PHYS 2212 Test 4
Fall 2014

Name(print) Jameis Winston (Key) **Lab Section** ________________

Lab section by day and time: Greco(N,P), Zangwill(Q)						
Monday	12:05-2:55pm	N01 or Q01	3:05-5:55pm	N02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	N03 or P03	3:05-5:55pm	Q03 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	N05 or P05	3:05-5:55pm	Q05 or P06	6:05-8:55pm	N04 or Q04
Thursday	12:05-2:55pm	P07 or N06	3:05-5:55pm	N07 or Q06	6:05-8:55pm	

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a\cdot b}{c\cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

____

Sign your name on the line above

Period 6, December 9th (Tue) at 2:50pm - 5:40pm

Every semester, someone receives a zero on the final because they missed the exam. Please don't let this happen to you!

Stressing over a conflict?

Did you complete "PHYS 2212 Final Exam Schedule" on WebAssign? If not, request and extension and complete the assignment today. The conflict exam schedule will be posted on Wednesday.

Are you an ADAPTS Student?

Don't forget to schedule your final with the ADAPTS office. Don't delay, spaces are limited.

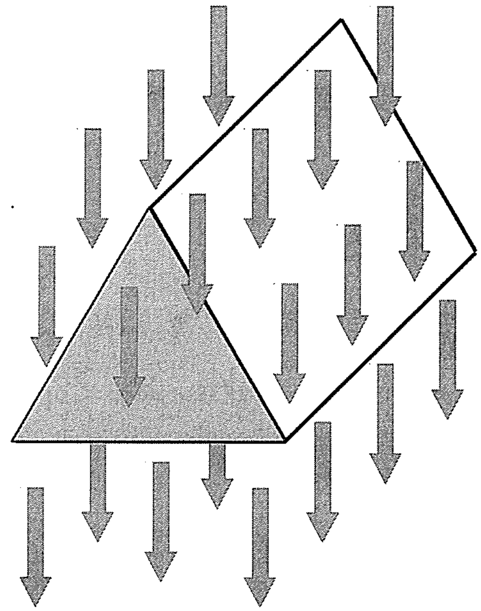
PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

Two equilateral triangles (each with sides of length d) are connected to three identical rectangles (each with sides of length d and L). These five surfaces form a tent which completely encloses the volume indicated in the diagram. At every point in space, there is a constant, downward-pointing electric field with magnitude E.



(a 5pts) Calculate the electric flux through the front surface of the tent (the shaded triangle). Show your work here and in all subsequent parts.

$$\boxed{\Phi = \int \bar{E} \cdot d\bar{a} = 0}$$

$\bar{E} \cdot d\bar{a} = da\, E \cos\theta = 0$
$(\theta = \frac{\pi}{2})$

5 All

(b 5pts) Calculate the electric flux through the back surface of the tent (the triangle not seen in the diagram).

$$\boxed{\Phi = \int \bar{E} \cdot d\bar{a} = 0}$$

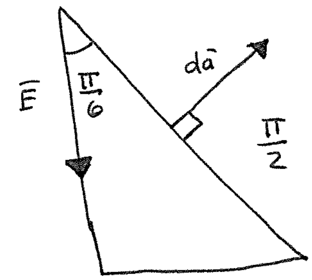
All 5

$\bar{E} \cdot d\bar{a} = E \cos\theta\, da = 0$
$(\theta = \frac{\pi}{2})$

(c 5pts) Calculate the electric flux through the top right rectangular surface of the tent (the white rectangle seen in the diagram).

$$\Phi = \int \bar{E} \cdot d\bar{a}$$
$$= \int E \cos(\theta)\, da$$
$$= E \cos\left(\tfrac{2\pi}{3}\right) dL$$
$$\boxed{= -\tfrac{1}{2} E\, dL}$$

All or (-2 for sign)



Angle between $\bar{E}$ and $d\bar{a}$ is $\frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}$

(d 5pts) Calculate the electric flux through the bottom rectangular surface of the tent (not seen in the diagram but connected to the flat bottom of both triangles).

$$\Phi = \int \bar{E} \cdot d\bar{a}$$
$$= \int E\, da$$
$$\boxed{= E\, dL}$$

All or (-2 for sign)

$\bar{E} \cdot d\bar{a} = E \cos\theta\, da = E\, da$

$\theta = 0$, $\bar{E}$ and $d\bar{a}$ point in same direction

(e 5pts) Calculate the total charge enclosed in the tent. To earn credit, you must show how you determined this.

