QUIZ 2 Name Solutions

Math 2551 D Steinbart Section ____ January 27, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(5) 1. Consider the curve $\mathbf{r}(t) = 2t\mathbf{i} + \frac{2}{3}t^{3/2}\mathbf{j} + (3-t)\mathbf{k}$. When $t = 0$, $\mathbf{r}(t) = \mathbf{r}(0) = 0\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$. Find the length of the curve from the point on the curve $P(0, 0, 3)$ to the point $Q(4, \frac{4\sqrt{2}}{3}, 1)$.

At Q, $\underline{r}(t) = 2t\underline{i} + \frac{2}{3}t^{3/2}\underline{j} + (3-t)\underline{k} = 4\underline{i} + \frac{4\sqrt{2}}{3}\underline{j} + 1\underline{k}$

So $2t = 4$ Check $t=2 \Rightarrow 2t = 4$ ✓

Need $\frac{2}{3}t^{3/2} = \frac{4\sqrt{2}}{3}$ $\frac{2}{3}t^{3/2}\Big|_{t=2} = \frac{2}{3}(2^{3/2}) = \frac{2}{3}2\sqrt{2} = \frac{4}{3}\sqrt{2}$ ✓

$3 - t = 1 \rightarrow t = 2$

$\underline{v}(t) = \underline{r}'(t) = 2\underline{i} + t^{1/2}\underline{j} - \underline{k}$

$|\underline{v}| = \sqrt{2^2 + (t^{1/2})^2 + (-1)^2} = \sqrt{t+5}$

Then the length of the curve from P to Q is

$$L = \int_0^2 |\underline{v}(t)|\,dt = \int_0^2 \sqrt{t+5}\,dt = \frac{2}{3}(t+5)^{3/2}\Big|_0^2 = \frac{2}{3}\left[7^{3/2} - 5^{3/2}\right] = \frac{2}{3}\left[7\sqrt{7} - 5\sqrt{5}\right]$$

$$= \frac{14}{3}\sqrt{7} - \frac{10}{3}\sqrt{5}$$

(5) 2. Consider the curve $\mathbf{r}(t) = 2t\mathbf{i} - \frac{1}{3}t^3\mathbf{j}$.

a. Find $\mathbf{T}$, the unit tangent vector at the point on the curve $P(2, -1/3)$.

b. Sketch $\mathbf{T}$ and $\mathbf{N}$ at the point P. (You do not need to compute $\mathbf{N}$).

a. $\underline{v}(t) = 2\underline{i} - t^2\underline{j}$

$|\underline{v}| = \sqrt{2^2 + (-t^2)^2} = \sqrt{4 + t^4}$

$\underline{r}(t) = 2t\underline{i} - \frac{1}{3}t^3\underline{j} = 2\underline{i} - \frac{1}{3}\underline{j}$

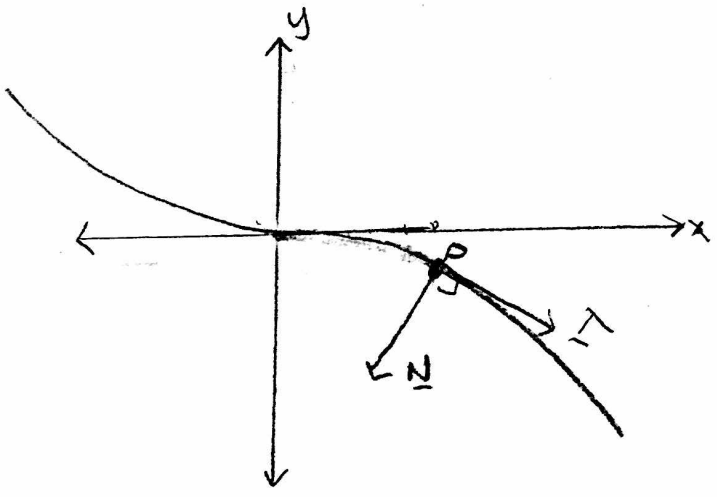
@ $\Rightarrow 2t = 2 \rightarrow t = 1$

$-\frac{1}{3}t^3 = -\frac{1}{3}$ if $t = 1$

$-\frac{1}{3}t = -\frac{1}{3}$ ✓

So $\underline{T}(t) = \frac{\underline{v}(t)}{|\underline{v}(t)|} = \frac{2}{\sqrt{4+t^4}}\underline{i} + \frac{-t^2}{\sqrt{4+t^4}}\underline{j}$

So $\underline{T}$ at P is $\underline{T}(1) = \frac{2}{\sqrt{5}}\underline{i} - \frac{1}{\sqrt{5}}\underline{j}$



Observe: $\underline{N} = \frac{-1}{\sqrt{5}}\underline{i} + \frac{2}{\sqrt{5}}\underline{j}$.

(?) How does one know this without computing $\frac{d\underline{T}/dt}{|d\underline{T}/dt|}$?