

(Please use the extra papers. Do not write your solutions on the back.)

Problem 1 (5 pts)

Water ($\rho = 1000 \text{ kg/m}^3$, $\mu = 1 \times 10^{-3} \text{ Ns/m}^2$, $g = 9.81 \text{ m/s}^2$) enters a two-dimensional channel of constant width, h , with uniform velocity, U . The channel makes a 90° bend that distorts the

flow to produce the velocity profile, $v = C \left(3.5 - \frac{x}{h} \right)$, shown at the exit. Find C .

cons. of mass

$$Q_1 = Q_2$$

$$\Rightarrow u h = \int_0^h C \left(3.5 - \frac{x}{h} \right) dx$$

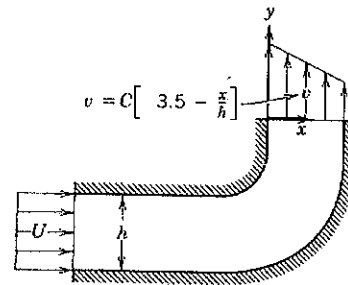
$$\Rightarrow u h = \int_0^h C \left(3.5 - \frac{x}{h} \right) dx$$

$$= C \left(3.5h - \frac{1}{h} \frac{1}{2} h^2 \right)$$

$$= C (3.5h - 0.5h)$$

$$\Rightarrow u h = C \cdot 3h$$

$$\Rightarrow C = u/3$$



Problem 2 (8 pts)

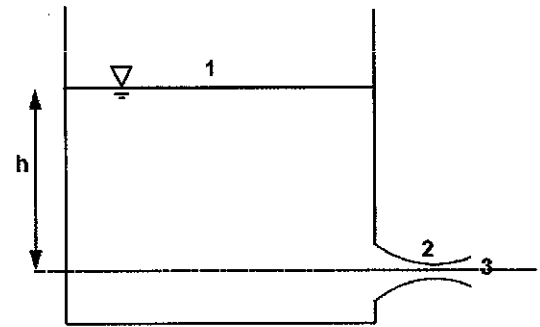
Water ($\rho = 1000 \text{ kg/m}^3$, $\mu = 1 \times 10^{-3} \text{ Ns/m}^2$, $g = 9.81 \text{ m/s}^2$) is discharging from a large tank through a convergent-divergent mouthpiece. The exit from the tank is rounded so that losses there may be neglected and the minimum diameter is 0.05m (i.e., the diameter at point 2). The head in the tank above the centre-line of the mouthpiece is $h=1.83\text{m}$. The absolute pressure at the minimum area (i.e., point 2) is 2.44m of water (Assume atmospheric pressure is 10m of water).

(a) What is the discharge?

(b) What is the diameter at the exit 3?

(c) What would the discharge be if the divergent part of the mouthpiece were removed?

(Hint: pressure as h meters of water means, $P = \gamma h$)



$$(a) \quad \frac{P_1}{\gamma} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{u_2^2}{2g} + z_2$$

$$1' \quad u_1 = 0, \quad P_1/\gamma = 10\text{m}, \quad z_1 = 1.83\text{m}$$

$$z_2 = 0, \quad P_2/\gamma = 2.44\text{m}$$

$$\Rightarrow 10 + 1.83 = 2.44 + \frac{u_2^2}{2g}$$

$$1' \Rightarrow u_2 = 13.57 \text{ m/s}$$

$$1' \Rightarrow Q = u_2 A_2 = 13.57 \text{ m/s} \cdot \frac{\pi}{4} (0.05\text{m})^2 = 0.02665 \text{ m}^3/\text{s}$$

$$(b) \quad \frac{P_1}{\gamma} + \frac{u_1^2}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{u_3^2}{2g} + z_3$$

$$1' \quad u_1 = 0, \quad P_1/\gamma = 10\text{m}, \quad z_1 = 1.83\text{m}$$

$$z_3 = 0, \quad P_3/\gamma = 10\text{m}$$

$$1' \Rightarrow 1.83\text{m} = \frac{u_3^2}{2g} \Rightarrow u_3 = 5.99 \text{ m/s}$$

$$\text{cons. of mass: } Q = u_3 A_3 = u_2 A_2 \Rightarrow 0.02665 \text{ m}^3/\text{s} = 5.99 \text{ m/s} \cdot \frac{\pi}{4} (d_3^2)$$

$$\Rightarrow d_3 = 0.0752\text{m} \quad 1'$$

(c) if the divergent part removed, $P_1 = P_2$

$$\Rightarrow \frac{P_1}{\gamma} + z_1 + \frac{u_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{u_2^2}{2g} \Rightarrow u_2 = \sqrt{2gz_1} = 5.99 \text{ m/s} \quad 1'$$

$$\Rightarrow Q = u_2 A_2 = 5.99 \text{ m/s} \cdot \frac{\pi}{4} (0.05\text{m})^2 = 0.0118 \text{ m}^3/\text{s} \quad 1'$$

Problem 3 (7pts)

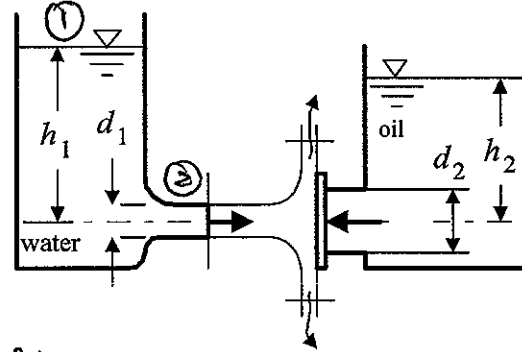
A water ($\rho = 1000 \text{ kg/m}^3$, $g = 9.81 \text{ m/s}^2$) jet is horizontally released from a large water tank and hits on a vertical plane, which covers the opening of an oil tank. The centerlines of the water jet and oil cover are at the same elevation. The water level is kept constant, $h_1 = 1.6 \text{ m}$. The diameters are $d_1 = 25 \text{ mm}$, and $d_2 = 50 \text{ mm}$. The specific gravity of oil is 0.8. Find the maximum oil level h_2 without oil leakage (head loss and weight of the vertical plane are negligible).

$$1' \quad \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

$$\Rightarrow V_2 = \sqrt{2gh_1}$$

$$= \sqrt{2 \cdot 9.81 \text{ m/s}^2 \cdot 1.6 \text{ m}}$$

$$1' \quad = 5.6 \text{ m/s}$$



$$1' \quad \Sigma F_x = \frac{\partial}{\partial t} \int_{CV} \rho u \, dV + \int_{CS} \rho u \, V \cdot dA$$

$$\Rightarrow R_x = -V_2 \rho Q = -5.6 \text{ m/s} \cdot 1000 \frac{\text{kg}}{\text{m}^3} \cdot 5.6 \text{ m/s} \cdot \frac{\pi}{4} (0.025 \text{ m})^2$$

$$= 15.4095 \text{ N}$$

$$1' \quad R_x = \gamma_o \cdot h_2 \cdot \frac{\pi}{4} d_2^2$$

$$\Rightarrow 15.4095 \text{ N} = 0.8 \cdot 9.81 \text{ m/s}^2 \cdot 1000 \frac{\text{kg}}{\text{m}^3} \cdot h_2 \cdot \frac{\pi}{4} (0.05 \text{ m})^2$$

$$1' \Rightarrow h_2 = 1 \text{ m}$$