} for $t > 0$ for the following circuit if (a) $v_s = 2e^{-3t}u(t)$ V; (b) $v_s = 3\cos(2t)u(t)$ V; (c) $v_s = 3u(t) - 1$ V.



34. The operational amplifier shown in the following figure is ideal. Find v_o and i_o.



35. The operational amplifier shown in the following figure is ideal. Find v_o.



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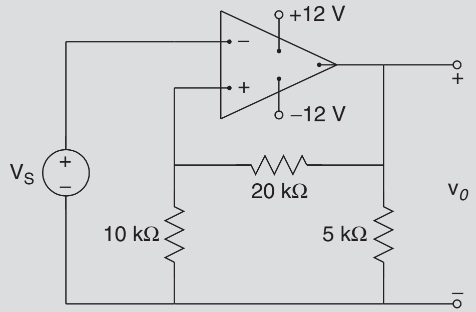
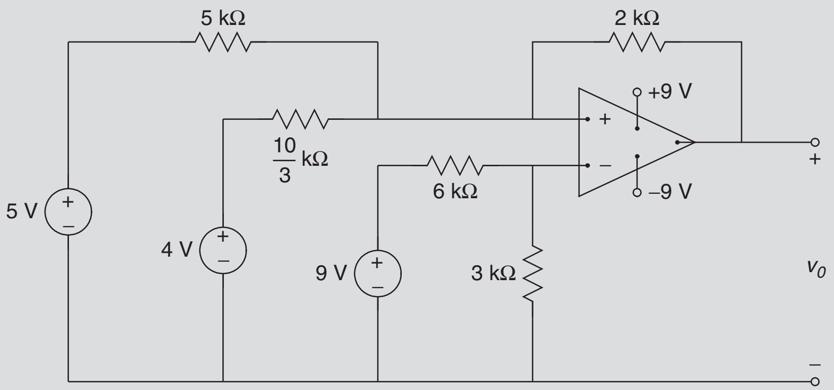
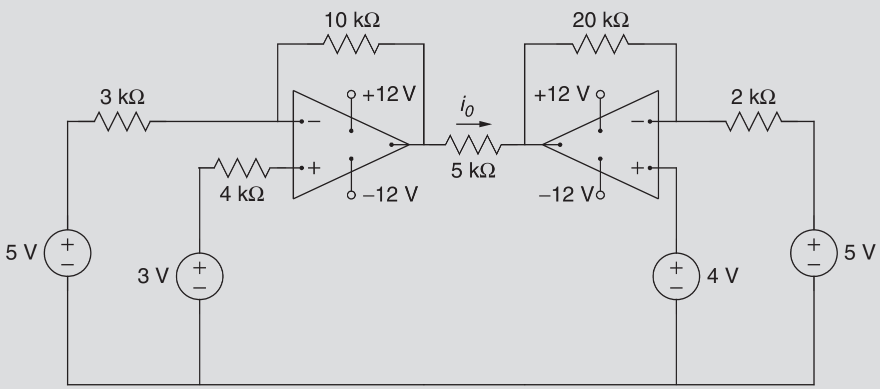
36. Find the overall gain for the following circuit if the operational amplifier is ideal. Draw a graph of v_o versus V_s if V_s varies from 0 to 10 V.



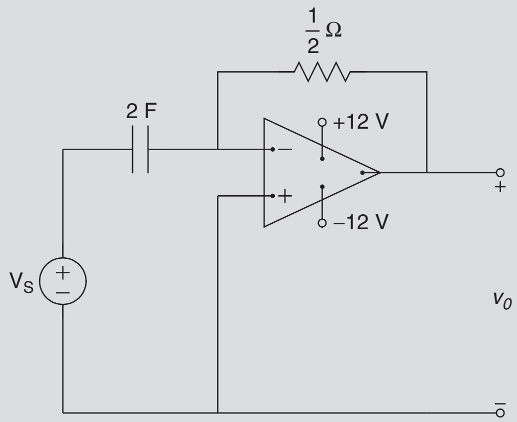
37. Find v_o in the following circuit if the operational amplifier is ideal.

