Name (Print)____________________________ ID#____________________

My signature *certifies* that I have taken this exam in accordance with the Georgia Tech Honor Code.

Signature____

There are 40 questions worth a total of 100 points. Choose the best option for each question. Use the no. 2 pencil to mark the answers on the Scantron form. Write your name and turn in **both the exam and Scantron** form together.

GRADING CREDIT GIVEN ONLY FOR THOSE ANSWERS MARKED ON THE SCANTRON FORM.

Choose the best answer for each question. Mark the answers on the Scantron form.

b_____1. If the average arrival rate for a queuing problem is 20 customers per hour, what is the average inter-arrival time between customers?

a. 20 minutes.
b. 3 minutes.
c. 3 hours.
d. 60 minutes.

a_____2. In order to achieve a steady state performance in a queuing system, the sum of the effective service rates ($m\mu$) of all servers, i.e. total capacity, must:

a. exceed the effective arrival rate (λ) of all customers.
b. equal the effective arrival rate (λ)of all customers.
c. be less than the effective arrival rate (λ) of all customers.
d. be exponentially distributed.

d_____3. In an *M/M/1* system, if the average arrival rate (λ) is 8 per hour and the average service rate (μ) is 12 per hour, what is the percentage that the server is busy?

a. 12%.
b. 8%
c. 33.33%.
d. 66.67%.

c_____4. Suppose a penny arcade worker is in charge of keeping **ten machines** in operation. The failure rate of each machine follows an exponential distribution with a mean time of ten hours and the time required to repair each machine follows an exponential distribution with a mean time of thirty minutes. Using the queuing formula, we can obtain P_0=0.538. Over the long run, approximately how many machines, on average, will be **operating**?

a. 0.297
b. 0.76
c. 9.24
d. 2

a_____5. Which of the following is **not** a basic necessary component of a queuing system?

a. Brick building.
b. Waiting queue (waiting line).
c. Server(s).
d. Customer calling population.

b_____6. Which of the following is not a common steady state performance measure for a queuing system?

a. The average time a customer has to wait in the queue.
b. The average number of customers who argue around the queue.
c. The probability all servers are idle.
d. The average time a customer spends in the system.

a_____7. In an *M/M/1* queuing system, $\lambda = 6$, and the system is idle 40% of the time. What is the average service rate, μ?

a. 10

b. 6
c. 4
d. 15

c_____8. "Balking" is a customer:
a. refusing service.
b. leaving the queue.
c. blocked and cannot enter the queue.
d. refusing to leave after service.

Use the following data to answer questions 9-13.

Customer arrives at Homestake Bank during the business hours according to a Poisson distribution at a mean rate of 30 per hour (λ). During the business hours the bank has **3 tellers** (i.e., 3 channels or m = 3) working. The average time a teller spends with a customer is 3 minutes, and service time follows an exponential distribution.

a_____9. What is the probability of 3 customers **arrive** in 5 minutes during the business hours?
a. 0.214.
b. 0.367.
c. 0.100.
d. 0.006.

b_____10. For a customer, what is the probability that the service time by the teller will take more than 5 minutes?
a. 0.109.
b. 0.189.
c. 0.811.
d. 0.577.

d_____11. What is the probability that there is no customer in the bank (P_0)?
a. 0.14286.
b. 0.2368.
c. 0.3158.
d. 0.2105.

a_____12. In average, how many customers are in the bank (L)?
a. 1.737.
b. 0.237.
c. 0.058.
d. 3.428.

c_____13. In average, how long a customer needs to wait in the line (W_q)?
a. 1.9285 hours.
b. 0.05789 hours.
c. 0.00789 hours.
d. 0.05789 hours.

b_____14. Cars arrive for servicing at Al's Repair Shop according to a Poisson distribution at a mean rate of 0.4 per **hour** (λ). The service time Al takes to work on a car is **exactly** 2 hours. Al is the only employee. Determine the average number of cars that will be waiting for Al to begin working on them (L_q)?

a. 4.
b. 1.6.
c. 3.2.
d. 0.8.

c_____15. Following the above question, what is the average time a customer will spend in Al's Repair Shop (W)?

a. 4 hours.
b. 2 hours.
c. 6 hours.
d. 10 hours.

