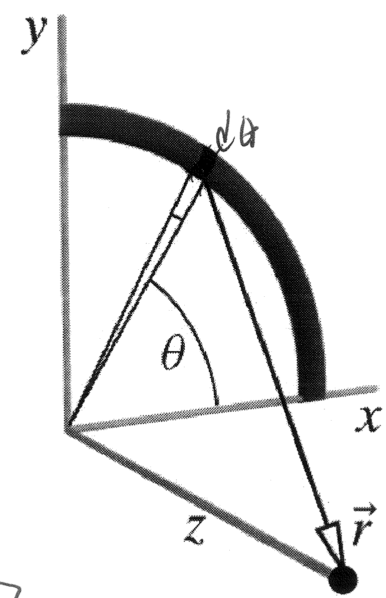
Physics 2212 Spring 2014 Lab Quiz #1

Name: KEY Section ________

Please show all of your work and box your final answers for full credit.

Consider a thin plastic rod bent into a quarter circular arc of radius R with center at the origin as indicated in the figure. The rod carries a uniformly distributed positive charge Q and is located in the x-y plane. Answer the following questions to determine the electric field at a point on the z-axis.



1. (*20 points*) Consider an infinitesimal slice of the bent rod with angular length $d\theta$ located at an angle θ from the right end of the rod as indicated in the figure. Find the position vector $\vec{r}$ that points from center of this slice to the observation location $\vec{A} = < 0, 0, z >$ on the z-axis.

$\vec{r} = \vec{r}_{obs} - \vec{r}_{source}$

$\vec{r}_{source} = \langle R\cos\theta, R\sin\theta, 0 \rangle$

$\vec{r} = \langle -R\cos\theta, -R\sin\theta, z \rangle$

2. (*20 points*) Determine an expression for the charge of the slice, dQ, in terms of the infinitesimal angular length $d\theta$ and relevant known variables.

linear charge density $\lambda = \frac{Q}{L}$ $L = R\theta_{max} = \frac{R\pi}{2}$

$\cancel{\frac{\pi}{2}R}$ $\lambda = \frac{2Q}{R\pi}$

$dQ = \lambda\, dL$

$dL = R\, d\theta$

$dQ = \frac{2Q}{\cancel{R}\pi}\cancel{R}\, d\theta = \boxed{\frac{2Q}{\pi}\, d\theta}$

3. (*40 points*) Derive an expression for the vector electric field $d\vec{E}$ of the slice of the rod at the observation location $\vec{A}$ in terms of the given variables and known constants.

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{dQ}{|\vec{r}|^3} \vec{r}$$

$$d\vec{E} = \frac{1}{\cancel{4}\pi\epsilon_0} \frac{2Q}{\pi} \frac{d\theta}{|\langle -R\cos\theta, -R\sin\theta, z\rangle|^3} \langle -R\cos\theta, -R\sin\theta, z\rangle$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi} \frac{d\theta}{(R^2\cos^2\theta + R^2\sin^2\theta + z^2)^{\frac{3}{2}}} \langle -R\cos\theta, -R\sin\theta, z\rangle$$

$$d\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi} \frac{d\theta}{(R^2 + z^2)^{3/2}} \langle -R\cos\theta, -R\sin\theta, z\rangle$$

4. (*20 points*) Integrate over the charge distribution to determine the z-component of the electric field E_z at observation location $\vec{A}$. You answer should only contain the given variables and known constants.

$$\vec{E}_z = \int_0^{\frac{\pi}{2}} \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi} \frac{z\, d\theta}{|\langle -R\cos\theta, -R\sin\theta, z\rangle|^3} = \int_0^{\frac{\pi}{2}} \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi} \frac{z\, d\theta}{(R^2 + z^2)^{3/2}}$$

$$\vec{E}_z = \frac{1}{4\pi\epsilon_0} \frac{2Q}{\pi} z \int_0^{\frac{\pi}{2}} \frac{d\theta}{(R^2+z^2)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{\cancel{2}Q}{\cancel{\pi}} z \left(\frac{\frac{\cancel{\pi}}{\cancel{2}}}{(R^2+z^2)^{3/2}} \right)$$

$$\boxed{\vec{E}_z = \frac{1}{4\pi\epsilon_0} \frac{Qz}{(R^2+z^2)^{\frac{3}{2}}}}$$