ISYE 3025 HW 1

Problem 1

Ans: Use F/P Factor

$$F = \$2500\,(F/P,\ 2.2\%,\ 2004-1958)$$

$$= \$2500\,(F/P,\ 2.2\%,\ 46)$$

$$= 2500\,(1+0.022)^{46}$$

$$= \$6803 \qquad \square$$

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2. $100{,}000 = 58{,}000\ (F/P, 6.5\%, N) = 58000(1+0.065)^N$

$$N = \frac{\ln\left(\frac{100{,}000}{58{,}000}\right)}{\ln(1.065)} = 8.65$$

N rounds up to 9

Question 3

$i = 3.5\%$ $N = 5$

$9000(P/F, i, N)$ $F = 9000$

$$P = \frac{9000}{(1+0.035)^5} = \frac{9000}{1.1877} = 7577.76$$

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4. Suppose interest rate is i

We know that having \$100 today and \$105 a year from now is indifferent. we can set the equation

$$100(1+i) = 105$$

$$i = 0.05$$

Then we can solve the problem using i

$$2000(1+i)^2 = x$$

$$x = 2000(1+0.05)^2$$

$$\boxed{x = 2205}$$

Problem 5

Ans: Use P/F Factor

Take 2018 as the reference point

$$P = \$2200\,(P/F,\ 6\%,\ 2020-2018)$$

$$= 2200\,(1+0.06)^{-2}$$

$$= 2200\,(0.88999)$$

$$= \$1958 \quad \square$$

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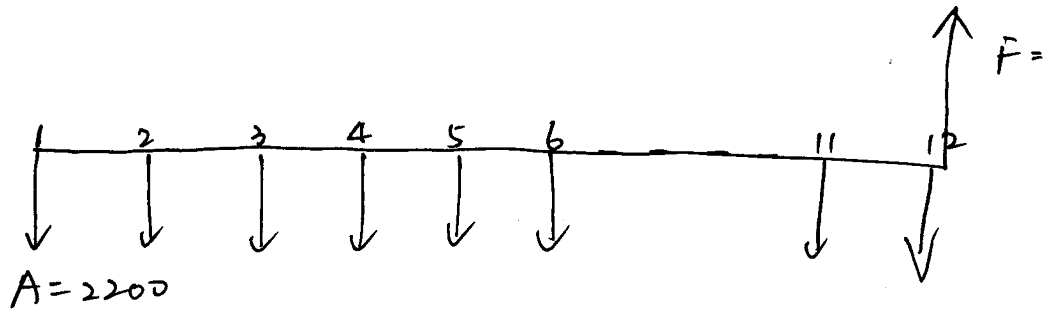
6.
Use F/P factor
Option A: F = 4500(F/P, 5%, 5)(F/P, 3%, 1)+1500(F/P, 3%, 6) =
4500*1.2763*1.03+1500*1.194 = 7706.65
Option B: F = 3800(F/P, 10%, 4)(F/P, 3%, 2)+2200(F/P, 3%, 6) =
3800*1.4641*1.0609+2200*1.194 = 8529.202
Option B will provide more money

Question 7

Monetary factors: Rent cost, transportation (commuting) cost, utility cost, tuition cost.

Non-monetary factors: Living space, home-cooked food (value of), friends vs. family, commuting time, extracurricular opportunities, weather.

8. Answer uses F/A Factor. $F = \$2200\,(F/A,\ 4.4\%,\ 12)$



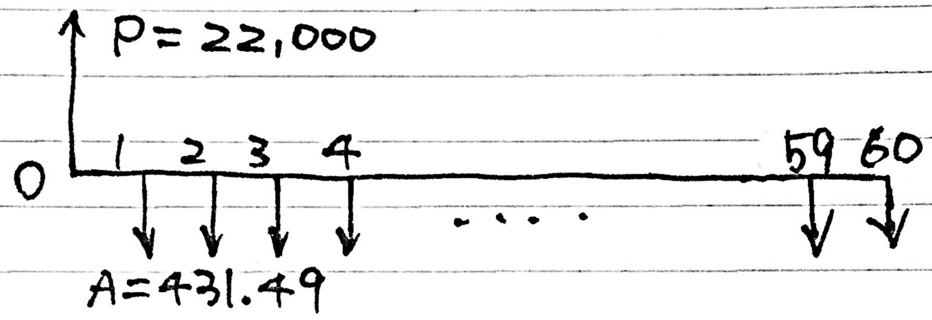
$$F = \frac{A[(1+i)^N - 1]}{i} = \frac{2200[(1+0.044)^{12} - 1]}{0.044} = \boxed{33825.47}$$