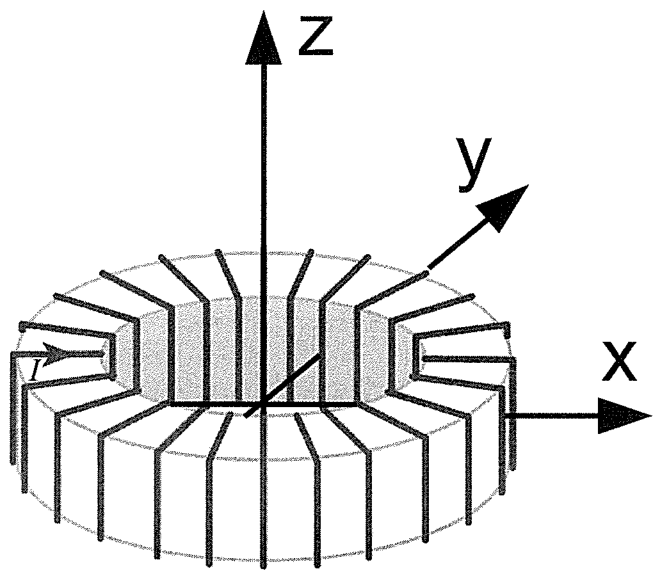
Add up the flux through all surfaces (a ... d) All or nothing, just ADD up previous parts

$$\Phi_{TOT} = \Phi_a + \Phi_b + 2\Phi_c + \Phi_d = -\frac{2}{2} E dL + E dL = 0$$

$$\Phi_{TOT} = \frac{Q_{enc}}{\epsilon_0} \Rightarrow \boxed{Q_{enc} = 0}$$

Problem 2 (25 Points)

A toroid, is a "doughnut" wrapped with loops of current that is essentially a solenoid bent around to make the ends meet. The toroid shown in the diagram has an inner radius R_i and an outer radius R_o and is centered at the origin as indicated in the diagram. The z-axis passes through the center of the doughnut hole. This toroid is wrapped with N loops of current I flowing up the outside surface of the toroid, radially inward, down the inner surface, and then radial outward. Assume that the magnetic field produced by this toroid has the form $\vec{B} = B(r, z)\hat{\phi}$ **at every point in space** where r is the perpendicular distance from the z-axis and $\hat{\phi}$ is a unit vector which "curls" around the z-axis, i.e., it is always tangent to any circle with rotational symmetry around the z-axis.



(a 5pts) Consider a z-axis centered Amperian loop in the plane of the toroid, at $z = 0$, with a radius $r < R_i$ and use it to find the magnitude of the magnetic field inside the inner radius of the toroid.

$$\int B \cdot dl = \mu_0 I_{enc}$$

$$B(2\pi r) = 0 \qquad (I_{enc} = 0)$$

$$|\bar{B}| = 0 \qquad \underline{All}$$

(b 5pts) Consider a z-axis centered Amperian loop in the plane of the toroid, at $z = 0$, with a radius $r > R_o$ and use it to find the magnitude of the magnetic field outside the outer radius of the toroid.

$$\int B \cdot dl = \mu_0 I_{enc}$$

$$= \mu_0 NI - \mu_0 NI$$

$$= 0$$

$$\therefore \quad |\bar{B}| = 0 \qquad \underline{All}$$

(c 10pts) Consider a z-axis centered Amperian loop in the plane of the toroid, at $z = 0$, with a radius $R_i < r < R_o$ and use it to find the magnitude of the magnetic field inside the loops of the toroid.

