Physics 2211A/B
Fall 2011

Test form $\boxed{851}$
Test 5

Name <u>Instructor Solutions</u>

A

1) Print your name, test form number (above), and nine-digit student number in the section of the answer card labeled "STUDENT IDENTIFICATION".

2) Bubble your test form number in columns 1-3, skip column 4, then bubble in your student number in columns 5-13.

3) For each free-response question, show all relevant work supporting your answer. **Clearly box or underline your final answer.** "Correct" answers which are not supported by adequate calculations and/or reasoning will be counted wrong.

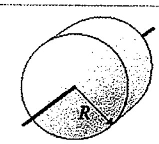
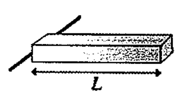
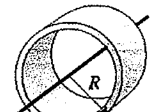
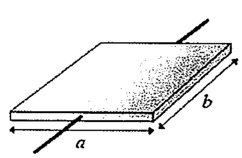
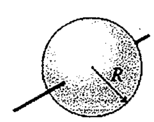
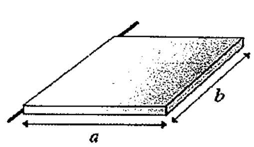
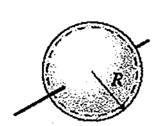
4) For each multiple-choice question, select the answer most nearly correct, **circle this answer on your test**, and bubble it in on your answer card. **Show all relevant work on your quiz.**

5) Be prepared to present your Buzzcard as you turn in your test. Scores will be posted to WebAssign after they have been been graded. **Quiz grades become final when the next quiz is given.**

6) You may use a simple scientific calculator capable of logarithms, exponentials, and trigonometric functions. **Programmable engineering calculators with text or graphical capabilities are not allowed. Wireless devices are prohibited.**

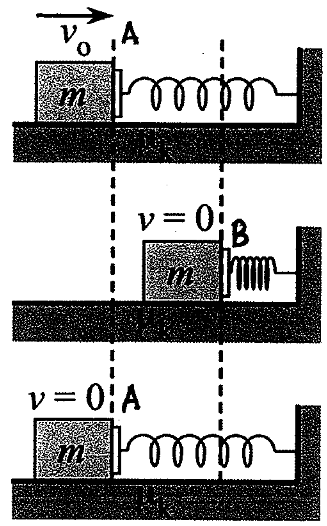
Your test form is: 851

TABLE 13.3 Moments of inertia of objects with uniform density

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod, about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod, about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

The following problem will be hand-graded. *<u>Show all your work for this problem.</u>* *Make no marks and leave no space on your answer card for it.*

[I] A mass m is traveling with initial speed v_0 along a rough surface (with coefficient of kinetic friction μ_k) just before it begins compressing a spring of elastic contant k. It momentarily stops after having compressed the spring by a distance d, then recoils and travels back in the opposite direction. On the return trip, the mass stops for good at the exact equilibrium position of the spring.



(A) *(10 points)* Find an expression for the maximum compression distance d, in terms of m, v_0, μ_k, and/or g.

Note that in round trip $A \to B \to A$, all mechanical energy is lost, dissipated by friction

→ total distance traveled is $s = 2d$

⇒ "Work by friction" (i.e. amount of energy "lost")

is $W_{diss} = -f_k(2d) = -\mu_k N(2d) = -\mu_k(mg)2d$

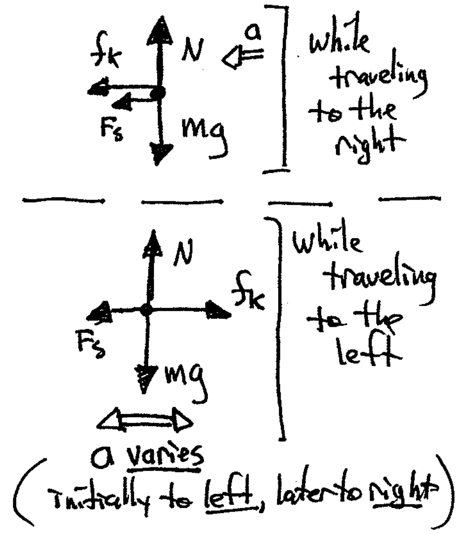
[since $\Sigma F_y = 0 = \langle +N \rangle + \langle -mg \rangle \Rightarrow N = mg$]

So, applying work-mechanical energy principle:

$$W_{diss} = \Delta E_{mech} = \Delta K + \cancel{\Delta U}$$ spring begins and ends uncompressed

$$-2\mu_k mgd = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \boxed{d = \frac{v_0^2}{4\mu_k g}}$$



(B) *(10 points)* Find an expression for the elastic constant of the spring, in terms of m, v_0, μ_k, and/or g.