Use the following data to answer questions 16-22.

Shelley's Supermarket is located in Virginia Beach, Virginia. It operates 16 hours a day, from 7:00 A.M. to 11:00 P.M. and has a maximum of four check-out stands. During the summer, volume at the supermarket increases dramatically due to the tourist traffic. The summer season runs approximately three months, from June 15 to September 15, although immediately prior to an immediately after this season, there is slightly more business than during the winter months. To simplify the analysis, however, Shelley's management assumes two seasons: peak (summer) and off-peak.

During the off-peak season, customers arrive according to a Poison process at an average rate of 14 per hour, compared to an arrival rate of 46 customers per hour during the peak season. Shelley's estimates that its average gross profit per customer is **$6.5 during the off-peak season** and **$4.5 during the peak season**. These gross profits are sales (revenue) minus materials, utilities, and overhead allocations but not clerk salaries and customer good wills. However, Shelley's estimates the following costs *per customer per hour in the check-out line* based on goodwill, and so on:

	Off-peak *Season*	**Peak Season**
While waiting to be served	$10/hour	$8/hour
While being served	$6/hour	$4/hour

Shelley's hires only union personnel as check-out clerks. Their pay, including benefits, averages $16 per hour. According to the union contract, these employees must be hired as permanent, year-round employees.

The union contract allows Shelley's management to hire part-time support personnel (baggers, stock clerks) to help deal with the store's increased traffic during the peak season, but there can never be more temporary employees than permanent employees working. These nonunion, temporary employees are paid $5 per hour.

Customer service time for each permanent check-out clerk follows an exponential distribution, with an average time of **four minutes** during the off-peak season. During the summer months (peak season), the average customer service time (exponential distribution) for each checker (in minutes) will equal x, where:

$$x = 4 - \frac{(1.5)(number\ of\ temporary\ employees)}{(number\ of\ permanent\ employees)}$$

For example, if Shelley's has three permanent employees and two temporary employees during the summer months, the average service time for each of the three checkers is 4 – (1.5)(2)/3 = 3 minutes.

Assume that the peak season lasts 90 days and that the off-peak seasons lasts 260 days (Shelley's is closed five days a year).

d_____16. What type of queuing system is Shelly's Supermarket's check out situation (assuming there are *m* checkout clerks)? Use Kendall's standard notation.

a. *M/M/1*
b. *M/G/1*
c. *M/M/1* with Finite Source
d. *M/M/m*

c_____17. During the off-peak season, if two union personnel are employed, what is the check-out utilization rate (ρ)?

a. 93.3%
b. 53.3%
c. 46.7%
d. 65%

c_____18. During the peak season, if three union personnel and three part-time workers are employed, what is the service rate (number of customers per hour) for each check out (μ)?

a. 15
b. 18.46
c. 24
d. 20

a_____19. During the peak season, if three union personnel and three part-time workers are employed, what is the checkout utilization rate (ρ)?

a. 63.9%
b. 36.1%
c. 46.7%
d. 95.8%

b_____20. During the off-peak season, if three union personnel are employed, L=0.967958, L_q=0.034625, what is the net hourly profit?

a. 14.35
b. 37.05
c. 43.80
d. 30.05

d_____21. During the peak season, if three union personnel and three part-time workers are employed, L=2.63507, L_q=0.718403, what is the net hourly profit?

a. 81.08
b. 96.20
c. 107.59
d. 130.58

c_____22. Based on the net hourly profits from the above two questions, what is the whole year net profit if three union personnel are employed to work for the whole year and three part-time workers are employed for the summer peak-season?

a. $502,938
b. $279,937
c. $342,163
d. $176,543

d_____23. Little's formula has been popularly used in industry to estimate the work-in-process (WIP) or order-in-process (OIP) in a business system. Let's say a business firm usually takes in an average of 6 business days to fill a customer order after the customer makes the order, i.e., enters the firm. If the customers issue in the rate of 1500 orders per business day to the firm, what is the estimate OIP in this business?

