GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 - BIOTRANSPORT

FIRST TERM TEST SPRING 2014 - KEMP

STUDENT NAME: Solution

GTID NUMBER: ______________________________

RECITATION SECTION: ______________________________

(Section A is Wednesdays at 12 noon; Section B is Wednesdays at 10 am)

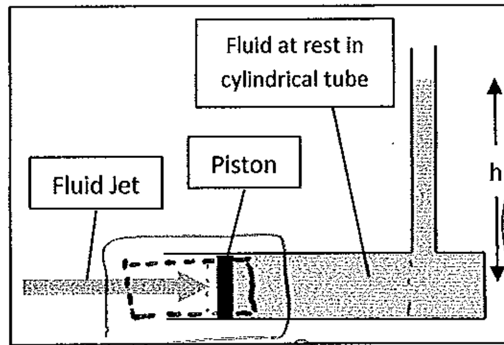
Open book
All non-communicating calculator types allowed
Time allotted: 50 minutes
Do all work in this booklet

Reminder: for questions requiring numerical answers, units are required and worth 50%

Question	Maximum Mark	Actual Mark
1	40	
2	60	
Total	**100**	

4 min

1. A fluid jet of cross-sectional area $A = 4\ cm^2$ travelling at constant speed V_0 strikes one side of a piston that can slide without friction in a cylindrical tube. After striking the piston, the fluid from the jet drops vertically downward and drains out of the tube (not shown). The diameter of the piston is $D = 5\ cm$ and no fluid leaks past it. The other side of the tube is filled with the same fluid at rest, connected to a fluid column of height h is 5 m. Note that $h \gg D$, so that hydrostatic pressure variations inside the cylinder on the right side of the piston can be neglected. Under steady conditions, what is the jet speed V_0? 10



y x

2

Do your GIM analysis here

- Steady 2
- hydrostatics on RHS of piston 3
- Use CV as shown, balance x-momentum 5
- Pressure in jet is atmospheric, no friction between piston & cylinder

C.V. around all fluid is no good.

must state direction

$\overset{+}{\rightarrow} \sum_i F_x = \dot{m} V_x|_{out} - \dot{m} V_x|_{in}$ 4 or integral form with unsteady crossed out.

$$-P_{right}\,\underbrace{\frac{\pi}{4}D^2}_{piston} = 0 - \underbrace{\rho A V_0^2}_{\dot{m}} \quad 4$$ no x-momentum out

But $P_{right} = \rho g h$ 4

$$\therefore \rho g h \frac{\pi}{4} D^2 = \rho A V_0^2 \quad 2$$

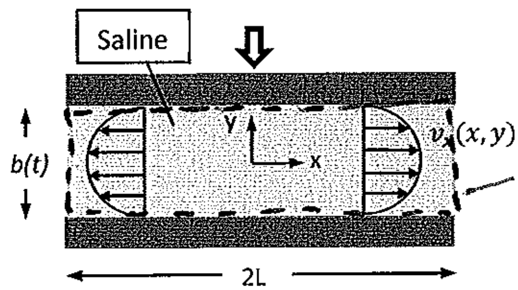
$$V_0^2 = \frac{\pi g h D^2}{4A} = \frac{\pi (981)(500)(25)}{4(4)} \quad \frac{cm}{s^2}\frac{cm^3}{cm^2} = 2.41 \times 10^6 \frac{cm^2}{s^2}$$

$$\therefore V_0 = 1552\ cm/s \quad 4$$

Must have correct CV to get full marks - else analysis makes no sense

6 min

2. A tissue sample mounted on a glass slide is to be protected by a coverslip. The sample is very thin and is covered with saline to keep it hydrated. This configuration can be modeled as two very long parallel plates of length $2L$ separated by a distance $b(t)$, with $b(t) \ll L$. Saline fills the space between the plates. The upper plate moves downward at a **non-constant** speed, which causes saline to be squeezed out between the plates. Using an (x, y) co-ordinate system centered on the saline between the plates, measurements show that the fluid velocity profile between the plates can be written as $v_x(x,y) = U\frac{x}{L}\left(1-\left(\frac{2y}{b}\right)^2\right)$, where U is a constant.



CV

3

   a. Show that this velocity profile for v_x satisfies the no-slip condition on both the top and bottom plates, where $y = \pm b/2$. **[5 marks]**
   b. If the viscosity of the flowing fluid is μ, use the given v_x to compute a formula for the shear stress exerted by the fluid on the lower plate. **[15 marks]**
   c. If the gap between the plates is b_0 at time $t = 0$, derive a formula for $b(t)$ in terms of b_0, U and L. Note that you do not need parts (a) and (b) to complete part (c). **[40 marks]**

Do your GIM analysis here
(a) do by direct substitution
(b) Use the fact that $\tau = \mu \, dv_x/dy$
(c) conserve mass of saline in CV shown 3
unsteady. 3
Work per unit depth into page

(a) No-slip requires ~~implies~~ $v_x = 0$ @ $y = \pm b/2$. Direct substitution

$$v_x(x,y) = \text{const}\left(1-\left(\frac{2y}{b}\right)^2\right)\Bigg|_{y=\pm b/2} = 0 \quad \text{QED.}$$

5

(b) For a Newtonian fluid

$$\tau = \mu \frac{dv_x}{dy} = \frac{\mu U x}{L}\left(-\frac{2 \cdot 4 \cdot y}{b^2}\right)$$

5 5

At the lower plate, we have

$$\tau_{lower} = -\frac{8\mu U x}{L b^2} y \Big|_{y=-b/2} = \frac{4\mu U x}{L b} .$$ (5) (15)

failure to sub in $y = -b/2$ (-5)

(c) $\underset{\text{in CV}}{\text{mass of saline}} = \rho \cdot 2L \cdot b$ (3) $\boxed{\text{See GIM}}$

$$\frac{d}{dt}\left(\underset{\text{in CV}}{\text{mass of saline}}\right) = 2\rho L \frac{db}{dt} .$$

$$0 = \frac{\partial}{\partial t}\int_{cv} \rho \, d\text{Vol} + \int_{cs} \rho \underline{v} \cdot \hat{n} \, dA$$ (3)

$$0 = 2\rho L \frac{db}{dt} + 2\rho \int_{-b/2}^{b/2} \frac{U x}{L}\left[1 - \left(\frac{2y}{b}\right)^2\right] dy .$$

Evaluate at $x = L$ because that is where CS is.

two ends of cv (two control surfaces)

(15)

integral is symmetric top/bottom

$$0 = \rho L \frac{db}{dt} + \rho U \cdot 2 \cdot \int_0^{b/2} \left[1 - \left(\frac{2y}{b}\right)^2\right] dy$$

failure to integrate V_x (-10)

$$-L\frac{db}{dt} = 2U \cdot \frac{b}{2} \int_0^1 (1-\eta^2) d\eta \qquad \eta = \frac{2y}{b}$$

wrong limits on integral (-4)

$$= bU\left(1 - \tfrac{1}{3}\right) = \frac{2bU}{3}$$

$$\therefore \quad \frac{db}{b} = -\frac{2}{3}\frac{U}{L} dt. \xrightarrow{\text{integrate}} b = b_0 e^{-\frac{2Ut}{3L}} .$$

(10)