NAME:

GRADE:

ISyE 3044 — Test #1

Fall 2012

No books and notes are allowed. You can use only the supplied formula sheet and tables, as well as a scientific calculator with single-variable statistical functions.

1. **[6 points]** Short Simio questions.

   (a) Two Server objects in a Simio model (`Server1` and `Server2`) are connected with a TimePath. The output buffer of `Server1` and the input buffer of `Server2` have zero capacity. Entities attempting to visit `Server2` after processing by `Server1` accumulate along the TimePath.
   True False

   (b) In order to conduct independent replications in Simio, we must define an experiment and hit the Run (Fast Forward) button.
   True False

   (c) The weight of a Simio Path can be a logical expression.
   True False

   (d) Simio Model Properties (defined in the Definitions tab) become Controls in Experiments.
   True False

   (e) A response of a Simio Experiment is the average time entities spend in a queue (buffer). Assume that practically each replicate starts in steady state. The resulting SMORE plot contains estimates of the steady-state mean time an entity spends in this queue.
   True False

2. **[8 points]** Consider the integral

$$\mu = \int_0^{2\pi} \sin\left(\sqrt{t}\right) dt.$$

(a) Use the pseudo-random numbers below

.10 .23 .58 .37 .72 .81 .97 .48 .04 .39

and the Monte Carlo method to compute a point estimate of μ. (Pay attention to the angular unit.)

ANSWER: ______________________

(b) Compute an approximate 95% confidence interval (CI) for μ.

ANSWER: ______________________

(c) Is the following statement correct? **Yes No**
The CI in part (b) contains μ with probability about 95%.

(d) Is the point estimate in part (a) unbiased? **Yes No**

3. **[8 points]** Entities arrive at a single-server facility with inter arrival times equal to 3 minutes. (The first entity arrives at time zero.) The service discipline is FIFO and the service times are i.i.d. from the discrete random variables taking values 2, 3 or 5 with probabilities 0.5, 0.3 and 0.2, respectively. Use the following pseudo-random numbers to generate the service times of the first ten entities:

.53 .89 .68 .03 .37 .91 .16 .44 .26 .72

(a) What is the average delay in queue for an entity (prior to service) during the simulated time window?

ANSWER: ____________________

(b) What is the average number of entities in the system?

ANSWER: ____________________

(c) What is the average queue length?

ANSWER: ____________________

(d) Is the system stable in the long run? Explain your answer.

ANSWER: ____________________

4. Short questions.

   (a) Entities arrive at a single-server queueing system as a Poisson process with a mean interarrival time of 5 minutes. The service times are i.i.d. from the normal distribution with a mean of 4 minutes and standard deviation 1.

   [2 pts.] Suppose that the 10th entity arrived before 10:00 a.m. What is the probability that the 12th entity will arrive before 10:10 a.m.?

   ANSWER: ____________________

   [2 pts.] Compute the mean time an entity spends in system in steady state.

   ANSWER: ____________________

   [1 pt.] What fraction of entities will find the server idle in steady state?

   ANSWER: ____________________

   (b) **[1 pt.]** Name the British scientist who used a simulation experiment for quality control problems in the early part of the 20th century.

   ANSWER: ____________________

   (c) **[1 pt.]** The nonstationary Poisson arrival model assumes that arrival counts in nonoverlapping time intervals are independent random variables. **True False**

   (d) **[1 pt.]** The (stationary) Poisson arrival model is reasonable for modeling arrivals at restaurants during a day. **True False**

ISyE 3044/Fall 2012: Solutions to Test #1

1. (a) True.
   (b) True.
   (c) True.
   (d) True.
   (e) True.

2. (a) Using the transformation $t = 2\pi u$, we write $\mu = \int_0^1 2\pi \sin(\sqrt{2\pi u})\, du$. The point estimate is
$$\hat{\mu} = \frac{1}{10}\sum_{i=1}^{10} X_i = 5.22; \quad X_i = 2\pi \sin\left(\sqrt{2\pi U_i}\right).$$
   (b) Using the sample variance
$$S_{10}^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \hat{\mu})^2 = 1.12^2$$
   and $t_{9,0.025} = 2.26$, we get the confidence interval
$$5.22 \pm 2.26\frac{1.12}{\sqrt{10}} = 5.22 \pm 0.80 = (4.42, 6.02).$$
   (c) No, this interval is a realization of a random interval. We discussed this extensively in class.
   (d) Yes.

