BMED 2210: Conservation Principles in BME
Spring 2014,

Exam 1
January 3, 2014
1:05 – 2:55 pm

Instructor: Edward Botchwey

Instructions: This is a closed book exam. The use of wireless devices is not permitted. The use of programmable calculators is only permitted if all relevant content has been erased from the calculator memory.

To receive full credit show ALL work and express numeric answers using the correct number of significant digits. If appropriate draw the system and write the full form of the accounting equation needed to solve the problem. Label all variables and equations, and present your solution clearly. Numerical answers without units or will not receive full credit.

Name: Solution Key + Rubric

GT ID: ______________________

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

Signature: ______________________

Problem 1: _____ / 10
Problem 2: _____ / 10
Problem 3: _____ / 10
Problem 4: _____ / 20
Problem 5: _____ / 20
Problem 6: _____ / 30

Total: _____ / 100 point

[redacted]

-0.5 pts answer
-1pt sigfigs
-1pt conversion
(half credit for only partially correct conversion)

(10pts)

1. Perform the following unit conversions observing the correct number of significant digits.

2 sig figs

a. 0.0020 ergs/min = 1.1×10^{-11} Btu/hr = 4.5×10^{-15} hp

$$0.0020 \frac{ergs}{min} \times \frac{9.486 \times 10^{-4}\ Btu}{10^7\ ergs} \times \frac{60\ min}{1\ hr} = 1.138 \times 10^{-11} \frac{Btu}{hr} \times \frac{1\ hr}{3600\ s} \times \frac{1.341 \times 10^{-3}\ hp}{9.486 \times 10^{-4}\ \frac{Btu}{s}}$$

$$= 4.46874 \times 10^{-15}\ hp$$

4 sig figs

b. 35.00×10^{-2} m Hg at 0°C = 4.666×10^5 dynes/cm² = -7.929 psig

$$35 \times 10^{-2}\ \cancel{m\ Hg} \times \frac{1000\ mm}{1\ m} \times \frac{1.01325 \times 10^6\ \frac{dyne}{cm^2}}{760\ mmHg} = 466628\ \frac{dynes}{cm^2} \times \frac{14.696\ psi}{101325 \times 10^6\ \frac{dynes}{cm^2}}$$

$P_{ABS} = P_{gauge} + P_{atm}$

$$= 6.76748 \approx 6.767\ psi - 14.696\ psi$$

$$= -7.929\ psig$$

(10pts) 2. Circle the correct answer(s) to the following questions? (Minus 0.5 points for circling the wrong answer)

-2.5pts for fully correct answer
-1.5 pts for partially correct answer
-0pts for incorrect answer
- minus 0.5 pts for wrong answer from one of the grades above

i. Which of the following is/are an intensive property?

a. Velocity
b. Entropy
c. Concentration
d. Thermal energy
e. Specific energy
f. Boiling point

ii. Which of the main extensive property(ies) and its/their sub-categories are always conserved?

a. Mass
b. Moles
c. Electrical charge
d. Momentum
e. None of the above

iii. Energy in a closed system can be transferred to and from a system through what mean(s)?

a. Heat
b. Gravity
c. Electrical current
d. Work
e. None of the above

iv. Reactions are associated typically with what term(s) of the accounting equation?

a. Inputs
b. Outputs
c. Generation
d. Consumption
e. Accumulation

(10pts) 3. Answer True (T) or False (F) to the following questions. (2pts each)

a. T The most common reference material for calculating specific gravity is water at 4°C.

b. F Counting quantities of intensive properties is the basis for all accounting equations.

c. T Species mass is NOT universally conserved.

d. F The differential form of the accounting equation is most appropriate when discrete quantities are involved.

e. F The integral accounting equation has dimensions of [extensive property]/ [time].

