COE 2001 Statics

Fall 2013

Exam 1

NAME Solution

The exam is closed books and closed notes. Scientific calculators are allowed. No ipods, cellphones, laptops etc.

Linearly document all steps and show all supporting work. Answers given without supporting work will be given zero credit. Write legibly and box all your final answers.

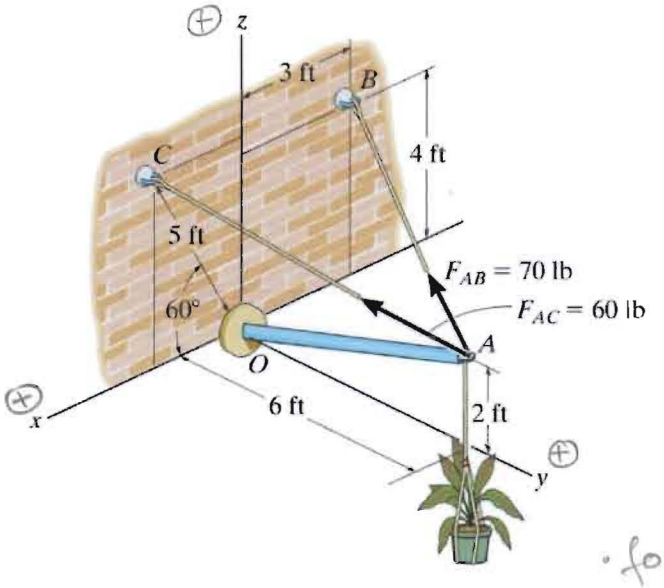
HONOR STATEMENT: I have red and strictly abided by all conditions set forth by Georgia Tech Honor Code and thus have neither given nor recived assistance of any type regarding the content or solution of the problems in this examination, nor will I discuss the content with other students until the exam has been graded and returned.

SIGNATURE:

Problem 1 *(25 points)*

For the force in the cable $\mathbf{F}_{AB}$ = 70 lb, determine the following:

1. Express force $\mathbf{F}_{AB}$ in its orthogonal components (10 pts)
2. Projection of the force $\mathbf{F}_{AB}$ along the strut OA, its magnitude and vector components (15 pts)



1) $F_{AB} = 70$ lb

· unit vector $\hat{u}_{AB}$

A(0, 6, 2) B(−3, 0, 4)

$$\hat{u}_{AB} = \frac{(-3-0)\hat{i} + (0-6)\hat{j} + (4-2)\hat{k}}{\sqrt{(-3)^2 + (-6)^2 + (2)^2}}$$

$$\hat{u}_{AB} = -\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

· force vector F_{AB}

$$F_{AB} = F_{AB}\hat{u}_{AB} = 70\left(-\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$$

$$\boxed{F_{AB} = -30\hat{i} - 60\hat{j} + 20\hat{k}\ \text{lb}}$$

2) Projection of the force F_{AB} along the strut OA

$$(F_{AB})_{AO} = (F_{AB} \cdot \hat{u}_{AO})\hat{u}_{AO} \quad ; \quad A(0,6,2) \quad O(0,0,0)$$

· unit vector $\hat{u}_{AO} = \dfrac{(0-0)\hat{i} + (0-6)\hat{j} + (0-2)\hat{k}}{\sqrt{(0)^2 + (-6)^2 + (-2)^2}} = -0.949\hat{j} - 0.316\hat{k}$

