HW 10) 1) 500 of 24", 200 of 18", 300 of 20"

roll is 70".

Original Problem:

$$\min \; x_1 + x_2$$
s.t.
$$\begin{pmatrix}1\\1\\1\end{pmatrix} x_1 + \begin{pmatrix}2\\0\\1\end{pmatrix} x_2 \geq \begin{pmatrix}500\\200\\300\end{pmatrix}$$

Opt Solution:
$x_1 = 200$
$x_2 = 150$
$z = 350$

Dual:

$$\max \; 500\pi_1 + 200\pi_2 + 300\pi_3$$
s.t.
$$\pi_1 + \pi_2 + \pi_3 \leq 1$$
$$2\pi_1 + 0\pi_2 + \pi_3 \leq 1$$

yields $\hat{\pi} = [1/2 \;\; 1/2 \;\; 0]$

Check pattern x_3, $a_3 = \begin{pmatrix}0\\2\\1\end{pmatrix}$

$\bar{c}_3 = 1 - 2\hat{\pi}_2 - \hat{\pi}_3 = 0$, not < 0, $\boxed{\text{don't use}}$

Identifying Best Pattern:

Look for pattern with best reduced cost:

$$\max \; z = a_i \pi_i$$
s.t. $24a_1 + 18a_2 + 20a_3 \leq 70$ where $\pi = [1/2 \;\; 1/2 \;\; 0]$
a_i int, $a_i \geq 0$

Yields solution $a_1 = 2$, $a_2 = 1$, $a_3 = 0$, $z = 1.5$

This would give reduced cost $\bar{c} = -0.5 < 0$ ✓

Add pattern $\begin{pmatrix}2\\1\\0\end{pmatrix}$ as pattern 3.

New optimal Solution:
$x_1 = 166\,2/3$
$x_2 = 133\,1/3$
$x_3 = 33\,1/3$
$z = 333\,1/3$

(problem not formulated as integer)

Optimal Solutions found in Excel.

2) As bin packing: lower bound $= \left\lceil \frac{\sum a_i}{b} \right\rceil$

where $b = 70''$, and for the total volume you use # needed · length

$\sum a_i = 500(24) + 200(18) + 300(20) = 21600$

$\left\lceil \frac{\sum a_i}{b} \right\rceil = \left\lceil \frac{21600}{70} \right\rceil = 309$

Continue with Cutting Stock:

Need to solve new dual:

$$\begin{aligned} \max\ & 500\pi_1 + 200\pi_2 + 300\pi_3 \\ \text{s.t. } & \pi_1 + \pi_2 + \pi_3 \le 1 \\ & 2\pi_1 + 0\pi_2 + \pi_3 \le 1 \\ & 2\pi_1 + \pi_2 + 0\pi_3 \le 1 \end{aligned}$$

Notice this is the same as previous with 1 new constraint, new pattern $\times \pi \le 1$. This is __always__ the case.

Dual solution: $[\tfrac{1}{3}\ \tfrac{1}{3}\ \tfrac{1}{3}]$

Now use this to find best new pattern:

$$\max\ \pi \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\text{s.t. } 24a_1 + 18a_2 + 20a_3 \le 70$$

$\boxed{\text{Same as before, but new } \pi.}$

Optimal Solution: $a_1 = 2,\ a_2 = 0,\ a_3 = 1,\ z = 1$

Note: reduced cost $= 1 - 1 = 0 \not< 0$, no new best exists

also, we already have this pattern.

Therefore our best cutting stock solution is

$x_1 = 166\tfrac{2}{3}$

$x_2 = 133\tfrac{1}{3}$ $\quad \boxed{z = 333\tfrac{1}{3}} \rightarrow$ lower bound on integer problem.

$x_3 = 33\tfrac{1}{3}$

(Optimal integer solution: $x_1 = 168,\ x_2 = 133,\ x_3 = 33,\ z = 334$)

3) Problem 5 on "Old Final"

Part 1) Node 4: No, if LP is infeasible, the IP is infeasible

Node 5: Yes, the z value is higher than your candidate solution

Node 6: No, you already have an integer solution. This is your candidate solution.

Node 7: No, any integer solution from this node will have $z^* \le 19.2$, you already have integer solution with $z^* = 22$.

Part 2) The lowest your optimal solution will be is 22, because you already have an integer solution with that value.

The upper bound is 24.3, which is the z-value for node 5. Any child nodes must have $z^* \le 24.3$, and 24.3 is the highest z^* of all unbranched nodes.

4) Problem 1 on "old final". Let c_{ij} = time it takes to make job j on machine i. Let k_i be the setup time of machine i. Let $x_{ij} = \begin{cases} 1 & \text{assign } j \text{ to } i \\ 0 & \text{o/w} \end{cases}$

Let $y_i = \begin{cases} 1 & \text{use machine } i \\ 0 & \text{o/w} \end{cases}$

Part 1)

$$\min \; w = \sum_{ij} c_{ij} x_{ij} + \sum_i k_i y_i$$

s.t.

$$\sum_i x_{ij} = 1 \quad \forall j \quad \text{(each job must be done)} \qquad (1)$$

$$\sum_j x_{ij} \le 4 y_i \quad \forall i \quad \text{(if any job on machine } i,\ y_i = 1) \qquad (2)$$

$$x_{ij} \in \{0, 1\}, \quad y_i \in \{0, 1\}$$

Note: here 4 is used in constraint 2, because at most all 4 jobs can be assigned to a machine

Part 2) From part 1, constraint 2, change the 4 on r.h.s to a 2. This will ensure if you use a machine, at most 2 jobs can be assigned to it.

4) Part 3) If machine 1 used, machine 3 used.
So if $y_1 = 1$, y_3 must $= 1$. $y_1 = 0$, $y_3 \in \{0, 1\}$

Add constraint: $y_1 \le y_3$ (3)

Part 4) Either 2 or 4 is used, but not both.

I read this as one of the two must be used.

$y_2 + y_4 = 1$ (could be $y_2 + y_4 \le 1$ if you can use neither)

↳ forces you to use one and only one

Part 5) If both m1 & m3, must not assign job 1 to m3.

Ignoring part 3: $x_{31} = 0$ if $y_1 = 1, y_3 = 1$ $(y_1 + y_3 = 2)$

$x_{31} \le (2 - y_1 - y_3)$ (if $y_1 = 1, y_3 = 0$ $x_{31} \le 1$; $y_1 = 0, y_3 = 1$ $x_{31} \le 1$)

If you assume part 3 as well, then if 1 is used, 3 also

$x_{31} \le y_1$ b/c $y_1 = 1$ represents both 1 & 3 if you keep constraint 3.