a. 1,500
b. 45,000
c. 345,000
d. 9,000

a_____24. Little's formula has been popularly used in industry to estimate the work-in-process (WIP) or order-in-process (OIP) in a business system. Amazon estimates that in average 1,500,000 orders are in different stages of the fulfillment process during the holiday season. If the customers issue in the rate of 300,000 orders per business day to Amazon, what is the average time each order will be with Amazon and send it out to the customer?

a. 5 days
b. 50 days
c. 2 days
d. 12 days

Use the following data to answer questions 25-28.

The following Power Point slides describe Three Hills Power Company's maintenance operations. The random number matching to the next generator breakdown and the repair time are listed in the middle tables. The first 15 simulation observations are listed at the last table. The 15th breakdown is repaired and finished at 10AM of the second day. That is, the total operation time is 34 hours for the 15 observations.

Three Hills Power Company

- Three Hills provides power to a large city through a series of almost 200 electric generators
- The company is concerned about generator failures because a breakdown costs about $75 per generator per hour
- Their four repair people earn $30 per hour and work rotating 8 hour shifts, i.e. always has one on-duty
- Management wants to evaluate the
  1. Service maintenance cost
  2. Simulated machine breakdown cost
  3. Total cost

Three Hills Power Company

- There are two important maintenance system components
- Time between successive generator breakdowns which varies from 30 minutes to three hours, a discrete distribution shown below
- The time it takes to repair the generators which ranges from one to three hours in one hour blocks, a discrete distribution shown below
- A next event simulation is constructed to study this problem

Three Hills Power Company

- Time between generator breakdowns at Three Hills Power

TIME BETWEEN RECORDED MACHINE FAILURES (HRS)	NUMBER OF TIMES OBSERVED	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
0.5	5	0.05	0.05	01 to 05
1.0	6	0.06	0.11	06 to 11
1.5	16	0.16	0.27	12 to 27
2.0	33	0.33	0.60	28 to 60
2.5	21	0.21	0.81	61 to 81
3.0	19	0.19	1.00	82 to 00
Total	100	1.00		

Three Hills Power Company

- Generator repair times required

REPAIR TIME REQUIRED (HRS)	NUMBER OF TIMES OBSERVED	PROBABILITY	CUMULATIVE PROBABILITY	RANDOM NUMBER INTERVAL
1	28	0.28	0.28	01 to 28
2	52	0.52	0.80	29 to 80
3	20	0.20	1.00	81 to 00
Total	100	1.00		

Three Hills Power Company

- Simulation of generator breakdowns and repairs

(1) BREAKDOWN NUMBER	(2) RANDOM NUMBER FOR BREAKDOWNS	(3) TIME BETWEEN BREAKDOWNS	(4) TIME OF BREAKDOWN	(5) TIME REPAIR-PERSON IS FREE TO BEGIN THIS REPAIR	(6) RANDOM NUMBER FOR REPAIR TIME	(7) REPAIR TIME REQUIRED	(8) TIME REPAIR ENDS	(9) NUMBER OF HOURS MACHINE DOWN
1	57	2	02:00	02:00	07	1	03:00	1
2	17	1.5	03:30	03:30	60	2	05:30	2
3	36	2	05:30	05:30	77	2	07:30	2
4	72	2.5	08:00	08:00	49	2	10:00	2
5	85	3	11:00	11:00	76	2	13:00	2
6	31	2	13:00	13:00	95	3	16:00	3
7	44	2	15:00	16:00	51	2	18:00	3
8	30	2	17:00	18:00	16	1	19:00	2
9	26	1.5	18:30	19:00	14	1	20:00	1.5
10	09	1	19:30	20:00	85	3	23:00	3.5
11	49	2	21:30	23:00	59	2	01:00	3.5
12	13	1.5	23:00	01:00	85	3	04:00	5
13	33	2	01:00	04:00	40	2	06:00	5
14	89	3	04:00	06:00	42	2	08:00	4
15	13	1.5	05:30	08:00	52	2	10:00	4.5
							Total	44

c_____25. Based on these 15 observations (breakdowns), what is the total operational cost including machine breakdown cost and service maintenance (repair personnel) cost?