Now, consider just the 1st leg of the trip: $A \to B$

→ if round trip dissipated all initial mechanical energy, then half the trip dissipates half the initial energy:

$$W_{diss} = -f_k d = -\mu_k mg d = \frac{1}{2}\left(-\frac{1}{2}mv_0^2\right) = -\frac{1}{4}mv_0^2$$

so, work-energy principle gives:

$$W_{diss} = \Delta E_{mech} = (U_B + \cancelto{0}{K_B}) - (\cancelto{0}{U_A} + K_A)$$

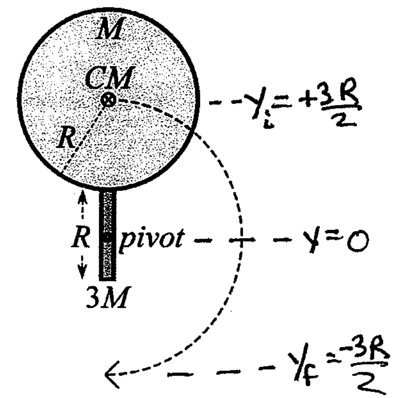
$$-\frac{1}{4}mv_0^2 = \frac{1}{2}kd^2 - \frac{1}{2}mv_0^2$$

$$\frac{1}{2}kd^2 = \frac{1}{4}mv_0^2$$

$$k = \frac{mv_0^2}{2d^2} \rightarrow k = \frac{mv_0^2}{2}\left(\frac{4\mu_k g}{v_0^2}\right)^2 = \boxed{\frac{8\mu_k^2 mg^2}{v_0^2}}$$

The following problem will be hand-graded. <u>*Show all your work for this problem.*</u> *Make no marks and leave no space on your answer card for it.*

[II] A solid disk of mass M and radius R is glued to the end of a thin rod of mass $3M$ and length R. The rod pivots without friction about an axle passing horizontally through its midpoint. The resulting object is positioned vertically in *unstable* equilibrium as shown in the figure, and a *very* gentle nudge causes it to start rotating about the pivot.



(A) *(10 points)* What will be the tangential speed of the disk's center of mass as it passes through the position of *stable* equilibrium, directly below the pivot?

① Find <u>total</u> moment of inertia <u>about pivot</u>

- rod $I_{rod,cm} = \frac{(3M)L^2}{12} = \frac{ML^2}{4}$ but $L=R$, so $I_{rod} = \frac{MR^2}{4}$
- disk: $I_{disk,cm} = \frac{1}{2}MR^2$ but <u>pivot</u> is <u>not</u> at disk's CM
  → parallel axis theorem: $I_{disk,\,pivot} = I_{cm} + Md^2 = \frac{1}{2}MR^2 + M\left(\frac{3R}{2}\right)^2 = \frac{11}{4}MR^2$

$$I_{TOT,pivot} = \frac{MR^2}{4} + \frac{11MR^2}{4} = \frac{12MR^2}{4} = \boxed{3MR^2}$$

Now – consider conservation of energy $K_i + U_i = K_f + U_f$

letting $y \equiv 0$ at the pivot:

- $y_{cm,\,rod}$ does not change
- $y_{cm,\,disk}$: $y_i = +\frac{3R}{2} \rightarrow y_f = -\frac{3R}{2}$

so:

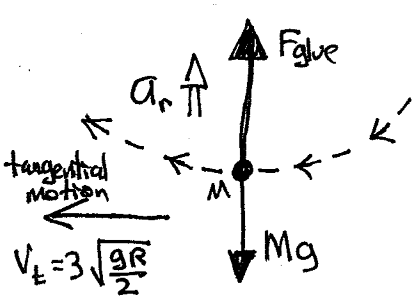
$$0 + Mg\cdot\frac{3R}{2} = \frac{1}{2}(3MR^2)\omega_{bottom}^2 + Mg\left(-\frac{3R}{2}\right) \rightarrow \frac{1}{2}(3MR^2)\omega_b^2 = 3MgR$$

$$\boxed{\omega_b = \sqrt{\frac{2g}{R}}} \rightarrow V_{cm,\,disk} = \frac{3}{2}R\cdot\omega_b$$

$$\boxed{V_{cm} = 3\sqrt{\frac{gR}{2}}}$$

(B) *(10 points)* At the moment described in Part A, what is the adhesive force exerted <u>on the disk</u> by the glue that attaches it to the rod? Keep in mind that at this moment, the disk is experiencing $\boxed{\text{circular motion}}$ about the pivot point! *[For this calculation, it suffices to consider the disk as a* $\boxed{\textit{pointlike}}$ *particle...]*

