

Circle your TA's name: D1 - Lacy Hardcastle D2 - Kyla Adams D3 - Kelly Robinson

1. Evaluate the following definite integrals. You don't have to simplify the answers completely.

$$\begin{aligned}
 \text{(a)} \quad \int_1^4 \sqrt{x} + \frac{1}{\sqrt{x}} + 2^x dx &= \int_1^4 x^{\frac{1}{2}} + x^{-\frac{1}{2}} + 2^x dx \\
 &= \left. \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2^x}{\ln 2} \right|_1^4 \\
 &= \left(\frac{4^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2^4}{\ln 2} \right) - \left(\frac{1^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1^{\frac{1}{2}}}{\frac{1}{2}} + \frac{2^1}{\ln 2} \right)
 \end{aligned}$$

(12 pts.)

$$\begin{aligned}
 &= \frac{2}{3}(8) + 2(2) + \frac{16}{\ln 2} - \frac{2}{3} - 2 - \frac{2}{\ln 2} \\
 &= \boxed{\frac{16}{3} + 4 + \frac{16}{\ln 2} - \frac{2}{3} - 2 - \frac{2}{\ln 2}}
 \end{aligned}$$

$$\text{(b)} \quad \int_0^1 x(3x^2-2)^4 dx$$

$$u = 3x^2 - 2$$

$$\frac{du}{dx} = 6x$$

$$\frac{du}{6} = x dx$$

$$x=0 \quad u = 3(0^2) - 2 = -2$$

$$x=1 \quad u = 3(1^2) - 2 = 1$$

$$\int_0^1 x(3x^2-2)^4 dx = \int_0^1 (3x^2-2)^4 x dx$$

$$= \int_{-2}^1 u^4 \frac{du}{6}$$

$$= \frac{1}{6} \int_{-2}^1 u^4 du = \frac{1}{6} \frac{u^5}{5} \Big|_{-2}^1$$

$$= \frac{1}{30} (1^5 - (-2)^5) = \frac{1}{30} (1 + 32)$$

$$= \boxed{\frac{33}{30}}$$

(15 pts.)

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1. Evaluate the following definite integrals. You don't have to simplify the answers completely.

$$\int_0^1 (x^4 + 2x^3 - 1) dx$$

x

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2. Evaluate $\int \frac{2}{9x+1} dx$. $= \int \frac{2}{u} \frac{du}{9} = \frac{2}{9} \int \frac{1}{u} du$

$$u = 9x+1$$

(10 pts.)

$$\frac{du}{dx} = 9$$

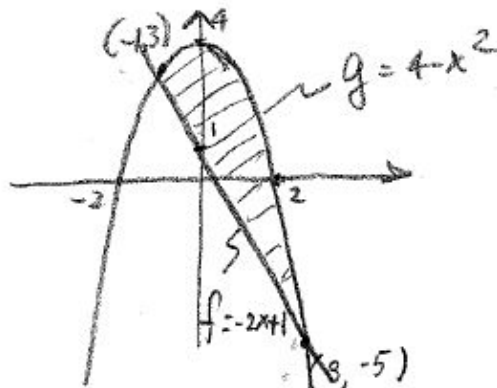
$$\frac{du}{9} = dx$$

$$= \frac{2}{9} \ln|u| + C$$

$$= \boxed{\frac{2}{9} \ln|9x+1| + C}$$

3. Sketch the graph and find the area of the region completely enclosed by the graphs of $f = -2x+1$ and $g = 4-x^2$.

(15 pts.)



Set $f = g$ to find the points of intersection.

$$-2x+1 = 4-x^2$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3, -1$$

$$\begin{aligned} \text{Area} &= \int_{-1}^3 (4-x^2) - (-2x+1) dx \\ &= \int_{-1}^3 4-x^2+2x-1 dx \\ &= \left[4x - \frac{x^3}{3} + \frac{2x^2}{2} - x \right]_{-1}^3 \\ &= \left(4(3) - \frac{3^3}{3} + 3^2 - 3 \right) - \left(4(-1) - \frac{(-1)^3}{3} + (-1)^2 - 1 \right) \end{aligned}$$

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MATH 1712 D1-D3 Test #3 Full (j

(rint) Page 2 of 4 Version A April 8, 2009

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3. Sketch the graph and find the area of the region completely
he region completely enclosed by the graphs of

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4. The temperature (in degree Fahrenheit) in Atlanta over a 12-hour period on a certain April day was given by

$$T = -0.6t^3 + 4.8t^2 + 45.6t + 67.2 \quad (0 \leq t \leq 12)$$

where t is measured in hours, with $t = 0$ corresponding to 10 a.m.