$$\int B \cdot dl = \mu_0 I_{enc}$$

$$\Rightarrow \quad B(2\pi r) = \mu_0 NI \qquad (I_{enc} = NI)$$

$$\Rightarrow \quad |\bar{B}| = \frac{\mu_0 NI}{2\pi r}$$

$\boxed{\begin{array}{l} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$

(d 5pts) Consider a z-axis centered Amperian loop far above the toroid $z >> R_o$, with a radius $R_i < r < R_o$ and use it to find the magnitude of the magnetic field far above the toroid.

$$\int B \cdot dl = \mu_0 I_{enc}$$

$$= 0$$

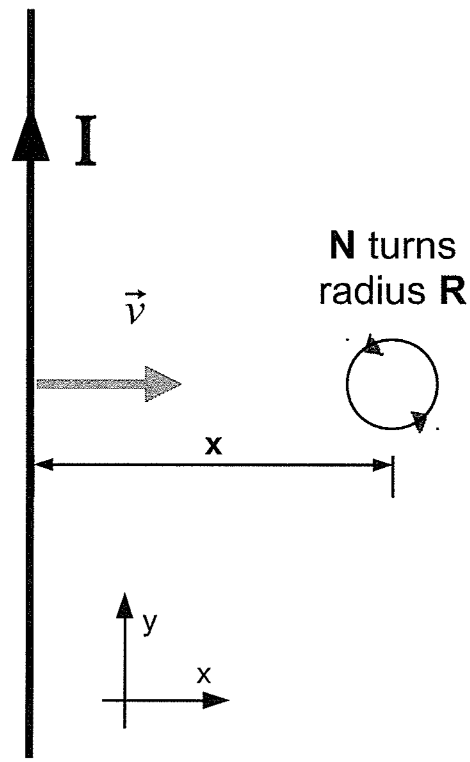
$I_{enc} = 0$, amperian loop does not have ~~cross through~~ a current pass through it.

$\therefore \boxed{|\bar{B}| = 0}$

All

Problem 3 (25 Points)

A long straight wire carrying current I is moving with speed v toward a small circular coil of radius R containing N turns. The long wire is in the plane of the coil. The coil is very small so that, at any fixed moment in time, you can neglect the spatial variation of the wire's magnetic field over the area of the coil.



(a 5pts) Is the induced current in the coil flowing clockwise or counterclockwise? Indicated this on the diagram. If the induced current is zero, state this explicitly.

Counterclockwise All

(b 15pts) At the instant when the long wire is a distance x from the center of the coil, determine the magnitude of the induced emf in the coil.

$$\mathcal{E}mf = -N\frac{d\Phi}{dt} \Rightarrow |\mathcal{E}mf| = \left|N\frac{d\Phi}{dt}\right|$$

$$B\cdot dl = \mu_0 I_{enc}$$
$$B(2\pi x) = \mu_0 I$$
$$B_{wire} = \frac{\mu_0 I}{2\pi x}$$

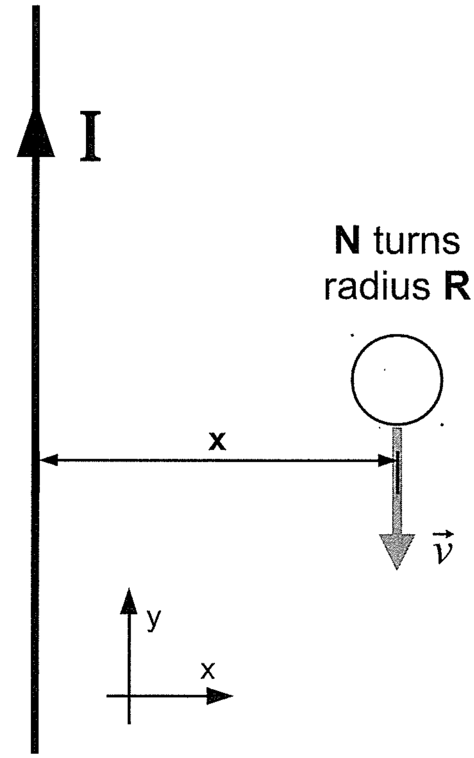
$$\Phi = \int \vec{B}\cdot d\vec{a} = B\pi R^2 = \frac{\mu_0 I \pi R^2}{2\pi x} = \frac{\mu_0 I R^2}{2x}$$

