MIDTERM II EXAM
Version 4 Solution

ISyE 2028 A
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1 A sample of 30 data observations has a sample mean $\bar{x} = 14.62$ and a sample standard deviation $s = 2.98$. Find the value of c for which $\mu \in (\infty, c)$ is a one-sided 95% t-interval for the population mean μ. Is it plausible that $\mu \leq 16$?

$\alpha = 0.05$
$c = \bar{x} + t_{\alpha,n-1} * \frac{s}{\sqrt{n}}$
$c = 14.62 + t_{0.05,29}\frac{2.98}{\sqrt{30}}$
$c = 14.52 + 1.699(0.54407)$
$c = 15.54$
Yes- it is plausible that $\mu \leq 15$ since the upper bound of the confidence interval is ≤ 16

2 A company is investigating how long it takes its drivers to deliver goods from its factory to a nearby port for export. Records reveal that with a standard specified driving route, the last $n = 48$ delivery times have a sample mean of $\bar{x} = 432.7$ minutes and a sample standard deviation of $s_x = 20.39$ minutes. A new driving route is proposed, and this has been tried $m = 10$ times with a sample mean of $\bar{y} = 403.5$ minutes and a sample standard deviation of $s_y = 15.62$ minutes. Is there evidence that the new route is quicker on average than the standard route?

The estimator we need is $\bar{x} - \bar{y}$ and we set up the hypothesis test
$H_0 : \bar{x} - \bar{y} = 0$
$H_A : \bar{x} - \bar{y} > 0$
$\bar{x} - \bar{y} \sim N(\bar{x} - \bar{y}, \frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}$
Test statistic: $t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_x^2}{n_x}+\frac{s_y^2}{n_y}}} = \frac{432.7-403.5}{\sqrt{\frac{20.39^2}{48}+\frac{15.62^2}{10}}} = \frac{29.2}{5.75} = 5.079$
Degrees of freedom: $N = \frac{(\frac{s_x^2}{n_x}+\frac{s_y^2}{n_y})^2}{\frac{s_x^4}{n_x^2(n_x-1)}+\frac{s_y^4}{n_y^2(n_y-1)}} = \frac{(\frac{20.39^2}{48}+\frac{15.62^2}{10})^2}{\frac{20.39^4}{48^2(47)}+\frac{15.62^4}{10^2(9)}} = \frac{1092.959}{67.74} = 16.13 \sim 17$
Choose $\alpha = 0.05$ - then $t_{\alpha,N} = t_{0.05,17} = 1.74$
Rejection rule: Reject H_0 if $t > t_{\alpha,N}$. We have $5.07 > 1.74$, so reject H_0 and conclude that there is evidence that the new route is faster.
P-value $= Pr(T_N \leq -|t|) = Pr(T_N \geq |t|) < 0.0005$ (for n=17 the t-table only goes to 3.965 which corresponds to 0.0005)
The small p-value also indicates that we should reject H_0.

3 Extracts of St. John's Wart are widely used to treat depression. An article in the Journal of the American Medical Association (April, 2001) titled "Effectiveness of St. John's Wart on Major Depression: A randomized controlled trial" compared the efficacy of a standard extract of St. John's Wart with a placebo in 200 outpatients diagnosed with major depression. Patients were randomly assigned to the two treatments (100 patients for the St. John's Wart extract and 100 for placebo). After eight weeks, 19 of the placebo-treated patients showed improvement, whereas 27 of those treated with St. John's Wart extract got treated. Is there evidence to believe that St. John's Wart extract is any different than the Placebo in treating depression? Compute the p-value and interpret your results.

This is an unpaired test.
$\widehat{p_p} = \frac{19}{100} = 0.19, \widehat{p}_{SJW} = \frac{27}{100} = 0.27$
$\widehat{p_p} - \widehat{p}_{SJW} \sim N(\widehat{p_p} - \widehat{p}_{SJW}, \frac{\widehat{p}_p(1-\widehat{p}_p)}{n_p} + \frac{\widehat{p}_{SJW}(1-\widehat{p}_{SJW})}{n_{SJW}})$
$H_0 : \widehat{p_p} - \widehat{p}_{SJW} = 0$
$H_A : \widehat{p_p} - \widehat{p}_{SJW} \neq 0$
Test statistic: $t = \frac{\widehat{p_p} - \widehat{p}_{SJW}}{\sqrt{\frac{\widehat{p}_p(1-\widehat{p}_p)}{n_p} + \frac{\widehat{p}_{SJW}(1-\widehat{p}_{SJW})}{n_{SJW}}}} = \frac{0.19 - 0.27}{\sqrt{\frac{0.19(1-0.19)}{100} + \frac{0.27(1-0.27)}{100}}} = \frac{-0.08}{0.0592} = -1.351$
Rejection rule: Reject H_0 if $t > z_{\frac{\alpha}{2}}$
$1.351 \not> 1.96$, and the P-Value $= 2Pr(Z < -|t|) = 2Pr(Z < -1.351) = 2(0.885) = 0.177$
Because the p-value is not small and the rejection rule failed, we do not reject H_0, so there is not evidence to believe that St. John's Wart extract is different than the Placebo in treating depression.