Write an integral expression that represent the average temperature on that day over the 6-hour period from noon to 6 p.m. Do not evaluate the integral.

(10 pts.) The average temperature from noon to 6 p.m. $= \frac{1}{8-2} \int_2^8 (-0.6t^3 + 4.8t^2 + 45.6t + 67.2) dt$

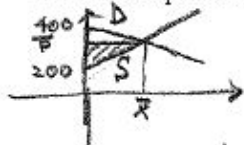
5. The quantity demanded x bicycles, per week, is related to the unit price p (in dollars) by the relation

$$D(x) = p = -x + 400$$

The quantity x bicycles that the supplier is willing to make available in the market is related to the unit price by the relation

$$S(x) = p = 3x + 200$$

If the market price is set at the equilibrium price, find the producers' surplus.



Set $D = S$ to find \bar{p} and \bar{x} .

$$-\bar{x} + 400 = 3\bar{x} + 200$$

(16 pts.)

$$200 = 4\bar{x}$$

$$\bar{x} = \frac{200}{4} = 50$$

$$\bar{p} = -50 + 400 = 350$$

$$\begin{aligned} PS &= \int_0^{\bar{x}} \bar{p} - S(x) dx = \int_0^{50} 350 - (3x + 200) dx \\ &= \int_0^{50} 150 - 3x dx = 150x - \frac{3x^2}{2} \Big|_0^{50} \\ &= 150(50) - \frac{3}{2}(50^2) - (150 \cdot 0 - \frac{3}{2}(0)^2) \\ &= \boxed{\$3750} \end{aligned}$$

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Page 3 of 4 Version A April 8, 2009

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4. The temperature (in degree Fahrenheit) in Atlanta over a 12-hour period on a certain April day where t is measured in hours, with $t = 0$ corresponding to 10 a.m. Write an integral expression that represent the average temperature on that day over the 6-hour period from noon to 6 p.m. Do g evaluate the integral.

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5. The quantity demanded x bicycles, per Week, is related to the unit price p (in dollars) by the relation

$p = -400x + 400$ The quantity x bicycles that the supplier is willing to make available in the market is related to

the unit price by the relation

$p = 3x + 200$ If the market price is set at the equilibrium price, find the producers' surplus.

Key
(Print)

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6. Determine whether each statement is true or false.
- Justify the answers.

- (a)
- $\int_a^b f(x) dx$
- is the area between the function
- f
- and the
- x
- axis from
- $x = a$
- to
- $x = b$
- .

(6 pts.)

False, the function f must be nonnegative between a & b for this statement to be true.

- (a) In a study conducted by the United Nations, it was found the Gini indices (or the coefficients of inequality) for the income distribution of Countries
- A
- and
- B
- are 0.481 and 0.435 respectively. Country
- A
- has a more equitable income distribution.

(6 pts.)

False, Country B has a more equitable income distribution because the Gini index is lower.

7. (a) What is the definition of an annuity?
- Give an example.

See pp 470-471.

(10 pts)

- (b) How do you derive the formula for the present value of an annuity using the formula for the present value of an income stream?

income stream
 $R(t)$

$$PV = \int_0^T R(t) e^{-rt} dt$$

present value

Annuity
(5 bonus pts.)
each payment = P
of payments per year = m

$$R(t) = PM$$

$$PV = \int_0^T PM e^{-rt} dt = PM \int_0^T e^{-rt} dt = PM \left[\frac{e^{-rt}}{-r} \right]_0^T = -\frac{PM}{r} (e^{-rT} - e^0) = \frac{PM}{r} (1 - e^{-rT})$$

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6. Determine whether each statement is true or false. Justify the answers.

(a) $\int_a^b f(x) dx$ is the area between the function f and the x -axis from $x = a$ to $x = b$.

(a) In a study conducted by the United Nations, it was found the Gini indices (or the coefficients of inequality) for the income distribution of Countries A and B are 0.481 and 0.435 respectively. Country A has a more equitable income distribution.