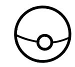
PHYS 2211 Test 4
Spring 2015

Name(print) ~~~ Test Key ~~~ **Lab Section** 

Greco (K or M), Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Tuesday	M01 N01	M02 N02	M03 N03
Tuesday	K03 K05	K04 K07	K06 K08
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Samuel Oak

Sign your name on the line above

Period 1, April 27th (Mon) at 8:00am - 10:50am

Every semester, someone receives a zero on the final because they missed the exam. Please don't let this happen to you!

Stressing over a conflict?

Did you complete "PHYS 2211 Final Exam Schedule" on WebAssign? If not, request and extension and complete it by April 14th.

Are you an ADAPTS Student?

Don't forget to schedule your final with the ADAPTS office. Don't delay, spaces are limited.

Please do not write on this page

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

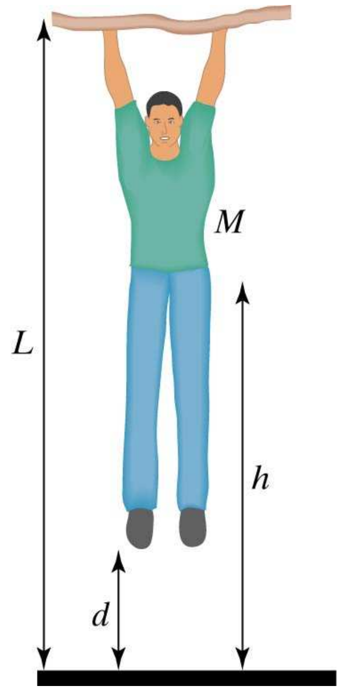
Problem 1 (25 Points)

Below is an incomplete code, very much like the one from your lab, to update the position of a ball hanging from a spring under the influence of gravity. The ball moves through a very thick fluid which exerts a drag on the ball $\vec{F}_{drag} = -b * \vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the ball.

```
from visual import *
## constants and data
g = 9.81 ## acceleration due to gravity m/s^2
mball = 0.2099 ## mass in kg of the ball used in lab
L0 = 0.3       ## the relaxed length (m) of the spring
k = 12         ## the spring constant (N/m^3)
b = 2          ## the drag coefficient  (N/(m/s))
deltat = 1e-3  ## the time step (s)
t = 0          ## start counting time at zero
ceiling = box(pos=(0,0,0), size = (0.2, 0.01, 0.2)) ## origin is at ceiling
ball = sphere(pos=(-0.1284, -0.1434, -0.1905), radius=0.025, color=color.orange)
spring = helix(pos=ceiling.pos, color=color.cyan, thickness=.003, coils=40, radius=0.015)
spring.axis = ball.pos - ceiling.pos
ball.v = vector(-0.17, -0.371, 0.258)
## calculation loop    W_fluid = 0
while t < 6.03:
## (a 15pts) Calculate the net force on the ball
    L = ball.pos - ceiling.pos
    S = mag(L) - L0                                      5pts
    Fspring = - K * S * (L/mag(L))
    Fgrav = vector(0, -mball * g, 0)   → 2pts
    Fdrag = - b * ball.v   → 3 pts
    Fnet = Fspring + Fgrav + Fdrag   → 5pts
## (b 5pts) Update the position of the ball
    ball.v = ball.v + (Fnet/mball) * deltat   → 2pts
    ball.pos = ball.pos + ball.v * deltat   → 2 pts
    spring.axis = ball.pos - ceiling.pos   → 1 pt
## (c 5pts) Calculate the total energy for the spring+ball system (Earth in surroundings)
    E = (0.5 * mball * mag(ball.v)**2) + (0.5 * K * S**2)   (2pts, 3pts)
## (Extra Credit 5pts) Calculate the total work done by the fluid on the spring+ball system
    W_fluid = W_fluid + dot(Fdrag, ball.pos)   → ok if W_fluid
                                                  is not initialized
    t = t + deltat ## update time
                                               → ok if W_field = ΔE - Wg
```

Problem 2 (25 Points)

You hang by your hands from a tree limb that is a height L above the ground, with your center of mass a height h above the ground and your feet a height d above the ground, as shown in the figure. You then let yourself fall. You absorb the shock by bending your knees, ending up momentarily at rest in a crouched position with your center of mass a height b above the ground. Your mass is M.