(20pts)

4. A patient undergoing surgery at standard conditions (1.0 atm, 25°C) is given an inhalational anaesthetic. A gas mask is placed over the patients face and a gas mixture is inhaled by the patient for the duration of the operation. The anesthetic contains by volume: 3.5% isoflurane ($C_3H_2ClF_5O$), 12% nitric oxide (NO), 21% oxygen, and nitrogen. Additionally the patient is breathing out an equivalent volume of carbon dioxide. } 2 sig figs

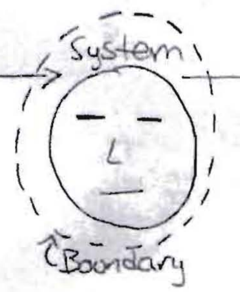
2pts a. Is the system open, closed, or isolated? (Circle one)

2pts b. Is the system dynamic or at steady? (Circle one)

2pts c. Are there any reactions or energy interconversions in the system? Yes or No (Circle one)

4pts d. Draw a complete picture of the system, labeling the system, boundary, surroundings, and all inputs and outputs.

IN → Anaesthetic
- $C_3H_2ClF_5O$
- NO
- O_2
- N_2



OUT →
- CO_2

Surroundings

-1pt - system
-0.5pts - Boundary
-0.5pts - Surroundings
-1pt INPUT
-1pt OUTPUT
half credit for not writing description

(4pts) e. Calculate the partial pressure of each gas in the anaesthetic in kPa.

1atm = 101.325 KPa $\quad P_i = X_i P_{Total} \quad X_i = \frac{n_i}{n_T} = \frac{V_i}{V_T}$

$N_2 = 100 - 3.5\% - 12\% - 21\% = 63.5\%$ (2pts)

$P_{C_3H_2ClF_5O} = 0.035(101.325\ KPa) = \boxed{3.5\ KPa - C_3H_2ClF_5O}$

$P_{NO} = 0.12(101.325\ KPa) = \boxed{12\ KPa - NO}$

$P_{O_2} = 0.21(101.325\ KPa) = \boxed{21\ KPa - O_2}$

$P_{N_2} = 0.635(101.325\ KPa) = \boxed{64\ KPa - N_2}$

2pts (0.5pts for each answer)

100.5 K Pa ← off due to rounding

(6pts) f. Calculate the total molecular weight and average molecular weight of isoflurane. (3 sig figs)

$C_3H_2ClF_5O$: $MW = \sum n_i MW_i = 3(12.011\frac{g}{mol}) + 2(1.00794\frac{g}{mol}) + 1(35.4527\frac{g}{mol}) + 5(18.9984\ g/mol) + 1(15.9994\ g/mol)$ (2pts)

$\boxed{MW = 184\ g/mol}$ (1pt answer)

half credit if wrong sig figs

12 moles total $\quad MW_{Avg} = \frac{MW}{n_T} = \frac{184}{12} \Rightarrow \boxed{MW_{Avg} = 15.3\ g/mol}$ (1pt answer)

(2pts)

OR $MW_{Avg} = \sum x_i MW_i = \frac{3}{12}(12.011\frac{g}{mol}) + \frac{2}{12}(1.00794\frac{g}{mol}) + \frac{1}{12}(35.4527\frac{g}{mol}) + \frac{5}{12}(18.9984\frac{g}{mol}) + \frac{1}{12}(15.9994\frac{g}{mol})$

$\Rightarrow \boxed{MW_{Avg} = 15.4\ g/mol}$

(20pts)

5. Two blood vessels join to form a larger vessel as shown below. During a surgical procedure a needle accidentally pokes a hole with a diameter of 0.60 cm, causing blood to flow out. Assume the system is at steady state.

