Dawson
ChBE 3200
Transport I

Exam II

10:05-10:55 AM (50-MINUTE EXAM)

To receive full credit on each problem, it is advised to write down all equations and work required to reach the final answer. Label all variables and equations. Include a brief word description to explain steps when necessary (e.g. $A_1=A_2=A$), stating all assumptions (e.g. incompressible).

Numerical answers without units or explanations (work required for solution) will not receive credit.

The use of wireless devices (e.g. cell phones, IR transmitters/receivers) is not permitted at any time.

NAME: Dawson
Write name on back of exam (top right corner)

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. **I pledge that I have abided by the above rules and the Georgia Tech Honor Code.**

Signed: ______________________________

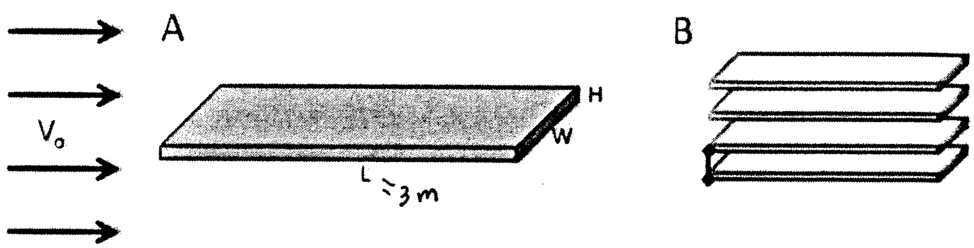
Problem I _____/ 30
Problem II _____/ 30
Problem III _____/ 40

Total _____/ 100

Make the following assumptions when necessary:
$g = 10 \text{ m s}^{-2} = 30 \text{ ft s}^{-2}$
$R = 8.31 \text{ J K}^{-1}\text{mol}^{-1} = 10.7 \text{ ft}^3 \text{ psi R}^{-1} \text{ lbm}^{-1}$
$P_{atm} = 1 \text{ atm} = 760 \text{ mm Hg} = 1.01\times10^5 \text{ Pa} = 14.7 \text{ psi}$

Problem I (30 points):

For steady flow of air over flat plate shown, assume that L= 3m, W = 1 m, H = 2 cm, and v_o=10 m/s. Air properties: ρ = 1.10 kg/m^3, μ = 2.03x10-5 Pa s.



Flat plate equations: laminar flow $\delta/x = 5\ Re_x^{-1/2}$; $C_{Fx} = 0.664\ Re_x^{-1/2}$; turbulent flow $\delta/x = 0.37\ Re_x^{-1/5}$; $C_{Fx} = 0.0576\ Re_x^{-1/5}$

A. Determine **drag force** on the surface of the plate.
B. If multiple plates (with same dimensions) are placed in parallel in the air stream (image B), what is the **minimum separation distance** (highlighted in B) that can be used to prevent boundary layers from mixing (overlapping)?

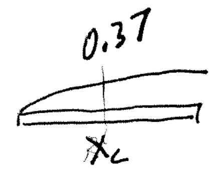
(A)

5

$$Re_L = \frac{\rho v L}{\mu} = \frac{(1.1\ kg/m^3)(10 m/s)(3m)}{2.03\times10^{-5}\ kg/ms} = \boxed{1.63\times10^6} < 3\times10^6 \quad \underline{MBL}$$

$$C_{F_L} = \frac{1}{L}\left[\int_0^{x_c} 0.664\left(\frac{\rho v}{\mu}\right)^{-1/2} x^{-1/2}\,dx + \int_{x_c}^{L} 0.0576\left(\frac{\rho v}{\mu}\right)^{-1/5} x^{-1/5}\,dx\right]$$

$Re_{x_c} = 200{,}000$



$$C_{F_L} = 1.328\ Re_{x_c}^{-1/2} + 0.072\ Re_L^{-1/5} - 0.072\ Re_{x_c}^{-1/5}$$

