ISyE 3232 Stochastic Manufacturing and Service Systems Fall 2013

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Homework 9

(Due: at the start of class on Tuesday/Wednesday, November 5/6)

1. Consider two stocks. Stock 1 always sells for \$10 or \$20. If stock 1 is selling for \$10 today, there is a .80 chance that it will sell for \$10 for tomorrow. If it is selling for \$20 today, there is a .90 chance that it will sell for \$20 tomorrow.

   Stock 2 always sells for \$10 or \$25. If stock 2 sells today for \$10 there is a .90 chance that it will sell tomorrow for \$10. If it sells today for \$25, there is a .85 chance that it will sell tomorrow for \$25. Let X_n denote the price of the 1st stock and Y_n denote the price of the 2nd stock during the nth day. Assume that $\{X_n : n \geq 0\}$ and $\{Y_n : n \geq 0\}$ are discrete time Markov chains.

   (a) What is the transition matrix for $\{X_n : n \geq 0\}$? Is $\{X_n : n \geq 0\}$ irreducible?

   (b) What is the transition matrix for $\{Y_n : n \geq 0\}$? Is $\{Y_n : n \geq 0\}$ irreducible?

   (c) What is the stationary distribution of $\{X_n : n \geq 0\}$?

   (d) What is the stationary distribution of $\{Y_n : n \geq 0\}$?

   (e) On January 1st, your grand parents decide to give you a gift of 300 shares of either Stock 1 or Stock 2. You are to pick one stock. Once you pick the stock you cannot change your mind. To take advantage of a certain tax law, your grand parents dictate that one share of the chosen stock is sold on each trading day. Which stock should you pick to maximize your gift account by the end of the year? (Explain your answer.)

2. Suppose each morning a factory posts the number of days worked in a row without any injuries. Assume that each day is injury free with probability 98/100. Furthermore, assume that whether tomorrow is injury free or not is independent of which of the preceding days were injury free. Let $X_0 = 0$ be the morning the factory first opened. Let X_n be the number posted on the morning after n full days of work.

   (a) Is $\{X_n, n \geq 0\}$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

   (b) Is the Markov chain irreducible? Explain.

   (c) Is the Markov chain periodic or aperiodic? Explain and if it is periodic, also give the period.

   (d) Find the stationary distribution.

   (e) Is the Markov chain positive recurrent? If so, why? If not, why not?