GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 - BIOTRANSPORT

FINAL EXAM SPRING 2013 - ETHIER

STUDENT NAME: Solution

GTID NUMBER: ____________________

Open book
All non-communicating calculator types allowed
Time allotted: 170 minutes
Do all work in this booklet

Reminder: for questions requiring numerical answers, units are <u>required</u> and worth 50%

	Question	Maximum Mark	Actual Mark
	Question	**Maximum Mark**	**Actual Mark**
R	1	18	6 min
E	2	18	6 –"–
R	3	30	11 –"–
E	4	18	6 –"–
R	5	20	5 –"–
E	6	36	13 –"–
	Total	**140**	

Note that total marks = 140. However, exam will be marked out of 100, i.e. 100 marks = 100%. All questions will be marked.

1. A porous sphere of radius 0.1 cm is initially saturated with distilled water. As part of a curing process it is placed in a large salt water bath, with salt concentration of 0.3 g/ml. The diffusion coefficient of the salt in the sphere is $D_{im} = 1 \times 10^{-3}\ cm^2/s$, and conditions in the bath are such that the mass transfer coefficient at the surface of the sphere for salt is $k_f = 0.001\ cm/s$. After 500 seconds, the sphere is placed in a different bath containing distilled water, where the external mass transfer coefficient for salt is now $k_f = 0.01\ cm/s$. What is the average salt concentration in the sphere after 4 seconds in the second bath?

Do your GIM analysis here

Unsteady diffusion problem with salt. 1
Bath 1: salt goes into sphere 1
Bath 2: salt comes out 1
will need to use charts &/or formulas 1

4 I

$R = 0.1$ cm
$\mathcal{D} = 10^{-3}$ cm²/s

Bath 1: $k_f = 10^{-3}$ cm/s

$$\therefore Bi = \frac{(R/3)k_f}{\mathcal{D}} = \frac{(0.1/3)(10^{-3})}{10^{-3}} \frac{cm^2}{s}\frac{s}{cm^2} = \frac{0.1}{3} < 0.1$$ 1 F

∴ Can use analytic sol'n.

$$Fo = \frac{\mathcal{D}t}{(R/3)^2} = \frac{(10^{-3})(500)}{(0.1/3)^2} \frac{cm^2}{cm^2} = 4.5 \times 10^2$$ 1 F

$$Bi\,Fo = \frac{45}{3} = 15$$ 1 S

$$\therefore \theta = e^{-Bi\,Fo} \approx 0$$ 2 S

∴ sphere is fully ~~loaded~~ loaded & $\rho \simeq \rho_\infty = \frac{0.3g}{ml}$. 2 S

using R instead of R/3 (-1).

Bath 2 k_f is larger so Bi will be larger. Will need to use chart.