(a 5pts) Starting from the energy principle, find your speed just before your feet touch the ground.

System: point particle
Initial: before falling
Final: at moment of impact with ground

$$\left.\begin{aligned}\Delta E = \Delta K &= W_{grav} \\ \frac{1}{2}m(v_f^2 - \cancel{v_i^2}_0) &= \vec{F}_g \cdot \Delta\vec{r}_{cm}\end{aligned}\right\} \text{3pts}$$

$\frac{1}{2}\cancel{m} v_f^2 = \cancel{m} g d$ ← positive b/c force and displacement are parallel

$\frac{1}{2}v_f^2 = gd$

$$\Rightarrow \boxed{v_f = \sqrt{2gd}} \rightarrow \text{2pts}$$

(b 10pts) Starting from the energy principle (point particle model) and assuming that the contact force of the ground on your feet is constant, find the magnitude of the contact force during your landing.

Initial: moment of impact

Final: crouching

$$\Delta E = \Delta K = \vec{F}_{net} \cdot \Delta \vec{r}$$

$$\frac{1}{2}m(\cancel{v_f^2} - \underbrace{v_i^2}_{\text{part A}}) = (F - F_g)\Delta y$$

$$-\frac{1}{\cancel{2}}m(\cancel{2}gd) = (F - mg)(b - (h-d))$$

$$\frac{-mgd}{b-h+d} = F - mg$$

$$F = mg - \frac{mgd}{b-h+d}$$

$$\boxed{F = mg\left(1 - \frac{d}{b-h+d}\right)}$$

−0.5
−1.5
−3.0
−8.0

(c 5pts) What is the (real) work done by the contact force?

$$W_c = \vec{F} \cdot \cancel{\Delta \vec{r}}^{\,0} \Rightarrow \boxed{W_c = 0}$$

(the ground doesn't move)

All

(d 5pts) Starting from the energy principle (real model), find the change in your internal energy during landing.

Initial: before falling

Final: crouching

from (c) but check for POE

$$\Delta E = \Delta K + \Delta E_{int} = W_{grav} + \cancel{W_c}^{\,0}$$

$$\frac{1}{2}m(\cancel{v_f^2}^{\,0} - \cancel{v_i^2}^{\,0}) + \Delta E_{int} = \vec{F}_g \cdot \Delta \vec{r}_{cm}$$

3pts

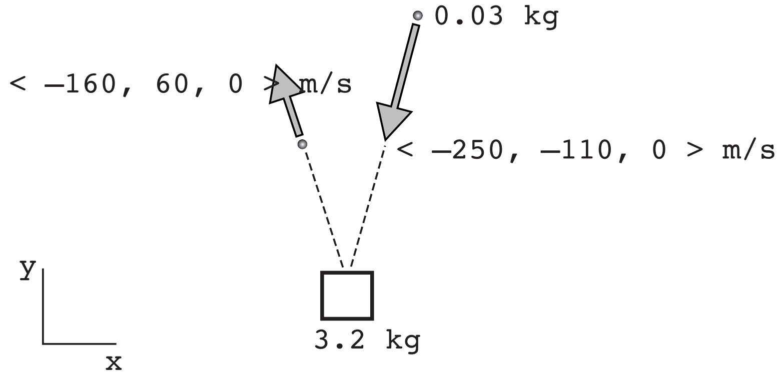
$$\Delta E_{int} = (-mg)(y_f - y_i)$$

$$\boxed{\Delta E_{int} = -mg(b-h) = mg(h-b)} \rightarrow 2 \text{ pts}$$

(positive b/c h>b)

Problem 3 (25 Points)

A block of steel with mass 3.2 kg sits on a low-friction surface. A bullet of mass 0.03 kg traveling horizontally bounces off the block as shown in the diagram, looking down from above. We take the plane of the motion to be the xy plane (with z pointing outward, toward the viewer).



