Solutions to Homework 7

1) $f(x) = \begin{cases} \frac{x}{8} & \text{for } 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$

$$E[X] = \int_0^4 x \frac{x}{8}\, dx = \int_0^4 \frac{x^2}{8}\, dx = \frac{x^3}{24}\Big|_0^4 = \frac{4^3}{24} = \frac{8}{3}$$

2) a) We need

$$\int_0^L Ax(L-x)\,dx = 1$$

$$\Rightarrow A\int_0^L x(L-x)\,dx = 1 \Rightarrow A\left(L\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^L = 1$$

$$\Rightarrow A\left(\frac{L^3}{2} - \frac{L^3}{3}\right) = 1$$

$$\Rightarrow A = \frac{6}{L^3}$$

b) Since the distance from one end is $L$, the distance from the other end is $L - X$. We will compute

$$E[X - (L - X)] = E[2X - L] = 2E[X] - L = 0$$

$$E[X] = \int_0^L \frac{6}{L^3} x^2 (L-x)\, dx = \frac{6}{L^3}\left(L\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^L$$

$$= \frac{6}{L^3}\left(\frac{L^4}{3} - \frac{L^4}{4}\right) = \frac{L}{2}$$

3) $P(X=0)=\frac{1}{2}$ $P(X=1)=\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$ $P(X=2)=\frac{1}{5}$ $P(X=3)=\frac{1}{10}$ $P(X=3.5)=\frac{1}{10}$

$$E[X]=0\times\frac{1}{2}+1\times\frac{1}{10}+2\times\frac{1}{5}+3\times\frac{1}{10}+3.5\times\frac{1}{10}=\frac{1+4+3+3.5}{10}=\frac{11.5}{10}=1.15$$