

Exam III - Spring 2016

10:05-10:55 AM (50-MINUTE EXAM)

To receive full credit on each problem, it is advised to write down necessary equations and work required to reach the final answer. Label variables and equations on diagrams when possible. Include a brief word description to explain steps (e.g. $A_1=A_2=A$), stating all assumptions (e.g. incompressible).

For problems involving long equations, you can circle the equations being used on the attached equation sheet, and simplify the equation on attached equation sheet (noting assumptions being applied), then write the final equation(s) on your exam.

Numerical answers without units or explanations (work required for solution) will not receive credit.

The use of wireless devices (cell phones, IR transmitters/receivers) is not permitted at any time during exam.

NAME: Dawson

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

Signed: _____

Problem I _____ / 30

Problem II _____ / 30

Problem III _____ / 40

Total _____ / 100

Make the following assumptions when necessary:

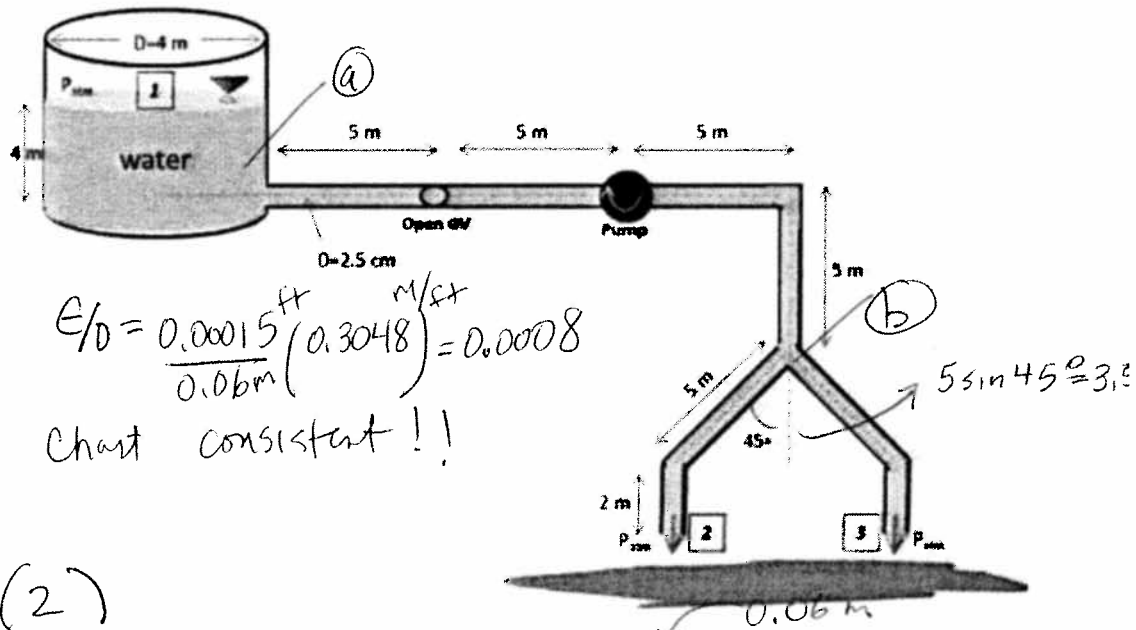
$P = 1 \text{ atm}$; $g = 10 \text{ m/s}^2$; $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$; $\rho_{\text{water}} = 1000 \text{ kg/m}^3$; $\mu_{\text{water}} = 0.01 \text{ Pa s}$

Problem 3 (40 points): Water at 300 K ($\rho_{\text{water}} = 1000 \text{ kg/m}^3$, $\mu_{\text{water}} = 0.01 \text{ Pa s}$) is pumped from a reservoir (region 1) through commercial stainless steel piping ($D = 6.0 \text{ cm}$, $\epsilon = 0.00015 \text{ ft}$) at mass flow rate of 25 kg/s to two pipe exits (regions 2-3) used to water grass field (piping diameter is the same throughout). Efficiency of pump is 85%. **Determine:**

(A) head loss at region 2.

(B) power to the fluid.

