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Solutions to Homework 5

1. (a) Using the notation in Section 1.5 of the newsvendor notes: $c_f = 1500$, $c_v = 50$, $h = 10$, $p = 100$. The optimal order-up-to quantity S follows from

$$F(S) = \frac{p - c_v}{p + h} = \frac{100 - 50}{100 + 10} = 0.455.$$

Hence, $S = 636$.

(b) Suppose the initial inventory level is less than 636. If we choose to make the order, the cost will be

$$C(x) = 1500 + (636 - x)40 + L(636) = 31731 - 40x. \tag{1}$$

We need to find x^* such that

$$L(x^*) = 31731 - 40x^*. \tag{2}$$

For $x \in [0, 500]$

$$\begin{aligned} L(x) &= pE[(D - x)^+] \\ &= 100 \int_x^{800} \frac{s - x}{300} ds = \frac{1}{6}x^2 - \frac{800}{3}x + \frac{320000}{3} \end{aligned}$$

We set

$$31731 - 40x = 100 \int_x^{800} \frac{s - x}{300} ds = \frac{1}{6}x^2 - \frac{800}{3}x + \frac{320000}{3}$$

No solution below 500.
For $x \in [500, 800]$

$$\begin{aligned} L(x) &= pE[(D - x)^+] + hE[(x - D)^+] \\ &= 100 \int_x^{800} \frac{s - x}{300} ds + 10 \int_{500}^{x} \frac{x - s}{300} ds = \frac{11}{60}x^2 - \frac{850}{3}x + \frac{332500}{3} \end{aligned}$$

We set

$$31731 - 40x = \frac{11}{60}x^2 - \frac{850}{3}x + \frac{332500}{3}$$

We have x^*= 569 or 758. Since we need $x^* \le S = 636$. We choose $x^* = 569$
Therefore, it is alway better to order if the inventory level is less than or equal to 569, and we order up to 636.

2. (a) The arrival rate, λ_A to machine A is 12 jobs/hour. The service rate of machine A is 15 jobs/hour and the service rate of machine B is 30 jobs/hour. The utilization of machine A is

$$\rho_A = 12/15 = 0.8.$$

Since $\rho_A < 1$ the arrival rate to machine B is equal to $\lambda_A = 12$ jobs/hour. Hence the utilization of machine B is

$$\rho_B = 12/30 = 0.4.$$

(b) Since the utilizations of both machines are less than 1, the throughout is equal to the arrival rate to the system which is equal to 12 jobs/hour.

(c) Using the Kingman's formula again

$$E[W_q] = \frac{c_a^2 + c_s^2}{2} \frac{\rho}{\mu - \lambda} = \frac{0.8}{3} = 16 \text{ mins}$$

(d) Similarly, we can use Kingman's formula to calculate the average waiting time at Machine B

$$E[W_{qB}] = \frac{c_a^2 + c_s^2}{2} \frac{\rho}{\mu - \lambda} = \frac{0.4}{30 - 12} = 1.3 \text{ mins}$$

Thus, the average time in the production is 16 + 1.3 + 3 + 2 = 22.3 Using the Little's Law

$$L = \lambda * W,$$

where λ is the arrival rate and is equal to 12 jobs/hour and $W = 22.3$ mins $= 0.37$ hrs . Hence,

$$L = 4.46 \text{ jobs}.$$

(e) If the arrival rate, λ_A to machine A is 60 jobs/hour then the utilization of machine A is

$$\rho_A = \min\{60/20, 1\} = 1.$$

Since $\rho_A = 1$ the arrival rate to machine B is equal to 20 jobs/hour. Hence the utilization of machine B is

$$\rho_B = 20/30 = 0.667.$$

Since the utilization of the first machine is equal to 1, the throughput is equal to the service rate of this machine which is equal to 20 jobs/hour.

3. (a) The mean waiting time of a customer is: $E(x) = \int_{s=0}^{\infty} s 4\lambda^2 s e^{-2\lambda s} ds = 3/2$. Therefore it is 1.5 mins.

(b) From (a) we know that $\mu = 40$, we also know $\lambda = 30$ Using Kingman's formula

$$E[W_q] = \frac{c_a^2 + c_s^2}{2} \frac{\rho}{\mu - \lambda} = \frac{0.75}{10} = 4.5 \text{ mins}$$

(c) E(W)= 4.5+1.5=6 mins

Little's law: L=$\lambda E(W)$=30*(1/10)=3