Midi-FreshML Typing Rules

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Γ is a list of (id, type) pairs representing the finite partial function from ids to types.
$\in$ is used to indicate list membership, @ to indicate list appending, and :: to indicate consing.
The empty list is denoted by [].
Σ is the set of all strings (used for string literal rule).

Expression type relation: $\Gamma \vdash e\ :\ t$

$$\text{(int)}\frac{}{\Gamma \vdash n : int}\ \text{n} \in \mathbb{N}$$

$$\text{(real)}\frac{}{\Gamma \vdash r : real}\ \text{r} \in \mathbb{R}$$

$$\text{(bool)}\frac{}{\Gamma \vdash b : bool}\ \text{b} \in \{\text{true, false}\}$$

$$\text{(string)}\frac{}{\Gamma \vdash \text{“}*\text{”} : string}\ * \in \Sigma \setminus \{\text{”}\}$$

$$\text{(unit)}\frac{}{\Gamma \vdash () : unit}$$

$$\text{(id)}\frac{(x,\ t) \in \Gamma}{\Gamma \vdash x : t}$$

$$\text{(fresh)}\frac{N \in \Gamma}{\Gamma \vdash (\text{fresh} : N) : N}$$

$$\text{(ctor)}\frac{(C,\ t \to D) \in \Gamma \quad \Gamma \vdash e : t}{\Gamma \vdash C\ e : D}$$

$$\text{(swap)}\frac{\begin{array}{c}\Gamma \vdash e_1 : N \\ \Gamma \vdash e_2 : N \quad \Gamma \vdash e_3 : t\end{array}}{\Gamma \vdash \text{swap}(e_1,\ e_2)\ \text{in}\ e_3 : t}$$

$$\text{(name abs)}\frac{\Gamma \vdash e_1 : N \quad \Gamma \vdash e_2 : t}{\Gamma \vdash \ll e_1 \gg e_2 : \ll N \gg t}$$

$$\text{(if)}\frac{\begin{array}{c}\Gamma \vdash e_1 : bool \\ \Gamma \vdash e_2 : t \quad \Gamma \vdash e_3 : t\end{array}}{\Gamma \vdash \text{if}\ e_1\ \text{then}\ e_2\ \text{else}\ e_3 : t}$$

$$\text{(pair)}\frac{\Gamma \vdash e_1 : t_1 \quad \Gamma \vdash e_2 : t_2}{\Gamma \vdash (e_1,\ e_2) : t_1 * t_2}$$

$$\text{(fun)}\frac{(x,\ t) :: \Gamma \vdash e : t_1}{\Gamma \vdash \text{fun}(x : t) \to e\ \ :\ \ t \to t_1}$$

$$\text{(app)}\frac{\Gamma \vdash e_1 : t_1 \to t_2 \quad \Gamma \vdash e_2 : t_1}{\Gamma \vdash e_1\ e_2 : t_2}$$

$$\text{(bin-op1)}\frac{\begin{array}{c}\Gamma \vdash e_1 : t \qquad \Gamma \vdash e_2 : t \\ op \in \{*, /, +, -\} \quad t \in \{int, real\}\end{array}}{\Gamma \vdash e_1\ op\ e_2 : t}$$

$$\text{(bin-op2)}\frac{\begin{array}{c}\Gamma \vdash e_1 : t \quad \Gamma \vdash e_2 : t \\ t \neq t_1 \to t_2\end{array}}{\Gamma \vdash e_1 = e_2 : bool}$$

$$\text{(bin-op3)}\frac{\begin{array}{c}\Gamma \vdash e_1 : t \quad \Gamma \vdash e_2 : t \\ t \in \{int, real, string\} \\ op \in \{>, \geq, <, \leq\}\end{array}}{\Gamma \vdash e_1\ op\ e_2 : bool}$$

$$\text{(bin-op4)}\frac{\Gamma \vdash e_1 : string \quad \Gamma \vdash e_2 : string}{\Gamma \vdash e_1 \hat{}\ e_2 : string}$$

