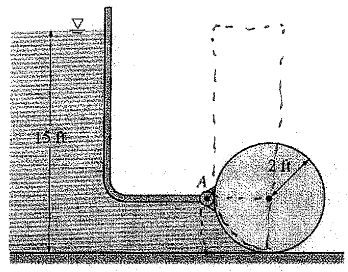
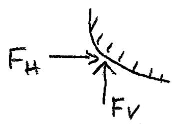
CEE 3040 Spring 2010 Final (Sample) Name:__________________

Problem 3 (25 pts)

A long, solid cylinder of radius 2 ft hinged at point A is used as an automatic gate, as shown. When the water level reaches 15 ft, the cylindrical gate opens by turning about the hinge at point A. Determine (a) the hydrostatic force per ft length acting on the cylinder and the angle at which it acts (relative to horizontal) when the gate opens, and (b) the weight of the cylinder per ft length of the cylinder. The properties of water are ρ = 1.94 slug/ft^3, μ = 2.0×10^{-5} lbs/ft^2, g = 32.2 ft/s^2.



(a) consider the forces acting on the cylinder surface

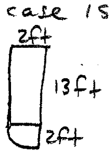


$F_H = P_{cv} A_V = \rho g h_c A = 1.94 \frac{slug}{ft^3}(32.2 \frac{ft}{s^2})(13ft + \frac{2ft}{2})(1ft \times 2ft)$

$= 1749 \text{ lb} \rightarrow$

$F_V = \gamma V$ acting upward

the volume in this case is an imaginary volume "above" the cylinder surface



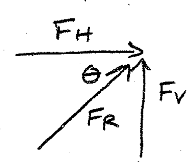
$V = (2ft \times 13ft \times 1ft) + \frac{1}{4}\pi(2ft)^2(1ft)$

$V = 29.1 \text{ ft}^3$

$F_V = 1.94 \frac{slug}{ft^3}(32.2 \frac{ft}{s^2})(29.1 ft^3) = 1818 \text{ lb} \uparrow$

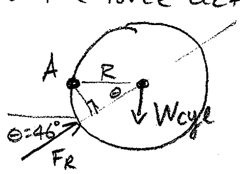
$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{(1749 \text{ lb})^2 + (1818 \text{ lb})^2} = 2523$ lb per ft

$\tan\theta = \frac{F_V}{F_H} = \frac{1818 \text{ lb}}{1749 \text{ lb}} \rightarrow \theta = 46°$



b) we know the force acts through center of cylinder

FBD



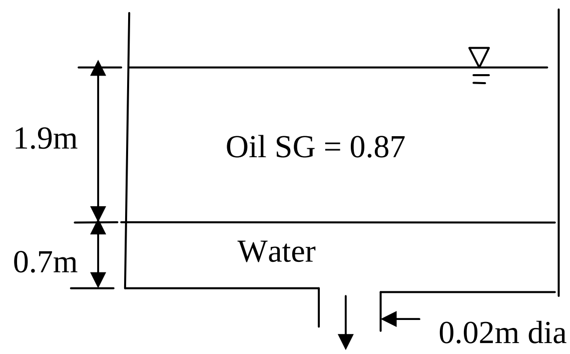
$\Sigma M_A = 0 = -W_{cyl} R + F_R (R \sin\theta)$

$0 = -(W_{cyl})(2ft) + 2523 \text{ lb}(2ft \sin 46°)$

$W_{cyl} = 1815$ lb per ft

Problem 2 (5pts)

Consider a rectangular oil tank (2.6m x 9.5m) with a water layer at the bottom. How long will it take for the water to drain through a 0.02m diameter drain hole?



$P_1 = P_o + \gamma_o h_o = 0 + 0.87\gamma_w(1.9m)$

Bernoulli Eqn from $1 \rightarrow 2$

$$\frac{P_1}{\gamma_w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma_w} + \frac{V_2^2}{2g} + z_2$$

$$\frac{0.87\cancel{\gamma_w}(1.9m)}{\cancel{\gamma_w}} + \cancel{\frac{V_1^2}{2g}}^{\text{small (i.e. quasi steady)}} + h(t) = 0 + \frac{V_2^2}{2g} + (0).$$

$$V_2 = \left[2(9.81\,m/s^2)\left(0.87(1.9m) + h\right)\right]^{1/2}$$

$$V_2 = 4.43\,(1.65 + h)^{\frac{1}{2}}$$

$$Q = V_2 A_2 = \frac{\pi}{4}(0.02m)^2\left[4.43\,(1.65+h)^{1/2}\right]$$

$$= 1.39\times10^{-3}(1.65+h)^{1/2}$$

$$Q = A_1(V_1) = A_1\left(-\frac{dh}{dt}\right) = (2.6m)(9.5m)\left(-\frac{dh}{dt}\right)$$

combine

$$-24.7\frac{dh}{dt} = 1.39\times10^{-3}(1.65+h)^{1/2}$$

$$\frac{dh}{(1.65+h)^{1/2}} = -5.63\times10^{-5}\,dt$$

integrate.

