MATH 2403 H1-H3. Differential Equations. Quiz 5. Feb 25, 2014.
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Student's Name: ______________________________ Section ______

Show all work to receive credit

1. Find the general solution of the system of equations

$$\overline{\mathbf{x}}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \overline{\mathbf{x}}.$$

$$p(\lambda) = (-2-\lambda)(4-\lambda) + 5 = \lambda^2 - 2\lambda - 8 + 5 = \lambda^2 - 2\lambda - 3$$
$$= (\lambda - 3)(\lambda + 1) = 0 \Rightarrow \lambda = +3 \text{ or } \lambda = -1$$

For $\lambda_1 = 3$: $\begin{pmatrix} -5 & 1 \\ -5 & 1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow -5v_1 + v_2 = 0$
$\Rightarrow \bar{v}_1 = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$

For $\lambda_2 = -1$: $\begin{pmatrix} -1 & 1 \\ -5 & 5 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow -v_1 + v_2 = 0$
$\Rightarrow \bar{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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$$\bar{x} = c_1 e^{3t}\begin{pmatrix} 1 \\ 5 \end{pmatrix} + c_2 e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2. Sketch a phase portrait for the system $\overline{\mathbf{x}}' = A\overline{\mathbf{x}}$ with the eigenvalues and eigenvectors of A

$$\lambda_1 = 0.3, \quad \overline{\mathbf{v}}_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}; \qquad \lambda_2 = 0.6, \quad \overline{\mathbf{v}}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$