PHYS 2211 Test 4
Spring 2014

Name(print) ~~ Test ~~ Key ~~ **Lab Section** KEY

Greco(K)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Wednesday	K03 K05	K04 K07	K06 K08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Regulus Arcturus Black

Sign your name on the line above

The final exam for this class is scheduled for:
Period 4, April 29th (Tue) at 8:00am - 10:50am

The conflict final exam for this class is TBD

ADAPTS Student will need to schedule their final exam with the **ADAPTS office as soon as possible.**

PHYS 2211

Do not write on this page!

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

A motorcycle (Hayabusa) of mass 250 kg accelerates from rest to 65 m/s in a distance of 402 m. Neglect air resistance and assume the wheels do not slip on the pavement, which means that the velocity of atoms in the wheel when they are in contact with the road is always zero. (This is similar to the case of walking or running without slipping; there's no displacement at the point of contact.)

(a 6pts) Use the point particle system to determine the average force that the road exerts on the motorcycle.

$$\underbrace{\Delta K_{trans} = K_f - \cancel{K_i}_{0} = \vec{F} \cdot \Delta \vec{r}_{cm} = F \Delta r}_{3pts}$$

$$\frac{1}{2} m v_f^2 = F \Delta r$$

$$\underbrace{F = \frac{m v_f^2}{2 \Delta r} = \frac{(250\text{ kg})(65\text{ m/s})^2}{(2)(402\text{ m})} =}_{2pts}$$

$$= \boxed{1314\text{ N}} \quad 1pt$$

(c 4pts) For the real system of the motorcycle, circle which of the following types of energy increase or decrease:

translational kinetic energy: decreases, stays the same, $\boxed{\text{increases}}$
rotational kinetic energy: decreases, stays the same, $\boxed{\text{increases}}$
thermal energy: decreases, stays the same, $\boxed{\text{increases}}$
chemical energy: $\boxed{\text{decreases,}}$ stays the same, increases

1pt each

(d 10pts) The moment of inertia for typical motorcycle wheel is $I_w = 1.5 kg \cdot m^2$. Calculate the change in chemical energy of the motorcycle as it starts from rest and accelerates through a distance of 402 meters. Assume Q and the change in thermal and rotational energy of engine is very small (zero).

$$\Delta E = \Delta K_{trans} + \Delta K_{rot} + \Delta E_{int} = \underset{0}{\cancel{W}} = 0$$

$$\Delta E_{int} = -\Delta K_{trans} - \Delta K_{rot} = \frac{-1}{2} m v_f^2 - 2\left(\frac{1}{2} I \omega^2\right) = \frac{-1}{2} m v^2 - \cancel{2}\left(\frac{1}{\cancel{2}} I \frac{v^2}{r^2}\right)$$

Radius of motorcycle wheel ⇒ 8.5 inches = 0.2159 m

$$\Rightarrow \Delta E_{int} = \left(\frac{-1}{2}\right)(250)(65^2) - \left(\frac{\cancel{2}}{\cancel{2}}\right)(1.5)\frac{(65^2)}{(0.2159)^2} =$$

$$= -528125 - 135960 =$$

$$\boxed{\Delta E_{int} = -664085 \text{ Joules}}$$

$$\boxed{\begin{array}{r} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$$

(approx. 49.6 mpg)

actual is 38 mpg ... drag

(e 5pts) While moving at 65 m/s, the driver hits the brakes and quickly comes to a stop (but there is no skidding; the wheels don't slip on the pavement). All of the kinetic energy of the motorcycle ends up as a thermal energy change in the brakes. The breaks contain 4 kg of high carbon steal. The heat capacity of this material is $0.50 J/(g \cdot C)$. Calculate the temperature change in the brakes?

$$\left.\Delta E_{th} + \Delta E_{int} = \underset{0}{\cancel{Q}} + \underset{0}{\cancel{W}} = 0\right\} \text{2pts}$$

$$\left.\begin{aligned} \Delta E_{th} &= -\Delta E_{int} = m c \Delta T \\ \frac{-\Delta E_{int}}{m c} &= \Delta T \end{aligned}\right\} \text{2pts}$$

$$\Delta T = \frac{-(-664085)}{(4000)(0.50)} = \boxed{332°C} \quad \text{1pt}$$

Problem 2 (25 Points)

