ISYE 3232A Fall 2015 Test 1 - A white

I, SHK, do swear that I abide by the Georgia Tech Honor Code. I understand that any honor code violations will result in a failure (an F).

Signature: ____________________

- You will have 1 hour 15 minutes.
- This quiz is closed book and closed notes. Calculators are not allowed. No scrap paper is allowed. Make sure that there is nothing on your desk except pens and erasers.
- If you need extra space, use the back of the page and indicate that you have done so.
- **Do not remove any page from the original staple**. Otherwise, there will be 5 points off.
- **Show your work on the test sheet**. If you do not show your work for a problem, we will give zero point for the problem even if your answer is correct.
- **We will not select among several answers**. Make sure it is clear what part of your work you want graded. If two answers are given, zero point will be given for the problem.
- Throughout, you will receive full credit (i) if the work is correct and (ii) if someone with no understanding of probability, set theory, and calculus could simplify your answer to obtain the correct numerical answer. **However, you must give a numerical answer where asked.**

Some formula that you may need:

- A continuous uniform random variable X from a to b has pdf $f(x) = \frac{1}{b-a}$ for $a \le x \le b$ and CDF $F(x) = \frac{x-a}{b-a}$.
- A discrete uniform random variable X in $\{x_1, x_2, \ldots, x_n\}$ has pmf $p(x) = 1/n$.
- An exponential r.v. X with rate λ has pdf $f(x) = \lambda e^{-\lambda x}$ for $x > 0$ and CDF $F(x) = 1 - e^{-\lambda x}$ for $x > 0$. Its mean is $1/\lambda$ and variance is $1/\lambda^2$.
- In a newsvendor problem, the expected cost $\mathsf{E}[\text{Cost}]$ with ordering quantity q is

$$\begin{aligned}
\mathsf{E}[\text{Profit}] &= c_p \mathsf{E}[D \wedge q] + c_s \mathsf{E}[(q-D)^+] - c_v q - p_u \mathsf{E}[(D-q)^+] \\
&= (c_p - c_v)\mathsf{E}[D \wedge q] - (c_v - c_s)\mathsf{E}[(q-D)^+] - p_u \mathsf{E}[(D-q)^+] \\
&= c_p \mathsf{E}[D] - \left\{(c_p + p_u)\mathsf{E}[(D-q)^+] + h\mathsf{E}[(q-D)^+] + c_v q\right\}.
\end{aligned}$$

- For a newsvendor problem, the optimal quantity that maximizes the expected profit (or minimizes the expected cost) is q^* such that $q^* = \min\{q : F(q) \ge \frac{c_p + p_u - c_v}{c_p + p_u - c_s}\}$ where $F(\cdot)$ is the CDF of demand.
- Kingman's formula: $w_q \approx m \frac{\rho}{1-\rho} \frac{c_a^2 + c_s^2}{2}$.

Table B.1. Right tail probabilities $1 - \Phi(a) = P(Z \geq a)$ for an $N(0, 1)$ distril random variable Z.

