MATH 1552 - SPRING 2016
QUIZ 6 - SHOW YOUR WORK

NAME: ________________________________ TA: ____________________________

1. (5 points) Does $\sum_{n=9}^{\infty} (-1)^n \dfrac{10^n}{n!}$ converge ? **DO NOT USE THE RATIO NOR THE ROOT TESTS**.

Use the AST. $a_n = \dfrac{10^n}{n!}$

a. $a_n = \dfrac{10^n}{n!} > 0$ b. $\lim_{n\to\infty} \dfrac{10^n}{n!} = 0$ (one of the common or important limits)

c. Show $a_{n+1} \le a_n$ $\dfrac{10^{n+1}}{(n+1)!} \le \dfrac{10^n}{n!}$ **iff** $\dfrac{10}{n+1} \le 1$ **iff** $n \ge 9$

By the AST, $\boldsymbol{\sum_{n=9}^{\infty} (-1)^n \dfrac{10^n}{n!}}$ **converges**

2. (8 points) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+\sqrt{n}}$ converge absolutely ? **Neither the Ratio nor the Root tests work.**

Does $\sum_{n=0}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ converge ?

Use the **Limit Comparison Test** with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ which diverges because it is a $p-series, p = \frac{1}{2} < 1$. So $a_n = \frac{1}{1+\sqrt{n}}$ & $b_n = \frac{1}{\sqrt{n}}$

$$\Rightarrow \lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n}}{1+\sqrt{n+1}} = 1 > 0$$

By the LCT since $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges, $\sum_{n=0}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{1+\sqrt{n}}$ diverges

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{1+\sqrt{n}}$ **does NOT converge absolutely**

3. (12 points) Consider the Power Series: $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$.

a. Find the radius of convergence. b. Find the interval of absolute convergence (written as $a < x < b$)

c. Determine if the PS converges or diverges at either of the endpoints. For part c, just state which test you are using.

** Use the ratio test: $\left|\frac{a_{n+1}}{a_n}\right| = \frac{|x-2|^{n+1}}{n+1} \quad \frac{n}{|x-2|^n} = |x-2| \quad \frac{n}{n+1} \to |x-2| < 1$

because $\frac{n}{n+1} \to 1$

** Use the root test: $\left[\frac{|x-2|^n}{n}\right]^{\frac{1}{n}} = \frac{|x-2|}{n^{\frac{1}{n}}} \to |x-2| < 1$ because $n^{\frac{1}{n}} \to 1$ (important limits)

In both cases you get $|x-2| < 1 \Rightarrow$

a. $R=1$ is the radius of convergence

b. $|x-2| < 1 \Rightarrow 1 < x < 3$ is the interval of absolute convergence

c. $x=1 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ which converges by the AST

$x=3 \Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$ which diverges b/c it is a $p-series, p=1$

4. (5 points) The series $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n$ is a geometric series (accept this as true). Find the sum of this series in terms of x and simplify your answer.

$$\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n = \sum_{n=0}^{\infty} \left[\frac{(3-x)}{2}\right]^n \text{ is a GS with } a = 1 \ \ \& \ \ r = \frac{(3-x)}{2}$$

$$\Rightarrow \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n (x-3)^n = \frac{1}{1-\frac{(3-x)}{2}} = \frac{2}{x-1}$$