Problem 9

Ans:

a) use A/P factor

loan Amount	Monthly Interest Rate	N month to repay
$22,000	0.55 %	60



$$A = \$22{,}000\,(A/P,\ 0.55\%,\ 60)$$

$$= 22{,}000 \cdot \frac{0.55\% \cdot (1+0.55\%)^{60}}{(1+0.55\%)^{60} - 1}$$

$$= \$431.49$$

b) use P/A factor

$$P = \$431.49\,(P/A,\ 0.55\%,\ 60-46)$$

$$= 431.49\ \frac{[(1+0.55\%)^{14} - 1]}{0.55\%\,(1+0.55\%)^{14}}$$

$$= \$\,5798.82$$

OR

Remaining balance

$$= \$22{,}000\,(F/P,\ 0.55\%,\ 46)$$

$$- \$431.49\,(F/A,\ 0.55\%,\ 46)$$

$$= 28{,}313.8 - 22515.21 = \$5799$$

10.

a) Each year, n, n=1,…40, the cumulative amount I have deposited will be greater than the amount my friend has deposited. Thus I will earn more interest on the deposits every year. Moreover, since I am earning more interest on the deposits, I also have more interest accumulated every year and thus will earn more interest on the interest every year.

b) My choice: Use F = 2000(F/A, 6%, 40) = $\frac{2000[(1+0.06)^{40}-1]}{0.06} = 309{,}523.93$

Friend's choice:

G + 2G + 3G …+39G = $\frac{(39G)40G}{2}$

780G = 80000

G = 4000/39

F = G$\frac{[1.06^{40}-0.06*40-1]}{0.06^2}$ = 196,174.3

So, my choice is better than my friend's.

Question 11

$N_1 = 22 \quad N_2 = 34$

$i = 4\%$

$$P = 50000 \, (P/A, i, N_1) \times (P/F, i, N_2)$$

$$P = 50000 \times \frac{\left[(1+i)^{N_1} - 1\right]}{i\,(1+i)^{N_1}} \times \frac{1}{(1+i)^{N_2}}$$

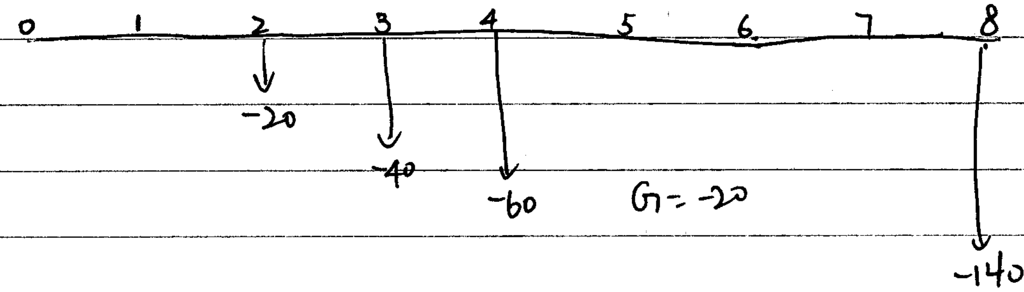
$$P = 50000 \times \frac{\left[(1.04)^{22} - 1\right]}{0.04 \times (1.04)^{22}} \times \frac{1}{1.04^{34}}$$

$$P = 50000 \times \frac{\left[2.3699 - 1\right]}{0.04 \times 2.3699} \times \frac{1}{3.794}$$

$$P = 50000 \times 14.45 \times 0.26355$$

$$P = 190431.08$$

12.