(a 10pts) What is the vector velocity of the block of steel just after the collision? Start from a fundamental principle, and show all your work.

✓ m, b ⇒ bullet

✓ M, B ⇒ block

−0.5
−1.5
−3.0
−8.0

$$(\vec{p}_b + \vec{p}_B)_i = (\vec{p}_b + \vec{p}_B)_f$$

$$m\vec{v}_{bi} + \cancel{M\vec{v}_{Bi}}^{\,0} = m\vec{v}_{bf} + M\vec{v}_{Bf}$$

$$m\vec{v}_{bi} - m\vec{v}_{bf} = M\vec{v}_{Bf}$$

$$\vec{v}_{Bf} = \frac{m\vec{v}_{bi} - m\vec{v}_{bf}}{M} = \frac{m}{M}(\vec{v}_{bi} - \vec{v}_{bf})$$

$$\vec{v}_{Bf} = \left(\frac{0.03}{3.2}\right)\left[\langle -250, -110, 0\rangle - \langle -160, 60, 0\rangle\right] =$$

$$= (9.375e-3)\langle -90, -170, 0\rangle$$

$$\boxed{\vec{v}_{Bf} = \langle -0.84375, -1.59375, 0\rangle \text{ m/s}}$$

(b 5pts) What was the transfer of energy Q (microscopic work) from the surroundings into the block+bullet system during the collision? (Remember that Q represents energy transfer due to a temperature difference between a system and its surroundings.)

$\boxed{Q = 0}$ b/c both system and surroundings are at the same temperature and $\Delta t_{coll} \ll 1$ so surroundings can be neglected

All

(c 5pts) What is the increase $\Delta E_{\text{thermal}}$ in the thermal energy of the block and bullet? Start from a fundamental principle, and show all your work.

$$\Delta E = \Delta E_{th} + \Delta K_{total} = 0$$

$$\Delta E_{th} = -\Delta K_{total} = -(\Delta K_b + \Delta K_B) =$$

3 pts

$$= (K_{bf} - K_{bi}) + (K_{Bf} - \cancel{K_{Bi}}^{\,0}) = \frac{1}{2}m(v_{bf}^2 - v_{bi}^2) + \frac{1}{2}M v_{Bf}^2$$

$$\checkmark\ v_{bf}^2 = (-160)^2 + (60)^2 + 0^2 = 29200$$

$$\checkmark\ v_{bi}^2 = (-250)^2 + (-110)^2 + 0^2 = 74600$$

$$\checkmark\ v_{Bf}^2 = (0.84375)^2 + (1.59375)^2 + 0^2 = 3.252$$

$$\Rightarrow \Delta E_{th} = -\left[\frac{1}{2}(0.03)(29200 - 74600) + \frac{1}{2}(3.2)(3.252)\right]$$

$$\boxed{\Delta E_{th} = 675.7968\ \text{J}} \longrightarrow 2\ \text{pts}$$

(d 5pts) Was this collision elastic or inelastic? Briefly explain how you know this.

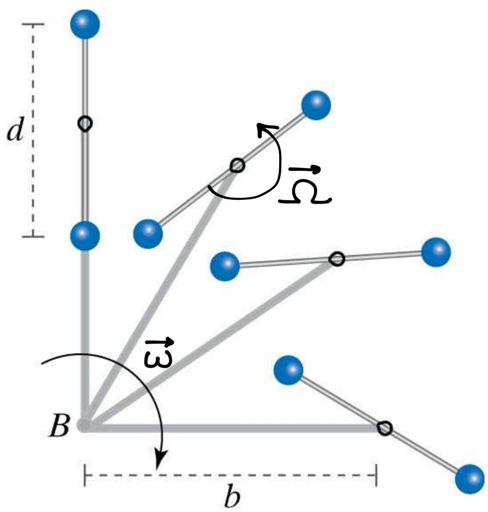
Inelastic, b/c the Kinetic energy changed.

All

* Note this only needs to match answer from (c).