$$\boxed{\begin{matrix} -1.0 \\ -2.0 \\ -4.5 \\ -12 \end{matrix}}$$

$$\frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{\mu_0 I R^2}{2x}\right) = -\frac{\mu_0 I R^2}{2x^2}\frac{dx}{dt} = -\frac{\mu_0 I R^2}{2x^2}v$$

$(x = x(t))$

$$|\mathcal{E}nf| = \frac{N\mu_0 I R^2 v}{2x^2}$$



Now consider the case where the wire is stationary and the small circular coil is moving down (parallel to the wire) with a constant speed v. As before, the long wire is in the plane of the coil and the coil is very small so that, at any fixed moment in time, you can neglect the spatial variation of the wire's magnetic field over the area of the coil.

(c 5pts) At the instant when the long wire is a distance x from the center of the coil, determine the magnitude of the induced emf in the coil.

All

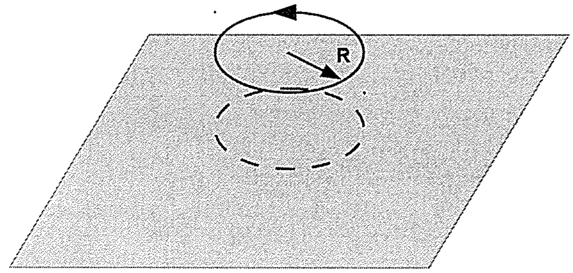
$$|\mathcal{E}mf| = \frac{N\mu_0 I R^2}{2x^2} \overbrace{\frac{dx}{dt}}^{=0} = 0$$

↳ X is no longer a function of time. That is, X is constant. $\left(\frac{dx}{dt} = 0\right)$

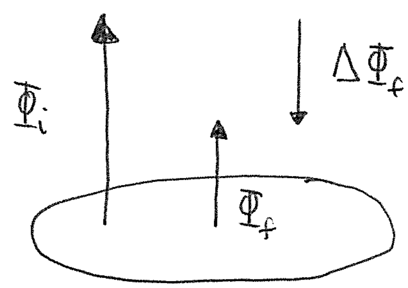
or $\frac{d\Phi}{dt} = 0 \Rightarrow \mathcal{E}mf = 0$

Problem 4 (25 Points)

A circular loop of radius R floats in space a very short distance above a flat sheet of conducting metal. The plane of the loop is parallel to the plane of the sheet and a current I flows counterclockwise around the loop when viewed from above. The dashed circle drawn on the surface of the sheet has radius R and lies directly below the loop. When the current in the loop is slowly decreased, currents appear in the conducting sheet in the form of closed loops in the plane of the sheet which are concentric with the original current loop.



(a 5pts) Focus on an induced current loop in the sheet with a radius **less** than R. Does this current flow clockwise or counterclockwise when viewed from above? You must explain your answer to get full credit here and in all subsequent parts.



$$\text{Emf} = -\frac{\Delta\Phi}{\Delta t}$$

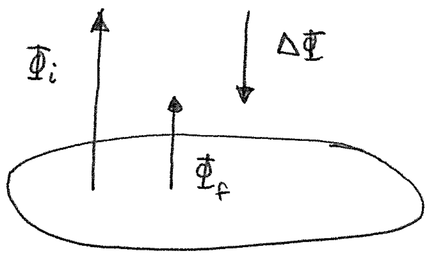
positive #

Since $-\Delta\Phi$ points ↑, then by the right hand rule the induced current is $\boxed{\text{CCW.}}$ All