a	0	1	2	3	4	5	6	7	8	9
0.0	5000	4960	4920	4880	4840	4801	4761	4721	4681	4641
0.1	4602	4562	4522	4483	4443	4404	4364	4325	4286	4247
0.2	4207	4168	4129	4090	4052	4013	3974	3936	3897	3859
0.3	3821	3783	3745	3707	3669	3632	3594	3557	3520	3483
0.4	3446	3409	3372	3336	3300	3264	3228	3192	3156	3121
0.5	3085	3050	3015	2981	2946	2912	2877	2843	2810	2776
0.6	2743	2709	2676	2643	2611	2578	2546	2514	2483	2451
0.7	2420	2389	2358	2327	2296	2266	2236	2206	2177	2148
0.8	2119	2090	2061	2033	2005	1977	1949	1922	1894	1867
0.9	1841	1814	1788	1762	1736	1711	1685	1660	1635	1611
1.0	1587	1562	1539	1515	1492	1469	1446	1423	1401	1379
1.1	1357	1335	1314	1292	1271	1251	1230	1210	1190	1170
1.2	1151	1131	1112	1093	1075	1056	1038	1020	1003	0985
1.3	0968	0951	0934	0918	0901	0885	0869	0853	0838	0823
1.4	0808	0793	0778	0764	0749	0735	0721	0708	0694	0681
1.5	0668	0655	0643	0630	0618	0606	0594	0582	0571	0559
1.6	0548	0537	0526	0516	0505	0495	0485	0475	0465	0455
1.7	0446	0436	0427	0418	0409	0401	0392	0384	0375	0367
1.8	0359	0351	0344	0336	0329	0322	0314	0307	0301	0294
1.9	0287	0281	0274	0268	0262	0256	0250	0244	0239	0233
2.0	0228	0222	0217	0212	0207	0202	0197	0192	0188	0183
2.1	0179	0174	0170	0166	0162	0158	0154	0150	0146	0143
2.2	0139	0136	0132	0129	0125	0122	0119	0116	0113	0110
2.3	0107	0104	0102	0099	0096	0094	0091	0089	0087	0084
2.4	0082	0080	0078	0075	0073	0071	0069	0068	0066	0064
2.5	0062	0060	0059	0057	0055	0054	0052	0051	0049	0048
2.6	0047	0045	0044	0043	0041	0040	0039	0038	0037	0036
2.7	0035	0034	0033	0032	0031	0030	0029	0028	0027	0026
2.8	0026	0025	0024	0023	0023	0022	0021	0021	0020	0019
2.9	0019	0018	0018	0017	0016	0016	0015	0015	0014	0014
3.0	0013	0013	0013	0012	0012	0011	0011	0011	0010	0010
3.1	0010	0009	0009	0009	0008	0008	0008	0008	0007	0007
3.2	0007	0007	0006	0006	0006	0006	0006	0005	0005	0005
3.3	0005	0005	0005	0004	0004	0004	0004	0004	0004	0003
3.4	0003	0003	0003	0003	0003	0003	0003	0003	0003	0002

A -1

1. (35 points) Next month's production at a manufacturing company will use a certain solvent for part of its production process. Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed and the solvent costs \$50 per liter. Due to short product life cycle, unused solvent cannot be used in following months. There will be a \$15 disposal charge for each liter of solvent left over at the end of the month. If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$85 per liter short.

   (a) (5 points) The demand has the following pmf:

Demand	300	500	700	900
Prob	0.2	0.4	0.3	0.1

   What is the optimal ordering quantity assuming there is no initial inventory? Show work.

$$\frac{85-50}{85+15} = \frac{35}{100} = 0.35 \quad \therefore \underline{q^* = 500}$$

   (b) (6 points) Now let's assume that we have 300 initial inventory and demand follows the pmf in part (a). Calculate the expected cost when we do not produce any additional solvent. Show full work.

$$85\left[200(0.4)+400(0.3)+600(0.1)\right] + 15(0)$$

or

$$85\left\{\left(300(0.2)+500(0.4)+700(0.3)+900(0.1)\right)-300\right\}$$

   (c) (9 points) We have 300 initial inventory and demand follows the pmf in part (a). If we decide to produce more, calculate the expected cost with production. show work.

$$1500+50(200)+85\left[200(0.3)+400(0.1)\right]+15\left(200(0.2)\right)$$

A-2

(d) (6 points) Now suppose that we want to come up with an exhaustive inventory policy for any initial inventory amount m. It is known that the critical point m^* is between 300 and 500. Give an equation for the expected cost without production assuming that the initial inventory is exactly equal to m^*.

$$85\left[(500-m^*)0.4+(700-m^*)0.3+(900-m^*)0.1\right]$$

$$+15\left[(m^*-300)0.2\right]$$

(e) (3 points) Now suppose that the demand is exponentially distributed with mean 800. What is the optimal ordering quantity if there is no initial inventory?

$$1-e^{-\frac{q}{800}}=0.35 \qquad \therefore \underline{q^*=-800\ln 0.65}$$

(f) (6 points) Suppose that answer for (e) is 350; demand is exponential with mean 800; and we have 300 initial inventory. Calculate the expected cost when we do not produce additional quantity. Leave your answer with integrals. (*If we cannot get a numerical answer when we enter your expression into a mathematical software package, you will be given a 0 point. For example, $\int_0^1 Ddx$ receives a 0 point because it is equal to D which is not a numerical answer.*)

$$85\int_{300}^{\infty}(x-300)\frac{1}{800}e^{-\frac{x}{800}}dx+15\int_{0}^{300}(300-x)\frac{1}{800}e^{-\frac{x}{800}}dx$$

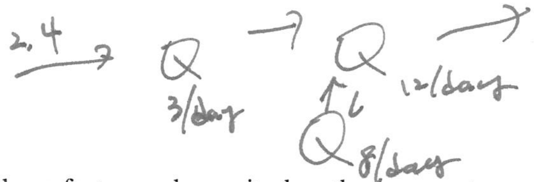
2. (5 points) The delivery time of an order for raw material is normally distributed with mean 20 days and standard deviation 4 days.

