PHYS 2211 Test 2
Spring 2013

Name(print) Key **Lab Section** 

Fenton(K), Curtis(N), Greco(HP/M)			
Day	12-3pm	3-6pm	6-9pm
Monday	M01 K01	M02 N01	
Tuesday	M03 N03	M04 K03	K02 N02
Wednesday	K05 N05	M05 N06	M06 K06
Thursday	K07 N07	M07 K08	M08 N08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Tom Marvolo Riddle

Sign your name on the line above

PHYS 2211

Do not write on this page!

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

An electron interacts with a negatively charged molecule with net charge $-10e$. Below is an incomplete VPython program to calculate the position of the electron moving near the molecule. Fill in the missing VPython statements below to update the position of the electron. You may assume that the molecule is massive enough that it will remain motionless. The electron and molecule are far from any other objects and we will assume that they only interact through the **electric force**.

```
from visual import *
# Objects
molecule = sphere(pos=vector(0,0,0),color=color.black,radius=5e-6)
electron = sphere(pos=vector(1e-10,6e-10,0),color=color.gray,radius=5e-8)

# Charge and Mass
epsilon0  = 8.85e-12 %Electric field constant
e         = 1.6e-19 %Charge of a proton
melectron = 9e-31 %Mass of an electron

# Initial values
pelectron = melectron*vector(2e4,-7e4,0)
deltat = 1e-3
t = 0
while = t<100
# (a 15pts) Update the position of the electron
 pinitial = mag(pelectron)        # For Part (b)
 r = electron.pos - molecule.pos
 rmag = mag(r)           } don't worry about syntAX
 rhat = norm(r)
 Fmag = (1/(4 * 3.14 * epsilon0))* (-10 * e) * (-e) / rmag**2
 Fnet = Fmas * rhat
 pelectron = pelectron + Fnet * deltat
 pfinal = mag(pelectron)          # For Part (b)
 electon.pos = electon.pos + (pelectron/melectron)* deltat
```

5pts

5pts

5pts

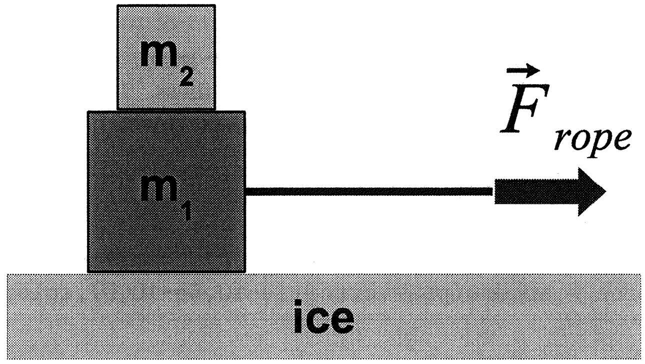
```
# (b 10pts) Calculate the components of the net force on the electron.
    Fnet_tangent = ((pfinal-pinitial)/deltat) * (pelectron/mag(pelectron))
    Fnet_perpendicular = Fnet - Fnet_tangent

    t = t + deltat
```

5pts

Problem 2 (25 Points)

A block of mass m_1 is pulled over ice (no friction) by a horizontal rope. A second block of mass m_2 sits on top of the first block as indicated in the figure. The coefficient of static friction between the two blocks is μ_s.



(a 10pts) Determine the maximum force that can be applied to the string such that the upper block of mass m_2 does not slide off of the lower block of mass m_1.

For the blocks to move together, they must have the same acceleration, $a_1 = a_2$. Since $F_{net} = ma$, then $a = F_{net}/m$, and:

$$\frac{F_{net1}}{m_1+m_2} = \frac{F_{net2}}{m_2}$$

because m_2 sits on top of m_1

F_N and F_g cancel out for each block, so $F_{net1} = F_{rope}$ and $F_{net2} = F_{friction}$, which points to the right.

