Midi-FreshML Typing Rules

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Γ is a list of (id, type) pairs representing the finite partial function from ids to types.
$\in$ is used to indicate list membership, @ to indicate list appending, and :: to indicate consing.
The empty list is denoted by [].
Σ is the set of all strings (used for string literal rule).

Expression type relation: $\Gamma \vdash e\,:\,t$

$$\text{(int)}\ \frac{}{\Gamma \vdash n : int}\ \text{n} \in \mathbb{N}$$

$$\text{(real)}\ \frac{}{\Gamma \vdash r : real}\ \text{r} \in \mathbb{R}$$

$$\text{(bool)}\ \frac{}{\Gamma \vdash b : bool}\ \text{b} \in \{\text{true, false}\}$$

$$\text{(string)}\ \frac{}{\Gamma \vdash \text{``}*\text{''} : string}\ * \in \Sigma \setminus \{\text{''}\}$$

$$\text{(unit)}\ \frac{}{\Gamma \vdash () : unit}$$

$$\text{(id)}\ \frac{(x,\ t) \in \Gamma}{\Gamma \vdash x : t}$$

$$\text{(fresh)}\ \frac{N \in \Gamma}{\Gamma \vdash (\text{fresh} : N) : N}$$

$$\text{(ctor)}\ \frac{(C,\ t \to D) \in \Gamma \qquad \Gamma \vdash e : t}{\Gamma \vdash C\ e : D}$$

$$\text{(swap)}\ \frac{\begin{array}{c}\Gamma \vdash e_1 : N \\ \Gamma \vdash e_2 : N \qquad \Gamma \vdash e_3 : t\end{array}}{\Gamma \vdash \text{swap}(e_1,\ e_2)\ \text{in}\ e_3 : t}$$

$$\text{(name abs)}\ \frac{\Gamma \vdash e_1 : N \qquad \Gamma \vdash e_2 : t}{\Gamma \vdash \ll e_1 \gg e_2 : \ll N \gg t}$$

$$\text{(if)}\ \frac{\begin{array}{c}\Gamma \vdash e_1 : bool \\ \Gamma \vdash e_2 : t \qquad \Gamma \vdash e_3 : t\end{array}}{\Gamma \vdash \text{if}\ e_1\ \text{then}\ e_2\ \text{else}\ e_3 : t}$$

$$\text{(pair)}\ \frac{\Gamma \vdash e_1 : t_1 \qquad \Gamma \vdash e_2 : t_2}{\Gamma \vdash (e_1,\ e_2) : t_1 * t_2}$$

$$\text{(fun)}\ \frac{(x,\ t) :: \Gamma \vdash e : t_1}{\Gamma \vdash \text{fun}(x : t) \to e\ \ :\ \ t \to t_1}$$

$$\text{(app)}\ \frac{\Gamma \vdash e_1 : t_1 \to t_2 \qquad \Gamma \vdash e_2 : t_1}{\Gamma \vdash e_1\ e_2 : t_2}$$

$$\text{(bin-op1)}\ \frac{\begin{array}{c}\Gamma \vdash e_1 : t \qquad \Gamma \vdash e_2 : t \\ op \in \{*, /, +, -\} \qquad t \in \{int, real\}\end{array}}{\Gamma \vdash e_1\ op\ e_2 : t}$$

$$\text{(bin-op2)}\ \frac{\begin{array}{c}\Gamma \vdash e_1 : t \qquad \Gamma \vdash e_2 : t \\ t \neq t_1 \to t_2\end{array}}{\Gamma \vdash e_1 = e_2 : bool}$$

$$\text{(bin-op3)}\ \frac{\begin{array}{c}\Gamma \vdash e_1 : t \qquad \Gamma \vdash e_2 : t \\ t \in \{int, real, string\} \\ op \in \{>, \geq, <, \leq\}\end{array}}{\Gamma \vdash e_1\ op\ e_2 : bool}$$

$$\text{(bin-op4)}\ \frac{\Gamma \vdash e_1 : string \qquad \Gamma \vdash e_2 : string}{\Gamma \vdash e_1\,\hat{}\,e_2 : string}$$

