ISyE 3232 Exam # 1
Spring 2013

Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Question 4
Total

(25) **1.** A department store sells Christmas cards during Christmas season for \$3 per pack. Any unsold cards after Christmas are sold at half-price. The cost of procurement of the cards is \$2.50 per pack. How many cards should the store stock for the season in order to minimize its expected cost if the demand for Christmas cards has **discrete** uniform distribution between 1 and 30?

b. What is the minimum expected cost (i.e. expected cost corresponding to Q^* that you computed above)?

(30) **2.** Let $\{X_n : n \geq 0\}$ be a Markov chain with state space $S = \{A, B, C, D\}$ and the following probability transition matrix.

$$P = \begin{bmatrix} 0.1 & 0.3 & 0.2 & 0.4 \\ 0.2 & 0.3 & 0.2 & 0.3 \\ 0.3 & 0.3 & 0.1 & 0.3 \\ 0.2 & 0.1 & 0.4 & 0.3 \end{bmatrix}$$

(a) (10) Compute $P\{X_1 = A, X_3 = C, X_4 = B, X_5 = A | X_0 = B\}$

(b) (20) Compute $E(X_2)$ if $P\{X_0 = A\} = 0.2$, $P\{X_0 = B\} = 0.4$, $P\{X_0 = C\} = 0.3$, and $P\{X_0 = D\} = 0.1$.

(20) **3.** In each game, a gambler wins the dollars he bets with probability 0.4 and loses them with probability 0.6. If he has less than \$3, he will bet all he has. Otherwise, since his goal is to have \$5, he will only bet the difference between \$5 and what he has. He continues to bet until he has \$0 or \$5. Let X_n be the amount he has after the nth bet. Is $\{X_n : n \geq 0\}$ a Markov chain? Why? If it is, write down the state space and the probability transition matrix.

(25) **4.** Consider a job shop with 5 stations in series in which the arrivals from outside come to the first station with respect to a Poisson process of rate 5/hr. Each station has a single server with exponentially distributed service times. The mean service times at stations 1, 2, 3, 4, and 5 are 3, 6, 5, 4, and 10 minutes, respectively. Compute

(a) (15) expected amount of time that each job spends in the system

(b) (10) expected number of jobs in the system.