

ISyE 3025, Fall 2015, HW1. This homework is for Learning Cycle 1, leading to Quiz 1

1. In 1958 your grandfather left your mother a \$2,500 U.S. government bond, to be used for your education. The annual interest rate was 2.2%. The annual interest was left to accumulate with the bond according to compound interest. Then your mother redeemed (cashed in) the bond. What were the *cash proceeds* many years later, in 2004?
2. You wish to join a club that has an initiation fee of \$100,000. You only have \$58,000 available for this right now. You can invest in a fund that pays 6.5% per year. If you invest your money in this fund, how many years must you wait until it will grow to the required amount for the initiation fee? [Assume the initiation fee does not change]
3. A certain investment is available that promises to return \$9,000 five years from now. If the investor's "time value of money" is 3.5% per year, find the *equivalent present value* of this proposed investment.
4. Using the assumptions made by our models. If you are indifferent between having \$100 today and \$105 a year from now, then you are indifferent between having \$2000 today and x dollars in two years. Find x.
5. You expect to receive \$2200 at the end of year 2020. However, you wish instead to receive this amount earlier, at the end of year 2018. At the annual interest rate of 6%, what is the *equivalent amount* at the end of Year 2018?
6. You are presented with two investment opportunities, A and B as described below. A choice of either one would require the initial investment now (if you select either investment opportunity, you cannot invest less or more in that opportunity, only the amount shown). In addition, you can *always* invest in a fund that pays 3% per year. You currently have \$6,000 to invest. If you wish to *maximize your cash amount* at the end of year 6, which of the two investments, A or B, is the better choice? You cannot select both. *Explain numerically.*

Investment opportunity	A	B
Initial investment needed	\$4,500	\$3,800
Annual interest rate	5%	10%
Length of investment	5 years	4 years
7. A high school student is considering the choice of college to attend. Several choices have been identified, and they are all ranked the same in academic quality. There are some differences with respect to other factors: some are private colleges and some are state universities, some are near to home while others are far away, some are in large cities and others are in small towns, some are in cold climates and others in warm climates, some have highly varied extracurricular activities while others have few organized clubs, etc. List *three* important *monetary factors* and *three* important *non-monetary factors* that should be considered by the student in the selection of which college to attend.
8. You wish to deposit \$2,200 at the end of each year into an account that pays 4.4% interest per year, for 12 years. How much money will you have accumulated in the account after the last deposit?
9. You arrange to borrow \$22,000 for an automobile purchase. The loan will be repaid in 60 equal, monthly payments, with end-of-month payments. Interest is compounded monthly at the rate of 0.55% per month.
  - a. What is the amount of the monthly payment?
  - b. After making the 46<sup>th</sup> payment you consider selling the car, and you wish to determine the remaining loan balance. What is the remaining debt (principal) on the loan?

10. In order to save money for your old age you have decided to save \$2000 a year for the next forty years. This money will be invested in an account earning interest. Your friend, wanting to live the good life now (and save more later) has decided to save the same amount of money ( $\$2000 \times 40 = \$80,000$ ) using an arithmetically increasing amount. That is 0G dollars in the first year, 1G dollars in the second year, 2G dollars in the third year. Where G is calculated so, in total, your friend saves \$80,000.
- Without performing any calculations (no need to find G), who will have more money at the end of 40 years, you or your friend? Explain.
  - Go ahead and perform the necessary calculations, using an interest rate of 6% to demonstrate numerically that your answer to part (a) is correct.
  - What is the amount of the monthly payment?
  - After making the 46<sup>th</sup> payment you consider selling the car, and you wish to determine the remaining loan balance. What is the remaining debt (principal) on the loan?
11. How much must be invested today in order to provide an annuity of \$50,000 per year for 22 years, with the first payment occurring exactly 35 years from now, if the interest rate is 4.0% per year?
12. Consider the sequence of cash flows given in the table. The interest rate is 5% per period.
- |                |     |     |     |    |    |    |    |   |
|----------------|-----|-----|-----|----|----|----|----|---|
| End of Period  | 1   | 2   | 3   | 4  | 5  | 6  | 7  | 8 |
| Cash flow (\$) | 140 | 120 | 100 | 80 | 60 | 40 | 20 | 0 |
- Convert the sequence to an *equivalent uniform sequence* over 8 periods.
  - Convert the sequence to an *equivalent uniform sequence* over 7 periods, with the first non-zero element at time 1.
13. You wish to save for retirement by depositing \$750 every 4 months for a period of 25 years. The first deposit occurs 4 months from now, and the last coincides with the end of year 25. The savings fund earns 0.5% per month, compounded monthly. How much will you have accumulated when you retire?
14. You wish to deposit a sum today in an account to provide for annual fuel expenses for the next 11 years. The fuel expenses are paid at the end of each year. The expenses at the end of the first year are expected to be \$5,000, and you expect that this amount will increase by 6% each year compared to the previous year. If the account pays 2.2% interest, what sum must you deposit now?
15. You wish to save money for retirement by making annual (end-of-year) deposits into an account for the next 25 years. The first deposit will be \$5,500 and each deposit thereafter will be 2.2% larger than the preceding deposit. The account will earn 1.5% compound interest. After the last deposit, you will leave the accumulated fund in the account for another 12 years (but without making deposits). How much will you have in the fund at the end?
16. You wish to compare two savings plans: one with monthly compounding (Plan 1) and one with quarterly compounding (Plan 2). These are at different banks. In either situation, you plan to make annual deposits, so you need to find the effective annual interest rates. The monthly interest rate for Plan 1 is 0.522% per month (0.00522) and the quarterly interest rate for plan 2 is 1.522% (0.01522). Use three digits of accuracy when answering the following questions.
- What is the equivalent annual interest rate for Plan 1?
  - What is the equivalent annual interest rate for Plan 2?
  - What would be the difference in accumulated amounts between the two plans if you deposited \$6,000 each year (end-of-year deposits) for twenty years? Any interest earned will be left in the account to accumulate.

17. You arrange to borrow \$3000 for some new furniture. The loan will be repaid in 22 equal monthly payments (end-of-month payments) with interest compounded monthly at the monthly interest rate of 3%. What is the amount of the monthly payment?
18. You are still negotiating for the same loan as in the previous problem, but the finance manager at the furniture store offers you a plan where you don't make the first payment until 4 months after you buy the items. However, interest will accumulate on the unpaid loan, so the monthly payment amount will be larger. The total number of payments remains the same as in the previous problem and the interest rate is the same. What is the amount of the monthly payment?
19. A company obtained a loan help finance the purchase of a new machine. The loan is for \$250,000 at an annual interest rate of 9% to be paid off over 5 year. The question is how to repay the loan. There are two popular options: equal principal payments and equal total payments (the total of principal and interest is the same each year). Prepare loan repayment schedules for these two methods.
20. You have to make a choice between two investments. Each investment is for \$100 for five years. Both offer a nominal interest rate of 4%. The first is compounded quarterly, the second is compounded monthly.
- Without performing any calculations, which investment is preferred? Explain.
  - Go ahead and perform the necessary calculations to demonstrate numerically that your answer to part (a) is correct.
  - Another investment opportunity has arisen and with the same conditions as the first two opportunities, but this time with continuous compounding.
    - Using a spreadsheet, demonstrate that even though your interest is compounded continuously, you will not be rich at the end of the five year.
    - Using a limit argument, calculate how much you will have after this five year investment. Be careful, the solution is not eeeeeeasy and is beyond what is necessary for this course (but it is fun).