Name (2 points): Dawson

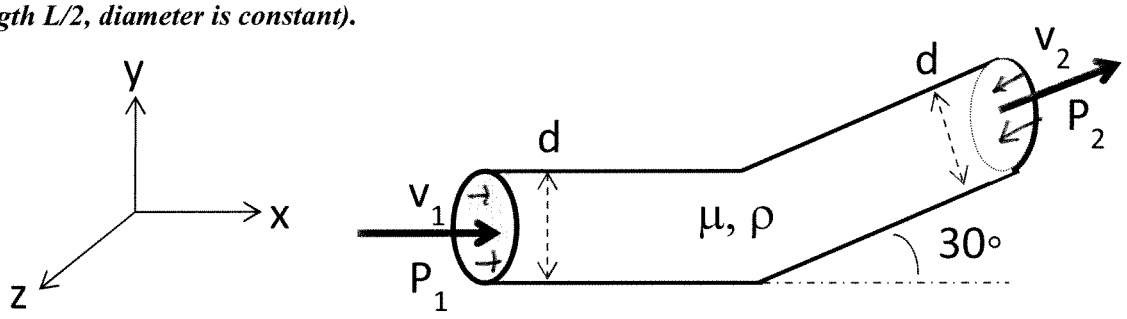
February 5, 2016
ChBE 3200
Quiz 3

Use this information to answer questions below:

$$\iint \rho(\boldsymbol{v}\cdot\boldsymbol{n})dA + \frac{\partial}{\partial t}\iiint \rho\, dV = 0$$

$$\iint v\,\rho(\boldsymbol{v}\cdot\boldsymbol{n})dA + \frac{\partial}{\partial t}\iiint v\,\rho\, dV = \sum F$$

System of interest: Steady flow of incompressible fluid in elbow shown below (total length is L with each segment having length L/2, diameter is constant).



Question 1 (2 points):

How does velocity $\mathbf{v}_1$ compare to velocity $\mathbf{v}_2$?

$\iint \rho(v\cdot n)\,dA = 0$

$A_1 = A_2 = \frac{\pi d^2}{4}$

$-\rho v_1 A_1 + \rho v_2 A_2 = 0$

$\boxed{v_1 = v_2} = v$

Question 2 (6 points):

Use conservation of momentum to find the **FORCE B** required for holding the fluid in place.

$$B_x + P_1 \pi d^2/4 - P_2 \frac{\pi d^2}{4}\cos 30 = -\rho v_1^2 \frac{\pi d^2}{4} + \rho v_2^2 \frac{\pi d^2}{4}\cos 30$$

$$B_x = -(P_1 - P_2\cos 30)\pi d^2/4 + \rho v^2 \pi d^2/4\,(\cos 30 - 1)$$

$$B_y - \rho g \frac{\pi d^2}{4} L - P_2 \frac{\pi d^2}{4}\sin 30 = \rho v_2^2 \frac{\pi d^2}{4}\sin 30$$

$$B_y = \frac{\pi d^2}{4}\left(\rho g L + \rho v^2 \sin 30 + P_2 \sin 30\right) \qquad B = B_x \vec{e}_x + B_y \vec{e}_y$$

Question 3 (2 points extra credit):

Use Bernoulli's equation to solve for the pressure difference **(P_1 - P_2).** Simplify as much as possible!

$$e_1 + \frac{P_1}{\rho} = e_2 + \frac{P_2}{\rho}$$

$$\frac{v_1^2}{2} + g y_1 + u_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + g y_2 + u_2 + \frac{P_2}{\rho}$$

$$P_1 - P_2 = \left[g(y_1 - y_2) + (u_2 - u_1)\right]\rho \Rightarrow \boxed{P_1 - P_2 = \rho g (y_2 - y_1) + \rho(\underbrace{u_2 - u_1}_{h_L})}$$

Now what if water fluid

$\rho = 1000\ kg/m^3$

$\mu = 1\ cP = 0.01\ \frac{gr}{cm\,s}\left(\frac{100\ cm}{1000\ gm}\right)$ ← $0.001\ \frac{kg}{m\,s}$ ← Table $\mu_f = 1.000 \times 10^{-6}\ Pa\,s$

$\dot{m} = 1\ kg/s$

$d = 10\ cm$

$L = 100\ cm$

$y_2 - y_1 = \frac{L}{2}\sin 30 = 25\ cm$ ($\sin 30 = 1/2$)

Q1

$$\dot{m} = \rho v A = 1\frac{kg}{s} = 1000\frac{kg}{m^3}\left(\frac{\pi (0.1m)^2}{4}\right) v$$

$$v = 0.127\ m/s$$

$v_1 = v_2 = v$

$$\frac{\pi d^2}{4} = 0.0079\ m^2$$

Q3

$$\frac{v_1^2}{2} + g y_1 + u_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + g y_2 + u_2 + \frac{P_2}{\rho}$$

$$P_1 - P_2 = \rho g (y_2 - y_1) + \rho g h_L$$

$$P_1 - P_2 = 1000\frac{kg}{m^3}\left(10\frac{m}{s^2}\right)(0.25m) + \rho g h_L$$

$$P_1 - P_2 = 2500\ \frac{N}{m^2} + \underbrace{\rho g h_L}_{\text{head loss in elbow.}}$$

Q2

$$B_x + (P_1 - P_2\cos 30)\pi d^2/4 = \dot{m}(-v_1 + v_2\cos 30)$$

$$B_x = 1\frac{kg}{s}\left(0.127\frac{m}{s}\right)(-0.134) - (P_1 - P_2\cos 30)(0.0079)$$

$$B_x = -0.017 - 0.0079(P_1 - P_2\cos 30)$$

$$B_y - \rho g \frac{\pi d^2}{4} L - P_2 \frac{\pi d^2}{4} \sin 30 = \underbrace{\rho V_2^2 \frac{\pi d^2}{4} \sin 30}_{\dot{m} V_2 \sin 30}$$

$$B_y = \underbrace{\rho g \frac{\pi d^2}{4} L}_{m} + P_2 \frac{\pi d^2}{4} \sin 30 + \underbrace{\dot{m} V_2 \sin 30}_{0.0635}$$

$$B_y = \underbrace{79}_{+W} + 0.0635 + 0.00395\, P_2$$

$$\boxed{B_y \sim W = 79 \text{ N}}$$

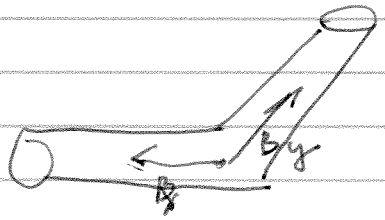
$P_2 \ll P_1$, P_2 is small

$$B_x = -0.017 - 0.0079 \underbrace{(P_1 - P_2 \cos 30)}$$

estimate by assuming $P_1 - P_2$ w/o headloss

$$B_x \approx -0.017 - 0.0079 (2500)$$

$$\boxed{B_x \sim -20 \text{ N}}$$



$$\boxed{\vec{B} = -20\, \vec{e}_x + 79\, \vec{e}_y}$$