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Solutions to Homework 4

1. (a) From the problem, we have $c_v = 40$, $h = 10$, $p = 100$, therefore, the optimal order-up-to quantity S is the smallest one such that

$$F(S) \geq \frac{p - c_v}{p + h} = \frac{100 - 50}{100 + 10} = 0.455.$$

Hence, $S = 600$.

(b) If we begin with x and do not order, the cost will be:

$$L(x) = 100\mathbb{E}(D - x)^+ + 10\mathbb{E}(x - D)^+$$

The order-up-to quantity as computed in (a) is 600. Therefore if we start with inventory $700 > 600$, we will not order for sure. For other cases, we will compare cost if we don't order $L(x)$ and cost if we order (up to 600) $C(x)$:
For all $x < 600$, if we do not order the cost will be

$$L(0) = 100\mathbb{E}(D - 0)^+ + 10\mathbb{E}(0 - D)^+ = 63750.$$
$$L(300) = 100\mathbb{E}(D - 300)^+ + 10\mathbb{E}(300 - D)^+ = 33750.$$
$$L(550) = 100\mathbb{E}(D - 550)^+ + 10\mathbb{E}(550 - D)^+ = 9437.5.$$

On the other hand, if we choose to make the order the cost will be

$$C(x) = 1500 + (600 - x)50 + L(600) = 36625 - 50x.$$

Then we have $C(0) = 36625 < L(0), C(300) = 21625 < L(300), C(550) = 9125 < L(550)$. Which means we will order when we begin with 0 , 300, or 550 , and the order amount is $600 - 0 = 600$, $600 - 300 = 300$, and $600 - 550 = 50$ respectively.

(c) Now $C(x) = 3000 + (600 - x)50 + L(600) = 38125 - 50x$, and we have $C(0) = 38125 < L(0), C(300) = 23125 < L(300), C(550) = 10625 > L(550)$. Therefore we'll only order when begin with 0 or 300.

2. (a) The waiting time of first 5 customers are:

$$0, 0, 0, 0, 7$$

The average is $7/5 = 1.4$.

(b) The system time of first 5 customers are:

$$1, 4, 2, 8, 10$$

The average is 5.

(c) There is a queue of size 1 (i.e. 1 customer is waiting in buffer) only during time $[18, 20]$. (You will see this by drawing a graph) So the average queue size in the first 20 minutes is

$$\frac{1 \times 2}{20} = 0.1.$$

(d) During time intervals $[2,3), [7,11), [14,16), [17,20]$, server is busy. (Again, you will see this by drawing a graph.) So the average utilization in the first 20 minutes is

$$\frac{1+4+2+3}{20} = 0.5.$$

(e) No. Little's law characterize quantities in the long-run average, in this problem, the given time is too short.

3.

$$E[W_q] = \frac{c_a^2 + c_s^2}{2} \frac{\rho}{\mu - \lambda},$$

where c_a^2 and c_s^2 are the squared coefficient of variations for the interarrival and the service times, respectively, λ is the arrival rate, μ is the service rate and ρ is the utilization of the server.

Since each interarrival time is constant $c_a^2 = 0$ and $\lambda = 30/7$ beams/hour.

For the human operator: $\mu = 36/7$ beams/hour, $c_s^2 = 900/4900 = 0.1837$, and $\rho = 30/36 = 5/6$.

For the automatic painter: $\mu = 36/7.4 = 4.865$ beams/hour, $c_s^2 = 150^2/740^2 = 0.041$, and $\rho = 0.88$.

Therefore, the average waiting time in the queue for the human operator is

$$E[W_{qh}] = \frac{0.1837}{2} \frac{5/6}{6/7} = 5.358 \text{ mins.}$$

The average waiting time in the queue for the automatic painter is

$$100E[W_{qa}] = \frac{0.041}{2} \frac{0.88}{0.58} = 1.86 \text{ mins.}$$

Using Little's Law, the average queue length for human operator is

$$E(L_h) = E(W_{qh})\lambda = 5.358 \times 1/14 = 0.383.$$

And the average queue length for automatic painter is

$$E(L_a) = E(W_{qa})\lambda = 1.86 \times 1/14 = 0.133.$$

Even though the service rate of the human operator is greater than the service rate of the automatic painter, because the service times of the automatic painter are less variable, the waiting time in the queue of the automatic painter is a lot smaller.