MATH 1711, Midterm 4

11/20/2013

Name: key 1 GTID:_____

Circle your section below

D1 TA: Katie Stocker

D2 TA: Maggie Ginn

D3 TA: Kayla McKenzie

Problem No.	Points
1	
2	
3	
4	

TOTAL:_____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (20 points).

Use the Simplex method to maximize the objective function $3x + y$ subject to the constraints

$$\begin{cases} x + 3y \leq 8 \\ 3x + 2y \leq 6 \\ x \geq 0, \quad y \geq 0. \end{cases}$$

$$\begin{cases} x + 3y + u = 8 \\ 3x + 2y + v = 6 \\ -3x - y + M = 0 \end{cases}$$

$x, y, u, v \geq 0$

M is as large as possible.

x	y	u	v	M	
1	3	1	0	0	8
(3)	2	0	1	0	6
-3	-1	0	0	1	0

$\uparrow$

$\frac{6}{3} < \frac{8}{1}$

x	y	u	v	M	
1	3	1	0	0	8
1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	2
-3	-1	0	0	1	0

$\Rightarrow$

x	y	u	v	M	
0	$\frac{1}{3}$	1	$-\frac{1}{3}$	0	6
1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	2
0	1	0	1	1	6

$\Rightarrow$

$$\begin{cases} x = 2 \\ y = 0 \end{cases}$$

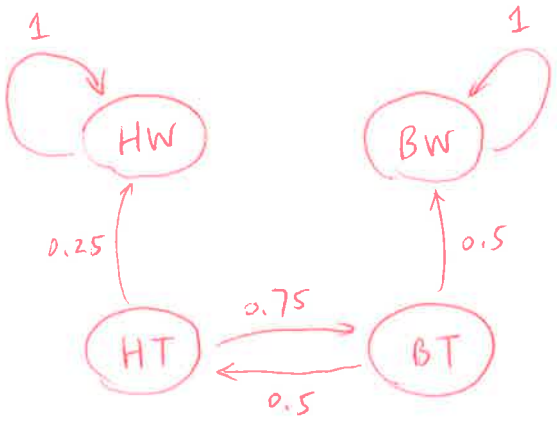
$3x + y = 6$

Turn over for more problems

Problem 2 (30 points).

Heather and Blake play a card game in which they take turns drawing a card from a standard deck of cards. Heather can win the game if she draws a heart and Blake can win the game if he draws a black card. When a player doesn't win on their turn, their card is returned to the deck, the deck is reshuffled, and it becomes the other players turn. The game has four states: Heather wins(HW), Blake wins(BW), Heather's turn(HT), and Blake's turn(BT).

(a) (5 points) Draw the transition diagram for this Markov process.



(b) (10 points) Set up an absorbing stochastic matrix for this Markov process.

	HW	BW	HT	BT
HW	1	0	0.25	0
BW	0	1	0	0.5
HT	0	0	0	0.5
BT	0	0	0.75	0

Turn over for more problems

(c) (15 points) Compute the fundamental matrix $(I-R)^{-1}$. Determine the expected number of turns if Heather goes first.

$$R = \begin{matrix} & \text{HT} & \text{BT} \\ \begin{matrix} \\ \\ \end{matrix} & \begin{bmatrix} 0 & 0.5 \\ 0.75 & 0 \end{bmatrix} \end{matrix}$$

$$(I-R)^{-1} = \begin{bmatrix} 1 & -0.5 \\ -0.75 & 1 \end{bmatrix}^{-1}$$

$$= \frac{1}{1-0.375} \begin{bmatrix} 1 & 0.5 \\ 0.75 & 1 \end{bmatrix}$$

$$= \frac{1}{0.625} \begin{bmatrix} 1 & 0.5 \\ 0.75 & 1 \end{bmatrix}$$

expected # of turns if Heather goes first is:

$$\frac{1+0.75}{0.625} \quad \text{or} \quad 2.8.$$

Turn over for more problems

Problem 3 (30 points).

Suppose that 60% of people who own a General Motors car buy a GM car as their next car and 90% of people who own a non-GM car buy a non-GM car as their next car.

(a) (10 points) Write the transition matrix for this Markov process.

$$\begin{array}{c} \\ \text{GM} \\ \text{Non-GM} \end{array} \begin{array}{c} \begin{array}{cc} \text{GM} & \text{non-GM} \end{array} \\ \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix} \end{array}$$

(b) (20 points) What will General Motors' market share be in the long run? (i.e., find the stable distribution)

Let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$$Ax = x \Rightarrow \begin{bmatrix} 0.6 & 0.1 \\ 0.4 & 0.9 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{cases} 0.6a + 0.1b = a \\ a + b = 1 \end{cases}$$

$$\Rightarrow \begin{cases} a = 0.2 \\ b = 0.8 \end{cases} \Rightarrow X = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

In the long run, GM market share will be 0.2.

Turn over for more problems

Problem 4 (20 points).

Suppose that R and C play a game by matching coins. On each play, C pays R the number of heads shown (0,1, or 2) minus *twice* the number of tails shown.

(a) (5 points) Set up a payoff matrix for this game.

R \ C	H	T
H	2	-1
T	-1	-4

(b) (10 points) What is the optimal pure strategy for each player?

R will choose Head.

C will choose Tail.

(c) (5 points) What is the value of the game?

-1

The End.