a. $1,020
b. $105
c. $4,320
d. $1,575

b_____26. Let's continue to simulate the next breakdown, i.e., observation/breakdown 16. If the random number generated to match the next breakdown (i.e., the time between 15th and 16th breakdowns) is 75, when does the 16th breakdown time start?

a. 2:30
b. 8:00
c. 9:00
d. 10:00

a_____27. For the 16th observation/breakdown, if the random number generated to match the repair time required 95, what is the repair time required for the 16th breakdown?

a. 3 hours
b. 2 hours
c. 1 hour
d. 10 hours

d_____28. For the 16th observation/breakdown, combine the breakdown starting time, the repair time required, and the repair finish time of the 15th breakdown, how long does the 16th breakdown remain not operating (waiting and under repair)?

a. 2 hours
b. 3 hours
c. 10 hour
d. 5 hours

a_____29. The service time follows the <u>exponential distribution</u> with an average time of 15 minutes per customer. Simulate this service process with a random number (R) of 0.3415. What is the simulated service time?

a. 6.27 minutes
b. 3.15 minutes
c. 13.11 minutes
d. 15.06 minutes

d_____30. A simple game works as follows: Each player flips an unbiased coin until the difference between the number of heads tossed and the number of tails tossed is three (3); pay $1 for each flip of coin; no quitting before the end of game is allowed; player receives $8 payoff at the end of game. If we map single digit random number 0-4 as H (head) and 5-9 as T (tail), what is the net return of the single game if the following random numbers are used to simulate this game starting with the first digit: 119818975126797859780537. Examples of game are below:

Game	Flips	Net result
HHH	3	$8-3=$5
THTTT	5	$8-5=$3
THHTHTHTTTT	11	$8-11=-$3

HHTTHTHHH	9	$8-9=-$1

a. Gain $4
b. Gain $1
c. Lose $5
d. Lose $1

Use the following data to answer questions 31-40.

Four Wheel Tire Shop is a single bay tire store located in Ames, Iowa. The inter-arrival time of cars in need of tires follows a <u>uniform distribution</u> with times between 10 and 50 minutes. Twenty percent of arriving customers want a single tire replaced, 40% want two tires replaced, 5% want three tires replaced, and 35% want all four tires replaced. Customer service time approximately follows a <u>uniform distribution</u> that varies with the number of tires that need to be replaced. The following statistics hold:

Number of Tires Needing Replacement	*Service Time Is Uniformly Distributed Between*
1	10 minutes and 20 minutes
2	15 minutes and 35 minutes
3	20 minutes and 40 minutes
4	25 minutes and 50 minutes

The owner of the store, Ben Stern, is considering leasing a new computerized tire balancing machine, which will reduce the average service time, resulting in the following uniform service distributions:

Number of Tires Needing Replacement	*Service Time Is Uniformly Distributed Between*
1	8 minutes and 18 minutes
2	12 minutes and 32 minutes
3	15 minutes and 35 minutes
4	20 minutes and 40 minutes

The computerized tire balancing machine lease cost averages $2.50 per hour. Mr. Stern estimates that the goodwill cost of a customer's being in the tire shop (either being served or waiting to be served) is $6.00 per hour.

c_____31. Use the inverse distribution function to simulate the customer arrivals. If the generated random number is 0.6506, what is the simulated inter-arrival time, i.e., next customer arrival time?

a. 15.78 minutes
b. 26.02 minutes
c. 36.02 minutes
d. 25.06 minutes

b_____32. Use the CDF to map to the number of tires needing replacement. If the generated random number is 0.3519, what is the number of tires needing replacement?

a. 1
b. 2
c. 3
d. 4

a_____33. Use the inverse distribution function to simulate the customer service time. If the generated random number is 0.5675, what is the simulated service time for replacing 2 tires before the computerized balance machine is used?