A muon is a particle that has the same charge as an electron but whose mass is 200 times larger ($q = -e$, $m = 200m_e$). The motion of this particle through the air, and it's interaction with diatomic nitrogen molecules N_2, is the trigger for lightning strikes. The program below, which is similar to your computer model from the Rutherford lab, updates the position of the muon as it moves near a N_2 ion (14 protons, 14 neutrons, 13 electrons). The program is missing several important lines of code. In the space provided, add the statements necessary to complete the code.

```
from visual import *
from visual.graph import *

## objects
muon = sphere(pos=(-4e-13,2e-14,0), radius=1e-15, color=color.red)
nitrogen = sphere(pos=(0,0,0), radius=28e-15, color=color.blue)

## constants
k = 9e9
deltat = 1e-23
t = 0

## Create energy graphs
gdisplay(width=500, height=250, x=524, y=500)
Kmuon_graph = gcurve(color=color.red)
Knitrogen_graph = gcurve(color=color.blue)
Ugraph = gcurve(color=color.green)
KplusUgraph = gcurve(color=color.yellow)
```

(a 4pts) Add the missing constants (mass and charge) listed below

1pt each

```
m_muon = 200 * 9e-31

m_nitrogen = (14 * 1.7e-27) + (14 * 1.7e-27)

q_muon = -1.6e-19

q_nitrogen = 1.6e-19
```

```
## initial values
muon.p = m_muon*vector(7.5e6,0,0)
nitrogen.p = vector(0,0,0)
```

THE PROGRAM CONTINUES ON THE NEXT PAGE

```
## calculation loop
while t < 1e-19:
```

(b 10pts) Add the necessary statements to update the momentum and position of the muon particle. You can assume the much more massive Nitrogen molecule remains motionless.

2pts
```
r = muon.pos - nitrogen.pos
rmag = mag(r)
rhat = norm(r)
```
2 `Fmag = k * q-muon * q-nitrogen /rmag ** 2`

2 `Fnet = Fmag * rhat`

2 `muon.p = muon.p + Fnet * deltat`

2 `muon.pos = muon.pos + (muon.p/m_muon) * deltat`

(c 5pts) Add the necessary statements to calculate the kinetic, potential and total energy of the muon-nitrogen system. You can assume the much more massive Nitrogen molecule remains motionless.

2pts
```
K_muon = (mag(muon.p))**2 / (2 * m_muon)
k_nitrogen = (mag(nitrogen.p)) **2 / (2 * m_nitrogen)
k_total = k_muon + k_nitrogen
```
2pts `U = k * q-muon * q-nitrogen /rmag`

1pt `E_total = K_total + U`

```
    t = t + deltat
```

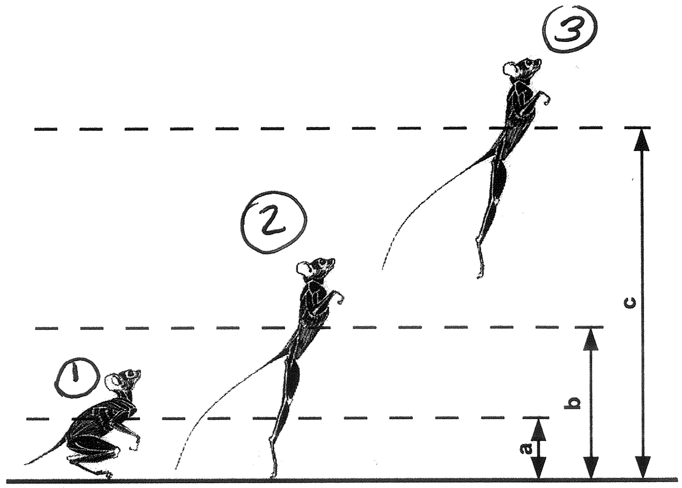
(d 5pts) Add the necessary statements to calculate the the scattering angle θ of the muon particle.

`angle = atan2(muon.p.y / muon.p.x) * (180/pi)`

All

Problem 3 (25 Points)

Stuck at the zoo one day, you take a video of a lemur crouching down and jumping straight up. When you get home you analyze the video to find three images: the lemur at rest in a crouched position, the lemur jumping up the moment his feet are leaving the ground, and the lemur at the top of his vertical trajectory. These images, along with the heights of the lemur's center of mass (a, b, c respectively) are shown in the diagram. The lemur has mass m.



