ISYE 3133 Homework 3 Summer 2015

Due Thursday, June 25 at 10am.
Problems with asterisks (*) will be graded for correctness.

1. Find the dual of the LP below. Find optimal solutions to both the primal and the dual.

$$\max z = 4x_1 + x_2$$
$$x_1 + 2x_2 = 6$$
$$x_1 - x_2 \geq 3$$
$$2x_1 + x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

Primal solution: $x_1 = \frac{14}{3}$, $x_2 = \frac{2}{3}$, $z = \frac{58}{3}$

Dual formulation:

$\min w = 6y_1 + 3y_2 + 10y_3$
s.t. $y_1 + y_2 + 2y_3 \geq 4$
$2y_1 - y_2 + y_3 \geq 1$
y_1 urs, $y_2 \leq 0$, $y_3 \geq 0$

Dual solution: $w = \frac{58}{3}$, $y_1 = \frac{-2}{3}$, $y_2 = 0$, $y_3 = \frac{7}{3}$

2. Find the dual of the LP and show it has the same feasible region as the primal.

$$\max z = -3x_1 + x_2 + 2x_3$$
$$x_2 + 2x_3 \leq 3$$
$$-x_1 + 3x_3 \leq -1$$
$$-2x_1 - 3x_2 \leq -2$$
$$x_1, x_{2,} x_{3,} \geq 0$$

Dual: $\min w = 3y_1 - y_2 - 2y_3$
s.t. $-y_2 - 2y_3 \geq -3$
$y_1 - 3y_3 \geq 1$
$2y_1 + 3y_2 \geq 2$
$y_1, y_2, y_3 \geq 0$

The constraints for the dual and primal are equivalent (which can be seen by multiplying each of them by -1 in either formulation). Therefore, the set of points satisfying the constraints (i.e. the feasible regions) are the same.

*3. Find the optimal value of the LP below without using the simplex method.

$$\max z = 5x_1 + 3x_2 + x_3$$
$$2x_1 + x_2 + x_3 \leq 6$$
$$x_1 + 2x_2 + x_3 \leq 7$$
$$x_1, x_2, x_3, \geq 0$$

The dual is

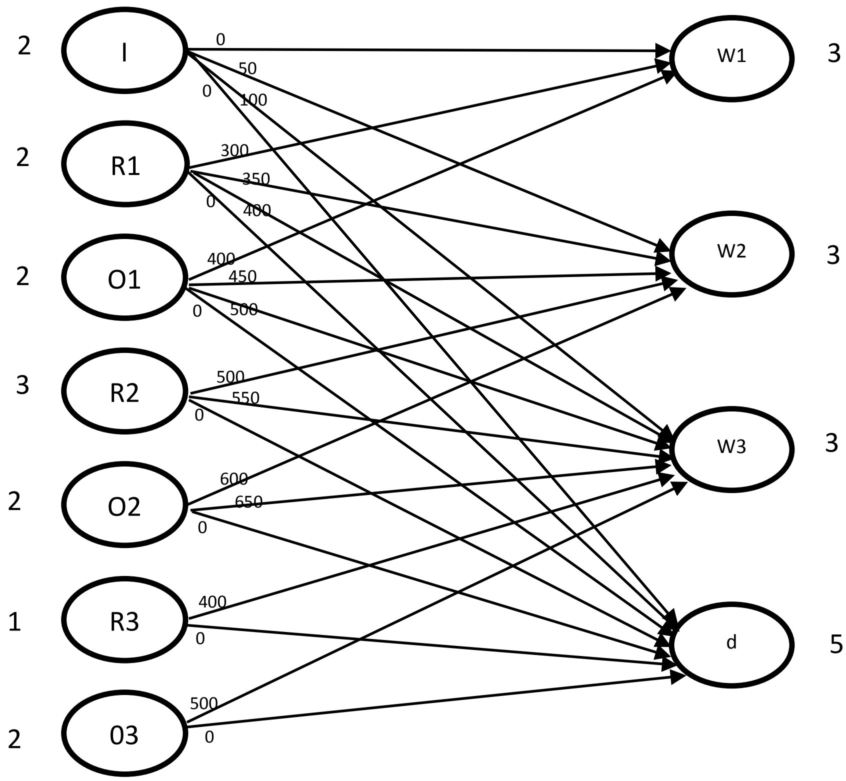
$$\min w = 6y_1 + 7y_2$$
$$\text{s.t.} \quad 2y_1 + y_2 \geq 5$$
$$y_1 + 2y_2 \geq 3$$
$$y_1 + y_2 \geq 1$$
$$y_1, y_2 \geq 0$$

Graphically we find the dual's optimal solution to be w = $\frac{49}{3}$, $y_1 = \frac{7}{3}$, $y_2 = \frac{1}{3}$.

*4. A company has agreed to supply its best customer with three widgets during *each* of the next 3 weeks, even though producing them will require some overtime work. The relevant production data are as follows:

Week	Maximum Production, Regular Time	Maximum Production, Overtime	Production Cost per Unit, Regular Time
1	2	2	$300
2	3	2	$500
3	1	2	$400

The cost per unit produced with overtime for each week is $100 more than for regular time. The cost of storage is $50 per unit for each week it is stored. There is already an inventory of two widgets on hand currently, but the company does not want to retain any widgets in inventory after the 3 weeks. Management wants to know how many units should be produced in each week to minimize the total cost of meeting the delivery schedule. Formulate this problem as a transportation problem and solve it using the transportation simplex method.



