**GEORGIA INSTITUTE OF TECHNOLOGY**

**COLLEGE OF ENGINEERING**

**BMED3300 – BIOTRANSPORT**

**QUIZ 3 (SPRING 2014) – ETHIER**

STUDENT NAME: Key

GTID NUMBER: ______________________

RECITATION SECTION: ______________________

(Section E is Wednesdays at 2 pm; Section F is Wednesdays at 1 pm)

*Closed Book*
*All non-communicating calculator types allowed*
*Time allotted: 15 minutes*
*Do all work in this booklet*

**Reminder: for questions that require numerical answers, units are required and worth 50%**

| Question | Maximum Mark | Actual Mark |
|---|---|---|
| 1 | 9 | |
| 2 | 3 | |
| Total | 12 | |

1) The mouse is now widely used as a model of cardiovascular disease in humans. Researchers decide to study blood flow patterns in the mouse aorta. To do so, they build a 5 times larger scale model of the aorta of known diameter D, attach it to a pulsatile pump to simulate the heart, and use a model fluid with the known properties of heart rate (HR), fluid viscosity $\mu$ ($kg \cdot m^{-1} \cdot s^{-1}$), fluid density $\rho$, and flow rate Q. Because it is known that disease development depends in part on the shear stress, $\tau$ ($kg \cdot m^{-1} \cdot s^{-2}$), exerted by flowing blood on the endothelial cells that line the arteries, they wish to measure this quantity.

a. Construct a $\pi$-matrix from the relevant parameters in this problem and confirm that 3 $\pi$-groups can be formed from these parameters.

| | D | HR | $\mu$ | $\rho$ | Q | $\tau$ |
|---|---|---|---|---|---|---|
| m | 0 | 0 | 1 | 1 | 0 | 1 |
| L | 1 | 0 | -1 | -3 | 3 | -1 |
| t | 0 | -1 | -1 | 0 | -1 | -2 |

+ 2

-1 for each

Rank = 3

6 variables − rank = 3 $\pi$ groups  + 1

If matrix wrong ↓ -4

b. Taking $\tau$, HR, and D as the core group of variables, find ~~three~~ one $\pi$-groups.

credit given for 1 $\pi$ group

$M^0L^0t^0 = \tau^a HR^b D^c \rho$

m: $0 = a + 1$

$a = -1$  + 1

L: $0 = -a + c - 3$

$0 = -2 + c$

$c = 2$  + 1

t: $0 = -2a - b$

$= 2 - b$

$b = 2$  + 1

$\pi_1 = \tau^{-1} HR^2 D^2 \rho$

$= \dfrac{HR^2 D^2 \rho}{\tau}$  + 3

$= \dfrac{t^2 \cdot L}{m} \cdot \dfrac{1}{t^2} \cdot L^2 \cdot \dfrac{m}{L^3}$ ✓

$M^0L^0t^0 = \tau^a HR^b D^c Q$

$= \left(\dfrac{m}{L \cdot t^2}\right)^a \left(\dfrac{1}{t}\right)^b (L)^c \cdot \left(\dfrac{L^3}{t}\right)$

M: $0 = a$

L: $0 = -a + c + 3$

$c = -3$

t: $0 = -2a - b - 1$

$b = -1$

$\pi_2 = \tau^0 HR^{-1} D^{-3} Q$

$= \dfrac{Q}{HR \cdot D^3}$

$= \dfrac{L^3 \cdot t}{t \; L^3}$ ✓

check $\pi$ group (dimensionless)

$M^0L^0t^0 = \tau^a HR^b$

$= \left(\dfrac{m}{L \cdot t^2}\right)^a \left(\dfrac{1}{t}\right)^b (L)$

m: $0 = a + 1$

$a = -1$

L: $0 = -a + c - 1$

$0 = 1 - 1 + c$

$c = 0$

t: $0 = -2a - b - 1$

$0 = 2 - 1 - b$

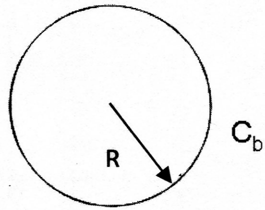
$b = 1$

$\pi_3 = \tau^{-1} HR^1 D^0 \mu$

$= \dfrac{HR \cdot \mu}{\tau}$

$= \dfrac{1}{t} \cdot \dfrac{m}{L \cdot t} \cdot \dfrac{L \cdot t^2}{m}$ ✓

2. Medical implants are capable of releasing drugs at a constant rate into the systemic circulation, a convenient alternative to oral drug administration when a constant blood level of drug is desired in the patient for extended periods of time. Several slow-release corticosteroid intraocular implants are undergoing clinical trials for treating macular edema. Perhaps more familiar is the use of Norplant™, an implantable contraceptive device which releases the steroid hormone levonorgestrel into the blood when implanted under the skin in the arm. A new device is being considered which does not contain any surface coating, i.e. the implant of **cylindrical geometry** consists of a single polymer gel material that the drug is imbedded in and can diffuse through. You may assume that only radial diffusion is taking place, as the two ends of the cylinder are impermeable to the drug. Consider the following definitions:




$c_b$ = concentration of drug in the body fluid at the surface of the device (assume constant)

$c_d$ = drug concentration in the implant

R = radius of the implant

$D_{d,i}$ = effective diffusion coefficient of the drug in the implant

a) What are the boundary conditions you would use for determining a solution?

at $r = R$, $c_d = c_b$ +1.5

at $r = 0$ $\frac{\partial c_d}{\partial r} = 0$ +1.5

0.5

1