MATH 2403 H1-H3. Differential Equations. Quiz 2. Jan 21, 2014.
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Student's Name: ____ Section ________

Show all work to receive credit

1. Use Euler's method to calculate the approximation to the solution of

$$\frac{dy}{dt} = 1 + t - 2y, \; y(0) = 1,$$

at the points $t = 1, 2, 3$.

Euler's Method: $y_{n+1} = y_n + h f(t_n, y_n)$.

$h = 1$.

t_n	y_n
0	1
1	$1 + 1(1 + 0 - 2(1)) = 0$
2	$0 + 1(1 + 1 - 2(0)) = 2$
3	$2 + 1(1 + 2 - 2(2)) = 1$

2. Solve the initial value problem

$$y' = 2y^2 + xy^2, \; y(0) = 1,$$

and determine where the solution attains its minimum value.

$$\frac{dy}{dx} = 2y^2 + xy^2 = (2+x)y^2 \Rightarrow \int \frac{dy}{y^2} = \int (2+x)dx$$

$$\Rightarrow -\frac{1}{y} = 2x + x^2/2 + C$$

$$y(0) = 1 \Rightarrow -1 = C \Rightarrow -\frac{1}{y} = 2x + \frac{x^2}{2} - 1$$

$$\therefore \; y = \frac{-1}{-1 + 2x + x^2/2}$$

The minimum is reached when $\frac{dy}{dx} = 0$. From the equation, this happens for $y = 0$ or $x = -2$. Since $y = 0$ cannot happen, then $\underline{x = -2}$