(C) NPSH given 5 kPa water vapor pressure.



$$\epsilon/D = \frac{0.00015 \text{ ft}}{0.06 \text{ m}} \left(\frac{0.3048 \text{ m}}{1 \text{ ft}} \right) = 0.0008$$

Chart consistent!!

$$\dot{m}_{in} = \dot{m}_{out} (2)$$

$$\dot{m}_{out} = \frac{\dot{m}_{in}}{2} = 12.5 \text{ kg/s}$$

$$A = \frac{\pi D^2}{4} = 0.003 \text{ m}^2$$

@ region a

$$Re_{Da} = \frac{4 \dot{m}_{in}}{\pi D \mu} = 53,000$$

$$V_A = \frac{\dot{m}_{in}}{\rho A} = 8.4 \text{ m/s}$$

$$f_{fA} = 0.006$$

$$\left(\frac{L}{D} \right)_A = \frac{20 \text{ m}}{0.06 \text{ m}} = 333$$

$$\sum K = 1.4 \left(\begin{array}{l} \text{Valve} \\ \text{Contraction} \\ 90^\circ \text{ Elbows} \end{array} \right)$$

@ region b

$$Re_{Db} = 26,526$$

$$V_B = 4.2 \text{ m/s}$$

$$f_{fB} = 0.0065$$

$$\left(\frac{L}{D} \right)_B = \frac{7}{0.06} = 117$$

$$\sum K = 0.70 \left(\begin{array}{l} 2 \text{ } 45^\circ \\ \text{Elbows} \end{array} \right)$$

if assumed constant \dot{m} (5)

$$\left(\frac{L}{D} \right)_{\text{form}} = 450, \sum K = 2.1$$

Exam 3, Problem 3

② h_{L2}

⑤ W_F

© NPSH

$$\textcircled{a} h_L = h_{L \rightarrow b} + h_{L \rightarrow 2}$$

$$h_L = \left(4 f_{f_1} \frac{L_1}{D} + \sum K \right) \frac{V_A^2}{2g} + \left(4 f_{f_2} \frac{L_2}{D} + \sum K \right) \frac{V_B^2}{2g}$$

$$h_L = (9.4) \frac{v_A^2}{2g} + (3.7) \frac{v_B^2}{2g} = 3.3 + 3.3$$

$$h_L = 36 \text{ m}$$

if m constant $\rightarrow h_L = 46 \text{ m}$

$$(b) -W_S = \overbrace{(m_b \cancel{2})}^{m_a} \left(\cancel{\frac{p_2}{\cancel{p}}} + \frac{v_2^2}{2} + g y_2 + \cancel{u_2} \right) - \ddot{m}_a \left(\cancel{\frac{p_1}{\cancel{p}}} + \frac{v_1^2}{2} + g y_1 + \cancel{u_1} \right)$$

$$-\frac{W_s}{m_a} = \frac{v_2^2}{2} + g \underbrace{(y_2 - y_1)}_{-14.5 \text{ m}} + g \underbrace{h_L}_{36 \text{ m}}$$

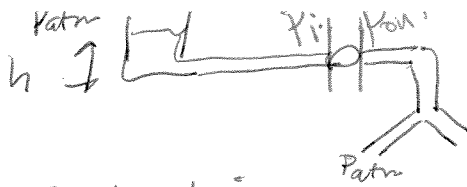
$$-\dot{W}_s = 25 \text{ kg/s} (223 \text{ m}^2/\text{s}^2) = 5596 \text{ W}$$

$$\dot{W}_F = \eta (-\dot{W}_S) = 4756 \text{ W}$$

$$\dot{W}_F = 4756 \text{ W}$$

$$\text{Constant } W_F = 7443 \text{ W}$$

See other alternative soln.



no h_L no ρV

(B)

