ChBE 3210 Spring 2014 Final Exam Solutions

Problem I/

A) False; Sh $\leftrightarrow$ Nu

B) True; $Sc = \frac{\nu}{D_{AB}}$

C) False; on L-side of equilibrium line

D) False; $\Delta T_H > \Delta T_C$ and $(\dot{m}\Delta T)_H = (\dot{m}\cdot\Delta T)_C$ (c_p same)

E) True

F) True;

G) True; $r^2 \cdot N_{A,r} = \text{constant}$

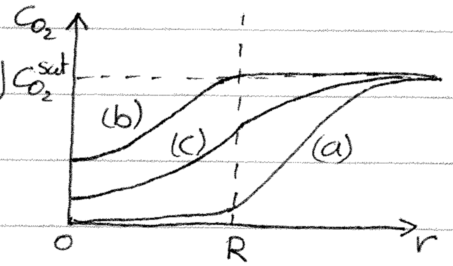
H) True; D_{AB} dominant $\Rightarrow$ no gradients

I) False

J) True; upward facing hot plate has higher h

Problem II/

A)

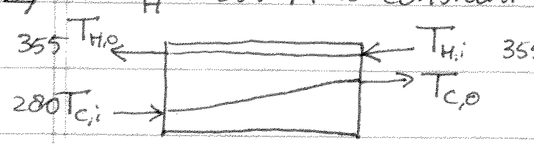


* for all cases: $\left.\frac{dc}{dr}\right|_{r=0} = 0$ (symmetry), $c \rightarrow c^{sat}$ for $r \rightarrow \infty$
* (a): very low conc. inside sphere
  (b): $c = c^{sat}$ at $r = R$
  (c): "in between" (a) and (b)

B) * spray tower; since resistance is mostly in gas phase, mixing of gas phase by droplets will enhance <u>overall</u> mass transfer.

C) * mass transfer is liquid-phase controlled, so k_G does not have real effect on <u>overall</u> coefficient K_L: $\frac{1}{K_L} = \frac{1}{k_L} + \frac{1}{mk_G}$ (k_G large) $\Rightarrow K_L \approx k_L$

Problem III/ * $T_H = 355$ K is constant (sat. steam at 0.51 bar) $\Rightarrow$ condenser behaves like ideal counterflow



A) $(\dot{m}\, c_p\, \Delta T_c)_C = \dot{m}_{steam} \cdot h_{fg} \Rightarrow \Delta T_c = \frac{\dot{m}_{steam}}{\dot{m}_c} \cdot \frac{h_{fg}}{c_{p,cold}} = \frac{1.5}{15} \cdot \frac{2304 \cdot 10^3\ J/kg}{4180\ J/kg \cdot K} = 55.1\ K$

h_{fg} from App.; $c_{p,cold}$ guess at 300K

$\Rightarrow T_{c,o} = 335.1\,K$ ($\Rightarrow T_{c,ave} \approx 307\,K$) close enough

B) $F = 1$ (T_H constant $\Rightarrow Z = 0$)

C) $UA = \left(\frac{1}{h_i A_i} + \frac{1}{h_o A_o}\right)^{-1}$

$A_i = A_o = A$ (thin-walled)

$\Rightarrow U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1}$ given

* h_i? Flow through pipe: $Re_D = \frac{\rho V D}{\mu} = \frac{4\dot m}{\pi D \underset{\hookrightarrow \text{ at } T_{c,ave} \approx 307K}{\mu}}$ $\left(\dot m = \rho V \frac{\pi}{4} D^2\right) = \frac{4 \cdot 15/100}{\pi \cdot 0.01 \cdot 700 \cdot 10^{-6}} = 2.73 \cdot 10^4$ (turbulent)

$\Rightarrow$ Dittus-Boelter: $Nu_D = 0.023 \cdot Re_D^{0.8} \cdot Pr^{0.4}$ (water being heated) $= 0.023 \cdot (2.73 \cdot 10^4)^{0.8} \cdot (5.0)^{0.4} = 155$

