Examples of Set-Builder Notation

$$A = \{a^0, \ a^1, \ldots, \ a^{100}\}$$
$$= \{a^n \mid n = 0, 1, \ldots, 100\}$$
$$= \{a^{n-1} \mid n = 1, 2, \ldots, 101\}$$

$$B = \{y_1, \ y_2, \ldots, \ y_n\}$$
$$= \{y_j \mid j = 1, 2, \ldots, n\}$$
$$= \{y_j\}_{j=1}^n$$

$$C = \{x_1, \ x_2, \ x_3, \ldots\}$$
$$= \{x_i \mid i \in \mathbb{N}\}$$
$$= \{x_i\}_{i \in \mathbb{N}}$$

$$D = \{A^0 B^4, \ A^1 B^3, \ A^2 B^2, \ A^3 B^1, \ A^4 B^0\}$$
$$= \{A^i B^j \mid i, j \in \mathbb{Z}_{\geq 0}, \ i + j = 4\}$$
$$= \{A^i B^{4-i} \mid 1 \leq i \leq 4\}$$

$$E = \{a_{11}, \ a_{12}, \ a_{13}, \ a_{14}, \ a_{21}, \ a_{22}, \ a_{23}, \ a_{24}\}$$
$$= \{a_{ij} \mid 1 \leq i \leq 2, 1 \leq j \leq 4\}$$
$$= \{a_{ij} \mid i, j \in \mathbb{N}, \ i \leq 2, j \leq 4\}$$

$$F = \{\ldots, \ (-2, -1, -3), \ (-1, 0, -1), \ldots, \ (3, 4, 7), \ (4, 5, 9), \ldots\}$$
$$= \{(x, y, z) \in \mathbb{Z}^3 \mid x \in \mathbb{Z}, \ y = x + 1, \ z = x + y\}$$
$$= \{(x, x + 1, 2x + 1) \mid x \in \mathbb{Z}\}$$

$$G = \{x_1, \ x_1 + x_3, \ x_1 + x_3 + x_5, \ldots\}$$
$$= \left\{ \sum_{i=1}^n x_{2i-1} \mid n \in \mathbb{N} \right\}$$

$$H = \{-1,\ 1,\ -1,\ 1,\ -1,\ 1, \ldots\}$$
$$= \{(-1)^j \mid j \in \mathbb{N}\}$$

$$I = \{1,\ -1,\ 1,\ -1,\ 1,\ -1, \ldots\}$$
$$= \{(-1)^{j+1} \mid j \in \mathbb{N}\}$$

$$J = \{-x,\ x^2,\ -x^3,\ x^4,\ -x^5, \ldots\}$$
$$= \{(-1)^k x^k \mid k \in \mathbb{N}\}$$
$$= \{(-x)^k \mid k \in \mathbb{N}\}$$

$$K = \{-1,\ x,\ -x^2,\ x^3,\ -x^4,\ x^5, \ldots\}$$
$$= \{(-1)^k x^{k-1} \mid k \in \mathbb{N}\}$$

$$L = \left\{\frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}, \ldots\right\}$$
$$= \left\{\frac{1}{2^k} \mid k \in \mathbb{N}\right\}$$

$$M = \left\{1,\ \frac{1}{2},\ \frac{1}{4},\ \frac{1}{8}, \ldots\right\}$$
$$= \left\{\frac{1}{2^{k-1}} \mid k \in \mathbb{N}\right\}$$

$$N = \left\{1,\ 1+\frac{1}{2},\ 1+\frac{1}{2}+\frac{1}{4},\ 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \ldots\right\}$$
$$= \left\{\sum_{k=1}^{1}\frac{1}{2^{k-1}},\ \sum_{k=1}^{2}\frac{1}{2^{k-1}},\ \sum_{k=1}^{3}\frac{1}{2^{k-1}},\ \sum_{k=1}^{4}\frac{1}{2^{k-1}}, \ldots\right\}$$
$$= \left\{\sum_{k=1}^{n}\frac{1}{2^{k-1}} \mid n \in \mathbb{N}\right\}$$

$$O = \{1,\ 2,\ 6,\ 24,\ 120,\ 720, \ldots\}$$
$$= \{1 \cdot 2 \cdot \cdots \cdot (n-1) \cdot n \mid n \in \mathbb{N}\}$$
$$= \left\{ \prod_{i=1}^{1} i,\ \prod_{i=1}^{2} i,\ \prod_{i=1}^{3} i,\ \prod_{i=1}^{4} i,\ \prod_{i=1}^{5} i,\ \prod_{i=1}^{6} i, \ldots \right\}$$
$$= \left\{ \prod_{i=1}^{n} i \mid n \in \mathbb{N} \right\}$$

$$P = \{x_1,\ x_1 x_2,\ x_1 x_2 x_3,\ x_1 x_2 x_3 x_4, \ldots\}$$
$$= \left\{ \prod_{j=1}^{1} x_j,\ \prod_{j=1}^{2} x_j,\ \prod_{j=1}^{3} x_j,\ \prod_{j=1}^{4} x_j, \ldots \right\}$$
$$= \left\{ \prod_{j=1}^{m} x_j \mid m \in \mathbb{Z}_{>0} \right\}$$

$$Q = \left\{ \ldots,\ \frac{-5\pi}{2},\ \frac{-3\pi}{2},\ \frac{-\pi}{2},\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{5\pi}{2},\ \frac{7\pi}{2}, \ldots \right\}$$
$$= \left\{ \frac{(2k-1)\pi}{2} \mid k \in \mathbb{Z} \right\}$$

$$R = \{a_{12},\ a_{13},\ a_{21},\ a_{23},\ a_{31},\ a_{32}\}$$
$$= \{a_{ij} \mid i, j \leq 3,\ i \neq j\}$$

$$S = \{\ldots,\ (-2, -1, 0),\ (-1, 0, 1), \ldots,\ (3, 4, 5),\ (4, 5, 6), \ldots\}$$
$$= \{(x, y, z) \in \mathbb{Z}^3 \mid x \in \mathbb{Z},\ y = x+1,\ z = y+1\}$$
$$= \{(x, x+1, x+2) \mid x \in \mathbb{Z}\}$$