Alternative Solution -

based on
Practice
Exam

$\eta = 0.85$ (across pump)

$$\dot{W}_F = \frac{\dot{m}}{\rho} (P_{out} - P_{in})$$

$$P_i = \left(NPSHA - \frac{v_i^2}{2g} \right) \rho g$$

assume this is small!!
or assume $v_i = 8.84 \text{ m/s}$

$$P_{out} = P_{atm} + \rho g h \quad (\text{based on diagram})$$

$$P_{out} = 1.013 \times 10^5 \text{ Pa} + \frac{1000 \text{ kg}}{\text{m}^3} (10 \text{ m}) (4 \text{ m})$$

$$P_{out} = 1.4 \times 10^5 \text{ Pa}$$

$$P_i \sim (NPSHA) \rho g = \frac{21}{31} (1000) (10)$$

$$P_i \sim 3.1 \times 10^5 \text{ Pa} \quad (2.7 \times 10^5)$$

$$\dot{W}_F = \frac{\dot{m}}{\rho} (P_i - P_o) = 4250 \text{ W}$$

OK!

$$P_{pump} = \frac{\dot{m}}{\rho \eta} (P_i - P_o) = 5000 \text{ W}$$

if this
instead
OK!

$$\eta = \frac{\dot{W}_F}{P_{pump}}$$

Note: answer is pretty similar
to result from energy balance!!

③ NPSH
 $P_v = 5 \times 10^3 \text{ Pa}$

Pool \rightarrow Pump (Bernoulli's Eqn)

$$\frac{P_{atm}}{\rho} + \frac{v_1^2}{2} + gy_1 + u_1 = \frac{P_i}{\rho} + \frac{v_i^2}{2} + gy_i + u_i$$

$$\frac{P_i}{\rho g} + \frac{v_i^2}{2g} = \frac{P_{atm}}{\rho g} + (y_1 - y_i) + h_{L1 \rightarrow i}$$

$$h_{L1 \rightarrow i} = \left(4 f + 9 \left(\frac{10}{0.06} \right) + 0.7 \right) \frac{V_A^2}{2g}$$

$$h_{L1 \rightarrow i} = 17 \text{ m}$$

$$y_1 - y_i = 4 \text{ m}$$

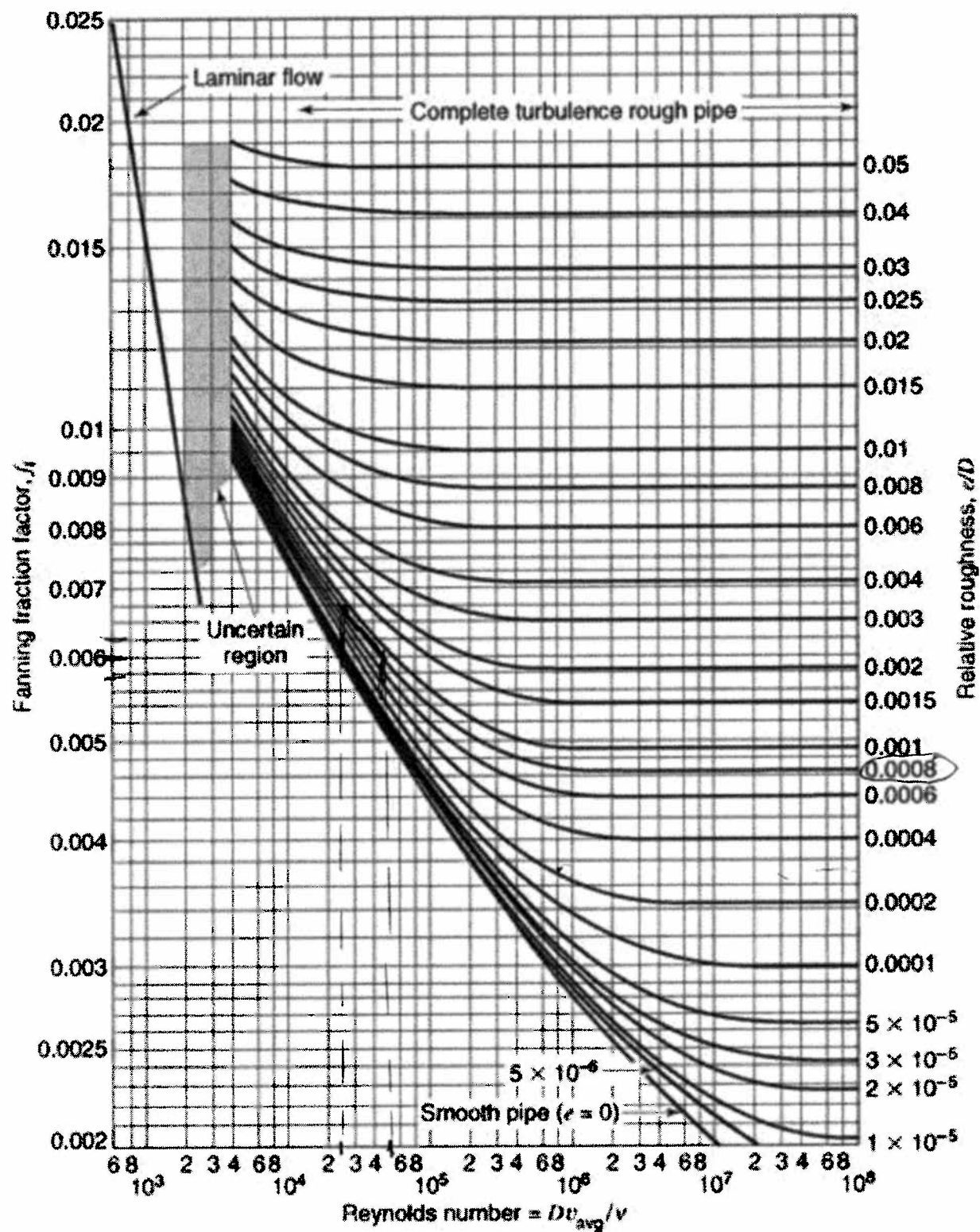
$$NPSHA = \frac{10^5}{10^4} + 4 \text{ m} + 17 \text{ m}$$