$$Bi = \frac{Rk_f}{D} = \frac{(0.1)(0.01)}{10^{-3}} = 1$$ ① F

F

$$Fo = \frac{Dt}{R^2} = \frac{10^{-3}(4)}{(0.1)^2} = 0.4$$ ①

From chart for avg conc in sphere $\theta \approx 0.4$ ③ S

$$\theta = \frac{\rho - \rho_\infty}{\rho_0 - \rho_\infty}$$

$\rho_\infty = 0$

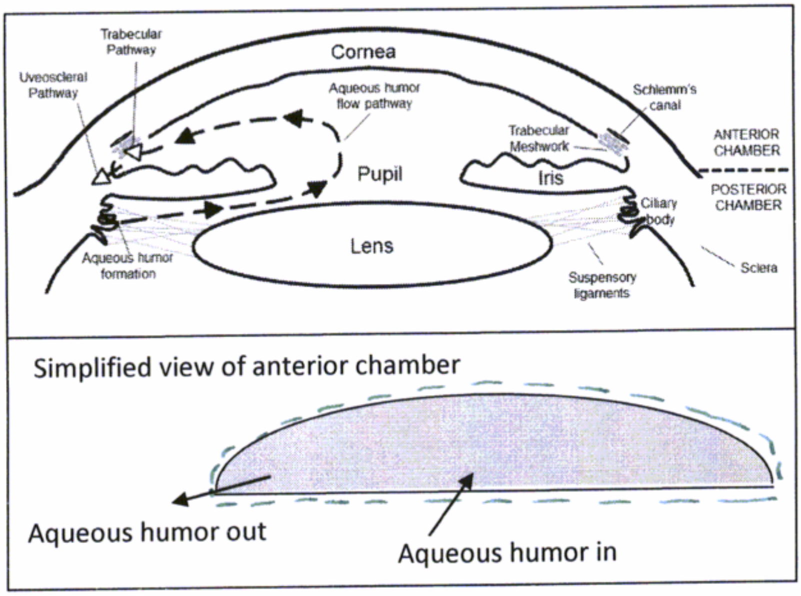
$\rho_0 = 0.3$ g/ml

$\therefore \rho = \theta \rho_0 = 0.12$ g/ml. ② S

18

I	F	S
4	4	10

2. The front part of your eye (the anterior chamber) is filled by a fluid called the aqueous humor. This fluid is produced just behind the anterior chamber and drains out of the eye at the edge of the anterior chamber (see arrows on figure[1]). It is decided to develop a contact lens system to deliver a drug into the aqueous humor. The drug-saturated contact lens is placed onto the cornea. It takes 30 minutes for the drug to traverse the cornea, after which time drug begins to enter the aqueous humor. You can treat the drug entry rate as a constant equal to 0.4 μg/min (total rate for the entire cornea). You may treat the anterior chamber as a well-mixed reservoir of fixed volume V = 125 μL. The aqueous humor inflow rate is steady and equal to 3 μL/min. What is the concentration of drug in the anterior humor 90 minutes after the contact lens is first applied to the cornea?



identify

Do your GIM analysis here

AH flow is steady (1)

Balance mass of drug (unsteady). (1) Use CV shown (1)

Account for 30 min time lag for drug to cross cornea. (1)

formulate

AH: $Q_{in} = Q_{out} = Q$ (1)

Drug: rate of accumulation $= \dot{m}_{in} - \dot{m}_{out}$

$$V \frac{d\rho}{dt} = \dot{m}_{in} - \rho Q \quad (5) \qquad \dot{m}_{in} = 0.4 \frac{\mu g}{min}$$

must identify $\dot{m}_{in}$ properly
If not, (3)

$$\therefore \frac{d\rho}{dt} = -\frac{Q}{V}\left[\rho - \frac{\dot{m}_{in}}{Q}\right]$$

[1] Image from Ito, Y. A., & Walter, M. A. (2013). Genetics and Environmental Stress Factor Contributions to Anterior Segment Malformations and Glaucoma.

$\therefore \quad \rho - \frac{\dot{m}_{in}}{\varphi} = \text{const}\, e^{-\varphi t/V}$ (2)

I solve.

when $t = 0$, $\rho = 0$, so const $= -\frac{\dot{m}_{in}}{\varphi}$ (1)

$\therefore \rho(t) = \frac{\dot{m}_{in}}{\varphi}\left[1 - e^{-\varphi t/V}\right]$ (2)

Plug in #'s. $\dot{m}_{in} = 0.4\ \mu g/min$
$\varphi = 3\ \mu L/min$
$t = 60$ min (90 min − 30 min for drug to cross cornea)
$V = 125\ \mu L$

$\therefore \rho(t) = \frac{0.4}{3}\ \frac{\mu g}{\mu L}\left[1 - e^{-180/125}\right]$

$= 0.102\ \mu g/\mu L.$ (3)

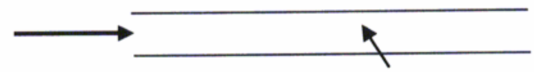
18

Wrong Eq Setup
but right
solution (− 3)

No in-flow: (− 9)

Identify (4)
Formulate (6)
Solve (8)

3. In class, we considered the pressure drop due to steady flow in straight tubes. However, many flows in the body are unsteady. Here we consider <u>unsteady</u> flow in a straight tube. We assume that the velocity at each location varies periodically, with oscillation period Γ. (See the diagram for other relevant parameters.)



