QUIZ 4

Name ~~Key~~ Key

Math 2551 D Steinbart

Section ______ February 17, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(7) 1. Let $f(x,y,z) = e^{x^2y} + 2xz^3 + 4y + 2$.

a. Find the directional derivative of the function $f(x,y,z)$ at the point $(2,0,-1)$ in the direction $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

b. Find the direction in which the function decreases most rapidly.

c. Find a unit vector in a direction in which the function is neither increasing or decreasing.

(a)

$\nabla f = \langle 2xy e^{x^2y} + 2z^3,\ x^2 e^{x^2y} + 4,\ 6xz^2 \rangle$

$\nabla f|_{(2,0,-1)} = \langle 0 + -2, 4+4, 12 \rangle = \langle -2, 8, 12 \rangle$

$|\underline{v}| = \sqrt{4^2 + (-2)^2 + (1)^2} = \sqrt{16+4+1} = \sqrt{21}$. So $\underline{u} = \frac{\underline{v}}{|\underline{v}|} = (4i - 2j + k)\left(\frac{1}{\sqrt{21}}\right) = \frac{4}{\sqrt{21}} i - \frac{2}{\sqrt{21}} j + \frac{1}{\sqrt{21}} k$ is a unit vector in the direction of $\underline{v}$.

$D_u f|_{(2,0,-1)} = \nabla f|_{(2,0,-1)} \cdot u = \langle -2, 8, 12 \rangle \cdot \left(\frac{4}{\sqrt{21}} i - \frac{2}{\sqrt{21}} j + \frac{1}{\sqrt{21}} k\right)$

$= \frac{1}{\sqrt{21}}(-8 - 16 + 12) = \boxed{\frac{-12}{\sqrt{21}}}$

(b) The function decreases most rapidly in the direction $-\nabla f|_{(2,0,-1)} = -\langle -2, 8, 12 \rangle = \boxed{\langle 2, -8, -12 \rangle}$

(c) We want a vector $\underset{w \neq 0}{\underline{w}}$ so $\nabla f \cdot \underline{w} = 0$. If $|\underline{w}| \neq 1$ then we can normalize it later.

$\nabla f \cdot \underline{w} = \langle -2, 8, 12 \rangle \cdot \langle w_1, w_2, w_3 \rangle$

$= -2w_1 + 8w_2 + 12w_3$. We can take $w_2 = 1, w_1 = 4, w_3 = 0$. Continued below.

So $\underline{w} = \langle 4, 1, 0 \rangle$

$|\underline{w}| = \sqrt{4^2 + 1^2 + 0^2} = \sqrt{17}$

$\boxed{\text{Let } u = \frac{\underline{w}}{|\underline{w}|} = \frac{1}{\sqrt{17}} \langle 4, 1, 0 \rangle = \left\langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}}, 0 \right\rangle}$

There are other correct answers!

(3) 2. Suppose that z is a differentiable function of x, y, that satisfies the equation $x^3z + yz^4 + xy^5 + 6y = 9$. Find $\frac{\partial z}{\partial y}$.

$\frac{\partial}{\partial y}\left[x^3 z + yz^4 + xy^5 + 6y\right] = \frac{\partial}{\partial y}[9]$

z depends on y. x does not depend on z

$x^3 \frac{\partial z}{\partial y} + y(4z^3)\frac{\partial z}{\partial y} + 1(z^4) + x\,5y^4 + 6 = 0$

$\frac{\partial z}{\partial y}\left[x^3 + 4yz^3\right] = -z^4 - 5xy^4 - 6$

$$\boxed{\frac{\partial z}{\partial y} = \frac{-z^4 - 5xy^4 - 6}{x^3 + 4yz^3}}$$