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Exam 1 *ISYE 3133* *Summer 2015*

Having read the Georgia Institute of Technology Academic Honor code, I understand and accept my responsibility as a member of the Georgia Tech community to uphold the Honor Code at all times. In addition, I understand my options for reporting honor violations as detailed in the code.

Signature_______________________________________

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1. (25 points) The following dictionaries were obtained from maximization LPs (maximizing z).

a)
$$\begin{array}{l}
x_1 = x_3 - 7x_5 + x_6 \\
x_2 = 1 - 9x_3 - x_5 - 7x_6 \\
x_4 = 3x_3 + 4x_5 + 4x_6 \\
\hline
z = 11 - 3x_5 - 2x_6
\end{array}$$

b)
$$\begin{array}{l}
x_1 = 2 + x_3 - 7x_5 + x_6 \\
x_2 = -9x_3 - 2x_5 + 5x_6 \\
x_4 = 5 + 3x_3 + 4x_5 + 4x_6 \\
\hline
z = 3 - x_3 + x_6
\end{array}$$

c)
$$\begin{array}{l}
x_1 = 2 - x_3 - 7x_5 - x_6 \\
x_2 = 2 - 9x_3 - 2x_6 \\
x_4 = 2x_3 + 4x_5 + 4x_6 \\
\hline
z = -4 + x_3 - 3x_5
\end{array}$$

d)
$$\begin{array}{l}
x_1 = x_3 - 7x_5 + x_6 \\
x_2 = 4 - 9x_3 - x_5 - 7x_6 \\
x_4 = 3x_3 + 4x_5 + 4x_6 \\
\hline
z = 11 - 3x_3 - 2x_6
\end{array}$$

e)
$$\begin{array}{l}
x_1 = 2 + x_3 - x_5 + x_6 \\
x_2 = -9x_3 - 2x_5 + 5x_6 \\
x_4 = -5 + 3x_3 + 4x_5 + 4x_6 \\
\hline
z = 3 - x_3 - x_6
\end{array}$$

f)
$$\begin{array}{l}
x_1 = -2 - x_3 - 7x_5 - x_6 \\
x_2 = 2 - 9x_3 - 2x_6 \\
x_4 = 2x_3 + 4x_5 + 4x_6 \\
\hline
z = -4 + x_3 - 3x_5
\end{array}$$

	a)	b)	c)	d)	e)	f)
1. Is the dictionary optimal?	Y	N	N	Y	N	N
2. Is the dictionary feasible?	Y	Y	Y	Y	N	N
3. Is the LP unbounded?	N	Y	N	N	N	N
4. If the dictionary is optimal, does the LP have a unique solution? (don't answer if dictionary not optimal)	N	-	-	N	-	-

(write **Y** for yes or **N** for no in the boxes)

2. (15 points) Short Answer.

a. Can a linear program can have exactly two optimal solutions? Explain.

No. If there are two optimal solutions, then there is an infinite number of linear combinations of those two solutions that give other optimal solutions.

b. When/why must we use the Big M or Two-Phase method?

If an LP has equality or $\geq$ constraints, then an initial bfs may be difficult to find. We use Big M or Two-Phase to find an initial bfs if one exists.

c. What does the shadow price of a constraint tell us?

The amount by which the objective function value will change for every unit increase in the resource represented by the constraint. We also interpret the shadow price as the value of the resource at optimality.

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3. (15 points) Buzz is the head of Stinger operations and needs to schedule the staffing of the bus drivers. Buses run from 8am until midnight each day. Buzz has monitored the usage of the buses at various times of the day and has determined the following numbers of drivers are required:

Shift	Time Period	Minimum Drivers Needed
1	8am - Noon	4
2	Noon - 4pm	8
3	4pm - 8pm	10
4	8pm - Midnight	6

Buzz can hire either full-time or part-time drivers. The full-time drivers work for 8 consecutive hours (either 8am - 4pm, noon - 8pm, or 4pm - midnight) and get paid $40 per hour. Part-time drivers can be hired to work any one of the four shifts listed above and are paid $30 per hour.