a. 26.35 minutes
b. 11.35 minutes
c. 15.00 minutes
d. 31.35 minutes

Simulate the first 20 customer arrivals before the new computerized balance machine is used and assuming the store starts with no customer. The following table shows the simulation and summarizes the related information:

Four Wheel Tires		Average Time in Shop =			29.6649									
			L =	0.9888										
		Hourly Cost =									Total			
							Service	Time Ser	Time Ser	Waiting	Shop			
Cust #	Rand #	IAT	Arrival Time	Rand #	# Tires	Rand #	Time	Begins	Ends	Time	Time		Cum Pr.	# Tires
1	0.6506	36.0240	36.0240	0.3338	2	0.2197	19.3940	36.0240	55.4180	0.0000	19.3940		0	1
2	0.8927	45.7080	81.7320	0.6320	3	0.1094	22.1880	81.7320	103.9200	0.0000	22.1880		0.2	2
3	0.1995	17.9800	99.7120	0.5971	2	0.5147	25.2940	103.9200	129.2140	4.2080	29.5020		0.6	3
4	0.3620	24.4800	124.1920	0.0582	1	0.2982	12.9820	129.2140	142.1960	5.0220	18.0040		0.65	4
5	0.8731	44.9240	169.1160	0.6037	3	0.4231	28.4620	169.1160	197.5780	0.0000	28.4620		1	
6	0.7761	41.0440	210.1600	0.9874	4	0.9834	49.5850	210.1600	259.7450	0.0000	49.5850			
7	0.8848	45.3920	255.5520	0.4630	2	0.7371	29.7420	259.7450	289.4870	4.1930	33.9350			
8	0.7971	41.8840	297.4360	0.6600	4	0.1296	28.2400	297.4360	325.6760	0.0000	28.2400			
9	0.5299	31.1960	328.6320	0.1374	1	0.9517	19.5170	328.6320	348.1490	0.0000	19.5170			
10	0.8177	42.7080	371.3400	0.2403	2	0.7443	29.8860	371.3400	401.2260	0.0000	29.8860			
11	0.6170	34.6800	406.0200	0.2631	2	0.6268	27.5360	406.0200	433.5560	0.0000	27.5360			
12	0.9089	46.3560	452.3760	0.8657	4	0.2809	32.0225	452.3760	484.3985	0.0000	32.0225			
13	0.3554	24.2160	476.5920	0.4814	2	0.7401	29.8020	484.3985	514.2005	7.8065	37.6085			
14	0.9983	49.9320	526.5240	0.8613	4	0.3199	32.9975	526.5240	559.5215	0.0000	32.9975			
15	0.6005	34.0200	560.5440	0.1879	1	0.9339	19.3390	560.5440	579.8830	0.0000	19.3390			
16	0.8800	45.2000	605.7440	0.9139	4	0.8305	45.7625	605.7440	651.5065	0.0000	45.7625			
17	0.2605	20.4200	626.1640	0.0030	1	0.5148	15.1480	651.5065	666.6545	25.3425	40.4905			
18	0.6300	35.2000	661.3640	0.1762	1	0.2499	12.4990	666.6545	679.1535	5.2905	17.7895			
19	0.5417	31.6680	693.0320	0.2607	2	0.7111	29.2220	693.0320	722.2540	0.0000	29.2220			
20	0.9892	49.5680	742.6000	0.3703	2	0.8408	31.8160	742.6000	774.4160	0.0000	31.8160			
	1)	Average customer inter-arrival time						37.1300	minutes					
	2)	Average number of tires needing replacement						2.3500	tires					
	3)	Average time a customer waiting						2.5931	minutes					
	4)	Average time a customer being in the tire shop						29.6649	minutes					
	5)	Average hourly relevant cost of the tire shop												

c_____34. Using the above simulation result, what is the average hourly relevant cost (only customer good will) before the computerized balance is used?

a. $2.35
b. $38.54
c. $5.93
d. $2.59

a_____35. Use the above result before the computerized balanced machine is used. The standard deviation (s) of the time a customer being in the tire shop from the above 20 simulation is 8.98 minutes. If we want to obtain the average time a customer being in the tire shop within +/- 0.5 minutes, how many observations are required for 95% confidence level?

