

Solutions to Homework 2

1) Define the following events

.. C : a person is a conservative

.. L : a person is a liberal

.. I : person is an independent

.. V : person is a voter

We need

$$P(L|V) = \frac{P(V|L)P(L)}{P(V|L)P(L) + P(V|C)P(C) + P(V|I)P(I)}$$

$$= \frac{0.8 \times 0.5}{0.8 \times 0.5 + \frac{2}{3} \times 0.3 + 0.5 \times 0.2} = \frac{0.4}{0.4 + 0.2 + 0.1} = \frac{4}{7}$$

2) Define the following events

.. R : red shows

D_1 : die 1 is picked

D_2 : die 2 is picked

We need $P(R)$.

$$P(R) = P(R|D_1)P(D_1) + P(R|D_2)P(D_2)$$

$$= \frac{4}{6} \cdot \frac{1}{2} + \frac{2}{6} \cdot \frac{1}{2} = \frac{1}{2}$$

3) We need to check if $P(A \cap B) = P(A)P(B)$

$$A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$$

$$B = \{(2,5), (5,2), (3,4), (4,3), (4,6), (6,1), (2,6), (6,2), (3,5), (5,3), (4,4), (3,6), (6,3), (4,5), (5,4), (5,5), (4,6), (6,4), (5,6), (6,5), (6,6)\}$$

$$A \cap B = \{(2,5), (5,2), (2,6), (5,3), (5,4), (5,5), (5,6)\}$$

$$\text{So, } P(A) = \frac{12}{36} = \frac{1}{3} \quad P(B) = \frac{21}{36} \quad P(A \cap B) = \frac{7}{36}$$

Since $P(A \cap B) = P(A)P(B)$, A and B are independent.

- 4) a) Define the following events
 G : George hits the target
 B : Bill hits the target
 T_1 : target is hit exactly once

We need $P(G|T_1)$

$$\begin{aligned} P(G|T_1) &= \frac{P(T_1|G)P(G)}{P(T_1|G)P(G) + P(T_1|B)P(B)} \\ &= \frac{0.3 \times 0.4}{0.3 \times 0.4 + 0.6 \times 0.7} = \frac{0.12}{0.54} = \frac{12}{54} = \frac{2}{9} \end{aligned}$$

- b) Now also define
 T : target is hit

We need
 $P(G|T)$

$$\begin{aligned} &= \frac{P(T \cap G)}{1 - P(T^c)} = \frac{P(T|G)P(G)}{1 - P(T^c)} \\ &= \frac{0.4}{1 - 0.3 \times 0.6} = \frac{0.4}{0.82} = \frac{40}{82} = \frac{20}{41} \end{aligned}$$