During each time period, the college requires that at least 2 full-time drivers are on duty for every part time driver on duty.

Formulate a linear program to help Buzz determine how many of each type of driver to hire to meet the requirements at a minimum cost.

Let $F_i =$ the number of full-time drivers who start in shift i,

$P_i =$ the number of part-time drivers who work shift i.

$$\min \quad (40)(8)(F_1 + F_2 + F_3) + (30)(4)(P_1 + P_2 + P_3 + P_4)$$

$$\text{s.t.} \quad F_1 + P_1 \geq 4 \ \text{(shift 1 minimum)}$$

$$F_1 + F_2 + P_2 \geq 8 \ \text{(shift 2 minimum)}$$

$$F_2 + F_3 + P_3 \geq 10 \ \text{(shift 3 minimum)}$$

$$F_3 + P_4 \geq 6 \ \text{(shift 4 minimum)}$$

$$2P_1 \leq F_1 \ \text{(2 full-time for every part-time in shift 1)}$$

$$2P_2 \leq F_1 + F_2 \ \text{(2 full-time for every part-time in shift 2)}$$

$$2P_3 \leq F_2 + F_3 \ \text{(2 full-time for every part-time in shift 3)}$$

$$2P_4 \leq F_3 \ \text{(2 full-time for every part-time in shift 4)}$$

$$\text{all variables } \geq 0$$

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4. (30 points) Consider the following LP:

maximize $3x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 100$ (constraint for resource 1)

$x_1 + x_2 \leq 80$ (constraint for resource 2)

$x_1 \leq 40$ (constraint for resource 3)

$x_1 \geq 0, x_2 \geq 0$

An optimal dictionary is as follows:

$$
\begin{aligned}
x_1 &= 20 - s_1 + s_2 \\
x_2 &= 60 + s_1 - 2s_2 \\
s_3 &= 20 + s_1 - s_2 \\
\hline
z &= 180 - s_1 - s_2
\end{aligned}
$$

a. For what values of c_2, the objective function coefficient of x_2, does the current basis remain optimal?

$1.5 \leq c_2 \leq 3$

b. For what values of b_1, the right-hand side of the first constraint, does the current basis remain optimal?

$80 \leq b_2 \leq 120$

c. Suppose a third product is proposed. This product requires only 1 unit of resource 1 and 2 units of resource 2 and sells for \$3.50. Should the product be produced?

Yes, cost of production only \$3.

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5. (15 points) A company is planning its aggregate production schedule for the next three months. Units may be produced in regular time or overtime. The relevant costs and capacities are shown in the table below. The demands for each month, that must be met, are also shown. Units produced in a particular month may be sold in that month or kept in inventory until sale in a later month. The inventory cost is $1 per unit per month. Sales may be backordered at a cost of $2 per unit per month. (Backorders represent production during the current month to satisfy demand in the previous month, and hence incur an additional cost.) Formulate a linear program to find the optimal production plan over the next 3 months.

	Capacity (units)		*Production Cost ($/unit)*		
Month	*Reg. Time*	*Overtime*	*Reg. Time*	*Overtime*	*Demand*
1	100	20	14	18	60
2	100	10	17	22	80
3	60	20	17	22	140

Let $R_i =$ the number of items produced in regular time in month i,

$E_i =$ the number of items produced in regular time in month i,

$I_i =$ the number of items on hand at the end of month i,

$B_i =$ the number of backordered from month i to use in month $i-1$.

$$\min \quad 14R_1 + 17R_2 + 17R_3 + 18E_1 + 22E_2 + 22E_3 + I_1 + I_2 + I_3 + 2B_2 + 2B_3$$

$$\text{s.t. } I_1 = R_1 + E_1 + B_2 - 60$$

$$I_2 = I_1 + R_2 + E_2 + B_3 - B_2 - 80$$

$$I_3 = I_2 + R_3 + E_3 - B_3 - 140$$

$$R_1 \leq 100$$

$$R_2 \leq 100$$

$$R_3 \leq 60$$

$$E_1 \leq 20$$

$$E_2 \leq 10$$

$$E_3 \leq 20$$

$$\text{all variables } \geq 0$$