10

$$C_{F_L} = 0.072\ Re_L^{-1/5} - 0.0033 = \boxed{8.22\times10^{-4}}$$

Note: $C_{F_{L\,turb\ only}} = 0.0041$ $\quad F_d = 1.35\ N / 0.676 N$

-4

$$F_d = C_{F_L} A_s \tfrac{1}{2}\rho v_o^2 =$$

$$A_s = 2LW = 6 m^2$$

$$F_d = 8.22\times10^{-4}(6m^2)(0.5)(1.10 kg/m^3)(10 m/s)^2 = 0.271\ kg\,m/s^2$$

5

* $\boxed{F_d = 0.271\ N}$ 2 surfaces $\quad \boxed{F_d = 0.136}$ top only

-2 wrong eqn

local instead of ave

(B) Sep. dist. $= 2\delta + (H/2)(2)$

$$\delta = (0.37\ Re_L^{-1/5})(L) = (0.37)(1.63\times10^6)^{-0.2}(3)\,m$$

5

$\boxed{\delta = 0.0635\ m}$ $\quad$ -3 MBL

$H = 2cm = 0.02\ m$

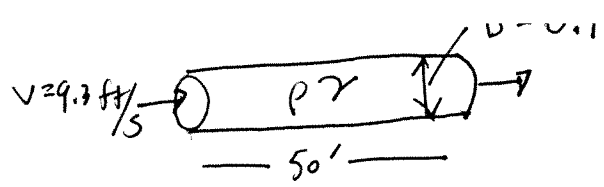
5

$$\text{Sep dist} = 2(0.0635m) + 0.02m = 0.147\ m$$

$\boxed{\text{sep. dist} = 14.7\ cm}$

28 OK

-2 calc. Error



Problem II (30 points):

Consider 50 foot long horizontal pipe of 1.2 inch inside diameter and roughness ε = 0.0002 ft. Water flows through the pipe at 9.3 ft/s, and the pressure drop in the pipe is 10 psi. You may assume water has kinematic viscosity $\nu = 0.93\times10^{-5}$ ft^2/s, $\rho = 62.4$ lbm/ft^3.

A. Calculate the **pressure loss from friction** per 50 ft of pipe length.

B. If the water needs to be pumped 10 ft up a hill, **how much work would be required?**

5

$$\boxed{\frac{\Delta P}{\rho} = \frac{10\ psi}{62.4\ lbm/ft^3} = 10\frac{lbf}{in^2}\ \frac{ft^3}{62.4\ lbm}\ \frac{32.2\ lbm\ ft}{lbf\ s^2} = 5.16\frac{ft^4}{in^2\ s^2}\left(\frac{144\ in^2}{ft^2}\right) = 743\frac{ft^2}{s^2}}$$

10

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu} = \frac{9.3\ ft/s\ (0.1\ ft)}{0.93\times10^{-5}\ ft^2/s} = \boxed{100{,}000\ \ \begin{matrix}TURB\\(>2300)\end{matrix}}$$

use L instead of D
−3

$$\frac{\epsilon}{D} = \frac{0.0002\ ft}{0.1\ ft} = \boxed{0.002}$$

$\frac{\epsilon}{D}=0.0002$ $f_f = 0.0046$ −2

$$\boxed{f_f = 0.00625}$$

$$h_L = 2f_f\frac{L}{D}\frac{V^2}{g} = 2(0.00625)(50/0.1)(9.3^2/30)\ ft = 18\ ft$$

$$\boxed{\frac{\Delta P_{HL}}{\rho} = g\,h_L = 30\ ft/s^2\ (18\ ft) = 540\ ft^2/s^2}$$

$\rho g h_L = 1046\ \frac{lbf}{ft^2}$

$\rho g h_L = 7.27\ psi$

5

$$\dot m = \rho V A = \rho V\frac{\pi D^2}{4} = 62.4\frac{lbm}{ft^3}\left(9.3\frac{ft}{s}\right)\left(\frac{\pi\,0.1^2}{4}ft^2\right)$$

$$\boxed{\dot m = 4.56\ lbm/s}$$

$$-\dot W_s = \dot m\left(\frac{P_2-P_1}{\rho} + g\,h_L + g\overbrace{\Delta y}^{y_2-y_1}\right)$$