(a) (2 points) What is the z value such that $\Pr(Z \leq z) = 0.9$ where Z represents a standard normal random variable?

$+1.28$

(b) (3 points) How many days are needed to ensure with 90% probability that an order will be received by then?

$r = 20+(1.28)(4)$



A-3

3. (30 points) Consider a factory whose site has the same setup as in the Littlefield game we played. There are three stations, each with 1 machine. Jobs arrive at Station A every exponential time with mean 10 hours and they are processed with constant processing time 8 hours. Jobs are moved to Station B whose processing time is constant 2 hours. Then they are sent to Station C for checking which takes exponential time with mean 4 hours. Finally jobs are sent back to Station B for the final processing and then leave the factory. Currently each station has one machine and one buffer with infinite capacity. *This factory runs 24 hours per day.*

(a) (2 points) What is the job arrival rate *per day* to Station A?

2.4/day

(b) (2 points) What is the service rate *per day* of a machine at Station A?

3/day

(c) (1 point) What is the steady-state utilization of a machine at Station A?

0.8

(d) (2 points) What is the square coefficient of variation of interarrival times to Station A?

1

(e) (2 points) What is the square coefficient of variation of service times of a machine at Station A?

0

(f) (1 point) What is the long-run average waiting time *in hours* in queue at Station A? (*If you are unsure of your answer in (c), then assume that you got 0.95 in (c); enter your initial here ________; and solve this problem.*)

$$8 \, \frac{0.8}{1-0.8} \, \frac{1+0}{2}$$

(g) (3 points) What is the overall job arrival rate *per day* to Station B (including new jobs from Station A and returning jobs from Station C)?

4.8/day

(h) (5 points) Suppose that the long-run average numbers of jobs at Stations A, B and C (including those waiting in buffer and in service) are 10, 25 and 10, respectively. Calculate the long-run average lead time *in days*?

$$\frac{10+25+10}{2.4}$$

(i) (3 points) What is the system throughput *per day*?

2.4/day

A -4

(j) (3 points) If job interarrival times are changed to exponential with mean 2 hours, what is the system throughput?

3/day

(k) (3 points) Suppose that after adding some number of machines to the systems, the manager found that Stations A, B, and C have 90%, 50%, and 85% utilization, respectively. If you want to add one additional machine, where would you add and why?

A because it has the highest utilization

(l) (3 points) Station A has 3 machines. What is the most appropriate queueing notation for this station?

M/G/3

4. (20 points) A store stocks a particular item. The demand for the product each day is 1 item with probability 1/6, 2 items with probability 1/2, and 3 items with probability 1/3. Assume that the daily demands are independent and identically distributed. Each evening if the remaining stock is less than 3 items (< 3), the store orders enough to bring the total stock up to 6 items. These items reach the store before the beginning of the following day. Assume that any demand is lost when the item is out of stock. Let Y_n be the amount in stock at the *beginning* of day n; assume that $Y_0 = 4$. It is known that Y_n is a Markov chain.

(a) (3 points) Give its state space.

$\{3, 4, 5, 6\}$

(b) (2 points) Give its initial distribution.

$[0 \quad 1 \quad 0 \quad 0]$

(c) (3 points) Calculate $\Pr(Y_{n+1} = 3 \mid Y_n = 6)$.

$\frac{1}{3}$

(d) (3 points) Calculate $\Pr(Y_{n+1} = 3 \mid Y_n = 5)$.

$\frac{1}{2}$

(e) (3 points) Calculate $\Pr(Y_{n+1} = 6 \mid Y_n = 4)$.

$\frac{1}{2} + \frac{1}{3}$

(f) (3 points) Calculate $\Pr(Y_{n+1} = 6 \mid Y_n = 3)$.

1

(g) (3 points) Calculate $\Pr(Y_{n+1} = 6 \mid Y_n = 2)$.

