MATH 2401 QUIZ 4
SECTION K

Name:

(1) Find the gradient of the function $f(x, y, z) = x^2 + y^2 - 2z^2 + z \ln x$ at point $(1, 1, 1)$.

$f_x = 2x + \frac{z}{x}$ $\quad f_x(1,1,1) = 3.$

$f_y = 2y$ $\quad f_y(1,1,1) = 2$

$f_z = -4z + \ln x$ $\quad f_z(1,1,1) = -4$

$\therefore \nabla f|_{(1,1,1)} = \langle 3, 2, -4 \rangle.$

3 points
1 point for gradient
1 point for computation at (1,1,1)
1 point for correct answers.

(2) Find the derivative of the function $f(x, y, z) = xy + yz + zx$ at point $P_0(1, -1, 2)$ in the direction $\mathbf{u} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.

$\nabla f = \langle y+z, x+z, x+y \rangle.$ $\quad \nabla f|_{(1,-1,2)} = \langle 1, 3, 0 \rangle.$

$\frac{u}{|u|} = \frac{3}{7} i + \frac{6}{7} j - \frac{2}{7} k.$

$P_u(f)|_{(1,1,1)} = \nabla f|_{(1,1,1)} \cdot \frac{u}{|u|} = \frac{3}{7} + \frac{18}{7} - 0 = 3.$

4 points
1 point for gradient
1 point for gradient at (1,-1,2)
1 point for unit vector associated to u
1 point for correct answer

(3) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial^2 z}{\partial x^2}$ where $x + y + z = e^z$.

$$1 + \frac{\partial z}{\partial x} = e^z \cdot \frac{\partial z}{\partial x} \quad \Rightarrow \quad \frac{\partial z}{\partial x} = \frac{1}{e^z - 1}$$

$$\frac{\partial^2 z}{\partial x^2} = e^z \cdot \frac{\partial^2 z}{\partial x^2} + e^z \cdot \left(\frac{\partial z}{\partial x}\right)^2 \quad \Rightarrow \quad \frac{\partial^2 z}{\partial x^2} = \frac{e^z \left(\frac{\partial z}{\partial x}\right)^2}{1 - e^z} = -\frac{e^z}{(e^z - 1)^3}$$

3 points
1 point for equation involving the derivative of z relative to x
1 point for derivative of z relative to z.
1 point for second derivative