

MATH 2401 QUIZ 5

Problem 1. Evaluate: $\iint_R xy e^{xy^2} dA$ where $R: 0 \leq x \leq 2 \quad 0 \leq y \leq 1$.

$$\begin{aligned} &= \int_0^2 \int_0^1 xy e^{xy^2} dy dx &= \int_0^2 \left[\frac{1}{2} e^{xy^2} \right]_0^1 dx &= \int_0^2 \frac{1}{2} e^x - \frac{1}{2} dx \\ &= \frac{1}{2} e^2 - 1 - \frac{1}{2} &= \frac{1}{2} e^2 - \frac{3}{2} \end{aligned}$$

Problem 2. Sketch the region of integration and evaluate: $\int_0^1 \int_y^{y^2} dx dy$

$$\begin{aligned} &= \int_0^1 [x]_y^{y^2} dy &= \int_0^1 y^2 - y dy &= \left[\frac{1}{3} y^3 - \frac{1}{2} y^2 \right]_0^1 \\ &= \frac{1}{3} - \frac{1}{2} &= -\frac{1}{6} \end{aligned}$$

Problem 3. Sketch the region of integration, reverse the order of integration, and evaluate the integral. (Note that no credit will be given if the integral is evaluated without the order of integration being reversed)

$$\begin{aligned} &\int_0^2 \int_x^2 2y^2 \sin(xy) dy dx \\ &= \int_0^2 \int_0^y 2y^2 \sin(xy) dx dy &= \int_0^2 [-2y \cos(xy)]_0^y dy &= \int_0^2 (2y - 2y \cos(y^2)) dy \\ &= [y^2 - \sin(y^2)]_0^2 &= 4 - \sin 4 \end{aligned}$$