· magnitude of the projection $(F_{AB})_{AO}$ – dot product

$$(F_{AB})_{AO} = F_{AB} \cdot \hat{u}_{AO} = (-30\hat{i} - 60\hat{j} + 20\hat{k}) \cdot (-0.949\hat{j} - 0.316\hat{k})$$

$$= (-30)(0) + (-60)(-0.949) + (20)(-0.316\hat{k})$$

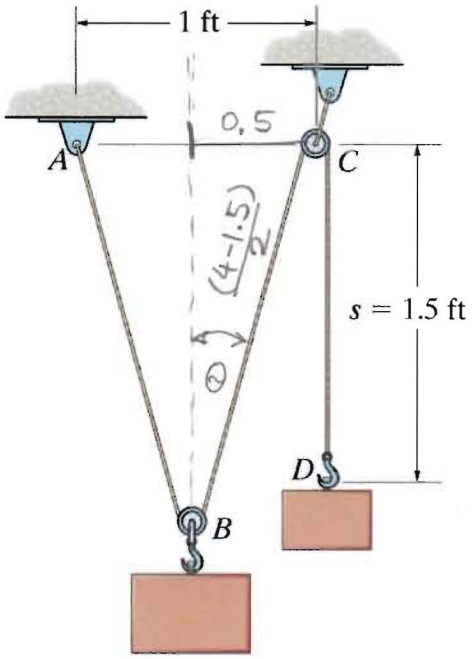
$$\boxed{(F_{AB})_{AO} = 50.60\ \text{lb}} \quad \text{magnitude}$$

· vector components $|F_{AB}|_{AO}\hat{u}_{AO}$

$$\boxed{(F_{AB})_{OA}\hat{u}_{OA} = 50.60(-0.949\hat{j} - 0.316\hat{k}) = -48\hat{j} - 16\hat{k}\ \text{lb}}$$

Problem 2 *(25 points)*

A "scale" is constructed with a 4-ft long cord and 10-lb block D. The cord is fixed to a pin at A and passes over two small pulleys at B and C. Determine the weight of the suspended block at B if the system is in equilibrium when s = 1.5 ft. Show FBD.



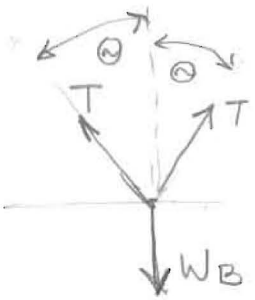
FBD

Block D

$\Sigma F_y = 0$

$T = W_D = 10\ lb$

Block B



· tension force T is the same throughout the cord $T = 10\ lb$

· from the geometry : $\sin\theta = \dfrac{0.5}{\frac{(4-1.5)}{2}} = \dfrac{0.5}{1.25}$

$\theta = \sin^{-1}\left(\dfrac{0.5}{1.25}\right) = 23.58^\circ$

· From the FBD for block B

$\pm\rightarrow \Sigma F_x = 0 \quad T\sin\theta - T\sin\theta = 0$ - satisfied

$+\uparrow \Sigma F_y = 0 \quad T\cos\theta + T\cos\theta - W_B = 0$

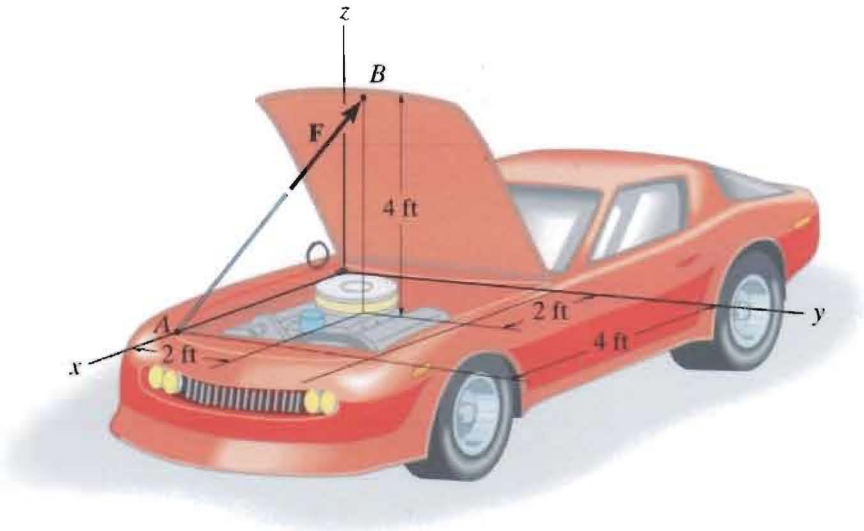
$2(10)\cos 23.58^\circ - W_B = 0$

$\boxed{W_B = 18.3\ lb}$

Problem 3 *(25 points)*

The hood of the automobile is supported by the strut AB, which exerts force F = 24 lb on the hood. Determine the following:

1. Moment of the force F about the origin O (15 pts)
2. Moment of the force about the hinged axis y (10 pts)



(1) $F = 24$ lb $A(4,0,0)$ $B(2,2,4)$

$$\hat{u}_{AB} = \frac{(2-4)\hat{i} + (2-0)\hat{j} + (4-0)\hat{k}}{\sqrt{(-2)^2 + (2)^2 + (4)^2}} = -0.408\hat{i} + 0.408\hat{j} + 0.816\hat{k}$$

$$F = (24)u_{AB} = -9.80\hat{i} + 9.80\hat{j} + 19.6\hat{k} \text{ lb}$$

$$M_O = r_{OA} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 0 & 0 \\ -9.8 & 9.8 & 19.6 \end{vmatrix} = 0\hat{i} - \left((4)(19.6)\right)\hat{j} + \left((4)(9.8)\right)\hat{k}$$

$r_{OA} = 4\hat{i}$

$$M_O = -78.4\hat{j} + 39.2\hat{k} \text{ lb·ft}$$

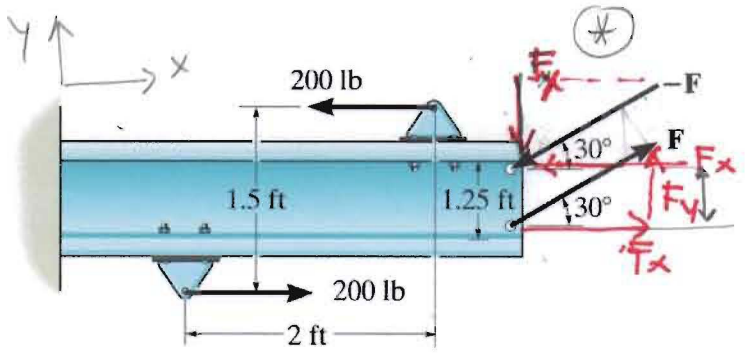
(2) $M_y = (M_O \cdot \hat{j})\hat{j}$

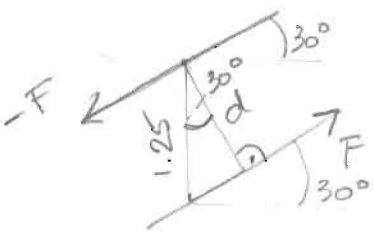
$$M_y = \left[(78.4\hat{j}) \cdot \hat{j}\right]\hat{j} = -78.4\hat{j} \text{ lb-ft}$$

$\hat{u}_y = \hat{j}$

Problem 4 (25 points)

Two couples act on the beam. If F = 125 lb, determine the moment of each couple and its direction. (20 pts)





$C_1 = (200)(1.5) = 300$ lb-ft

C_1 direction $+\hat{\imath}$ (out of the paper)

$C_2 = F \cdot d = (125)(1.082) = 135.32$ lb-ft $(+\hat{\imath})$

$\frac{d}{1.25} = \cos 30°$

$d = 1.082$ ft

(out of the paper)

Where on the beam is the couple moment the largest? (5 pts)

Couple moment is the same @ any point on the beam.

(*) 125 lb couple can be resolved into their horizontal and vertical components (F_x and F_y) as shown

F_y components cancel each other, and do not contribute to the couple moment, distance between them = 0

$F_x = 125 \cos 30°$ and $C_2 = F_x \cdot (1.25)$

$C_2 = 125 \cos 30° (1.25) = 135.32$ lb-ft $(+\hat{\imath})$