10

(a ~~30~~pts) Starting from the energy principle, find lemur's speed the moment his feet are leaving the ground. Your answer should be in terms of the given variables and known constants.

✓ Initial : @ 2 ⇒ $v_i = ?$, $h_i = b$

✓ Final : @ 3 ⇒ $v_f = 0$, $h_f = c$

$\Delta E = \Delta K + \Delta U = 0$

$\cancel{K_f} - K_i + U_f - U_i = 0$

0

$-\frac{1}{2}mv_i^2 + mg\,\Delta h = 0$

$-\frac{1}{2}mv_i^2 + mg(c-b) = 0$

$\cancel{m}g(c-b) = \frac{1}{2}\cancel{m}v_i^2$

$g(c-b) = \frac{1}{2}v_i^2$

$v_i^2 = 2g(c-b)$

$$\boxed{v_i = \sqrt{2g(c-b)}}$$

-0.5
-1.5
-3.0
-8.0

5

(b ~~40~~pts) Assuming that the contact force of the ground on the lemur's feet is constant, find the magnitude of the contact force during the jump.

Point-particle system

Initial: @ 1 ⇒ $v_i = 0$, $h_i = a$

Final: @ 2 ⇒ v_f = from part A, $h_f = b$

$$\Delta K = F_{net}\, \Delta r_{cm} = (F - mg)(b - a) \rightarrow \text{2pts}$$

$$\left.\begin{aligned} K_f - \cancel{K_i}^{\,0} &= (F - mg)(b - a) \\ \tfrac{1}{2} m v_f^2 &= (F - mg)(b - a) \end{aligned}\right\} \text{1pt}$$

$$\tfrac{1}{\cancel{2}} m (\cancel{2}g)(c - b) = (F - mg)(b - a) \rightarrow \text{1pt}$$

$$\frac{mg(c-b)}{(b-a)} = F - mg$$

$$F = mg + mg\left(\frac{c-b}{b-a}\right) = \boxed{mg\left(1 + \frac{c-b}{b-a}\right)} \quad \text{1pt}$$

(d ~~30~~ 10 pts) Find the change in the lemur's internal energy during the jump.

Real system → same initial & final as in Part B.

$$\Delta K + \Delta E_{int} = W = (-mg)(b-a)$$

$$\tfrac{1}{2} m v^2 + \Delta E_{int} = -mg(b-a)$$

$$\tfrac{1}{\cancel{2}} m (\cancel{2}g)(c-b) + \Delta E_{int} = -mg(b-a)$$

$$mg(c-b) + \Delta E_{int} = -mg(b-a)$$

$$\Delta E_{int} = -mg(b-a) - mg(c-b) =$$

$$= -mg\left[(b-a) + (c-b)\right] =$$

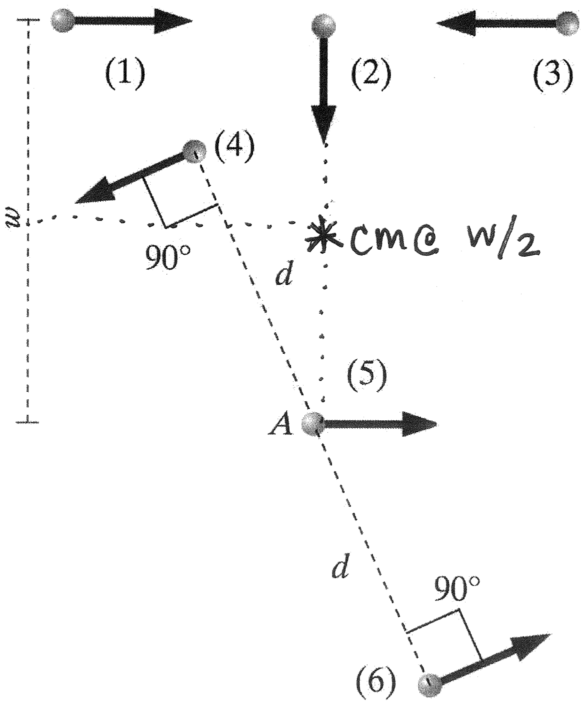
$$= -mg(\cancel{b} - a + c - \cancel{b}) = -mg(-a + c)$$

$$= \boxed{-mg(c-a)}$$

−0.5
−1.5
−3.0
−8.0

Problem 4 (25 Points)

In the diagram below, six identical particles of mass m and speed v are moving relative to a point A, the current location of particle (5). The distance of these particles from point A is indicated in the diagram. As usual, x is to the right, y is up and z is out of the page, towards you.



