Solutions to Homework 3

1. Data: $p = 18$, $c_v = 3$, $s_v = 1$, and the distribution of demand is:

$D = d$	10	11	12	13	14
$P\{D = d\}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	$\frac{1}{10}$

   - (a) $E(D)$= 12
   - (b) Since $E[min\{D, 11\}] = 10.9$ and $E[max\{11 - D, 0\}]$= 0.1 it follows that expected profit under the policy $q = 11$ is:
     $g(11)$= $15E[min\{D, 11\}]$ - $2E[max\{11 - D, 0\}]$= 163.3.
   - (c) The smallest q satisfying $F(q) \geq \frac{p-c_v}{p-s_v} = \frac{15}{17}$ is $q = 13$. It is the best amount.
   - (d) This is optimal provided that the environment does not change and that the game is played many times, it is the long run average.
   - (e) From tables of the Gaussian (i.e., normal) distribution or by using a calculator: $F(q) = P\{D < q\} = \frac{15}{17} \Rightarrow q = 1017$

2. Data: $p = 20$, $c_v = 8$, and the demand distribution is:

b	15	16	17	18	19	20
$P\{D = d\}$	$\frac{1}{20}$	$\frac{4}{20}$	$\frac{5}{20}$	$\frac{5}{20}$	$\frac{2}{20}$	$\frac{3}{20}$

   The appropriate decision rule is to choose the smallest q such that $F(q) \geq \frac{p-c_v}{p} = \frac{12}{20}$. So for $q = 18$ we have $\frac{15}{20} > \frac{12}{20}$. For $q < 18$ $F(q) < \frac{p-c_v}{p} = \frac{12}{20}$.Thus, the answer is y^*= 18. When y^*= 18, the expected sale is E min(D,18) = 17.2. So the weekly profit is 17.2*20-18*8 = 200.

3. Data: unit retail price is 550, unit delivery charge is 50; consequently the unit penalty cost is $p = 600$, unit variable cost is $c_v = 400$, unit inventory holding cost is $h = 25$, and the distribution of demand is:

$D = d$	10	11	12	13	14	15
$P\{D = d\}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

   - (a) Expected shortage cost: $pE[max\{D - 12, 0\}] = p = 600$ and expected inventory cost: $hE[max\{12 - D, 0\}] = 12.5$. The expected total cost is 612.5.
   - (b) q is smallest number such that $F(q) \geq \frac{p-c_v}{p+h} = \frac{200}{625} = \frac{8}{25} \Rightarrow \quad q = 11$.
   - (c) Now q solves $F(q) = \frac{p-c_v}{p+h} = \frac{200}{625} = \frac{8}{25} \Rightarrow \quad q = 953$; so order 953 cameras.