$$\text{(un-op)}\ \frac{\Gamma \vdash e : t \qquad t \in \{int, real\}}{\Gamma \vdash {\sim} e : t}$$

$$\text{(match)}\ \frac{\Gamma \vdash e : t_1 \qquad \Gamma \vdash branch : t_1 \to t_2}{\Gamma \vdash \text{match}\ e\ \text{with}\ branch : t_2}$$

$$\text{(local let)}\ \frac{\Gamma \vdash dec \rightsquigarrow (\Gamma', \_) \qquad \Gamma'\ @\ \Gamma \vdash e : t}{\Gamma \vdash \text{let}\ dec\ \text{in}\ e : t}$$

$$\text{(global let)}\ \frac{\Gamma \vdash dec \rightsquigarrow (\Gamma',\ t)}{\Gamma \vdash \text{let}\ dec : t}$$

Branch type relation: $\Gamma \vdash \mathit{branch} : D \rightarrow t$

$$\text{(single)}\ \frac{\Gamma \vdash \mathit{pattern} : t_1 \rightsquigarrow \Gamma' \qquad \Gamma' \mathbin{@} \Gamma \vdash e : t_2}{\Gamma \vdash \mathit{pattern} \rightarrow e \ : \ t_1 \rightarrow t_2}
\qquad
\text{(multiple)}\ \frac{\Gamma \vdash \mathit{pattern} \rightarrow e : t_1 \rightarrow t_2 \qquad \Gamma \vdash \mathit{branch} : t_1 \rightarrow t_2}{\Gamma \vdash (\mathit{pattern} \rightarrow e \mid \mathit{branch}) : t_1 \rightarrow t_2}$$

Pattern type relation: $\Gamma \vdash \mathit{pattern} : t \rightsquigarrow \Gamma'$

$$\text{(don't care)}\ \frac{}{\Gamma \vdash _ : t \rightsquigarrow []}
\qquad
\text{(id)}\ \frac{}{\Gamma \vdash x : t \rightsquigarrow [(x,\ t)]}$$

$$\text{(literal)}\ \frac{t \in \{\mathit{int}, \mathit{real}, \mathit{bool}, \mathit{string}\} \qquad l \text{ is a literal of type } t}{\Gamma \vdash l : t \rightsquigarrow []}
\qquad
\text{(ctor)}\ \frac{(C,\ t \rightarrow D) \in \Gamma \qquad \Gamma \vdash p : t \rightsquigarrow \Gamma'}{\Gamma \vdash C\ p : D \rightsquigarrow \Gamma'}$$

$$\text{(name abs)}\ \frac{\Gamma \vdash p : t \rightsquigarrow \Gamma' \qquad x \notin \mathit{dom}(\Gamma')}{\Gamma \vdash \ll x \gg p : \ll N \gg t \rightsquigarrow (x,\ N) :: \Gamma'}
\qquad
\text{(unit)}\ \frac{}{\Gamma \vdash () : \mathit{unit} \rightsquigarrow []}$$

$$\text{(pair)}\ \frac{\Gamma \vdash p_1 : t_1 \rightsquigarrow \Gamma' \qquad \Gamma \vdash p_2 : t_2 \rightsquigarrow \Gamma'' \qquad \mathit{dom}(\Gamma') \cap \mathit{dom}(\Gamma'') = []}{\Gamma \vdash (p_1,\ p_2) : t_1 * t_2 \rightsquigarrow \Gamma' \mathbin{@} \Gamma''}$$

Declaration type relation: $\Gamma \vdash \mathit{dec} \rightsquigarrow (\Gamma',\ t)$

$$\text{(val bind)}\ \frac{\Gamma \vdash e : t \qquad \Gamma \vdash p : t \rightsquigarrow \Gamma'}{\Gamma \vdash p = e \rightsquigarrow (\Gamma',\ t)}
\qquad
\text{(rec fun)}\ \frac{(f,\ t_1 \rightarrow t_2) :: (x,\ t_1) :: \Gamma \vdash e : t_2}{\Gamma \vdash f(x : t_1) : t_2 = e \rightsquigarrow ([(f,\ t_1 \rightarrow t_2)],\ t_1 \rightarrow t_2)}$$