$\Rightarrow h_i = \frac{k}{D} Nu_D = \frac{0.62}{0.01} \cdot 155 = 9608 \ W/m^2K$

$\Rightarrow U = \left(\frac{1}{5000} + \frac{1}{9608}\right)^{-1} = 3289 \ W/m^2 \cdot K$

D) $UA\,\Delta T_{LM} \cdot F = (\dot m h_{fg})_{steam} \Rightarrow A = 100 \cdot \pi D L = \frac{\dot m h_{fg}}{U \Delta T_{LM}}$ $\quad \Delta T_{LM} = \frac{75-20}{\ln(75/20)} = 41.6 \ K$

$\Rightarrow L = \frac{1.5 \cdot 2304 \cdot 10^3}{3289 \cdot 41.6 \cdot 100 \cdot \pi \cdot 0.01} = 8.04 \ m$

E) $\dot m$ changes for steam, but also $T_{c,o}$ (and thus ΔT_{LM})

$\Rightarrow$ new situation: $\Delta T_c = \frac{1.0}{15} \cdot \frac{2304 \cdot 10^3}{4180} = 36.7 \ K \Rightarrow T_{c,o} = 316.7 \ K \Rightarrow \Delta T_{LM} = \frac{75 - 38.3}{\ln(75/38.3)} = 54.6 \ K$

$\Rightarrow U = \frac{\dot m h_{fg}}{100 \pi D L \cdot \Delta T_{LM}} = \frac{1.0 \cdot 2304 \cdot 10^3}{100 \cdot \pi \cdot 0.01 \cdot 8.04 \cdot 54.6} = 1671 \ W/m^2K$

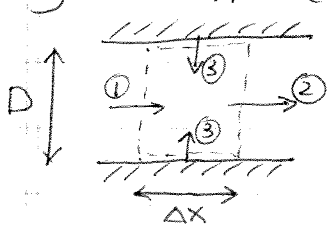
Problem IV/A) k_c? Flow in pipe $\Rightarrow Re_D = \frac{V_{ave} \cdot D}{\nu_{\rightarrow 300K\,air}} = \frac{0.3 \cdot 0.05}{1.57 \cdot 10^{-5}} = 955$ laminar!

$$Sc = \frac{\nu}{D_{AB}} = \frac{1.57 \cdot 10^{-5}}{10^{-5}} = 1.57$$

$$\Rightarrow Sh_D = \frac{k_c D}{D_{AB}} = 1.86 \cdot \left(\frac{D}{L} \cdot Re \cdot Sc\right)^{1/3} = 1.86 \cdot \left(\frac{0.05}{6} \cdot 955 \cdot 1.57\right)^{1/3} = 4.32$$

$$\Rightarrow k_c = 4.32 \cdot \frac{10^{-5}}{0.05} = 8.63 \cdot 10^{-4} \ m/s = 0.863 \ mm/s$$

B) Flux $N_A = k_c (C_A^{sat} - C_A)$ but C_A varies along tube $\Rightarrow$ differential mass balance



$$\underbrace{V_{ave} \frac{\pi}{4} D^2 C_A|_x}_{1} + \underbrace{\pi D \Delta x}_{A_{wall}\ 2} N_A = \underbrace{V_{ave} \frac{\pi}{4} D^2 C_A|_{x+\Delta x}}_{3}$$

$$\Rightarrow V_{ave} \frac{D}{4} \left[\frac{C_A|_{x+\Delta x} - C_A|_x}{\Delta x}\right] = k_c (C_A^{sat} - C_A)$$

$$\Rightarrow \frac{dC_A}{dx} = \frac{4k_c}{V_{ave} D} (C_A^{sat} - C_A) \Rightarrow \frac{dC_A}{C_A^{sat} - C_A} = \frac{4 k_c dx}{V_{ave} D}$$

