Full name: Math 2551, Section B___

Quiz 1 — §12.1-4

Please ***clearly*** show all work. Scientific calculators are allowed, but no graphing calculators!

(1) Let $\mathbf{u}$ and $\mathbf{v}$ be two non-zero vectors. Is $(\text{proj}_{\mathbf{v}}\mathbf{u}) \cdot (\mathbf{u} \times \mathbf{v})$ positive, negative, or zero? You must justify your answer to receive credit. [4 points]

By definition, $\text{proj}_{\mathbf{v}}\mathbf{u}$ is parallel to $\mathbf{v}$. On the other hand $\mathbf{u} \times \mathbf{v}$ is perpendicular to $\mathbf{v}$. Therefore $\mathbf{u} \times \mathbf{v}$ is perpendicular to $\text{proj}_{\mathbf{v}}\mathbf{u}$ as well, which shows that $(\text{proj}_{\mathbf{v}}\mathbf{u}) \cdot (\mathbf{u} \times \mathbf{v}) = 0$.

(2) Using an equation or a set of equations, describe the circle of radius 2 centered at $(-3, 4, 1)$ lying in the plane parallel to the

(a) xy-plane (b) xz-plane (c) yz-plane

[6 points]

(a) $(x+3)^2 + (y-4)^2 = 4$, $z = 1$
(b) $(x+3)^2 + (z-1)^2 = 4$, $y = 4$
(c) $(y-4)^2 + (z-1)^2 = 4$, $x = -3$

(3) Let R be the parallelogram in space with vertices $A = (0,0,0)$, $B = (3,2,4)$, $C = (5,1,4)$, and $D = (2,-1,0)$. Compute the ratio

$$\frac{\text{Perimeter}(R)}{\text{Area}(R)}$$

[10 points]

The parallelogram R is determined by the vectors $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{AD}$,

$$\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k} \qquad \mathbf{v} = 2\mathbf{i} - \mathbf{j}.$$

The perimeter of R is $2|\mathbf{u}| + 2|\mathbf{v}|$, so we must compute the magnitudes of $\mathbf{u}$ and $\mathbf{v}$:

$$|\mathbf{u}| = \sqrt{3^2 + 2^2 + 4^2} = \sqrt{29} \qquad |\mathbf{v}| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

giving $\text{Perim}(R) = 2\sqrt{29} + 2\sqrt{5}$. The area of R is the magnitude of $\mathbf{u} \times \mathbf{v}$:

$$\mathbf{u} \times \mathbf{v} = \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 4 \\ 2 & -1 & 0 \end{pmatrix} = 4\mathbf{i} + 8\mathbf{j} - 7\mathbf{k} \qquad |\mathbf{u} \times \mathbf{v}| = \sqrt{4^2 + 8^2 + 7^2} = \sqrt{129}$$

Combining these, we have

$$\frac{\text{Perimeter}(R)}{\text{Area}(R)} = \boxed{\frac{2\sqrt{29} + 2\sqrt{5}}{\sqrt{129}}}$$