MATH 2551, FAll 2015
Midterm I, practice

Name: _____ Section: _____

Guideline: Please read the following carefully.
Print your name first. Remember to show all your work; including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. Correct answers without major steps will only receive minor portion of the credits of the whole problem. You may use one side of a letter sized sheet paper for formulas. No calculator is allowed in this exam. You have 50 minutes.

Problem	*Actual points*
1	
2	
3	
4	
Total	

Problem 1(30 points). Calculations.

(a)(5 pt) $\dfrac{d}{dt}[(e^t\mathbf{i} + \sqrt{t}\mathbf{j}) \bullet (e^{-t}\mathbf{i} - 3\sqrt{t}\mathbf{j})]$.

(b) (5 pt) $\dfrac{d}{dt}[(t^2\mathbf{i} + \mathbf{j}) \times (t^2\mathbf{i} - 3t\mathbf{j})]$.

(c)(10 pt) Let $h(r, \theta, t) = r^2 e^{2t} sin(\theta - t)$, calculate h_r, h_t and h_{rt}.

(d)(10 pt) Set $f(x, y) = \frac{x-y^3}{x^3-y^3}$. Determine whether or not f has a limit at $(1, 1)$.

Problem 2(30 pt) An object moves so that

$$\mathbf{r}(t) = 4\mathbf{i} + (1 + 3t)\mathbf{j} + (9 - t^2)\mathbf{k},\ t \geq 0.$$

(a)(6 pt) Compute the velocity, the acceleration and the speed of the ball at an arbitrary time t.

(b) (6 pt) Find the time $t_1 > 0$ and the coordinates of the point P where the object hits the xy plane.

(c)(6 pt) Set up a definite integral equal to the length of the arc of the trajectory from $\mathbf{r}(0)$ to the point P. Do not evaluate the integral.

(d)(6 pt) Find the equation of the line tangent to the trajectory at P.

(e)(6 pt) Find the curvature of the trajectory at P.

Problem 3(40 pt) At each point $P(x(t), y(t), z(t))$ of its motion, an object of mass m is subject to a force:

$$\mathbf{F}(t) = -m(sint\mathbf{i} + cost\mathbf{j} + (sint + cost)\mathbf{k}).$$

Given that $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$, and $\mathbf{r}(0) = \mathbf{j} + 3\mathbf{k}$. Find the following:

(a) (8 pt) The velocity $\mathbf{v}(t)$.

(b) (4 pt) The speed $v(\pi)$.

(c) (8 pt) The position function $\mathbf{r}(t)$.

(d) (10 pt) The tangential and normal components of the acceleration $\mathbf{a}(\pi)$.

(e) (10 pt) The osculating plane at $\mathbf{r}(\pi)$.