(b 5pts) Is the direction of the electric field vector parallel or anti-parallel to the direction of the current flow in the induced loop in part (a)?

parallel All

(c 5pts) Focus on an induced current loop in the sheet with a radius **bigger** than R. Does this current flow clockwise or counterclockwise when viewed from above?



$$\mathcal{E}mf = \frac{-\Delta\Phi}{\Delta t}$$

← positive #

Again, $-\Delta\Phi$ points ↑, then by the right hand rule the induced current is CCW.

All

(d 5pts) Is the direction of the electric field vector parallel or anti-parallel to the direction of the current flow in the induced loop in part (c)?

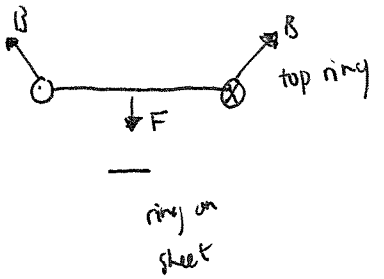
parallel

All

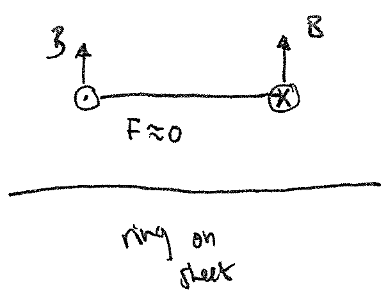
(e 5pts) Consider the direction of the net electromagnetic force acting on the loop of radius R floating above the space? Is this force towards the sheet, away from the sheet, zero or unable to be determined?

Consider a small and larger ring. ~~AM~~ $\Delta\vec{F} = I\Delta\vec{L} \times \vec{B}_{ext}$

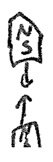
r<R



r>R



From the two comparisons, we can extend the process to all the rings to determine the net electromagnetic force is towards the sheet.



note: up if cw (a) & (B) → full credit.

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \dfrac{d\vec{v}}{dt}$

$\Delta U_{el} = q\Delta V$

$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$

$\oint \vec{E} \bullet \hat{n} dA = \dfrac{\sum q_{inside}}{\epsilon_0}$

$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left| \dfrac{d\Phi_{mag}}{dt} \right|$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \dfrac{d}{dt} \int \vec{E} \bullet \hat{n} dA \right]$

$\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\dfrac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x \Delta x + E_y \Delta y + E_z \Delta z)$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$

$\oint \vec{B} \bullet \hat{n} dA = 0$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r\sqrt{r^2 + (L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$, $\vec{p} = \alpha\,\vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{qz}{(z^2 + R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{(z^2 + R^2)^{1/2}}\right]$ ($z > 0$ along axis)

$\left|\vec{E}_{disk}\right| \approx \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{R}\right] \approx \dfrac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \dfrac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \dfrac{Q/A}{\epsilon_0}\left(\dfrac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{\ell} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \dfrac{\mu_0}{4\pi}\dfrac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$

$\left|\vec{B}_{loop}\right| = \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$)

$\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \dfrac{1}{4\pi\epsilon_0}\dfrac{-q\vec{a}_\perp}{c^2 r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \dfrac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\, nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\, nu$

$J = \dfrac{I}{A} = \sigma E$

$R = \dfrac{L}{\sigma A}$

$E_{dielectric} = \dfrac{E_{applied}}{K}$

$\Delta V = \dfrac{q}{4\pi\epsilon_0}\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right]$ due to a point charge

$I = \dfrac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$

$\mathrm{K} \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\frac{d\vec{p}}{dt}_{\perp}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\, dx = ax + c \quad \int ax\, dx = \frac{a}{2}x^2 + c \quad \int ax^2\, dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N · m^2/C^2)$^{-1}$
Magnetic constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T · m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T · m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T