MATH 2403 H1-H3. Differential Equations. Quiz 6. Mar 4, 2014.
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**Show all work to receive credit**

1. Find the general solution of the system of equations

$$\bar{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \bar{x}.$$

Then find the solution with initial condition $\bar{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$p(\lambda) = (1-\lambda)(-7-\lambda)+16 = \lambda^2+6\lambda-7+16 = \lambda^2+6\lambda+9 = (\lambda+3)^2$

$\Rightarrow \lambda = -3$ repeated.

Eigenvector:

$$\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \rightarrow v_1 = v_2 \Rightarrow \bar{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Generalized eigenvector:

$$\begin{pmatrix} 4 & -4 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow 4w_1 - 4w_2 = 1$$

Two ways:

1) $w_1 = \frac{1}{4} + w_2 \Rightarrow \bar{w} = \begin{pmatrix} \frac{1}{4}+w_2 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1/4 \\ 0 \end{pmatrix} + w_2\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

or 2) $w_2 = -\frac{1}{4} + w_1 \Rightarrow \bar{w} = \begin{pmatrix} w_1 \\ -1/4 + w_1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1/4 \end{pmatrix} + w_1\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

General solution:

1) $\bar{x} = c_1 e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t}\left(t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}\right)$

or 2) $\bar{x} = c_1 e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2 e^{-3t}\left(t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1/4 \end{pmatrix}\right)$

For the initial condition: $\bar{x}(0) = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

1) $\bar{x}(0) = \begin{pmatrix} c_1 + c_2/4 \\ c_1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 2 \\ c_2 = -12 \end{matrix}$

$$\Rightarrow \bar{x} = 2e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + (-12)e^{-3t}\left(t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}\right) = \begin{pmatrix} e^{-3t}(-1-12t) \\ e^{-3t}(2-12t) \end{pmatrix}$$

Cont →

2)

$$\bar{x}(0) = \begin{pmatrix} c_1 \\ c_1 - \frac{1}{4}c_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = -1 \\ c_2 = -12 \end{matrix}$$

$$\Rightarrow \bar{x} = -e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + (-12)e^{-3t}\left( t\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ -1/4 \end{pmatrix}\right) = \begin{pmatrix} e^{-3t}(-1-12t) \\ e^{-3t}(2-12t) \end{pmatrix}$$