$$\frac{F_{rope}}{m_1+m_2} = \frac{F_{friction}}{m_2}$$

$$F_{rope} = \frac{m_1+m_2}{m_2} F_{friction}$$

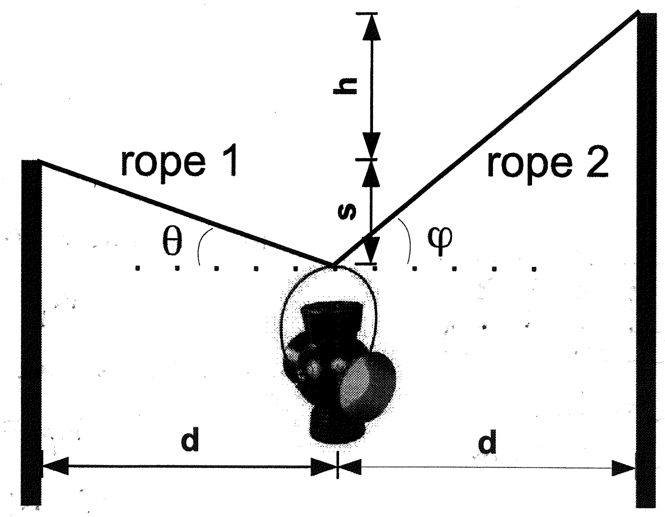
At maximum, $F_{friction} = \mu_s F_N = \mu_s m_2 g$ (top block), so:

$$F_{rope} = \frac{m_1+m_2}{m_2} F_{friction} = \frac{m_1+m_2}{\cancel{m_2}} \mu_s \cancel{m_2} g$$

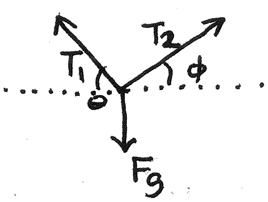
$$\boxed{F_{rope} = \mu_s (m_1+m_2) g}$$

-0.5
-1.5
-3.0
-8.0

A lantern of mass M hangs motionless from two ropes as seen in the figure. Each rope is attached to a tent pole a horizontal distance d away. The tent pole on the right is longer than the pole on the left by an amount h. The lantern is a vertical distance s below the top of the left tent pole. Each rope makes an angle θ and ϕ, respectively, with the horizontal.



(b 15pts) Determine the magnitude of the tension in rope 1 and 2.



System is at rest, so $F_{net} = 0$ for both the x- and y-components.

X-components:

$$0 = -T_{1x} + T_{2x}$$

$$T_{1x} = T_{2x}$$

$$T_1 \cos\theta = T_2 \cos\phi$$

$$\boxed{T_1 = T_2 \frac{\cos\phi}{\cos\theta}} \quad \text{Eq 1}$$

y-components:

$$0 = -F_g + T_{1y} + T_{2y}$$

$$\boxed{Mg = T_1 \sin\theta + T_2 \sin\phi} \quad \text{Eq 2}$$

Now substitute Eq 1 into Eq 2:

$$Mg = \left(T_2 \frac{\cos\phi}{\cos\theta}\right) \sin\theta + T_2 \sin\phi$$

$$Mg = T_2 \left(\frac{\cos\phi \sin\theta}{\cos\theta} + \sin\phi\right)$$

$$Mg = T_2 \left(\frac{\cos\phi \sin\theta + \sin\phi \cos\theta}{\cos\theta}\right)$$

$$\boxed{T_2 = \frac{Mg \cos\theta}{\cos\phi \sin\theta + \sin\phi \cos\theta}}$$

Eq 3

Finally, plug back Eq 3 into Eq 1:

$$T_1 = T_2 \frac{\cos\phi}{\cos\theta}$$

$$T_1 = \frac{Mg \cancel{\cos\theta}}{\cos\phi \sin\theta + \sin\phi \cos\theta} \frac{\cos\phi}{\cancel{\cos\theta}}$$

$$\boxed{T_1 = \frac{Mg \cos\phi}{\cos\phi \sin\theta + \sin\phi \cos\theta}}$$

Eq 4

Final solution:

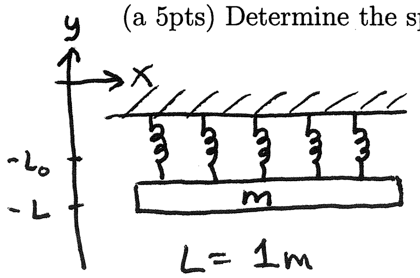
The tensions T_1 and T_2 are given by Eqs 4 and 3, respectively.