$\Rightarrow$ Integrate from $x = 0$ ($C_A = 0$) to $x = L$ ($C_{A,out}$):

$$-\ln\left(\frac{C_A^{sat} - C_{A,out}}{C_A^{sat}}\right) = -\ln\left(1 - \frac{C_{A,out}}{C_A^{sat}}\right) = \frac{4 k_c L}{V_{ave} D}$$

$$\Rightarrow 1 - \frac{C_{A,out}}{C_{A,sat}} = 1 - \frac{P_{A,out}}{P_{A,sat}} = \underbrace{\exp\left(-\frac{4k_cL}{V_{ave}D}\right)}_{0.251} \Rightarrow P_{A,out} = P_A^{sat}\left(1 - \exp\left(-\frac{4k_cL}{V_{ave}D}\right)\right)$$

$$= 0.0112 \ atm$$

C) All solvent that leaves wall has to get out of the pipe

$$M_A \cdot C_{A,out} \cdot \frac{\pi}{4} D^2 \cdot V_{ave} = M_A \frac{P_{A,out}}{RT} \frac{\pi}{4} D^2 V_{ave} = \underset{kg/mol}{85 \cdot 10^{-3}} \cdot \frac{0.0112\,atm}{\underset{\frac{m^3 \cdot atm}{mol \cdot K}}{8.206 \cdot 10^{-5}}} \cdot \frac{\pi}{4} \underset{m^2}{(0.05)^2} \cdot \underset{m/s}{0.3}$$

$$= 2.28 \cdot 10^{-5} \ kg/s$$

D) * Required heat flow $q = \dot{m} \cdot h_{fg} = 2.28 \cdot 10^{-5}\,kg/s \cdot 300\,kJ/kg = 6.83 \cdot 10^{-3}\,kJ/s = 6.83\,W$

* Air temp change: $q = \dot{m}\, c_p\, \Delta T = \underbrace{\rho V_{ave} \tfrac{\pi}{4} D^2 c_p}_{\text{at } 300K} \Delta T$

$$\Rightarrow \Delta T = \frac{6.83}{1.177 \cdot 1006 \cdot 0.3 \cdot \frac{\pi}{4}(0.05)^2} = 9.8\,K$$

* Average temp difference between liquid and bulk air:

$$\Delta T = \frac{q}{h_{ave}\,\pi D L}$$

h_{ave} from Chilton-Colburn: $h_{ave} = \rho c_p k \left(\frac{Sc}{Pr}\right)^{2/3} = 1.177 \cdot 1006 \cdot 0.863 \cdot 10^{-3} \left(\frac{1.57}{0.708}\right)^{2/3}$

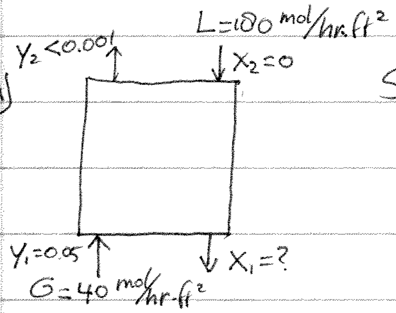
$= 1.74\ W/m^2 \cdot K$

$$\Rightarrow \Delta T = \frac{6.83}{1.74 \cdot \pi \cdot 0.05 \cdot 6} = 4.2\,K$$

E) * Coldest spot in liquid is near outlet $\sim 286\,K$

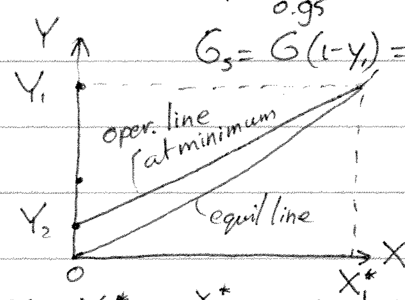
* Average bulk air temp ($\sim 295\,K$) and film temp ($\sim 293\,K$) are not really affected by evaporation

