PHYS 2212 Test 2
Spring 2015

Name(print) Key Lab Section ____________

Lab section by day and time: Kim(P), Ballantyne(Q)				
Monday	1:05-3:55pm	P01 or Q01	4:05-6:55pm	Q02 or P02
Tuesday	12:05-2:55pm	Q03 or P03	3:05-5:55pm	Q04 or P04
Wednesday	1:05-3:55pm	Q05 or P05	4:05-6:55pm	Q06 or P06
Thursday	12:05-2:55pm	Q07 or P07	3:05-5:55pm	Q08 or P08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5\times10^{4}$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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Sign your name on the line above

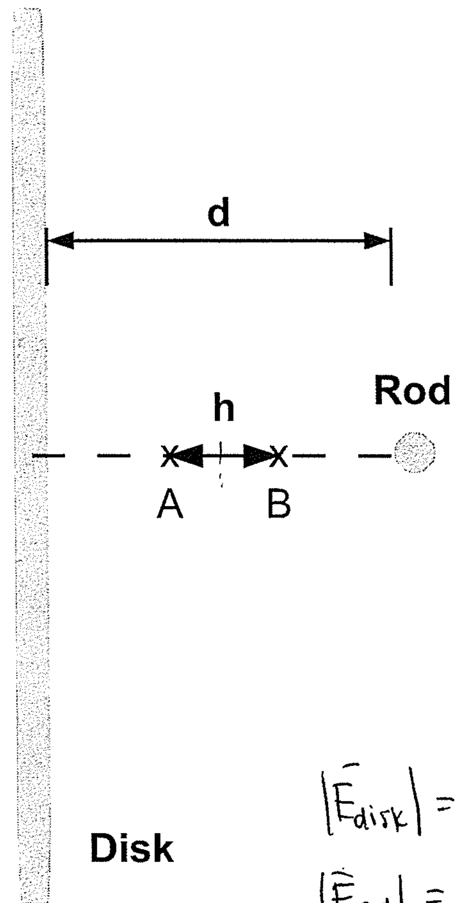
PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

cleric	−1.0
minor	−4.0
major	−7.5
BTN	−20



A very long, thin glass rod of length L carries a uniformly distributed charge $+q$. A very large plastic disk of radius R, carrying a uniformly distributed charge $-Q$, is located a distance d from the rod, where d << L and d<< R. Calculate the potential difference $V_B - V_A$ near the center of the disk. Location A is a distance $(d-h)/2$ from the surface of the disk. To earn full credit you must show your work.

$$|\bar{E}_{disk}| = \frac{Q/A}{2\epsilon_0}$$

$$|\bar{E}_{rod}| = \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r}$$

$$\Delta V = -\int \bar{E}\cdot d\bar{l}$$

$$= -\int \bar{E}_{disk}\cdot d\bar{l} - \int \bar{E}_{rod}\cdot d\bar{l}$$

$$= -\int \frac{(-Q)}{2\epsilon_0 A}dx - \frac{1}{4\pi\epsilon_0}\int \frac{2Q}{xL}dx$$

$$= \int_{\frac{d-h}{2}}^{\frac{d+h}{2}} \frac{Q}{2\epsilon_0 A}dx - \frac{1}{2\pi\epsilon_0 L}\frac{Q}{} \int_{-\frac{d+h}{2}}^{-\frac{d-h}{2}} \frac{1}{x}dx$$

$$= \frac{Q}{2\epsilon_0 A}\left[\left(\frac{d+h}{2}\right)-\left(\frac{d-h}{2}\right)\right] - \frac{Q}{2\pi\epsilon_0 L}\ln\left[\frac{-\frac{(d-h)}{2}}{-\frac{(d+h)}{2}}\right]$$