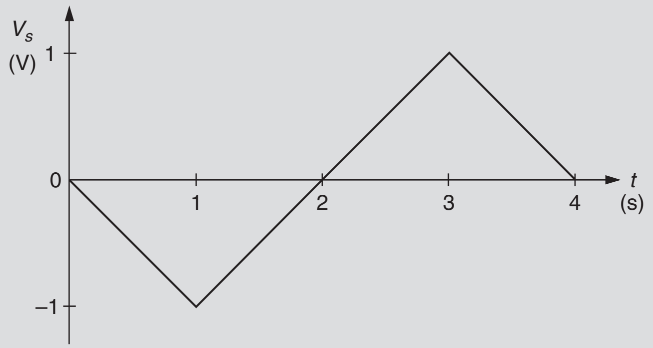


38. Find i_o in the following circuit if the operational amplifiers are ideal.

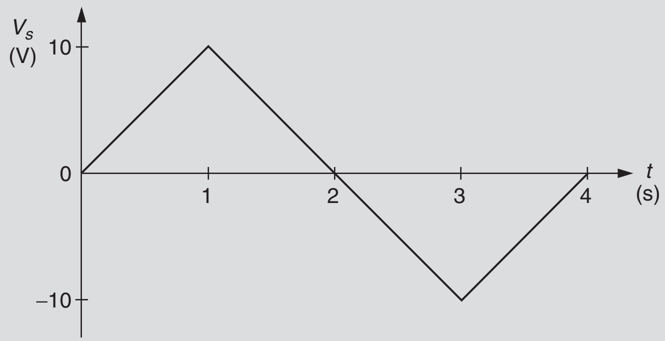


39. Suppose the input V_s is given as a triangular waveform as shown in the following figure. If there is no stored energy in the following circuit with an ideal operational amplifier, find v_o.





40. Suppose the input V_s is given in the following figure. If there is no stored energy in the following circuit with an ideal operational amplifier, find v_o.



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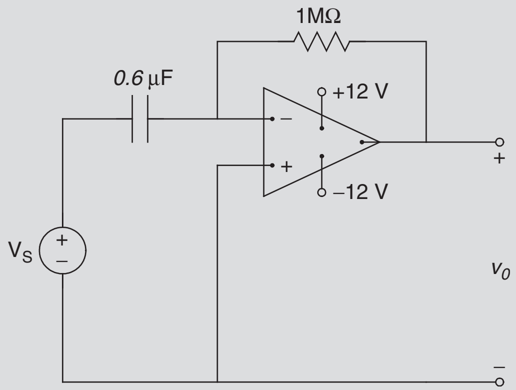
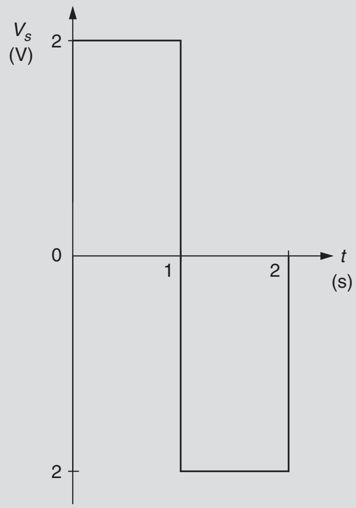
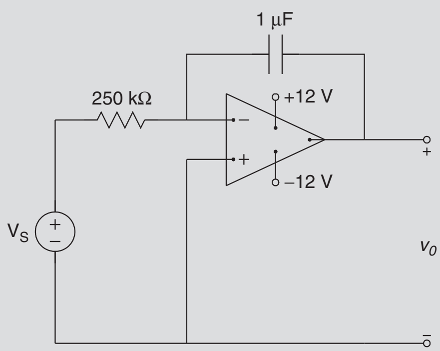
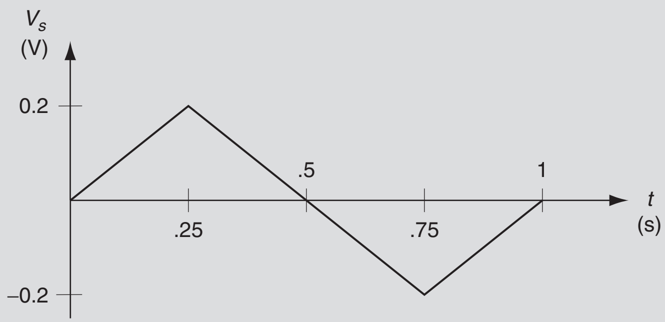


