GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 - BIOTRANSPORT

FIRST TERM TEST SPRING 2014 - ETHIER

STUDENT NAME: Solution

GTID NUMBER: ______________________________

RECITATION SECTION: ______________________________

(Section E is Wednesdays at 2 pm; Section F is Wednesdays at 1 pm)

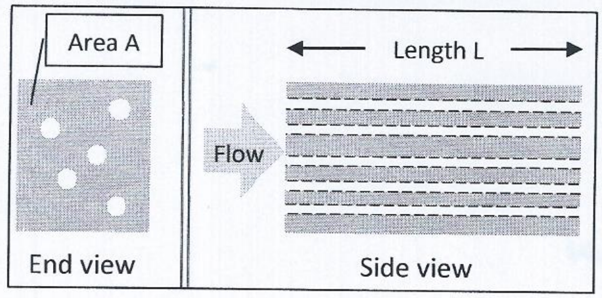
Open book
All non-communicating calculator types allowed
Time allotted: 50 minutes
Do all work in this booklet

Reminder: for questions requiring numerical answers, units are required and worth 50%

Question	Maximum Mark	Actual Mark
1	40	
2	60	
Total	**100**	

5 min

1. Flow in connective tissues, e.g. cartilage, occurs through complicated flow passages. One way to analyze such flows is to approximate these passages as a large number of cylindrical, straight tubes of radius R. Consider a block of cartilage of cross-sectional area A and length L. Suppose that the porosity of the cartilage is denoted by ϵ, where porosity is defined to be the volume of tubes in the tissue divided by the total tissue volume.



   a. Show that the number of tubes in the tissue, N, can be written as $N = \frac{\epsilon A}{\pi R^2}$. **[10 marks]**
   b. Derive a formula for the total flow rate, Q, of a fluid of viscosity μ through this connective tissue if a pressure drop Δp is applied across it. Your expression for the flow rate should only depend on μ, R, Δp, L, A and ϵ. **[30 marks]**

Do your GIM analysis here — this is for part b.

This is tubes in parallel. A

Treat using Poiseuille flow. A

for (b)

Assume tubes are long so flow is fully-developed. Assume steady.

(a) Volume of one tube $= \pi R^2 L$

— " — N tubes $= N\pi R^2 L$

— " — tissue $= AL$

$$\epsilon = \frac{\text{volume of N tubes}}{\text{volume of tissue}} = \frac{N\pi R^2 L}{AL} = \frac{N\pi R^2}{A}$$

$$\therefore \quad N = \epsilon A / \pi R^2 \qquad \underline{\text{QED}}.$$

10

(b) For one tube in Poiseuille flow $R = \frac{8\mu L}{\pi R^4}$ 10 A

We have N tubes in parallel so

$$\mathcal{R}_{tot} = \frac{8\mu L}{N\pi R^4} = \frac{8\mu L}{\epsilon A R^2} \quad \text{from part A.}$$

10 4

Then $$Q = \frac{\Delta p}{\mathcal{R}_{tot}} = \frac{\Delta p \, \epsilon A R^2}{8\mu L}$$ 4

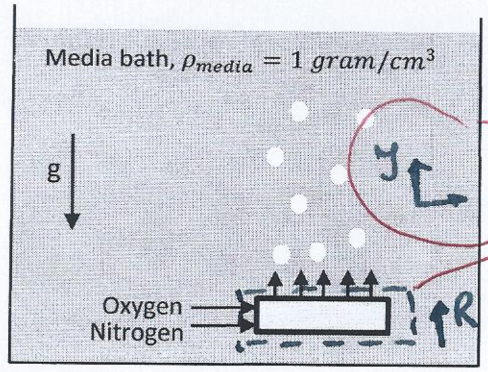
30

check units : $\frac{\cancel{M}}{\cancel{L}T\cancel{T}} \frac{L^2 L^2}{\cancel{M}} \frac{\cancel{L}\cancel{T}}{\cancel{L}}$ ok.