$$\int_{0.7m}^{h=0}\frac{dh}{(1.65+h)^{1/2}} = -5.63\times10^{-5}\int_{t=0}^{t_{final}}dt$$

$$2(1.65+h)^{1/2}\Big|_{0.7}^{0} = -5.63\times10^{-5}\,t_f$$

$$t_f = 8.83\times10^3\,s = 2.45\,hr$$

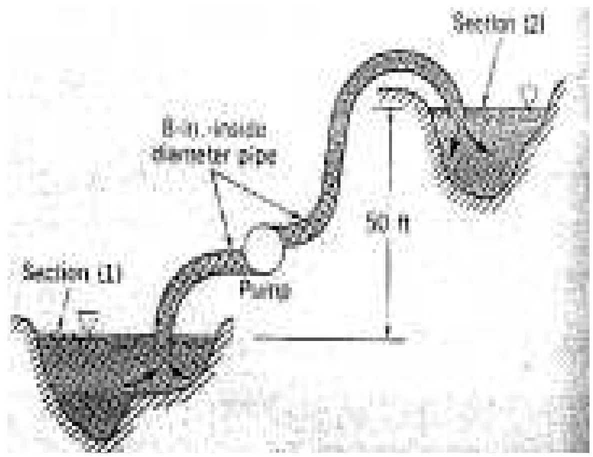
Problem 3 (5 pts)

Water is moved from one large reservoir to another at 50ft higher elevation as shown. The head loss associated with $Q=2.5ft^3/s$ is $h_L=61V^2/(2g)$, where V is the average velocity of water in the 8-in diameter pipe. Fine:

(a) the average velocity in the pipe, V

(b) the pump head, h_p

(c) the pump work rate, W_p (in ftlb/s)



(a) $V = \frac{Q}{A} = \frac{2.5\ ft^3/s}{\left(\frac{8}{12}\right)^2 \frac{\pi}{4}} = 7.16\ ft/s$

(b) energy eqn between ① & ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$h_p = 50ft + 61\frac{(7.16\ ft/s)^2}{2(32.2\ ft/s^2)}$$

$$h_p = 98.6\ ft$$

(c) $$\dot{W}_p = \gamma Q h_p$$

$$= \left(1.94\ \frac{slug}{ft^3}\right)(32.2\ ft/s^2)(2.5\ ft^3/s)(98.6\ ft)$$

$$= 15,400\ \frac{lb\,ft}{s}$$

Problem 4 (5pts)

The pressure rise, Δp, across a pump can be expressed as Δp=f(D,ρ,ω,Q) where D is the impeller diameter, ρ the fluid density, ω the rotational speed, and Q the flowrate. Determine a suitable set of dimensionless parameters.

1. $\Delta p = f(D, \rho, \omega, Q)$

2. $\Delta p[ML^{-1}T^{-2}] = f(D[L], \rho[ML^{-3}], \omega[T^{-1}], Q[L^3T^{-1}])$

3. Dimensions = 3 , 5 − 3 = 2 dimensionless groups

4. repeating variables: D, ρ, ω

5. $\pi_1 = \Delta p D^a \rho^b \omega^c$

$$= [ML^{-1}T^{-2}][L]^a[ML^{-3}]^b[T^{-1}]^c = M^0L^0T^0$$

for M $\Rightarrow$ $1 + b = 0 \Rightarrow b = -1$

for T $\Rightarrow$ $-2 - c = 0 \Rightarrow c = -2$

for L $\Rightarrow$ $-1 + a - 3b = 0 \Rightarrow a = -1 - 3 = -2$

$$\Rightarrow \pi_1 = \frac{\Delta p}{D^2 \rho \omega^2}$$

6. $\pi_2 = Q D^a \rho^b \omega^c$

$$= [L^3T^{-1}][L]^a[ML^{-3}]^b[T^{-1}]^c = M^0L^0T^0$$

for M $\Rightarrow$ $b = 0$

for T $\Rightarrow$ $-1 - c = 0 \Rightarrow c = -1$

for L $\Rightarrow$ $3 + a - 3b = 0 \Rightarrow a = -3$

$$\Rightarrow \pi_2 = \frac{Q}{D^3 \omega}$$

7. check.

