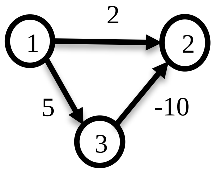
ISYE 3133 Homework 4 Summer 2015

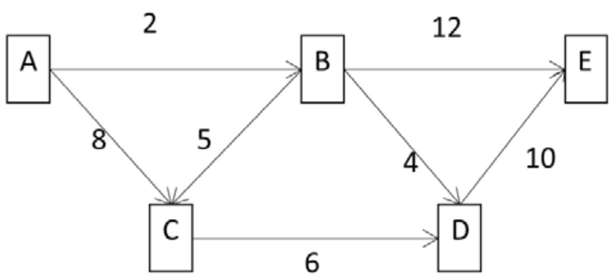
Due Tuesday, July 7th at 10am.
Problems with asterisks (*) will be graded for correctness.

1. Give an example to show that Dijkstra's algorithm can give incorrect results if negative costs are allowed.



Node 1 gets permanently labeled 0, node 2 gets temp label 2, node 3 gets temp label 5
Node 2 gets permanently labeled 2
Node 3 gets permanently labeled 5
The shortest path from node 1 to node 2 is -5, but Dijkstra's algorithm gives a solution of length 2.

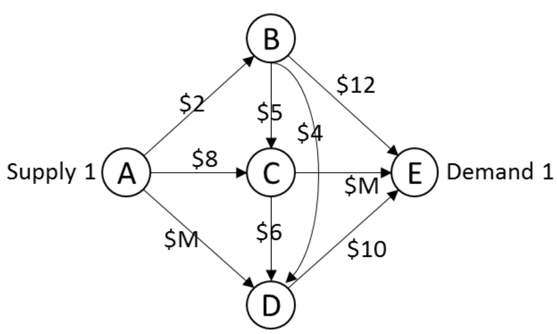
2. Consider the following network:



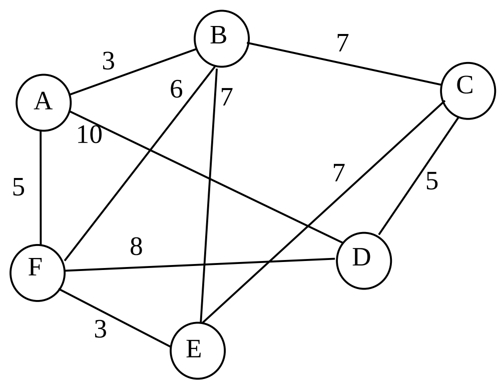
a. Find the shortest path from A to E.

A – B – D – E (length 16)

b. Formulate the problem as a transportation/transshipment problem.



*3. Consider the following network:



a. Find a minimum spanning tree for the network.
MST = {(A,B), (E,F), (A,F), (C,D), (B,C)} value = 23

b. Formulate a linear program that will solve the MST problem for the above network. (Hint: each arc/edge will have its own binary variable indicating whether or not it is in the MST.)

$\min 3x_{AB} + 10x_{AD} + 5x_{AF} + \cdots + 5x_{CD}$
$x_{AB} + x_{AD} + x_{AF} + \cdots + x_{CD} = 5$ (spanning tree for a graph with 6 nodes has 5 arcs)
$x_{AB} + x_{AD} + x_{BF} \leq 2$ (can't have cycle A-B-F)
$x_{BF} + x_{BE} + x_{EF} \leq 2$ (can't have cycle B-E-F)
$x_{BF} + x_{BC} + x_{EF} + x_{CE} \leq 3$ (can't have cycle B-C-E-F)
$\vdots$
(include for all cycles)

4. The following puzzles are taken from Raymond Smullyan's "The Lady or the Tiger." In each puzzle a prisoner is faced with a decision where he must open one of two doors. Behind each door is either a lady or a tiger. They may be both tigers, both ladies, or one of each. If the prisoner opens a door to find a lady he will marry her, and if he opens a door to find a tiger he will be eaten alive. Of course, the prisoner would prefer to be married than eaten alive. Each of the doors has a sign bearing a statement that may be either true or false.
Formulate an integer program for each puzzle to help the prisoner avoid being eaten alive by a tiger.

a) The First Trial
The statement on door one says, "In this room there is a lady, and in the other room there is a tiger."
The statement on door two says, "In one of these rooms there is a lady, and in one of these rooms there is a tiger."
The prisoner is informed that one of the statements is true and one is false.