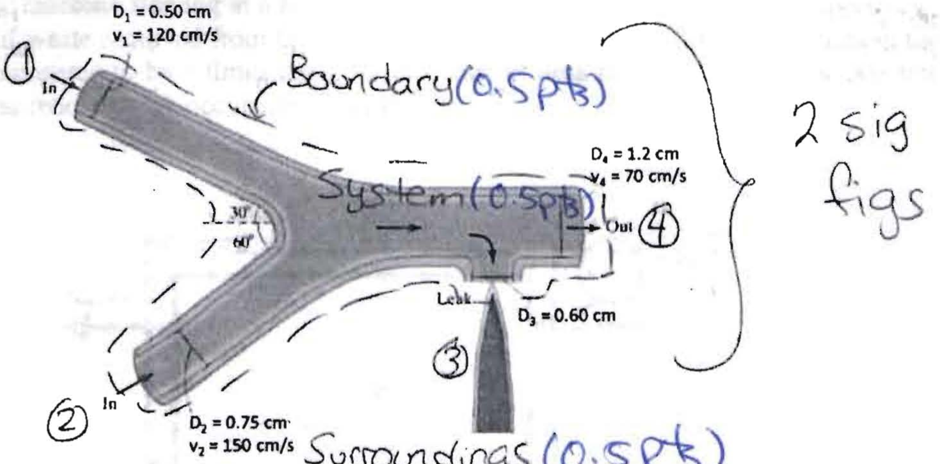
2pts a. Is the system open, closed, or isolated? (Circle one)

2pts b. What is the extensive property of this system? Mass of Blood

2pts c. Are there any reactions or energy interconversions in the system? Yes or No (Circle one)

2pts d. What property of liquids assumes density of the fluid is always constant? Incompressible

1.5pts e. Labeling the system, boundary, and surroundings.



(5.5pts)

2.5pts f. Write and simplify the appropriate accounting equation for the system. Then write the appropriate mass conservation equation and simplify.

(0.5pts each part)

Accounting: $\dot{\Phi}_{in} - \dot{\Phi}_{out} + \dot{\Phi}_{gen} - \dot{\Phi}_{con} = \dot{\Phi}_{acc}$ (No Rxn: $\dot{\Phi}_{gen} = 0$, $\dot{\Phi}_{con} = 0$; S.S.: $\dot{\Phi}_{acc} = 0$) $\Rightarrow \boxed{\dot{\Phi}_{in} = \dot{\Phi}_{out}}$

Mass Conservation: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 + \dot{m}_4$ (1pt)

blood is incompressible $\Rightarrow \boxed{\dot{V}_1 + \dot{V}_2 = \dot{V}_3 + \dot{V}_4}$ (2pts)

(5pts) g. Determine the blood's velocity at the leak.

$\dot{V} = Av$ (1pt) $\Rightarrow v_3 = \dfrac{v_1 A_1 + v_2 A_2 - v_4 A_4}{A_3}$ (2pts)

$$= \frac{\left(120\,\frac{cm}{s}\right)\frac{\pi}{4}(0.50\,cm)^2 + \left(150\,\frac{cm}{s}\right)\frac{\pi}{4}(0.75\,cm)^2 - \left(70\,\frac{cm}{s}\right)\frac{\pi}{4}(1.2\,cm)^2}{\frac{\pi}{4}(0.60\,cm)^2}$$

