Relations

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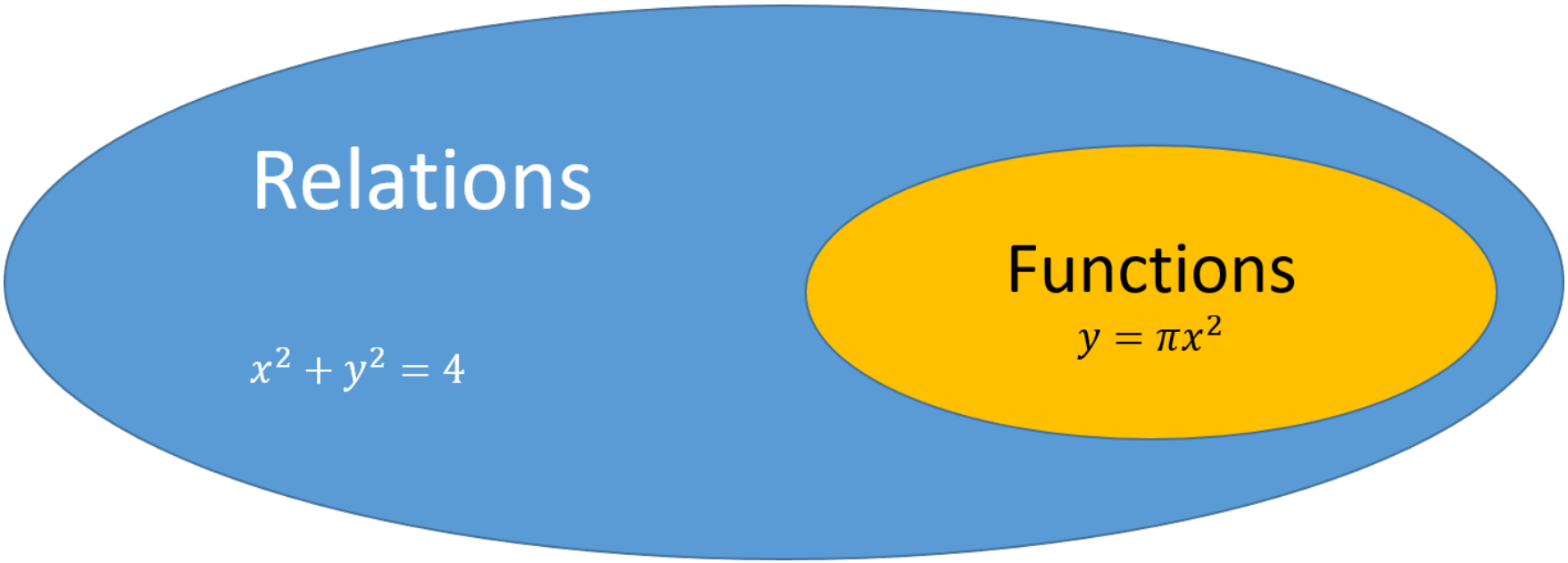

Relations
$x^2 + y^2 = 4$
Functions
$y = \pi x^2$

Relations

$$R \subset A \times B$$

Equivalent:

- a is related to b
- $(a, b) \in R$
- aRb

Domain: $\text{dom}(R) = \{x \in A \mid (x, y) \in R\}$
Range: $\text{ran}(R) = \{y \in B \mid (x, y) \in R\}$

Relations - Alternate notation

$$\sim \subset A \times B$$

Equivalent:

- a is related to b
- $(a, b) \in \sim$
- $a \sim b$

Equivalent:

- a is not related to b
- $(a, b) \notin \sim$
- $a \nsim b$

How are relations defined?

Example

Relation on $\mathbb{Z}$:

$$\sim \ = \{(3,4),\ (-2,4),(1,5),(1,4)\}$$

$\text{dom}(\sim) = \{3,-2,1\}$
$\text{ran}(\sim) = \{4,5\}$

Example

$\sim$ = has the same birthday month as

A = persons

If persons x and y were both born in October, then $x \sim y$.

Example

$\sim$ = is half of
$A = \mathbb{R}$

$$\begin{aligned} \sim &= \{(1,2),(2,4),(\pi,2\pi),(-1.6,-3.2),\ldots\} \\ &= \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid x = y/2\} \\ &= \{(x,y) \in \mathbb{R} \times \mathbb{R} \mid y = 2x\} \\ &= \{(x,2x) \mid x \in \mathbb{R}\} \end{aligned}$$

Note this represents the function $f(x) = 2x$.

Relation Properties

Relation $\sim$ on a set A

- Reflexive: $\forall a \in A$,

$$a \sim a$$

- Symmetric: $\forall a, b \in A$,

$$a \sim b \implies b \sim a$$

- Transitive: $\forall a, b, c \in A$,

$$a \sim b \text{ and } b \sim c \implies a \sim c$$

Note: Other properties exist.

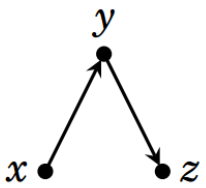
A relation is **reflexive** if for each point x ...

...there is a loop at x:

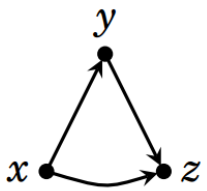
A relation is **symmetric** if whenever there is an arrow from x to y ...

...there is also an arrow from y back to x:

A relation is **transitive** if whenever there are arrows from x to y and y to z ...



...there is also an arrow from x to z:



(If $x = z$, this means that if there are arrows from x to y and from y to x ...

...there is also a loop from x back to x.)

Relations on $\mathbb{Z}$:	$<$	$\leq$	$=$	$\mid$	$\nmid$	$\neq$
Reflexive	no	yes	yes	yes	no	no
Symmetric	no	no	yes	no	no	yes
Transitive	yes	yes	yes	yes	no	no