a. 1239
b. 2367
c. 356
d. 2147

d_____36. Use the inverse distribution function to simulate the customer service time. If the generated random number is 0.4396, what is the simulated service time for replacing 4 tires if the **<u>new computerized balance machine</u>** is used?

a. 26.35 minutes
b. 17.584 minutes
c. 8.792 minutes
d. 28.792 minutes

d_____37. Simulate the first 20 customer arrivals if the new computerized balance machine is used and assuming the store starts with no customer. The following table shows the simulation and summarizes the related information:

Four Wheel Tires			Average Time in Shop =			23.3057							
With Balance Machine			L =	0.7769									
			Hourly Cost =										
Cust #	Rand #	IAT	Arrival Time	Rand #	# Tires	Rand #	Service Time	Time Ser Begins	Time Ser Ends	Waiting Time	Total Shop Time	Cum Pr.	# Tires
1	0.2128	18.5102	18.5102	0.2031	2	0.1542	15.0848	18.5102	33.5950	0.0000	15.0848	0	1
2	0.3521	24.0825	42.5927	0.0897	1	0.7637	15.6366	42.5927	58.2293	0.0000	15.6366	0.2	2
3	0.1836	17.3458	59.9385	0.6100	3	0.1865	18.7307	59.9385	78.6693	0.0000	18.7307	0.6	3
4	0.2994	21.9765	81.9151	0.8407	4	0.4754	29.5084	81.9151	111.4235	0.0000	29.5084	0.65	4
5	0.9683	48.7323	130.6474	0.1640	1	0.4027	12.0274	130.6474	142.6748	0.0000	12.0274	1	
6	0.7892	41.5665	172.2139	0.9542	4	0.8298	36.5969	172.2139	208.8108	0.0000	36.5969		
7	0.3262	23.0484	195.2624	0.6020	3	0.5206	25.4120	208.8108	234.2228	13.5484	38.9605		
8	0.3243	22.9701	218.2325	0.1262	1	0.3457	11.4566	234.2228	245.6795	15.9904	27.4470		
9	0.4658	28.6318	246.8643	0.0178	1	0.6063	14.0629	246.8643	260.9272	0.0000	14.0629		
10	0.9239	46.9542	293.8185	0.0340	1	0.8466	16.4660	293.8185	310.2845	0.0000	16.4660		
11	0.0212	10.8492	304.6677	0.1891	1	0.7771	15.7706	310.2845	326.0550	5.6168	21.3873		
12	0.9931	49.7231	354.3909	0.0657	1	0.8412	16.4117	354.3909	370.8026	0.0000	16.4117		
13	0.9598	48.3928	402.7836	0.0274	1	0.5401	13.4014	402.7836	416.1851	0.0000	13.4014		
14	0.3567	24.2664	427.0500	0.5125	2	0.4025	20.0510	427.0500	447.1010	0.0000	20.0510		
15	0.8732	44.9274	471.9774	0.1133	1	0.3783	11.7833	471.9774	483.7607	0.0000	11.7833		
16	0.4397	27.5871	499.5646	0.4337	2	0.2393	16.7851	499.5646	516.3497	0.0000	16.7851		
17	0.8683	44.7339	544.2985	0.6728	4	0.9309	38.6184	544.2985	582.9168	0.0000	38.6184		
18	0.1364	15.4548	559.7533	0.0993	1	0.0692	8.6918	582.9168	591.6086	23.1636	31.8553		
19	0.3630	24.5200	584.2733	0.4364	2	0.5669	23.3380	591.6086	614.9467	7.3353	30.6733		
20	0.0691	12.7634	597.0367	0.4155	2	0.5358	22.7164	614.9467	637.6630	17.9099	40.6263		

1)	Average customer inter-arrival time	29.8518	minutes
2)	Average number of tires needing replacement	1.9000	tires
3)	Average time a customer waiting	4.1782	minutes
4)	Average time a customer being in the tire shop	23.3057	minutes
5)	Average hourly relevant cost of the tire shop		

Based on the above simulation results, what is the average time a customer <u>waiting</u> in the tire shop before getting the service if the new computerized balance is used?

a. 29.85 minutes
b. 1.9 minutes
c. 23.31 minutes
d. 4.18 minutes

b_____38. Using the above simulation result, what is the average hourly relevant cost (including customer good will and balance machine costs) if the computerized balance machine is used?

a. $3.95
b. $7.16
c. $16.67
d. $15.64

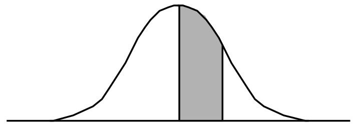
b_____39. Based on the simulation result in (37) with the new computerized balance machine, if the shop opens 10 hours a day, 300 days a year, and if each tire is estimated to have $20 in profit, about how much the annual profit the shop expects to have a year in average?

a. $11400
b. $228,000
c. $1,000,000
d. $56700

b_____40. Use the above result after the new computerized balanced machine is used. The standard deviation (s) of the time a customer being in the tire shop from the above 20 simulation is 7.22 minutes. If we want to obtain the average time a customer being in the tire shop within +/- 0.5 minutes, how many observations are required for 99% confidence level?

a. 800
b. 1388
c. 356
d. 2067

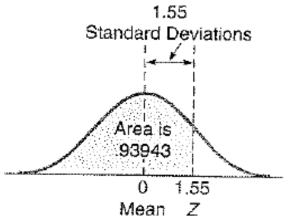
Areas of the Standard Normal Distribution



An entry in the table is the proportion under the entire curve which is between Z = 0 and a positive value of z. Areas for negative values of z are obtained by symmetry.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

APPENDIX A: AREAS UNDER THE STANDARD NORMAL CURVE



Example: To find the area under the normal curve, you must know how many standard deviations that point is to the right of the mean. Then the area under the normal curve can be read directly from the normal table. For example, the total area under the normal curve for a point that is 1.55 standard deviations to the right of the mean is .93943.

	00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73536	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97784	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950

	00	.01	.02	.03	.04	.05	.06	.07	.08	.09
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99998	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

KEY EQUATIONS For Queuing Models

λ = mean number of arrivals per time period

μ = mean number of people or items served per time period

Equations 14-2 through 14-8 describe operating characteristics in the single-channel model that has Poisson arrival and exponential service rates. **(M/M/1)**

(14-2) L = average number of units (customers) in the system

$$= \frac{\lambda}{\mu - \lambda}$$

(14-3) W = average time a unit spends in the system (Waiting time + Service time)

$$= \frac{1}{\mu - \lambda}$$

(14-4) L_q = average number of units in the queue $= \dfrac{\lambda^2}{\mu(\mu - \lambda)}$

(14-5) W_q = average time a unit spends waiting in the queue

$$= \frac{\lambda}{\mu(\mu - \lambda)}$$

(14-6) ρ = utilization factor for the system $= \dfrac{\lambda}{\mu}$

(14-7) P_0 = probability of 0 units in the system (that is, the service unit is idle)

$$= 1 - \frac{\lambda}{\mu}$$

(14-8) $P_{n>k}$ = probability of more than k units in the system

$$= \left(\frac{\lambda}{\mu}\right)^{k+1}$$

Equations 14-14 through 14-19 describe operating characteristics in multichannel models that have Poisson arrival and exponential service rates, where m = the number of open channels. **(M/M/m)**

(14-14) $$P_0 = \frac{1}{\left[\displaystyle\sum_{n=0}^{n=m-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{m!}\left(\frac{\lambda}{\mu}\right)^m \frac{m\mu}{m\mu - \lambda}}$$

for $m\mu > \lambda$

Probability that there are no people or units in the system.

(14-15) $$L = \frac{\lambda\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{\lambda}{\mu}$$

Average number of people or units in the system.