Fluid in
- Density ρ
- Viscosity μ
- Average velocity V

Tube
- Diameter D
- Roughness ϵ
- Length L

a) We want to know the pressure drop due to unsteady blood flow in an arterial segment, and so build a 4x scale model using a working fluid with $\rho_{model} = 2\ g/cm^3$ and $\nu_{model} = 0.7\ cm^2/s$. Show that results from the 4x scale model will be valid if the working fluid is infused at an average velocity which is 5 times larger than that present in the real blood vessel, and with an oscillatory period Γ_{model} that is 80% of that in the real blood vessel. The density and kinematic viscosity of blood are $\rho_{blood} = 1.05\ g/cm^3$ and $\nu_{blood} = 0.035\ cm^2/s$. Hint: what are the relevant pi-groups? **[12 marks]**

b) Using the parameters above, a time-averaged pressure drop of 9×10^5 dynes/cm^2 is measured in the scale model (from inlet to outlet) for an time-averaged inlet fluid velocity of $V_{model} = 250\ cm/s$. What time-averaged pressure drop does this correspond to in the arterial segment? (Although this flow is unsteady, you may use the ME equation so long as V and p are taken as time-average values.) **[8 marks]**

c) If the real arterial segment is 7 cm long and has diameter 0.2 cm, how much higher is the average pressure drop during unsteady flow compared to the pressure drop that would occur for <u>steady</u> flow at the same average velocity in the arterial segment? **[10 marks]**

Do your GIM analysis here

This is Buckingham Π-theorem/similitude. Like tube flow with 1 extra parameter, Γ. ② for (a)

Part (c): use ME eq'n. ① for (c)

(a) Parameters ρ, μ, V, D, ϵ, L & Γ. We know Π-groups will be f, Re_D, ϵ/D and a new group Π_4. ② Take ρ, μ, D as core $\Pi_4 = \rho^a \mu^b D^c \Gamma$ $[\Gamma] \sim T$

M: $a + b = 0$
L: $-3a - b + c = 0$
T: $-b + 1 = 0$

$b = 1$
$a = -1$
$c = -2$

② $\boxed{\therefore \Pi_4 = \dfrac{\mu \Gamma}{\rho D^2}}$ ✓ or $\dfrac{V\Gamma}{D}$

Check M T L / L T M

For similitude, match: ① ∈ ① [occurs due to model] ②

② $Re_D \Rightarrow \frac{V_m D_m}{\nu_m} = \frac{V_b D_b}{\nu_b}$ ① $\Rightarrow V_m = V_b\left(\frac{\nu_m}{\nu_b}\right)\left(\frac{D_b}{D_m}\right) = V_b\left(\frac{0.7}{0.035}\right)\left(\frac{1}{4}\right)$

$= 5V_b$ ✓

③ $\Pi_4 \Rightarrow \frac{\nu_m \Gamma_m}{D_m^2} = \frac{\nu_b \Gamma_b}{D_b^2} \Rightarrow \Gamma_m = \Gamma_b\left(\frac{D_m}{D_b}\right)^2 \frac{\nu_b}{\nu_m} = \Gamma_b \cdot 16 \cdot \frac{0.035}{0.7}$ ① ②

$= 0.8\,\Gamma_b$ ✓

Failing to identify Π_4 (-7) || wrong units for Γ (-2) 10/12

(b) If ①, ②, ③ match, then $f_m = f_b$. ② Use ME eq'n from inlet to outlet, note $V_1 = V_2$, $z_1 = z_2$, $\Delta p = p_1 - p_2$ ∴ ②

$$\Delta p = \rho g\left[f\frac{L}{D}\frac{V^2}{2g}\right] \Rightarrow f = \frac{\Delta p\, D}{L(\frac{1}{2}\rho V^2)}$$ ①

$$f_m = f_b \Rightarrow \frac{\Delta p_m}{\rho_m V_m^2} = \frac{\Delta p_b}{\rho_b V_b^2} \Rightarrow \Delta p_b = \Delta p_m\left(\frac{\rho_b}{\rho_m}\right)\left(\frac{V_b}{V_m}\right)^2$$

$$= 9\times10^5 \frac{\text{dyne}}{\text{cm}^2}\ \frac{1.05}{2}\left(\frac{1}{25}\right)$$

$$= 1.89\times10^4\ \text{dyne}/\text{cm}^2$$ ③ 8

(c) Check $Re_b = \frac{V_b D_b}{\nu_b} = \frac{\left(50\ \frac{\text{cm}}{\text{s}}\right)(0.2\,\text{cm})}{0.035\ \text{cm}^2/\text{s}}$

