PHYS 2212 Test 3
Spring 2014

Name(print) KEY Lab Section ____

Lab section by day and time: Curtis(H), Ballantyne(Q), Kim(P)						
Monday	12:05-2:55pm	H01 or Q01	3:05-5:55pm	H02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	Q03 or P03	3:05-5:55pm	Q04 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	H03 or Q05	3:05-5:55pm	P05 or Q06	6:05-8:55pm	H04 or P06
Thursday	12:05-2:55pm	H05 or Q07	3:05-5:55pm	Q08 or H06	6:05-8:55pm	H07 or P07

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Sign your name on the line above

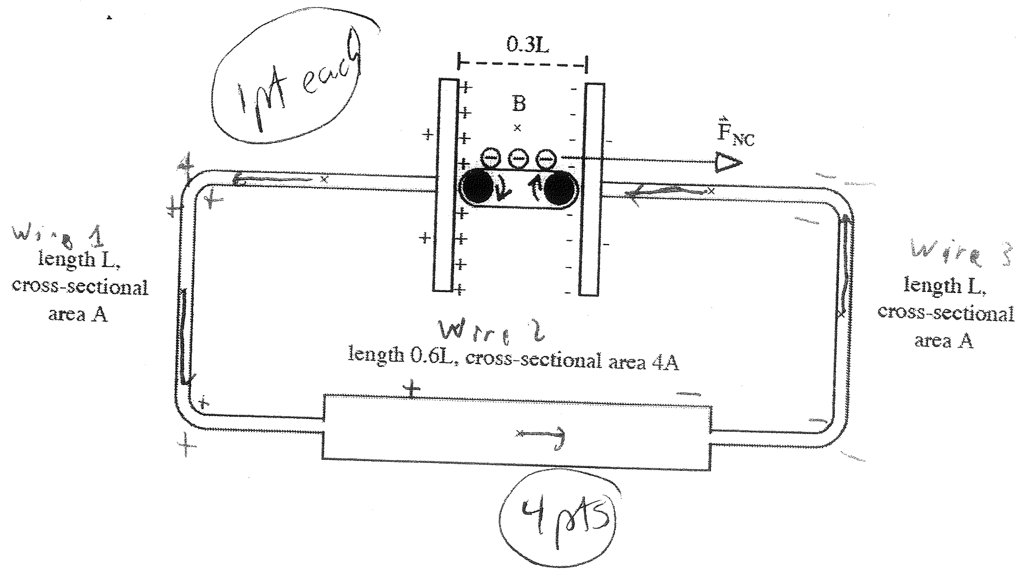
PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		CAPTAIN AMERICA
Problem 2 (25 pts)		IRONMAN
Problem 3 (25 pts)		THOR
Problem 4 (25 pts)		HULK

em 1 (25 Points)

A circuit contains an ideal mechanical battery which exerts a Non-Coulomb force F_{NC} to move electrons through the battery (with negligible internal resistance). The end plates of the battery are very large compared to the distance 0.3L between the plates (plates not drawn to scale). Two thin nichrome wires of length L and cross-sectional area A connect the battery to a thick nichrome wire of length 0.6L and cross-sectional area 4A. The mobility of the nichrome is u, and there are n mobile electrons per cubic meter in the nichrome.



(a 8pts) Show the electric field at the six locations marked with x (including location B between the plates). Pay attention to the relative magnitudes of the six vectors that you draw.

(b 5pts) Show the approximate distribution of charge on the surface of the nichrome wires. Make sure that your distribution is compatible with the electric fields that you drew in part (a). All ✓

(c 7pts) Calculate the number of electrons that leave the battery every second, in terms of the given quantities L, A, n, u, and F_{NC} (and fundamental constants). Be sure to show all of your work.

$$|\vec{E}_{NC}| = \frac{|\vec{F}_{NC}|}{|q_e|} \qquad \mathcal{E}_{mf} = 0.3L\,|\vec{E}_{NC}| = 0.3L\frac{|\vec{F}_{NC}|}{|q_e|}$$

$$R_{Eff} = R_{wire1} + R_{wire2} + R_{wire3}$$

$$R_{wire1} = \frac{L}{\sigma A} = \frac{L}{|q_e|nuA} \qquad \sigma = |q_e|nu \qquad R_{wire2} = \frac{0.6L}{|q_e|nu(4A)} \qquad R_{wire3} = \frac{L}{|q_e|nuA}$$

