PHYS 2211 Test 3
Spring 2014

Name(print) ~~~ Test ~ Key ~~~ **Lab Section** Key

Greco(K)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Wednesday	K03 K05	K04 K07	K06 K08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Finnick Odair

Sign your name on the line above

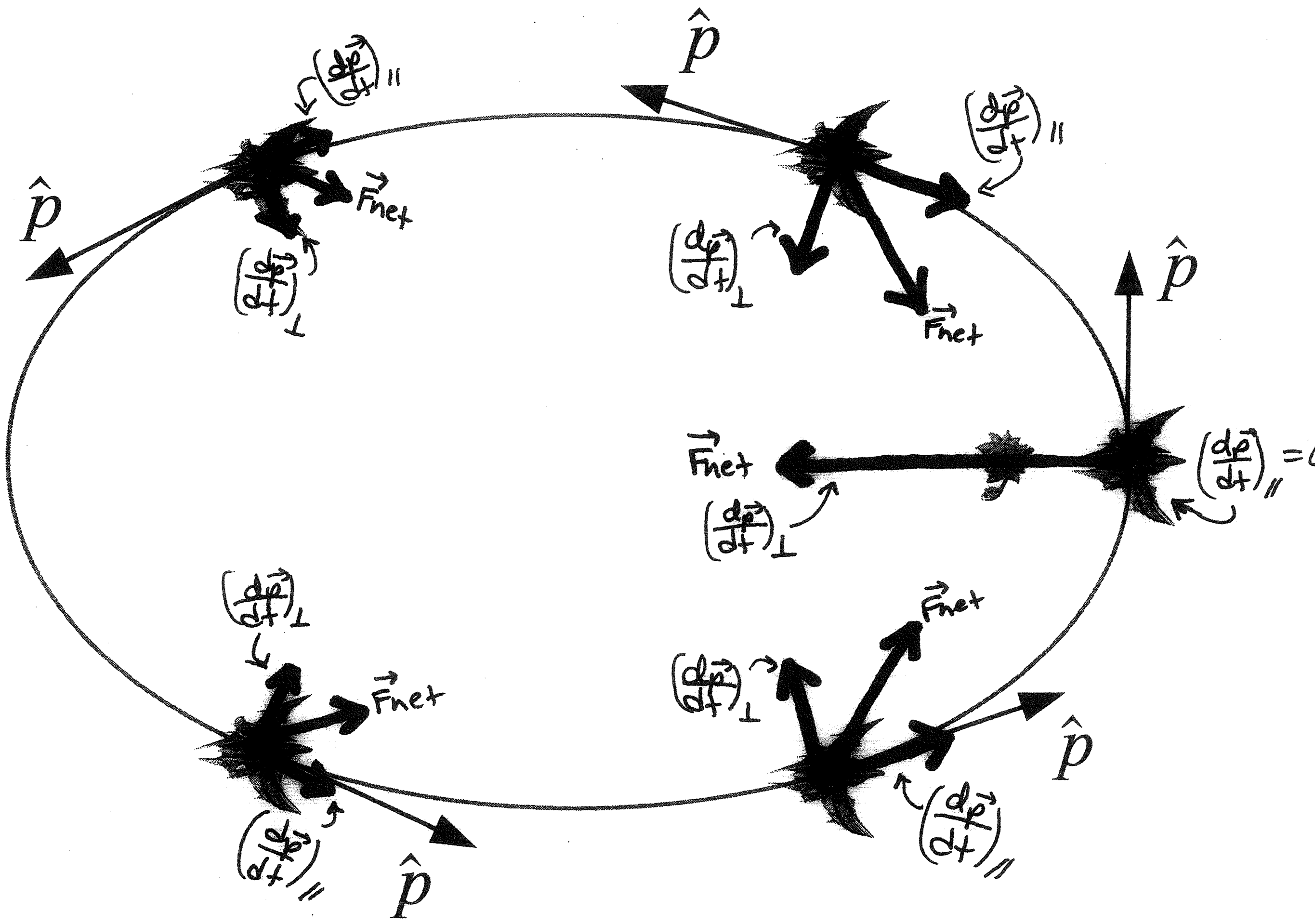
PHYS 2211

Do not write on this page!

