ISyE 2027C Probability with Applications

HINTS for Word Problems for class January 22 and homework due Thursday January 29, 2015

be ready for a quiz on THURSDAY January 29

Problems. Define the pertinent events. State the given information and the desired values in terms of probabilities involving those events.

1. You have two cards, one red on both sides, the second blue on one side and red on the other. While your eyes are closed, your friend picks one card at random and places it on a table without taking color into account. You open your eyes and see red. What is the probability that the other side of the card is red?

   2/3

2. A game contestant chooses between a yellow urn and a purple urn. The yellow urn contains 10 white balls and 10 black balls. The purple urn contains 6 white balls and 6 black balls. The contestant draws two balls without replacement from the chosen urn without looking inside the urn. If the contestant draws two white balls, what the probability that the contestant chose the purple urn?

   You must assume that the contestant chooses randomly between the yellow and purple urns. 95/194

3. 60% of 70 year olds reach the age of 80. 35% of 70 year olds diagnosed with cancer reach 80. 68% of 70 yeare olds not so diagnosed reach 80. John is 70. What is the probability that he is diagnosed with cancer?

   8/33

4. Suppose the gender of a child is female w.p. (with probability) .5 and male w.p. .5, independent of the genders of other children in the family. The Spohr family has 3 children. At least one of the children is a boy. What is the probability that the other two are girls?

   3/7

5. Suppose the gender of a child is female w.p. (with probability) .5 and male w.p. .5, independent of the genders of other children in the family. The Hummel family has 3 children. At least one of the children is a boy. What is the probability that the other two are girls?

   1/4

6. Suppose that 50% of couples are physiologically predisposed such that each of their children is male w.p. .6 and female w.p. .4 independent of each other. Suppose that the other couples are predisposed the other way, so that each of their children is female w.p. .6 independent of each other. The Czerny family has 3 children. At least one of the children is a boy. What is the probability that the other two are girls?

   18/43

   Suppose that 50% of couples are physiologically predisposed such that each of their children is male w.p. .6 and female w.p. .4 independent of each other. Suppose that the other couples are predisposed the other way, so that each of their children is female w.p. .6 independent of each other. The Albinoni family has 3 children. The youngest is a boy. What is the probability that the other two are girls?

   .24

7. With your eyes closed, you draw 3 M&Ms without replacement from a bowl containing 10 orange, 15 green, 20 blue, and 25 yellow candies. You win \$20 if you get three oranges, and \$10 if you get one each of orange, green, and blue. What is the probability that you win money? If the first candy you draw is orange, what is the probablity that you win money?

   $18,720/(70 \cdot 69 \cdot 68)$; $672/(69 \cdot 68)$

8. You flip two coins and roll a 6-sided die. You win if the two coins come up the same, or if the die shows a 1. Given that you win, what is the probability that you rolled a 1? Given that you win, what is the probability that the first coin came up heads?

   2/7; 1/2

9. Todd is playing bridge, a 4-person game played with a standard 52-card deck. At the start of the game, each player is dealt 13 cards. If Todd's first card is an ace, what is the probability that he has two or more aces? If Todd's first card is the ace of spades, what is the probability that he has two or more aces? If Todd has an ace, what is the probability that he has at least two aces? If Todd has the ace of spades, what is the probability that he has at least two aces?

   $1 - \frac{39 \cdot 38 \cdot 37}{51 \cdot 50 \cdot 49}$; same; $(6\binom{48}{11} + 4\binom{48}{10} + \binom{48}{9})/\binom{52}{13}(1 - \binom{40}{4}/\binom{52}{4})$; $((\binom{48}{9} + .75\binom{4}{3}\binom{48}{10} + .5\binom{4}{2}\binom{48}{11})/\binom{52}{13}(\frac{1}{4})$.

10. Todd is playing bridge again. His first two cards are hearts. What is the probability that he is void in diamonds (he has no cards in the diamond suit)?

    $\binom{37}{11}/\binom{50}{11}$.

Problems. Define the pertinent variables. State the given information in terms of your variables.

1. You are drunk, standing on the 2nd rung of a 3-rung ladder. No matter which rung you stand on, you step up w.p. 1/3 and down w.p. 2/3. If you try to step up from the 3rd (top) rung, you will fall and break your arm. If you try to step down from the 1st rung, you will stumble away safely. What is the probability that you break your arm?

   Hint: Let x_i be the probability that you will break your arm if you start on the ith rung, $i = 1, 2, 3$.

2. You buy one unit of a mutual fund that tracks the S&P, at a price of 1980. Each day the price goes up by 10, stays the same, or goes down by 10, each with probability 1/3 independent of what happens on other days. You give orders to sell if it reaches 2000 and to sell if it reaches 1950. What is the chance you will make a profit?

   Hint: There are two ways to do this. The straightforward way is to define x_1 = chance the price reaches 2000 before it reaches 1950 if the price starts at 1990; x_2 = chance the price reaches 2000 before it reaches 1950 if the price starts at 1980; and so on to x_4 for starting at 1960. The sneaky way is to see that on average you must break even.

Problems to solve completely.

1. You flip a fair coin until it comes up heads. What is the probability that you flip the coin an odd number of times?

   4/7

2. You roll a 6-sided die repeatedly until you get the same number twice in a row. What is the probability that you will roll exactly twice? Exactly three times? Exactly 4 times?

   1/6; 5/36; 25/216.

3. You roll a 4-sided die repeatedly until you have seen each number at least once. What is the probability that you will roll exactly 4 times? Exactly 5 times?

   3/32; 9/64

4. You roll a 4-sided die repeatedly until you have seen the number 4 four times. What is the probablility that you will roll exactly 4 times? Exactly 5 times?

   1/256; 9/1024.