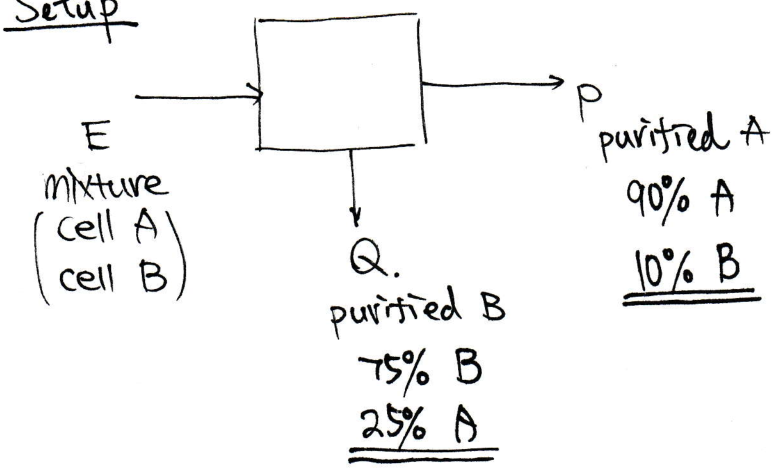
Name: Solution - Exam 1

Problem #1 (25 points)

A mixture of 2 cells types (A and B) is passed through a cell separator to yield enriched fractions of "A" and "B" cells. It is found that the final purity of the "A" fraction is 90%, whereas the "B" fraction is only 75%. If the volume of the purified "A" sample is half of the purified "B" sample, what was the ratio of A:B cells in the original mixture?

Setup



Given:
- 90% of P is cell type A
- 75% of Q is cell type B
- $2P = Q$.

Assumption:
- Steady-state
- No accumulation of either cell types.
- Given fractions are by volume.
- Mixture consists of A & B only.

Basis: total volume of $E = 1L$.
(it doesn't matter which unit of volume you pick)

Assume the fraction of cell type A in E is x

then the fraction of cell type B in E is $(1-x)$

Balance equations (F_A^E means the fraction of A in E)

- cell type A:

$$E \cdot F_A^E = P \cdot F_A^P + Q \cdot F_A^Q$$

and $2P = Q$

- cell type B:

$$E \cdot F_B^E = P \cdot F_B^P + Q \cdot F_B^Q$$

Problem #1 (cont'd)

Substitution

$F_A^E = x$, $F_B^E = 1-x$

$F_A^P = 0.9$, $F_B^P = 0.1$

$F_A^Q = 0.25$, $F_B^Q = 0.75$

$E = 1$.

$\Rightarrow \begin{cases} P = 10/3 \ , \ Q = 20/3 \\ x = 0.47 \sim \text{fraction of A in E} \\ 1-x = 0.53 \sim \text{fraction of B in E} \end{cases}$

$\therefore$ In the original mixture (E)

$$\frac{A}{B} = \frac{0.47}{0.53} = \boxed{47/53}$$

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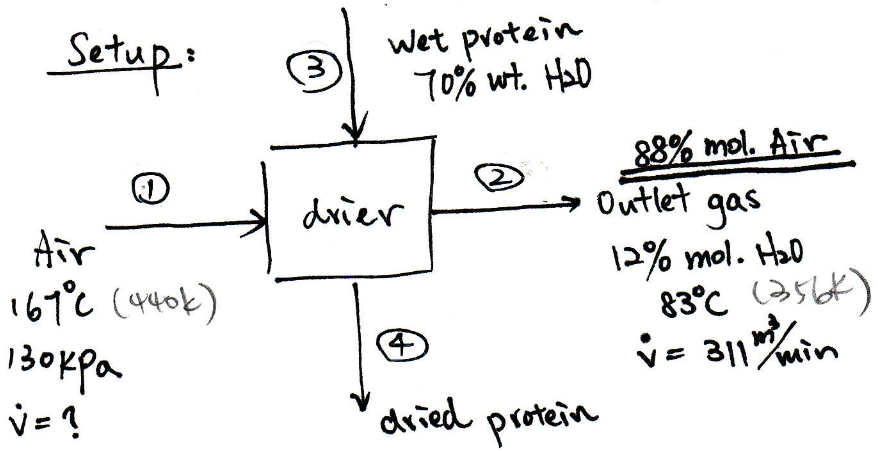
Problem #2 (25 points)

