

## Chapter 0

(Judson)

### Functions

1. Be familiar with the terminology and symbols of functions.
2. Determine if a proposed function is well-defined or not.
3. Know how to take the composition of functions.
4. Given a defined function, determine its domain, codomain, and image.
5. Given a subset  $S$  of the domain, find its image set,  $f(S)$ .
6. Given a subset  $T$  of the image, find its preimage set,  $f^{-1}(T)$ .
7. Prove whether or not a function is injective or surjective.
8. If a defined function is bijective, determine its inverse.

### Relations

1. Know definitions: relation, equivalence relation, equivalence class, congruence mod  $n$ , divides, divisor, multiple.
2. List elements in a relation; list elements not in a relation.
3. Determine if a relation satisfies the reflexive property.
4. Determine if a relation satisfies the symmetric property.
5. Determine if a relation satisfies the transitive property.
6. Determine if a relation is an equivalence relation.
7. Determine equivalence classes of a given relation.

## Congruence

1. Memorize the proof that congruence mod  $n$  is an equivalence relation.
2. Compute modulus operations.
  - $16 \bmod 2 \equiv ?$
  - $31 \bmod 4 \equiv ?$
  - $50 \bmod 19 \equiv ?$
3. Understand the equivalence classes for the congruence mod  $n$  relation.
  - In  $\mathbb{Z}_7$ , define  $\bar{3}$ .
  - In  $\mathbb{Z}_{10}$ , is  $30 \in \bar{2}$ ? Is  $32 \in \bar{2}$ ?
  - List all the distinct equivalence classes for  $\mathbb{Z}_8$ .
  - In  $\mathbb{Z}_9$ , is  $\overline{-10} = \overline{80}$ ?
4. Know the elements of  $\mathbb{Z}_n$ , the integers modulo  $n$ . Perform operations (addition, subtraction, multiplication) in  $\mathbb{Z}_n$ .

*The following chapter references are from the Pinter textbook. Remember that some notes are in my slides. For more references, Judson covers some of these same topics.*

## Chapter 2: Operations

1. Be familiar with the terminology and symbols in this section. Know the definitions.
2. Given an operation and a set, determine if it has closure and associativity, and if it satisfies the identity and inverse properties.
3. Know how to complete an operation table.
4. Be able to determine certain facts about closure, the identity and inverses by looking at an operation table.
5. Favorite sets: All the common sets of numbers and the common sets without 0,  $\mathbb{Z}_n$ ,  $U(n)$ ,  $\mathbb{Z}_n \times \mathbb{Z}_m$ ,  $M_2(\mathbb{R})$ .
6. Know when two numbers are relatively prime. Use that to find elements of  $U(n)$ .
7. Evaluate the Euler totient function  $\phi(n)$  for  $n \geq 2$ .

8. Solve linear congruence equations of the form  $\bar{a}x = \bar{b}$  using the inverse of  $\bar{a}$ .  
Solve this linear congruence mod 10.

$$\begin{aligned}\bar{7}x &= \bar{9} \\ \bar{7}^{-1}\bar{7}x &= \bar{7}^{-1}\bar{9} \\ \bar{3} \cdot \bar{7}x &= \bar{3}^{-1} \cdot \bar{9} \\ \bar{21}x &= \bar{27} \\ \bar{1}x &= \bar{7} \\ x &= \bar{7}\end{aligned}$$

## Chapter 3: Groups

1. Be familiar with the terminology and symbols in this section.
2. Determine if a given set with an operation is a group.
3. Determine if a given set with an operation is an abelian group.
4. Know how to complete an Cayley table (i.e. the operation table for a group.)
5. Be able to determine certain facts about the identity and inverses and commutativity looking at a Cayley table.
6. Know about permutations (see slides on Top Hat.)

## Chapter 4: Elementary Properties of Groups

1. Be familiar with the terminology and symbols in this section.
2. Know how to prove the identity of a group is unique.
3. Know how to prove the inverse for a given element is unique.
4. Know and understand Theorems 1, 2, and 3 in Chapter 4.
5. Know what is meant by multiplicative notation versus additive notation. Know the exponential laws of multiplying elements in a group.
6. Know how to show a group is abelian.
7. Understand the following as groups with their respective operations:  $\mathbb{Z}_n$ ,  $U(n)$ ,  $\mathbb{Z}_n \times \mathbb{Z}_m$ ,  $GL_2(\mathbb{R})$ ,  $SL_2(\mathbb{R})$ ,  $S_n$ .
8. Understand the structure of  $\mathbb{Z}_n \times \mathbb{Z}_m$  where  $n, m \geq 2$ .

## Chapter 5: Subgroups

1. Be familiar with the terminology and symbols in this section.
2. Know how to determine whether a given subset is a subgroup or not.

## Matrices

1. Know how to do matrix operations: addition, multiplication by a scalar, multiply two matrices.
2. Find the determinant and inverse of a  $2 \times 2$  matrix.
3. Understand when matrix multiplication is valid.
4. Know the identity matrix for matrix addition. Know the identity matrix for matrix multiplication.