10

$$\dot W_s = \dot m\left(\frac{\Delta P_{drop}}{\rho} - \frac{\Delta P_{HL}}{\rho} - g\,\Delta y\right)$$

$$\dot W_s = 4.56\frac{lbm}{s}\left(743\frac{ft^2}{s^2} - 540\frac{ft^2}{s^2} - 30\frac{ft}{s^2}(10\ ft)\right)\frac{s^2\ lbf}{32.2\ lbm\ ft}$$

$$\boxed{\dot W_s = -13.7\ \frac{lbf\ ft}{s}}\ \frac{1.341\times10^{-3}\ s^2\ hp}{0.7376\ lbf\ ft} = -0.025\ hp$$

overshot ΔP

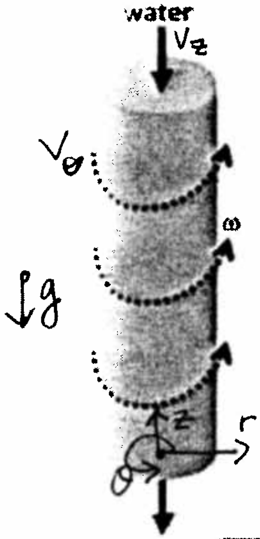
no work

not required

$\rho, \mu = \text{constant}$ $\quad v_r = 0\ ;\ v_z, v_\theta = f(r)$

$\frac{d}{d\theta} = 0$ Symmetry $\quad g_\theta, g_r = 0\ ;\ g_z = -g$

(5)



Problem III (40 points):

Assuming steady state laminar flow of water (an incompressible Newtonian fluid) down through a vertical tube (diameter = 20 cm, length = 1 m), which rotates at ω = 60 revolutions per minute, use Navier Stokes equations to answer the questions below.

A. Determine the **velocity profile**

B. Determine the **shear profile**

C. Evaluate the **pressure gradient**

2 velocity components (v_θ, v_z)

BCs for V_θ

#1 $V_\theta = 0 \quad r = 0$

#2 $V_\theta = R\omega \quad r = R$

BCs for V_z

#3 $V_z = 0 \quad r = R$

#4 $\frac{dV_z}{dr} = 0 \quad r = 0$

(5)

NS Cyl. EQNS

r: $\frac{dP}{dr} = \frac{V_\theta^2}{r}$

z: $\frac{dP}{dz} = -\rho g + \mu\left[\frac{1}{r}\frac{d}{dr} r \frac{dV_z}{dr}\right]$

θ: $\cancel{\frac{dP}{d\theta}} = \mu r \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r V_\theta)\right)$

$\frac{dP}{dr}, \frac{dP}{dz}$

(4) this is soln to c

Velocity Profile (V_θ)

$\frac{d}{dr}\frac{1}{r}\frac{d}{dr} r V_\theta = 0$

$\frac{1}{r}\frac{d}{dr} r V_\theta = c_1$

$\frac{d}{dr} r V_\theta = c_1 r$

$r V_\theta = \frac{c_1 r^2}{2} + c_2$

$$\boxed{V_\theta = \frac{c_1 r}{2} + \frac{c_2}{r}}$$

(5)

BC#1 $\quad c_2 = 0$

BC#2 $\quad R\omega = \frac{c_1 R}{2}$

$c_1 = 2\omega$

(4)

$$\boxed{V_\theta = r\omega}\ \vec{e}_\theta$$

Shear on circumference

$\frac{dV_\theta}{dr} = \omega \Rightarrow \boxed{\tau_{r\theta} = \mu\omega}$ θ direction

Velocity Profile (V_z)

$\frac{d}{dr} r \frac{dV_z}{dr} = \left(\frac{dP}{dz} + \rho g\right)\frac{r}{\mu}$

