Math 2602 K1-K3 **Name (Print):** ____________________

Spring 2014

Midterm 2 practice

2/27/14

Time Limit: 80 Minutes Section ____________________

This exam contains 5 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes and calculators on this exam.

You are required to show your work on each problem on this exam.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

1. (10 points) Using Euclidean Algorithm find the greatest common divisor of 135 and 102.

$$135 = 102 \cdot 1 + 33$$
$$102 = 33 \cdot 3 + \boxed{3}$$
$$33 = 3 \cdot 11 + 0$$

2. (10 points) For any natural number k show that $gcd(5k + 3, 3k + 2) = 1$.

Using Euclidean Alg

$$5k+3 = (3k+2) \cdot 1 + 2k+1$$
$$3k+2 = (2k+1) \cdot 1 + k+1$$
$$2k+1 = (k+1) \cdot 1 + k$$
$$k+1 = k \cdot 1 + \boxed{1}$$
$$k = 1 \cdot k + 0$$

$$gcd(5k+3, 3k+2) = 1$$

3. (10 points) Find all integers x satisfying $5x = 2 \mod 11$.

As $\gcd(11, 5) = 1$

$11 + 5\cdot(-2) = 1$

$5(-2) = 1 \mod 11$

$5\cdot(-2\cdot 2) = 2 \mod 11$

$x \equiv -4 \mod 11 \equiv 7 \mod 11.$ ∎

4. (10 points) Find $29^{70} \mod 7$ in terms of the least non-negative residue.

$29 \equiv 1 \mod 7$

So $29^{70} \mod 7 \equiv 1^{70} \mod 7 \equiv$

$1 \mod 7.$ ∎

5. (10 points) Find smallest non-negative integer x that satisfies the system of congruences:

$$x \equiv 3 \mod 5$$
$$x \equiv 5 \mod 12$$

Method 1:

$5 \cdot 5 + 12(-2) = 1$

$X \equiv 5 \cdot 5 \cdot 5 + 3 \cdot 12(-2) \mod 5 \cdot 12$

$X \equiv 125 - 72 \mod 60$

$X \equiv 53 \mod 60$

Smallest non-negative X is 53.

Method 2:

Check $x = 5, 5+12, 5+24, 5+36, 5+48,$
for $X \equiv 3 \mod 5$, $5+48 = 53$ Works

6. (10 points) Use mathematical induction to show that for all natural numbers n, $8^n - 3^n$ is divisible by 5.

1) Base case: $n = 1$ $8^1 - 3^1 = 5$ ✓

2) Assume for $n = k$ $8^k - 3^k$ is divible by 5.

for $n = k+1$ we have

$8^{k+1} - 3^{k+1} = 8 \cdot 8^k - 3 \cdot 3^k =$

$= 5 \cdot 8^k + 3 \cdot 8^k - 3 \cdot 3^k = 5 \cdot 8^k + 3 \cdot (8^k - 3^k)$.

Both of them are divisibly by 5, so their sum is also divisable by 5. Hence $8^n - 3^n$ is divisibly by 5.

7. (10 points) Let $a_1, a_2, a_3, \ldots$ be the sequence defined by $a_1 = 0$ and $a_n = 3a_{n-1} + 2$ for $n > 1$. Guess the formula for a_n and use mathematical induction to prove that the formula is correct.

$a_1 = 0$, $a_2 = 2$, $a_3 = 8$, $a_4 = 26$,

we notice that $a_n = 3^{n-1} - 1$.

Let's prove it by induction.

1) Base Case: $n = 1$ $a_1 = 3^0 - 1 = 0$ ✓

2) Assume $a_k = 3^{k-1} - 1$ then for $n = k+1$

$a_{k+1} = 3a_k + 2 = 3(3^{k-1} - 1) + 2 = 3^k - 3 + 2 =$

$= 3^k - 1$. Hence $a_n = 3^{n-1} - 1$

holds for all $n \geq 1$. ■

8. (10 points) Solve the recurrence relation $a_{n+1} = 2a_n + 3a_{n-1}$, $n \geq 1$, given $a_0 = 0, a_1 = 8$.

$x^2 = 2x + 3$

$x_1 = -1$, $x_2 = 3$

$a_n = c_1(-1)^n + c_2 3^n$

Plug in $n = 0, 1$

$0 = c_1 + c_2$

$+$

$8 = -c_1 + 3c_2$

$8 = 4c_2$

$c_2 = 2$ $a_n = -2(-1)^n + 2 \cdot 3^n$

$c_1 = -2$

■