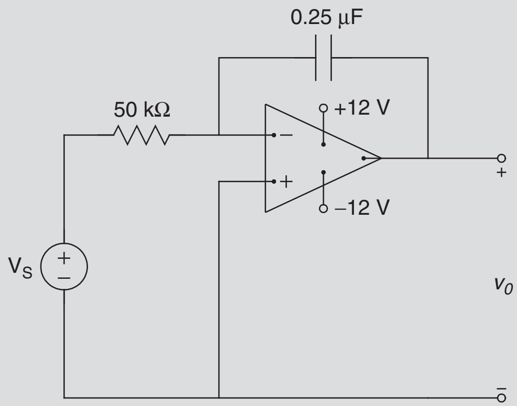
41. Suppose the input V_s is given in the following figure. If there is no stored energy in the following circuit with an ideal operational amplifier, find v_o.



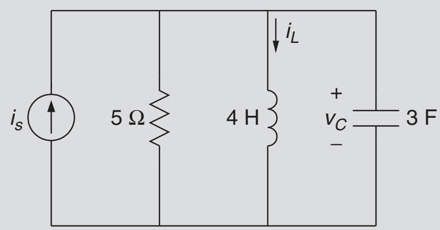


42. Suppose the input V_s is given in the following figure. If there is no stored energy in the following circuit with an ideal operational amplifier, find v_o.





43. The following circuit is operating in the sinusoidal steady state. Find the steady-state expression for i_L if $i_s = 30 \cos 20t$ A.



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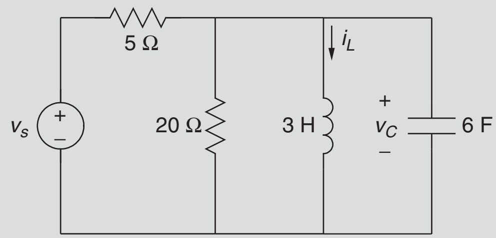
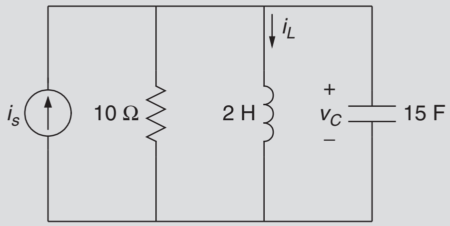
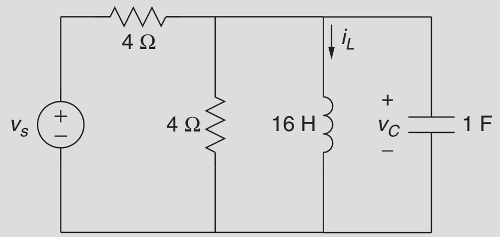
44. The following circuit is operating in the sinusoidal steady state. Find the steady-state expression for v_c if $v_s = 10\sin 1000t$ V.



45. The following circuit is operating in the sinusoidal steady state. Find the steady-state expression for i_L if $i_s = 5\cos 500t$ A.



46. The following circuit is operating in the sinusoidal steady state. Find the steady-state expression for v_c if $i_s = 25\cos 4000t$ V.



