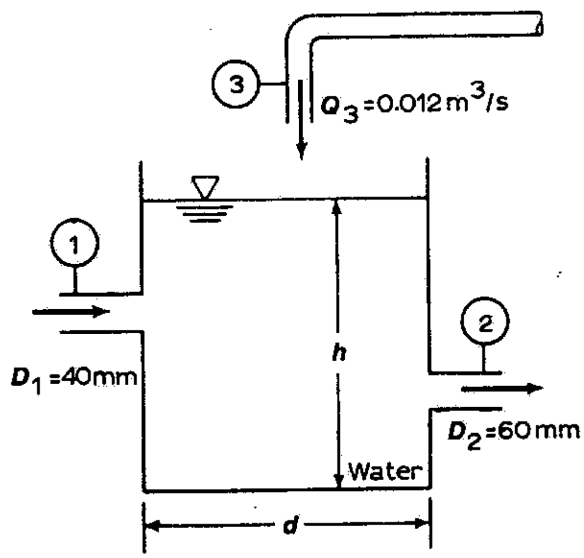
NAME:____________________

This is a closed book exam. 1 additional sheet of US-Letter paper with personal notes/equations on both sides are allowed. Method of computation (calculator, nomograph, etc.) is up to you, but keep in mind that I cannot give partial credit if you used an equation solver in your calculator and come up with the wrong answer. Hence provide all necessary solving steps to follow through to the result. Show all work on attached pages and/or add additional pages if necessary.

1. (10 pts) **Water tank:** $d = 1.0\,m$ (cylindrical tank)
   The water in a tank shown below is being filled through section (1) at $v_1 = 5$ m/s, through section (3) at $Q_3 = 0.012$ m^3/s.
   (a) If the water level h is constant, determine exit velocity v_2.
   (b) If the water level varies and v2 = 8 m/s, find rate of change in water level dh/dt.



(a) $Q_1 + Q_3 = Q_2$

$$Q_2 = v_1 A_1 + Q_3$$

$$Q_2 = \left(5\,\tfrac{m}{s}\right)\cdot\frac{\pi(0.04m)^2}{4} + 0.012\,\frac{m^3}{s}$$

$$Q_2 = 0.01828\,\frac{m^3}{s}$$

$$\underline{\underline{v_2}} = \frac{Q_2}{A_2} = \frac{0.01828\,\frac{m^3}{s}}{\frac{\pi(0.06m)^2}{4}} = \underline{\underline{6.47\,\frac{m}{s}}}$$

(b) CONSERVATION OF MASS:

$$\frac{d}{dt}\int_{CV} \rho\, dV + \int_{CS} \rho \cdot \underset{\sim}{v} \cdot d\underset{\sim}{A} = 0$$

$$\cancel{\rho}\cdot A\,\frac{dh}{dt} + \cancel{\rho}\cdot v_2 \cdot A_2 - \cancel{\rho}\cdot v_1 A_1 - \cancel{\rho}\, Q_3 = 0$$

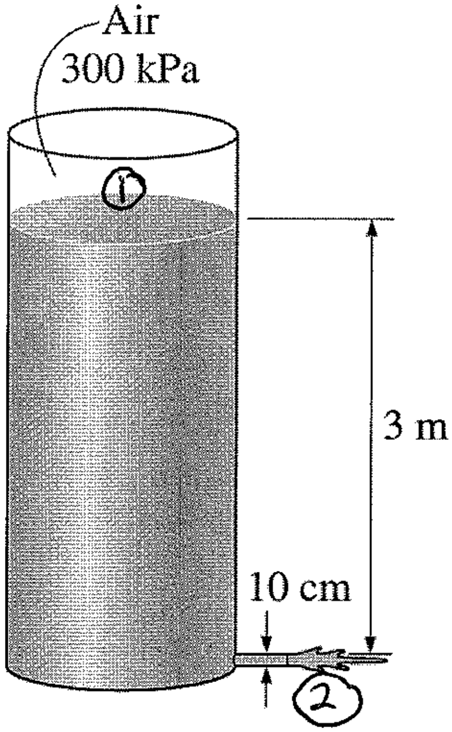
$$Q_1 + Q_3 = Q_2 + \frac{dh}{dt}\,\frac{\pi d^2}{4}$$