(a 15pts) Calculate the total angular momentum of the system of particles with respect to location A. Be sure to show your work to earn full credit.

$\vec{L}_{1A} = \vec{r}_{1A} \times \vec{p}_1 = -wmv\,\hat{z}$

$\vec{L}_{2A} = \vec{r}_{2A} \times \vec{p}_2 = r_{2A} P_2 \sin \cancel{(180°)}^{0} = 0$

$\vec{L}_{3A} = \vec{r}_{3A} \times \vec{p}_3 = wmv\,\hat{z}$

$\vec{L}_{4A} = \vec{r}_{4A} \times \vec{p}_4 = d p_4 = dmv\,\hat{z}$

$\vec{L}_{5A} = \vec{r}_{5A}^{\,0} \times \vec{p}_5 = 0$

$\vec{L}_{6A} = \vec{r}_{6A} \times \vec{p}_6 = d p_6 = dmv\,\hat{z}$

2pts each

$\vec{L}_{A,total} = \sum_i \vec{L}_{iA} = (-\cancel{wmv} + \cancel{wmv} + dmv + dmv)\,\hat{z} =$

$= \boxed{2dmv\,\hat{z}}$ 3pts

(b 5pts) Determine the translational angular momentum of the system of particles with respect to location A. Be sure to show your work to earn full credit.

✓ $\vec{r}_{cm} = \langle 0, W/2, 0 \rangle$ (using point A as the "origin" → 1pt

✓ $\vec{P}_{total} = \sum_i \vec{p}_i = (\cancel{\vec{P}_1 + \vec{P}_3})^0 + (\cancel{\vec{p}_4 + \vec{p}_6})^0 + \vec{P}_2 + \vec{P}_5 = \vec{P}_2 + \vec{P}_5 =$

$= m_2\vec{V}_2 + m_5\vec{V}_5 = 2m(\vec{V}_2 + \vec{V}_5) = 2m[\langle 0, -v, 0\rangle + \langle v, 0, 0\rangle] =$

$= 2m\langle v, -v, 0\rangle = \langle 2mv, -2mv, 0\rangle$ → 1pt

✓ $\vec{L}_{trans} = \vec{r}_{cm} \times \vec{P}_{total} = \langle 0, W/2, 0\rangle \times \langle 2mv, -2mv, 0\rangle =$

$$\begin{vmatrix} x & y & z \\ 0 & W/2 & 0 \\ 2mv & -2mv & 0 \end{vmatrix}$$

x: $\left(\frac{W}{2}\right)(0) - (0)(-2mv) = 0$

y: $(0)(2mv) - (0)(0) = 0$

z: $(0)(-2mv) - \left(\frac{W}{2}\right)(2mv) = \left(\frac{-W}{\cancel{2}}\right)(\cancel{2}mv) = -wmv$

3pts

$\Rightarrow \boxed{\vec{L}_{trans} = -wmv\ \hat{z}}$

(c 5pts) Determine the rotational angular momentum of the system of particles with respect to location A. Be sure to show your work to earn full credit.

$\vec{L}_{total} = \vec{L}_{rot} + \vec{L}_{trans} \Rightarrow \vec{L}_{rot} = \vec{L}_{total} - \vec{L}_{trans}$ 3pts

$\Rightarrow \vec{L}_{rot} = \underbrace{2dmv\ \hat{z}}_{\text{part A}} - \underbrace{(-wmv)\ \hat{z}}_{\text{part B}} =$ → 1 pt

$= (2dmv + wmv)\ \hat{z} =$

$= \boxed{mv(2d + w)\ \hat{z}}$ 1pt

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

$$\Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

$$\text{prob}(E) \propto \Omega\,(E)\,e^{-\frac{E}{kT}}$$

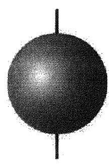
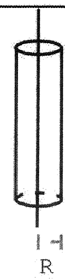
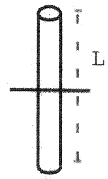
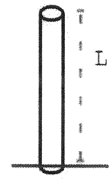
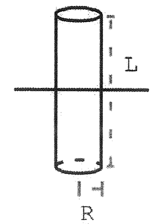
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule · second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule · second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}