

Problem 1:

Apply ratio of index values,  $235000 \times (244.8/187.6) = 235000 \times (1.304904051) = 306652.452$

# ISYE 3025 HW 4

2.

a) Obtain sum of expenses 24,800 and subtract from 55,000 to obtain discretionary income of 30,200.

b) Obtain inflation multiplier of  $(1.032)^{(2005-1980)} = (1.032)^{25} = 2.198$ .

Then apply this multiplier to each expense category.

Category of necessity	Expenses in 2005
Rent and utilities	15,385
Groceries	18,681
Clothing	3,297
Transportation	4,835
Medical	12,308

c) Obtain sum of expenses 54506 in 2005 and subtract from 111,000 to obtain discretionary income of 56,494. Then convert this amount to 1980 dollars, in the amount of 25,705. This is Mary's discretionary income measured in 1980 dollars, so her father John was better off in 1980.

d)

Solution: Obtain ratio of 2005 expenses to 1980 expenses for each category, and then obtain the 25<sup>th</sup> root. Values are in table.

Category of necessity	Expenses in 2005	Expenses in 1980	Multiplier over 27 years	Annual inflation rate
Rent and utilities	24,000	7,000	3.429	5.053%
Groceries	19,000	8,500	2.235	3.269%
Clothing	8,000	1,500	5.333	6.925%
Transportation	13,000	2,200	5.909	7.364%
Medical	13,000	5,600	2.321	3.425%



3. A company projects revenues and expenses as shown in the table (values are in thousands), but the projection is in constant dollars based on time 0. There are no depreciation expenses. The forecasts for inflation rates are as follows: general inflation rate of 3% which applies to revenue, rate of 2% for labor, and rate of 4% for materials. The general rate of 3% also applies to MARR; MARR is 12% for zero inflation.

a.

- Apply inflation factor  $(1+f)^N$  for each item of cash flow, using the relevant inflation rate for each type of item.
- Obtain taxable income (revenue less expenses)
- Income Tax from the Federal Corporate Tax Table, range 4, so apply formula. In this case, apply 0.34 in years 1 – 4 (marginal rate is same as effective rate for this range)
- Inflated MARR =  $(1.12)*(1.03) - 1 = 15.36\%$
- To obtain NPV after tax, discount the cash flow after tax using  $(1.1536)^N$

Values in constant dollars	Values in actual dollars	Tax. calc.	Cash flow	NPV
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Revenue	Labor	Materials	Revenue	Labor	Materials	Income	Tax	after tax	after tax
800	300	120	824	306	125	393	134	259	225
810	320	130	859	333	141	385	131	254	191
830	330	140	907	350	157	400	136	264	172
830	340	150	934	368	175	391	133	258	146
			3%	2%	4%			Total	734

b.

In the US there is no effect of inflation on the depreciation expenses. So, the taxable income would be higher, which leads to higher taxes. Again, this causes the cash flow after tax to be worth less than the one with no inflation in effect.  
[In some countries with high inflation rates, adjustment of depreciation expense is allowed.]

4.

Scenarios	1	2	3	4	5	6	Initial Investment	500000
Price/Unit	100	100	115	115	140	140	Useful life	10
Quantity	5500	7500	5500	7500	5500	7500	Salvage value	40000
Probability	0.1	0.15	0.3	0.2	0.15	0.1	Variable cost/unit	99
Annual Profit	5500	7500	88000	120000	225500	307500	MARR	15%
EUV	-92155.9	-90155.9	-9655.9	22344.1	127844.1	209844.1	Expected EUV	18994.05
EUV * Prob	-9215.6	-13523	-2897	4468.8	19176.6	20984.4		

- Lowest EUV is in Scenario 1 with -92155.9 (from  $5500 \times (100 - 33 - 66) + 40000 \times (A/F, 15\%, 10) - 500000 \times (A/P, 15\%, 10) = \text{EUV}$ )
- Highest EUV is in Scenario 6 with 209844.1 (from  $7500 \times (140 - 33 - 66) + 40000 \times (A/F, 15\%, 10) - 500000 \times (A/P, 15\%, 10) = \text{EUV}$ )
- Expected EUV is 18994.05 (from Sum of (EUV \* Prob) in each scenario)
- The EUV is greater than 0 in Scenarios 4,5,6 with probability 0.45 (0.2 + 0.15 + 0.1)
- Using What If in Excel we are able to determine that at a MARR of 19.74% the Expected EUV is near 0.