$$NPSHA = 31 \text{ m}$$

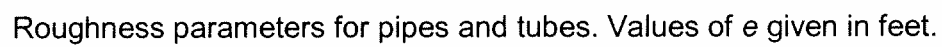
$$NPSH = NPSHA - \frac{P_v}{\rho g}$$

$$\frac{P_v}{\rho g} = \frac{5 \times 10^3}{10^4} = 0.5$$

$$NPSH = 30.5 \text{ m}$$



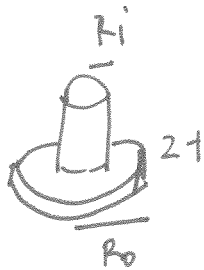
The Fanning friction factor as a function of Re and D/e



Problem 2 (30 points): External flow of air ($T=300$ K) is used to cool the surface of a steel pipe ($T_{\text{surface}} = 400$ K, 10 m long, 3 cm in diameter). Aluminum fins, which fit like washers (with outer diameter of 5 cm, thickness of 3 mm) are added to the surface to increase the rate of cooling. The convective transfer coefficient is $h = 15$ W/m²K, the thermal conductivity of steel is 17 W/mK, and aluminum is 30 W/mK.



How many fins are needed to increase the heat transfer rate by 4-fold?



$$r_i = 0.015 \text{ m}$$

$$r_o = 0.025 \text{ m}$$

$$L = 10 \text{ m}$$

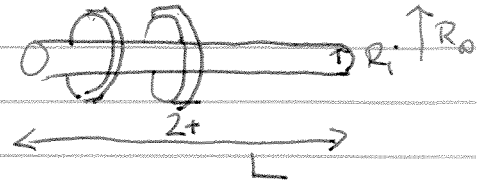
$$2t = 0.003 \text{ m}$$

$$t = 0.0015 \text{ m}$$

$$q_{wF} = 5652 \text{ W} = 4 q_{\text{loof}}$$

Exam 3, Problem 2

How many fins needed?



