MATH 1552 QUIZ 1, FALL 2015, GRODZINSKY

Print Your Name: Key-1

T.A.: (circle one) Miheer Brandon Stephen Kabir

1. A small town has a population that is changing at a rate of $f(t) = 11t - 3t^2$ citizens, where t is time in years.

(a) (6 points) Set up an integral to find the population of the town after two years.

$$P(t) = \int_0^2 (11t - 3t^2)\,dt$$

(b) (20 points) Using a general Riemann Sum, evaluate your integral in part (a). You may choose x_i^* to be the right-hand endpoint of each subinterval. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}, \text{ so } x_i^* = a + i\Delta x$$

$$\Rightarrow x_i^* = \frac{2i}{n}$$

Then $f(x_i^*) = 11\left(\frac{2i}{n}\right) - 3\left(\frac{2i}{n}\right)^2$

$$= \frac{22i}{n} - \frac{12i^2}{n^2}$$

So: $\displaystyle\int_0^2 (11t - 3t^2)\,dt = \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(x_i^*)$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{22i}{n} - \frac{12i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n} \left[\frac{22}{n} \sum_{i=1}^{n} i - \frac{12}{n^2} \sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \frac{2}{n} \left[\frac{22}{n} \cdot \frac{n(n+1)}{2} - \frac{12}{n^2} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

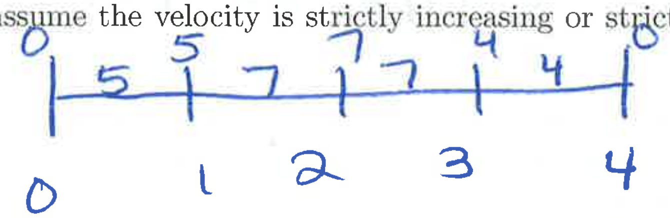
$$= \lim_{n\to\infty} \left[\frac{22n+22}{n} - \frac{4(n+1)(2n+1)}{n^2}\right]$$

$$= 22 - 8 = \boxed{14} \text{ citizens}$$

2. (12 points) The velocity of a model train is measured in inches over a four second interval, and results in the following table:

Time (sec)	Velocity (in/sec)
0	0
1	5
2	7
3	4
4	0

Estimate the distance traveled by the train by using an **upper** sum with time intervals of 1 second. You may assume the velocity is strictly increasing or strictly decreasing along each subinterval.



$$U_f = \Delta x\,[f(1) + f(2) + f(2) + f(3)]$$
$$= 1\cdot[5+7+7+4] = 23 \text{ inches}$$

3. (12 points) Given that:

$$\int_1^5 f(x)dx = 10, \quad \int_1^2 f(x)dx = -4,$$

find the **total** area bounded by the curve $y = f(x)$, $x = 1$, $x = 5$, and the x-axis.

Note $\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx$

$$= 10 - (-4) = 14$$

Total area $= \left|\int_1^2 f(x)\,dx\right| + \left|\int_2^5 f(x)\,dx\right|$

$$= 4 + 14 = 18 \text{ units}^2$$

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1. (12 points) Given that:

$$\int_2^7 f(x)dx = 8, \quad \int_5^7 f(x)dx = -3,$$

find the **total** area bounded by the curve $y = f(x)$, $x = 2$, $x = 7$, and the x-axis.

Note $\int_2^5 f(x)dx = \int_2^7 f(x)dx - \int_5^7 f(x)dx$

$= 8-(-3) = 11$

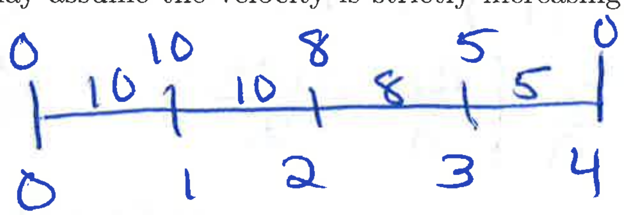
Total area $= \left|\int_2^5 f(x)dx\right| + \left|\int_5^7 f(x)dx\right|$

$= 11 + |-3| = \boxed{14}$ units^2

2. (12 points) The velocity of a model train is measured in inches over a four second interval, and results in the following table:

Time (sec)	Velocity (in/sec)
0	0
1	10
2	8
3	5
4	0

Estimate the distance traveled by the train by using an **upper** sum with time intervals of 1 second. You may assume the velocity is strictly increasing or strictly decreasing along each subinterval.



$U_f = \Delta x\,[f(1) + f(1) + f(2) + f(3)]$

$= 1\cdot[10 + 10 + 8 + 5]$

$= \boxed{33}$ inches

3. A small town has a population that is changing at a rate of $f(t) = 10t - 3t^2$ citizens, where t is time in years.

(a) (6 points) Set up an integral to find the population of the town after two years.

$$P(t) = \int_0^2 (10t - 3t^2)\,dt$$

(b) (20 points) Using a general Riemann Sum, evaluate your integral in part (a). You may choose x_i^* to be the right-hand endpoint of each subinterval. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\Delta x = \frac{2-0}{n} = \frac{2}{n}, \quad x_i^* = a + i\Delta x = \frac{2i}{n}$$

$$f(x_i^*) = 10\left(\frac{2i}{n}\right) - 3\left(\frac{2i}{n}\right)^2$$

$$= \frac{20i}{n} - \frac{12i^2}{n^2}$$

So $$\int_0^2 (10t - 3t^2)\,dt = \lim_{n\to\infty} \Delta x \sum_{i=1}^{n} f(x_i^*)$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{20i}{n} - \frac{12i^2}{n^2}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{20}{n}\sum_{i=1} i - \frac{12}{n^2}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{20}{n}\cdot\frac{n(n+1)}{2} - \frac{12}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty}\left[\frac{20n+20}{n} - \frac{4(n+1)(2n+1)}{n^2}\right]$$

$$= 20 - 8 = 12 \text{ citizens}$$