

GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 – BIOTRANSPORT

QUIZ 4 (SPRING 2014) – ETHIER

STUDENT NAME: Key

GTID NUMBER: _____

RECITATION SECTION: _____

(Section E is Wednesdays at 2 pm; Section F is Wednesdays at 1 pm)

Closed Book

All non-communicating calculator types allowed

Time allotted: 15 minutes

Do all work in this booklet

Reminder: for questions that require numerical answers, units are required and worth 50%

Question	Maximum Mark	Actual Mark
1	2	
2	8	
Total	10	

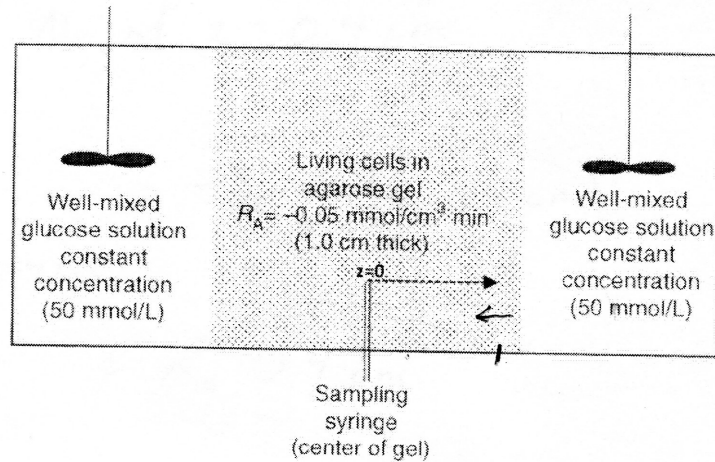
Consider the experiment shown below where a slab of the cell-immobilized gel of 1.0 cm thickness is placed within a well-mixed aqueous solution of glucose maintained at a concentration of $c_0 = 50$ mM. The glucose consumption within the cell-immobilized gel proceeds by a zero-order process given by:

$$R_A = -0.05 \text{ mM/min}$$

The solubilities of the glucose in both the water and the gel are the same; that is, the concentration of glucose on the water side of the water-gel interface is equal to the concentration of glucose on the gel side of the water-gel interface.

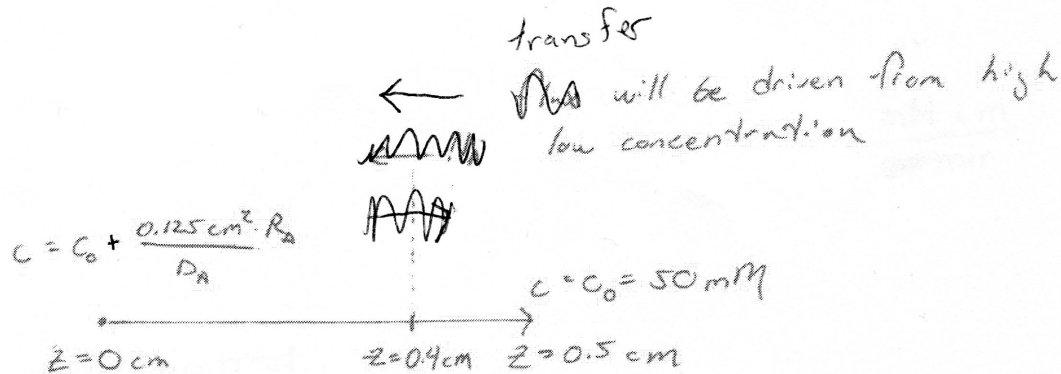
You are given a known steady-state concentration profile of glucose that arises from this geometry:

$$c_A(z) = c_0 - \frac{R_A}{D_A} \left(\frac{z^2}{2} - 0.125 \text{ cm}^2 \right)$$



- 1) Draw a diagram of the system and indicate the directionality of the glucose flux inside the gel 1 mm away from the gel-liquid interface.

1.0 mm



2) Derive an expression for the flux of glucose inside the gel ^{10 mm} 1 mm away from the gel-liquid interface.

$$N_A = -D \frac{\partial c}{\partial z} \quad \text{Fick's 1st Law} \quad +2$$

$$\text{if } c_A = c_0 - \frac{R_A}{D_A} \left(\frac{z^2}{2} - 0.125 \text{ cm}^2 \right)$$

then first find $\frac{\partial c}{\partial z}$

$$\frac{\partial c}{\partial z} = -\frac{R_A}{D_A} z \quad +4$$

evaluating N_A at $z = 0.4 \text{ cm}$:

$$N_A \Big|_{z=0.4 \text{ cm}} = -D_A \cdot \left(-\frac{R_A}{D_A} \cdot 0.4 \text{ cm} \right)$$

$$= R_A \cdot 0.4 \text{ cm}$$

$$= -0.05 \frac{\text{mM}}{\text{min}} \cdot 0.4 \text{ cm} = -0.02 \frac{\text{mM} \cdot \text{cm}}{\text{min}} \quad +2$$

or

$$\frac{\text{mM} \cdot \text{cm}}{\text{min}}$$

-2 if sign incorrect.

Definition of ∇ and ∇^2 operators in different coordinate systems:

Cartesian:

$$\nabla = \mathbf{e}_x \frac{\partial}{\partial x} + \mathbf{e}_y \frac{\partial}{\partial y} + \mathbf{e}_z \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Cylindrical:

$$\nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \mathbf{e}_z \frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Spherical:

$$\nabla = \mathbf{e}_r \frac{\partial}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + \mathbf{e}_\varphi \frac{1}{r \sin \theta} \frac{\partial}{\partial \varphi}$$

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2}$$