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Name Blue Key

Exam 1 ISyE 4301

Please read the following: This is a closed-book and closed-note exam. In addition, calculators and/or other devices such as phones are not allowed during the exam and must be cleared from your desk. By signing the following, you are agreeing to these terms and acknowledging that all of the work on this exam is your own.

_____(*Signature*)

The following multiple-choice questions are worth 5 points each. Clearly mark your answer.

1. A store uses an (*R,Q*) policy with an order quantity of 100. The annual demand faced by the store is normally distributed with a mean of 5200 and standard deviation of 700. The replenishment leadtime is 2 weeks. If they desire a service level of 95%, what should the value of *R* be?
   a. R=100+ 700*sqrt(2/52)*1.645
   b. R=100 + 700/12*1.645
   c. R=5200*sqrt(2/52)+700*sqrt(2/52)*1.645
   d. R=100+ 700*sqrt(2/52)*0.3289
   e. None of the above

$\sigma_L = \sqrt{\frac{2}{52}}(700)$

$Z_{.95} = 1.645$

$\mu_L = 200$

2. Suppose the percent change in quantity demanded decreases by 3% when the percent change in price increases by 1%. Which of the following is true?
   a. Demand is inelastic
   b. Demand is elastic
   c. Demand is inelastic
   d. We don't have enough information to determine

3. A store has a single customer that orders on average 40 items per week. Half of the time the customer orders 0 and half of the time they order 80 (but the store cannot predict which). The store replenishes every 2 weeks, and replenishment leadtime is zero. The store maintains a 100% service level. How much average inventory (pipeline plus cycle plus safety) does the store hold?
   a. Average inventory is 160
   b. Average inventory is 140
   c. Average inventory is 100
   d. Average inventory is 60
   e. None of the above

pipeline = 0

cycle = Avg/2 = 80/2 = 40

safety = 80 ← since could get 160 total demand over 2 weeks

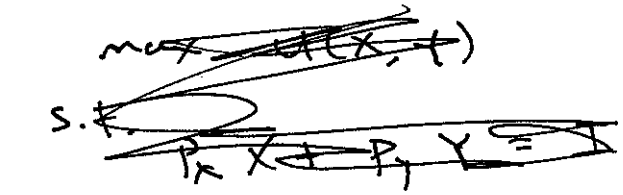
total = 120

4. A flight has a capacity of ~~100~~ 80 seats in coach class (all seats are equivalent). The expected marginal seat revenue heuristic came up with protection levels for the three ticket types (1, 2, and 3, which 1 being highest) of y1=20 and y2=30. Which of the following is true?
   a. At most 20 tickets can be sold of ticket type 1
   b. At least 30 tickets are held to be sold for ticket type 1 and 2
   c. At most 50 tickets can be sold for ticket type 3
   d. a. and c. are both true
   e. b. and c. are both true
   f. a., b., and c. are all true

5. A firm uses two inputs (Y_1 and Y_2) to produce units of output (Q). Suppose we triple the inputs and the output goes up by a factor of 3.2 times. Which is the best answer?
   a. The firm is showing decreasing returns to scale
   b. The firm is showing increasing returns to scale
   c. The marginal rate of technical substitution (Y_1 for Y_2) is 3.2
   d. a. and c. are both true
   e. b. and c. are both true
   f. None of the above

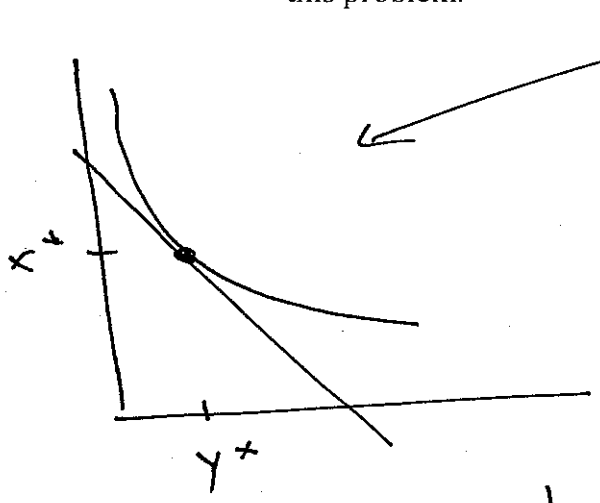
The two following concept questions are worth 10 points each. Please make your answers are clear and concise.

6. In class we discussed an individual's utility optimization problem. Do the following:
   a. For two items X and Y (with unit costs p_X and p_Y), write the individual's optimization problem to achieve a given utility U_0.



$$\min \; P_x X + P_y Y$$
$$\text{s.t.} \quad U(x,y) = U_0$$

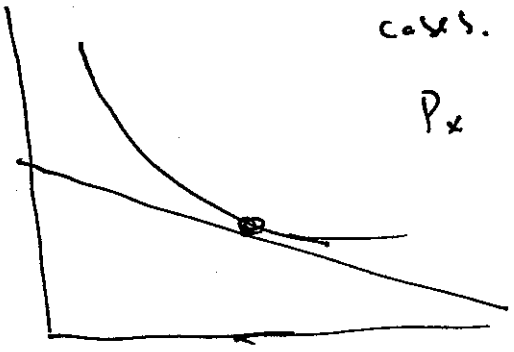
   b. Explain how the individual's quantity demanded for X is derived from this problem.



