Full name: Math 2551, Section B___

Quiz 3 — §13.5, 14.1

Please ***clearly*** show all work. Scientific calculators are allowed, but no graphing calculators!

(1) Let

$$\mathbf{r}(t) = (\sin t - t\cos t)\mathbf{i} + (\cos t + t\sin t)\mathbf{j}.$$

Find the curvature κ of the curve parameterized by $\mathbf{r}(t)$ as a function of $t > 0$. [8 points]

The velocity of the parameterization is

$$\mathbf{v}(t) = t\sin t\,\mathbf{i} + t\cos t\,\mathbf{j} \quad \Longrightarrow \quad |\mathbf{v}(t)| = |t| = t,$$

the last equality being because we are assuming $t > 0$. The unit tangent vector is therefore

$$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \sin t\,\mathbf{i} + \cos t\,\mathbf{j}.$$

Its derivative is then

$$\frac{d\mathbf{T}}{dt} = \cos t\,\mathbf{i} - \sin t\,\mathbf{j} \quad \Longrightarrow \quad \left|\frac{d\mathbf{T}}{dt}\right| = 1.$$

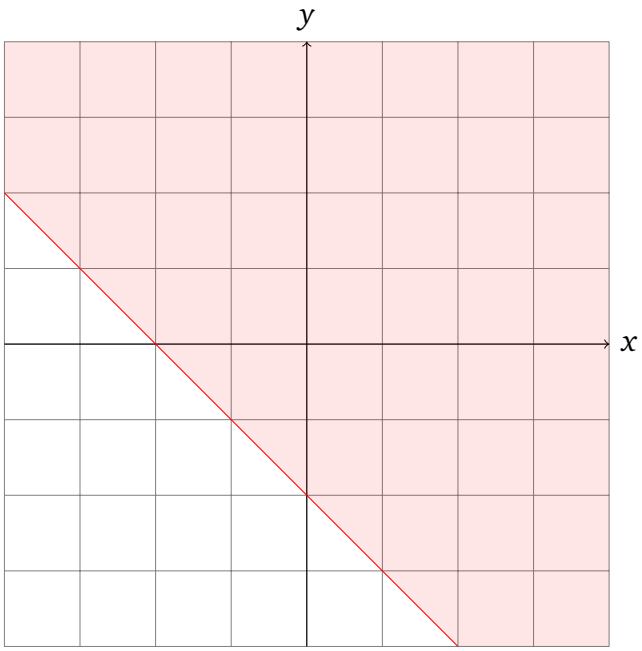
We can now compute the curvature κ:

$$\kappa = \frac{1}{|\mathbf{v}|}\left|\frac{d\mathbf{T}}{dt}\right| = \boxed{\frac{1}{t}}$$

(2) Let $f(x,y) = \sqrt{2+x+y}$.

(a) What is the domain of f? Sketch it. [4 points]

The domain is going to consist of all points (x,y) such that $2+x+y \geq 0$, or equivalently all points (x,y) such that $y \geq -2-x$.



(b) What is the range of f? [4 points]

The range of f is the interval $[0,\infty)$. It is impossible for f to take any values less than 0 since we are (implicitly) taking the positive square root in the definition of f. On the other hand, every value ≥ 0 can be attained by choosing x and y appropriately.

(c) Describe the level curves of f. [4 points]

The level curve $f^{-1}(c)$ has equation $\sqrt{2+x+y} = c$, which implies that $2+x+y = c^2$. This is the equation for a line, written more traditionally as $y = c^2 - 2 - x$. The level curves are therefore lines with slope -1 whose y-intercepts are ≥ -2.