$r\frac{dV_z}{dr} = A\frac{r^2}{2\mu} + c_1$

$\frac{dV_z}{dr} = \frac{Ar}{2\mu} + \frac{c_1}{r}$

$$\boxed{V_z = \frac{Ar^2}{4\mu} + c_1 \ln r + c_2}$$

(5)

Bc#3 $\quad \frac{dV_z}{dr} = 0 \Rightarrow c_1 = 0$

Bc#4 $\quad 0 = \frac{AR^2}{4\mu} + c_2 \Rightarrow c_2 = -\frac{AR^2}{4\mu}$

$$\boxed{V_z = -\frac{AR^2}{4\mu}\left(1 - \frac{r^2}{R^2}\right)}\ \vec{e}_z$$

(4)

$A = \frac{dP}{dz} + \rho g$

$\frac{dV_z}{dr} = \frac{Ar}{2\mu} \quad \boxed{\tau_{rz} = \frac{Ar}{2}}$ z direction

Ho #2

ChBE 3200 **Transport Processes I**

SS $v_r=0$ P.S. $\frac{d}{d\theta}=0$ PD $\frac{d}{dz}=0$ $g_r=g_\theta=0$

in cylindrical coordinates: $v_r=0$ $\frac{d}{d\theta}=0$ $v_r=0$

r: $$\rho\left(\cancel{\frac{\partial v_r}{\partial t}}+\cancel{v_r\frac{\partial v_r}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial\theta}}-\frac{v_\theta^2}{r}+\cancel{v_z\frac{\partial v_r}{\partial z}}\right)=-\frac{\partial P}{\partial r}+\cancel{\rho g_r}+\mu\left[\cancel{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right)}+\cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial\theta^2}}-\cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}}+\cancel{\frac{\partial^2 v_r}{\partial z^2}}\right]$$

θ: $$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}}+\cancel{v_r\frac{\partial v_\theta}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial\theta}}+\frac{v_rv_\theta}{r}+\cancel{v_z\frac{\partial v_\theta}{\partial z}}\right)=-\frac{1}{r}\frac{\partial P}{\partial\theta}+\cancel{\rho g_\theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial\theta^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial\theta}}+\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right]$$

z: $$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}+\cancel{v_r\frac{\partial v_z}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial\theta}}+\cancel{v_z\frac{\partial v_z}{\partial z}}\right)=-\frac{\partial P}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial\theta^2}}+\cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

- Viscosity: $\tau_{yx}=\mu\frac{dv_x}{dy}$ (Cartesian)

  or $\tau_{r\theta}=\mu r\frac{d}{dr}\left(\frac{v_\theta}{r}\right)$, $\tau_{rz}=\mu\frac{dv_z}{dr}$ (Cylindrical)

- Drag force: $\frac{F}{A_P}=C_D\frac{\rho v_\infty^2}{2}$

- Head loss in pipe flow: $h_L=2f_f\frac{L}{D}\frac{v^2}{g}$

- Head loss in pipe fittings: $h_L=K\frac{v^2}{2g}$, table of K-values provided below

UNITS:

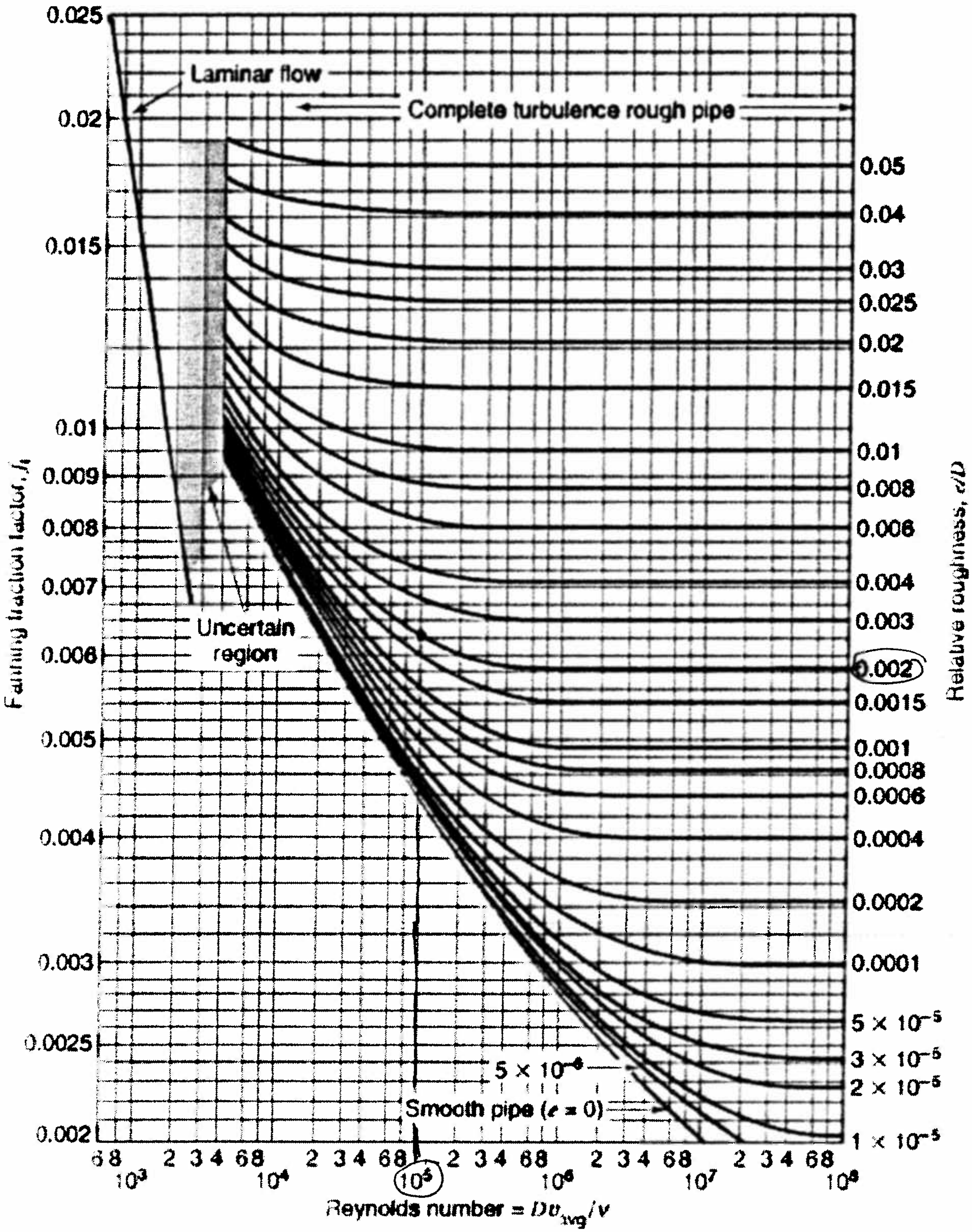
In the imperial system the conversion factor $g_c=32.2\ \frac{lb_m\cdot ft}{lb_f\cdot s^2}$

CONSTANTS:

Gas law constant: R = 8314.5 (kg m^2)/(s^2 kg-mol K)
R = 49686 (lb$_m$ ft^2)/(s^2 lb-mol °F)