Solution gives x^*, y^*

change p_x and resolve to get new x^*

Do for several cases. Plot of P_x versus X is individual's demand



7. There are currently two car dealers (D1 and D2) in the Georgia Tech area that sell Chevy Volts; demand for Volts from each dealer is normally distributed with the same mean and variance. They are considering location pooling. They use a (R,Q) system to manage inventory.
   a. Explain how location pooling would work and the potential benefit that would be achieved from its implementation. Estimate the amount of inventory savings (use general expression).

originally, each manage inventory seperately. ss total safety stock = $2 z_{1-\alpha} \sigma_L$

If pool two dealers $\sigma_p^2 = (\sigma_1^2 + \sigma_2^2) \rightarrow \sigma_p = \sqrt{2}\,\sigma$ $(\sigma_1 = \sigma_2)$

So under location pooling the total safety stock goes to $\sqrt{2}\, z_{1-\alpha} \sigma_L$

   b. Suppose demand for Volts at the two dealers are positively correlated. What would the impact be?

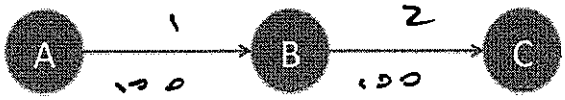
Recall $Var[D_1 + D_2] = \sigma_1^2 + \sigma_2^2 + 2\rho_{12}\sigma_1\sigma_2$

So if positively correlated, the σ_L term goes up. Therefore the benefit of location pooling would be reduced.

   c. What factors other than those mentioned in a. and b. should be considered if using location pooling?

The key factor is how to do the pooling. If there is a physical location for example, then there are costs associated with this. If it is virtual, it still needs to be organized and may incur lateral shipments (increasing logistics costs)

8. (20 points) Consider the flight network shown below. Capacity on each leg is 100. There are 7 possible tickets, with origin-destination (price; expected demand) 1. A-B ($160;20), 2. A-B ($110;60), 3. A-C ($450;20), 4. A-C ($300;40), 5. A-C ($200;50), 6. B-C ($220;30), and 7. B-C ($140;45).



a. Write the primal and dual formulations. Explain in words what the dual variables represent.

Primal

$$\max \quad 160x_1 + 110x_2 + 450x_3 + 300x_4 + 200x_5 + 220x_6 + 140x_7$$

s.t.

$$x_1 + x_2 + x_3 + x_4 + x_5 \le 100$$
$$x_3 + x_4 + x_5 + x_6 + x_7 \le 100$$
$$x_1 \le 20 \quad x_2 \le 60 \quad x_3 \le 20 \quad x_4 \le 40$$
$$x_5 \le 50 \quad x_6 \le 30 \quad x_7 \le 45 \qquad x_i \ge 0$$

Dual

$$\min \quad 100y_1 + 100y_2 + 20v_1 + 60v_2 + 20v_3 + 40v_4 + 50v_5 + 30v_6 + 45v_7$$

s.t.

$$y_1 + v_1 \ge 160 \qquad y_1 + y_2 + v_5 \ge 200$$
$$y_1 + v_2 \ge 110 \qquad y_2 + v_6 \ge 220$$
$$y_1 + y_2 + v_3 \ge 450 \qquad y_2 + v_7 \ge 140$$
$$y_1 + y_2 + v_4 \ge 300$$

$$y_i, v_i \ge 0$$

The y_i variables represent the value of the capacity on the corresponding leg.

A - C = 310

b. Suppose you find that the bid price for A-B is $120 and for B-C is $190. Which tickets should not be offered?

Close 2., 4., 5., 7.

c. Rewrite the dual formulation assuming we sold 5 tickets of B-C ($140;45). So Demand for 7 = 40
capacity of leg 2 = 95

$$\min \quad 100y_1 + 95y_2 + 20v_1 + 60v_2 + 20v_3 + 40v_4 + 50v_5 + 30v_6 + 40v_7$$

Rest is the same.

9. (20 points) A manufacturer produces flu vaccines for the flu season that they sell through CVS (assume newskid applies). It costs the producer $10 per unit and they sell to CVS at an intermediate price of $30 per unit. Demand for vaccines is normall distributed. CVS sells the vaccine through their stores at $40 per unit. Any units left over are salvaged at $5 each.
    a. Explain why if CVS orders the vaccines based on their analysis that it would lead to decreased profits for the entire supply chain (provide as much detail as you can to make the argument).

For coordinated

$$CR_C = \frac{40 - 10}{40 - 5} = \frac{30}{35}$$

For decentralized

$$CR_D = \frac{40 - 30}{40 - 5} = \frac{10}{35}$$

$$\therefore \quad Q_D < Q_C$$

This is due to downside risk at CVS (high cost of left overs). Since CVS orders less, CVS also sells less from coordinated and both are made worse off.

b. How could a revenue-sharing contract be set up to improve profitability for both the manufacturer and CVS as compared to a? Explain how to determine the components of the contract.

set

$$\frac{40-c}{40-5} = CR_c$$

& solve for c.

using this c will lead CVS to order the optimal amount. In this case it will be actual cost.

~~for order~~

To make up for lost profit at mfg, CVS will need to give enough of its profit back so that mfg is made at least as well off.

10. (15 points) A concert venue faces two types of demand. Demand 1 is given by $Q_1=120-P_1$ and demand 2 is given by $Q_2=100-2P_2$. The marginal cost of a unit is \$20. The firm can segment the market (e.g., checking the age of the person on their driver's license). In addition, the capacity of the venue is C. Write out the firm's optimization problem to determine the best prices to charge.

$P_1 = 120 - Q_1$ $\quad P_2 = \frac{1}{2}(100 - Q_2)$

$$\max_{P_1, P_2} \; P_1(120 - P_1) + P_2(100 - 2P_2) - 20((120 - P_1) + (100 - 2P_2))$$

s.t.

$$(100 - 2P_2) + (120 - P_1) \leq C$$

$$100 - 2P_2 \geq 0$$

$$120 - P_1 \geq 0$$