Problem V/ A)



Solute free basis: $Y_1 = \frac{0.05}{0.95} = 0.0526$, $Y_2 = \frac{0.001}{0.999} \approx 0.001$, $X_2 = 0$

$G_s = G(1-y_1) = 38\ mol/hr \cdot ft^2$ $\quad L_s = L$



* At minimum L_s: $\left.\frac{L_s}{G_s}\right|_{min} \Rightarrow X_1 = X_1^* = \frac{x_1^*}{1-x_1^*}$ with $x_1^* = \frac{y_1}{2.7} = 0.0185 \Rightarrow X_1^* = 0.0188$

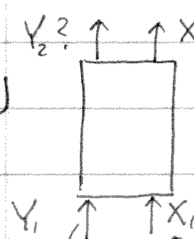
$\Rightarrow \left.\frac{L_s}{G_s}\right|_{min} = \frac{Y_1 - Y_2}{X_1^*} = \frac{0.053 - 0.001}{0.0188} = 2.76$ $\quad \left.\frac{L_s}{G_s}\right|_{actual} = \frac{180}{38} = 4.74 > 2.76$

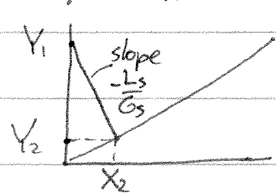
$\Rightarrow$ goal can be achieved

B) * $\frac{1/k_y}{1/k_y + c/k_x}$ with equil. constant $c = \frac{y}{x} = 2.7$

$$\Rightarrow \frac{1/k_y a}{1/k_y a + c/k_x a} = \frac{1/15}{1/15 + 2.7/50} = 0.552 \Rightarrow 55.2\% \text{ in gas phase}$$

C)





Operating line: $Y = Y_1 - \overbrace{\frac{L_s}{G_s}}^{4.74} X$ $\quad$ Equil. line: $Y = 2.7 \cdot X$

$\Rightarrow$ at intersection (X_2, Y_2): $Y_1 - \frac{L_s}{G_s} X_2 = 2.7\, X_2 \Rightarrow X_2 = \frac{Y_1}{7.44}$

$\Rightarrow X_2 = 0.00707 \Rightarrow Y_2 = 2.7 \cdot X_2 = 0.0191$

Not low enough!

D) (Y_2, X_2) in C achieved in very tall co-flow tower

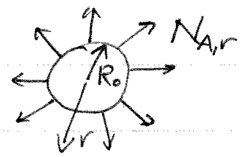
Problem VI/ A) * Hirschfelder equation for napthalene (A) in air (B)

$M_A = 128\ g/mol$, $M_B = 29\ g/mol$, $T = 303K$, $P = 1atm$, $\sigma_A = 6.2\ \mathring{A}$, $\sigma_B = 3.617\ \mathring{A}$

$\frac{\varepsilon_A}{\kappa} = 550\ K$, $\frac{\varepsilon_B}{\kappa} = 97 K \Rightarrow \sigma_{AB} = \frac{6.2+3.617}{2} = 4.91\ \mathring{A}$, $\frac{\varepsilon_{AB}}{\kappa} = \sqrt{550 \cdot 97} = 231\ K$

$\Rightarrow D_{AB} = \frac{0.001858 \cdot (303)^{3/2} \cdot (1/128 + 1/29)^{1/2}}{1 \cdot (4.91)^2 \cdot 1.273} = 0.066\ cm^2/s = 6.6 \cdot 10^{-6} m^2/s$ $\quad \left(\Rightarrow \frac{\kappa T}{\varepsilon_{AB}} = 1.312 \Rightarrow \Omega_{AB} = 1.273 \right)$

B) *



* CV: air outside sphere $\Rightarrow$ spherical coordinates
* 1D diffusion ($N_{A,r}$ only), pseudo steady state, no bulk reaction

