Name: Solution Key Handout # ________

Transport II ChBE 3210

Exam

Monday, July 1, 2015

Remember, to receive full credit on each problem:

- Write down relevant relationships/equations
- Label variables
- State all assumptions
- Show intermediate steps
- Present solutions clearly with appropriate units

Problem	Possible Points	Score
1	10	
2	8	
3	16	
4	16	
5	18	
6	10	
7	16	
8	6	
Total		

1. [10 pts] Identify how the following sets of diffusivities generally compare, using **<, >, =,** or **N/A** (not enough information to determine).

For e.g. $\mathcal{D}_{AB,liquid}$	>	$\mathcal{D}_{AB,solid}$
A. $\mathcal{D}_{AB,liquid}$	<	$\mathcal{D}_{AB,gas}$
B. $\mathcal{D}_{AB,gas\ at\ 1\ atm}$	>	$\mathcal{D}_{AB,gas\ at\ 10\ atm}$
C. $\mathcal{D}_{AB,gas}$	=	$\mathcal{D}_{BA,gas}$
D. $\mathcal{D}_{AB,liquid}$	N/A	$\mathcal{D}_{BA,liquid}$
E. $\mathcal{D}_{AB,liquid\ at\ 300K}$	<	$\mathcal{D}_{AB,liquid\ 350\ K}$

2. [8 pts] Explain how the following conditions/ assumptions modify the Molar Flux (Fick's) equation:

A. Equimolar counterdiffusion $\vec{N}_A = -\vec{N}_B$

for binary mixture $y_A(\vec{N}_A - \vec{N}_A = 0$

B. Dilute solution

$y_A \approx 0 \quad y_A \Sigma \vec{N}_i = 0$

C. No bulk motion

$C_A \vec{V} = 0$

D. Reaction A + 3B → 2C $\vec{N}_A = \frac{1}{3}\vec{N}_B = \frac{-1}{2}\vec{N}_C$

$y_A \Sigma \vec{N}_i = y_A(\vec{N}_A + 3\vec{N}_A - 2\vec{N}_A) = y_A(2\vec{N}_A)$

yong fund -3

3. The MW of water is 18.02 g/mole and methanol (CH_3OH) is 32.04 g/mole. At 300K, the viscosity of methanol is 0.549 cP.

A. [6 pts] Determine the diffusivity of dilute methanol in water at 300 K using the Wilke-Chang equation.

Methanol = A

1 $V_A = 14.8 + 4(3.7) + 7.4 = 37$

Water = B

$\mu_B = 855 \times 10^{-6}\ N \cdot s/m^2$

1 $= 0.855\ mPa \cdot s$

1 $\Phi_B = 2.6$

1 $M_B = 18.02$ g/mole

1 $$\mathcal{D}_{AB} = \frac{T}{\mu_B} \frac{7.4 \times 10^{-8} (\Phi_B \cdot M_B)^{1/2}}{V_A^{0.6}}$$

$$\frac{300K}{0.855} \left(7.4 \times 10^{-8} \right) \left(\frac{(2.6 \times 18.02)^{1/2}}{(37)^{0.6}} \right)$$

1 $\mathcal{D}_{AB} = 2.036 \times 10^{-5}\ cm^2/s$

B. [4 pts] Compare this to the experimental value (Table J.2 in exam packet) adjusted to 300K.

2 At 288 $\mathcal{D}_{AB} = 1.28 \times 10^{-5}\ cm^2/s$ at 288 $\mu_B = 1.138\ mPa \cdot s$

1 $$\mathcal{D}_{AB\,300} = \mathcal{D}_{AB\,288} \left(\frac{\mu_{B\,288}}{T_{288}} \right) \left(\frac{T_{300}}{\mu_{B\,300}} \right)$$

$$= 1.28 \times 10^{-5} \left(\frac{1.138}{0.855} \right) \left(\frac{300}{288} \right) = 1.775 \times 10^{-5}\ cm^2/s$$ 1

C. [6 pts] Determine the diffusivity, if the methanol is 20 mol% (i.e. not dilute).

$$\mathcal{D}_{BA} = \frac{300}{0.549} \left(7.4 \times 10^{-8} \right) \left(\frac{(1.9 \times 32.04)^{1/2}}{(18.9)^{0.6}} \right)$$

$V_B = 18.9$

$= 5.409 \times 10^{-5}\ cm^2/s$ 2

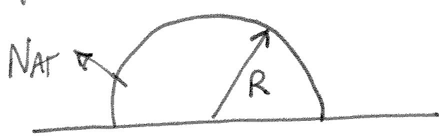
2 $$\mathcal{D}_{AB} = \mathcal{D}_{AB}^{x_B} \mathcal{D}_{BA}^{x_A}$$

$$= (2.036 \times 10^{-5})^{0.8} (5.409 \times 10^{-5})^{0.2} = 2.475 \times 10^{-5}\ cm^2/s$$

4. A hemispherical droplet of liquid water, lying on a flat surface, evaporates by molecular diffusion through still air surrounding the droplet. Initially, the droplet has a radius R. As the liquid water slowly evaporates, the droplet shrinks slowly with time, but the flux of the water is at a nominal steady state. The temperature of the droplet and the surrounding still air are constant. The air contains water vapor of fixed concentration, $y_{A,\infty}$, at an infinitely long distance.