$= 286$ ∴ laminar ②

$\therefore f = \frac{64}{Re_D} = 0.224$ ②

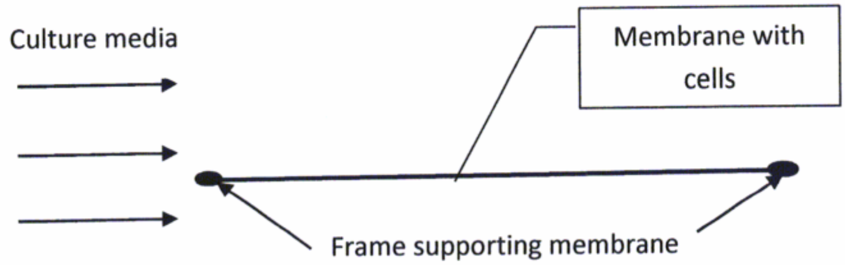
$\therefore \Delta p_b$ for steady $= f\frac{L}{D}\cdot\frac{1}{2}\rho V^2 = (0.224)\left(\frac{7.0}{0.2}\right)\left(\frac{1.05}{2}\right)(50)^2\ \frac{\text{g}}{\text{cm}^3}\frac{\text{cm}^2}{\text{s}^2}$

$= 1.029\times10^4$ dyne/cm² ⑤

Unsteady Δp is 84 % higher 10

Not showing laminar (-2) 30

4. Endothelial cells are cultured on a very thin membrane that is 3 cm long and 3 cm wide. The membrane is supported on its edges by a thin frame, and is placed in flowing culture media, with density $\rho_{media} = 1\ g/cm^3$ and viscosity $\mu_{media} = 0.01\ g/(cm\ s)$. The total frictional drag force on the cell-covered membrane (both sides) due to fluid flow is measured to be 1000 dynes. A substance with Schmidt number = 1000 is being transferred from the cells to the flowing media. Determine the Sherwood number for this mass transfer process at a location 1 cm from the leading edge of the membrane. You may neglect the effects of the frame and may assume laminar flow.



Do your GIM analysis here

Mass & momentum b. layer. ① Get b. layer characteristics ① from overall drag force (use average C_f). Then ① compute local Sh_x. Steady ①

4

not accounting for 2 sides -1 need global formula, -2 if local

Momentum: $F_D = 2(wL) \cdot \frac{1}{2}\rho V^2 \cdot 1.328\ Re_L^{-1/2}$ -1 4.

$$\frac{F_D}{\rho w L (1.328)} = V^2 \left(\frac{VL}{\nu}\right)^{-1/2} = V^{3/2}\left(\frac{L}{\nu}\right)^{-1/2}$$ 4

$$\therefore V^3 = \frac{L}{\nu}\left[\frac{F_D}{\rho w L(1.328)}\right]^2 = \frac{3}{0.01}\left[\frac{10^3}{(1)(9)(1.328)}\frac{g\,cm^2}{s^2\,g}\right]^2 \frac{cm}{cm^2/s}$$

$$= 2.1\times10^6 \quad \frac{cm^4}{s^4}\frac{s}{cm}$$ 2

$V = 128.1\ cm/s$

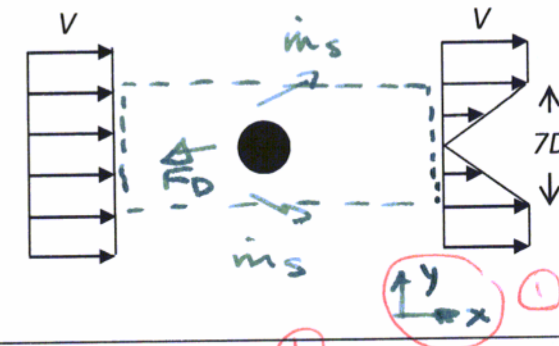
Mass: $Sh_x = 0.332\ Re_x^{1/2}\ Sc^{1/3}$ 1.

$$= 0.332\left(\frac{(28.1)(1)}{0.01}\right)^{1/2}(10^3)^{1/3}$$ ①

②

$$= 375.7$$

5. Incompressible fluid moving at uniform speed V flows steadily over a cylinder of diameter D, producing the velocity profile shown. What is the drag coefficient for this cylinder?



