Solve the following LPs using the simplex algorithm. Use the two-phase simplex algorithm when appropriate. If the LP has an optimal solution, state it, and state the optimal value. Otherwise, state clearly whether the LP is infeasible or unbounded.

1.
$$\begin{aligned}
\max \quad & 2x_1 + x_2 - 3x_3 + 5x_4 \\
\text{s.t.} \quad & x_1 + 2x_2 + 2x_3 + 4x_4 \le 40 \\
& 2x_1 - x_2 + x_3 + 2x_4 \le 8 \\
& 4x_1 - 2x_2 + x_3 - x_4 \le 10 \\
& x_1 \ge 0, x_2 \ge 0, x_3 \ge 0, x_4 \ge 0
\end{aligned}$$

2.
$$\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & -x_1 + x_2 \le 2 \\
& x_1 - x_2 \le 2 \\
& x_1 \ge 0, x_2 \ge 0
\end{aligned}$$

3.
$$\begin{aligned}
\max \quad & 3x_1 - x_2 + 2x_3 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 \le 4 \\
& 2x_1 + x_2 - x_3 \ge 1 \\
& -x_2 + x_3 \ge 1 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}$$

4.
$$\begin{aligned}
\min \quad & 3x_1 \\
\text{s.t.} \quad & 2x_1 + x_2 + x_3 \ge 6 \\
& 3x_1 + 2x_2 + x_3 = 4 \\
& x_1 \ge 0, x_2 \ge 0, x_3 \ge 0
\end{aligned}$$