

8-4 a) 95% CI for  $\mu$ ,  $n = 10$ ,  $\sigma = 20$   $\bar{x} = 1000$ ,  $z = 1.96$

$$\begin{aligned}\bar{x} - z\sigma / \sqrt{n} &\leq \mu \leq \bar{x} + z\sigma / \sqrt{n} \\ 1000 - 1.96(20/\sqrt{10}) &\leq \mu \leq 1000 + 1.96(20/\sqrt{10}) \\ 987.6 &\leq \mu \leq 1012.4\end{aligned}$$

b) .95% CI for  $\mu$ ,  $n = 25$ ,  $\sigma = 20$   $\bar{x} = 1000$ ,  $z = 1.96$

$$\begin{aligned}\bar{x} - z\sigma / \sqrt{n} &\leq \mu \leq \bar{x} + z\sigma / \sqrt{n} \\ 1000 - 1.96(20/\sqrt{25}) &\leq \mu \leq 1000 + 1.96(20/\sqrt{25}) \\ 992.2 &\leq \mu \leq 1007.8\end{aligned}$$

c) 99% CI for  $\mu$ ,  $n = 10$ ,  $\sigma = 20$   $\bar{x} = 1000$ ,  $z = 2.58$

$$\begin{aligned}\bar{x} - z\sigma / \sqrt{n} &\leq \mu \leq \bar{x} + z\sigma / \sqrt{n} \\ 1000 - 2.58(20/\sqrt{10}) &\leq \mu \leq 1000 + 2.58(20/\sqrt{10}) \\ 983.7 &\leq \mu \leq 1016.3\end{aligned}$$

d) 99% CI for  $\mu$ ,  $n = 25$ ,  $\sigma = 20$   $\bar{x} = 1000$ ,  $z = 2.58$

$$\begin{aligned}\bar{x} - z\sigma / \sqrt{n} &\leq \mu \leq \bar{x} + z\sigma / \sqrt{n} \\ 1000 - 2.58(20/\sqrt{25}) &\leq \mu \leq 1000 + 2.58(20/\sqrt{25}) \\ 989.7 &\leq \mu \leq 1010.3\end{aligned}$$

e) When  $n$  is larger, the CI is narrower. The higher the confidence level, the wider the CI.

8-7 a) Find  $n$  for the length of the 95% CI to be 40.  $Z_{\alpha/2} = 1.96$

$$\begin{aligned}1/2 \text{ length} &= (1.96)(20) / \sqrt{n} = 20 \\ 39.2 &= 20\sqrt{n} \\ n &= \left(\frac{39.2}{20}\right)^2 = 3.84\end{aligned}$$

Therefore,  $n = 4$ .

b) Find  $n$  for the length of the 99% CI to be 40.  $Z_{\alpha/2} = 2.58$

$$\begin{aligned}1/2 \text{ length} &= (2.58)(20) / \sqrt{n} = 20 \\ 51.6 &= 20\sqrt{n} \\ n &= \left(\frac{51.6}{20}\right)^2 = 6.66\end{aligned}$$

Therefore,  $n = 7$ .

8-8 Interval (1):  $3124.9 \leq \mu \leq 3215.7$  and Interval (2):  $3110.5 \leq \mu \leq 3230.1$

Interval (1): half-length =  $90.8/2=45.4$  and Interval (2): half-length =  $119.6/2=59.8$

a)  $\bar{x}_1 = 3124.9 + 45.4 = 3170.3$

$\bar{x}_2 = 3110.5 + 59.8 = 3170.3$  The sample means are the same.

b) Interval (1):  $3124.9 \leq \mu \leq 3215.7$  was calculated with 95% confidence because it has a smaller half-length, and therefore a smaller confidence interval. The 99% confidence level widens the interval.

8-14 a) 95% Two-sided CI on the true mean life of a 75-watt light bulb

For  $\alpha = 0.05$ ,  $z_{\alpha/2} = z_{0.025} = 1.96$ , and  $\bar{x} = 1014$ ,  $\sigma = 25$ ,  $n=20$

$$\bar{x} - z_{0.025} \left( \frac{\sigma}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + z_{0.025} \left( \frac{\sigma}{\sqrt{n}} \right)$$

$$1014 - 1.96 \left( \frac{25}{\sqrt{20}} \right) \leq \mu \leq 1014 + 1.96 \left( \frac{25}{\sqrt{20}} \right)$$

$$1003 \leq \mu \leq 1025$$

b) 95% one-sided CI on the true mean piston ring diameter

For  $\alpha = 0.05$ ,  $z_{\alpha} = z_{0.05} = 1.65$  and  $\bar{x} = 1014$ ,  $\sigma = 25$ ,  $n=20$

$$\bar{x} - z_{0.05} \frac{\sigma}{\sqrt{n}} \leq \mu$$

$$1014 - 1.65 \left( \frac{25}{\sqrt{20}} \right) \leq \mu$$

$$1005 \leq \mu$$

The lower bound of the one-sided confidence interval is greater than the lower bound of the two-sided interval even though the level of significance is the same. This is because for a one-sided confidence interval the probability in the left tail ( $\alpha$ ) is greater than the probability in the left tail of the two-sided confidence interval ( $\alpha/2$ ).