3. The service times are 3, 5, 3, 2, 2, 5, 2, 2, 2, and 3 and the respective departure times are 3, 8, 11, 13, 15, 20, 22, 24, 26, and 30.
   (a) $8/10 = 0.8$.
   (b) $37/30 = 1.23$.
   (c) $8/30 = 0.27$.
   (d) Yes because the mean service time is $E(S) = 2(0.5) + 3(0.3) + 5(0.2) = 2.9 < 3$.

4. (a) Let $N(t)$ be the number of arrivals in an interval of length t. It follows that $N(t)$ has the Poisson distribution with mean $\frac{1}{5}t$. The probability that the 12th entity will arrive before 10:10 a.m. is
$$\Pr\{N(10) \geq 2\} = \Pr\{\text{Poisson}(2) \geq 2\} = 1 - \Pr\{\text{Poisson}(2) < 2\} = 1 - e^{-2} - 2e^{-2} = 0.594.$$
   We have an M/G/1 queue with $\lambda = 1/5$, $E(S) = 4$, and $\text{Var}(S) = 1$. Using the Pollaczek-Khinchin formula we have
$$W = \frac{\frac{1}{5}(4^2 + 1)}{2(1 - \frac{4}{5})} + 4 = 12.5.$$
   The long-run fraction of entities that find the server idle in steady state is $1 - \frac{4}{5} = 0.2$.
   (b) Dr. Who.
   (c) True.
   (d) False. The rate of arrivals varies over time...

NAME →

SCORE (Max. 30): ______

ISyE 3044 — Test 1

Summer 2012

No books and notes are allowed. You can use only the supplied formula sheet and statistical table(s).

1. **[12 points]** Entities arrive at single-server facility as a Poisson process with a rate of 1/4 per minute. The service times are i.i.d. uniform 3 ± 1 minutes.

   (a) What is the distribution of the number of arrivals during a 10-minute time window? Give the parameter(s) of this distribution.

   ANSWER: ______________________

   (b) Three entities arrived between 10:00 and 10:10 a.m. What is the probability that the next entity will arrive before 10:15 a.m?

   ANSWER: ______________________

   For parts (c)–(g) assume that the queue in front of the server has infinite capacity and the entities are served in FIFO order.

   (c) Give the notation of this queueing system.

   ANSWER: ______________________

   (d) Compute the mean waiting time of an entity prior to service in steady state.

   ANSWER: ______________________

   (e) Compute the mean number of entities in service in steady state.

   ANSWER: ______________________

(Problem 1 continued.)

(f) We develop a Simio model with a Source object (`Source1`), a Server object (`Server1`), and a Sink object (`Sink1`). Write the expression for the interarrival time in `Source1`.

ANSWER: ____________________________

(g) Write the expression for the processing time in `Server1`.

ANSWER: ____________________________

(h) Suppose that the system can accommodate only 3 entities (including the one in service). Arriving entities that find the system full depart immediately. To estimate the fraction of arrivals that are blocked from entry, we add a `Sink2` and connect it to `Source1` with a single Connector. Give the expression for the weight of the latter connector.

ANSWER: ____________________________

2. **[6 points]** Consider the definite integral

$$\mu = \int_0^2 e^{\sqrt{t}}\, dt.$$

(a) Use Monte Carlo integration with the following pseudo-random random numbers to compute a point estimate of μ: .83, .29, .13, .28, .44, .08, .66, .72, .98.

ANSWER: ________________________

(b) Compute an approximate 95% confidence interval for μ.

ANSWER: ________________________

(c) Is the following statement correct? "If we had used 10,000 random numbers, the confidence interval from part (b) would contain the unknown μ with probability about 95%." Circle the answer.

Yes No

3. **[6 points]** Short questions.

   (a) A Simio model contains the function `Random.Discrete(2,0.35,4,0.6,7,1.0)`. What value will this function return given the uniform random number 0.55?

   ANSWER: ______________________________

   (b) Who discovered the t distribution?

   ANSWER: ______________________________

   Who was his/her employer?

