ChBE 3200
Transport Phenomena I
Fall 2013

Exam I
September 20, 2013

This exam is closed-book, closed-notes. Some equations and other relevant information are provided. The use of wireless devices (e.g. cell phones, IR transmitters/receivers) is not permitted. The use of programmable calculators is only allowed if all relevant content has been erased from the calculator memory.

To receive full credit on each problem, it is advised to start with the appropriate full form of the balance equation(s) needed to solve the problem. Label all variables and equations. Include a brief word description to explain each step in your problem if appropriate. State all your assumptions clearly. Present your solution clearly. Numerical answers without units or explanations will not receive credit.

Name: ______________________________
(PLEASE WRITE YOUR NAME ALSO ON THE BACK OF THE EXAM.)

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

Signed: ______________________________

Problem I __________/20

Problem II __________/50

Problem III __________/30

Total __________/100

PLEASE SCAN THROUGH THE ENTIRE EXAM BEFORE WORKING ON IT.

Problem I (20 points)

1. Power law can be used to describe non-Newtonian fluids; therefore it is inappropriate to describe a Newtonian fluid using power law. TRUE/FALSE
(If false, please provide explanation.) (3 pts)

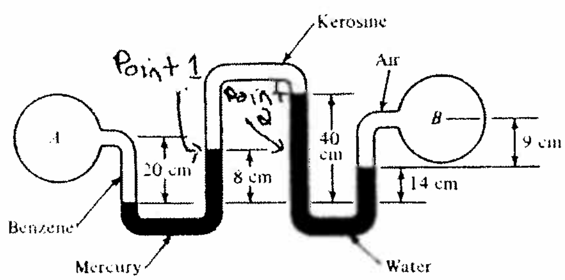
   FALSE; Newtonian fluid is a special case of power-law fluid where n = 1, and m = viscosity

2. Steady state flow means that the density is not a function of space (e.g. x, y, z). TRUE/FALSE
(If false, please provide explanation.) (3 pts)

   FALSE; steady state means no time dependence, but density can still be a function of space; density independent of space implies that the fluid is incompressible.

3. Observe the manometer below. The pressure at point 1 equals the pressure at point 2. TRUE/FALSE
(If false, please provide explanation.) (3 pts)

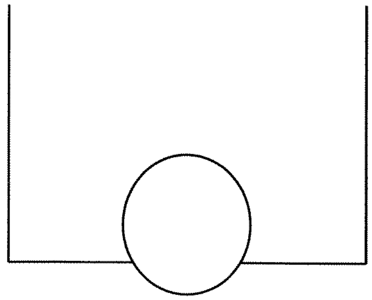
   FALSE: although Point 1 and Point 2 are the same vertical difference, there are two different manometer fluids with different densities in the tube connecting the two points.



4. Normal stresses are defined as positive in tension. TRUE/FALSE
(If false, please provide explanation.) (3 pts)
TRUE; Sign definition used by your text: normal stresses are defined as positive in tension and negative in compression.

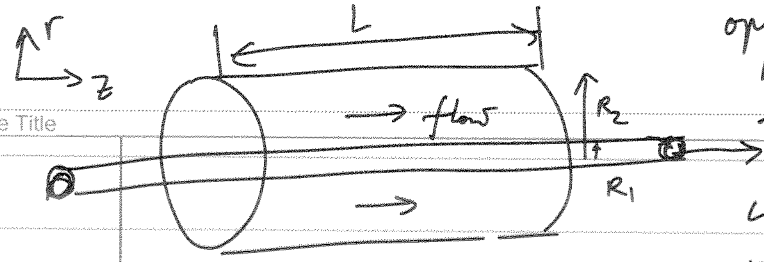
5. All stresses in a static fluid are zero. TRUE/FALSE
(If false, please provide explanation.) (3 pts)
FALSE; There is no shear stress in a fluid at rest; the only forces acting on the fluid are due to gravity and pressure (i.e. normal forces).