8-25 a)  $t_{0.025,12} = 2.179$  b)  $t_{0.025,24} = 2.064$  c)  $t_{0.005,13} = 3.012$

d)  $t_{0.0005,15} = 4.073$

8-26 a)  $t_{0.05,14} = 1.761$  b)  $t_{0.01,19} = 2.539$  c)  $t_{0.001,24} = 3.467$

8-27 a)  $\text{Mean} = \frac{\text{sum}}{N} = \frac{251.848}{10} = 25.1848$

$\text{Variance} = (\text{stDev})^2 = 1.605^2 = 2.5760$

b) 95% confidence interval on mean

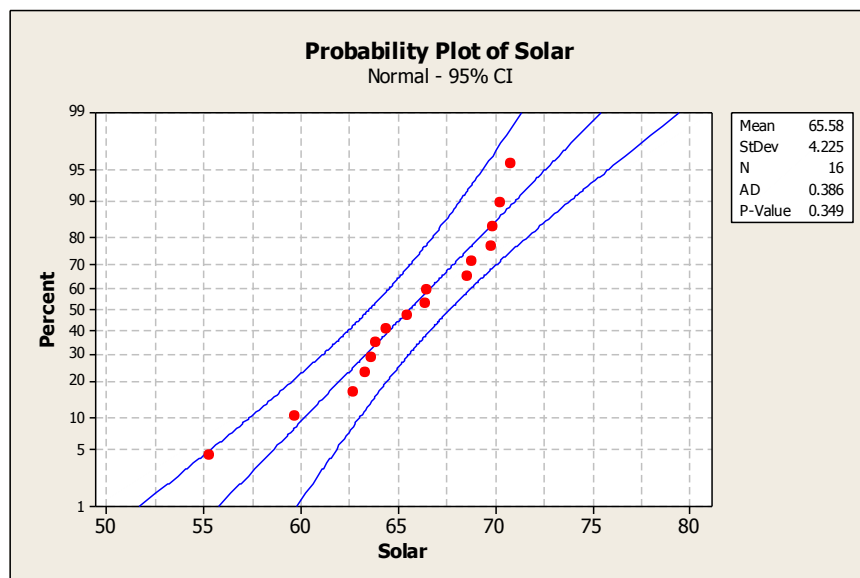
$n = 10 \quad \bar{x} = 25.1848 \quad s = 1.605 \quad t_{0.025,9} = 2.262$

$$\bar{x} - t_{0.025,9} \left( \frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025,9} \left( \frac{s}{\sqrt{n}} \right)$$

$$25.1848 - 2.262 \left( \frac{1.605}{\sqrt{10}} \right) \leq \mu \leq 25.1848 + 2.262 \left( \frac{1.605}{\sqrt{10}} \right)$$

$$24.037 \leq \mu \leq 26.333$$

8-36 The data appear to be normally distributed based on the normal probability plot below.



95% confidence interval on mean solar energy consumed

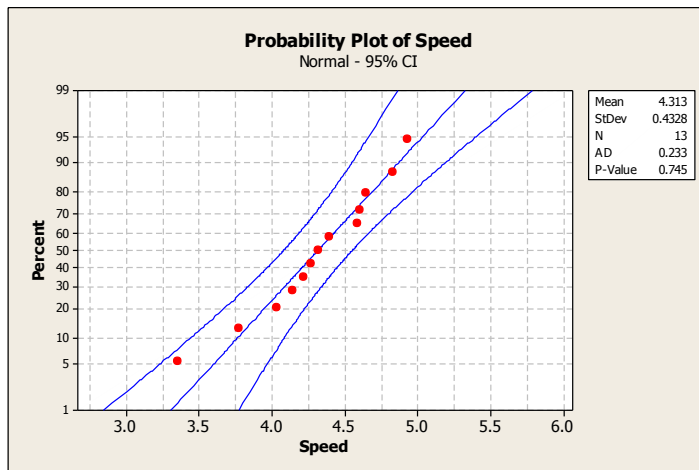
$n = 16 \quad \bar{x} = 65.58 \quad s = 4.225 \quad t_{0.025,15} = 2.13$

$$\bar{x} - t_{0.025,15} \left( \frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025,15} \left( \frac{s}{\sqrt{n}} \right)$$

$$65.58 - 2.131 \left( \frac{4.225}{\sqrt{16}} \right) \leq \mu \leq 65.58 + 2.131 \left( \frac{4.225}{\sqrt{16}} \right)$$

$$63.329 \leq \mu \leq 67.831$$

8-41. a) The data appear to be normally distributed based on examination of the normal probability plot below.



b) 95% confidence interval on mean speed-up

$$n = 13 \quad \bar{x} = 4.313 \quad s = 0.4328 \quad t_{0.025,12} = 2.179$$

$$\bar{x} - t_{0.025,12} \left( \frac{s}{\sqrt{n}} \right) \leq \mu \leq \bar{x} + t_{0.025,12} \left( \frac{s}{\sqrt{n}} \right)$$

$$4.313 - 2.179 \left( \frac{0.4328}{\sqrt{13}} \right) \leq \mu \leq 4.313 + 2.179 \left( \frac{0.4328}{\sqrt{13}} \right)$$

$$4.051 \leq \mu \leq 4.575$$

c) 95% lower confidence bound on mean speed-up

$$n = 13 \quad \bar{x} = 4.313 \quad s = 0.4328 \quad t_{0.05,12} = 1.782$$

$$\bar{x} - t_{0.05,12} \left( \frac{s}{\sqrt{n}} \right) \leq \mu$$

$$4.313 - 1.782 \left( \frac{0.4328}{\sqrt{13}} \right) \leq \mu$$

$$4.099 \leq \mu$$