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

In the not too distant future, an alien spacecraft is observed orbiting our Sun. the only force acting on the spacecraft is the gravitational force of the Sun. As indicated on the diagram the spacecraft orbits the sun counter-clockwise. At each location on the diagram, sketch three vectors for the spacecraft: $(\frac{d\vec{p}}{dt})_{\parallel}$, $(\frac{d\vec{p}}{dt})_{\perp}$, and $\vec{F}_{net}$. Please pay careful attention to the relative length of your vectors.



$(\frac{d\vec{p}}{dt})_{\parallel} \Rightarrow$ 1 pt mag 1 pt direction

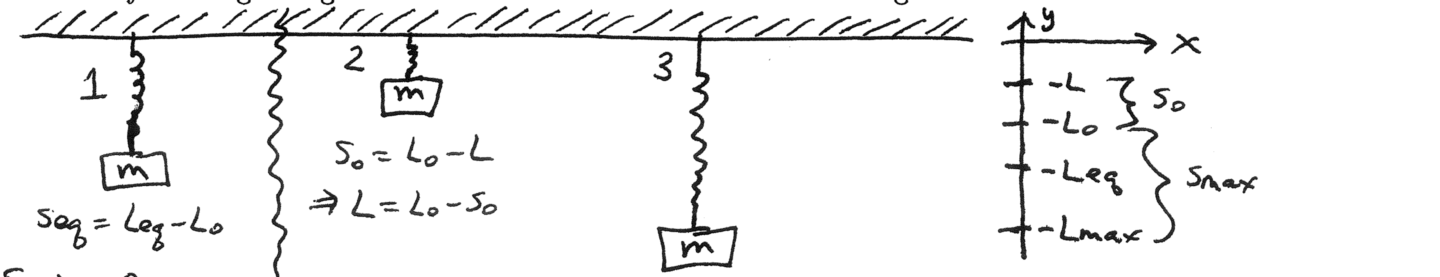
$(\frac{d\vec{p}}{dt})_{\perp} \Rightarrow$ 1 pt mag 1 pt direction

$(\vec{F}_{net}) \Rightarrow$ 1 pt

Problem 2 (25 Points)

A mass m hangs motionless from a vertical spring with stiffness k_s in the lab room. The spring has a relaxed length L_0. You lift the mass so that the spring compresses an amount s_0 and let go, giving the mass no initial velocity.

(a 5pts) Calculate the maximum length of the spring during the oscillations of the mass. It may help you to start by drawing a diagram. Your answer should be in terms of the given variables and known constants.



$S_{eq} = L_{eq} - L_o$

$F_{net_y} = 0$

$mg = k\,S_{eq}$

$S_{eq} = \frac{mg}{K}$

$S_o = L_o - L$

$\Rightarrow L = L_o - S_o$

$S_{max} = L_{max} - L_o$

$\Rightarrow L_{max} = S_{max} + L_o$

$$\left.\begin{aligned}\Delta U_g &= mg\,\Delta h = mg(h_3 - h_2) = mg\,(-L_{max} - -L) = \\ &= mg\,(-L_{max} + L) = mg\,(-s_{max} - \cancel{L_o} + \cancel{L_o} - S_o) = \\ &= mg\,(-S_{max} - S_o) = -mg\,(S_{max} + S_o)\end{aligned}\right\} \text{1pt}$$

$$\left.\begin{aligned}&\Delta E = 0 \Rightarrow E_f - E_i = 0 \\ &U_{sf} - U_{si} + \Delta U_g = 0\end{aligned}\right\} \text{1pt}$$

$$\left.\begin{aligned}&\tfrac{1}{2}k\,S_{max}^2 - \tfrac{1}{2}k\,S_o^2 + -mg\,(S_{max} + S_o) = 0 \\ &\tfrac{1}{2}k\,(S_{max}^2 - S_o^2) - mg\,(S_{max} + S_o) = 0 \\ &S_{max}^2 - S_o^2 - \frac{2mg}{K}S_{max} - \frac{2mg}{K}S_o = 0 \\ &S_{max}^2 - \frac{2mg}{K}S_{max} - S_o^2 - \frac{2mg}{K}S_o = 0\end{aligned}\right\} \text{2pt}$$

uadratic: $S_{max} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ith $a = 1$; $b = \dfrac{-2mg}{K}$; $c = -S_o^2 - \dfrac{2mg}{K}S_o$

o:

$$S_{max} = \frac{-\left(\frac{-2mg}{K}\right) \pm \sqrt{\frac{4m^2g^2}{K^2} - 4\left(-S_o^2 - \frac{2mg}{K}S_o\right)}}{2}$$

sing only the positive root (we want max stretch):

$$S_{max} = \frac{\frac{2mg}{K} + \sqrt{\frac{4m^2g^2}{K^2} + 4S_o^2\left(1 - \frac{2mg}{KS_o}\right)}}{2}$$

$$= \frac{mg}{K} + \sqrt{\left(\frac{mg}{K}\right)^2 + S_o^2\left(1 - \frac{2mg}{K}\frac{1}{S_o}\right)}$$