5.

- a. At a price of 105 the breakeven sales quantity is  $13334 = (80000 / (105 - 99))$
- b. At a price of 135 the breakeven sales quantity is  $2222 = (80000 / (135 - 99))$
- c. At a sales quantity of 4500 the breakeven price is  $117 = (80000 / 4500 + 99)$
- d. At a sales quantity of 7000 the breakeven price is  $110 = (80000 / 7000 + 99)$

6.

Initial Investment	500000
Useful life	10
Salvage value	40000
Variable cost/unit	99
MARR	15.00%
Expected EUV	5505.971
Tax	22.00%

	Depr	Tax Effect	PV of Tax Effect
1	83636.36	-18400	16000
2	75272.73	-16560	12521.74
3	66909.09	-14720	9678.639
4	58545.45	-12880	7364.182
5	50181.82	-11040	5488.831
6	41818.18	-9200	3977.414
7	33454.55	-7360	2766.897
8	25090.91	-5520	1804.498
9	16727.27	-3680	1046.086
10	8363.636	-1840	454.8199
NPV			61103.1
NAV			12174.92

Scenarios	1	2	3	4	5	6
Price/Unit	100	100	115	115	140	140
Quantity	5500	7500	5500	7500	5500	7500
Probability	0.1	0.15	0.3	0.2	0.15	0.1
Annual Gross Profit	5500	7500	88000	120000	225500	307500
EUV	-81191.0	-79631.0	-16841.0	8119.0	90409.0	154369.0
EUV * Prob	8119.102909	11944.65436	5052.308727	1623.79418	13561.34564	15436.89709

- Lowest EUV is in Scenario 1 with -81191 (from  $5500 \times (100 - 33 - 66) \times (1 - 0.22) + 12174.9 + 40000 \times (A/F, 15\%, 10) - 500000 \times (A/P, 15\%, 10) = \text{EUV}$ )
- Highest EUV is in Scenario 6 with 209844.1 (from  $7500 \times (140 - 33 - 66) \times (1 - 0.22) + 12174.9 + 40000 \times (A/F, 15\%, 10) - 500000 \times (A/P, 15\%, 10) = \text{EUV}$ )
- Expected EUV is 5505.97 (from Sum of (EUV \* Prob) in each scenario)
- The EUV is greater than 0 in Scenarios 4, 5, 6 with probability 0.45 (0.2 + 0.15 + 0.1)
- Using What If in the Excel we are able to determine that at a MARR of 16.44% the Expected EUV is near 0.

Problem 7:

- a) Use the highest revenues and lowest costs; if annual revenue is positive, use the longest project life; otherwise use the shortest life.

$$\text{Max NPV} = (1,100,000 - 440,000 - 100,000) * (P/A, 5\%, 22) = 7,371,281.44$$

- b) Use the lowest revenue and highest costs; if the annual revenue is positive, use the shortest project life, if the annual revenue is negative, use the longest project life.

$$\text{Min NPV} = (600,000 - 660,000 - 500,000) * (P/A, 5\%, 22) = (-560,000) * (13.16) = -7,371,281.44$$

- c) Obtain expected revenue first and then use medium values for other items.

$$\text{Expected revenue} = 600,000 * 0.10 + 700,000 * 0.10 + 1,100,000 * 0.80 = 1,010,000$$

$$\text{Expected NPV} = (1,010,000 - 550,000 - 400,000) * (P/A, 0.05, 10) = 60,000 * (7.72) = 463,304.10$$

- d) Total number of scenarios that could occur:  $3 * 3 * 3 * 3 = 81$

8.

a) When MARR = 0%, Let x = miles/day

Fuel cost/year (A)

$$= (364 \text{ days})(x \text{ miles/day})(\text{gallon}/9 \text{ miles})(\$2.22/\text{gallon})$$

$$= (\$89.787)(x)$$

Fuel cost/year (B)

$$= (364 \text{ days})(x \text{ miles/day})(\text{gallons}/7 \text{ miles})(\$2.22/\text{gallon})$$

$$= (\$115.44)(x)$$

To find the breakeven point, equate the total costs.