Standard free body diagram for disk at bottom of arc, <u>as a point object</u>



(don't forget – the radius of the disk's trajectory is $r = \frac{3R}{2}$!)

- Disk following circular trajectory experiences <u>upward</u> radial acceleration:

$$\vec{a}_r = \langle +V_t^2/r \rangle = \left\langle +\frac{9gR}{2} \Big/ \frac{3R}{2} \right\rangle = \langle +3g \rangle$$

So, 2nd law for radial direction gives:

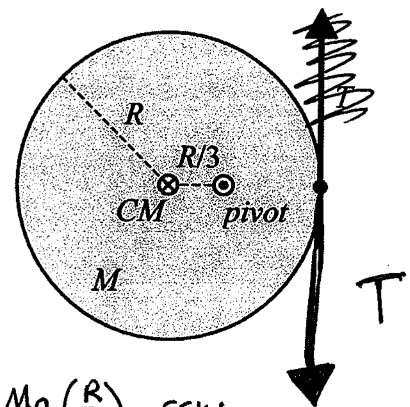
$$\langle +F_{glue} \rangle + \langle -Mg \rangle = M\langle +3g \rangle$$

$$\boxed{F_{glue} = 4Mg}$$

(Hope you used Superglue™!)

The following problem will be hand-graded. *Show all your work for this problem.* *Make no marks and leave no space on your answer card for it.*

[III] A disk of mass M and radius R is supported by a horizontal axle through a pivot point that is a distance $R/3$ from the center of the disk. A cord is attached to the rim of the disk at the point nearest the pivot, and the disk is held as shown at right, with the cord vertical and the center of mass level with the pivot.



(A) (*10 points*) If the disk is to remain in rotational equilibrium in this orientation, what must be the tension T? Express your answer in terms of M, R, and/or g.

require $\sum \tau_{pivot} = 0$

• Forces acting are
① Gravity at CM → generates torque $\vec{\tau}_g = Mg\left(\frac{R}{3}\right)$, ccw
② Tension at rim → " " $\vec{\tau}_T = T\left(\frac{2R}{3}\right)$, cw
③ Force by axle at pivot → generates no torque : moment arm $\equiv 0$

so: letting ccw = positive (ie out of page = positive)

$$\sum \tau = \left\langle + \frac{MgR}{3} \right\rangle + \left\langle - \frac{2TR}{3} \right\rangle = 0$$

$$\Rightarrow \boxed{T = \frac{mg}{2}}$$

(B) (*10 points*) Suppose that the cord is pulled with a tension T that exactly equaly the weight of the disk Mg. What will be the resulting angular acceleration $\vec{\alpha}$ (if any) for the disk? Express your answer in terms of M, R, and/or g—*and be sure to indicate the* direction *of* $\vec{\alpha}$!

If $T = mg$, results of (A) clearly indicate non-equilibrium
→ cw torque due to tension exceeds ccw torque due to gravity

$$\sum \tau = \left\langle + \frac{MgR}{3} \right\rangle + \left\langle -\frac{2}{3}(mg)R \right\rangle = \left\langle - \frac{mgR}{3} \right\rangle$$

Now, to apply 3rd Law, compute $I_{pivot} = I_{cm} + Md^2$ (where $d = \frac{R}{3}$)

$$\rightarrow I_p = \frac{1}{2}MR^2 + \frac{1}{9}MR^2 = \frac{11}{18}MR^2$$

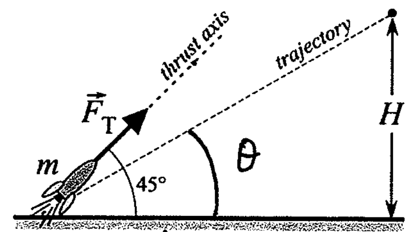
$$\Rightarrow \sum \vec{\tau} = I\vec{\alpha}$$

$$\left\langle - \frac{mgR}{3} \right\rangle = \frac{11}{18}MR^2 \left\langle -\alpha \right\rangle \rightarrow \boxed{\alpha = \frac{6}{11}\frac{g}{R}, \text{ clockwise}}$$

