Name: Key

ChBE 2120, Numerical Methods, Paravastu Section, Fall 2015
Quiz 6: 20 points possible

1) (10 points) Setup the matrix equation necessary to perform a second order polynomial regression ($y = a_0 + a_1x + a_2x^2$) on the following data. Use the General least squares regression approach. You do not need to solve this question. $z_0 = 1;\ z_1 = x;\ z_2 = x^2$ (+2)

x	y	x^2
2	1	4
4	50	16
6	100	36
8	200	64

$$[Z] = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \\ 1 & 8 & 64 \end{bmatrix} \quad (+2) \qquad [Z]^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \\ 4 & 16 & 36 & 64 \end{bmatrix}$$

General Least Squares
Best-fit function: $y = a_0z_0 + a_1z_1 + a_2z_2 + \cdots + a_mz_m$, where $z_i's$ are any functions of x.
Minimization of S_r: $[Z]^T[Z][A] = [Z]^T[Y]$

$$[Z] = \begin{bmatrix} z_{01} & \cdots & z_{m1} \\ \vdots & \ddots & \vdots \\ z_{0n} & \cdots & z_{mn} \end{bmatrix}, z_{ij} = z_i(x_j), [Y] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, [A] = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

$$[A] = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \quad (+1) \qquad [Y] = \begin{bmatrix} 1 \\ 50 \\ 100 \\ 200 \end{bmatrix} \quad (+1)$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \\ 4 & 16 & 36 & 64 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \\ 1 & 8 & 64 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 4 & 6 & 8 \\ 4 & 16 & 36 & 64 \end{bmatrix} \begin{bmatrix} 1 \\ 50 \\ 100 \\ 200 \end{bmatrix}$$

(+4)

2) (10 points) Derive the matrix equation used to calculate the slope and intercept of the least-squares best-fit line for a data set. Recall that least-squares regression involves minimizing $S_r = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}(y_i - a_0 - a_1x_i)^2$.

$$\frac{\partial S_r}{\partial a_0} = 0 = -2\sum_{i=1}^{n}(y_i - a_0 - a_1x_i) = \sum y_i - na_0 - a_1\sum x_i$$
(+2) (+2) (+1)

$$\frac{\partial S_r}{\partial a_1} = 0 = -2\sum_{i=1}^{n}x_i(y_i - a_0 - a_1x_i) = \sum x_iy_i - a_0\sum x_i - a_1\sum x_i^2$$

$$n a_0 + (\sum x_i)a_1 = \sum y_i \quad (+1)$$
$$(\sum x_i)a_0 + (\sum x_i^2)a_1 = \sum x_iy_i$$

$$\Rightarrow \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_iy_i \end{bmatrix}$$

(+2)