−1.0
−2.0
−4.5
−12

Problem 3 (25 Points)

A 100 kg block hangs at rest 1 meter from a ceiling. The block hangs connected to five identical springs. The springs are connected in parallel and each individual un-stretched spring has a length of 55 cm.

(a 5pts) Determine the spring constant for each of these springs.



$L = 1m$

$L_0 = 0.55m$

For springs in parallel,

$$k_p = \sum_i k_i \quad (1pt)$$

Since there are five identical springs,

$$k_p = 5k_i$$

$$\Rightarrow k_i = \frac{1}{5}k_p$$

Using only y-components (since this is a one-dimensional problem):

$$F_{net} = F_s + F_g = -k_p \Delta L - mg$$
$$= -k_p(L - L_0) - (100)(9.8)$$
$$= -k_p(-1 - (-0.55)) - 980$$
$$= -k_p(-0.45) - 980$$

$$F_{net} = 0.45k_p - 980 = 0$$ because system is at rest

$$0.45k_p = 980$$
$$k_p = \frac{980}{0.45}$$
$$k_p = 2177.78 \text{ N/m}$$

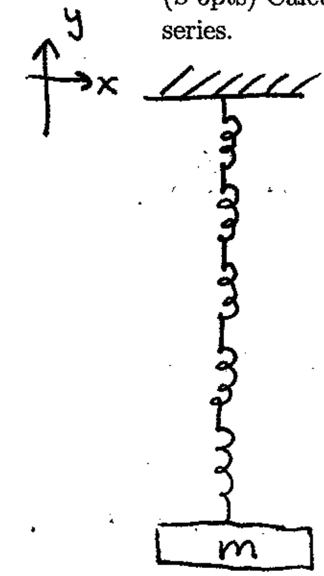
(3pts)

Going back to the relationship between k_i and k_p...

$$k_i = \frac{1}{5}k_p = \frac{1}{5}(2177.78) \quad (1pt)$$

$$\boxed{k_i = 435.56 \text{ N/m}}$$

(b 5pts) Calculate how far from the ceiling the block would hang if the five springs had been connected in series.



Again, using only y-components:

$$F_{net} = F_s + F_g = -k_s \Delta L - mg$$
$$= -k_s(L - L_0) - (100)(9.8)$$
$$= -87.11(L - (-0.55 \cdot 5)) - 980$$
$$= -87.11L - (-87.11)(-2.75) - 980$$
$$= -87.11L - 239.55 - 980$$
$$F_{net} = -87.11L - 1219.55 = 0$$ because system is at rest

(3pts)

$$-87.11L = 1219.55$$
$$L = \frac{-1219.55}{87.11}$$
$$\boxed{L = -14.0\text{ m}}$$

☆ The block hangs 14 m below the ceiling. ☆

Springs in series:

$$\frac{1}{k_s} = \sum_i \frac{1}{k_i}$$

Since there are five springs:

$$\frac{1}{k_s} = \frac{5}{k_i}$$
$$\Rightarrow k_s = \frac{k_i}{5}$$
$$k_s = \frac{435.56}{5}$$
$$k_s = 87.11\text{ N/m}$$

(2pts)

(c 5pts) The five springs in series are replaced with a single spring that has an equivalent stiffness. Determine the period of oscillation for the block connected to this single spring.

(2pt) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{100}{87.11}}$ (2pts)

☆ $\boxed{T = 6.73\text{ sec}}$ ☆

(1pt)

(d 10pts) A copper wire with a square cross-sectional area of 1e-6 m^2 has an un-stretched length of 2 meters. Copper has a density of and 8.96 $g{\cdot}cm^{-3}$ and atomic mass of 63.546. When the block is attached to this wire it stretches 15.1 mm. Calculate the speed of sound in the wire.

