MA303 Set Theory and Logic - Matrix Worksheet

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Part I: First example

$$A = \begin{pmatrix} 3 & 2 \\ -2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 8 \\ 1 & -4 \end{pmatrix}$$

1. Find $\det A$ and $\det B$.
2. Find A^{-1} and B^{-1}.
3. Find $\det A^{-1}$ and $\det B^{-1}$.
4. Compare $\det(A)$ and $\det(A^{-1})$.
5. Compare AB and BA.
6. Compare $\det(A)$ and $\det(B)$ and $\det(AB)$.
7. Compare $(AB)^{-1}$ and $A^{-1}B^{-1}$ and $B^{-1}A^{-1}$.
8. Compare A^T and $(A^T)^T$.
9. Compare $(A^T)^{-1}$ and $(A^{-1})^T$.

Part I: Second example

$$A = \begin{pmatrix} 3 & 1 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & -1 & 3 \\ -2 & 0 & 4 \\ 1 & 0 & 0 \end{pmatrix}$$

10. Find $\det A$ and $\det B$.
11. Compare AB and BA.
12. Compare $\det(A)$ and $\det(B)$ and $\det(AB)$.
13. Compare $(AB)^{-1}$ and $A^{-1}B^{-1}$ and $B^{-1}A^{-1}$.
14. Compare A^T and $(A^T)^T$.

Part I: Conjectures

15. For any two matrices A and B, what can we say about AB and BA?

16. For any matrix A, what must be true of the determinant in order for A^{-1} to exist?

17. What is the relationship between $\det(A)$ and $\det(A^{-1})$?

18. What is equivalent to $(AB)^{-1}$?

19. What is the relationship between $\det(A)$ and $\det(B)$ and $\det(AB)$?

20. What is the relationship between A^T and $(A^T)^T$?

21. What is the relationship between $(A^T)^{-1}$ and $(A^{-1})^T$?

Part II: First example

$$C = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 2 \\ 6 & 3 \end{pmatrix}$$

22. Find $\det C$ and $\det D$.

23. Find C^{-1} and D^{-1}.

24. Find CD and DC.

25. Compare $(CD)^T$ and $C^T D^T$ and $D^T C^T$.

Part II: Second example

$$C = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 0 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

26. Find $\det C$ and $\det D$.

27. Find CD and DC.

28. Compare $(CD)^T$ and $C^T D^T$ and $D^T C^T$.

Part II: Conjectures

29. For any matrix A, what must be true of its determinant in order for A^{-1} to exist?

30. For any matrix A, what must B look like if $AB = BA$?

31. Is $\det(AB) = \det(BA)$?

32. Simplify $\det(ABA^{-1})$.

33. What is $(AB)^T$ equivalent to?