Tue 5 7:30-11pm

QUIZ 3 Name Solution's

Math 2551 D Steinbart Section ____ February 4, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(5) 1. Consider the curve $\mathbf{r}(t) = 2e^t\mathbf{i} + (t+6)\mathbf{j} + e^{2t}\mathbf{k}$.

a. Find $\mathbf{v}(t)$ and $\mathbf{a}(t)$.

b. Find the normal component of acceleration a_N at the point on the curve $P(2,6,1)$.

c. Find the curavture κ at P.

$$\boxed{\begin{aligned} \underline{v}(t) &= \frac{d}{dt}(\underline{r}(t)) = 2e^t\underline{i} + \underline{j} + 2e^{2t}\underline{k} \\ \underline{a}(t) &= \frac{d}{dt}(\underline{v}(t)) = 2e^t\underline{i} + 4e^{2t}\underline{k} \end{aligned}}$$

Aside: $a_T = \frac{d}{dt}(|v|) = \frac{d}{dt}\left(\sqrt{(2e^t)^2 + 1^2 + (2e^{2t})^2}\right)$

$= \frac{d}{dt}\sqrt{4e^{2t} + 1 + 4e^{4t}} = \frac{1}{2}(4e^{2t} + 1 + 4e^{4t})^{-1/2}(8e^{2t} + 0 + 16e^{4t})$

$a_T\big|_{t=0} = \frac{1}{2}(4(1) + 1 + 4(1))^{-1/2}(8+16)$

$= \frac{1}{2}\cdot\frac{1}{3}\, 24 = \frac{12}{3} = 4.$ ②

(b) $a_N = \sqrt{|\underline{a}|^2 - a_T^2}$ ①

$P(2,6,1)$ corresponds to the position vector $2\underline{i} + 6\underline{j} + \underline{k}$. $\underline{r}(t) = 2\underline{i} + 6\underline{j} + \underline{k}$ when

$2e^t = 2 \Rightarrow t = 0$

$t + 6 = 6$ Check $t+6\big|_{t=0} = 6$ ✓

$e^{2t} = 1$ $e^{2t}\big|_{t=0} = 1$ ✓

So $t=0$ when the curve is at $P(2,6,1)$

At $t=0$, $\underline{a}(t) = \underline{a}(0) = 2\underline{i} + 4\underline{k}$.

So $|\underline{a}|^2 = 2^2 + 4^2 = 20$ ③

Combining ①, ②, ③: $a_N = \sqrt{20 - 4^2} = \sqrt{4} = \boxed{2}$

(c) $\kappa = \frac{|\underline{v} \times \underline{a}|}{|\underline{v}|^3} = \frac{6}{3^3} = \boxed{\frac{2}{9}}$

at $t=0$ $|v| = \sqrt{2^2 + 1^2 + 2^2} = 3$

Aside: $\underline{v} \times \underline{a}$ at $t=0$ $= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 2 \\ 2 & 0 & 4 \end{vmatrix} = \underline{i}(4) - \underline{j}(8-4) + \underline{k}(-2) = 4\underline{i} - 4\underline{j} - 2\underline{k}$

So $|\underline{v} \times \underline{a}| = \sqrt{4^2 + (-4)^2 + (-2)^2} = \sqrt{16+16+4} = \sqrt{36} = 6.$

(5) 2. Let $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-9}}$.

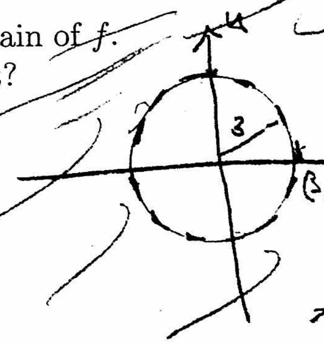
a. Find (express in symbols) and sketch the domain of f.

b. Is the domain of f bounded? Why or why not?

(a) f will be defined when $x^2+y^2-9 > 0$

$x^2 + y^2 > 9$

Domain of $f = \{(x,y) \mid x^2+y^2 > 9\}$



Domain is the region outside the circle $x^2+y^2 = 9$

Point on the circle are not in the domain of f.

(b) The domain of f is not bounded

Given any $R_0 > 0$ there are points in the domain of f which do not lie in the disk $x^2+y^2 \le R_0^2$

The point $(R_0+3, 0)$ will be a point in the domain, but it is not in the disk $x^2+y^2 \le R_0^2$.