A. [4 pts] Sketch the system. Determine the appropriate coordinate system.
B. [4 pts] List four reasonable assumptions for the mass transfer process.
C. [4 pts] Develop the simplified Flux (Fick's) Equation (mole basis)
D. [4 pts] Develop the simplified Differential Equation for mass transfer (mole basis)

(A) Spherical coordinates



(B)
1) T constant → D_{AB} constant
2) No reaction $R_A = 0$
3) Mass transfr in r direction only
4) Stagnant air $N_B = 0$
5) Steady state
6) Constant c

(C)

$$\vec{N}_A = -cD_{AB}\vec{\nabla}y_A + y_A \Sigma \vec{N}_i$$

$$N_{Ar} = -cD_{AB}\frac{dy_A}{dr} + y_A(N_{Ar})$$

$$N_{Ar} = \frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dr}$$

(D)

$$\vec{\nabla}\cdot\vec{N}_A + \frac{\partial c_A}{\partial t} - R_A = 0$$

$$\vec{\nabla}\cdot\vec{N}_A = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 N_{Ar}) = 0$$

(E) At $r = \infty$ $C_A = C_{A\infty}$

At $r = R$ $C_A = C_{AS} = \frac{p_A}{RT}$

5. The diffusivity of carbon tetrachloride (CCl_4) is being determined by observing the steady-state evaporation of CCl_4 in an Arnold Diffusion Cell containing oxygen (assume constant flowing O_2 at top of tube, but stagnant gas inside). After steady state has been attained, the system is observed for 10 hours. It was found that 0.0208 cm^3 of CCl_4 evaporated and the liquid level change was negligible. The flux path length, (z_2-z_1) was 17.1 cm. The total pressure on the system is 755 mmHg at 0°C. At this temp, at the surface of the liquid, the partial pressure of CCl_4 is 33.0 mmHg, and its density is 1.629 g/cm^3. Molecular weight is 153.81 g/mole. The cross-sectional area inside the tube is 0.82 cm^2.

A. [4 pts] Determine the molar flux, N_A of CCl_4.
B. [8 pts] Determine the simplified Flux (Fick's) equation and state all relative assumptions/ conditions.
C. [2 pts] Determine the boundary conditions: y_{A1} at the liquid surface and y_{A2} at the top of the tube/cell.
D. [4 pts] Determine the CCl_4-O_2 diffusivity, $\mathcal{D}_{AB}$ using the developed flux equation.

(A)

$$N_A = \frac{\text{moles}}{\text{cm}^2\cdot\text{s}} = \frac{0.0208\ \text{cm}^3\ CCl_4}{} \cdot \frac{1.629\ \text{g}}{\text{cm}^3} \cdot \frac{\text{mole}}{153.81\ \text{g}} \cdot \frac{}{0.82\ \text{cm}^2} \cdot \frac{}{10\ \text{hr}} \cdot \frac{1\ \text{hr}}{60(60)}$$

$$\boxed{7.462\times10^{-9}\ \text{mol/cm}^2\cdot\text{s}}$$

2.686×10^{-5} mol/cm^2 hr

(B) $\vec{N}_A = -c\mathcal{D}_{AB}\vec{\nabla}y_A + y_A\sum\vec{N}_i$

One directional flux in z direction

Stagnant gas $N_B = 0$

$\vec{N}_A = -c\mathcal{D}_{AB}\vec{\nabla}y_A + y_A\vec{N}_A \Rightarrow N_{Az} = -c\mathcal{D}_{AB}\frac{dy_A}{dz} + y_A N_{Az}$

$$\boxed{N_{Az} = \frac{-c\mathcal{D}_{AB}}{1-y_A}\frac{dy_A}{dz}}$$

$\xrightarrow{\text{integrate}}$ $N_{Az}(z_2-z_1) = c\mathcal{D}_{AB}\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right)$

2 (c) At liquid surface (1) $y_{A1} = \frac{p_A}{P} = \frac{33}{755} = 0.0437$

At top of tube (1) $y_{A2} = 0$

4 (D) $$D_{AB} = \left[\frac{N_{Az}(z_2 - z_1)}{C}\right]\left[\frac{1}{\ln\left(\frac{1-y_{A2}}{1-y_{A1}}\right)}\right] \qquad C = \frac{P}{RT}$$