Young's Modulus

$$Y = \frac{F/A}{\Delta L/L_o} = \frac{F L_o}{A \Delta L} = \frac{(100)(9.8)(2)}{(1e-6)(15.1e-3)} = \boxed{1.298e11 \ N/m^2}$$

Density:

$$\rho = \frac{8.96 \cancel{g}}{\cancel{cm^3}} \cdot \frac{1 \text{ kg}}{1000 \cancel{g}} \cdot \frac{10^6 \cancel{cm^3}}{1 m^3} = \frac{(8.96)(10^6)}{1000} = \boxed{8960 \ kg/m^3}$$

The speed of sound is $v = d\sqrt{\frac{k_{si}}{m_a}}$, so we need d and k_{si}.

$$\rho = \frac{m_a}{V_a} = \frac{m_a}{d^3} \Rightarrow d^3 = \frac{m_a}{\rho} \Rightarrow \boxed{d = \left(\frac{m_a}{\rho}\right)^{1/3}}$$

$$Y = \frac{k_{si}}{d} \Rightarrow k_{si} = Yd \Rightarrow \boxed{k_{si} = Y\left(\frac{m_a}{\rho}\right)^{1/3}}$$

Putting it all together:

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$v^2 = d^2 \frac{k_{si}}{m_a} = \left(\frac{m_a}{\rho}\right)^{2/3} \frac{1}{m_a} Y \left(\frac{m_a}{\rho}\right)^{1/3}$$

$$v^2 = \frac{Y}{m_a}\left(\frac{m_a}{\rho}\right)^{3/3} = \frac{Y}{\cancel{m_a}} \frac{\cancel{m_a}}{\rho}$$

$$v^2 = Y/\rho$$

$$\boxed{v = \sqrt{\frac{Y}{\rho}}}$$

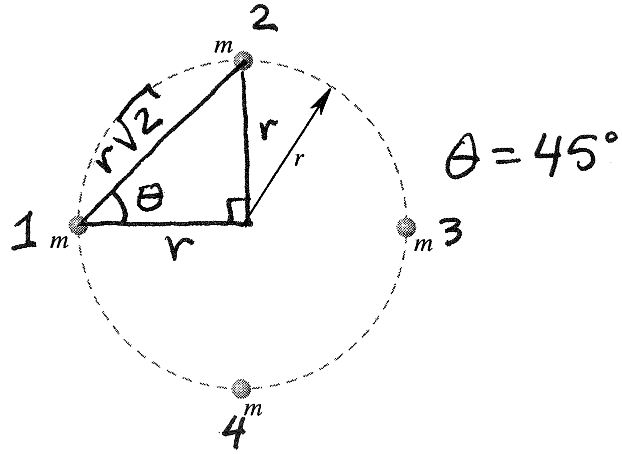
−0.5
−1.5
−3.0
−8.0

Plugging in the numbers...

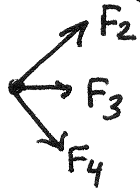
$$v = \sqrt{\frac{1.298e11}{8960}} \Rightarrow ☆ \ \boxed{v = 3806 \ m/s} \ ☆$$

Problem 4 (25 Points)

Consider four stars, each of mass m, which interact via the gravitational force. The net force of the masses on one another results in uniform circular motion of all four masses at a constant speed. Calculate how long it takes for one of the masses to make a complete revolution. You can assume $v << c$. Hint: consider the parallel and perpendicular components of all the forces.



Forces acting on Mass 1:



−1.0
−4.0
−7.5
−20

Perpendicular component of the net force:

$$F_{net,\perp} = F_3 + F_2 \cos\theta + F_4 \cos\theta$$

$$= \frac{Gmm}{(2r)^2} + \left(\frac{Gmm}{(r\sqrt{2})^2} + \frac{Gmm}{(r\sqrt{2})^2}\right)\cos\theta$$

$$= \frac{Gm^2}{4r^2} + \frac{Gm^2}{r^2}\cos\theta$$

$$\boxed{F_{net,\perp} = \frac{Gm^2}{r^2}\left(\frac{1}{4} + \cos\theta\right)}$$

When there's a circular motion, $F_{net,\perp} = \frac{|p||v|}{R}$. In this case, $R = r$, so:

$$\frac{pv}{r} = \frac{Gm^2}{r^2}\left(\frac{1}{4} + \cos\theta\right)$$

$$\frac{\cancel{m}vv}{\cancel{r}} = \frac{Gm^{\cancel{2}}}{r^{\cancel{2}}}\left(\frac{1}{4} + \cos\theta\right) \Rightarrow v^2 = \frac{Gm}{r}\left(\frac{1}{4} + \cos\theta\right)$$