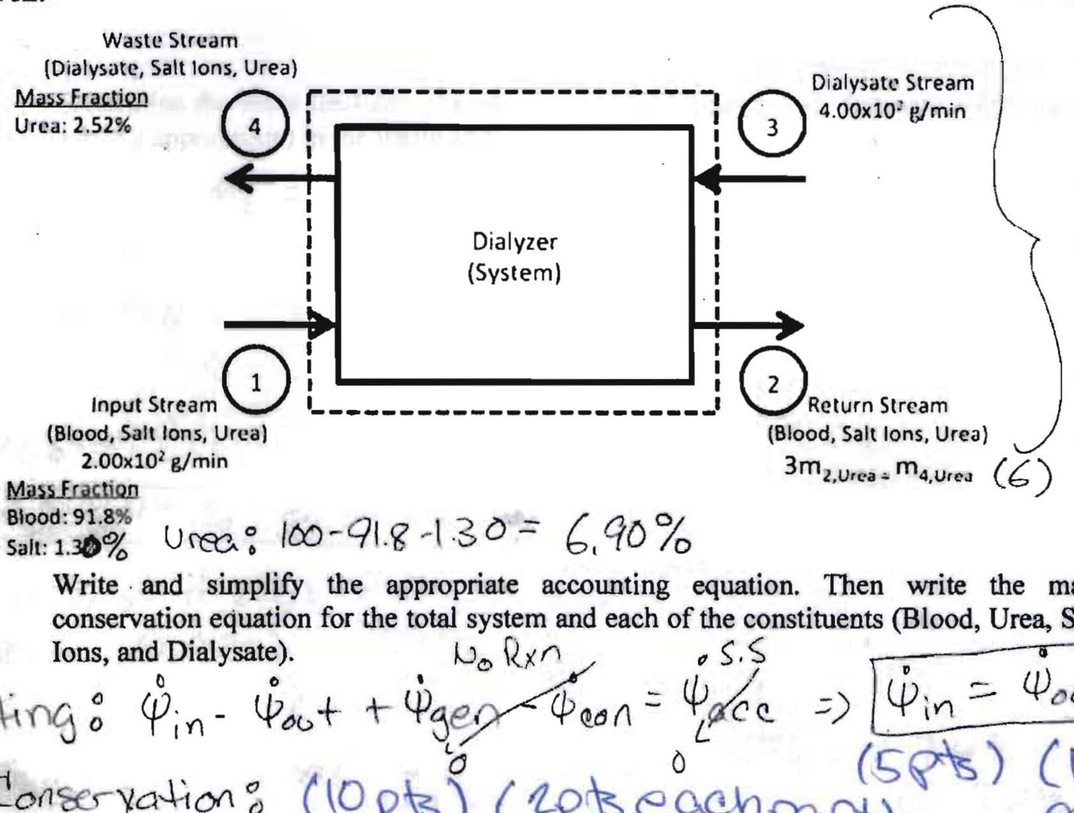
$= 37.7\,\frac{cm}{s} \Rightarrow \boxed{v_3 = 38\,\frac{cm}{s}}$

(2pts answer)
(-1pt for wrong sig figs)

6. One role of the kidneys is to remove toxins that build up as a result of metabolism. When people experience kidney failure, a machine called a dialyzer must remove toxins. In the dialyzer, the blood passes through a thin-walled membrane in one direction while the dialysate flows along the outside of the tubes in the opposite direction. Small pores in the tubes allow small molecules to pass back and forth between the two streams, but prevent larger molecules (such as proteins and cells) from passing through.

Though the machine is made of multiple components the dialyzer will be examined as a whole. The dialyzer is designed such that blood cannot enter the waste stream, and dialysate cannot enter the return stream. If blood carrying contaminates (urea and salt ions), and pure dialysate enter the machine flowing at a rate of 2.00×10^2 g/min and 4.00×10^2 g/min respectively, find the amount of waste removed from the blood. Additionally, the mass flow rate of urea in the waste stream is measured to be 3 times the mass flow rate of urea in the return stream. Assume steady state and no reactions are occurring inside the machine.

Given:



Urea: $100 - 91.8 - 1.30 = 6.90\%$

a. Write and simplify the appropriate accounting equation. Then write the mass conservation equation for the total system and each of the constituents (Blood, Urea, Salt Ions, and Dialysate). (15pts)

No Rxn, S.S

Accounting: $\dot{\varphi}_{in} - \dot{\varphi}_{out} + \cancel{\dot{\varphi}_{gen}}^{0} - \cancel{\dot{\varphi}_{con}}^{0} = \cancel{\dot{\varphi}_{acc}}^{0} \Rightarrow \boxed{\dot{\varphi}_{in} = \dot{\varphi}_{out}}$ (5pts) (1pt for each part)

Mass Conservation: (10 pts) (2pts each part)