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11/20/2013

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Circle your section below

D1 TA: Katie Stocker

D2 TA: Maggie Ginn

D3 TA: Kayla McKenzie

Problem No.	Points
1	
2	
3	
4	

TOTAL: _____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (20 points).

Use the Simplex method to maximize the objective function $x + 3y$ subject to the constraints

$$\begin{cases} 3x + y \leq 8 \\ 2x + 3y \leq 6 \\ x \geq 0, \quad y \geq 0. \end{cases}$$

$$\begin{cases} 3x + y + u = 8 \\ 2x + 3y + v = 6 \\ -x - 3y + M = 0 \end{cases}$$

$x, y, u, v \geq 0$

M as large as possible.

x	y	u	v	M	
3	1	1	0	0	8
2	(3)	0	1	0	6
-1	-3	0	0	1	0

$\frac{6}{3} < \frac{8}{1}$

$\Rightarrow$

x	y	u	v	M	
3	1	1	0	0	8
$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	2
-1	-3	0	0	1	0

$\Rightarrow$

x	y	u	v	M	
$\frac{7}{3}$	0	1	$-\frac{1}{3}$	0	6
$\frac{2}{3}$	1	0	$\frac{1}{3}$	0	2
1	0	0	1	1	6

$\begin{cases} x = 0 \\ y = 2 \end{cases}$

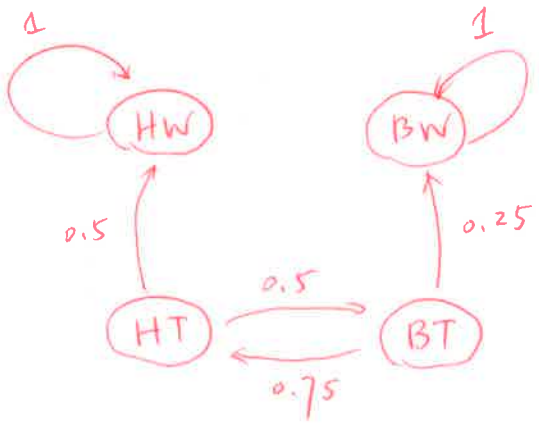
$x + 3y = 6.$

Turn over for more problems

Problem 2 (30 points).

Heather and Blake play a card game in which they take turns drawing a card from a standard deck of cards. Heather can win the game if she draws a red card and Blake can win the game if he draws a diamond. When a player doesn't win on their turn, their card is returned to the deck, the deck is reshuffled, and it becomes the other players turn. The game has four states: Heather wins(HW), Blake wins(BW), Heather's turn(HT), and Blake's turn(BT).

(a) (5 points) Draw the transition diagram for this Markov process.



(b) (10 points) Set up an absorbing stochastic matrix for this Markov process.

	HW	BW	HT	BT
HW	1	0	0.5	0
BW	0	1	0	0.25
HT	0	0	0	0.75
BT	0	0	0.5	0

Turn over for more problems

(c) (15 points) Compute the fundamental matrix $(I-R)^{-1}$. Determine the expected number of turns if Heather goes first.

$$R = \begin{matrix} & \begin{matrix} HT & BT \end{matrix} \\ \begin{matrix} \\ \\ \end{matrix} & \begin{bmatrix} 0 & 0.75 \\ 0.5 & 0 \end{bmatrix} \end{matrix}$$

$$(I-R)^{-1} = \begin{bmatrix} 1 & -0.75 \\ -0.5 & 1 \end{bmatrix}^{-1}$$

$$= \frac{1}{1-0.375} \begin{bmatrix} 1 & 0.75 \\ 0.5 & 1 \end{bmatrix}$$

$$= \frac{1}{0.625} \begin{bmatrix} 1 & 0.75 \\ 0.5 & 1 \end{bmatrix}$$

expected # of turns if Heather goes first is:

$$\frac{1+0.5}{0.625} \quad \text{or} \quad 2.4$$

Turn over for more problems

Problem 3 (30 points).

Suppose that 60% of people who own a General Motors car buy a GM car as their next car and 80% of people who own a non-GM car buy a non-GM car as their next car.

(a) (10 points) Write the transition matrix for this Markov process.

$$\begin{array}{c} \\ \text{GM} \\ \text{Non-GM} \end{array} \begin{array}{c} \begin{array}{cc} \text{GM} & \text{Non-GM} \end{array} \\ \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \end{array}$$

(b) (20 points) What will General Motors' market share be in the long run? (i.e., find the stable distribution)

let $X = \begin{bmatrix} a \\ b \end{bmatrix}$

$$AX = X \Rightarrow \begin{bmatrix} 0.6 & 0.2 \\ 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{cases} 0.6a + 0.2b = a \\ a + b = 1 \end{cases}$$

$$\Rightarrow \begin{cases} a = \frac{1}{3} \\ b = \frac{2}{3} \end{cases}$$

In the long run, GM market share will be $\frac{1}{3}$.

Turn over for more problems

Problem 4 (20 points).

Suppose that R and C play a game by matching coins. On each play, C pays R the number of tails shown (0,1, or 2) minus *twice* the number of heads shown.

(a) (5 points) Set up a payoff matrix for this game.

R \ C	H	T
H	-4	-1
T	-1	2

(b) (10 points) What is the optimal pure strategy for each player?

R will choose Tail

C will choose Head

(c) (5 points) What is the value of the game?

-1

The End.

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