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ISyE 3232
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Homework 13

1. Consider a call center that is staffed by K agents with three phone lines. Call arrivals follow a Poisson process with rate 1 per minute. An arrival call that finds all lines busy is lost. Call processing times are exponentially distributed with mean 2 minutes.

   (a) Find the throughput and average waiting time when K = 1.

   (b) Find the throughput and average waiting time when K = 2.

   (c) Find the throughput and average waiting time when K = 3.

2. Consider a service system with a single server whose service time is exponentially distributed with rate μ and infinite capacity. The arrivals come to the system following a Poisson process of rate λ but an arrival finding i people in the system will enter the system with probability $1/(i+1)$. For example at the time of your arrival if there are 2 people in the system you will enter with probability 1/3. What is the expected value of the number of customers in the system in the long-run?

3. Calls arrive to DFK Ambulance Services according to a Poisson process with rate $\lambda = 5$ per hour. DFK Ambulance Services operates 2 ambulances, and data suggests that the amount of time that an ambulance devotes to each call is exponentially distributed with a mean of 15 minutes. The service time begins when the ambulance is dispatched to the call and ends when the ambulance is available for dispatch to another call. DFK has at most 4 calls waiting or in service at any point in time; if there are already 4 calls in the system, additional calls are rerouted to another ambulance service. We will let $X(t)$ denote the number of calls in the system at time t. Thus, the state space is $S = \{0, 1, 2, 3, 4\}$. Then $X = \{X(t) : t \geq 0\}$ is a continuous time Markov chain.

   (a) Find the generator of the CTMC.

   (b) Find the stationary distribution of the Markov chain.

   (c) What is the steady state probability that an arriving call will be accepted by DFK?

   (d) What is the steady state probability that an ambulance is immediately dispatched to an arriving call; i.e., no waiting?

   (e) What is the average number of calls waiting for service in the queue in steady state?

   (f) Of those calls that are accepted, what is the steady state average length of time the call waits in the queue until an ambulance is dispatched to that call?

   (g) Suppose Caller X enters the system, and there are 3 calls already in the system. What is the probability that an ambulance will be dispatched to Caller X within 20 minutes?