−0.5
−1
−2
−5.5

$$R_{eff} = \frac{2.15L}{|q_e|nuA}$$

$$\Delta V = IR$$

$$\mathcal{E}_{mf} = I R_{eff}$$

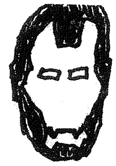
$$I = \frac{\mathcal{E}_{mf}}{R_{eff}} = \frac{0.3\cancel{L}\frac{|\vec{F}_{NC}|}{\cancel{|q_e|}}\,\cancel{|q_e|}nuA}{2.15\cancel{L}}$$

$$i = \frac{I}{|q_e|} = 0.14\frac{|\vec{F}_{NC}|}{|q_e|}nuA$$

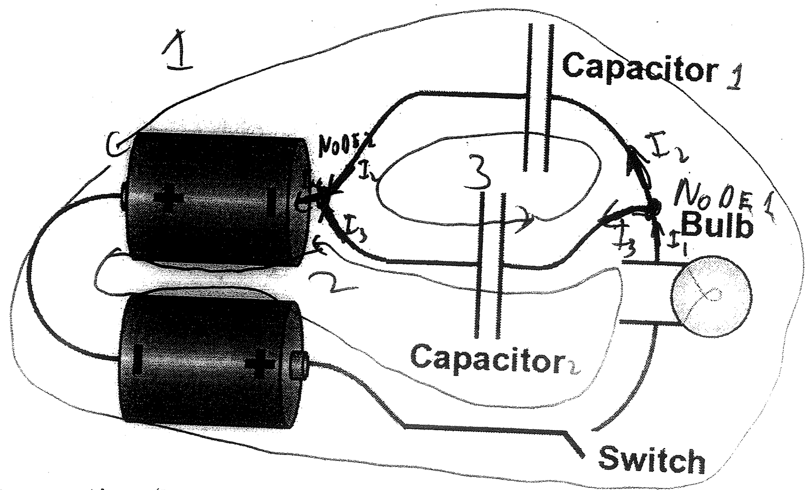
(d 5pts) Calculate the magnitude of the electric field between the plates of the battery (location B):

$$|\vec{E}_{NC}| = \frac{|\vec{F}_{NC}|}{|q_B|} \quad \underline{A\checkmark}$$

m 2 (25 Points)



Two batteries, two capacitors, and a light bulb are connected by wires as indicated in the diagram. These wires and circuit components are just like the ones you used in lab. The batteries each have a potential difference of Emf, the capacitors both have capacitance C, and the bulb has resistance R. Initially the capacitors are uncharged and the switch has just been closed.



(a 9pts) Write down the three energy conservation (loop rule) equations for this circuit. Be sure to label any quantities you use on your diagram.

3 Loop 1 $2\,Emf + \Delta V_{Bulb} + \Delta V_{Capacitor_1} = 0$

3 Loop 2 $2\,Emf + \Delta V_{Bulb} + \Delta V_{Capacitor_2} = 0$

3 Loop 3 $\cancel{2\,Emf + \Delta}\ \Delta V_{Capacitor_1} - \Delta V_{Capacitor_2} = 0$

(b 3pts) Write down the charge conservation (node rule) equation for this circuit you would need to determine the current through the bulb and both capacitors. Be sure to label the currents in your diagram.

NODE 1 $I_1 = I_2 + I_3$ 3pt

NODE 2 $I_4 = I_2 + I_3$

(c 3pts) Using your results from part (a) and (b), determine the initial current passing through the bulb.

