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Solutions to Homework 10

1. (a) From the transition matrix P, we know that brand loyalty is strong, since there were 6 million beer drinkers in 1978 divided equally among three brands, we know that there are about $6 \times \frac{1}{3}$=2 million prefer Miller products.

   (b) P^{∞} is

$$\begin{bmatrix} 0.0720 & 0.8852 & 0.0428 \\ 0.0720 & 0.8852 & 0.0428 \\ 0.0720 & 0.8852 & 0.0428 \end{bmatrix}$$

   Therefore, in the long run, $6 \times 0.0720 = 0.4320$ million customers prefer Anheuser-Busch.

2. (a) The Markov Chain is aperiodic since $P_{ii} > 0$ for each state i.

   (b) The Markov Chain is irreducible since the states are communicate with each other.

   (c) The flow balance equations are:

$$\begin{aligned} \pi_0 p &= \pi_1 q \\ \pi_1 p &= \pi_2 q \\ &\ldots \\ \pi_i p &= \pi_{i+1} q \\ &\ldots \end{aligned}$$

   and $\sum_{i=0}^{\infty} \pi_i = 1$.
   Solving above equations we have: $\pi_i = (\frac{p}{q})^i (1 - \frac{p}{q})$ for each $i = 0, 1, 2, \cdots$
   Since the chain is irreducible and aperiodic, the stationary distribution exists and it is unique.

   (d) The Markov Chain is positive recurrent since $p < q$.

   (e) The Markov Chain is not positive recurrent since $p > q$, the chain will eventually drift to infinity.

3.(b-d) The recurrent states are {b,d,f} {c} are recurrent and irreducible sets, and both are aperiodic, {a,e} is transient.

   (e) P^{∞} is

$$\begin{bmatrix} 0 & 0.5405 & 0 & 0.2703 & 0 & 0.1892 \\ 0 & 0.5405 & 0 & 0.2703 & 0 & 0.1892 \\ 0 & 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0.5405 & 0 & 0.2703 & 0 & 0.1892 \\ 0 & 0.2703 & 0.5000 & 0.1351 & 0 & 0.0946 \\ 0 & 0.5405 & 0 & 0.2703 & 0 & 0.1892 \end{bmatrix}$$

4. (a) The whole chain is irreducible and finite, so every state is recurrent.

(b) P^∞ is

$$\begin{bmatrix} 0.2617 & 0.1386 & 0.1373 & 0.2746 & 0.1878 \\ 0.2617 & 0.1386 & 0.1373 & 0.2746 & 0.1878 \\ 0.2617 & 0.1386 & 0.1373 & 0.2746 & 0.1878 \\ 0.2617 & 0.1386 & 0.1373 & 0.2746 & 0.1878 \\ 0.2617 & 0.1386 & 0.1373 & 0.2746 & 0.1878 \end{bmatrix}$$

(c) Since the chain is irreducible and aperiodic, we know that $\pi = (0.2617, 0.1386, 0.1373, 0.2746, 0.1878)$ and $\lim_{n\to\infty} P_{ii}^{(n)} = \pi_i$