→ magnitude
→ direction = cw

Question value 10 points

(1) A missile of mass m = 100 kg generates a thrust of magnitude F_T = 3,000 N. The rocket is lauched with it's nose directed at an angle of 45° above the horizontal direction (the *thrust axis*). What is the speed of the rocket after it has reached an altitude H = 100 m? (Don't forget that there is also a gravitational force acting on the rocket!)



(a) 64 m/s

(b) 92 m/s (*)

(c) 101 m/s

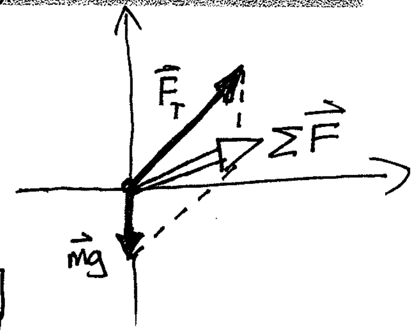
(d) 81 m/s (*)

(e) 44 m/s

$$\sum \vec{F}_x = +F_T \cos 45° = \boxed{2121\text{N}}$$

$$\sum F_y = +F_T \sin 45° - Mg$$

$$= \boxed{1141\text{N}}$$

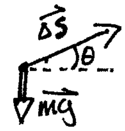


⇒ Net Force is directed at angle

$$\theta = \tan^{-1}\left(\frac{1141\text{N}}{2121\text{N}}\right) = 28.3° \text{ above horizontal}$$

⇒ accel $\vec{a}$ – and hence, $\boxed{\text{trajectory is } 28.3° \text{ above horiz.}}$

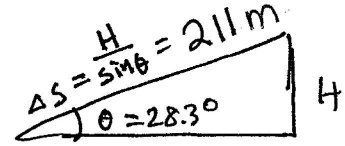
• Work by gravity:



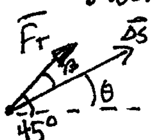
$$W_g = (Mg)(s)(\cos\beta) = Mgs\cos(\theta + 90°)$$

$$= -Mgs\sin\theta$$

$$= -MgH \text{ (duh)} = -98{,}000\text{ J}$$



• Work by Thrust:

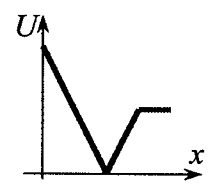


$$W_T = (F_T)(s)(\cos\beta) = F_T s \cos(45° - \theta) = +606{,}000\text{ J}$$

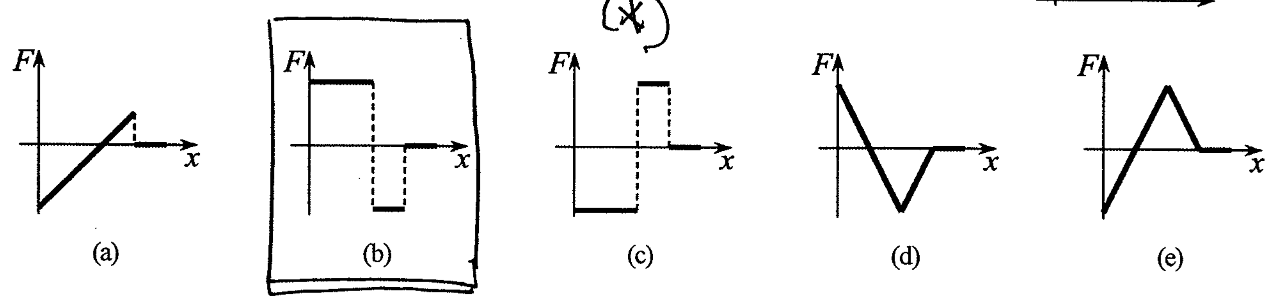
$$W_{TOT} = +508{,}000\text{ J} = \Delta K = \tfrac{1}{2}mV_f^2 \rightarrow \boxed{V_f = 100.8\text{ m/s}}$$

Question value 10 points

(2) A particle moves along the x-axis subject to a conservative force $F(x)$. The potential energy function for this force is shown in the figure. Which of the graphs below best characterizes the force F as a function of position?