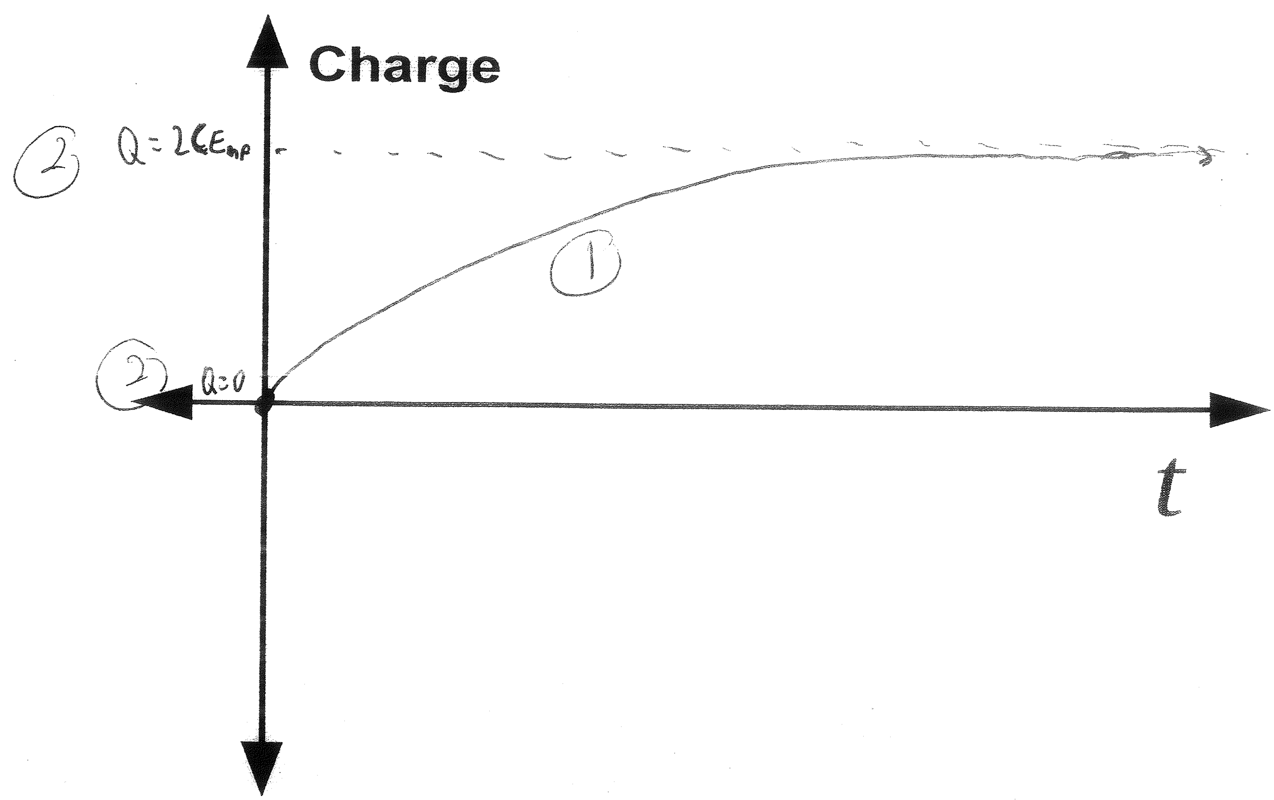
When first on, $\Delta V_{capacitor\,1} = \Delta V_{capacitor\,2} = 0$

So, loop 1 $\quad 2E_{mf} + \Delta V_{bulb} + \cancel{\Delta V_{capacitor\,1}}^{\,0} = 0$ (2pt)

$$2E_{mf} = IR$$

$$I = \frac{2E_{mf}}{R}$$ (1pt)

(d 5pts)) Make a qualitative sketch of the **charge vs. time** on the top capacitor in this circuit (starting from time $t = 0$). Be sure to label this curve and indicate the starting and final values for the charge. You do not need to solve a differential equation. Think about the initial and final charge on each capacitor and what type of curve should connect these values based on our discussion in class.



(e 5pts) After fully charging the capacitors, a sheet of plastic whose dielectric constant is K is inserted into one of the capacitors such that it completely fills the gap. What is the final equilibrium charge on the positive plate of each capacitor after inserting the plastic?

$\vec{E}_{Now} = \frac{\vec{E}_{old}}{K}$, but charges back to $\vec{E}_{old}$ so

~~Q_{NEW}~~

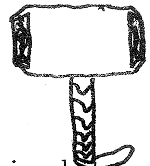
$|\vec{E}_{cap}| = \frac{Q_{old}}{A\epsilon_0} = \frac{Q_{NEW}}{KA\epsilon_0}$

$Q_{NEW} = K Q_{OLD}$

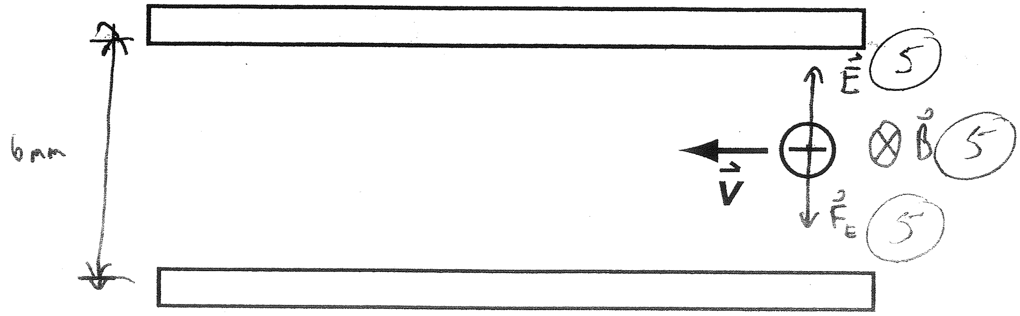
$Q_{NEW} = 2 \mathcal{E}_{mf} KC$

Other capacitor still at $Q = C\Delta V = 2\mathcal{E}_{mf} C$

Problem 3 (25 Points)



An electron moving with a velocity $\vec{v}$ enters a region between two charged parallel plates that are 6 mm apart. The electron is deflected toward the bottom plate.



