Relations

1. Fill in the following table.

   Let $\mathcal{L}$ be the set of lines in the coordinate plane; let $\mathcal{S}$ be the set of nonempty sets.

Set	Relation	Reflexive	Symmetric	Transitive	Equivalence Relation?
$\mathbb{R}$	$x = y$				
$\mathbb{Q}$	$x < y$				
$\mathbb{Z}$	$x \geq y$				
$\mathcal{S}$	$A \subset B$				
$\mathcal{S}$	$A \cap B \neq \emptyset$				
$\mathcal{L}$	$\ell_1 \| \ell_2$				
$\mathcal{L}$	$\ell_1 \perp \ell_2$				
$\mathbb{Z}$	$a - b$ is a mult. of 3				

2. For which integers $x \in [-10,\ 10]$ is $x - 0$ a multiple of 3?

3. For which integers $x \in [-10,\ 10]$ is $x - 1$ a multiple of 3?

4. For which integers $x \in [-10,\ 10]$ is $x - 2$ a multiple of 3?

   The congruence modulus n relation is defined by $x \equiv a \mod n$ if and only if $x - a$ is a multiple of n. The equivalence class for the class representative $a \in \mathbb{Z}$ for this relation is the set

$$[a] = \{x \in \mathbb{Z} \mid x \equiv a \mod n\} = \{x \in \mathbb{Z} \mid x - a \text{ is a multiple of } n\}.$$

5. Write at least 7 elements in each equivalence class; these sets are infinite, so only test integers x where $-10 \leq x \leq 10$. Which equivalence classes seem to be equal? What do you notice about their class representatives?

   (a) $[0] = \{x \in \mathbb{Z} \mid x \equiv 0 \mod 3\}$:

   (b) $[1] = \{x \in \mathbb{Z} \mid x \equiv 1 \mod 3\}$:

   (c) $[2]$:

   (d) $[3]$:

   (e) $[4]$: