

Examples of Set-Builder Notation

$$\begin{aligned} A &= \{a^0, a^1, \dots, a^{100}\} \\ &= \{a^n \mid n = 0, 1, \dots, 100\} \\ &= \{a^{n-1} \mid n = 1, 2, \dots, 101\} \end{aligned}$$

$$\begin{aligned} B &= \{y_1, y_2, \dots, y_n\} \\ &= \{y_j \mid j = 1, 2, \dots, n\} \\ &= \{y_j\}_{j=1}^n \end{aligned}$$

$$\begin{aligned} C &= \{x_1, x_2, x_3, \dots\} \\ &= \{x_i \mid i \in \mathbb{N}\} \\ &= \{x_i\}_{i \in \mathbb{N}} \end{aligned}$$

$$\begin{aligned} D &= \{A^0B^4, A^1B^3, A^2B^2, A^3B^1, A^4B^0\} \\ &= \{A^iB^j \mid i, j \in \mathbb{Z}_{\geq 0}, i + j = 4\} \\ &= \{A^iB^{4-i} \mid 1 \leq i \leq 4\} \end{aligned}$$

$$\begin{aligned} E &= \{a_{11}, a_{12}, a_{13}, a_{14}, a_{21}, a_{22}, a_{23}, a_{24}\} \\ &= \{a_{ij} \mid 1 \leq i \leq 2, 1 \leq j \leq 4\} \\ &= \{a_{ij} \mid i, j \in \mathbb{N}, i \leq 2, j \leq 4\} \end{aligned}$$

$$\begin{aligned} F &= \{\dots, (-2, -1, -3), (-1, 0, -1), \dots, (3, 4, 7), (4, 5, 9), \dots\} \\ &= \{(x, y, z) \in \mathbb{Z}^3 \mid x \in \mathbb{Z}, y = x + 1, z = x + y\} \\ &= \{(x, x + 1, 2x + 1) \mid x \in \mathbb{Z}\} \end{aligned}$$

$$\begin{aligned} G &= \{x_1, x_1 + x_3, x_1 + x_3 + x_5, \dots\} \\ &= \left\{ \sum_{i=1}^n x_{2i-1} \mid n \in \mathbb{N} \right\} \end{aligned}$$

$$\begin{aligned}
H &= \{-1, 1, -1, 1, -1, 1, \dots\} \\
&= \{(-1)^j \mid j \in \mathbb{N}\}
\end{aligned}$$

$$\begin{aligned}
I &= \{1, -1, 1, -1, 1, -1, \dots\} \\
&= \{(-1)^{j+1} \mid j \in \mathbb{N}\}
\end{aligned}$$

$$\begin{aligned}
J &= \{-x, x^2, -x^3, x^4, -x^5, \dots\} \\
&= \{(-1)^k x^k \mid k \in \mathbb{N}\} \\
&= \{(-x)^k \mid k \in \mathbb{N}\}
\end{aligned}$$

$$\begin{aligned}
K &= \{-1, x, -x^2, x^3, -x^4, x^5, \dots\} \\
&= \{(-1)^k x^{k-1} \mid k \in \mathbb{N}\}
\end{aligned}$$

$$\begin{aligned}
L &= \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \right\} \\
&= \left\{ \frac{1}{2^k} \mid k \in \mathbb{N} \right\}
\end{aligned}$$

$$\begin{aligned}
M &= \left\{ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots \right\} \\
&= \left\{ \frac{1}{2^{k-1}} \mid k \in \mathbb{N} \right\}
\end{aligned}$$

$$\begin{aligned}
N &= \left\{ 1, 1 + \frac{1}{2}, 1 + \frac{1}{2} + \frac{1}{4}, 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}, \dots \right\} \\
&= \left\{ \sum_{k=1}^1 \frac{1}{2^{k-1}}, \sum_{k=1}^2 \frac{1}{2^{k-1}}, \sum_{k=1}^3 \frac{1}{2^{k-1}}, \sum_{k=1}^4 \frac{1}{2^{k-1}}, \dots \right\} \\
&= \left\{ \sum_{k=1}^n \frac{1}{2^{k-1}} \mid n \in \mathbb{N} \right\}
\end{aligned}$$

$$\begin{aligned}
O &= \{1, 2, 6, 24, 120, 720, \dots\} \\
&= \{1 \cdot 2 \cdot \dots \cdot (n-1) \cdot n \mid n \in \mathbb{N}\} \\
&= \left\{ \prod_{i=1}^1 i, \prod_{i=1}^2 i, \prod_{i=1}^3 i, \prod_{i=1}^4 i, \prod_{i=1}^5 i, \prod_{i=1}^6 i, \dots \right\} \\
&= \left\{ \prod_{i=1}^n i \mid n \in \mathbb{N} \right\}
\end{aligned}$$

$$\begin{aligned}
P &= \{x_1, x_1x_2, x_1x_2x_3, x_1x_2x_3x_4, \dots\} \\
&= \left\{ \prod_{j=1}^1 x_j, \prod_{j=1}^2 x_j, \prod_{j=1}^3 x_j, \prod_{j=1}^4 x_j, \dots \right\} \\
&= \left\{ \prod_{j=1}^m x_j \mid m \in \mathbb{Z}_{>0} \right\}
\end{aligned}$$

$$\begin{aligned}
Q &= \left\{ \dots, \frac{-5\pi}{2}, \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots \right\} \\
&= \left\{ \frac{(2k-1)\pi}{2} \mid k \in \mathbb{Z} \right\}
\end{aligned}$$

$$\begin{aligned}
R &= \{a_{12}, a_{13}, a_{21}, a_{23}, a_{31}, a_{32}\} \\
&= \{a_{ij} \mid i, j \leq 3, i \neq j\}
\end{aligned}$$

$$\begin{aligned}
S &= \{\dots, (-2, -1, 0), (-1, 0, 1), \dots, (3, 4, 5), (4, 5, 6), \dots\} \\
&= \{(x, y, z) \in \mathbb{Z}^3 \mid x \in \mathbb{Z}, y = x + 1, z = y + 1\} \\
&= \{(x, x + 1, x + 2) \mid x \in \mathbb{Z}\}
\end{aligned}$$