Name: __Key_____ Section: _____

Show all work clearly and in order, and box your final answers.

A plastic sphere of radius R has hollow center of radius $R/2$. This thick spherical shell has a charge $+Q$ distributed uniformly throughout the plastic. For the following three regions, determine the magnitude and direction of the electric field at an observation location a distance r from the center of the sphere. Be sure to provide a briefly explanation to earn full credit.
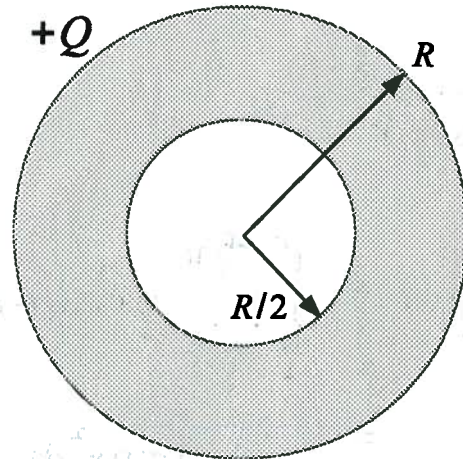
1. (*20 points*) $r < R/2$ (inside the hollow center)

$$|\vec{E}| = 0 \qquad \{ 10 pts$$

Electric field inside a spherical shell is zero. $\qquad \{ 10 pts$

2. (*20 points*) $r > R$ (outside the plastic shell)

$$|\vec{E}| = \frac{Q}{4\pi \epsilon_0 r^2} \qquad \text{radially outward} \quad \{ \begin{array}{l} 10 pts \\ (5 mag \\ 5 dir) \end{array}$$

Electric field outside of a shell looks like a point charge
Electric field outside a spherical shell reduces to a point charge $\quad \} 10 pts$

3. (*60 points*) $R/2 < r < R$ (in the plastic)

origin: center of shell

coordinates: spherical coordinates.

$$\begin{bmatrix} \text{clerical} & -3.0 \\ \text{minor} & -9.0 \\ \text{major} & -18 \\ \text{BTN} & -48 \end{bmatrix}$$

$$|dE| = \frac{1}{4\pi\epsilon_0} \frac{dQ}{r^2}$$

$$Q = \int \rho \, dV = \int_{\frac{R}{2}}^{r} \int_{0}^{2\pi} \int_{0}^{\pi} \rho \, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \int_{\frac{R}{2}}^{r} 4\pi \rho r^2 \, dr$$

$$Q = \frac{\pi}{6}\left(8r^3 - R^3\right)\rho$$

Charge up
to shell of
radius $\frac{R}{2} \to r$

Note: ρ is related to total charge Q_{TOT}.

$$Q_{TOT} = \int_{\frac{R}{2}}^{R} 4\pi \rho r^2 \, dr$$

Total charge inside
a ~~sphere~~ shell of radius $\frac{R}{2} \to R$.

$$Q_{TOT} = \frac{7}{6}\pi\rho R^3 \quad\Rightarrow\quad \rho = \frac{6\, Q_{TOT}}{7\pi R^3}$$

$$|\bar{E}| = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0 r^2} \frac{\pi}{6}\left(8r^3 - R^3\right)\rho$$

$$= \frac{1}{4\pi\epsilon_0 r^2} \frac{\pi}{6}\left(8r^3 - R^3\right)\left(\frac{6\, Q_{TOT}}{7\pi R^3}\right)$$

$$|\bar{E}| = \frac{Q_{TOT}}{28\pi\epsilon_0 r^2}\left(\frac{8r^3 - R^3}{R^3}\right) \qquad \text{radially outward.}$$