ISYE 3232A Spring 2016 Quiz 7

April, 2016

Name: ______________________________

Suppose we agree to deliver an order in one day. The contract states that if we deliver the order within one day we receive \$1000. However, if the order is late, we lose money proportional to the tardiness until we receive nothing if the order is two days late. The length of time for us to complete the order is exponentially distributed with mean 1 day.

1. What is the probability that we are late in delivering the order (i.e., the probability that we deliver after 1 day)?

   Let T represent the delivery time. Then

$$\Pr(T > 1) = e^{-\lambda} = e^{-1} \approx 0.37.$$

   About 37% of orders are late.

2. What is the probability that we receive nothing for an order?

$$\Pr(T > 2) = e^{-2\lambda} = e^{-2}.$$

   Or if you interpreted "we receive nothing if the order is two days late" as "we receive nothing if the deliver time is 3 or more", then the answer should be

$$\Pr(T > 3) = e^{-3\lambda} = e^{-3}.$$

3. Calculate the expected revenue per order. Leave your answer with integrals.

$$\begin{aligned} \mathsf{E}[\text{ profit }] &= \int_0^1 \$1000\lambda e^{-\lambda t} dt + \int_1^2 \$(2000 - 1000t)\lambda e^{-\lambda t} dt \\ &= \int_0^1 \$1000 e^{-t} dt + \int_1^2 \$(2000 - 1000t) e^{-t} dt \quad (\approx \$767.456) \end{aligned}$$

   Similar to (b), if you interested "2 days late" differently, then the answer should be

$$\begin{aligned} \mathsf{E}[\textit{ profit }] &= \int_0^1 \$1000\lambda e^{-\lambda t} dt + \int_1^3 \$(1500 - 500t)\lambda e^{-\lambda t} dt \\ &= \int_0^1 \$1000 e^{-t} dt + \int_1^3 \$(1500 - 500t) e^{-t} dt \quad (\approx \$840.954) \end{aligned}$$