Quiz 8 (25 mins), MA1501 Calculus I, 10/24/2013 PRINTED NAME: ________________GT ID:____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

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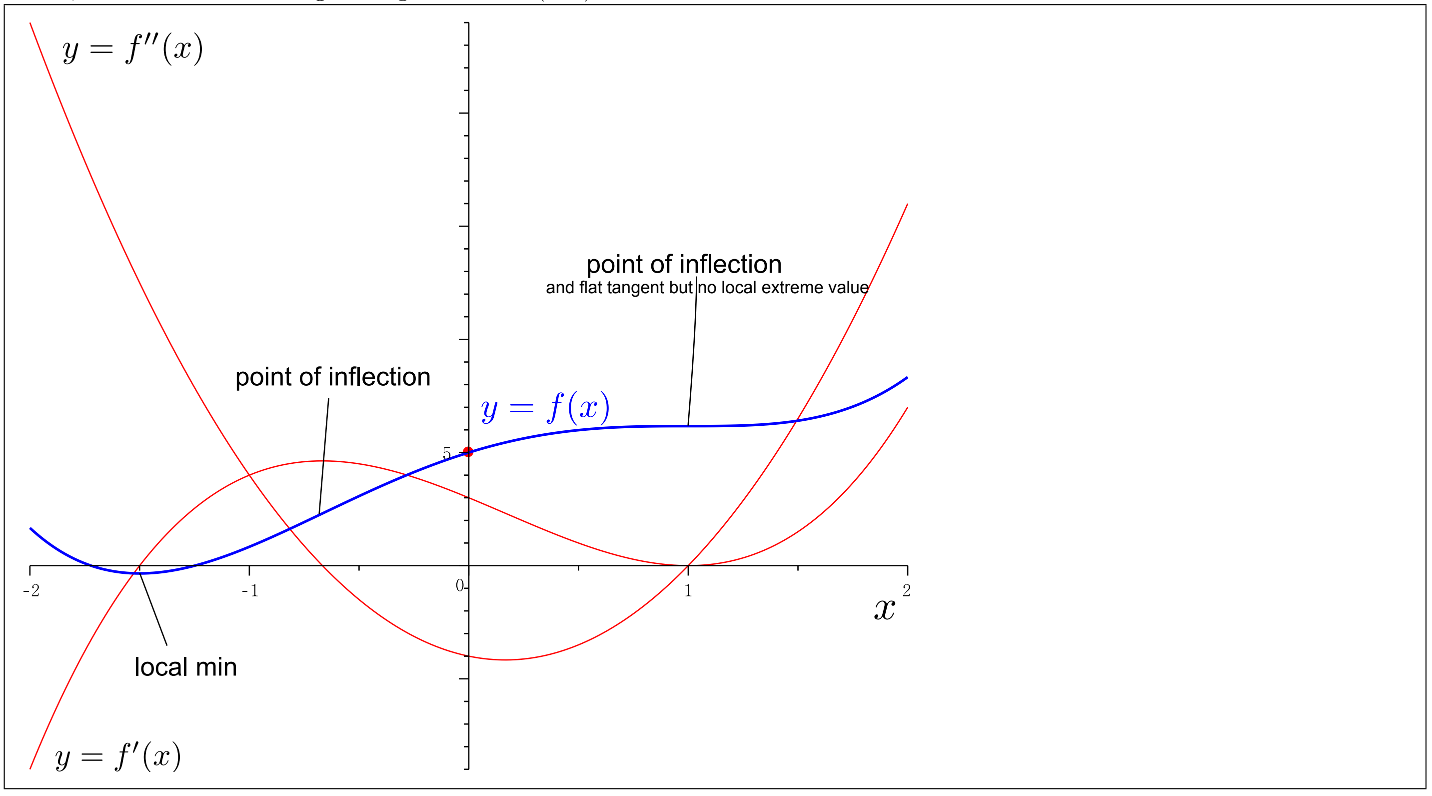
1. (30 pts) Find the global (absolute) maximum value of $f(x) = x^2 \ln(1/x)$ and say where it is assumed.

Answer: $f'(x) = 2x\ln(\frac{1}{x}) + x^2 \cdot \frac{1}{\frac{1}{x}} \cdot (\frac{1}{x})' = 2x\ln(\frac{1}{x}) + x^3 \cdot (-\frac{1}{x^2}) = 2x\ln(\frac{1}{x}) - x = x(-2\ln(x) - 1)$.
$f'(x) = 0 \implies x = 0$ and $\ln(x) = -\frac{1}{2}$. Since $x = 0$ is not in the domain of f, $x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$. Also, $f'(x) > 0$ for $0 < x < \frac{1}{\sqrt{e}}$ and $f'(x) < 0$ for $x > \frac{1}{\sqrt{e}}$. Therefore, $f(\frac{1}{\sqrt{e}}) = \frac{1}{e}\ln\ \sqrt{e} = \frac{1}{2e}\ln\ e = \frac{1}{2e}$ is the global (absolute) maximum value of f assumed at $x = \frac{1}{\sqrt{e}}$.

2. (30 pts) The plot shows the graphs of the first and second derivatives of a function $y = f(x)$. Sketch the approximate graph of f which passes through the given point $(0, 5)$.



3. (40 pts) You are planning to close off a corner of the first quadrant with a line segment 20 units long running from $(a, 0)$ to $(0, b)$. Find a and b so that the area of the triangle enclosed by the segment is largest.

Answer: The are of the triangle is $A = \frac{1}{2}ba = \frac{b}{2}\sqrt{400 - b^2}$, where $0 \leq b \leq 20$. Then

$$\frac{dA}{db} = \frac{1}{2}\sqrt{400 - b^2} - \frac{b^2}{2\sqrt{400 - b^2}} = \frac{200 - b^2}{\sqrt{400 - b^2}} = 0.$$

The critical point on the domain is $b = 10\sqrt{2}$.
(method 1: comparing critical point values with endpoint values) For the endpoint values, when $b = 0$ or 20, the area is zero. Therefore, $A(10\sqrt{2})$ is the maximum area.
(method 2: using the first derivative) If $0 \leq b < 10\sqrt{2}$, $\frac{dA}{db} > 0$. If $10\sqrt{2} < b \leq 20$, $\frac{dA}{db} < 0$. Therefore, $A(10\sqrt{2})$ is the maximum area.
(method 3: using the second derivative) The second derivative is

$$\frac{d^2A}{db^2} = \frac{b(-600 + b^2)}{(400 - b^2)^{3/2}}$$

and $\frac{d^2A}{db^2} < 0$ when $b = 10\sqrt{2}$. Therefore, A attains a local maximum at $b = 10\sqrt{2}$.

Conclusion: When $a^2 + b^2 = 400$ and $b = 10\sqrt{2}$, the value of a is also $10\sqrt{2}$. The maximum area occurs when $a = b = 10\sqrt{2}$.