   ANSWER: ______________________________

   (c) Give the last name of the scientist who came up with the term *Monte Carlo* simulation.

   ANSWER: ______________________________

   (d) A Simio model contains a `DefaultEntity` and a single sink object (`Sink1`), where all entity instances are destroyed. An experiment aims at estimating the distribution of the average entity time-in-system during a finite time window. We define the response `AvgTimeInSystem` as `Sink1.TimeInSystem.Average`. The default SMORE plot for `AvgTimeInSystem` contains estimates for the x, y, and z and quantiles of the `AvgTimeInSystem`.

   $x =$ _______________

   $y =$ _______________

   $z =$ _______________

4. **[6 points]** A single-server queueing system has room for up to 2 customers (including the one in service). Construct a detailed table to simulate the first 10 customers using the following interarrival times:

$$0, 3, 4, 3, 9, 5, 7, 2, 4, 7$$

and as many of the following service times as necessary:

$$5, 6, 3, 3, 10, 9, 2, 5, 3, 4$$

Hint: To check the system's capacity upon arrival, look at the departure times of customers who entered earlier. Do not generate data beyond the arrival times for customers who find the system full and don't enter.

(a) When did the 10th customer leave the system?

ANSWER: ____________________

(b) What is the average waiting time (prior to service) for customers who entered the system?

ANSWER: ____________________

(c) Compute the average number of customers in the system during the simulated time window.

ANSWER: ____________________

ISyE 3044/Summer 2012: Solutions to Test #1

1. (a) Poisson$(10 \times \frac{1}{4}) = $ Poisson(2.5).
   (b) By the memoryless property, the answer is $1 - e^{-5/4} = 0.71$.
   (c) M/G/1 queue.
   (d) We have $\mathrm{E}(S) = 3$, $\mathrm{Var}(S) = \frac{2^2}{12} = \frac{1}{3}$. Hence $\mathrm{E}(S^2) = \frac{1}{3} + 9 = \frac{28}{3}$. The Pollaczek-Khintchine formula yields
$$W_Q = \frac{\frac{1}{4}(\frac{28}{3})}{2(1-\frac{3}{4})} = \frac{14}{3} = 4.66.$$
   (e) The traffic intensity (server utilization), 3/4.
   (f) `Random.Exponential(4)`.
   (g) `Random.Uniform(2,4)`.
   (h) `Server1.InputBuffer.Contents.NumberWaiting == 2`.

2. (a) Using the transformation $t = 2u$, we write $\mu = \int_0^1 2\exp(\sqrt{2u})\,du$. The Monte Carlo estimate is
$$\hat{\mu} = \frac{1}{9}\sum_{i=1}^{9} X_i = 5.36; \quad X_i = 2\exp(\sqrt{2U_i}).$$
   (b) We have $S^2 = \frac{1}{8}\sum_{i=1}^{9}(X_i - \hat{\mu})^2 = 1.80^2$. Using $t_{8,0.025} = 2.31$, we get the confidence interval
$$5.36 \pm 2.31\frac{1.80}{\sqrt{9}} = 5.36 \pm 1.38 = (3.98, 6.74).$$
   (c) No! The confidence interval in part (b) is a realization of a random interval.

3. (a) 4.
   (b) Dr. Who.
   (c) A distant cousin of Dr. Who.
   (d) $x = 0.25$ (lower quartile), $y = 0.50$ (median), and $z = 0.75$ (upper quartile).

4. We have the following table. Customers 4 and 8 don't enter because they find two customers in the system.

Customer	Interarrival Time	Arrival Time	Time Service Starts	Delay	Service Time	Departure Time	Time In System
1	0	0	0	0	5	5	5
2	3	3	5	2	6	*11*	8
3	4	7	11	4	3	*14*	7
4	3	*10*	–	–	–	–	–
5	9	19	19	0	3	22	3
6	5	24	24	0	10	*34*	10
7	7	31	34	3	9	*43*	12
8	2	*33*	–	–	–	–	–
9	4	37	43	6	2	45	8
10	7	44	45	1	5	50	6

(a) The last car leaves at time 50.

(b)

$$\frac{\text{Sum of delays}}{8} = \frac{16}{8} = 2.$$

(c)

$$\frac{\text{Sum of times in system}}{50} = \frac{59}{50} = 1.18.$$