(2) $a \sim b$ in $\mathbb{R}$ if $a - b \in \mathbb{Z}$

Reflexive:

WTS: $\forall a \in \mathbb{R}$, $a \sim a$

i.e. $\forall a \in \mathbb{R}$, $a - a \in \mathbb{Z}$

Let $a \in \mathbb{R}$. Then $a - a = 0 \in \mathbb{Z}$.

Symmetric.

WTS: $\forall a, b \in \mathbb{R}$, $a \sim b \rightarrow b \sim a$

i.e. $\forall a, b \in \mathbb{R}$, if $a - b \in \mathbb{Z}$ then $b - a \in \mathbb{Z}$

Let $a, b \in \mathbb{R}$ such that $a - b \in \mathbb{Z}$.

Then there is an integer t such that $a - b = t$.

So $b - a = -(a - b) = -t$ is still an integer.

Transitive:

WTS: $\forall a, b, c \in \mathbb{R}$, $a \sim b \wedge b \sim c \rightarrow a \sim c$

i.e. $\forall a, b, c \in \mathbb{R}$, if $a - b \in \mathbb{Z}$ and $b - c \in \mathbb{Z}$, then $a - c \in \mathbb{Z}$.

Let $a, b, c \in \mathbb{R}$ such that $a - b \in \mathbb{Z}$ and $b - c \in \mathbb{Z}$.

Then $a - b = t$ and $b - c = s$ for some $r, s \in \mathbb{Z}$.

Adding both sides of these two equations, we get

$$(a - b) + (b - c) = t + s$$

$$a - c = t + s.$$

~~But $t+s \in \mathbb{R}$~~, So $a - c \in \mathbb{Z}$ since t and s are integers.

③ $a \sim b$ in $\mathbb{Z}$ if $3a+b$ is a multiple of 4

Reflexive:

WTS: $\forall a \in \mathbb{Z}$, $a \sim a$

i.e. $\forall a \in \mathbb{Z}$, $3a+a$ is a multiple of 4

Let $a \in \mathbb{Z}$. Then $3a+a = 4a$ is a multiple of 4.

Symmetric:

WTS: $\forall a, b \in \mathbb{Z}$, $a \sim b \rightarrow b \sim a$

i.e. $\forall a, b \in \mathbb{Z}$, if $3a+b$ is a multiple of 4, then $3b+a$ is a multiple of 4.

Let $a, b \in \mathbb{Z}$ such that $3a+b$ is a multiple of 4.

So $3a+b = 4k$ for some integer k. By applying algebraic techniques we have

$$3a+b = 4k$$
$$3(3a+b) = 3(4k)$$
$$9a+3b = 12k$$
$$8a+(a+3b) = 12k$$
$$a+3b = 12k-8a = 4(3k-2a).$$

Hence $3b+a$ is a multiple of 4.

Transitive:

WTS: $\forall a, b, c \in \mathbb{Z}$, $a \sim b \wedge b \sim c \rightarrow a \sim c$

i.e. $\forall a, b, c \in \mathbb{Z}$, if $3a+b \in 4\mathbb{Z}$ and $3b+c \in 4\mathbb{Z}$, then $3a+c \in 4\mathbb{Z}$.

Let $a, b, c \in \mathbb{Z}$ such that $3a+b \in 4\mathbb{Z}$ and $3b+c \in 4\mathbb{Z}$.

Thus $3a+b = 4t$ and $3b+c = 4s$ for some $t, s \in \mathbb{Z}$.

Adding both sides of the two equations, we get

$$(3a+b)+(3b+c) = 4t+4s$$
$$3a+4b+c = 4(t+s)$$
$$3a+c = 4(t+s-b).$$

Hence $3a+c$ is a multiple of 4.