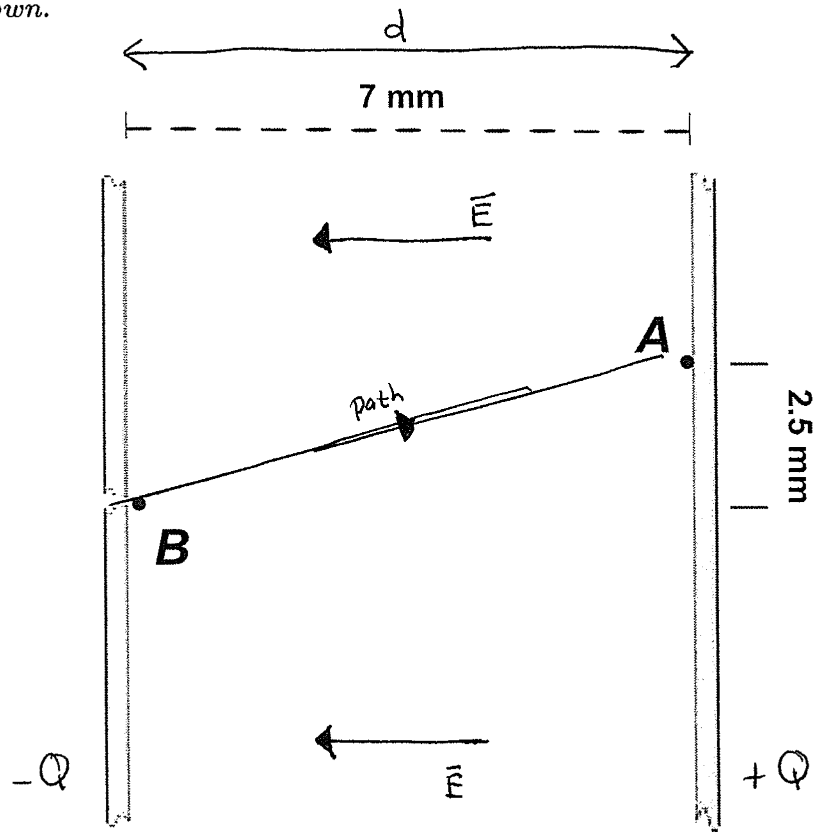
$$= \frac{Q}{2\epsilon_0 A}h - \frac{Q}{2\pi\epsilon_0 L}\underbrace{\ln\left(\frac{d-h}{d+h}\right)}_{<0} \quad > 0 \quad \text{as it should be.}$$

$$= \frac{Qh}{2\epsilon_0 A} - \frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{d-h}{d+h}\right)$$

Problem 2 (25 Points)

A capacitor consists of two very large metal disks of radius 4 m placed 7 mm apart. The plates are charged and then disconnected. One of the plates has a charge of $+Q$ and the other has a charge of $-Q$. An electron moving at a speed of 3.8×10^6 m/s enters through a hole in the left plate at location B. It moves across the gap, and when it reaches the right plate, at location A, its speed is 7.2×10^6 m/s.

Note: The diagram is not to scale. The plates are much larger and the separation distance is much smaller than shown.



(a 5pts) On the diagram, indicate which plate is positive and which is negative. All

(b 5pts) Draw arrows to represent the electric field vector at two different locations in the region between the plates and label them $\vec{E}$. ***The relative lengths of the arrows must be correct.***

→ BAd direction but same Arrows (may too) } −3 pts

→ only one arrow } − 3 pts

→ BAd may but same correct direction } −3 pts (c 5pts) Calculate the potential difference $V_A - V_B$.

$$\Delta V = -\int \bar{E}_{cap} \cdot d\bar{l}$$

$$= \int_0^d \frac{Q}{A\epsilon_0} dl$$

$$= \frac{Q}{A\epsilon_0} d$$

$$= 106.3 \text{ V} \quad \text{(using Q from part d)}$$

$$|\bar{E}_{cap}| = \frac{Q}{A\epsilon_0}$$

$$\bar{E}_{cap} \cdot d\bar{l} = |E_{cap}||dl| \cos\theta$$
$$= -|E_{cap}||dl| \qquad \theta = \pi$$

All or $\begin{cases} -2 \text{ for sign} \\ -1 \text{ for units} \end{cases}$

(d 5pts) Calculate the magnitude of the charge on each plate.

$$\frac{1}{2}mv_i^2 + e\Delta V = \frac{1}{2}mv_f^2 \quad \Rightarrow \quad \frac{eQd}{A\epsilon_0} = \frac{1}{2}m(v_f^2 - v_i^2)$$

from $\Delta K + \Delta U = 0$

$$\Rightarrow \quad Q = \frac{1}{2}\frac{mA\epsilon_0}{ed}(v_f^2 - v_i^2)$$

$$= 6.75 \times 10^{-6} \text{ C}$$

note: students will likely correctly do (c) & (d) in diff order

(e 5pts) Now a plastic slab of dielectric constant 2.8 with a thickness of 3.5mm is placed into the gap. The plastic fills the gap from the surface of the left plate to halfway between the plates. Calculate $V_A - V_B$.