40

(10 min)

2. A bubble generator for a culture system is immersed in a very large bath of media. It has two inlets: one for oxygen and one for nitrogen. Each gas is supplied at 5 gram/second. The oxygen-nitrogen mixture escapes from the device via a series of small holes on the top at an effective velocity of 40 cm/s (averaged over the entire top surface of the device).



   a. What vertical force is required to keep the bubble generator in place? The inlet tubing is flexible and exerts no force on the device. The mass of the device (including the internal gases) is 85 gram and its volume is 80 cm^3. **[30 marks]**

   b. As the bubbles leave the device they rise up through the media bath. It is known that gas is lost from the bubble to the surrounding media according to the relationship $\dot{m}_{bubble} = 4\pi k R^2$ where k is a constant, R is the radius of the bubble, and $\dot{m}_{bubble}$ is the rate at which gas is lost from a bubble. Assuming that the pressure and temperature in the bubbles remains constant, and that the perfect gas law holds for each bubble ($pV = m_{gas}\mathbb{R}_{gas}T$), derive an expression for bubble radius as a function of time, $R(t)$. Assume that the bubble radius at time zero is R_0. **[30 marks]**

Do your GIM analysis here for part (a)

This is a c.v. problem. Use cv as shown. Steady (2)
Balance mass of gas mixture (2) need to state what you are balancing mass of
Momentum balance in vertical direction ~~(4)~~ (3) must state in vertical (or y) direction

Conserve mass: $\sum \dot{m}_{in} = \dot{m}_{out}$

$$\dot{m}_{top} = \dot{m}_{oxy} + \dot{m}_{Nitrogen} = 10\,g/s$$ (5)

y-momentum:

$$+\uparrow \sum F_y = \dot{m} V_y \big|_{out} - \cancel{\dot{m} V_y \big|_{in}}^{\,0}$$ (3)

Call R the force. Then

$$R - \underset{\substack{\uparrow \\ \text{weight}}}{mg} + \underset{\substack{\uparrow \\ \text{buoyancy}}}{V_{disp}\,\rho_{media}\,g} = \dot{m}_{top} V_{y\,top}$$ (8)

(-3) for neglecting each of buoyancy, weight

$$R = \dot{m}_{top} V_{y|top} + g\,(m - V_{disp}\,\rho_{medium})$$

$$= \left(10\,\frac{g}{s}\right)\left(40\,\frac{cm}{s}\right) + 981\,\frac{cm}{s^2}\left(85\,g - (80)(1)\,g\right) \quad (2)$$

$$= 5305\ g\,cm/s^2 = \underline{5305\ dyne} = 0.0531\ N$$

30

Do your GIM analysis here for part (b)

Look at a single bubble. Balance mass of gas mixture. (3) Unsteady (3)

CV as shown (4)

Perfect gas law: $m_{gas} = \dfrac{pV}{\mathcal{R}_{gas}T} = \dfrac{P}{\mathcal{R}_{gas}T}\,\dfrac{4}{3}\pi R^3$ (4)

Conserve mass $\quad 0 = \dfrac{\partial}{\partial t}\displaystyle\int_{cv} \rho\, dVol + \int_{cs} \rho\, \underline{v}\cdot\hat{n}\, dA$ (3)

$$0 = \frac{d}{dt}\left(\frac{P}{\mathcal{R}T}\,\frac{4}{3}\pi R^3\right) + 4\pi k R^2 \quad (3)$$

$$0 = \frac{P}{\mathcal{R}T}\cdot\cancel{4\pi R^2}\,\frac{dR}{dt} + \cancel{4\pi}\,k\cancel{R^2}$$

Const.

$$\underline{-\frac{k\mathcal{R}T}{P} = \frac{dR}{dt}} \quad (7) \longrightarrow \boxed{R = R_0 - \left(\frac{k\mathcal{R}T}{P}\right)t}$$

(3)

30

60