Air is introduced into a spray drier at 167°C and 130 kPa to dry a partially hydrated protein powder that initially contains 70% water by mass. If the outlet gas containing 12% (mol) water leaves the drier at 83°C at a rate of 311 m^3/min, what is the volumetric flow rate of the inlet air?

Setup:



Given:

- $T_1 = 167°C$
- $P_1 = 130\ kPa$
- 70% wt. in ③ is H_2O
- $T_2 = 83°C$
- $\dot{V}_2 = 311\ m^3/min$
- 12% mol. in ② is H_2O

Basis: 1 min

Assumptions:

- air can be treated as ideal gas
- No accumulation
- Pure air in ①
- P_2 = 1atm (contact with open air)
- ③ and ④ don't contain any air.

Balance equations:

To solve this problem, all you need to do is to apply balance equation on air between ① and ②

$$\Rightarrow n^1_{air} = n^2_{air}$$

$$\Rightarrow n^1_{air} = n^1_{total} = n^2_{air} = 0.88 n^2_{total}$$

$$\Rightarrow \frac{P_1 V_1}{\cancel{R} T_1} = 0.88 \frac{P_2 V_2}{\cancel{R} T_2}$$

Substitution

$P_1 = 130\ kPa$	$P_2 = 1atm = 101.3\ kPa$
$V_1 = ?$	$V_2 = 311\ m^3$
$T_1 = 167°C = 440K$	$T_2 = 83°C = 356K$

$\therefore\ V_1 = 263.58\ (m^3) \Rightarrow$ flow rate of inlet air = $\boxed{263.58\ m^3/min}$

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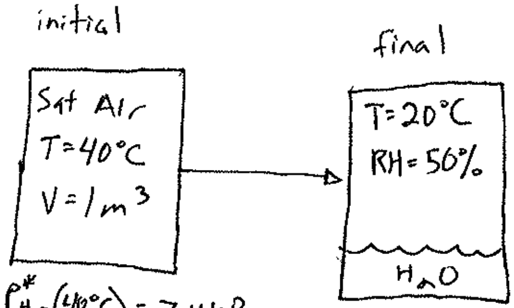
(3)



$p^*_{H_2O}(40°C) = 7.4\ kPa$ $\qquad$ $p^*_{H_2O}(20°C) = 2.3\ kPa$

Basis: initial conditions

Assume: Initially at 1 atm
isovolumetric ($\Delta V = 0$) process
air at end is 50% saturated

$$P_{TOT} V = n_{TOT} R T$$
$$(101.3)(1) = n_{TOT}(8.314)(40+273)$$
$$n_{TOT} = 0.03893 \text{ kg mol}$$

$$\frac{p^*_{H_2O}}{P_{Air}} = \frac{n_{H_2O}}{n_{Air}}$$
$$\frac{p^*_{H_2O}}{P_{TOT} - p^*_{H_2O}} = \frac{n_{H_2O}}{n_{TOT} - n_{H_2O}}$$
$$\frac{7.4}{101.3 - 7.4} = \frac{n_{H_2O}}{0.03893 - n_{H_2O}}$$
$$\Rightarrow n_{H_2O} = 0.00284 \text{ kg mol}$$

$$\cancel{P_f V = n_{TOT} R T_f}$$
$$\cancel{P_f (1) = (0.03893)(8.314)(20+273)}$$
$$\cancel{P_f = 94.8277\ kPa}$$