$$\Delta V = -\int \bar{E} \cdot d\bar{l} \qquad \Delta V_{ins} = \frac{\Delta V_{vacuum}}{K}$$

All or $\begin{cases} -2 \text{ for sign} \\ -1 \text{ for units} \end{cases}$

$$= -\int \bar{E}_{cap} \cdot d\bar{l} - \int \bar{E}_{cap} \cdot d\bar{l}$$

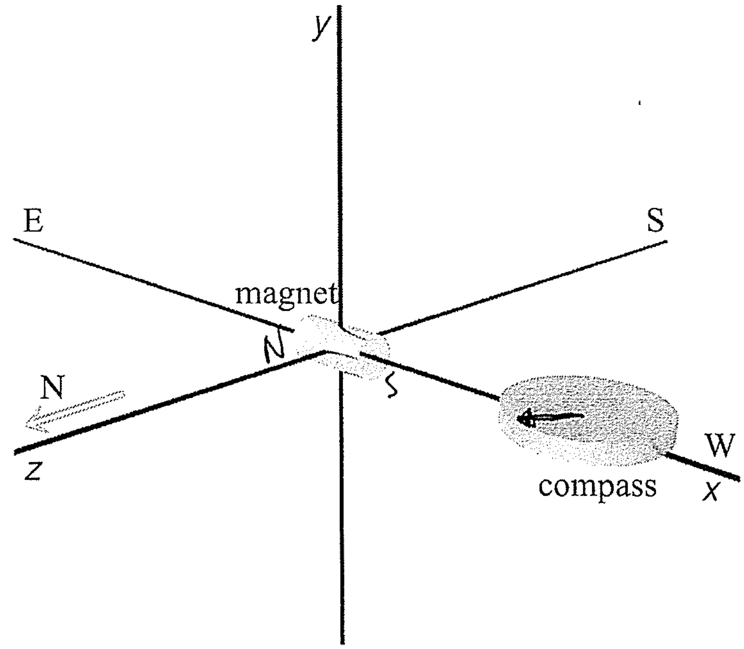
$$= \frac{1}{K}\int_0^{\frac{d}{2}} \frac{Q}{\epsilon_0 A} dl + \int_{\frac{d}{2}}^{d} \frac{Q}{\epsilon_0 A} dl$$

$$= \frac{1}{K}\frac{Q}{\epsilon_0 A}\frac{d}{2} + \frac{Q}{\epsilon_0 A}\frac{d}{2}$$

$$= \frac{Qd}{\epsilon_0 A 2}\left(\frac{1}{K} + 1\right) = 72.15 \text{ V}$$

Problem 3 (25 Points)

A bar magnet, with its center at the origin, is oriented along the x-axis. The Earth's magnetic field points North, in the +z direction as indicated in the diagram. A compass placed at location $\langle 0.06, 0, 0\rangle$ m deflects 40° to the East.



(a 5pts) Label the "north" and "south" poles of the bar magnet on the diagram at right.

(b 5pts) What is the magnetic dipole moment of the bar magnet? Show your work.

$$|B_{mag}| = |B_{Earth}| \tan\theta \qquad |\vec{B}_{dipole,axis}| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$$

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{2\mu}{r^3} = |B_E|\tan\theta \qquad |B_E| \approx 2\times10^{-5}\,T$$

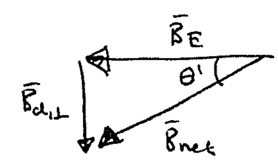
$$\Rightarrow \mu = \frac{2\pi r^3}{\mu_0}|B_E|\tan\theta$$

$$= 0.0181\ A\cdot m^2$$

All or −1 for units

(c 5pts) What compass deflection (magnitude and direction) would you expect to see if the compass were moved to location $\langle 0, 0.06, 0\rangle$ m?

