

Date: February 17, 2016

Last Name (Print): _____ First Name (Print): _____

Instructions Please print your name at the top of this page. To obtain maximum marks show all your work, carefully justifying your answers.

1. (8 points) Let $a, b \in \mathbb{Z} \setminus \{0\}$, show that if $d \mid a$ and $d \mid b$ then $d \mid \gcd(a, b)$.

Solution: Let $g = \gcd(a, b)$. By a theorem from class (Theorem 15) there exist $m, n \in \mathbb{Z}$ such that $g = ma + nb$. Now, d divides any linear combination of a and b by a result from class (Proposition 13), therefore $d \mid ma + nb$ or $d \mid g$, as desired.

2. (8 points) Find $\text{lcm}(1575, -231)$ using the expression that relates gcd to lcm.

Solution: Let us begin by computing $\gcd(1575, 231) = \gcd(1575, -231)$. We do so by using the Extended Euclidean Algorithm.

	a	b
1575	1	0
231	0	1
189	1	-6
42	-1	7
21	5	-34

Thus $\gcd(1575, -231) = 21$ and $\text{lcm}(1575, -231) = \frac{231 \cdot 1575}{21} = 77 \cdot 225 = 17325$

3. (8 points) Decide whether each of the following statements is true or false. Justify your answer.
- (a) (4 points) $\overline{-25} = \overline{25} \pmod{17}$

Solution: False. Note that $-25 \in \overline{9}$ and $25 \in \overline{8}$. Thus $\overline{-25} = \overline{9} \neq \overline{8} = \overline{25}$.

- (b) (4 points) $\overline{-20} \cap \overline{79} = \emptyset$ modulo 11.

Solution: False. We have that $-20 \in \overline{2}$ and $79 \in \overline{2}$. Thus $\overline{-20} = \overline{2} = \overline{79}$.