Quiz 10 (25 mins), MA1501 Calculus I, 11/7/2013 PRINTED NAME: ________________ GT ID: ____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

---

1. For the function

$$f(x) = x + x^2 \text{ over the interval } [0,1].$$

(a) (20 pts) Find a formula for the Riemann sum obtained by dividing the interval $[a, b]$ into n equal subintervals and using the **right-hand endpoint** for each c_k.

(b) (20 pts) Give a limit of the Riemann sum as $n \to \infty$.

Answer: (a) The subintervals all have equal width $\Delta x = \frac{b-a}{n} = \frac{1}{n}$, and the partition is chosen by

$$P = \{0, \Delta x, 2\Delta x, \cdots, (n-1)\Delta x, n\Delta x\}$$

where $n\Delta x = 1$. We form the Riemann sum as

$$\sum_{k=1}^{n} f(c_k)\Delta x = \sum_{k=1}^{n} f(k\Delta x)\,\Delta x.$$

c_k's are chosen as the right-hand endpoint for each subinterval.

$$\begin{aligned}
\sum_{k=1}^{n} f(k\Delta x)\,\Delta x &= \sum_{k=1}^{n} \left(k\Delta x + (k\Delta x)^2\right)\Delta x \\
&= \sum_{k=1}^{n} \left(\frac{k}{n} + \frac{k^2}{n^2}\right)\frac{1}{n} \\
&= \sum_{k=1}^{n} \left(\frac{k}{n^2} + \frac{k^2}{n^3}\right) \\
&= \frac{1}{n^2}\sum_{k=1}^{n} k + \frac{1}{n^3}\sum_{k=1}^{n} k^2 \\
&= \frac{n(n+1)}{2n^2} + \frac{n(n+1)(2n+1)}{6n^3}
\end{aligned}$$

(b) The limit of the Riemann sum as $n \to \infty$ is

$$\lim_{n\to\infty}\left(\frac{n(n+1)}{2n^2} + \frac{n(n+1)(2n+1)}{6n^3}\right) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

2. (30 pts) Evaluate the sum:

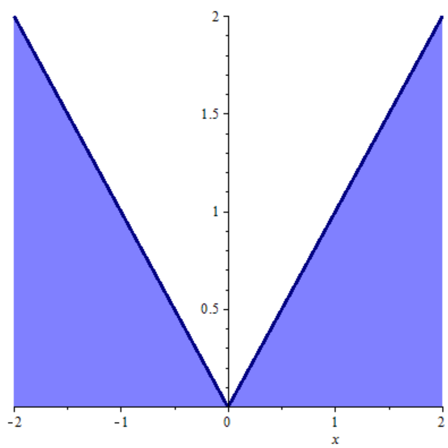
$$\sum_{k=-1}^{10}(3-k)$$

Answer:

$$\begin{aligned}\sum_{k=-1}^{10}(3-k) &= 4+3+\sum_{k=1}^{10}(3-k)\\ &= 7+30-\sum_{k=1}^{10}k\\ &= 37-\frac{10\cdot 11}{2}\\ &= 37-55=-18\end{aligned}$$

3. (30 pts) Graph the integrand and use areas to evaluate the integrals

$$\int_{-2}^{2}|x|dx.$$



Answer:
The area between $|x|$ and x-axis over $[-2,2]$ is $(2\cdot 2+2\cdot 2)\cdot\frac{1}{2}=4$.