Since $S_{eq} = \frac{mg}{K}$, then:

$$S_{max} = S_{eq} + \sqrt{S_{eq}^2 + S_o^2\left(1 - \frac{2S_{eq}}{S_o}\right)}$$

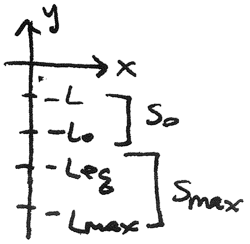
Finally, since $L_{max} = S_{max} + L_o$, then we have that:

$$\boxed{L_{max} = L_o + S_{eq} + \sqrt{S_{eq}^2 + S_o^2\left(1 - \frac{2S_{eq}}{S_o}\right)}}$$

1pt

(b 10pts) At the maximum length of the spring, how much work was done by the Earth on the mass? Your answer should be in terms of the given variables and known constants.

$W = \vec{F} \cdot d\vec{r} \Rightarrow$ positive because $\vec{F}$ points down & mass moves down



$$W = \vec{F} \cdot d\vec{r} = \boxed{mg\,(S_o + S_{max})}$$

$\Rightarrow$ total distance moved:

$S_o + S_{max}$

-0,5
-1,5
-3.0
-8.0

(c 10pts) Determine the maximum speed of the mass during it's oscillations. Your answer should be in terms of the given variables and known constants.

Initial: $K_i = 0$, $U_{si} = \frac{1}{2} k S_o^2$

Final: $K_{max} = \frac{1}{2} m v_{max}^2$ (@ L_{eq}), $U_{sf} = \frac{1}{2} k S_{eq}^2$

$\Delta U_g = mg\,(h_f - h_i) = mg(-L_{eq} - -L) = mg(-L_{eq} + L) =$

$= mg(-S_{eq} - \cancel{L_o} + \cancel{L_o} - S_o) = mg(-S_{eq} - S_o) = -mg\,(S_{eq} - S_o)$

$\Delta E = 0 \Rightarrow E_f - E_i = 0$

$K_f - \cancel{K_i} + U_{sf} - U_{si} + \Delta U_g = 0$

$\frac{1}{2} m v_{max}^2 + \frac{1}{2} k\left(S_{eq}^2 - S_o^2\right) - mg\,(S_{eq} - S_o) = 0$

$\frac{1}{2} m v_{max}^2 = mg\,(S_{eq} - S_o) - \frac{1}{2} k\,(S_{eq}^2 - S_o^2)$

$$\boxed{V_{max} = \sqrt{2g\,(S_{eq} - S_o) - \frac{k}{m}\left(S_{eq}^2 - S_o^2\right)}}$$

-0.5
-1.5
-3.0
-8.0

Problem 3 (25 Points)

(a 15pts) Imagine that you are standing on a spherical asteroid deep in space far from other objects. You pick up a small rock and throw it straight up from the surface of the asteroid. The asteroid has a radius of 5×10^3 m and the rock you threw has a mass of 0.149 kg. You notice that if you throw the rock with a velocity less than 44.7 m/s it eventually comes crashing back into the asteroid. Calculate the mass of the asteroid.