$$|B_{dipole,\perp}| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3} = \frac{1}{2}|B_E|\tan\theta$$



$$|\vec{B}_{mag}| = |B_E|\tan\theta'$$

$$\Rightarrow \frac{1}{2}|B_E|\tan\theta = |B_E|\tan\theta'$$

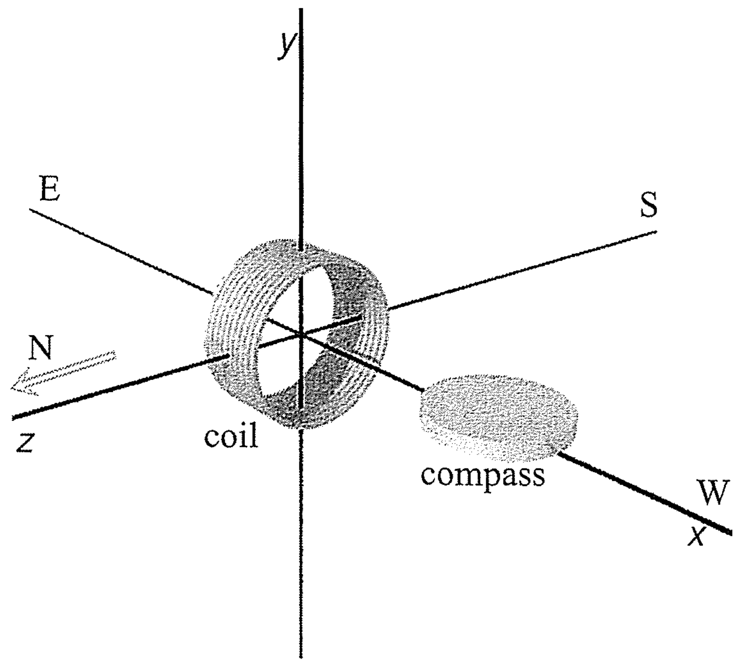
$$\Rightarrow \theta' = \tan^{-1}\left(\frac{1}{2}\tan\theta\right)$$

$$\approx 22.8^\circ \qquad \text{Westward}$$

if $\theta' = \frac{\theta}{2}$ − 4 } else All

if direction is wrong then − 3 pts }

The compass is moved back to its original location at $\langle 0.06, 0, 0\rangle$ m. The magnet is removed, and replaced by a small coil composed of 15 turns of wire, with a radius of 0.02 m. The coil is connected to a battery, by wires not shown in the diagram, and current flows in the coil. The compass deflection is 40° to the East.



(d 5pts) Calculate the conventional current in the wire.

$$|B_{coil}| = |B_E| \tan\theta \qquad |B_E| \approx 2\times10^{-5}\,T$$

$$= \frac{\mu_0 N}{4\pi}\frac{2I\pi R^2}{x^3}$$

$$\Rightarrow \quad I = \frac{2x^3}{\mu_0 N R^2}|B_E|\tan\theta$$

All or -1 for units

$$= 0.961 \text{ A}$$

(e 5pts) What compass deflection (magnitude and direction) would you expect to see if the compass were moved to location $\langle 0.18, 0, 0\rangle$ m?

$$|B_{coil}| = \frac{\mu_0 N}{4\pi}\frac{2I\pi R^2}{\bar{x}^3}$$

$$= \frac{\mu_0 R^2 N}{2\bar{x}^3}\left(\frac{2x^3}{\mu_0 N R^2}\right)|B_E|\tan\theta$$

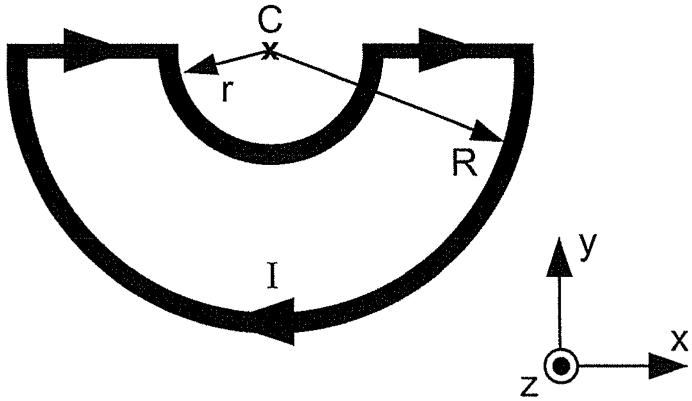
$$= \frac{x^3}{\bar{x}^3}|B_E|\tan\theta$$