$$5 \left[\frac{q_{wF}}{q_{\text{loof}}} = 4 = \frac{(A_f \eta_f + A_o) h \Delta T}{A h \Delta T} \right. \quad \begin{array}{l} R_i = 0.015 \text{ m} \\ R_o = 0.025 \text{ m} \\ L = 10 \text{ m} \\ 2t = 0.003 \text{ m} \end{array}$$

$$A = 2\pi R_i L$$

$$q_{wF} = (15)(0.942)(400-300) = 1413 \text{ W}$$

$$A_o = 2\pi R_i L - N_F 2\pi R_i (2t)$$

$$A_f = N_F (2(\pi R_o^2 - \pi R_i^2) + 2\pi R_o (2t)) = 0.003 N_F$$

$$A = 0.942 \text{ m}^2$$

$$A_o = (0.942 - 2.83 \times 10^{-4} N_F) \text{ m}^2$$

$$A_f = N_F (0.0025 + 4.71 \times 10^{-4}) = 0.003 N_F$$

$$R_o/R_i = 1.7$$

$$8 \left[\frac{(R_o - R_i) \left(\frac{h}{k t} \right)^{1/2}}{0.010} = 0.183 \right.$$

$$\eta_f = 0.98$$

$$4(0.942) - 0.942 = 0.00266 N_F$$

$$N_F = 1062$$

Calc. error
-2

$$q = h A \Delta T = 1413 \text{ W}$$

$$q_w = 4(1413) = 5652 \text{ W}$$

6

Friction factors of other pipe fittings

Fitting

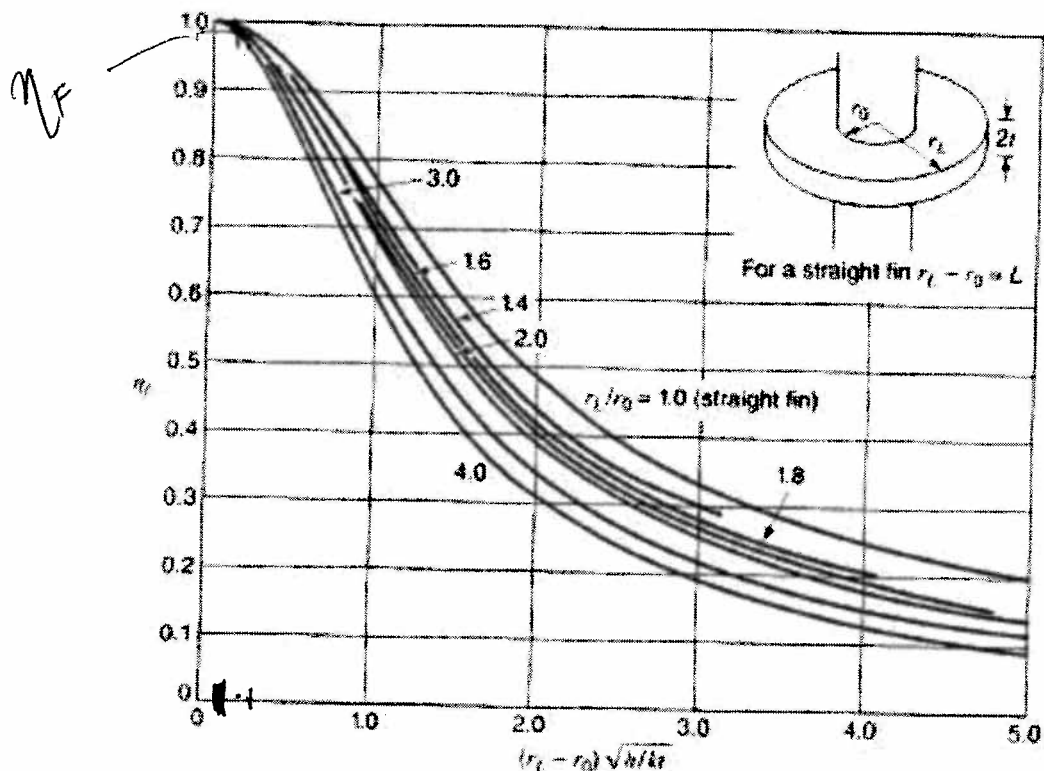
Tee, through side outlet	$K=1.5$
Tee straight through	$K=0.4$