$z_2 - z_1 = 17.1\ cm$

$$D_{AB} = \frac{7.462\times10^{-9}\ \text{moles}}{cm^2 \cdot s} \Bigg| (17.1\ cm) \Bigg| \frac{}{755\ mmHg} \Bigg| \frac{760\ mmHg}{1\ atm} \Bigg| \frac{8.206\times10^{-5}\ m^3\ atm}{mol\ K}$$

$$\times\ 273\ K \times \left[\ln\left(\frac{1}{1-\left(33/755\right)}\right)\right]^{-1} \times \left(\frac{(100\ cm)^3}{m^3}\right)$$

$D_{AB} = 6.438 \times 10^{-2}\ cm^2/s$

231.8 cm^2/hr

6. [10 pts] A system at steady state is described by the following combined equation, where the total molar concentration and diffusivity are constant.

$$\frac{d}{dz}\left(\frac{-c\mathcal{D}_{AB}}{1-y_A}\frac{dy_A}{dz}\right) = 0$$

Determine the concentration profile for the following boundary conditions:
At z = 0, y_A = 0.55 and at z = L, y_A = 0

$$\frac{d}{dz}\left[\left(\frac{-1}{1-y_A}\right)\frac{dy_A}{dz}\right] = 0 \quad \text{integrate}$$

(2) $$-\frac{1}{1-y_A}\frac{dy_A}{dz} = C_1 \Rightarrow \int \frac{-1}{1-y_A}\,dy_A = \int C_1\,dz$$

(2) $$\ln(1-y_A) = C_1 z + C_2$$

B.C.1 (1) $\ln(1-0.55) = C_1(0) + C_2$ $\qquad C_2 = -0.799$ (1)

B.C.2 (1) $\ln(1) = C_1 L + C_2$

$0 = C_1 L - 0.799$ $\qquad C_1 = 0.799/L$ (1)

(2) $$\ln(1-y_A) = \left(\frac{0.799}{L}\right)z - 0.799$$

or

$$1-y_A = \exp\left[\left(0.799/L\right)z - 0.799\right]$$

$$y_A = 1 - \exp\left[\left(0.799/L\right)z - 0.799\right]$$

7. A solid rectangular slab of agar gel at 5°C has a thickness (x direction) of 8 mm and contains a uniform concentration of urea of 0.1 kmol/m³. The rod is suddenly immersed in pure water where the surface resistance to mass transfer is negligible (i.e. very large convective mass transfer coefficient). The ends of the slab are sealed such that diffusion occurs only in the x direction. The diffusivity of urea in agar is 4.72 x 10⁻¹⁰ m²/s.

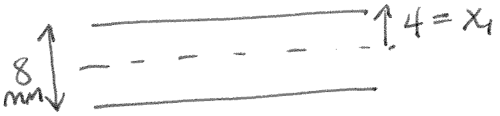
   A. [10 pts] After 10 hours, determine the concentration at the midpoint. Show all relevant intermediate calculations/values.

   B. [~~1~~6 pts] After 10 hours, determine the concentration at a point 1 mm from the surface of the slab.

A Utilize Heissler Charts

$$Y = \frac{C_{A1} - C_A}{C_{A1} - C_{A0}} \Rightarrow \frac{C_{A\infty} - C_A}{C_{A\infty} - C_{A0}} = \frac{C_A}{C_{A0}} = \frac{C_A}{0.1\ kmol/m^3}$$

2 $\boxed{m \approx 0}$ $k_c \gg D_{AB}$



2 $\boxed{n = 0/4 = 0}$ midpoint

2 $X = \frac{D_{AB}\, t}{x_1^2} = \frac{4.72\times10^{-10}\ m^2}{s} \Big| \frac{10\ hr}{} \Big| \frac{60(60)\,s}{1\ hr} \Big| \frac{1}{(0.004\ m)^2} = \boxed{1.062}$

2 $Y = 0.10$

2 $C_A = 0.10\,(0.1\ kmol/m^3) = 0.01\ kmol/m^3$

B $x_1 \overset{0.004}{=} 4$ $x = 0.003\,m$ m, X unchanged 2

1 $n = 3/4 = 0.75$

1 $Y = 0.04$

2 $C_A = 0.04\,(0.1\ kmol/m^3) = 0.004\ kmol/m^3$

8. [6 pts] When making an analogy between heat and mass transfer, the system must meet appropriate conditions. For the three listed analogies (Reynolds, Von Karman, and the j factor), indicate the required conditions by placing a check in the appropriate box.

Conditions	Reynolds	Von Karman	$j_D = j_H$
Schmidt and Prandtl numbers must equal unity	✓		
No form drag	✓	✓	
No heat generation	✓	✓	✓
No homogeneous reaction	✓	✓	✓