$$\frac{\pi(0.04m)^2}{4}\left(5\,\frac{m}{s}\right) + 0.012\,\frac{m^3}{s} = \frac{\pi(0.06m)^2}{4}\left(8\,\frac{m}{s}\right) + \frac{dh}{dt}\left[\frac{\pi\cdot(1.0m)^2}{4}\right]$$

$$\underline{\underline{\frac{dh}{dt} = -5.52\cdot 10^{-3}\,\frac{m}{s}}} \quad \text{(i.e. falling)}$$

2. (10 pts) **Water discharge from pressure tank:**

A pressurized tank of water has a 10-cm-diameter orifice at the bottom, where water discharges to the atmosphere. The water level is 3 m above the outlet. The tank air pressure above the water level is 300 kPa (absolute pressure) while the atmospheric pressure is 100 kPa. Neglecting frictional effects, determine the initial discharge rate of water from the tank.



BERNOULLI:

$$\frac{p_1}{\rho g} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$$\frac{V_2^2}{2g} = \frac{p_1 - p_2}{\rho g} + z_1$$

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho} + 2 g z_1}$$

$$V_2 = \sqrt{\frac{2(300-100)\,kPa}{1000\ kg/m^3}\left(\frac{1000\ N/m^2}{1\ kPa}\right)\left(\frac{1\,kg\cdot m/s^2}{1\,N}\right) + 2(9.81\,\tfrac{m}{s^2})(3\,m)}$$

$$V_2 = 21.4\ \frac{m}{s}$$

$$Q_2 = A_2 \cdot V_2 = \frac{\pi d^2}{4} V_2 = \frac{\pi (0.10\,m)^2}{4}\left(21.4\ \frac{m}{s}\right) = 0.168\ \frac{m^3}{s}$$

3. (10 pts) **Siphoning Out Gasoline from a Fuel Tank:**

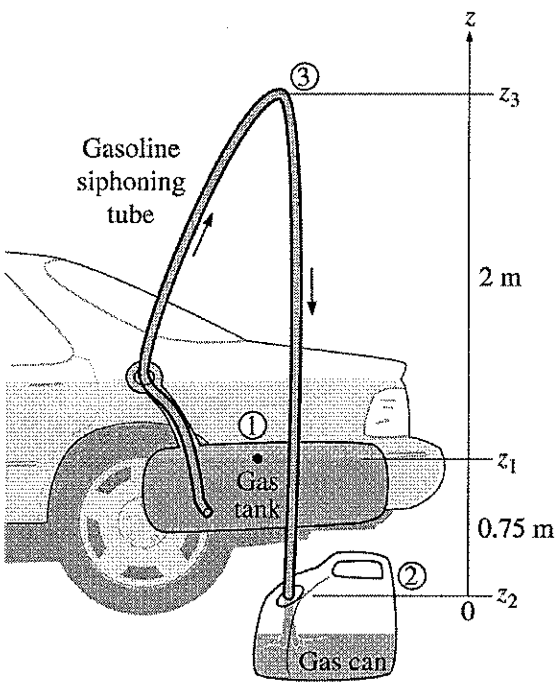
During a trip to the beach (P_{atm} = 1 atm = 101.3 kPa), a car runs out of gasoline, and it becomes necessary to siphon gas out of the car of a Good Samaritan (Figure below). The siphon is a small-diameter hose, and to start the siphon it is necessary to insert one siphon end in the full gas tank, fill the hose with gasoline via suction, and then place the other end in a gas can below the level of the gas tank. The difference in pressure between point 1 (at the free surface of the gasoline in the tank) and point 2 (at the outlet of the tube) causes the liquid to flow from the higher to the lower elevation. Point 2 is located 0.75 m below point 1 in this case, and point 3 is located 2 m above point 1. The siphon diameter is 5 mm, and frictional losses in the siphon are to be disregarded. Determine:

(*a*) the minimum time to withdraw 4 L of gasoline from the tank to the can and

(*b*) the pressure at point 3 (highest point in the system).

