ISyE 4803 Final Exam
Summer 2010

Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Question 4
Total

(30) **1.** Consider a two station closed queueing network. Assume that there are three customers in the system. When a customer departs from a station, he either goes back to the same station or to the other station with equal probabilities. The service rate at station 1 is 4/hr and the service rate at station 2 is 6/hr. All service times are exponentially distributed. Compute the expected number of customers at each station in the long run.

(30) **2.** At registration at a very small college, students arrive at the English table with respect to a Poisson process of rate 10/hr and the Math table with respect to a Poisson process of rate 5/hr. A student who completes service at the English table goes to the Math table with probability 1/4 and to the cashier with probability 3/4. A student who completes service at the Math table goes to the English table with probability 2/5 and to the cashier with probability 3/5. Students who reach the cashier leave the system after they pay. Suppose that the service times for the English table, Math table, and cashier are exponentially distributed with rates 25/hr, 30/hr, and 20/hr, respectively.

(10) **a.** Does the stationary joint distribution of the number of students at the English table, Math table, and cashier exist? If it does, compute it.

(10) **b.** What is the expected number of students in the system in the long-run?

(10) **c.** What is the expected time that a student spends while registering for the classes?

(20) **3.** Consider a model with $S = \{s_1, s_2\}$, set of actions in state s_1 as $A_{s_1} = \{a_{1,1}, a_{1,2}\}$ and set of actions in state s_2 as $A_{s_2} = \{a_{2,1}\}$. We have $r^{a_{1,1}}(s_1) = 1$, $r^{a_{1,2}}(s_1) = 0$ and $r^{a_{2,1}}(s_2) = 2$, and $p^{a_{1,1}}(s_1, s_1) = 1$, $p^{a_{1,2}}(s_1, s_2) = 1$, and $p^{a_{2,1}}(s_2, s_2) = 1$. Suppose that you want to maximize the discounted infinite horizon expected reward.

(10) **a.** Compute the optimal policy when the discount factor $\alpha = 0.4$.

(10) **b.** Compute the optimal policy when the discount factor $\alpha = 0.6$.

(20) **4.** Consider the server allocation problem that we discussed in class. Suppose the size of the buffer between stations is 1. Suppose $\mu_{11} = 8$, $\mu_{12} = 12$, $\mu_{21} = 6$, and $\mu_{22} = 9$. Is the optimal server assignment policy unique? Justify your answer.