Material Properties (use if not given)

Stainless Steel (25° C)	$k=16 \text{ W/mK}$, $C_p=460.8 \text{ J/kgK}$
Stainless Steel (100° C)	$k=17.3 \text{ W/mK}$, $C_p=460.8 \text{ J/kgK}$
Mild Steel (25° C)	$k=42.9 \text{ W/mK}$, $C_p=473.3 \text{ J/kgK}$
Mild Steel (100° C)	$k=42.9 \text{ W/mK}$, $C_p=473.3 \text{ J/kgK}$
Water (25° C)	$k=0.613 \text{ W/mK}$, $C_p=4179 \text{ J/kgK}$
Water (100° C)	$k=0.680 \text{ W/mK}$, $C_p=4217 \text{ J/kgK}$
Air (25° C)	$k=0.026 \text{ W/mK}$, $C_p=1006 \text{ J/kgK}$
Air (100° C)	$k=0.036 \text{ W/mK}$, $C_p=1014 \text{ J/kgK}$

Figure 17.11

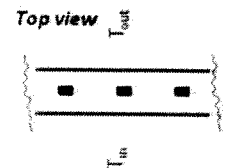
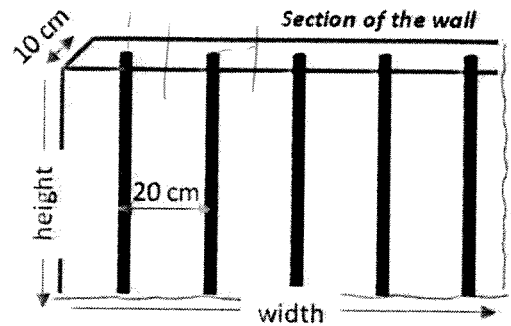
Fin efficiency for straight and circular fins of constant thickness



Problem 1 (30 points): Rectangular steel bars ($X_{SA}=1.5\text{cm} \times 1.5\text{cm}$) aligned vertically (6 m long) are used to reinforce a concrete wall (6 m high, 10 m width, 10 cm thick). The steel bars at the center of the concrete slab (10 cm thick) are spaced 20 cm apart along the width of the wall (10 m wide). Room temperature inside the concrete wall is 295 K and outside the concrete wall 250 K. Corresponding convective heat transfer coefficients are $h_{in} = 20 \text{ W/m}^2\text{K}$ and $h_{out} = 15 \text{ W/m}^2\text{K}$, respectively.

$k_{\text{steel}} = 16 \text{ W/mK}$, $k_{\text{concrete}} = 0.15 \text{ W/mK}$, $k_{\text{air}} = 0.026 \text{ W/mK}$

Determine: (A) temperature at outside surface of the wall, (B) overall resistance to heat transfer, and (C) overall rate of heat transfer.



$$\begin{aligned} L_i &= 1.5 \text{ cm} \\ L &= 4.25 \text{ cm} \end{aligned}$$

50 sections

$$\begin{aligned} H &= 6 \text{ m} \\ W_i &= 0.2 \text{ m} \\ W &= 50W_i = 10 \text{ m} \end{aligned}$$

Total

$$\begin{aligned} A &= 60 \text{ m}^2 \\ W &= 10 \text{ m} \\ W_s &= 0.75 \text{ m} \\ W_c &= 9.25 \text{ m} \\ A_s &= 4.5 \text{ m}^2 \\ A_c &= 55.5 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A_i &= 1.2 \text{ m}^2 \\ \left[\begin{aligned} W_i &= 0.2 \text{ m} \\ W_{is} &= 0.015 \text{ m} \\ W_{ic} &= 0.185 \text{ m} \end{aligned} \right. \\ A_{s,i} &= 0.09 \text{ m}^2 \\ A_{c,i} &= 1.11 \text{ m}^2 \end{aligned}$$

section Adjusted Resistances

$$\begin{aligned} R_i &= 0.042 \\ R_o &= 0.056 \\ R_c &= 0.236 \\ R_{c,i} &= 0.090 \\ R_{s,i} &= 0.010 \\ R_{eq} &= 0.009 \\ \Sigma R &= 0.579 \frac{\text{K}}{\text{W}} \end{aligned}$$

Solved using total

If solved using individual
Multiply q by 50.