Time for one revolution:

$$T = \frac{2\pi r}{v} \Rightarrow T^2 = \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^2\, r}{Gm\left(\frac{1}{4} + \cos\theta\right)}$$

$$T^2 = \frac{4\pi^2 r^3}{Gm\left(\frac{1}{4} + \cos\theta\right)} \Rightarrow$$ Interesting note: $T^2 \propto r^3$ is Kepler's 3rd law of Planetary Motion ☺!!

☆ $$\boxed{T = 2\pi\sqrt{\frac{r^3}{Gm\left(\frac{1}{4} + \cos\theta\right)}}}$$ ☆ $\Rightarrow$

And since $\theta = 45°$, $\cos\theta = 0.707$, so this also equals:

$$T = 2\pi\sqrt{\frac{r^3}{0.957\,Gm}}$$

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2-(pc)^2=\left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt}=\frac{d|\vec{p}|}{dt}\hat{p}+|\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel}=\frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp}=|\vec{p}|\frac{d\hat{p}}{dt}=|\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav}=-G\frac{m_1m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav}=-G\frac{m_1m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right|\approx mg \text{ near Earth's surface} \qquad \Delta U_{grav}\approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec}=\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right|=k_ss \qquad U_{spring}=\frac{1}{2}k_ss^2$$

$$U_i\approx\frac{1}{2}k_{si}s^2-E_M \qquad \Delta E_{thermal}=mC\Delta T$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+\dots}{m_1+m_2+\dots} \qquad I=m_1r_{1\perp}^2+m_2r_{2\perp}^2+\dots$$

$$K_{tot}=K_{trans}+K_{rel} \qquad K_{rel}=K_{rot}+K_{vib}$$

$$K_{rot}=\frac{L_{rot}^2}{2I} \qquad K_{rot}==\frac{1}{2}I\omega^2$$

$$\vec{L}_A=\vec{L}_{trans,A}+\vec{L}_{rot} \qquad \vec{L}_{rot}=I\vec{\omega}$$

$$\omega=\sqrt{\frac{k_s}{m}} \qquad v=d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y=\frac{F/A}{\Delta L/L}\text{ (macro)} \qquad Y=\frac{k_{si}}{d}\text{ (micro)}$$

$$\Omega=\frac{(q+N-1)!}{q!\,(N-1)!} \qquad S\equiv k\ln\Omega$$

$$\frac{1}{T}\equiv\frac{\partial S}{\partial E} \qquad \Delta S=\frac{Q}{T}\text{ (small } Q\text{)}$$

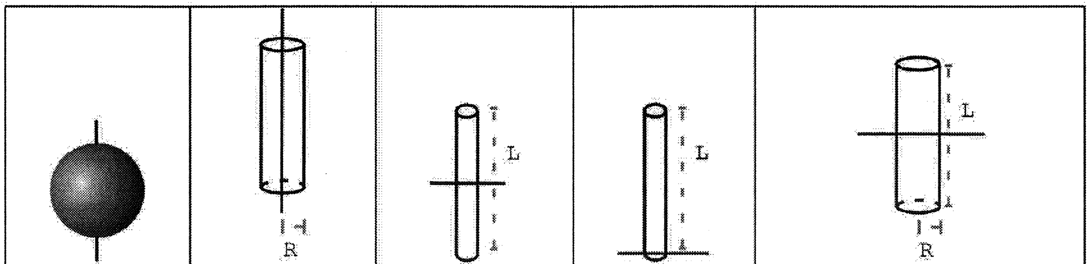
$$\text{prob}(E)\propto\Omega(E)\,e^{-\frac{E}{kT}} \qquad E_N=-\frac{13.6\text{eV}}{N^2}\text{ where } N=1,2,3\dots$$

$$E_N=N\hbar\omega_0+E_0 \text{ where } N=0,1,2\dots \text{ and } \omega_0=\sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_y B_z - A_z B_y, A_z B_x - A_x B_z, A_x B_y - A_y B_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}