

ISyE 3232 Exam # 2
Fall 2012

Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Question 4
Total

(25) **1.** A professor has two light bulbs in his garage. When both are burned out, they are replaced and the next day starts with two working light bulbs. Suppose that when both are working, one of the two will go out with probability 0.2 and we ignore the possibility of losing two on the same day. However, when only one is there, it will burn out with probability 0.05. What is the long run fraction of time that there is exactly one bulb working? ?

(25) **2.** Roll a fair die repeatedly. Let X_n be the number of values we have seen in the first n rolls for $n \geq 1$.

(a) (15) Is $\{X_n : n \geq 1\}$ a Markov chain? If it is, provide the state space and the probability transition matrix.

(b) (10) Classify the states of $\{X_n : n \geq 1\}$ if it is a Markov chain. That is specify the irreducible sets, recurrent and transients states and the period of each state.

(30) **3.** (a) (15) Suppose that in a hardware store you must first go to server 1 to get your goods and then to server 2 to pay for them. Suppose that the times for two activities are exponentially distributed with mean 6 minutes and 3 minutes, respectively. Compute the expected amount of time that it will take Bob to get his goods and pay if when he comes in there is one customer named Al with server 1 and no one at server 2.

(b) (15) A flashlight needs two batteries to be operational. You start with four batteries. Whenever a battery fails it is replaced by one of the remaining ones (if there is one). Suppose that the lifetime of each battery is exponential with mean 100 hours. Let T be the time until flashlight can no longer operate. Compute $E[T]$.

(20) 4. Let $\{N(t) : t \geq 0\}$ be a Poisson process with rate 3/hr. Let S_n be the time of the n th arrival

(a) (5) Compute $E[S_{12}]$.

(b) (10) Suppose that 5 customers came in the first two hours. What is the expected value of S_{12} in this case?

(c) (5) Suppose that 5 customers came in the first two hours. What is the expected number of arrivals in the first five hours in this case?