Bad direction then -3pts else All

$$\Rightarrow \quad |B_E|\tan\theta' = \frac{x^3}{\bar{x}^3}|B_E|\tan\theta \Rightarrow \theta' = \tan^{-1}\left(\frac{x^3}{\bar{x}^3}\tan\theta\right) \approx 1.78° \text{ East}$$

Problem 4 (25 Points)

A conventional current I runs clockwise around a loop of wire that lies in the x-y plane. As seen in the diagram, the top half of the wire is bent into a half circle of radius r and the bottom half is bent into a circle of radius R. Determine the magnitude and direction of the magnetic field at point C, the center of the circular arcs. To earn full credit for this problem you need to show your work.



$$B = \frac{\mu_0}{4\pi} \int \frac{I\, d\bar{l} \times \hat{r}}{r^2}$$

$$I\, d\bar{l} \times \hat{r} = I\, dl = I \underbrace{r\, d\theta}_{\text{arc length}}$$

$$B_{inner} = \frac{\mu_0}{4\pi} \int \frac{I\, d\bar{l} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \int_0^{\pi} \frac{I r\, d\theta}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r} \int_0^{\pi} d\theta$$

$$= \frac{\mu_0 I}{4r} \hat{z}$$

$$B_{outer} = \frac{\mu_0}{4\pi} \int \frac{I\, d\bar{l} \times \hat{r}}{R^2}$$

$$= \frac{\mu_0}{4\pi} \int_0^{\pi} \frac{I R\, d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi R} \int_0^{\pi} d\theta$$

$$= -\frac{\mu_0 I}{4R} \hat{z}$$

$$\bar{B}_{net} = \frac{\mu_0 I}{4} \left[\frac{1}{r} - \frac{1}{R} \right] \hat{z}$$

clevic − 1.0
minor − 4.0
major − 7.5
BTN − 20

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \dfrac{d\vec{v}}{dt}$

$\dfrac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\dfrac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta U_{el} = q\Delta V$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x\Delta x + E_y\Delta y + E_z\Delta z)$

$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$

$\oint \vec{E} \bullet \hat{n} dA = \dfrac{\sum q_{inside}}{\epsilon_0}$

$\oint \vec{B} \bullet \hat{n} dA = 0$

$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left|\dfrac{d\Phi_{mag}}{dt}\right|$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \dfrac{d}{dt}\int \vec{E} \bullet \hat{n} dA\right]$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r\sqrt{r^2 + (L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$, $\vec{p} = \alpha \vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \dfrac{1}{4\pi\epsilon_0}\dfrac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \dfrac{1}{4\pi\epsilon_0}\dfrac{qz}{(z^2 + R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{(z^2 + R^2)^{1/2}}\right]$ ($z > 0$ along axis)

$\left|\vec{E}_{disk}\right| \approx \dfrac{Q/A}{2\epsilon_0}\left[1 - \dfrac{z}{R}\right] \approx \dfrac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \dfrac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \dfrac{Q/A}{\epsilon_0}\left(\dfrac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\Delta\vec{\ell} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \dfrac{\mu_0}{4\pi}\dfrac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$

$\left|\vec{B}_{loop}\right| = \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \dfrac{\mu_0}{4\pi}\dfrac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$)

$\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \dfrac{\mu_0}{4\pi}\dfrac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \dfrac{1}{4\pi\epsilon_0}\dfrac{-q\vec{a}_\perp}{c^2 r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \dfrac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\, nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\, nu$

$J = \dfrac{I}{A} = \sigma E$

$R = \dfrac{L}{\sigma A}$

$E_{dielectric} = \dfrac{E_{applied}}{K}$

$\Delta V = \dfrac{q}{4\pi\epsilon_0}\left[\dfrac{1}{r_f} - \dfrac{1}{r_i}\right]$ due to a point charge

$I = \dfrac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$

$\text{K} \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\frac{d\vec{p}}{dt}_{\perp}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\, dx = ax + c \quad \int ax\, dx = \frac{a}{2}x^2 + c \quad \int ax^2\, dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N $\cdot$ m^2/C^2)$^{-1}$
Magnetic constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T $\cdot$ m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T $\cdot$ m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T