ISyE 2027 Exam # 2
Fall 2014

Name KEY

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Question 4
Total

(25) **1.** Suppose X has a density function $f(x) = x/2$ for $0 < x < 2$ and 0 otherwise.

(a) (15) Compute $E[2X - X^2]$.

$$E[2X - X^2] = 2E[X] - E[X^2] = \frac{8}{3} - 2 = \frac{2}{3}$$

$$E[X] = \int_0^2 x \cdot \frac{x}{2}\,dx = \frac{x^3}{6}\Big|_0^2 = \frac{8}{6} = \frac{4}{3}$$

$$E[X^2] = \int_0^2 x^2 \cdot \frac{x}{2}\,dx = \frac{x^4}{8}\Big|_0^2 = \frac{16}{8} = 2$$

(b) (10) Compute $Var(2X - 5)$.

$$Var(2X-5) = 4\,Var(X) = 4 \cdot \frac{2}{9} = \frac{8}{9}$$

$$Var(X) = 2 - \frac{16}{9} = \frac{2}{9}$$

(25) **2.** (a) (15) Suppose X has the probability density function

$$f(x) = c(3-|x|) \quad -3 < x < 3$$
$$f(x) = 0 \text{ otherwise}$$

Compute c.

$$f(x) = \begin{cases} c(3+x) & \text{if } -3<x<0 \\ c(3-x) & \text{if } 0 \le x < 3 \\ 0 & \text{otherwise} \end{cases}$$

We need $\int_{-3}^{3} f(x)dx = 1 \Rightarrow c\int_{-3}^{0}(3+x)dx + c\int_{0}^{3}(3-x)dx = 1$

$$\Rightarrow c\left(3x+\frac{x^2}{2}\Big|_{-3}^{0} + \left(3x-\frac{x^2}{2}\right)\Big|_{0}^{3}\right) = 1 \Rightarrow c\left(9-\frac{9}{2}+9-\frac{9}{2}\right) = 1 \Rightarrow c = \frac{1}{9}$$

(b) (10)Compute $E[X]$

$$E[X] = \frac{1}{9}\int_{-3}^{0}(3x+x^2)\,dx + \frac{1}{9}\int_{0}^{3}(3x-x^2)dx$$

$$= \frac{1}{9}\left[\frac{3x^2}{2}+\frac{x^3}{3}\Big|_{-3}^{0} + \left(\frac{3x^2}{2}-\frac{x^3}{3}\right)\Big|_{0}^{3}\right]$$

$$= \frac{1}{9}\left(-\frac{27}{2}+\frac{27}{3}+\frac{27}{2}-\frac{27}{3}\right) = 0$$

(25) **3.** Suppose two dice are rolled and let X be the larger of the two numbers that appear. Compute the mean and the variance of X.

$P(X=1) = \frac{1}{36}$ $\quad P(X=2) = \frac{3}{36}$ $\quad P(X=3) = \frac{5}{36}$ $\quad P(X=4) = \frac{7}{36}$

$P(X=5) = \frac{9}{36}$ $\quad P(X=6) = \frac{11}{36}$

$$E[X] = \frac{1}{36} \times (1\times1 + 2\times3 + 3\times5 + 4\times7 + 5\times9 + 6\times11) = \frac{161}{36}$$

$$E[X^2] = \frac{1}{36} + (1^2\times1 + 2^2\times3 + 3^2\times5 + 4^2\times7 + 5^2\times9 + 6^2\times11) = \frac{791}{36}$$

$$Var(X) = \frac{791}{36} - \left(\frac{161}{36}\right)^2 = \frac{28476 - 25921}{1296} = \frac{2555}{1296}$$

(25) **4.** (10) (a) Can we have a random variable with $E[X] = 3$ and $E[X^2] = 8$? Justify your answer.

No. If X were a random variable, it would have a variance of -1. So, X cannot be a random variable.

(b) (15) Suppose X has a density function $x^{-1/2}/2$ for $0 < x < 1$, 0 otherwise. Compute its median.

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^{1/2} & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

We $F(q_{0.5}) = \frac{1}{2} \Rightarrow (q_{0.5})^{1/2} = \frac{1}{2}$

$$q_{0.5} = \frac{1}{4}$$