(1) Total: $\dot{m}_1 + \dot{m}_3 = \dot{m}_2 + \dot{m}_4$

(2) Blood: $\dot{m}_1 \omega_{1,B} + \cancel{\dot{m}_3 \omega_{3,B}}^{0} = \dot{m}_2 \omega_{2,B} + \cancel{\dot{m}_4 \omega_{4,B}}^{0}$

(3) Dialysate: $\cancel{\dot{m}_1 \omega_{1,D}}^{0} + \dot{m}_3 \omega_{3,D} = \cancel{\dot{m}_2 \omega_{2,D}}^{0} + \dot{m}_4 \omega_{4,D}$

(4) Salt Ions: $\dot{m}_1 \omega_{1,S} + \cancel{\dot{m}_3 \omega_{3,S}}^{0} = \dot{m}_2 \omega_{2,S} + \dot{m}_4 \omega_{4,S}$

(5) Urea: $\dot{m}_1 \omega_{1,U} + \cancel{\dot{m}_3 \omega_{3,U}}^{0} = \dot{m}_2 \omega_{2,U} + \dot{m}_4 \omega_{4,U}$

(6pts) b. Determine the flow rate of the waste and return streams.

Eq (5) + (6): $\dot{m}_1 \omega_{1,U} = \underbrace{\dot{m}_2 \omega_{2,U}}_{\frac{\dot{m}_4 \omega_{4,U}}{3}} + \dot{m}_4 \omega_{4,U}$ (2.5pts) e.q.

$(200 \frac{g}{min})(0.069) = \frac{\dot{m}_4 (0.0252)}{3} + \dot{m}_4 (0.0252) \Rightarrow \boxed{\dot{m}_4 = 411 \frac{g}{min}}$ (1pt answer)

Eq (1): $\dot{m}_1 + \dot{m}_3 = \dot{m}_2 + \dot{m}_4$ (1.5pts) eq.

$200\,g/min + 400\,g/min = \dot{m}_2 + 411\,g/min \Rightarrow \boxed{\dot{m}_2 = 189 \frac{g}{min}}$ (1pt answer)

(half off for wrong sigfigs)

c. Determine the mass fractions of each of the constituents (blood, dialysate, salt ions, and urea as appropriate) in the waste and return streams.

Eq (6): $3\omega_{2,U} \dot{m}_2 = \omega_{4,U} \dot{m}_4$ (1.5pts) eq.

$3\omega_{2,U}(189\,g/min) = 0.0252(411\,g/min) \Rightarrow \omega_{2,U} = 0.0183 = 1.83\%$ (0.5pts answer)

Eq (2): $\dot{m}_1 \omega_{1,B} = \dot{m}_2 \omega_{2,B}$ (1.5pts) eq.

$(200\,g/min)(0.918) = (189 \frac{g}{min}) \omega_{2,B} \Rightarrow \omega_{2,B} = 0.971 = 97.1\%$ (0.5pts) answer

$\omega_{2,S} = 1 - 0.0183 - 0.971 \Rightarrow \omega_{2,S} = .0107$ (1pt) eq. (0.5pt) answer

$\boxed{\omega_{2,B} = 97.1\%,\ \omega_{2,D} = 0\%,\ \omega_{2,S} = 1.07\%\ \omega_{2,U} = 1.83\%}$

Eq (4): $\dot{m}_1 \omega_{1,S} = \dot{m}_2 \omega_{2,S} + \dot{m}_4 \omega_{2,S}$ (1.5pts) eq.

$(200\,g/min)(0.0130) = (189\,g/min)(0.107) + (411\,g/min)\omega_{4,S}$

$\omega_{4,S} = 0.00141 = 0.141\%$ (0.5pts) answer

$\omega_{4,D} = 1 - 0.0252 - 0.00141 = 0.974 = 97.4\%$ (1pt) eq. (0.5pts answer)

$\boxed{\omega_{4,B} = 0\%,\ \omega_{4,D} = 97.4\%,\ \omega_{4,S} = 0.141\%,\ \omega_{2,U} = 2.52\%}$