MA 376, Abstract Algebra, Spring 2015
Rigid motions, (a.k.a. Symmetries) of the Square

In front of you is a square, with each of its corners numbered. An action where one picks up the square, moves it in some way (or not), then puts the square back into the space it came from is called a *rigid motion* or *symmetry* of the square. Our first goal is to figure out all the different symmetries of the square.

Let D_4 denote the set of rigid motions we've named above. So,

$$D_4 = \{R_0, R_{90}, R_{180}, R_{270}, V, H, D, D'\}.$$

Now that we have names for all the possible rigid motions of the square, lets see what happens if we start to combine them.

1. What rigid motion corresponds to doing R_{90} twice?

2. What rigid motion corresponds to undoing R_{90}?

3. What rigid motion corresponds to doing H and then R_{90}? We will denote this combination of motions by $R_{90}H$ (remember, this means first do H, then R_{90}.)

4. If you do $(R_{180}V)D$, is that different from doing $R_{180}(VD)$?

Note that if we do any motion and then do R_0, we haven't really changed anything. Likewise, if we first do R_0 and then any motion. This special element which leaves others unchanged is an *identity* element for our binary operation of doing one motion and then another.

To undo R_{90} we can simply do R_{270}. So, $R_{270}R_{90} = R_{90}R_{270} = R_0$. Motions which undo one another are called *inverses*. Note that each element of D_4 has a unique inverse element.

5. Cayley Table (records the combinations of these rigid motions)

	R_0	R_{90}	R_{180}	R_{270}	H	V	D	D'
R_0								
R_{90}		R_{180}			D'			
R_{180}								
R_{270}								
H								
V								
D								
D'								

Note that every entry of the Cayley Table is another element of D_4. This means that D_4 is *closed* under the binary operation of combining motions. The set of rigid motions of the square D_4, together with the operation of doing one motion after another, forms what is called a *group*. In particular, it is called the *dihedral group of size 8*, or the *group of symmetries of the square.* The special properties we discussed above (closure, identity, inverses, associativity) are the required properties of a group.

6. Let x denote an unknown member of D_4. Solve the equation $R_{270}x = D$. What is the "physical" interpretation of this question?

7. Solve $xR_{270} = D$. Compare to your answer for #6.

In general, for a regular polygon with n sides, the group of symmetries of that polygon is denoted D_n and is called the *dihedral group of size* $2n$. (Can you see why a regular n-gon will have $2n$ distinct symmetries?)

Homework:
1. Solve the following equations for x in D_4.
(a) $xR_{90} = R_{270}$ (b) $Vx = H$ (c) $Vx = D'$ (d) $xV = D'$.

2. Consider D_3, the group of symmetries of an equilateral triangle. Come up with your own names for the the 6 members of D_3 and write a Cayley Table for D_3.

References:

Chapter 7 in your text, *A Book of Abstract Algebra*, by Charles C. Pinter

Gallian, Joseph A., *Contemporary Abstract Algebra.* 4th Ed. Houghton Mifflin, 1998.