Quiz 12 (25 mins), MA1501 Calculus I, 11/26/2013 PRINTED NAME: ________________ GT ID: ____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

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1. (30 pts) Evaluate the integral using integration by parts

$$\int x(\ln x)^2 dx$$

Answer:

$$\begin{aligned}
\int x(\ln x)^2 dx &= \frac{1}{2}x^2(\ln x)^2 - \int \frac{1}{2}x^2 \cdot 2(\ln x) \cdot \frac{1}{x} dx \\
&= \frac{1}{2}x^2(\ln x)^2 - \int x \ln x dx \\
&= \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx\right) \\
&= \frac{1}{2}x^2(\ln x)^2 - \left(\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x dx\right) \\
&= \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2 \ln x + \frac{1}{4}x^2 + C
\end{aligned}$$

2. (30 pts) Evaluate the following integral

$$\int \sin^3 x \cos^3 x \, dx$$

Answer: There are two ways to evaluate the integral.
Method 1) Using $\sin^2 x = 1 - \cos^2 x$,

$$\begin{aligned}
\int \sin^3 x \cos^3 x \, dx &= \int (1 - \cos^2 x) \sin x \cos^3 x \, dx \\
&= \int (\cos^3 x - \cos^5 x) \sin x \, dx.
\end{aligned}$$

Let $u = \cos x$ and $du = -\sin x dx$.

$$\begin{aligned}
\int \sin^3 x \cos^3 x \, dx &= \int (u^3 - u^5)(-1) du \\
&= \int (-u^3 + u^5) du \\
&= -\frac{1}{4}u^4 + \frac{1}{6}u^6 + C \\
&= -\frac{1}{4}\cos^4 x + \frac{1}{6}\cos^6 x + C
\end{aligned}$$

Method 2) Using $\cos^2 x = 1 - \sin^2 x$,

$$\begin{aligned}
\int \sin^3 x \cos^3 x \, dx &= \int \sin x^3 (1 - \sin^2 x) \cos x \, dx \\
&= \int (\sin^3 x - \sin^5 x) \cos x \, dx.
\end{aligned}$$

Let $u = \sin x$ and $du = \cos x dx$.

$$\begin{aligned}
\int \sin^3 x \cos^3 x \, dx &= \int (u^3 - u^5) du \\
&= \frac{1}{4}u^4 - \frac{1}{6}u^6 + C \\
&= \frac{1}{4}\sin^4 x - \frac{1}{6}\sin^6 x + C
\end{aligned}$$

3. (40 pts) Using a trigonometric substitution, evaluate

$$\int \frac{dx}{\sqrt{9+x^2}}.$$

Answer: We set

$$x = 3\tan\theta, \qquad dx = 3\sec^2\theta d\theta, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2},$$

$$9 + x^2 = 9\sec^2\theta.$$

Then

$$\begin{aligned} \int \frac{dx}{\sqrt{9+x^2}} &= \int \frac{3\sec^2\theta d\theta}{\sqrt{9\sec^2\theta}} = \int \frac{\sec^2\theta d\theta}{|\sec\theta|} \\ &= \int \sec\theta d\theta \quad (\because \sec\theta > 0, -\frac{\pi}{2} < \theta < \frac{\pi}{2}) \\ &= \ln|\sec\theta + \tan\theta| + C \end{aligned}$$

From $\tan\theta = \frac{x}{3}$, we consider a reference triangle with opposite$=3$, adjacent$=x$ and hypotenuse$=\sqrt{9+x^2}$. Therefore, we have

$$\int \frac{dx}{\sqrt{9+x^2}} = \ln\left|\frac{\sqrt{9+x^2}}{3} + \frac{x}{3}\right| + C.$$