Let t_i = 1 if statement on door i is true, 0 otherwise.
Let x_{ij} = 1 if item i is in room j, 0 otherwise, where 1 = lady, 2 = tiger

Max x_{11}
s.t.
$t_1 + t_2 = 1$ (one statement is false)
$x_{11} + x_{21} = 1$ (room 1 has either a lady or a tiger)
$x_{12} + x_{22} = 1$ (room 2 has either a lady or a tiger)

[if $t_1 = 1$ ($t_1 > 0$), then $x_{11} + x_{22} = 2$ ($x_{11}+x_{22} - 2 \geq 0$)]
$t_1 \leq Mz$
$2-x_{11}-x_{22} \leq M(1-z)$
$z = 0,1$
[if $x_{11} + x_{22} = 2$ ($x_{11}+x_{22} - 1 > 0$), then $t_1 = 1$ ($t_1 \geq 1$)]
$x_{11}+x_{22} - 1 \leq Mz_2$
$1-t_1 \leq M(1-z_2)$

[if $t_2 = 1$ ($t_2 > 0$), then $x_{11} + x_{12} = 1$ ($x_{11}+x_{12} - 1 \geq 0$) and $x_{21} + x_{22} = 1$]
$t_2 \leq Mz_3$
$1-x_{11}-x_{12} \leq M(1-z_3)$
$1-x_{21}-x_{22} \leq M(1-z_3)$

***b) The Second Trial**
The statement on door one says, "At least one of these rooms contains a lady."
The statement on door two says, "A tiger is in the other room."
The statements are either both true or both false.

Max x_{11}
s.t.
$t_1 = t_2$ (either both true or both false)
$x_{11} + x_{21} = 1$ (room 1 has either a lady or a tiger)
$x_{12} + x_{22} = 1$ (room 2 has either a lady or a tiger)

[if $t_1 = 1$ ($t_1 > 0$), then $x_{11}+x_{12} \geq 1$ ($x_{11}+x_{12} - 1 \geq 0$) and $x_{21} = 1$ ($x_{21} \geq 1$)]
$t_1 \leq Mz_1$
$1-x_{11}-x_{12} \leq M(1-z1)$
$1-x_{21} \leq M(1-z_1)$

[if $t_1 = 0$ ($t_1 < 1$ or $1-t_1 > 0$), then $x_{11}+x_{12} = 0$ ($x_{11}+x_{12} \leq 0$ or $-x_{11}-x_{22} \geq 0$) and $x_{21}=0$]
$1-t_1 \leq Mz_2$
$x_{11}+x_{12} \leq M(1-z_2)$
$x_{21} \leq M(1-z_2)$

c) The Third Trial
The statement on door one says, "Either a tiger is in this room or a lady is in the other room."

The statement on door two says, "A lady is in the other room."
The statements are either both true or both false.

Max x_{11}
s.t.
$t_1 = t_2$ (either both true or both false)
$x_{11} + x_{21} = 1$ (room 1 has either a lady or a tiger)
$x_{12} + x_{22} = 1$ (room 2 has either a lady or a tiger)

[if $t_1 = 1$ ($t_1>0$), then $x_{21}+x_{12} \geq 1$ ($x_{21}+x_{12} - 1 \geq 0$) and $x_{11} = 1$ ($x_{11} \geq 1$)]

$t_1 \leq Mz_1$
$1-x_{21}-x_{12} \leq M(1-z_1)$
$1-x_{11} \leq M(1-z_1)$

[if $x_{11} = 1$, then $t_1 = 1$]

$x_{11} \leq t_1$

[if $x_{21}+x_{12} \geq 1$, then $t_1=1$]

…

*5. A child lost all of his toys when his suitcase went missing on a flight out of Atlanta. The airline will replace the toys, but they need to know how many there were. The child doesn't remember the exact number, but he (of course) remembers that when he divided the number of toys by 2, 3, 4, 5, or 6, there was always exactly one toy leftover. When he divided them by 7, there were no toys leftover. Formulate an integer program that will determine the minimum number of toys the boy had.

min x
s.t.
$x = 2y_2+1$
$x = 3y_3+1$
$x = 4y_4+1$
$x = 5y_5+1$
$x = 6y_6+1$
$x = 7y_7$
x, y integer