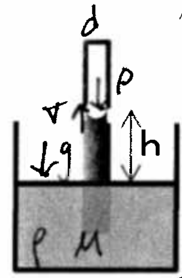
Name (2 points): Dawson

February 29, 2016
ChBE 3200
Quiz 5

Question 1 (4 points):
Use dimensional analysis to determine the number of dimensionless parameters required to describe the capillary effects on an incompressible Newtonian fluid (shown below). **(1) Label the variables describing the system, (2) specify fundamental dimensions, and (3) write equations for dimensionless parameters in terms of the product of the recurring and non-recurring variables.** Don't solve for constants.



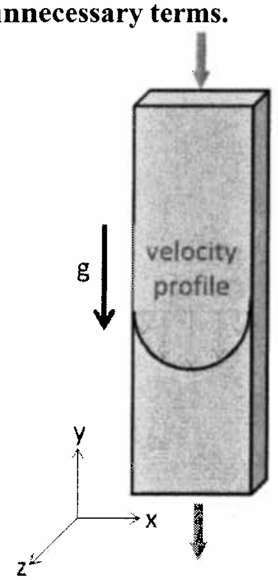
① Fluid properties ρ, μ or ν
Char length d
Forces v (up), g (down), P (down)
at least one of these → ok if both.
Capillary effect h

Variables 7 ρ, μ, d, v, g, P, h
② FDs 3 M, L, t

RVs: ρ, μ, d (covers M, L, t)
NRVs: v, g, P, h

③
$$\boxed{\begin{aligned}\Pi_1 &= \rho^a \mu^b d^c v\\ \Pi_2 &= \rho^a \mu^b d^c g\\ \Pi_3 &= \rho^a \mu^b d^c P\\ \Pi_4 &= \rho^a \mu^b d^c h\end{aligned}}$$

Question 2 (4 points):
For steady flow of an incompressible Newtonian fluid down the inner surface of a rectangular conduit (as shown below) specify **(1) which of the Navier-Stokes equations listed below apply, (2) list your assumptions, and (3) eliminate unnecessary terms.**



①

SS $\quad v_x=0 \quad v_x=0 \quad v_z=0 \quad =0$

$$\rho\left(\cancel{\frac{\partial v_x}{\partial t}} + \cancel{v_x\frac{\partial v_x}{\partial x}} + \cancel{v_y\frac{\partial v_x}{\partial y}} + \cancel{v_z\frac{\partial v_x}{\partial z}}\right) = -\frac{\partial P}{\partial x} + \cancel{\rho g_x} + \mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}} + \cancel{\frac{\partial^2 v_x}{\partial y^2}} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}\right)$$

$v_y \neq f(y)$ $\qquad$ $v_y = f(y)$; uniform z

$$\rho\left(\cancel{\frac{\partial v_y}{\partial t}} + \cancel{v_x\frac{\partial v_y}{\partial x}} + \cancel{v_y\frac{\partial v_y}{\partial y}} + \cancel{v_z\frac{\partial v_y}{\partial z}}\right) = -\frac{\partial P}{\partial y} + \rho g_y + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \cancel{\frac{\partial^2 v_y}{\partial y^2}} + \cancel{\frac{\partial^2 v_y}{\partial z^2}}\right)$$

$=0$

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_x\frac{\partial v_z}{\partial x}} + \cancel{v_y\frac{\partial v_z}{\partial y}} + \cancel{v_z\frac{\partial v_z}{\partial z}}\right) = -\frac{\partial P}{\partial z} + \cancel{\rho g_z} + \mu\left(\cancel{\frac{\partial^2 v_z}{\partial x^2}} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right)$$

② expected
- SS ($\frac{d}{dt}=0$)
- FD ($v_y \neq f(y)$)
- Incomp./Newtonian (ρ, μ const.)
- Uniform in z ($\frac{d}{dz}=0$)
- Velocities ($v_x, v_z = 0$; $v_y = f(x)$)

helpful
- $g_x, g_z = 0$
- g_y acts down (neg)

③
$$\boxed{\frac{dP}{dy} + \rho g_y = \mu\frac{d^2 v_y}{dx^2}}$$