0

(work space begins)

Y_n	D_n	End	Y_{n+1}
4	1	3	3
	2	2	6
	3	1	6
3	1	2	6
	2	1	6
	3	0	6
6	1	5	5
	2	4	4
	3	3	3
5	1	4	4
	2	3	3
	3	2	6

(work space ends)

A-5

5. (10 points) It is the beginning of 2015 winter season. A high-end designer store is trying to determine how many new season bags should be ordered. Each bag costs the store \$250. The demand for the new season bag has the probability distribution as follows:

demand	25	30	35	40	45	50	55	60
probability	.10	.20	.30	0.20	0.05	0.05	0.05	0.05

Each bag is sold for \$1,000. If the demand for the season bags falls short, the store can sell out any left-over bags in an end-of-season sale for \$200.

(a) (5 points) If the demand for new season bags exceeds the number ordered at the beginning of the winter season, then the unfulfilled demand will be lost. When the manager orders 45 bags, what is the expected ~~profit for the season~~? # sold

$25(0.1) + 30(0.2) + 35(0.3) + 40(0.2) + 45(0.2)$

(b) (5 points) It turns out that when the demand exceeds the number ordered at the beginning of the season, the store can reorder them at a cost of \$400 per bag for expedite production and shipping because customers are willing to wait. How many should be ordered at the beginning of 2015 winter season? Show work.

$$\frac{400-250}{400-200} = \frac{150}{200} = 0.75$$

$$\therefore q^* = 40$$

Green

ISYE 3232A Fall 2015 Test 1 - B

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Some formula that you may need:

- A continuous uniform random variable X from a to b has pdf $f(x) = \frac{1}{b-a}$ for $a \le x \le b$ and CDF $F(x) = \frac{x-a}{b-a}$.
- A discrete uniform random variable X in $\{x_1, x_2, \ldots, x_n\}$ has pmf $p(x) = 1/n$.
- An exponential r.v. X with rate λ has pdf $f(x) = \lambda e^{-\lambda x}$ for $x > 0$ and CDF $F(x) = 1 - e^{-\lambda x}$ for $x > 0$. Its mean is $1/\lambda$ and variance is $1/\lambda^2$.
- In a newsvendor problem, the expected cost $\mathsf{E}[\text{Cost}]$ with ordering quantity q is

$$\begin{aligned}\mathsf{E}[\text{Profit}] &= c_p\mathsf{E}[D \wedge q] + c_s\mathsf{E}[(q-D)^+] - c_v q - p_u\mathsf{E}[(D-q)^+] \\ &= (c_p - c_v)\mathsf{E}[D \wedge q] - (c_v - c_s)\mathsf{E}[(q-D)^+] - p_u\mathsf{E}[(D-q)^+] \\ &= c_p\mathsf{E}[D] - \left\{(c_p + p_u)\mathsf{E}[(D-q)^+] + h\mathsf{E}[(q-D)^+] + c_v q\right\}.\end{aligned}$$

- For a newsvendor problem, the optimal quantity that maximizes the expected profit (or minimizes the expected cost) is q^* such that $q^* = \min\{q : F(q) \ge \frac{c_p+p_u-c_v}{c_p+p_u-c_s}\}$ where $F(\cdot)$ is the CDF of demand.
- Kingman's formula: $w_q \approx m\frac{\rho}{1-\rho}\frac{c_a^2+c_s^2}{2}$.

Table B.1. Right tail probabilities $1 - \Phi(a) = P(Z \geq a)$ for an $N(0, 1)$ distri random variable Z.

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1.1	1357	1335	1314	1292	1271	1251	1230	1210	1190	1170
1.2	1151	1131	1112	1093	1075	1056	1038	1020	1003	0985
1.3	0968	0951	0934	0918	0901	0885	0869	0853	0838	0823
1.4	0808	0793	0778	0764	0749	0735	0721	0708	0694	0681
1.5	0668	0655	0643	0630	0618	0606	0594	0582	0571	0559
1.6	0548	0537	0526	0516	0505	0495	0485	0475	0465	0455
1.7	0446	0436	0427	0418	0409	0401	0392	0384	0375	0367
1.8	0359	0351	0344	0336	0329	0322	0314	0307	0301	0294
1.9	0287	0281	0274	0268	0262	0256	0250	0244	0239	0233
2.0	0228	0222	0217	0212	0207	0202	0197	0192	0188	0183
2.1	0179	0174	0170	0166	0162	0158	0154	0150	0146	0143
2.2	0139	0136	0132	0129	0125	0122	0119	0116	0113	0110
2.3	0107	0104	0102	0099	0096	0094	0091	0089	0087	0084
2.4	0082	0080	0078	0075	0073	0071	0069	0068	0066	0064
2.5	0062	0060	0059	0057	0055	0054	0052	0051	0049	0048
2.6	0047	0045	0044	0043	0041	0040	0039	0038	0037	0036
2.7	0035	0034	0033	0032	0031	0030	0029	0028	0027	0026
2.8	0026	0025	0024	0023	0023	0022	0021	0021	0020	0019
2.9	0019	0018	0018	0017	0016	0016	0015	0015	0014	0014
3.0	0013	0013	0013	0012	0012	0011	0011	0011	0010	0010
3.1	0010	0009	0009	0009	0008	0008	0008	0008	0007	0007
3.2	0007	0007	0006	0006	0006	0006	0006	0005	0005	0005
3.3	0005	0005	0005	0004	0004	0004	0004	0004	0004	0003
3.4	0003	0003	0003	0003	0003	0003	0003	0003	0003	0002

B-1

1. (35 points) Next month's production at a manufacturing company will use a certain solvent for part of its production process. Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed and the solvent costs \$50 per liter. Due to short product life cycle, unused solvent cannot be used in following months. There will be a \$20 disposal charge for each liter of solvent left over at the end of the month. If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$80 per liter short.

   (a) (5 points) The demand has the following pmf:

Demand	400	500	700	900
Prob	0.1	0.1	0.5	0.3

   What is the optimal ordering quantity assuming there is no initial inventory? Show work.

$$\frac{80-50}{80+20} = \frac{30}{100} = 0.3 \qquad \therefore q^* = 700$$

   (b) (9 points) We have 400 initial inventory and demand follows the pmf in part (a). If we decide to produce more, calculate the expected cost with production. show work.

$$1500 + 50(300) + 80\left[200(0.3)\right] + 20\left[300(0.1) + 200(0.1)\right]$$

   (c) (6 points) Now let's assume that we have 400 initial inventory and demand follows the pmf in part (a). Calculate the expected cost when we do not produce any additional solvent. Show full work.

$$80\left[100(0.1) + 300(0.5) + 500(0.3)\right] + 20(0)$$

or

$$80\left[\{400(0.1) + 500(0.1) + 700(0.5) + 900(0.3)\} - 400\right] + 20(0)$$

(d) (6 points) Now suppose that we want to come up with an exhaustive inventory policy for any initial inventory amount m. It is known that the critical point m^* is between 400 and 500. Give an equation for the expected cost without production assuming that the initial inventory is exactly equal to m^*.

$$80\left[(500-m^*)(0.1)+(700-m^*)(0.5)+(900-m^*)(0.3)\right]$$

$$+20\left[(m^*-400)(0.1)\right]$$

(e) (3 points) Now suppose that the demand is exponentially distributed with mean 600. What is the optimal ordering quantity if there is no initial inventory?

$$1-e^{-\frac{q}{600}}=0.3 \qquad \therefore q^* = -600\ln 0.7$$

(f) (6 points) Suppose that answer for (e) is 700; demand is exponential with mean 600; and we have 400 initial inventory. Calculate the expected cost when we do not produce additional quantity. Leave your answer with integrals. (*If we cannot get a numerical answer when we enter your expression into a mathematical software package, you will be given a 0 point. For example,* $\int_0^1 D\,dx$ *receives a 0 point because it is equal to D which is not a numerical answer.*)

$$80\int_{400}^{\infty}(x-400)\frac{1}{600}e^{-\frac{x}{600}}dx+20\int_{0}^{400}(400-x)\frac{1}{600}e^{-\frac{x}{600}}dx$$

2. (5 points) The delivery time of an order for raw material is normally distributed with mean 15 days and standard deviation 9 days.

(a) (2 points) What is the z value such that $\Pr(Z \leq z) = 0.8$ where Z represents a standard normal random variable?

0.84

(b) (3 points) How many days are needed to ensure with 80% probability that an order will be received by then?

$15+(0.84)9$

B-3

3. (30 points) Consider a factory whose site has the same setup as in the Littlefield game we played. There are three stations, each with 1 machine. Jobs arrive at Station A every exponential time with mean 5 hours and they are processed with constant processing time 4 hours. Jobs are moved to Station B whose processing time is constant 1 hours. Then they are sent to Station C for checking which takes exponential time with mean 3 hours. Finally jobs are sent back to Station B for the final processing and then leave the factory. Currently each station has one machine and one buffer with infinite capacity. *This factory runs 24 hours per day.*

(a) (2 points) What is the job arrival rate *per day* to Station A?