6. Under what circumstance can an air balloon be used to plug a hole on the bottom of a tank filled with water? (5 pts)



The balloon will have to be pressed down by enough water "head". In other words, the pressure force ($\iint_{surface\ of\ the\ balloon\ exposed\ to\ water} P(x,y,z)dA$) will have to exceed the buoyancy that the balloon experiences ($\rho_{water} g V_{balloon\_in\_water}$). (The excess force will be balanced by the normal force exerted by the bottom of the tank, so that everything is stationary and zero net force.)

Problem II

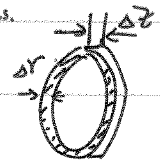


open to air
Newtonian fluid
v_0, F

Note Title 9/17/2013

What is μ?
neglect g

cylindrical coordinates.
pick C.V.



mom. bal.

$$P_z \cdot 2\pi r \Delta r - P_{z+\Delta z}\, 2\pi r \Delta r - \left(\tau_{rz}\, 2\pi r \Delta z\right)_r + \left(\tau_{rz}\, 2\pi r \Delta z\right)_{r+\Delta r} = 0$$

$$\left(\text{no } g, \ \frac{\partial}{\partial t}\cdots = 0, \ \iint_{cs} v_z \rho (\vec{v}\cdot\vec{n})\, dA = 0, \text{ fully dev. s.s.}\right)$$

div. by $2\pi \Delta r \Delta z$

$$\frac{r\left(P_z - P_{z+\Delta z}\right)}{\Delta z} + \frac{\left(\tau_{rz} r\right)_{r+\Delta r} - \left(\tau_{rz} r\right)_r}{\Delta r} = 0$$

$\lim_{\Delta r \Delta z \to 0}$

$$-r\frac{dP}{dz} + \frac{d}{dr}\left(r\tau_{rz}\right) = 0$$

system is open so $\frac{dP}{dz} = 0$

$$\Rightarrow \frac{d}{dr}\left(r\tau_{rz}\right) = 0$$

$$\Rightarrow r\tau_{rz} = C_1 \qquad \tau_{rz} = \frac{C_1}{r}$$

B.C. ① at $r = R_1$, $F = -\tau_{rz}\big|_{R_1} 2\pi R_1 L$

$$\tau_{rz}\Big|_{R_1} = \frac{-F}{2\pi R_1 L} = \frac{C_1}{R_1}$$

$$\Rightarrow C_1 = \frac{-F}{2\pi L} \Rightarrow \boxed{\tau_{rz}(r) = \frac{-F}{2\pi L r}}$$

Newtonian

$$\tau_{rz} = \mu \frac{dv_z}{dr} = \frac{-F}{2\pi L r}$$

$$\frac{dv_z}{dr} = \frac{-F}{2\pi \mu L r}$$

$$v_z = \frac{-F}{2\pi\mu L}\ln r + C_2$$

B.C. ② $r = R_2$ $v = 0$

$$\Rightarrow \quad 0 = \frac{-F}{2\pi\mu L} \ln R_2 + C_2$$

$$C_2 = \frac{F}{2\pi\mu L} \ln R_2$$

$$\Rightarrow \quad \boxed{v_z = \frac{F}{2\pi\mu L} \ln\left(\frac{R_2}{r}\right)}$$