$$RH = 0.5 = \frac{P_{H_2O}}{P^*_{H_2O}} = \frac{P_{H_2O}}{2.3\ kPa}$$
$$\Rightarrow P_{H_2O} = 1.15\ kPa$$
$$P_{H_2O} V = n_{H_2O} R T$$
$$(1.15)(1) = n_{H_2O}(8.314)(273+20)$$
$$n_{H_2O} = 0.000472 \text{ kg mol}$$

$$H_2O_{liq} = H_2O_{vap,init} - H_2O_{vap,final}$$
$$= 0.00284 \text{ kg mol} - 0.000472 \text{ kg mol}$$
$$= 0.00237 \text{ kg mol}$$

$$0.00237 \text{ kg mol } H_2O \times \frac{1000 \text{ gmol}}{\text{kg mol}} \times \frac{18.0\ g}{1\ \text{gmol}} \times \frac{mL}{1\ g} = \boxed{42.68\ mL\ H_2O \text{ at bottom of tank}}$$

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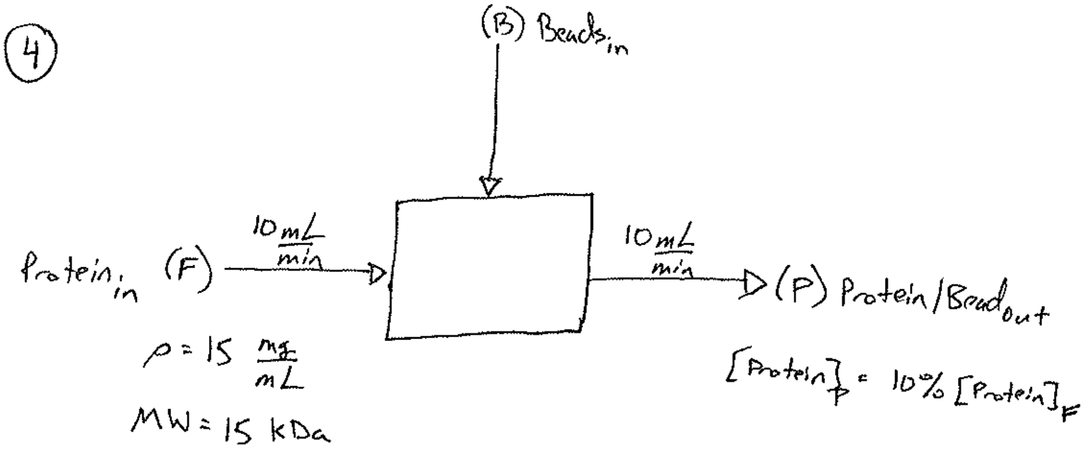
Basis: 1 min



$$\text{Protein}_{in} = 1\ \text{min} \times \frac{10\ \text{mL}}{\text{min}} \times \frac{15\ \text{mg}}{\text{mL}} \times \frac{\text{g}}{1000\ \text{mg}} \times \frac{\text{kg}}{1000\ \text{g}} \times \frac{\text{mol}}{15\ \text{kg}} = 1.00\text{e-}5\ \text{mol Protein in}$$

$$\text{Protein}_{out} = 10\%\ \text{Protein}_{in} = (0.1)(1.00\text{e-}5) = 1.00\text{e-}6\ \text{mol protein}_{out}$$

$$\text{Protein}_{absorbed} = \text{Protein}_{in} - \text{Protein}_{out} = (1.00\text{e-}5) - (1.00\text{e-}6) = 9.00\text{e-}6\ \text{mol protein absorbed by spheres}$$

$$9.00\text{e-}6\ \text{mol Protein absorbed} \times \frac{1\ \text{bead}}{1200\ \text{proteins absorbed}} = \boxed{7.5\text{ e-}9\ \frac{\text{mol beads}}{\text{min basis}}}$$