(a 5pts) Draw and label the electric field vector due to the parallel plates at the location of the electron. Label it $\vec{E}$.

(b 5pts) Draw and label the electric force vector acting on the electron. Label it $\vec{F}_E$.

(c 5pts) If one were to apply a magnetic field, B, that allows the electron to travel between the plates without any deflection, draw the direction of that magnetic field. Label it $\vec{B}$.

(d 10pts) If the magnitude of this magnetic field is 2.2×10^{-3} T and the potential difference between the two plates is 180 V, what is the speed of the electron when it enters the region between the two plates? ***Show all steps in your work.***

No Deflection $\quad \vec{F}_E = -\vec{F}_B$

$q_e = -1.6\times10^{-19}$ C

$\vec{F}_E = q_e \vec{E}$ $\quad$ For constant $\hat{E}$, $\vec{E} = \frac{\Delta V}{d}\hat{E}$

$\hat{E} = \langle 0, 1, 0 \rangle$

$\Delta V = 180$ V

$d = 0.006$ m

$\vec{F}_B = q_e \vec{v} \times \vec{B}$

$\vec{v} = \langle$ [illegible] $, 0 \rangle$

$\vec{F}_B = -q_e |\vec{v}| |\vec{B}| \langle 0, 1, 0 \rangle$

$\vec{B} = \langle 0, 0, -2.2\times10^{-3} \rangle$ T

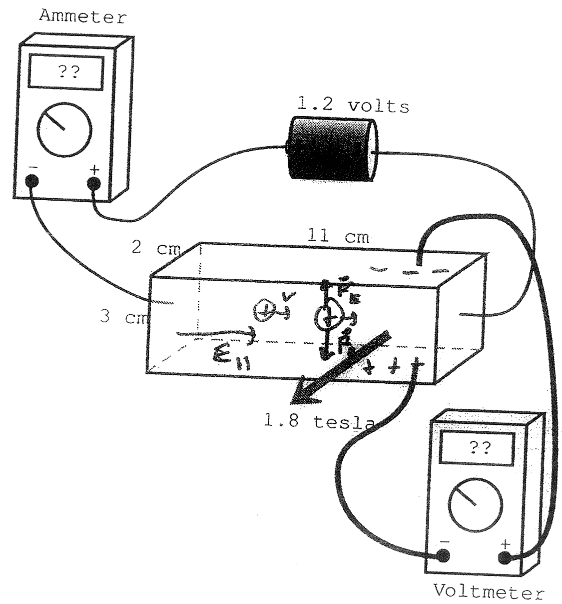
$$q_e \frac{\Delta V}{d} \langle 0, 1, 0 \rangle = +q_e |\vec{v}| |\vec{B}| \langle 0, 1, 0 \rangle$$

$$|\vec{v}| = \frac{\Delta V}{d|\vec{B}|} = 1.36\times10^{7}\ \frac{m}{s}$$

-0.5
-1.5
-3.0
-8.0

Problem 4 (25 Points)

Drawn below is a circuit that include a bar 11 cm long with a rectangular cross section 3 cm high and 2 cm deep, connected to a 1.2-volt battery and an ammeter. The resistance of the copper connecting wires and the ammeter, and the internal resistance of the battery, are all negligible compared to the resistance of the bar. Using large coils not shown on the following diagram, a uniform magnetic field of 1.8 tesla is applied perpendicular to the bar (out of the page, as shown). A voltmeter is connected across the bar, with the connections across the bar carefully placed directly across from each other. The mobile charges in the bar have charge $+e$, their density is 7×10^{25} per cubic meter, and their mobility is $3 \times 10^{-5}\ \dfrac{\text{m/s}}{\text{V/m}}$.



~~-0.5~~
~~-1.5~~
~~-3.0~~

-1
-2
-4.5
-12

(a 15pts) Predict the reading of the voltmeter, including sign. Explain carefully, using diagrams to support your explanation. Remember that a voltmeter reads positive if the "+" terminal is connected to higher potential.