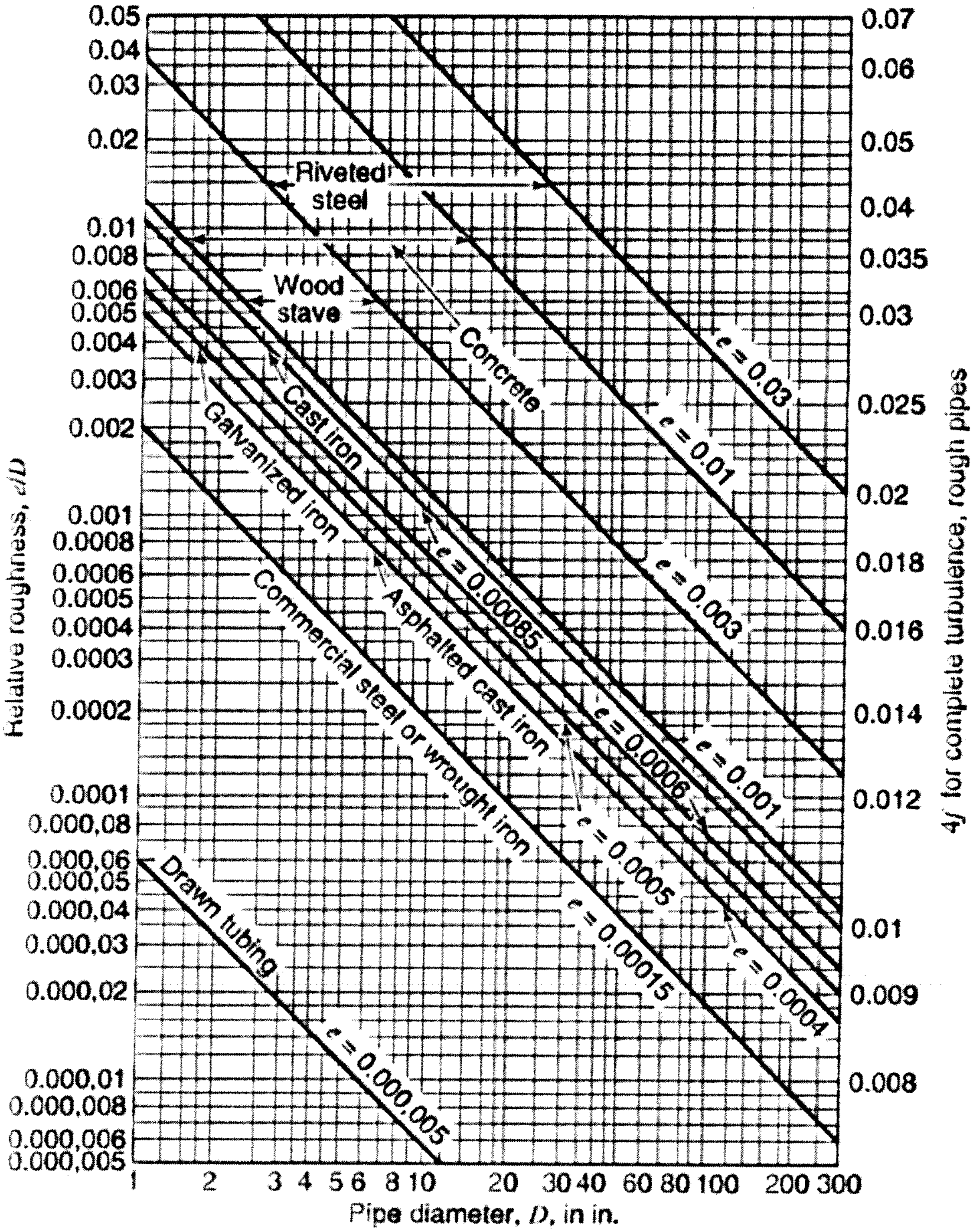
FRICTION FACTORS OF PIPE FITTINGS:

Fitting	***K***	***L$_{eq}$/D***
Globe valve, wide open	7.5	350
Angle valve, wide open	3.8	170
Gate valve, wide open	0.15	7
Gate valve, half open	4.4	200
Standard 90° elbow	0.7	32
Standard 45° elbow	0.35	15
180° Bend	1.6	75
Contraction (prefactor)	0.55	
Expansion (prefactor)	1.0	



The Fanning friction factor as a function of Re and *D/e*

Ho #5



Roughness parameters for pipes and tubes. Values of *e* given in feet.