47. Design a low-pass filter with a magnitude of 10 and a cutoff frequency of 250 $\frac{\text{rad}}{\text{s}}$.

48. Design a high-pass filter with a magnitude of 20 and a cutoff frequency of 300 $\frac{\text{rad}}{\text{s}}$.

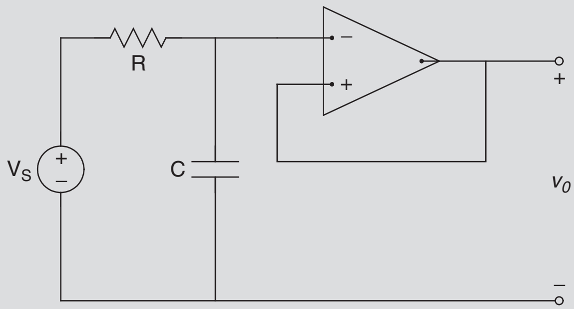
49. Design a band-pass filter with a gain of 15 and passthrough frequencies from 50 to 200 $\frac{\text{rad}}{\text{s}}$.

50. Design a low-pass filter with a magnitude of 5 and a cutoff frequency of 200 $\frac{\text{rad}}{\text{s}}$.

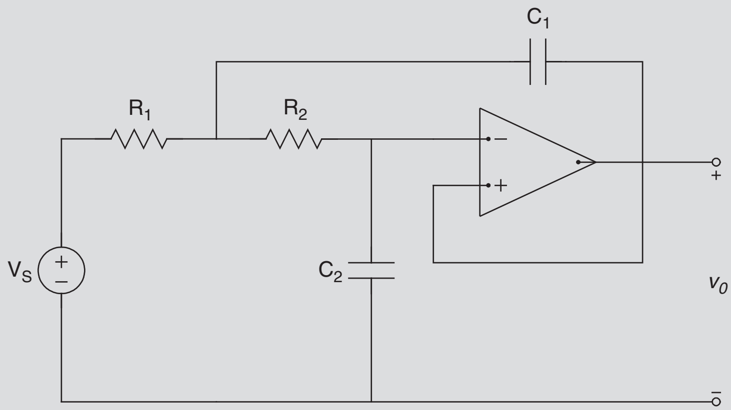
51. Design a high-pass filter with a magnitude of 10 and a cutoff frequency of 500 $\frac{\text{rad}}{\text{s}}$.

52. Design a band-pass filter with a gain of 10 and passthrough frequencies from 20 to 100 $\frac{\text{rad}}{\text{s}}$.

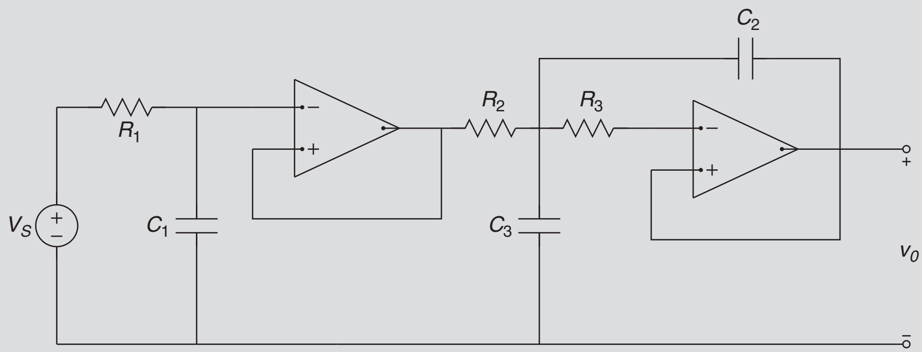
53. Suppose the operational amplifier in the following circuit is ideal. (The circuit is a low-pass first-order Butterworth filter.) Find the magnitude of the output v_o as a function of frequency.



54. With an ideal operational amplifier, the following circuit is a second-order Butterworth low-pass filter. Find the magnitude of the output v_o as a function of frequency.



55. A third-order Butterworth low-pass filter is shown in the following circuit with an ideal operational amplifier. Find the magnitude of the output v_o as a function of frequency.



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