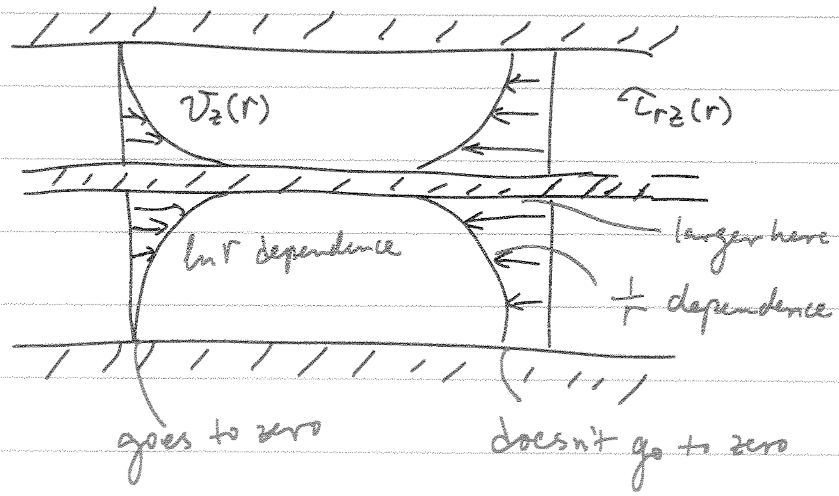
use the other piece of info given

$r = R_1$, $v_z = v_0$

$$\Rightarrow \quad v_0 = \frac{F}{2\pi\mu L} \ln\left(\frac{R_2}{R_1}\right)$$

$$\Rightarrow \quad \boxed{\mu = \frac{F}{2\pi v_0 L} \ln\left(\frac{R_2}{R_1}\right)}$$

Bonus points



Problem III

a) Mass Balance: $\iint_{C.S} \rho(\vec{V}\cdot\vec{n})\,dA + \frac{\partial}{\partial t}\iiint_{C.V.} \rho\, dV = 0$ Steady state

$-\rho_1 v_1 A_1 = \rho v_2 A_2$ Density of constant: $\rho_1 = \rho_2$

$\therefore v_2 = v_1 \frac{A_1}{A_2}$ $A_1 = \frac{\pi}{4}(.1m)^2 = .0079m^2$ $A_2 = \frac{\pi}{4}(.075m)^2 = .0044m^2$

$v_2 = (4m/s)\left(\frac{.0079m^2}{.0044m^2}\right) = \boxed{7.11 m/s}$

b) Energy Balance reduces to Bernouilli's

- Flow is incompressible, steady and inviscid
- No heat transfer or internal energy change occurs
- No work

$y_1 + \frac{V_1^2}{2g} + \frac{P_1}{\rho g} = y_2 + \frac{V_2^2}{2g} + \frac{P_2}{\rho g}$

#1 = pipe inlet
#2 = pipe outlet
no gravitational effects => $y_1 = y_2$

$\therefore P_2 = \frac{\rho}{2}\left[V_1^2 - V_2^2\right] + P_1$

$= \left(\frac{880 kg/m^3}{2}\right)\left[(4)^2 - (7.11)^2 \frac{m^2}{s^2}\right] + 300000 \frac{N}{m^2}$

$P_2 = 284797\ N/m^2 = \boxed{284.797\ KPa}$

C. $\Sigma F_x = F_{xp} + R_x = \iint V_x \rho(\vec{V}\cdot\vec{n})\,dA + \frac{\partial}{\partial t}\iiint \rho \cancel{V_x}\, dV$ steady state

$R_x + P_1 A_1 + P_2 A_2 \cos 60 =$ ~~$V_{x1}\rho(-V_{x1})A_1$~~

~~$V_{x1}\rho_1(-V_{x1})A_1 + V_{x2}\rho_2(V_{x2}$~~

$V_1\rho_1(-V_1)A_1 + (-V_2\cos\theta)\rho_2(V_2)A_2$

$R_x = -P_1A_1 - P_2A_2\cos 60 - V_1^2\rho A_1 - V_2^2\cos\theta\, \rho_2 A_2$

P_1 and P_2 must be gauge pressure

$R_x = -(300000 - 101000\ N/m^2)(.0079m^2) - (284797 - 101000\ N/m^2)(.0044m^2)\cos 60$
$- (4m/s)^2(880 kg/m^3)(.0079m^2) - (7.11m/s)^2(\cos 60)(880\frac{kg}{m^3})(.0044m^3)$

$\boxed{R_x = -2185\ N}$