Problem 4 (25 Points)

A barbell consists of two small balls, each with mass m, at the ends of a very low mass rod of length d. The center of mass for the barbell is mounted on the end of a low mass rigid rod of length b. As shown in the diagram, the rod rotates clockwise with angular speed ω. In addition, the barbell rotates counterclockwise about its own center, with an unknown angular speed.



(a 10pts) Determine the translational angular momentum $\vec{L}_{trans,B}$ (magnitude and direction) for the barbell.

Method #1

$$I = Mr^2 = 2mb^2$$

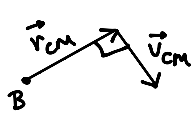
$$\vec{L}_{trans,B} = I\vec{\omega} = 2mb^2\omega \quad \text{(direction: into the page)}$$

$$\Rightarrow \boxed{\vec{L}_{trans,B} = -2mb^2\omega\,\hat{z}}$$

−0.5
−1.5
−3.0
−8.0
(either method)

Method #2

$$\vec{L}_{trans,B} = \vec{r}_{cm} \times M_{sys}\vec{V}_{cm}$$
$$= r_{cm} M_{sys} V_{cm} \, (-\hat{z})$$



always 90° and by right-hand-rule, $\vec{r} \times \vec{v}$ points into the page $(-\hat{z})$

✓ $M_{sys} = 2m$
✓ $V_{cm} = \omega r_{cm}$

$$\Rightarrow \vec{L}_{trans,B} = (r_{cm})(2m)(\omega r_{cm})(-\hat{z}) = 2m\omega (r_{cm})^2 (-\hat{z})$$

$$\Rightarrow \boxed{\vec{L}_{trans,B} = \langle 0, 0, -2mb^2\omega \rangle}$$

(b 5pts) If the total angular momentum for this system (about the point B) is zero, calculate the rotational angular momentum for the barbell about its center of mass. $\vec{L}_{rot,cm}$ (magnitude and direction)

$$\left.\begin{aligned}\vec{L}_{total,B} &= \vec{L}_{trans,B} + \vec{L}_{rot,CM} = 0 \\ \vec{L}_{rot,cm} &= -\vec{L}_{trans,B}\end{aligned}\right\} \text{3pts}$$

$$\vec{L}_{rot,cm} = -(-2mb^2\omega\,\hat{z})$$

$$\boxed{\vec{L}_{rot,CM} = 2mb^2\omega\,\hat{z}} \longrightarrow \text{2pts}$$

(c 5pts) Calculate the moment of inertia I for the barbell about its center of mass.

$$I_{barbell} = (2m)(d/2)^2 = (\cancel{2}m)\left(\tfrac{1}{\cancel{4}}d^2\right)$$

$$\boxed{I_{barbell} = \tfrac{1}{2}md^2} \qquad \text{All}$$

(d 5pts) Determine the unknown angular speed of the barbell about its center of mass.

$$\left.\begin{aligned}&\checkmark\ \vec{L}_{rot,cM} = I_{barbell}\vec{\Omega} = \tfrac{1}{2}md^2\vec{\Omega} = \tfrac{1}{2}md^2\Omega\,\hat{z}\ \text{(b/c counterclockwise)} \\ &\checkmark\ \vec{L}_{rot,cm} = 2mb^2\omega\,\hat{z}\ \text{(from Part b)} \longrightarrow \text{Check for PDE}\end{aligned}\right\} \text{3pts}$$

$$\Rightarrow \tfrac{1}{2}\cancel{m}d^2\Omega\,\hat{z} = 2\cancel{m}b^2\omega\,\hat{z}$$

$$\boxed{\Omega = \frac{4b^2\omega}{d^2}} \longrightarrow \text{2pts}$$

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s \qquad U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M \qquad \Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots} \qquad I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel} \qquad K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I} \qquad K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot} \qquad \vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}} \qquad v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E} \qquad \Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

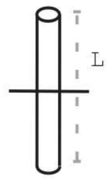
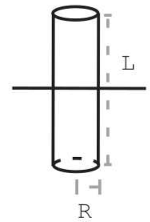
$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}} \qquad E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}