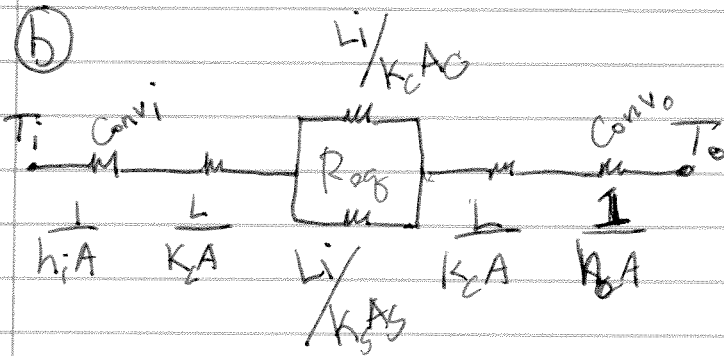
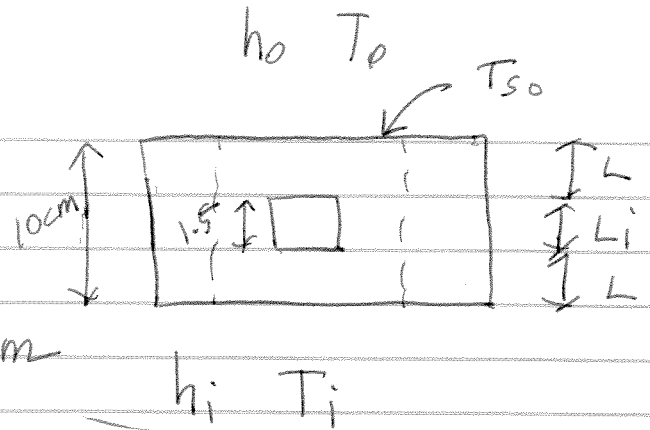
$$q = 50 \frac{\Delta T}{\Sigma R} \approx 3886 \text{ W}$$

$q = 78 \text{ W per section}$

Exam 3 Problem #1

- (a) T_{so}
 (b) ΣR
 (c) q

$$L = 4.25 \text{ cm}$$



$$A = HW = 6(10) = 60 \text{ m}^2$$

$$A_s = N_s H W_s$$

$$A_c = H(W - N_s W_s)$$

$$N_s = \frac{W}{0.20 \text{ m}} = 50$$

$$W_s = 0.015 \text{ m}$$

$$H = 6 \text{ m}$$

$$W = 10 \text{ m}$$

$$A = 60 \text{ m}^2 = A_i = A_o$$

$$A_s = 4.5 \text{ m}^2$$

$$A_c = 55.5 \text{ m}^2$$

$$\frac{L}{h_i A_i} = 8.3 \times 10^{-4} \quad \frac{L}{h_o A_o} = 0.001$$

$$\frac{L}{k_c A} = 0.005 \quad \frac{L_i}{k_c A_s} = 0.002$$

$$\frac{L_i}{k_s A_s} = 2.1 \times 10^{-4}$$

$$R_{reg} = \frac{(L_i/k_s A_s)(L/k_c A_c)}{L_i/k_s A_s + L/k_c A_c} = 1.9 \times 10^{-4}$$

$$\Sigma R = 8.3 \times 10^{-4} + 0.005 + 1.9 \times 10^{-4} + 0.005 + 0.001$$

$$\Sigma R = 0.012 \text{ K/W}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{(295 - 250) \text{ K}}{0.012 \text{ K}} \text{ W}$$

$$q = 3750 \text{ W}$$

a)

$$q = h_o A_o (T_s - T_o)$$

$$q = \frac{(T_s - T_o)}{0.001 \text{ K}} u$$

$$T_o = 250 \text{ K} \quad , \quad q = 3750 \text{ W}$$

$$T_s = T_o + 3.75$$

$$T_s = 254 \text{ K}$$