ISyE 2027C Probability with Applications

HINTS for Homework 7 for Quiz March 31 **March 24, 2015**

Simple Calculation Problems

1. Random variable X_j has exponential distribution with mean $4j$ for $j = 1, 2, 3$. The variables are jointly independent. Find $P(1 \le X_2 \le 12)$. Find $P((X_1 \ge 8) \cap (X_2 \le 4))$. Find $P(\min_j X_j \le 16)$. Use independence.

2. Random variable Y has cdf $F(t) \equiv P(Y \le t) = 1 - 1/t^3 : 1 \le t < \infty$. Find the pdf of Y. Find $E[Y]$. Find $\sigma^2(Y)$. $\frac{3}{4}$

3. Random variables W_1, W_2, W_3 are jointly independent each uniformly distributed on $[0, 12]$. Let $W = \max_{1 \le j \le 3} W_j$. Find $P(1 \le W \le 3)$. Find the pdf of W and find $E[W]$. .01505

Poisson Process Problems

1. 100 light bulbs in a chandelier have independent lifetimes each exponentially distributed with mean 2 months. 20 of the bulbs are blue; 30 are red; 50 are white. What is the probability that the first bulb to burn out is red? What is the probability that the first two bulbs to burn out are blue? What is the probability that the first 6 burnouts are red, white, blue, red, white, blue in that order? What is the expected number of working white bulbs after 1 month? Conditioned on there being 80 working bulbs after 2.5 months, what is the expected number of working blue bulbs? .3, 19/495, $\frac{(30)(29)}{(99)(97)(96)}$, 30.33, 16.

2. Consider the same chandelier as in the previous problem. What is the expected amount of time until there are only 99 bulbs working? 98? 1? 0? Leave your answers for 1 and 0 as summation formulae. If you number the bulbs from 1 to 100 now, what is the expected amount of time until bulb 55 burns out? What is the expected number of bulbs that will burn out more than 2 months but less than 3 months from now? 14 hours 24 minutes (treating a month as 30 days); 1 day 4 hours 56 minutes and 43.6 seconds. For 0 bulbs working, $2\sum_{j=1}^{100} \frac{1}{j}$ months.

3. Now consider two chandeliers. The first is identical to the one in the previous problems. The second is like the first, but each bulb has exponentially distributed lifetime with mean 4 months. What is the probability that the 1st bulb to burn out is from the first chandelier? The 1st chandelier arrival of failures is initially a Poisson process with rate $50/month$; the 2nd has rate $25/month$. Together they make a Poisson process with rate 75/month. The chance an arrival from such a combined process comes from the 1st is $\frac{50}{50+25} = 2/3$.

4. A mcg of U-238 emits particles according to a Poisson process at rate 2/sec. A mcg of U-235 emits particles according to a Poisson process at rate 10/sec. Ali H.K. receives a gift of 1 mcg uranium, which is U-238 with probability .5 and U-235 with probability .5.

   (a) Ali observes his gift for one second. What is the probability that no particles are emitted? .06769034158318, approximately.

   (b) Suppose that the gift is U-238 and Ali carries it in his pocket for 11 hours. Write as a sum the probability that not more than 39,000 particles are emitted during that time. Use Chebyshev's inequality to bound that probability. Estimate square roots or use a calculator.

   (c) Ali observes that 15 particles are emitted from his mcg of uranium during a 3 second period. Conditioned on that observation, what is the probability that his uranium is U-235?

   (d) The next day, Ali receives a second gift of 1 mcg uranium, of the type he didn't receive previously. He observes both gifts simultaneously. The first particle emitted by either one comes from the first. What is the probability that the first is U-235? If the first two particles emitted come from the first, what is the probability that the first is U-235? What is the probability that no particles are emitted (from either one) during the first 2 seconds?

5. Calls from Maryland arrive at a call center at a rate of 15/minute. Calls from Georgia arrive at the same center at a rate of 5/minute. What is the mean time between calls if both arrival processes are independent Poisson processes? Suppose each call from Maryland is silly with probability .3 and each

call from Georgia is silly with probability .25. What is the mean time between silly calls? Suppose a silly call arrives at exactly 1:23. What is the expected number of seconds until the next non-silly call?

Combine the calls into one Poisson process with rate 20/minute; the silly Maryland calls are a Poisson process independent of the silly Georgia calls – combine the two processes; use the memoryless property.