In equilibrium $\quad \vec{F}_{E\perp} = -\vec{F}_B \quad$ Constant $\vec{E} \rightarrow \vec{F}_E = q\dfrac{|\Delta V_\perp|}{h}\hat{E}$

~~From diagram and positive~~ particles, ~~positive charge~~

Positive charges move in $+\hat{x}$ direction from diagram

$q = +1.6\times10^{-19}\,C$

$h = 0.03\,m$

$w = 0.02\,m$

$\vec{B} = \langle 0,0,1.8\rangle T$

$\vec{F}_B = q\,\vec{v}\times\vec{B} \qquad \vec{v} = \langle v,0,0\rangle$

$v = \dfrac{I}{|q|nA} \qquad A = hw$

$L = 0.11\,m$

$\Delta V_{\parallel} = 1.2\,V$

$I = \dfrac{\Delta V_{\parallel}}{R} \qquad R = \dfrac{L}{\sigma A} \qquad \sigma = |q|nu$

$n = 7\times10^{25}\,m^{-3}$

$u = 3\times10^{-5}\,\dfrac{m/s}{V/m}$

$I = \dfrac{\Delta V_{\parallel}}{L}|q|nu\,hw \Rightarrow v = \dfrac{\Delta V_{\parallel}\cancel{|q|}\cancel{n}u\,\cancel{hw}}{L\cancel{|q|}\cancel{n}\cancel{hw}} = \dfrac{\Delta V_{\parallel}u}{L}$

$\vec{F}_B = q\dfrac{\Delta V_{\parallel}u}{L}|\vec{B}|\langle 0,-1,0\rangle$

$\hat{E} = \langle 0,1,0\rangle$

$\cancel{q}\dfrac{\Delta V_{\parallel}u}{L}|\vec{B}|\langle 0,1,0\rangle = \cancel{q}\dfrac{|\Delta V_\perp|}{h}\hat{E}$

$|\Delta V_\perp| = \dfrac{\Delta V_{\parallel}u}{L}|\vec{B}|h = 1.76\times10^{-5}\,V$

$\Rightarrow \boxed{\Delta V \quad 1.76\times10^{-5}\,V}$

(b 10pts) Predict the reading of the ammeter, including sign. Remember that an ammeter reads positive if conventional current enters the "+" terminal.

From part A

$\Delta V_{11} = 1.2\ V$

$L = 0.11m$

$|I| = \frac{|\Delta V_{11}|}{L} |q| n u h w$

$|q| = 1.6 \times 10^{-19}\ C$

$n = 7 \times 10^{25}\ m^{-3}$

$|I| = 2.2\ A$

$u = 3 \times 10^{-5}\ \frac{m/s}{V/m}$

$h = 0.03m$

$w = 0.02m$

~~Am~~ Ammeter reading $= +2.2A$ because conventional current flows into "+" terminal

$\boxed{\begin{array}{l} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \dfrac{d\vec{v}}{dt}$

$\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\dfrac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta U_{el} = q\Delta V$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x\Delta x + E_y\Delta y + E_z\Delta z)$

$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$

$\oint \vec{E} \bullet \hat{n} dA = \dfrac{\sum q_{inside}}{\epsilon_0}$

$\oint \vec{B} \bullet \hat{n} dA = 0$

$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left|\dfrac{d\Phi_{mag}}{dt}\right|$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \dfrac{d}{dt} \int \vec{E} \bullet \hat{n} dA\right]$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r\sqrt{r^2 + (L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$, $\vec{p} = \alpha\,\vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{qz}{(z^2 + R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{(z^2 + R^2)^{1/2}}\right]$ (z along axis)

$\left|\vec{E}_{disk}\right| \approx \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{R}\right] \approx \dfrac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \dfrac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \dfrac{Q/A}{\epsilon_0}\left(\dfrac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{\ell} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \dfrac{\mu_0}{4\pi}\dfrac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$

$\left|\vec{B}_{loop}\right| = \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$)

$\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \dfrac{1}{4\pi\epsilon_0}\dfrac{-q\vec{a}_\perp}{c^2 r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \dfrac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\,nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\,nu$

$J = \dfrac{I}{A} = \sigma E$

$R = \dfrac{L}{\sigma A}$

$E_{dielectric} = \dfrac{E_{applied}}{K}$

$\Delta V = \dfrac{q}{4\pi\epsilon_0}\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right]$ due to a point charge

$I = \dfrac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$

$K \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\frac{d\vec{p}}{dt}_{\perp}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\, dx = ax + c \quad \int ax\, dx = \frac{a}{2}x^2 + c \quad \int ax^2\, dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N $\cdot$ m^2/C^2)$^{-1}$
Magnetic constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T $\cdot$ m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T $\cdot$ m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T