

Equivalence class of $a \in A$: $[a] = \{x \in A \mid x \sim a\}$

16b

$$[5] = \{x \in \mathbb{R} \mid x \sim 5\}$$

$$= \{x \in \mathbb{R} \mid x - 5 \in \mathbb{Z}\}$$

$$= \{x \in \mathbb{R} \mid x - 5 = t, t \in \mathbb{Z}\}$$

$$= \{x \in \mathbb{R} \mid x = 5 + t, t \in \mathbb{Z}\}$$

$$= \{5 + t \mid t \in \mathbb{Z}\}$$

$$[5.5] = \{x \in \mathbb{R} \mid x \sim 5.5\}$$

$$= \{x \in \mathbb{R} \mid x - 5.5 \in \mathbb{Z}\}$$

$$= \{x \in \mathbb{R} \mid x - 5.5 = t, t \in \mathbb{Z}\}$$

$$= \{x \in \mathbb{R} \mid x = 5.5 + t, t \in \mathbb{Z}\}$$

$$= \{5.5 + t \mid t \in \mathbb{Z}\}$$

General:

$$[a] = \{x \in \mathbb{R} \mid x \sim a\}$$

$$= \{x \in \mathbb{R} \mid x - a = t, t \in \mathbb{Z}\}$$

$$= \{x \in \mathbb{R} \mid x = a + t, t \in \mathbb{Z}\}$$

$$= \{a + t \mid t \in \mathbb{Z}\}$$

25b (Judson)

$$[a] = \{m \in \mathbb{Z} \mid m \sim a\}$$

$$= \{m \in \mathbb{Z} \mid ma > 0\}$$

If $a > 0$, $[a]$ is the set of positive integers.

If $a < 0$, $[a]$ is the set of negative integers.

17b

$$[0] = \{a \in \mathbb{Z} \mid a \sim 0\}$$

$$= \{a \in \mathbb{Z} \mid 3a + 0 \text{ is a multiple of } 4\}$$

$$= \{a \in \mathbb{Z} \mid 3a \text{ is a mult. of } 4\}$$

$$= \{a \in \mathbb{Z} \mid 3a = 4k, k \in \mathbb{Z}\}$$

$$= \{a \in \mathbb{Z} \mid a = 4 \cdot \frac{k}{3}, k \in \mathbb{Z}\}$$

$$= \{\frac{4k}{3} \mid k \text{ is a multiple of } 3\}$$

$$[2] = \{a \in \mathbb{Z} \mid a \sim 2\}$$

$$= \{a \in \mathbb{Z} \mid 3a + 2 \text{ is a mult. of } 4\}$$

$$= \{a \in \mathbb{Z} \mid 3a + 2 = 4k, k \in \mathbb{Z}\}$$

$$= \{a \in \mathbb{Z} \mid a = \frac{4k - 2}{3}, k \in \mathbb{Z}\}$$

$$= \{\frac{4k - 2}{3} \mid 4k - 2 \text{ is a multiple of } 3, k \in \mathbb{Z}\}$$

General:

$$[b] = \{a \in \mathbb{Z} \mid a \sim b\}$$

$$= \{a \in \mathbb{Z} \mid 3a + b = 4k, k \in \mathbb{Z}\}$$

$$= \{a \in \mathbb{Z} \mid a = \frac{4k - b}{3}, k \in \mathbb{Z}\}$$

$$= \{\frac{4k - b}{3} \mid 4k - b \text{ is a multiple of } 3, k \in \mathbb{Z}\}$$