$$\text{(un-op)}\frac{\Gamma \vdash e : t \quad t \in \{int, real\}}{\Gamma \vdash \sim e : t}$$

$$\text{(match)}\frac{\Gamma \vdash e : t_1 \quad \Gamma \vdash branch : t_1 \to t_2}{\Gamma \vdash \text{match}\ e\ \text{with}\ branch : t_2}$$

$$\text{(local let)}\frac{\Gamma \vdash dec \leadsto (\Gamma', \_) \quad \Gamma'\ @\ \Gamma \vdash e : t}{\Gamma \vdash \text{let}\ dec\ \text{in}\ e : t}$$

$$\text{(global let)}\frac{\Gamma \vdash dec \leadsto (\Gamma',\ t)}{\Gamma \vdash \text{let}\ dec : t}$$

Branch type relation: $\Gamma \vdash branch : D \to t$

$$\text{(single)}\frac{\Gamma \vdash pattern : t_1 \rightsquigarrow \Gamma' \quad \Gamma' \mathbin{@} \Gamma \vdash e : t_2}{\Gamma \vdash pattern \to e \;:\; t_1 \to t_2}$$

$$\text{(multiple)}\frac{\Gamma \vdash pattern \to e : t_1 \to t_2 \quad \Gamma \vdash branch : t_1 \to t_2}{\Gamma \vdash (pattern \to e \mid branch) : t_1 \to t_2}$$

Pattern type relation: $\Gamma \vdash pattern : t \rightsquigarrow \Gamma'$

$$\text{(don't care)}\frac{}{\Gamma \vdash \_ : t \rightsquigarrow []}$$

$$\text{(id)}\frac{}{\Gamma \vdash x : t \rightsquigarrow [(x,\ t)]}$$

$$\text{(literal)}\frac{t \in \{int, real, bool, string\} \quad l \text{ is a literal of type } t}{\Gamma \vdash l : t \rightsquigarrow []}$$

$$\text{(ctor)}\frac{(C,\ t \to D) \in \Gamma \quad \Gamma \vdash p : t \rightsquigarrow \Gamma'}{\Gamma \vdash C\ p : D \rightsquigarrow \Gamma'}$$

$$\text{(name abs)}\frac{\Gamma \vdash p_1 : t_1 \rightsquigarrow \Gamma' \quad \Gamma \vdash p_2 : t_2 \rightsquigarrow \Gamma'' \quad dom(\Gamma') \cap dom(\Gamma'') = []}{\Gamma \vdash \ll p_1 \gg p_2 : \ll t_1 \gg t_2 \rightsquigarrow \Gamma' \mathbin{@} \Gamma''}$$

$$\text{(pair)}\frac{\Gamma \vdash p_1 : t_1 \rightsquigarrow \Gamma' \quad \Gamma \vdash p_2 : t_2 \rightsquigarrow \Gamma'' \quad dom(\Gamma') \cap dom(\Gamma'') = []}{\Gamma \vdash (p_1,\ p_2) : t_1 * t_2 \rightsquigarrow \Gamma' \mathbin{@} \Gamma''}$$

$$\text{(unit)}\frac{}{\Gamma \vdash () : unit \rightsquigarrow []}$$

Declaration type relation: $\Gamma \vdash dec \rightsquigarrow (\Gamma', t)$

$$\text{(val bind)}\frac{\Gamma \vdash e : t \quad \Gamma \vdash p : t \rightsquigarrow \Gamma'}{\Gamma \vdash p = e \rightsquigarrow (\Gamma',\ t)}$$

$$\text{(rec fun)}\frac{(f,\ t_1 \to t_2) :: (x,\ t_1) :: \Gamma \vdash e : t_2}{\Gamma \vdash f(x : t_1) : t_2 = e \rightsquigarrow ([(f,\ t_1 \to t_2)],\ t_1 \to t_2)}$$