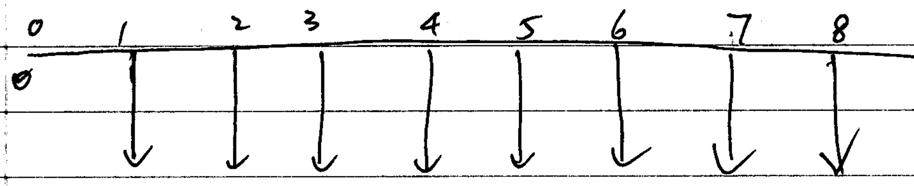


a. Answers uses A/G factor

$$EUV = 140 - 20(A/G, 5\%, 8)$$

$$= 140 - \frac{20[(1+0.05)^8 - 0.05 \cdot 8 - 1]}{0.05(1+0.05)^8 - 0.05}$$

$$= 75.11$$



$A = -64.89$

Diagram (above) is only for the linear gradient conversion, since the contained uniform series does not need to be converted.

b. First find the equivalent value at time 0, then apply $(A/P, i, 7)$

① $P = 140(P/A, 5\%, 8) + (-20)(A/G, 5\%, 8)(P/A, 5\%, 8)$

$$= \frac{140[(1+0.05)^8 - 1]}{0.05(1+0.05)^8} + (-20) \cdot \frac{[(1+0.05)^8 - 0.05 \cdot 8 - 1]}{0.05(1+0.05)^8 - 0.05} \cdot \frac{(1+0.05)^8 - 1}{0.05(1+0.05)^8}$$

$$= 485.45$$

First find the equivalent value at time 0, then Apply $(A/P, i, 7)$

$485.45 (A/P, 5\%, 7)$

$$= 485.45 \frac{0.05(1+0.05)^7}{(1+0.05)^7 - 1} = \boxed{83.9}$$

Problem 13

Ans:

Effective interest rate:

$$1 + i_{4\text{-month}} = (1 + i_{month})^4 = 1.005^4 = 1.020151$$

$$\Rightarrow \quad i_{4\text{-month}} = 1.020151 - 1 = 0.020151$$

$$\Rightarrow \quad F = \$750\,(F/A, 2.0151\%, 75)$$

$$= \$750 \left[\frac{(1 + 2.0151\%)^{75} - 1}{2.0151\%} \right]$$

$$= \$128969 \quad \square$$

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14. N = 11, i=2.2%, g=6%
Use geometric gradient series:
P = 5000(P/g, i=2.2%, g=6%,11) = 5000(1.3005) = 65024.47

Question 15

$N_1 = 25$ $\quad N_2 = 12$

$i = 1.5\%$ $\quad g = 2.2\%$

$$F = 5500 \times (F/g,i, i, N_1, g) \times (F/P, i, N_2)$$

$$F = 5500 \times \frac{(1+i)^{N_1} - (1+g)^{N_1}}{i-g} \times (1+i)^{N_2}$$

$$F = 5500 \times \frac{(1.015)^{25} - (1.022)^{25}}{0.015 - 0.022} \times 1.015^{12}$$

$$F = 5500 \times \frac{1.4509 - 1.7229}{0.015 - 0.022} \times 1.1956$$

$$F = 5500 \times \frac{-0.272}{-0.007} \times 1.1956$$

$$F = 5500 \times 38.8576 \times 1.1956$$

$$F = 255524$$ //

16. c. $1 + i_{annual} = (1 + i_{mo})^{12}$

$= (1 + 0.00522)^{12}$

$i_{annual} = 0.0645$

d. $1 + i_{annual} = (1 + i_{quart})^{4}$

$= (1 + 0.01522)^{4}$

$i_{annual} = 0.0623$

e. Plan 1: $F = \$6000\,(F/A, 6.45\%, 20) = 231693.614$

Plan 2: $F = \$6000\,(F/A, 6.23\%, 20) = 226249.083$

Plan 1 makes \$5444.531 more Plan 2

Problem 17

Ans: use A/P Factor.