8. $$\frac{\Delta p}{D^2 \rho \omega^2} = F\left(\frac{Q}{D^3 \omega}\right)$$

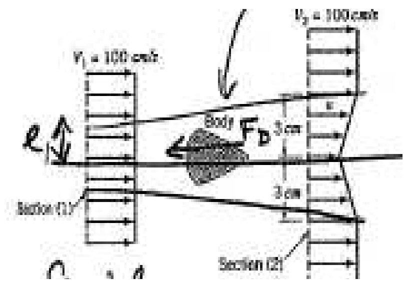
Problem 5 (5pts)

The drag force on a yellowfin tuna model is measured in a water (ρ=1000 kg/m3, μ=10-3Ns/m2, g=9.81m/s2) channel. The approaching flow is uniform at 100cm/s. The downstream velocity can be described as

$$u = 100 - 30\left(1 - \frac{|y|}{3}\right), \quad |y| < 3cm$$

$$u = 100, \qquad |y| > 3cm$$

where u is the velocity in cm/s and y is the distance from the centerline in cm. Assume the flow is uniform into the paper. Find the drag force on the model per unit length into the paper.



use cons of mass to find ℓ_1

$$-\ell_1 u_1 + \int_0^{3cm} 100 - 30\left(1 - \frac{y}{3}\right) dy = 0$$

$$-\ell_1 (100\,cm/s) + \left[100y - 30y + 30\frac{y^2}{2}\frac{1}{3}\right]_0^{3cm} = 0$$

$$-\ell_1 (100\,cm/s) + \left[70(3cm) + 30\frac{(3cm)^2}{6}\right] = 0$$

$$\ell_1 = 2.55cm$$

cons of momentum

$$\sum F_x = \cancel{\frac{d}{dt}\int_{cv} u\rho\, dV}^{\text{steady}} + \int_{cs} u\rho\, V \cdot dA$$

b = width into paper

$$-\frac{F_D}{b} = 2\int_0^{3cm} u_2 \rho u_2\, dy - 2u_1\rho u_1 \ell_1$$

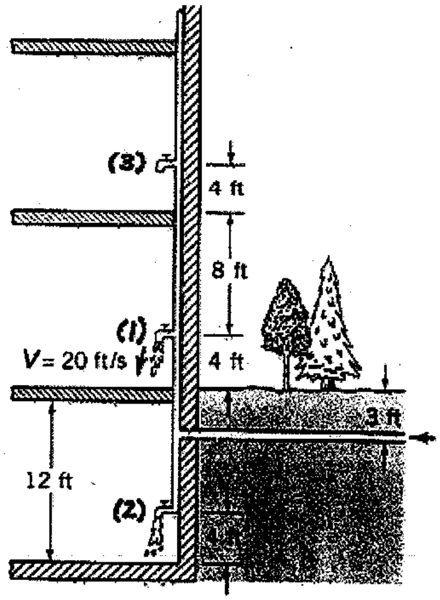
$$-\frac{F_D}{b} = 2\rho\int_0^{3cm} (70+10y)^2 dy - 2\left(100\frac{cm}{s}\right)^2\left(0.001\frac{kg}{cm^3}\right)(2.55cm)$$

$$-\frac{F_D}{b} = 2\left(0.001\frac{kg}{cm^3}\right)\frac{1}{3(10)}(70+10y)^3\Big|_0^{3cm} - 51\frac{N}{m}$$

$$\frac{F_D}{b} = 7.2\frac{N}{m}$$

Problem 6 (5pts)

Water (ρ = 1.94 slug/ft^3, g = 32.2 ft/s^2) flows from the faucet on the first floor of the building shown in the figure with a maximum velocity of 20 ft/s. For steady inviscid flow, (a) determine the maximum water velocity from the basement faucet and from the faucet on the second floor (assume each floor is 12 ft tall); (b) a pump will be necessary if the maximum velocity on the second floor also needs to reach 20 ft/s. What is power of the pump (diameter of the faucet is 1 in)?



(a) $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$

1'

$P_1 = P_2 = 0$, $V_1 = 20\,ft/s$, $z_1 = 4\,ft$, $z_2 = -8\,ft$

$\Rightarrow \frac{(20\,ft/s)^2}{2\cdot(32.2\,ft/s^2)} + 4\,ft = \frac{V_2^2}{2(32.2\,ft/s)^2} + (-8\,ft)$

1' $\Rightarrow V_2 = 34.2\,ft/s$

(b) $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3$

1'

$P_1 = P_3 = 0$, $V_1 = 20\,ft/s$, $z_1 = 4\,ft$, $z_3 = 16\,ft$

$\Rightarrow \frac{(20\,ft/s)^2}{2(32.2\,ft/s^2)} + 4\,ft = \frac{V_3^2}{2(32.2\,ft/s^2)} + 16\,ft$

1' $\Rightarrow V_3 = \sqrt{-373}$ $\Rightarrow$ no flow

(c) $\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 + h_p = \cancel{\frac{P_3}{\gamma}} + \cancel{\frac{V_3^2}{2g}} + z_3$

1' $\Rightarrow h_p = z_3 - z_1 = 12\,ft$

1' $\Rightarrow \dot{W}_p = \dot{m} g h_p = Q \gamma h_p = 20\,ft/s \cdot \frac{\pi}{4}\left(\frac{ft}{12}\right)^2 (32.2\,ft/s^2)(1.94\,slug/ft^3)\cdot 12\,ft$

$= 81.77\ slug\cdot ft^2/s^2$