4.8/day

(b) (2 points) What is the service rate *per day* of a machine at Station A?

6/day

(c) (1 point) What is the steady-state utilization of a machine at Station A?

0.8

(d) (2 points) What is the square coefficient of variation of service times of a machine at Station A?

0

(e) (2 points) What is the square coefficient of variation of interarrival times to Station A?

1

(f) (1 point) What is the long-run average waiting time *in hours* in queue at Station A? (*If you are unsure of your answer in (c), then assume that you got 0.75 in (c); enter your initial here ________; and solve this problem.*)

$$4 \, \frac{0.8}{1-0.8} \, \frac{0+1}{2}$$

(g) (3 points) What is the overall job arrival rate *per day* to Station B (including new jobs from Station A and returning jobs from Station C)?

9.6/day

(h) (5 points) Suppose that the long-run average numbers of jobs at Stations A, B and C (including those waiting in buffer and in service) are 15, 20 and 15, respectively. Calculate the long-run average lead time *in days*?

$$\frac{15+20+15}{4.8}$$

(i) (3 points) What is the system throughput *per day*?

4.8/day

B-4

(j) (3 points) If job interarrival times are changed to exponential with mean 2 hours, what is the system throughput?

6/day

(k) (3 points) Suppose that after adding some number of machines to the systems, the manager found that Stations A, B, and C have 50%, 90%, and 85% utilization, respectively. If you want to add one additional machine, where would you add and why?

B because it has the highest utilization

(l) (3 points) Station A has 2 machines. What is the most appropriate queueing notation for this station?

M/G/2

4. (20 points) A store stocks a particular item. The demand for the product each day is 0 item with probability 1/3, 1 item with probability 1/6, and 2 items with probability 1/1. Assume that the daily demands are independent and identically distributed. Each evening if the remaining stock is less than 2 items (< 2), the store orders enough to bring the total stock up to 5 items. These items reach the store before the beginning of the following day. Assume that any demand is lost when the item is out of stock. Let Y_n be the amount in stock at the *beginning* of day n; assume that $Y_0 = 4$. It is known that Y_n is a Markov chain.

0	1	2
1/3	1/6	1/2

(a) (3 points) Give its state space.

{2, 3, 4, 5}

(b) (2 points) Give its initial distribution.

[0 0 1 0]

(c) (3 points) Calculate $\Pr(Y_{n+1} = 6 \mid Y_n = 2)$.

0

(d) (3 points) Calculate $\Pr(Y_{n+1} = 6 \mid Y_n = 3)$.

$\frac{1}{6} + \frac{1}{2}$

(e) (3 points) Calculate $\Pr(Y_{n+1} = 3 \mid Y_n = 4)$.

$\frac{1}{6}$

(f) (3 points) Calculate $\Pr(Y_{n+1} = 3 \mid Y_n = 5)$.

$\frac{1}{2}$

(g) (3 points) Calculate $\Pr(Y_{n+1} = 5 \mid Y_n = 6)$.

$\frac{1}{3}$

(work space begins)

Y_n	D_n	End	Y_{n+1}
4	0	4	4
	1	3	3
	2	2	2
3	0	3	3
	1	2	2
	2	1	5
2	0	2	2
	1	1	5
	2	0	5
5	0	5	5
	1	4	4
	2	3	3

(work space ends)

B-5

5. (10 points) It is the beginning of 2015 winter season. A high-end designer store is trying to determine how many new season bags should be ordered. Each bag costs the store \$300. The demand for the new season bag has the probability distribution as follows:

demand	25	30	35	40	45	50	55	60
probability	.10	.20	.30	0.20	0.05	0.05	0.05	0.05

Each bag is sold for \$800. If the demand for the season bags falls short, the store can sell out any left-over bags in an end-of-season sale for \$100.

(a) (5 points) If the demand for new season bags exceeds the number ordered at the beginning of the winter season, then the unfulfilled demand will be lost. When the manager orders 40 bags, what is the expected ~~profit for the season?~~ # of bags sold?

$\cancel{800}\left(25(0.1)+30(0.2)+35(0.3)+40(0.4) \right)$

(b) (5 points) It turns out that when the demand exceeds the number ordered at the beginning of the season, the store can reorder them at a cost of \$500 per bag for expedite production and shipping because customers are willing to wait. How many should be ordered at the beginning of 2015 winter season? Show work.

$$\frac{500-300}{500-100} = \frac{200}{400} = 0.5$$

$$\therefore q^* = 35$$