$$(66,000 - 7,000)/8 + 89.787x = (22,000 - 5,000)/8 + 115.44x$$

$$x = 204.65 \text{ miles/day}$$

b) When MARR = 15%, Let x = miles/day

$$66,000(A/P, 15\%, 8) - 7,000(A/F, 15\%, 8) + 89.787x = 22,000(A/P, 15\%, 8) - 5,000(A/F, 15\%, 8) + 115.44x$$

$$(A/P, 15\%, 8) = 0.223$$

$$(A/F, 15\%, 8) = 0.07285$$

$$\Rightarrow 66000(0.223) - 7000(0.07285) + 89.787x = 22000(0.223) - 5000(0.07285) + 115.44x$$

$$\Rightarrow 14208.1 + 89.787x = 4541.75 + 115.44x$$

$$\Rightarrow x = 349.56 \text{ miles/day}$$

c) 100 miles/day, MARR = 0%, and let y = price/gallon

Fuel cost/year (A)

$$= (364 \text{ days})(100 \text{ miles/day})(\text{gallon}/9 \text{ miles})(y/\text{gallon})$$

$$= (4044.4)(y)$$

Fuel cost/year (B)

$$= (364 \text{ days})(100 \text{ miles/day})(\text{gallons}/7 \text{ miles})(y/\text{gallon})$$

$$= (5200)(y)$$

To find the breakeven point, equate the total costs.

$$(66,000 - 7,000)/8 + 4044.4y = (22,000 - 5,000)/8 + 5200y$$

$$y = 4.543/\text{gallon}$$

d) When MARR = 15%, Let y = price/gallon

$$66,000(A/P, 15\%, 8) - 7,000(A/F, 15\%, 8) + 4044.4y = 22,000(A/P, 15\%, 8) - 5,000(A/F, 15\%, 8) + 5200y$$

$$(A/P, 15\%, 8) = 0.223$$

$$(A/F, 15\%, 8) = 0.07285$$

$$\Rightarrow 66000(0.223) - 7000(0.07285) + 4044.4y = 22000(0.223) - 5000(0.07285) + 5200y$$

$$\Rightarrow 14208.1 + 4044.4y = 4541.75 + 5200y$$

$$\Rightarrow y = 8.36/\text{gallon}$$



9.

a.

NPV of Cost =  $77,000,000 + 4,000,000(P/A, 10\%, 30) = 77,000,000 + 4,000,000(9.4269) = \$114,707,600$   
Expected annual revenue =  $330[0.2(4400*4 + 100*8) + 0.5(6000*4 + 1500*8) + 0.3(9000*4 + 1800*8)] = 330[0.2(18,400) + 0.5(36,000) + 0.3(50,400)] = 330[36,800] = \$12,144,000$   
NPV of Revenue =  $\$12,144,000(P/A, 10\%, 30) = \$12,144,000(9.4269) = \$114,480,449.29$   
so NPV =  $\$114,480,449.29 - \$114,707,600 = \$-227,151$

b.

Let  $x$  = number of auto crossings, NPV of Cost = NPV of Revenues  
 $\$114,707,600 = (\$4/\text{auto crossing})(x \text{ auto crossings})(330 \text{ days/year})(P/A, 10\%, 30)$ ,  
 $\$114,707,600 = (4)(x)(330)(9.4269) = 12443.508x$ , so  $x = 9218.27$  or 9218 auto crossings.  
In practice, it's easier to do the math for an EUV of zero, which gives the same result for the breakeven value.

$$\begin{aligned}
 10 \text{ (a)} \quad NPV &= 33,000 [P/A, 2.5\%, 16] + 35,000 [P/A, 2.5\%, 35] \\
 &= 33,000 [13.055] + 35,000 [23.145] \\
 &= 430,815 + 810,081 \\
 &= 1,240,896
 \end{aligned}$$

This amount needs to be deposited

(b) Solving the problem using constant dollars

The amount needed for food & clothing is unchanged. This is because the expenses & the market rate are equally affected by inflation. Thus 810,081 is needed

The mortgage payments are constant in actual dollars, so they go down in constant (year 0) dollars. They go down every year by a factor of  $(1/1.03)$  with the first payment being  $33,000/(1.03)$

The amount that needs to be deposited for the mortgage payments is

$$\begin{aligned}
 & (33,000/1.03) [P/A, i, n = \frac{1}{1.03} - 1, i = 2.5\%, N = 16] \\
 &= (33,000/1.03) [10.71975] = 343,448
 \end{aligned}$$

Thus a total of  $343,448 + 810,081$   
 $= 1,153,529$  is required.

(c) As inflation increases the mortgage goes down in terms of year 0 dollars. The other payments remain the same. Since the market rate is fully adjusted for inflation, increased inflation means less money needs to be deposited.