Initial state: $v_i = v_{esc} = 44.7 \text{ m/s}, \; r_i = 5e3 \text{ m}$

Final state: $r \to \infty, \; v_f \to 0$

$$\Delta E = \Delta K + \Delta U = 0$$

$$K_f - K_i + U_f - U_i = 0$$

$$\frac{1}{2} m(\cancel{v_f^2} - v_i^2) + \frac{-GM\cancel{m}}{\cancel{r_f}} - \frac{-GMm}{r_i} = 0$$

$$-\frac{1}{2} m v_{esc}^2 + \frac{GMm}{r_i} = 0$$

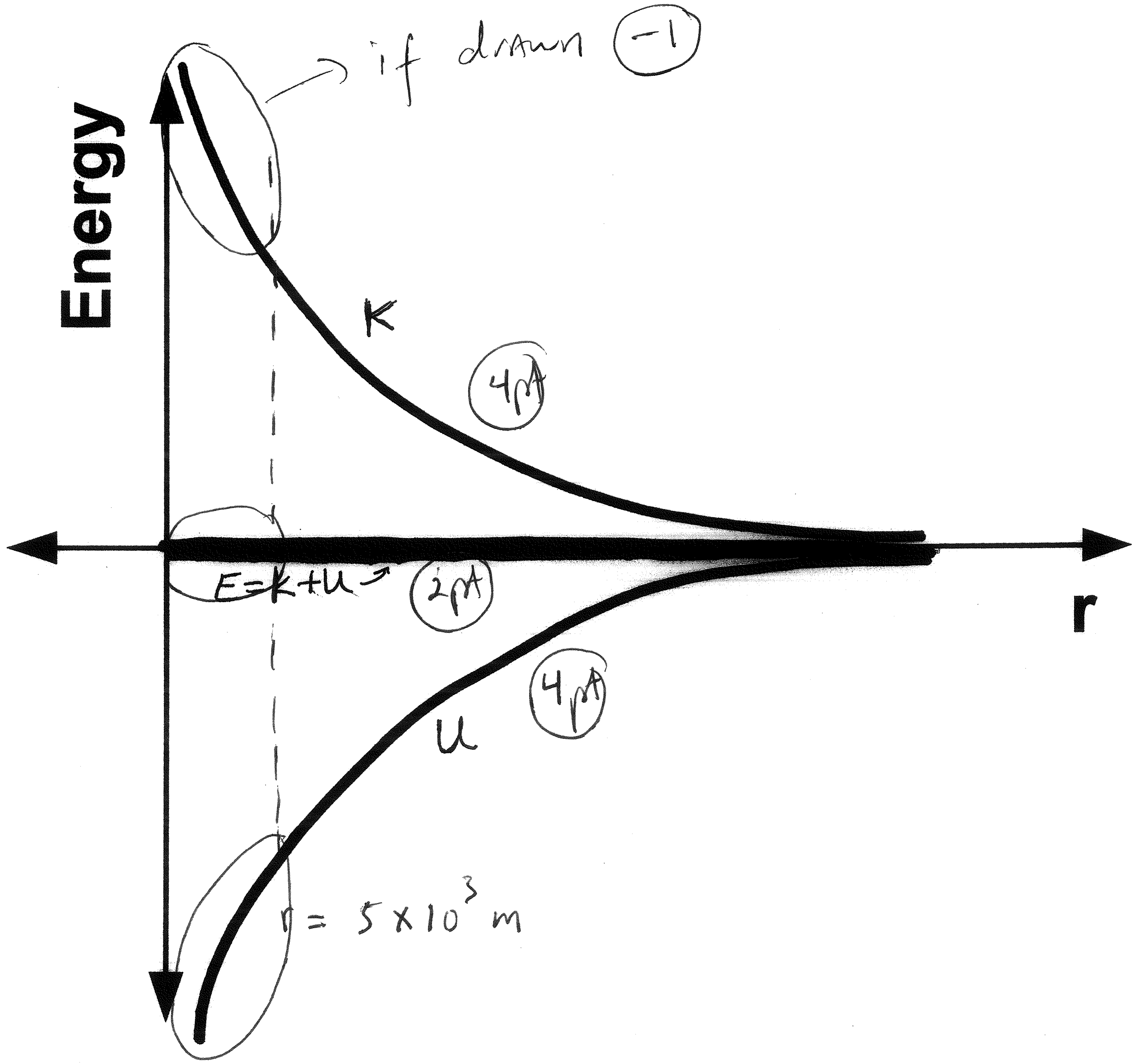
$$\frac{GM\cancel{m}}{r_i} = \frac{1}{2}\cancel{m} v_{esc}^2$$

$$\frac{GM}{r_i} = \frac{1}{2} v_{esc}^2$$

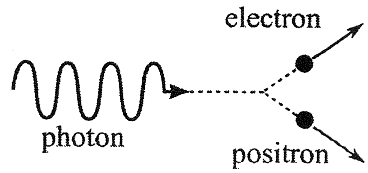
$$M = \frac{r_i v_{esc}^2}{2G} = \frac{(5e3\,\cancel{m})(44.7\,\cancel{m}/\cancel{s})^2}{(2)(6.7e{-11}\,\cancel{m^3}/\cancel{s^2}kg)} = \boxed{7.46e16 \text{ kg}}$$

−1.0
−2.0
−4.5
−12

(b 10pts) On the graph below plot the kinetic, potential and total energy (K,U,E) for the rock+asteroid system when the rock is thrown at 44.7 m/s. Please be sure to label your graphs.



