

MATH 1711, Midterm 2

10/02/2013

Name: key GTID: _____

Circle your section below

D1 TA: Katie Stocker

D2 TA: Maggie Ginn

D3 TA: Kayla McKenzie

Problem No.	Points
1	
2	
3	
4	
5	
6	
7	
8	

TOTAL: _____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (5+5 points).

A pair of dice is rolled, and the sum of the numbers on the two uppermost faces is observed.

1. What is the sample space of this experiment?
2. Find two mutually exclusive events in this sample space.

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$\text{e.g. } E_1 = \{2\}$$

$$E_2 = \{12\}$$

Problem 2 (10 points).

Six people are chosen at random. What is the probability that at least two of them were born in the same month? Assume that each month is as likely as any other. You do *not* need to simplify your final answer.

$$\Pr(\geq 2 \text{ were born in the same month})$$

$$= 1 - \Pr(\text{all born in different months})$$

$$= 1 - \frac{\binom{12}{6} \cdot 6!}{12^6}$$

$$= 1 - \frac{P(12, 6)}{12^6}$$

Turn over for more problems

Problem 3 (10 points).

Bob, Sue, and Jack are playing a game. The probability that Bob will win is 0.3, and the probability that Jack will lose is 0.4. What are the *odds* that Sue will win?

$$\Pr(\text{Bob win}) = 0.3$$

$$\Pr(\text{Bob or Sue win}) = \Pr(\text{Jack lose}) = 0.4$$

$$\Rightarrow \Pr(\text{Sue win}) = 0.4 - 0.3 = 0.1 = \frac{1}{10} = \frac{1}{1+9}$$

the odds that Sue will win are 1:9

Problem 4 (15 points).

When a pair of dice is rolled, what is the probability that the sum of the dice is 8, given that none of the dice shows 3?

~~Pr~~

$$E_1 = \text{sum is 8 and no 3} = \{(2,6), (4,4), (6,2)\}$$

$$E_2 = \text{no 3} = \{ \dots \}$$

$$\Pr(\text{sum is 8} \mid \text{no 3})$$

$$= \frac{\Pr(\text{sum is 8 and no 3})}{\Pr(\text{no 3})}$$

$$= \frac{|E_1|}{|E_2|}$$

$$= \frac{3}{25}$$

Turn over for more problems

Problem 5 (15 points).

An urn contains two white balls and four red balls. A ball is withdrawn at random (without being replaced), and then a second ball is drawn. Are the events "the first ball is red" and "the second ball is white" independent? Justify your answer.

Not independent.

$$\Pr(\text{1st is red}) = \frac{4}{6} = \frac{2}{3}$$

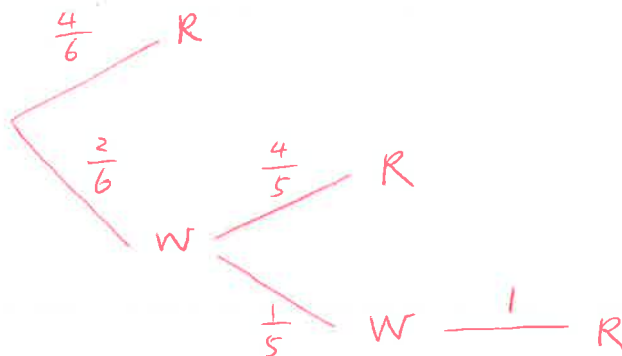
$$\Pr(\text{2nd is white}) = \frac{2}{6} = \frac{1}{3}$$

$$\Pr(\text{1st is red and 2nd is white}) = \frac{4}{6} \cdot \frac{2}{5} = \frac{4}{15}$$

$$\frac{4}{15} \neq \frac{2}{3} \cdot \frac{1}{3}$$

Problem 6 (10+5 points).

An urn contains two white balls and four red balls. Balls are selected one at a time (without replacement) until a red ball is selected. Draw a tree diagram for this experiment and label all the appropriate probabilities on the tree. Then find the probability that at least two balls are selected. You do *not* need to simplify your final answer.



$$\Pr(\geq 2 \text{ balls are selected}) = \frac{2}{6} \cdot \frac{4}{5} + \frac{2}{6} \cdot \frac{1}{5} \cdot 1$$

Turn over for more problems

$$\text{or } 1 - \frac{4}{6}$$

Problem 7 (15 points).

A retail store carries a product that is supplied by three manufactures, A, B, and C, and 30% from A, 20% from B and 50% from C. It is known that 2% of the products from A are defective, 3% from B are defective, and 5% from C are defective. If a defective product is found, what is the probability that it was from A? You do *not* need to simplify your final answer.

by Bayes' theorem.

$$Pr = \frac{0.3 \times 0.02}{0.3 \times 0.02 + 0.2 \times 0.03 + 0.5 \times 0.05}$$

Problem 8 (10 points).

A basketball player makes free throws with probability 0.8. What is the probability of making at least three out of five free throws? You do *not* need to simplify your final answer.

$$\begin{aligned} & Pr(X \geq 3) \\ &= Pr(X=3) + Pr(X=4) + Pr(X=5) \\ &= \binom{5}{3} \cdot 0.8^3 \cdot 0.2^2 + \binom{5}{4} \cdot 0.8^4 \cdot 0.2 + \binom{5}{5} \cdot 0.8^5 \end{aligned}$$

The End.

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