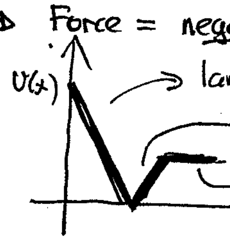
(*)



(a) (b) (c) (d) (e)

$$F_x = -\frac{dU}{dx}$$

→ Force = negative of slope of $U(x)$



→ large negative slope → large positive F_x

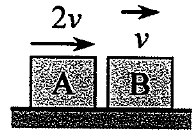
→ large positive slope → large negative F_x

→ zero slope → $F_x = 0$

$K_i = \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$

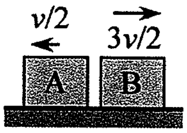
Question value 10 points

(3) The figure at right shows blocks A and B having identical masses m, just before they experience a perfectly elastic collision. Which of the figures below best characterizes their motion just after the collision?

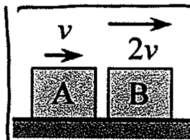


$\vec{P}_i = \langle +3mv \rangle$

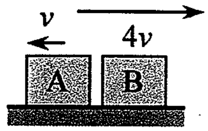
Collision: $\vec{P}$ = conserved "perfectly elastic": K is also conserved



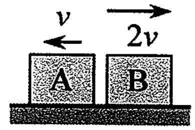
(a)



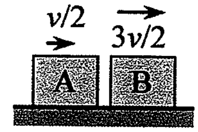
$\boxed{\text{(b)}}$



(c) (*)



(d) (*)



(e)

(a): $\vec{P}_f = \langle -m\frac{v}{2} \rangle + \langle m\frac{3v}{2} \rangle = \langle +mv \rangle$ NOT conserved

$K_f = \frac{1}{2}m\frac{v^2}{4} + \frac{1}{2}m\frac{9v^2}{4} = \frac{5}{4}mv^2$ NOT conserved

(b): $\vec{P}_f = \langle +mv \rangle + \langle 2mv \rangle = \langle 3mv \rangle$ $\boxed{\text{CONSERVED}}$

$K_f = \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$ $\boxed{\text{conserved}}$

(c): $\vec{P}_f = \langle -mv \rangle + m\langle 4v \rangle = \langle +3mv \rangle$ $\boxed{\text{conserved}}$

$K_f = \frac{1}{2}mv^2 + \frac{1}{2}m(4v)^2 = \frac{17}{2}mv^2$ NOT conserved

(d): $\vec{P}_f = \langle -mv \rangle + \langle +2mv \rangle = \langle +mv \rangle$ NOT conserved

$K_f = \frac{1}{2}mv^2 + \frac{1}{2}m(2v)^2 = \frac{5}{2}mv^2$ $\boxed{\text{Conserved}}$

(e): $\vec{P}_f = \langle +\frac{mv}{2} \rangle + \langle +\frac{3mv}{2} \rangle = \langle +2mv \rangle$ NOT conserved

$K_f = \frac{1}{2}m(\frac{v}{2})^2 + \frac{1}{2}m(\frac{3v}{2})^2 = \frac{5}{4}mv^2$ NOT conserved

Question value 10 points

(4) A sled of mass m = 100 kg is being pulled horizontally by a constant horizontal force of magnitude F = 250 N. The coefficient of kinetic friction is μ_k = 0.20. During time interval Δt = 8.6 seconds, the sled moves a distance s = 20 m, starting from rest. Find the average power P_{avg} delivered to the sled $\boxed{\text{by the force } F.}$

(a) 456 W

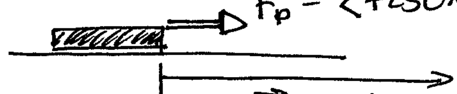
(b) 126 W

(c) 2280 W

(d) 746 W

$\boxed{\text{(e) 581 W}}$

$P_{avg} = \frac{W}{\Delta t}$



$\vec{F}_p = \langle +250N \rangle \hat{i}$

$\vec{\Delta s} = \langle +20m \rangle \hat{i}$

where W = work by force F_p

$= \vec{F}_p \cdot \vec{\Delta s} = 5000\ J$

so $P_{avg} = \frac{F_p \Delta s}{\Delta t} = \boxed{581\ W}$