(1)

Do your GIM analysis here (1) Steady flow. Balance (1) mass, then (1) momentum. Use CV (1) shown. Get F_D, then compute C_D. Note there is "leakage" flow, $\dot m_s$ (1)

(6)

Cylinder length into page = L

Mass: $\dot m_{in} = \dot m_{out}$. (2)

$$7DL\rho V = 2\dot m_s + \frac{7}{2}DL\rho V$$

$$\therefore \dot m_s = \frac{7}{4}DL\rho V$$ (1)

Momentum: $\sum_i F_x = \dot m V_x|_{out} - \dot m V_x|_{in}$ (3)

$$-F_D = 2\dot m_s V + 2\int_0^{3.5D} \rho\left(\frac{Vy}{3.5D}\right)^2 L\,dy - 7DL\rho V^2$$ (3)

$$-F_D = \frac{7}{2}DL\rho V^2 - 7DL\rho V^2 + 7D\rho LV^2\int_0^1 \eta^2 d\eta \qquad \eta = \frac{y}{3.5D}$$

$$= DL\rho V^2\left[-\frac{7}{2} + \frac{7}{3}\right]$$

$$= -DL\rho V^2\left[\frac{21-14}{6}\right]$$

$$\therefore F_D = \rho V^2 LD \cdot \left(\frac{7}{6}\right)$$ (2)

Get $\frac{14}{3}\rho V^2 LD$ if neglect top & bottom.

Then $C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 L D} = \dfrac{7}{6} \cdot \dfrac{2}{1} = \dfrac{7}{3}$ //.

1

2

20

-1 for expressing C_D per unit length of cylinder, or inconsistent dimensions

using wrong area of cylinder in C_D calc -2

Not accounting ~~for~~ for in top/bottom -3 or -4, depending on severity

6. Culture media flows inside a cylindrical tube lined with cells, which secrete a cytokine into the flowing media. The tube diameter is $D_{tube} = 0.2\ cm$, and culture medial flows at $Q = 0.314\ ml/s$. The kinematic viscosity of the culture media is $\nu = 0.01\ cm^2/s$ and the diffusion coefficient of the cytokine in culture media is $\mathfrak{D}_{im} = 2 \times 10^{-4}\ cm^2/s$.
   a) If the cells produce a cytokine at a constant rate of $n_i = 2\ ng/(s\ cm^2)$, compute the mixing cup concentration of cytokine at a location 2 cm from the tube inlet. There is no cytokine in the media when it enters the tube. **[6 marks]**
   b) What is the surface concentration of the cytokine at this same location? **[10 marks]**
   c) It is realized that the cells are not actually secreting the cytokine at a constant rate, but instead according to the relationship

$$n_i = \Lambda(\rho_{sat} - \rho_{i,s})$$

where $\Lambda = 0.02\ cm/s$ and $\rho_{sat} = 100\ ng/cm^3$ are both constants, and $\rho_{i,s}$ is the local surface concentration of the cytokine. What is $\rho_{i,m}$ at a location 2 cm from the tube inlet? For this part of the question only, you may take the local Sherwood number in the entire tube as a constant, equal to 15. **[20 marks]**

part b　　part a

Do your GIM analysis here　Steady mass X fer in a tube. ①.

Parts (a) & (b) are constant wall flux. ① Part (c) is neither const wall flux or const concentration. Will need k_f for parts (b) & (c). ② (part c).

(a) Given $P n_i = Q \dfrac{d\rho_{i,m}}{dx}$ ②　　$\rho_{i,m} = 0 \ @\ x = 0$.