Assumptions: 1 The flow is steady and incompressible. 2 Neglect frictional losses through the pipe. 3 The change in the gasoline surface level inside the tank is negligible compared to elevations z_1 and z_2 during the siphoning period.

Properties The density of gasoline is given to be 750 kg/m^3.



(a) BERNOULLI:

$$\cancel{\frac{p_1}{\rho g}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + z_1 = \cancel{\frac{p_2}{\rho g}}^{0} + \frac{V_2^2}{2g} + \cancel{z_2}^{0}$$

$$V_2 = \sqrt{2g\,z_1} = \sqrt{2(9.81\tfrac{m}{s^2})(0.75\,m)}$$

$$V_2 = 3.84\ m/s$$

$$A_{TUBE} = A_2 = \frac{\pi D^2}{4} = \frac{\pi}{4}\left(5\times10^{-3}\,m\right)^2$$

$$A_2 = 1.96\times10^{-5}\,m^2$$

$$Q = V_2 \cdot A_2 = 3.84\,\frac{m}{s} \times 1.96\times10^{-5}\,m^2$$

$$Q = 7.53\times10^{-5}\,\frac{m^3}{s} = 0.0753\,\frac{L}{s}$$

$$\underline{t} = \frac{V}{Q} = \frac{4\ L}{0.0753\,\frac{L}{s}} = \underline{\underline{53.1\ s}}$$

(b) $$\frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + \cancel{z_2}^{0} = \frac{P_3}{\rho g} + \cancel{\frac{V_3^2}{2g}} + z_3$$

BERNOULLI BETWEEN PTS. 2 and 3

NOTE CONTINUITY: $V_2 \equiv V_3$

$P_2 = P_{ATM}$

$$\frac{P_{ATM}}{\rho g} = \frac{P_3}{\rho g} + z_3$$

$$P_3 = P_{ATM} - \rho g z_3$$

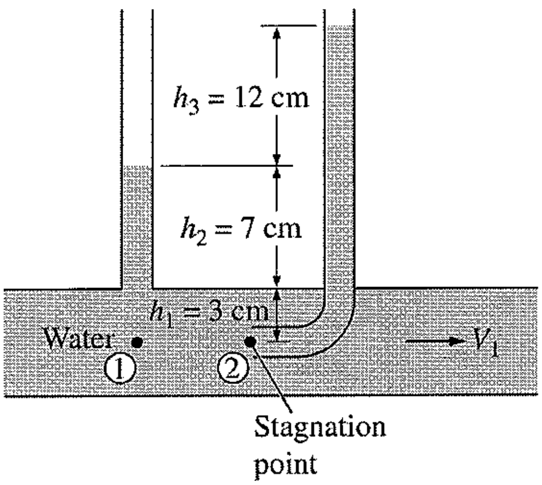
$$P_3 = 101.3\text{ kPa} - \left(750 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)(2.75\,m)\left(\frac{1N}{1 kg \cdot \frac{m}{s^2}}\right)\left(\frac{1\text{ kPa}}{1000\ N/m^2}\right)$$

$$\underline{\underline{P_3 = 81.1\text{ kPa}}}$$

4. (10pts) **Velocity Measurement by a Pitot Tube:**

A piezometer and a Pitot tube are tapped into a horizontal water pipe, as shown in figure below, to measure static and stagnation (static + dynamic) pressures. For the indicated water column heights, determine the velocity at the centerline of the pipe.

Assumptions: **1** The flow is steady and incompressible. **2** Points 1 and 2 are close enough together that the irreversible energy loss between these two points is negligible.



$$P_1 = \rho g (h_1 + h_2)$$
$$P_2 = \rho g (h_1 + h_2 + h_3)$$

BERNOULLI:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}}^{\,0} + \cancel{z_2}$$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g (h_1 + h_2 + h_3) - \rho g (h_1 + h_2)}{\rho g}$$

$$\frac{V_1^2}{2g} = h_3 \Rightarrow V_1 = \sqrt{2 g h_3} = \sqrt{2 \cdot (9.81 \tfrac{m}{s^2})(0.12 m)}$$

$$\underline{\underline{V_1 = 1.53 \tfrac{m}{s}}}$$