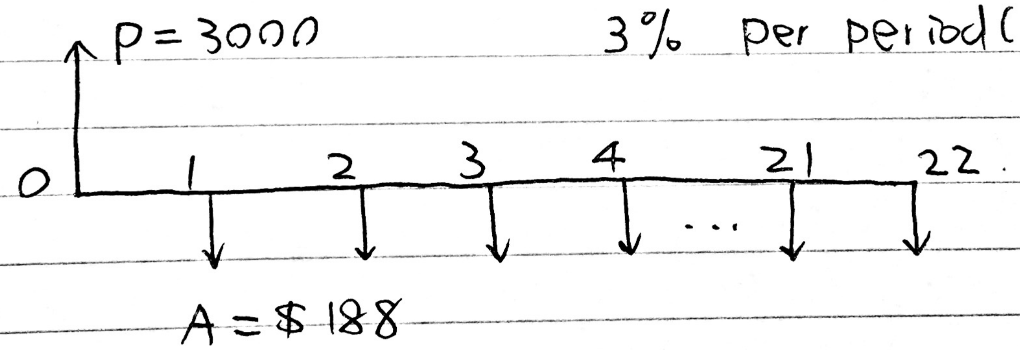
Loan Amount	Monthly Interest Rate	N months to repay
\$ 3000	3 %	22

$$A = \$3000\,(A/P,\ 3\%,\ 22)$$

$$= 3000\left(\frac{3\%\,(1+3\%)^{22}}{(1+3\%)^{22}-1}\right)$$

$$= \$188$$

3% per period (per month)



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18. Need answer from question 17:
Use A/P factor: A = 3000(A/P, 3%, 22) = 188.24

4 months before 1st payment, we can think of each payment delayed by 3 months.
F = 188.24(F/P, 3%, 3) = 205.695

Question 19

Principal = 250000 $i = 9\%$ $N = 5$

a. Equal principal payment = $\frac{\text{Principal}}{N} = \frac{250\,000}{5} = 50\,000$

Year	Principal Payment	Interest payment	Total Payment	Principal Balance
0	0	0	0	250000
1	50000	250 000 × 9% = 22500	72500	200 000
2	50000	200 000 × 9% = 18 000	68 000	150 000
3	50000	150 000 × 9% = 13500	63 500	100 000
4	50000	100 000 × 9% = 9000	59 000	50 000
5	50000	50000 × 9% = 4500	54 500	0

b. Equal payments $= 250\,000 \times (A/P, i, N) = 250\,000 \times \frac{i(1+i)^N}{(1+i)^N - 1}$

$$= 250\,000 \times \frac{0.09(1.09)^5}{1.09^5 - 1} \approx 64\,273$$

Year	Principal Payment	Interest Payment	Total Payment	Principal Balance
0	0	0	0	250 000
1	41773	22 500	64273	208 227
2	45533	18 740	64273	162 694
3	49631	14 642	64273	113063
4	54097	10176	64 273	58 966
5	58966	5307	64273	0

20.

a. Compound monthly. Because with all else being equal, the more frequent the compounding, the better the return on your savings

b. Compound quarterly.

$i = (1+(0.04/4))^4 - 1 = 0.0406 = 4.06\%$

compound monthly

$i = (1+(0.04/12))^{12} - 1 = 0.0407 = 4.07\%$

$\because 4.07\% > 4.06\%$ $\therefore$ The answer to question a is correct.

C. i. Compounding Formula

$1 =$ Annual $r = (1+\frac{0.04}{1})^1 - 1 = 4.0000\%$

$4 =$ Quarterly $r = (1+\frac{0.04}{4})^4 - 1 = 4.0604\%$

$12 =$ Monthly $r = (1+\frac{0.04}{12})^{12} - 1 = 4.07415\%$

$52 =$ Weekly $r = (1+\frac{0.04}{52})^{52} - 1 = 4.07947\%$

$365 =$ Daily $r = (1+\frac{0.04}{365})^{365} - 1 = 4.08084\%$

$8760 =$ Hourly $r = (1+\frac{0.04}{8760})^{8760} - 1 = 4.08088\%$

∞ = Continuously $r = e^{0.04} - 1 = 4.08108\%$

There is no big increase in interest rate

ii $\lim_{n\to\infty} (1+\frac{1}{n})^n = e.$

~~P = 100~~

$\lim_{n\to\infty} P(1+\frac{r}{n})^{nt}$ $x = \frac{n}{r}$ $n = xr$

$P\lim_{n\to\infty} (1+\frac{1}{x})^{xrt} = P\left(\lim_{x\to\infty} (1+\frac{1}{x})^{x}\right)^{rt}$

$= Pe^{rt}.$

$r = (1+0.04/365)^{365} - 1 = 0.0408$

$P = 100$ $t = 5$

$100 \cdot e^{0.0408\cdot 5} = 122.63$