$\therefore \rho_{i,m} = \rho_{i,m}\big|_{x=0} + \dfrac{P n_i}{Q} x$ ②

$P = \pi D$

$n_i = 2 \dfrac{ng}{s\ cm^2}$

$Q = 0.314\ cm^3/s$

$x = 2\ cm$

⑤

$\rho_{i,m} = \dfrac{\pi(0.2)(2)(2)}{0.314} \dfrac{cm\ cm\ ng\ s}{s\ cm^2\ cm^3}$

$= 8\ ng/cm^3$ //. ①

(b) $n_i = k_f(\rho_{i,s} - \rho_{i,m})$. What is k_f? Use charts.

$Pe_D = \dfrac{VD}{\mathfrak{D}} = \dfrac{4Q}{\pi D \mathfrak{D}} = \dfrac{4(0.314)}{\pi(0.2)(2\times10^{-4})} \dfrac{cm^3\ s}{s\ cm^3} = 10^4$. ②

$$\frac{x}{D} = \frac{2\,cm}{0.2\,cm} = 10$$

$$\frac{1}{Pe_D}\frac{x}{D} = 10^{-3}.$$ (1) $\Rightarrow$ use chart (p. V.41) (1) for const wall flux to read local

$$Sh_D = 15 = \frac{k_f D}{\mathcal{D}}$$ (1)

$$\therefore k_f = \frac{15\,\mathcal{D}}{D} = \frac{(15)(2\times10^{-4})}{0.2}\ \frac{cm^2}{s\,cm} = 0.015\ cm/s$$ (1)

Then $\rho_{i,s} = \rho_{i,m} + \frac{n_i}{k_f} = 8\,\frac{ng}{cm^3} + \frac{2}{0.015}\ \frac{ng\ s}{s\ cm^2\ cm}$ (2)

$$= 141.3\ ng/cm^3$$ (1)

(c) Return to basic eq'n : $Pn_i = Q\frac{d\rho_{i,m}}{dx}$. Note $n_i \neq$ const, so we need an expression for n_i in terms of $\rho_{i,m}$.

$$n_i = k_f(\rho_{i,s} - \rho_{i,m}) = \Lambda(\rho_{sat} - \rho_{i,s})$$ (3)

$$\therefore (k_f + \Lambda)\rho_{i,s} = k_f\rho_{i,m} + \Lambda\rho_{sat}$$

$$\rho_{i,s} = \frac{k_f\rho_{i,m} + \Lambda\rho_{sat}}{k_f + \Lambda}$$

$$n_i = \frac{k_f}{k_f+\Lambda}\left[\cancel{k_f\rho_{i,m}} + \Lambda\rho_{sat} - \cancel{k_f\rho_{i,m}} - \Lambda\rho_{i,m}\right]$$

$$n_i = \frac{k_f\, \Delta}{k_f + \Delta}\left(\rho_{sat} - \rho_{i,m}\right)$$ (4)

Plug in to basic eq'n

$$\frac{P\, k_f\, \Delta}{q\,(k_f + \Delta)}\left(\rho_{sat} - \rho_{i,m}\right) = \frac{d\rho_{i,m}}{dx}$$ (4)

Call $B = \dfrac{P\, k_f\, \Delta}{q\,(k_f+\Delta)}$. Then $\dfrac{d\rho_{i,m}}{dx} = -B\left(\rho_{i,m} - \rho_{sat}\right)$

$$\therefore\ \rho_{i,m} - \rho_{sat} = \text{Const}\ e^{-Bx}$$ (2)

$\rho_{i,m} = 0$ @ $x = 0$, $\therefore$ const $= -\rho_{sat}$

$$\therefore\ \rho_{i,m} = \rho_{sat}\left[1 - e^{-Bx}\right]$$ (2)

Plug in #'s

$$Bx = \frac{\pi(0.2)(0.015)(2)(0.02)}{(0.314)(0.035)} \qquad \text{cm}\ \frac{\text{cm}}{\text{s}}\ \text{cm}\ \frac{\text{s}}{\text{cm}^3}$$

$$= 0.0343$$

$$\rho_{i,m} = 100\ \frac{\text{ng}}{\text{cm}^3} \times 0.0337 = 3.372\ \text{ng}/\text{cm}^3.$$ (3)

right idea / 2

36