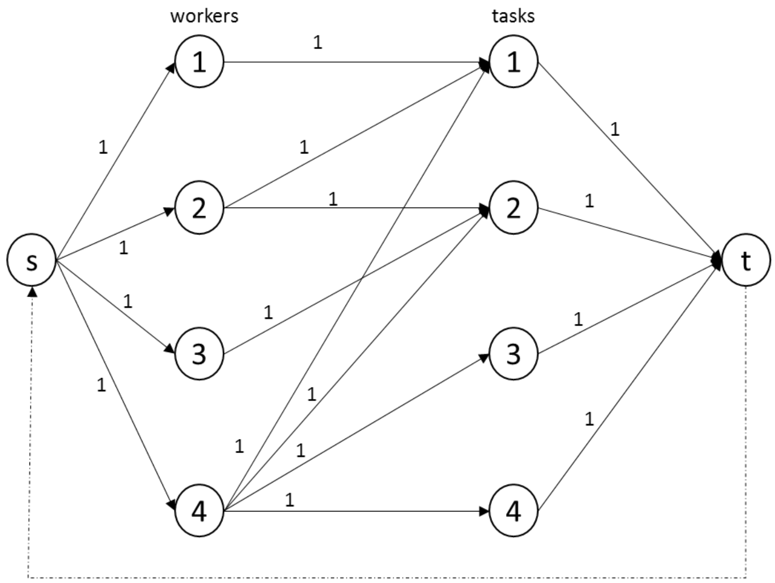
COSTS	1	2	3	d	supply
I	0	50	100	0	2
R1	300	350	400	0	2
O1	400	450	500	0	2
R2	M	500	550	0	3
O2	M	600	650	0	2
R3	M	M	400	0	1
O3	M	M	500	0	2
demand	3	3	3	5	

initial BFS	1	2	3	d
I	2			
R1	1	1		
O1		2	0	
R2			3	0
O2				2
R3				1
O3				2

optimal	1	2	3	d
I		2		
R1	1	1		
O1	2			
R2				3
O2				2
R3			1	
O3			2	

*5. Four workers are available to perform tasks 1-4. However, worker 1 can't do tasks 2, 3, or 4. Also, worker 2 can't do tasks 3 or 4 and worker 3 can't do tasks 1, 3, or 4. Worker 4 can do any task. Each worker can do at most one task, and each task should be performed at most once.

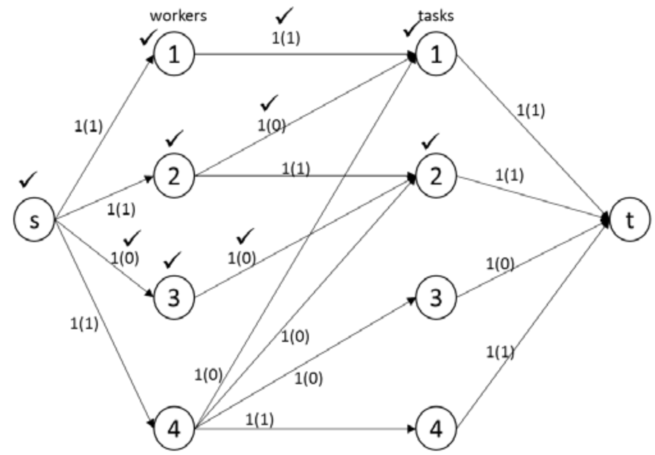
a. Draw the network for the maximum flow problem that can be used to determine whether all tasks can be assigned to a suitable worker.



b. Formulate this problem as a linear/integer program. Clearly define all variables.

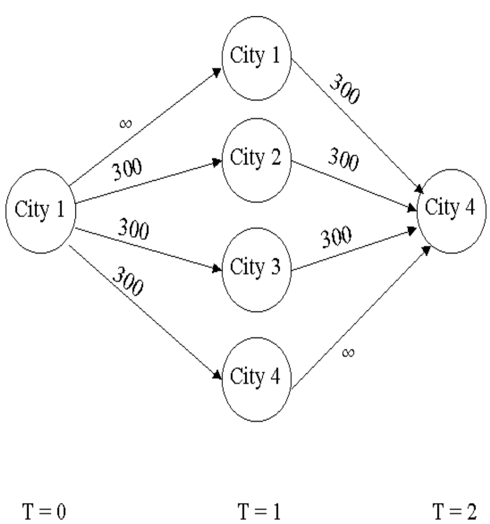
$$
\begin{aligned}
\max \quad & x_{ts} \\
s.t. \quad & x_{ij} \leq 1 \qquad \forall i,j \quad [0 \leq flow \leq capacity] \\
& x_{s,w1} = x_{w1,t1} \\
& x_{s,w2} = x_{w2,t1} + x_{w2,t2} \qquad [flow\ in = flow\ out] \\
& \vdots \\
& x_{ij} \geq 0
\end{aligned}
$$

c. Solve this problem using the Ford-Fulkerson method. You must start with 0 flow. Clearly label all networks you use, particularly the optimal network, and explain how you know the network is optimal.



Sink cannot be labeled ➡ solution is optimal.

6. Suppose up to 300 cars per hour can travel between any two of the cities 1, 2, 3, and 4. Set up a maximum flow problem that can be used to determine how many cars can be sent in the next two hours from **city 1** to **city 4**. (*Hint*: Have portions of the network represent t = 0, t = 1, and t = 2.) Do not solve.



7. During the next 4 months, a construction firm must complete 3 projects. Project 1 must be completed within 3 months and requires 8 months of labor. (8 workers working for 1 month = 8 months of labor.) Project 2 must be completed within 4 months and requires 10 months of labor. Project 3 must be completed in 2 months and requires 12 months of labor. Each month, 8 workers are available. During a given month, no more than 6 workers can work on a single job. Formulate a maximum flow problem (draw the network and write the LP) to determine whether all 3 projects can be completed on time.