Problem 4 (25 Points)



When a high-energy photon passes near the nucleus of an atom, it may be converted into an electron and a positron ("pair production"), as seen in the figure shown above. That is, the photon is transformed into the electron and positron; the original photon ceases to exist. A positron has the same mass as an electron (9.1×10^{-31} kg), but the opposite charge (electron charge= -1.6×10^{-19} coulombs, positron charge= $+1.6 \times 10^{-19}$ coulombs).

(a 10pts) A photon of energy 1.8×10^{-13} J produces an electron-positron pair. When the electron and positron are 1.0×10^{-14} m from each other, what is the total kinetic energy of the electron-positron system?

Initial state: photon; $E_\gamma = 1.8e{-13}$ J

Final state: e+ & e− @ r = 1e-14 m; U_{elec}, K_{sys}, E_{rest+}, E_{rest-}

$\Delta E = 0 \Rightarrow E_i = E_f$

$E_\gamma = U_{elec} + K_{sys} + E_{rest+} + E_{rest-}$

$E_\gamma = \frac{kq_+q_-}{r} + K_{sys} + 2E_{rest}$

$E_\gamma = \frac{-kq^2}{r} + K_{sys} + 2m_ec^2$

$K_{sys} = E_\gamma + \frac{kq^2}{r} - 2m_ec^2$

$\rightarrow K_{sys} = (1.8e{-13}) + \frac{(9e9)(1.6e{-19})^2}{1e{-14}} - (2)(9.1e{-31})(3e8)^2$

$\boxed{K_{sys} = 3.924e{-14} \text{ Joules}}$

(around 245 keV)

$\boxed{\begin{array}{l} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$

(b 10pts) What is the total kinetic energy of the electron-positron system when the electron and positron are very far away from each other?

Initial state: e+ & e− @ r = 1e-14 m; U_i, K_i, E_{rest+}, E_{rest-}

Final state: e+ & e− @ $r \to \infty$; U_f, K_f, E_{rest+}, E_{rest-}

$\Delta E = 0 \Rightarrow E_i = E_f$

$U_i + K_i + \cancel{E_{rest+}} + \cancel{E_{rest-}} = U_f + K_f + \cancel{E_{rest+}} + \cancel{E_{rest-}}$

$\frac{-kq^2}{r_i} + K_i = \cancel{\frac{-kq^2}{r_f}}^{0} + K_f$

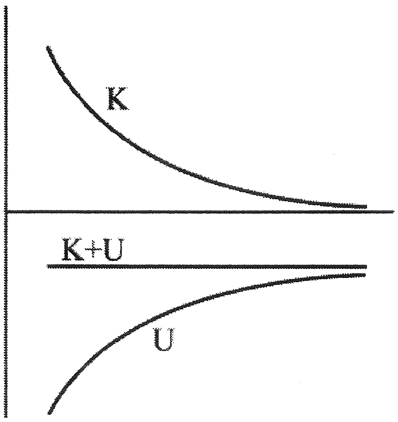
$K_f = K_i - \frac{kq^2}{r_i} = (3.924e{-14}) - \frac{(9e9)(1.6e{-19})^2}{1e{-14}} = \boxed{1.62e{-14} \text{ Joules}}$

(around 100 keV)

$\boxed{\begin{array}{l} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$

(c 5pts) Consider the initial state of the system to be when the electron and positron are 1.0×10^{-14} m from each other, and the final state when they are far away from each other. Below, choose (circle one) the correct graph that plots the the potential energy U, total kinetic energy K, and total K+U vs. separation for the electron-positron system