(14-16) $$W = \frac{\mu(\lambda/\mu)^m}{(m-1)!(m\mu - \lambda)^2} P_0 + \frac{1}{\mu} = \frac{L}{\lambda}$$

Average time a unit spends in the waiting line or being serviced (namely, in the system).

(14-17) $$L_q = L - \frac{\lambda}{\mu}$$

Average number of people or units in line waiting for service.

(14-18) $$W_q = W - \frac{1}{\mu} = \frac{L_q}{\lambda}$$

Average time a person or unit spends in the queue waiting for service.

(14-19) $$\rho = \frac{\lambda}{m\mu}$$

Utilization rate.

Equations 14-20 through 14-23 describe operating characteristics in single-channel models that have Poisson arrivals and constant service rates. **(M/D/1)**

(14-20) $$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$$

Average length of the queue.

(14-21) $$W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$$

Average waiting time in the queue.

(14-22) $$L = L_q + \frac{\lambda}{\mu}$$

Average number of customers in the system.

(14-23) $$W = W_q + \frac{1}{\mu}$$

Average waiting time in the system.

Equations 14-24 through 14-29 describe operating characteristics in single-channel models that have Poisson arrivals and exponential service rates and a finite calling population. **(M/M/1//N)**

(14-24) $$P_0 = \frac{1}{\displaystyle\sum_{n=0}^{N} \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n}$$

Probability that the system is empty.

(14-25) $$L_q = N - \left(\frac{\lambda + \mu}{\lambda}\right)(1 - P_0)$$

Average length of the queue.

(14-26) $$L = L_q + (1 - P_0)$$

Average number of units in the system.

(14-27) $$W_q = \frac{L_q}{(N - L)\lambda}$$

Average time in the queue.

(14-28) $$W = W_q + \frac{1}{\mu}$$

Average time in the system.

(14-29) $$P_n = \frac{N!}{(N-n)!}\left(\frac{\lambda}{\mu}\right)^n P_0 \quad \text{for} \quad n = 0, 1, \ldots, N$$

Probability of n units in the system.

Little's Formula

Equations 14-30 to 14-32 are Little's Flow Equations, which can be used when a steady state condition exists.

(14-30) $L = \lambda W$

(14-31) $L_q = \lambda W_q$

(14-32) $W = W_q + 1/\mu$

General Cost Function

Equations 14-9 through 14-13 are used for finding the costs of a queuing system.

(14-9) Total service cost = mC_s

where

m = number of channels

C_s = service cost (labor cost) of each channel

(14-10) Total waiting cost per time period = $(\lambda W)C_w$

C_w = cost of waiting

Waiting time cost based on time in the system.

(14-11) Total waiting cost per time period = $(\lambda W_q)C_w$

Waiting time cost based on time in the queue.

(14-12) Total cost = $mC_s + \lambda W C_w$

Waiting time cost based on time in the system.

(14-13) Total cost = $mC_s + \lambda W_q C_w$

Waiting time cost based on time in the queue.

For the situation that customer goodwill costs are different when waiting and in service, let's define

- C_s: Average unit time cost for each server
- g_w: Customer goodwill cost per time unit while waiting
- g_s: Customer goodwill cost while in service per time unit

The total cost per time unit can be expressed as follows:

$$TC = m\,C_s + L_q\,g_w + (L - L_q)\,g_s$$

The Poisson Arrival Process

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

Where

P(X=k) = the probability of k customers arrive during t time units.

λ = mean arrival rate per time unit.

t = the length of the interval.

k = the number of customers arrive during t interval.

e = 2.7182818 (the base of the natural logarithm).

k! = k (k -1) (k -2) (k -3) ... (3) (2) (1).

The Exponential Service Time Distribution (PDF)

$f(t) = \mu e^{-\mu t}$, t is service time and $t \geq 0$

μ = the average number of customers who can be served per time period. Therefore, **$1/\mu$ = the mean service time**.

The probability that the service time X is less than some "t."

$$P(X \leq t) = 1 - e^{-\mu t}$$

The probability that the service time X is more than some "t."

$$P(X \geq t) = 1 - (1 - e^{-\mu t}) = e^{-\mu t}$$