$\Rightarrow$ Gen. diff. equation: $\vec{\nabla} \cdot \vec{N}_A + \cancel{\frac{\partial c_A}{\partial t}} - \cancel{R_A} = 0 \Rightarrow \frac{1}{r^2}\left(\frac{d}{dr}(r^2 N_{A,r})\right) = 0$

$\Rightarrow r^2 N_{A,r} = \text{constant}$

* stagnant medium and $y_A << 1$ (max $y_A = 0.0015$)

$\Rightarrow$ Fick's rate equation: $N_{A,r} = -cD_{AB}\frac{dy_A}{dr} + \cancel{y_A}(N_{A,r} + \cancel{N_{B,r}})$

C) $\int_{r=R_0}^{\infty} \frac{r^2 N_{A,r}}{r^2} dr = -\int_{y_A^{sat}}^{0} c D_{AB}\, dy_A \Rightarrow \underbrace{(r^2 N_{A,r})}_{\text{constant}} \left[-\frac{1}{r}\right]_{R_0}^{\infty} = -c D_{AB} \left[y_A\right]_{y_A^{sat}}^{0}$

$\Rightarrow (r^2 N_{A,r}) \frac{1}{R_0} = c D_{AB}\, y_A^{sat} \Rightarrow (r^2 N_{A,r}) = \frac{P_A^{sat}}{RT} D_{AB} R_0$

* total molar flow $W_A = 4\pi r^2 N_{A,r} = 4\pi \frac{P_A^{sat}}{RT} D_{AB} R_0$

$\Rightarrow$ mass flow $W_A \cdot M_A = 4\pi M_A \frac{P_A^{sat}}{RT} D_{AB} R_0 = 4\pi \cdot \underset{kg/mol}{0.128} \frac{0.0015 atm}{8.206 \cdot 10^{-5} \cdot 303} \cdot 6.6 \cdot 10^{-6} \cdot 0.005$
$= 3.21 \cdot 10^{-9} kg/s$
$= 0.015\ g/hr$

D) • With convection around sphere

$Re_D = \frac{vD}{\nu} = \frac{0.2 \cdot 0.01}{1.57 \cdot 10^{-5}} = 127.4$
$Sc = \frac{\nu}{D_{AB}} = \frac{1.57 \cdot 10^{-5}}{6.6 \cdot 10^{-6}} = 2.38$

Eqn. 30-9: $Sh_D = 2 + 0.552 \cdot (127.4)^{1/2} \cdot (2.38)^{1/3} = 10.32$

$\Rightarrow k_c = \frac{Sh_D \cdot D_{AB}}{D} = \frac{10.32 \cdot 6.6 \cdot 10^{-6}}{0.01} = 0.00681\ m/s$

$\Rightarrow N_A|_{conv} = k_c \cdot C_A^{sat}$

* From C): $N_A|_{diff\ r=R_0} = C_A^{sat} \cdot \frac{D_{AB}}{R_0} \Rightarrow$ enhancement $\frac{N_A|_{conv}}{N_A|_{diff}} = \frac{k_c R_0}{D_{AB}} = \frac{0.00681 \cdot 0.005}{6.6 \cdot 10^{-6}} = 5.2$

E) • Mothball contains $\frac{\frac{4}{3}\pi R_0^3 \rho}{M_A} = \frac{\frac{4}{3}\pi (0.005)^3 1140}{0.128} = 4.66 \cdot 10^{-3}$ mol naphthalene

with half distributed across closet $C_{A,bulk} = \frac{2.33 \cdot 10^{-3}}{81} = 2.88 \cdot 10^{-5} mol/m^3$

* $C_A^{sat} = \frac{P_A^{sat}}{RT} = \frac{0.0015}{8.206 \cdot 10^{-5} \cdot 303} = 0.060\ mol/m^3 \Rightarrow C_{A,bulk} << C_A^{sat}$