MATH 2602, Midterm 2

July 2nd, 2012

Name: ____________________________ **GTID:**____________________________

Section: ___________________________

Problem	*Points*
1	
2	
3	
4	
5	

TOTAL:________________

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (24 points).

Determine whether each of the following statements is true-or-false. If the statement is true, circle the "**T**"; if false, circle the "**F**".

[**T** \ **F**] A general graph is Eulerian if and only if every vertex of the graph is even.

[**T** \ **F**] An Eulerian graph is Hamiltonian, but a Hamiltonian graph is not necessarily Eulerian.

[**T** \ **F**] A graph that contains a proper cycle cannot be Hamiltonian.

[**T** \ **F**] Any edge added to a tree must produce a cycle.

[**T** \ **F**] The complete graph K_4 has four vertices and four edges.

[**T** \ **F**] If A is the adjacency matrix of the graph K_5, then the (2, 4)-entry of A^2 is 4.

[**T** \ **F**] A tree with more than one vertex has at most two leaves.

[**T** \ **F**] If a graph G has a unique spanning tree, then G is a tree.

Problem 2 (21 points).

A die is tossed ten times and the sequence of the outcomes is observed.

1. How many different sequences are possible?
2. How many of these sequences contain exactly two 1's?
3. How many of these sequences contain at most two 1's?

Problem 3 (15 points).

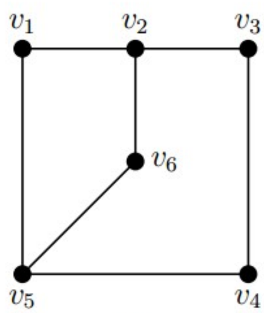
Find the coefficient of x^6 in the binomial expansion of

$$\left(2x + \frac{3}{x^2}\right)^{18}$$

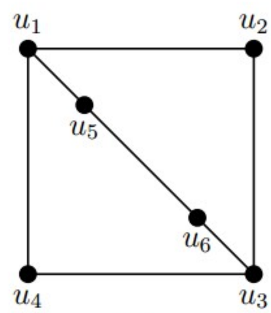
Problem 4 (28 points).

Given the following two graphs G_1 and G_2

1. Find the adjacency matrix of G_1;
2. Explain why G_1 is not Hamiltonian;
3. Explain why G_2 is not Eulerian;
4. Show that G_1 and G_2 are isomorphic.



G_1



G_2

Problem 5 (12 points).

Do **ONE** of the following two problems.

a) In a group of $2n$ people, each person has at least n friends. Show that the group can be seated in a circle, each person next to at least one friend.
Hint: Consider a graph G on $2n$ vertices. $v_a v_b$ is an edge in graph G if and only if person a and person b are friends.

b) Prove that

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$

Hint: Consider drawing n balls from a box with $2n$ balls.