1



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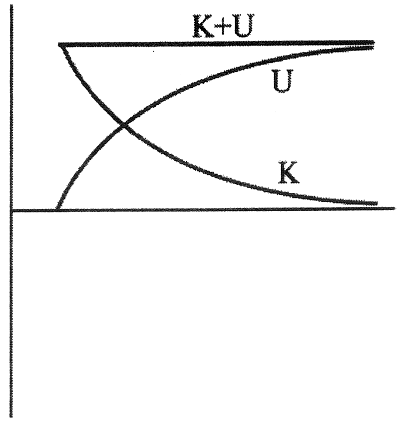

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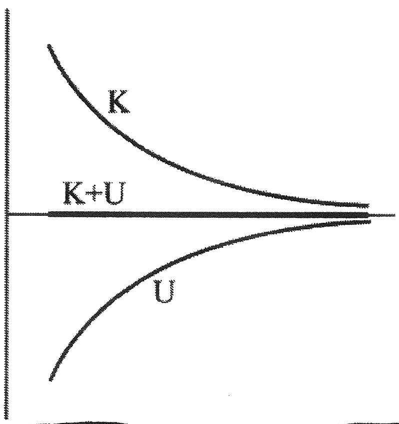

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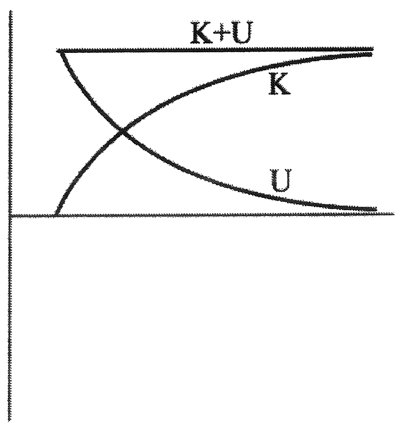

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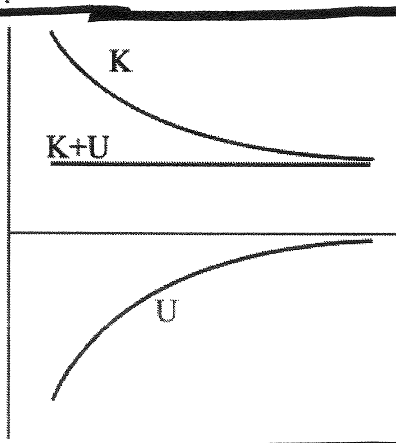

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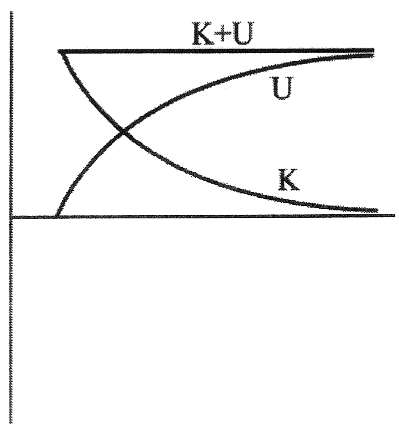


All

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$\left|\vec{F}_{grav}\right| \approx mg$ near Earth's surface

$\Delta U_{grav} \approx mg\Delta y$ near Earth's surface

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

$$\Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

$$\text{prob}(E) \propto \Omega(E)\,e^{-\frac{E}{kT}}$$

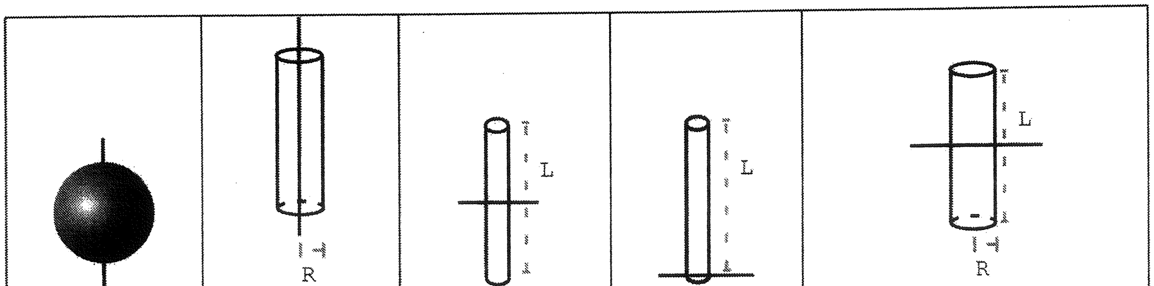
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}