Full name: Solutions Math 2551, Section B___

Quiz 2 — §12.5-13.1

Please ***clearly*** show all work. Scientific calculators are allowed, but no graphing calculators!

(1) Write down a parametric equation for the line L that is the intersection of the two planes $x+2y-2z=5$ and $5x-2y-z=0$. [8 points]

First we find a point on the line L by finding a solution to the simultaneous system

$$\begin{cases} x+2y-2z=5 \\ 5x-2y-z=0 \end{cases}$$

One example is the point $P_0=(0,5/6,-5/3)$; this is the solution you get by imposing the extra condition $x=0$. Next we need a direction vector $\mathbf{m}$ for L. Such a direction vector is given by the cross product of the normal vectors of the planes:

$$\mathbf{m}=\mathbf{n}_1\times\mathbf{n}_2=(\mathbf{i}+2\mathbf{j}-2\mathbf{k})\times(5\mathbf{i}-2\mathbf{j}-\mathbf{k})=-6\mathbf{i}-9\mathbf{j}-12\mathbf{k}.$$

We can now write a parametric equation for L:

$$\begin{cases} x=-6t \\ y=\frac{5}{6}-9t \\ z=-\frac{5}{3}-12t \end{cases} \qquad -\infty<t<\infty$$

(2) Find the velocity and acceleration at time $t = \pi/2$ of a particle whose position vector $\mathbf{r}(t)$ is $\mathbf{r}(t) = \cos^2 t\,\mathbf{i} + \ln t\,\mathbf{j} + 4t\mathbf{k}$. [6 points]

$$\mathbf{v}(t) = -2\cos t \sin t\,\mathbf{i} + \frac{1}{t}\mathbf{j} + 4\mathbf{k} \quad \Longrightarrow \quad \mathbf{v}(\pi/2) = \frac{2}{\pi}\mathbf{j} + 4\mathbf{k}$$

$$\mathbf{a}(t) = (2\sin^2 t - 2\cos^2 t)\mathbf{i} - \frac{1}{t^2}\mathbf{j} \quad \Longrightarrow \quad \mathbf{a}(\pi/2) = 2\mathbf{i} - \frac{4}{\pi^2}\mathbf{j}$$

(3) Sketch the surface in space defined by $z = 2 - x^2 - y^2$. [6 points]

The horizontal slices where $z = c$ are the circles $x^2 + y^2 = 2 - c$. The "profile" of the surface can be seen by taking a vertical slice, say where $x = 0$: this is the parabola $z = 2 - y^2$. Combining this information we easily draw the surface: