Name Key (Yellow)

Exam 2 ISyE 4301

Please read the following: This is a closed-note exam. In addition, only calculators that *do not* have the capability to send or receive data may be used (e.g., phones are not allowed). By signing the following, you are agreeing to these terms and acknowledging that all of the work on this exam is your own.

______________________________(*Signature*)

The following multiple-choice questions are worth 5 points each. Clearly mark your answer.

1. The output from a data envelopment analysis (DEA) linear program (assuming constant returns to scale) firm 3 (with 8 total firms) is $\lambda_1=0.0$, $\lambda_2=0.3$, $\lambda_3=0.0$, $\lambda_4=0.2$, $\lambda_5=0.0$, $\lambda_6=0.0$, $\lambda_7=0.0$, $\lambda_8=0.0$, $\theta_3=0.84$. Which is the best answer?
   a. Firm 3 is efficient.
   b. Firm 1 is efficient.
   c. Firm 2 is efficient. — $\lambda_2 > 0$
   d. a. and b. are both true
   e. a. and c. are both true
   f. None of the above

2. A risk-averse individual has a utility of wealth equal to the square root of the wealth. Consider two possibilities: a) \$60 or b) a gamble where a fair coin is tossed and they get \$0 for tails and \$120 for heads. The individual is given b). How much would they be willing to pay to trade b) for a). $p = .5$
   $\sqrt{60} = 7.75$
   $.5\sqrt{120} = 5.48$
   diff $= 2.27 \rightarrow \$ = 2.27^2 = 5.15$
   a. Approximately \$2.27.
   b. Approximately \$5.15.
   c. Approximately \$7.75.
   d. They wouldn't pay anything since they prefer b) to a).
   e. We don't have enough information to determine.

3. Market demand is P=100 – Q, where Q is the sum of outputs from firms 1 and 2. Each firm has a unit cost c=0. If firm 1 is the leader in a Stackelberg game, what is the profit of firm 1 equal to?
   a. $(100-((100-q_1)/2))q_1$
   b. $(100-2q_1)q_1$
   c. $(100-(q_1+(100-q_1)/2))q_1$
   d. Profit would be infinite since there is no unit cost.
   e. None of the above

For 2

$$\pi_2 = (100-(q_1+q_2))q_2$$

$$\pi_2' = 100 - q_1 - 2q_2 \equiv 0$$

$$q_2 = \frac{100-q_1}{2}$$

$$\pi_1 = \left(100 - \left(q_1 + \frac{100-q_1}{2}\right)\right)q_1$$

4. You are asked to rank suppliers based on 2 attributes (cost and quality) using the Borda count method and rank quality over cost in importance. Also, the higher the value, the better. The values for the 3 suppliers are: S1 = 12, 10; S2 = 9, 14; S3 = 18, 5. What is the ranking?
   a. S1, S2, S3
   b. S1, S3, S2
   c. S2, S1, S3
   d. S2, S3, S1
   e. S3, S1, S2
   f. S3, S2, S1

cost ← quality

quality = 2 → $w_Q = \frac{2}{3} = .67$
cost = 1 → $w_C = \frac{1}{3} = .33$

S1 = .33(12) + .67(10) = 10.66

S2 = 12.35 S3 = 9.29 → S2, S1, S3

5. Two US firms compete on price and face a market demand of P=300-2Q, where Q is the sum of the outputs of each firm. The unit cost for firm 1 is \$8, and for firm 2 is \$13. What is the Bertrand equilibrium?
   a. Firm 1 sells at a price of \$8.01 and firm 2 sells at a price of \$13.01.
   b. Both firms sell at a price of \$8.01.
   c. Firm 1 sells at a price of \$8.01, and firm 2 sells nothing.
   d. There is no Bertrand equilibrium in this case.
   e. None of the above

Firm 1 sells at 12.99
Firm 2 sells nothing.

6. Consider a simultaneous move 2-person game. Each player can choose action A_1 or A_2. The payoffs for the 4 outcomes are: $(A_1,A_1) = (8, -2)$; $(A_1, A_2) = (4, -1)$; $(A_2, A_1) = (7, 7)$; $(A_2, A_2) = (3, 6)$. Which outcomes (if any) are NE?
   a. (A_2, A_1)
   b. (A_2, A_2)
   c. (A_2, A_1) and (A_2, A_2)
   d. (A_1, A_2) and (A_2, A_1)
   e. None of the above

8, -2	4, -1
7, 7	3, 6

NE (A_1, A_2)

7. A milk supplier has a vendor-managed contract with a grocery store chain. In this contract, they are allowed to determine the quantity of milk to stock in each store, and are paid on delivery. As a result, they keep each store completely filled with milk, whether store needs it or not. This is an example of:
   a. Incentive compatibility
   b. Exploitable quasi-rents
   c. Participation
   d. Adverse selection
   e. None of the above

This is moral hazard

8. (*10 points*) A phone company sells data plans to two types of users: high data users and low data users. The high data user requires approximately 10GB of data per month, and the low data user only requires 2GB per month. There are roughly equal numbers of each type of user. The phone company is considering pricing their plan as if average consumption of data is 6GB. What is a potential problem with this pricing strategy? What might be a better approach?

If the company doesn't know in advance how much a customer will use and they price at the average, it is possible they will price out low use customers and then they wind up only selling to high use customers, but at the wrong price (i.e., adverse selection would occur).

One way to fix this is to ~~offer two options~~ price based on actual usage. They would also offer two options: A low fixed cost with high per GB usage fee and high fixed cost with low GB usage fee. Each will then self-select.

9. (*10 points*) A large multinational firm (F, which is risk-neutral) wants to hire a risk-averse contract manufacturer (M) for one of their products (utility = sqrt(wage) - effort). M has an alternative offer with utility U, and if they take the contract can choose one of two actions: A_1 (which has effort e_1 for the agent) and A_2 (which has effort e_2 for the agent), though F wants M to choose A_1. F cannot directly observe M's actions, but only the resulting outcomes (O_1 to O_3). Write out F's optimization problem and also explain how the resulting solution would work given the probabilities in the table below. Also, define any variables that you use.

	O_1	O_2	O_3
A_1	0.8	0.1	0.1
A_2	0.3	0.5	0.2

Need three wages, one for each outcome: w_1, w_2, w_3

so

$$\min \ .8w_1 + .1w_2 + .1w_3$$

s.t.

$$.8\sqrt{w_1} + .1\sqrt{w_2} + .1\sqrt{w_3} - e_1 \geq U$$

$$.8\sqrt{w_1} + .1\sqrt{w_2} + .1\sqrt{w_3} - e_1 \geq .3\sqrt{w_1} + .5\sqrt{w_2} + .2\sqrt{w_3}$$

$$w_i \geq 0 \ \forall i$$

so the way this works is that the principal offers the 3 wages to the agent. Once they observe the outcome, they pay the agent the corresponding wage.

10. (*10 points*) Two firms (A and B) compete and have the same unit costs. In the first case the firms compete in a Cournot game and in the second case the firms compete in a Stackleberg game (with A being the leader). Which case would firm B prefer and why? (There is no need to solve either game to answer this question).

For Stackleberg game, since A is the leader, they get a first mover advantage (and hence a larger share of the profit) while B gets lower share of profit.

In cournot, their profits would be the same.

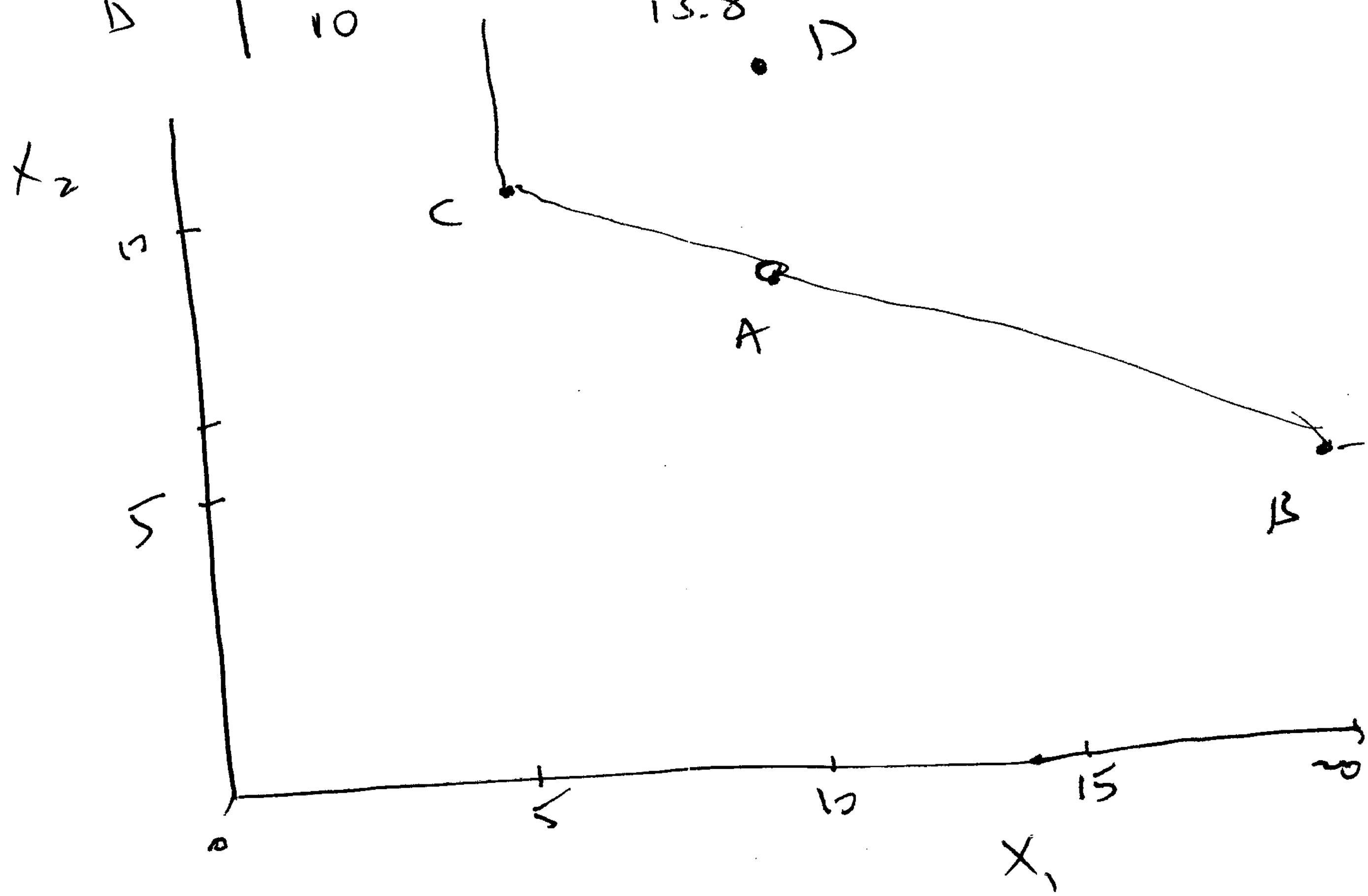
B, therefore, would prefer the cournot game.

11. (*10 points*) Given the following data for 4 firms below (X_1 and X_2 are inputs and Y is an output). Answer the following:

Firm	X_1	X_2	Y
A	5	4	10
B	15	4	15
C	6	10	20
D	8	11	16

a. Assuming constant returns to scale, determine the inputs for which all firms are scaled to an output of 20 (show in a table). Plot these points and show the efficient frontier.

Firm	X_1'	X_2'
A	$= \frac{20}{10}(5) = 10$	8
B	20	5.3
C	6	10
D	10	13.8



b. Looking at the plot from a, estimate the efficiency of each firm.

C and B both have $\theta = 1$ (on frontier)

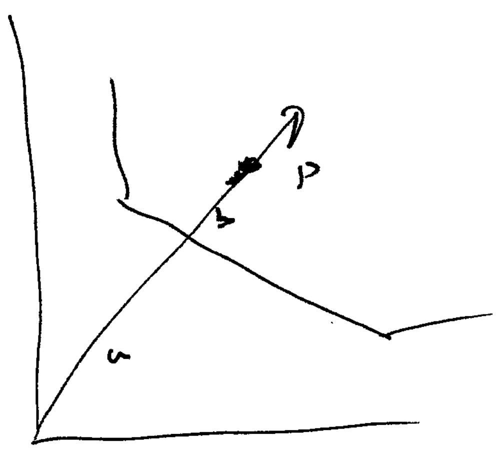
A is very close to frontier ("eyeballing" it)

so $\theta \approx 1$.

Firm D is away from frontier. If

we look at $\frac{a}{a+b}$ it looks to be

roughly $\theta = .7$

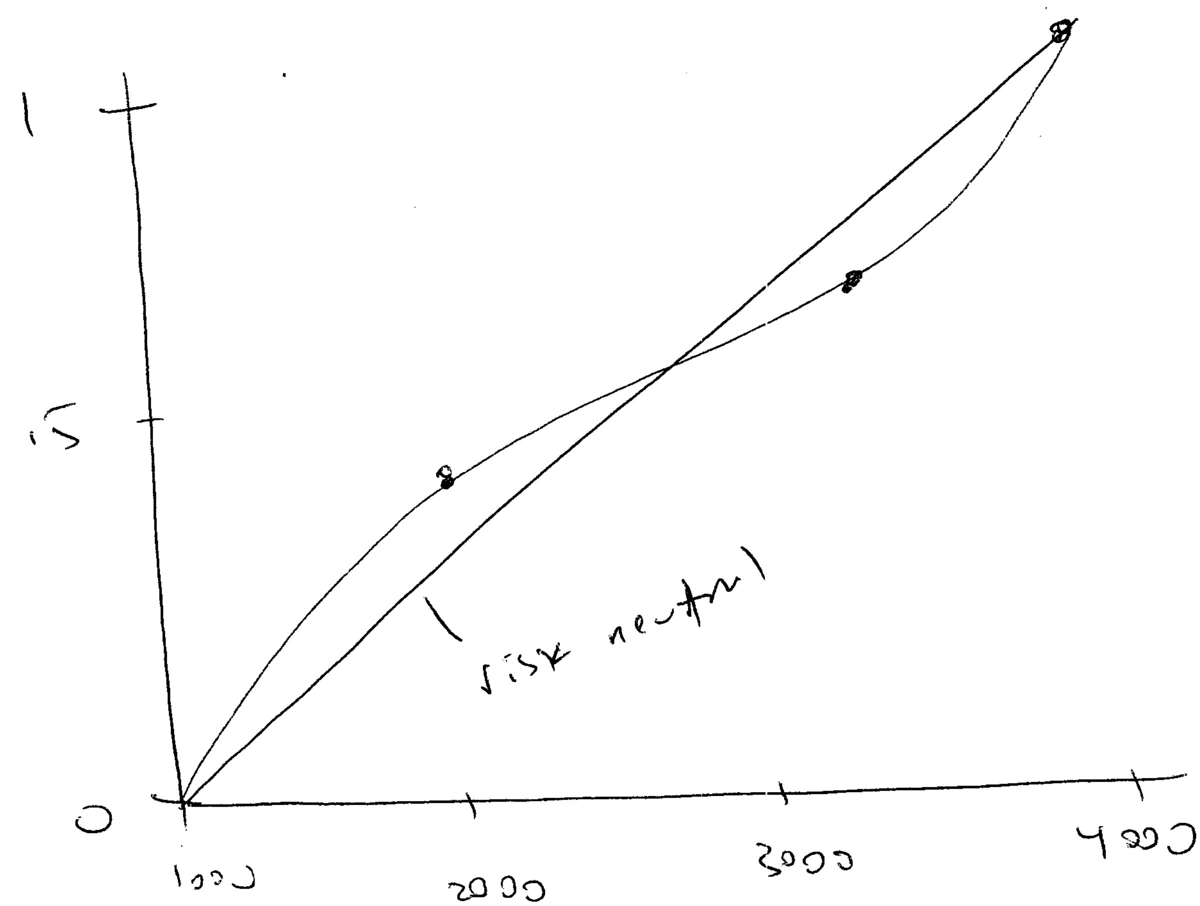


12. (*10 points*) A firm has four possible outcomes: \$1000, \$2000, \$3000, and \$4000. Given two options: i) earning \$3000 with certainty and ii) earning \$4000 with probability p and \$1000 with probability 1-p; they are indifferent between the options with p=0.6. Further, given two options: i) earning \$2000 with certainty and ii) earning \$3000 with probability p and \$1000 with probability 1-p; they are indifferent between the options with p=0.7. Plot a curve with outcomes on the x-axis and utility on the y-axis for their utility (it can be between 0 and 1). Show what their utility curve would look like if they were risk neutral on this same plot.

let $u(1000) = 0$ $\quad$ $u(4000) = 1$

the $u(3000) = .6\,u(4000) + .4\,u(1000) = .6$

$u(2000) = .7\,u(3000) + .3\,u(1000) = .42$



13. (*15 points*) Two firms are considering the joint development of a grocery delivery service. Firm A will pick the items in the stores, and Firm B will deliver them to the customers It will be a partnership where the total revenue is split between them. The market demand for the service per period is P=400 – 4Q, where Q is the number of standard orders provided. For firm A, they have two choices: hire 6 pickers at a cost of $5 per order and an output of 20 orders per period or hire 10 pickers at a cost of $7 per order and an output of 35 orders per period. For firm B, they have two choices: hire 3 drivers at a cost of $3 per order and an output of 20 orders per period or hire 6 drivers at a cost of $6 per order and an output of 35 orders per period. Assuming that both firms know each other's possible choices and associated costs, what would the equilibrium choice for each firm be? Is there a Pareto improving outcome?

There are 4 possible outcomes:

1. Both choose 20 → $P = 320$ $Q = 20$ Rev = 6400
   $\pi_A = .5(6400) - 20(5) = 3100$
   $\pi_B = .5(6400) - 20(3) = 3140$
2. A chooses 20, B chooses 35 → $Q = 20$ $P = 320$ Rev = 6400
   $\pi_A = .5(6400) - 20(5) = 3100$
   $\pi_B = .5(6400) - 20(6) = 3080$
3. A chooses 35, B chooses 20 → $Q = 20$ $P = 320$ Rev = 6400
   $\pi_A = .5(6400) - 20(7) = 3060$
   $\pi_B = .5(6400) - 20(3) = 3140$
4. Both choose 35 → $Q = 35$ $P = 260$ Rev = 9100
   $\pi_A = .5(9100) - 35(7) = 4305$
   $\pi_B = .5(9100) - 35(6) = 4340$

		B: 20	B: 35
A	20	3100 3140	3100 3080
	35	3060 3140	4305 4340

Two NE:
(20, 20) (35, 35)

The Pareto outcome is
(35, 35)
(as compared to (20, 20))

Name Key (Blue)

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   a. Firm 3 is efficient.
   b. Firm 1 is efficient.
   c. Firm 2 is efficient.
   d. a. and b. are both true
   e. a. and c. are both true
   f. None of the above

2. A risk-averse individual has a utility of wealth equal to the square root of the wealth. Consider two possibilities: a) $60 or b) a gamble where a fair coin is tossed and they get $0 for tails and $120 for heads. The individual is given b). How much would they be willing to pay to trade b) for a).
   a. Approximately $2.27.
   b. Approximately $5.15.
   c. Approximately $7.75.
   d. They wouldn't pay anything since they prefer b) to a).
   e. We don't have enough information to determine.

$p = .5$

$\sqrt{60} = 7.75$

$.5\sqrt{120} = 5.48$

diff $= 2.27$

$\$ = 2.27^2 = 5.15$

3. You are asked to rank suppliers based on 2 attributes (cost and quality) using the Borda count method and rank quality over cost in importance. Also, the higher the value, the better. The values for the 3 suppliers are: S1 = 12, 10; S2 = 9, 14; S3 = 18, 5. What is the ranking?
   a. S1, S2, S3
   b. S1, S3, S2
   c. S2, S1, S3
   d. S2, S3, S1
   e. S3, S1, S2
   f. S3, S2, S1

Q = 2 C = 1

$w_Q = \frac{2}{3} = .67$ $w_c = .33$

S1: $.67(10) + .33(12) = 10.66$

S2 $.67(14) + .33(9) = 12.35$

S3 = $.67(5) + .33(18) = 9.29$

S2 S1 S3

4. Two US firms compete on price and face a market demand of P=300-2Q, where Q is the sum of the outputs of each firm. The unit cost for firm 1 is \$8, and for firm 2 is \$13. What is the Bertrand equilibrium?
   a. Firm 1 sells at a price of \$8.01 and firm 2 sells at a price of \$13.01.
   b. Both firms sell at a price of \$8.01.
   c. Firm 1 sells at a price of \$8.01, and firm 2 sells nothing.
   d. There is no Bertrand equilibrium in this case because the cost differ.
   e. None of the above

Firm 1 sells at \$12.99
Firm 2 sells nothing

5. Consider a simultaneous move 2-person game. Each player can choose action A_1 or A_2. The payoffs for the 4 outcomes are: $(A_1,A_1) = (8, -2)$; $(A_1, A_2) = (4, -1)$; $(A_2, A_1) = (7, 7)$; $(A_2, A_2) = (3, 6)$. Which outcomes (if any) are NE?
   a. (A_2, A_1)
   b. (A_2, A_2)
   c. (A_2, A_1) and (A_2, A_2)
   d. (A_1, A_2) and (A_2, A_1)
   e. None of the above

8, -2	4, -1
7, 7	3, 6

NE

6. A milk supplier has a vendor-managed contract with a grocery store chain. In this contract, they are allowed to determine the quantity of milk to stock in each store, and are paid on delivery. As a result, they keep each store completely filled with milk, whether store needs it or not. This is an example of:
   a. Incentive compatibility
   b. Exploitable quasi-rents
   c. Participation
   d. Adverse selection
   e. None of the above
   f. We don't have enough information to determine.

moral hazard

7. Market demand is P=100 – Q, where Q is the sum of outputs from firms 1 and 2. Each firm has a unit cost c=0. If firm 1 is the leader in a Stackelberg game, what is the profit of firm 1 equal to?
   a. $(100-((100-q_1)/2))q_1$
   b. $(100-2q_1)q_1$
   c. $(100-(q_1+(100-q_1)/2))q_1$
   d. Profit would be infinite since there is no unit cost.
   e. None of the above

for 2: $\pi_2 = (100 - (q_1 + q_2))q_2$

$\pi_2' = 100 - q_1 - 2q_2 = 0$

$\rightarrow q_2 = \frac{100 - q_1}{2}$

$\pi_1 = (100 - (q_1 + \frac{100 - q_1}{2}))q_1$

8. (*10 points*) A pizza restaurant is opening an all you can eat buffet. Market research has shown they will have two types of customers: big eaters and small eaters. Big eaters eat 8 slices of pizza, and the small eaters eat 2 slices of pizza. There are roughly equal numbers of each type of customer. The restaurant is considering pricing their plan as if average number of slices is 5. What is a potential problem with this pricing strategy? What might be a better approach?

If price buffet at average, it may be above valuation for low eaters and so they don't come. only big eaters come (adverse selection) and so the pricing would be too low.

A restaurant could offer all you can eat at one price and a la carte at another. This way both types would come.

9. (*10 points*) A large multinational firm (F, which is risk-neutral) wants to hire a risk-averse contract manufacturer (M) for one of their products (utility = wage$^{0.8}$ - effort). M has an alternative offer with utility 23,000, and if they take the contract can choose one of two actions: A_1 (which has effort e_1 for the agent) and A_2 (which has effort e_2 for the agent), though F wants M to choose A_2. F cannot directly observe M's actions, but only the resulting outcomes (O_1 to O_3). Write out F's optimization problem and also explain how the resulting solution would work given the probabilities in the table below. Also, define any variables that you use.

	O_1	O_2	O_3
A_1	0.8	0.1	0.1
A_2	0.3	0.5	0.2

Let w_i be wage paid for outcome i

$$\min \ .3w_1 + .5w_2 + .2w_3$$

s.t.

$$.3w_1^{.8} + .5w_2^{.8} + .2w_3^{.8} - e_2 \geq 23000$$

$$.8w_1^{.8} + .1w_2^{.8} + .1w_3^{.8} - e_1 \leq .3w_1^{.8} + .5w_2^{.8} + .2w_3^{.8} - e_2$$

$$w_i \geq 0 \quad \forall i$$

So contract would be offered as a set of wages and then after effort, wages paid based on outcome.

10. (*10 points*) Two firms (A and B) compete and have the same unit costs. In the first case the firms compete in a Cournot game and in the second case the firms compete in a Stackelberg game (with A being the leader). Which case would firm A prefer and why? (There is no need to solve either game to answer this question).

In cournot game (for equal firms) equilibrium quantities (and hence price) are the same.

In Stackelberg, A gets a 1st mover advantage.

They will choose a higher quantity which forces B to choose a lower one.

So A prefers Stackelberg.

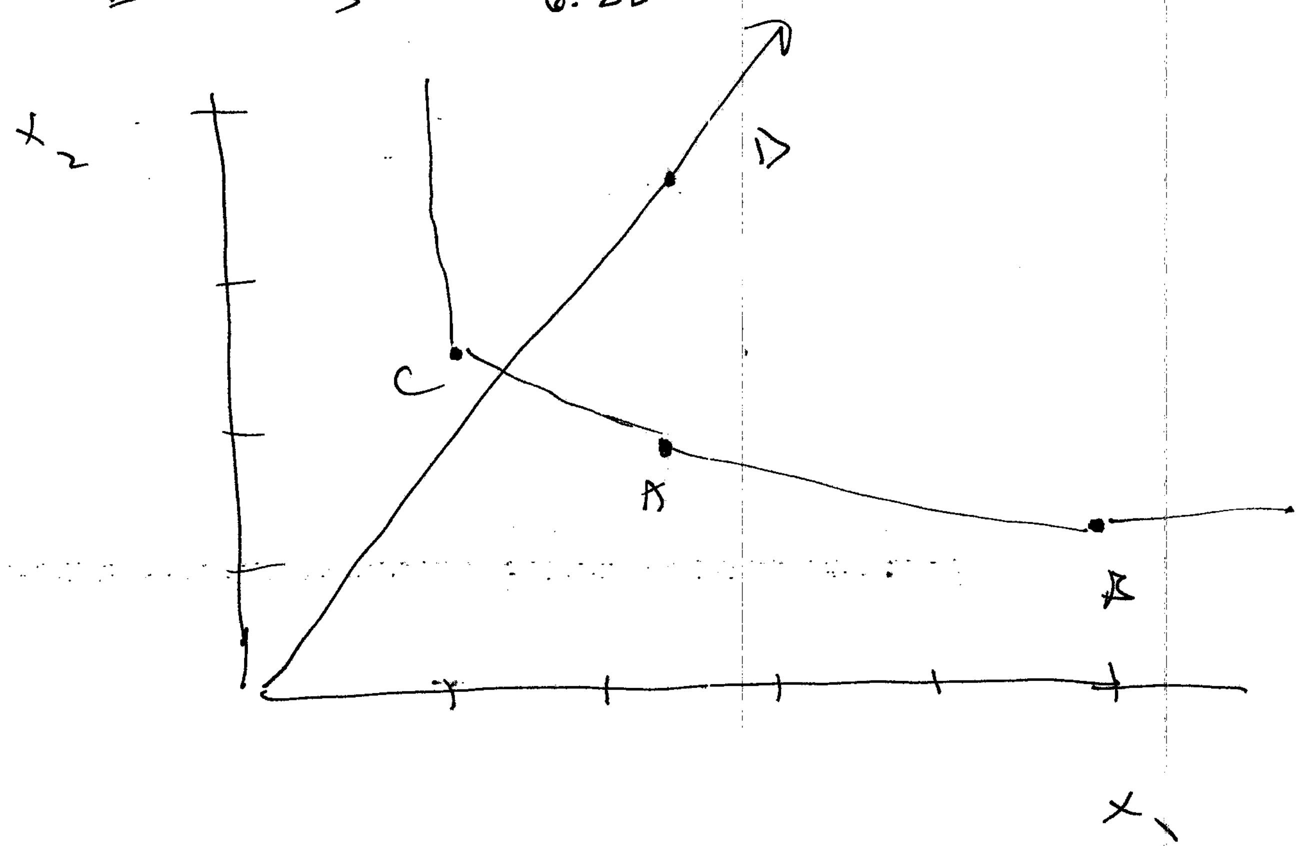
11. (*10 points*) Given the following data for 4 firms below (X_1 and X_2 are inputs and Y is an output). Answer the following:

Firm	X_1	X_2	Y
A	5	4	10
B	15	4	15
C	6	10	20
D	8	11	16

a. Assuming constant returns to scale, determine the inputs for which all firms are scaled to an output of 10 (show in a table). Plot these points and show the efficient frontier.

at $y = 10$

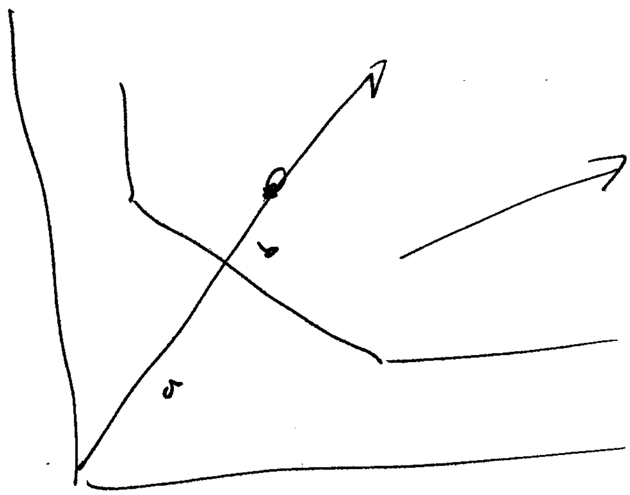
Firm	x_1	x_2
A	5	4
B	10	2.67
C	3	5
D	5	6.88



b. Looking at the plot from a, estimate the efficiency of each firm.

$\theta_A = \theta_B = \theta_3 = 1$ (on frontier)

$\theta_D \approx .67$



$$\theta = \frac{a}{a+b}$$

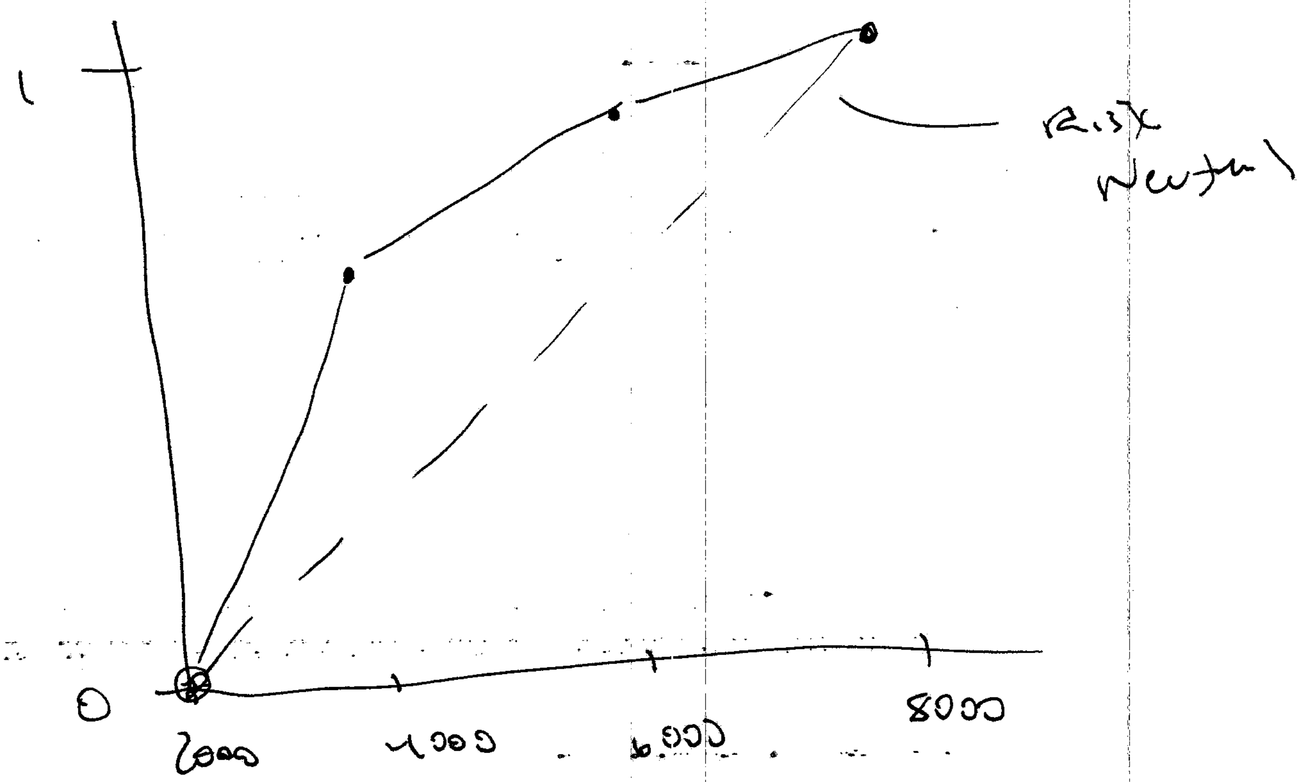
12. (*10 points*) A firm has four possible outcomes: \$2000, \$4000, \$6000, and \$8000. Given two options: i) earning \$4000 with certainty and ii) earning \$8000 with probability p and \$2000 with probability 1-p; they are indifferent between the options with p=0.6. Further, given two options: i) earning \$6000 with certainty and ii) earning \$8000 with probability p and \$4000 with probability 1-p; they are indifferent between the options with p=0.6. Plot a curve with outcomes on the x-axis and utility on the y-axis for their utility (it can be between 0 and 1). Show what their utility curve would look like if they were risk neutral on this same plot.

$u(2000) = 0$

$U(8000) = 1.0$

$\rightarrow u(4000) = .6\,u(8000) + .2\,u(2000)$
$= .6$

$u(6000) = .6\,u(8000) + .4\,u(4000)$
$= .6(1) + .4(.8) = .84$



13. (*15 points*) Two firms are considering the joint development of a grocery delivery service. Firm A will pick the items in the stores, and Firm B will deliver them to the customers It will be a partnership where the total revenue is split between them. The market demand for the service per period is P=300 – 2Q, where Q is the number of standard orders provided. For firm A, they have two choices: hire 6 pickers at a cost of \$5 per order and an output of 20 orders per period or hire 10 pickers at a cost of \$7 per order and an output of 35 orders per period. For firm B, they have two choices: hire 3 drivers at a cost of \$3 per order and an output of 20 orders per period or hire 6 drivers at a cost of \$6 per order and an output of 35 orders per period. Assuming that both firms know each other's possible choices and associated costs, what would the equilibrium choice for each firm be? Is there a Pareto improving outcome?

If $Q = 20 \rightarrow$ revenue $= (300 - 2(20))\,20 = 5200$

$Q = 35 \rightarrow$ revenue $= (300 - 2(35))\,35 = 8050$

4 cases: $A = 20$ $B = 20 \rightarrow Q = 20$

$\pi_A = 5200/2 - 5(20) = 2500$ $\quad \pi_B = 5200/2 - 3(20) = 2540$

$A = 20$ $B = 35 \rightarrow Q = 20$

$\pi_A = 5200/2 - 5(20) = 2500$ $\quad \pi_B = 5200/2 - 6(20) = 2480$

$A = 35$ $B = 20 \rightarrow Q = 20$

$\pi_A = 5200/2 - 7(20) = 2460$ $\quad \pi_B = 5200/2 - 3(20) = 2540$

$A = 35$ $B = 35 \rightarrow Q = 35$

$\pi_A = 8050/2 - 7(35) = 3780$ $\quad \pi_B = 8050/2 - 6(35) = 3815$

		B: 20	B: 35
A	20	2500 2540	2500 2480
	35	2460 2540	3780 3815

(20, 20) and (35, 35) are both NE. However (35, 35) is Pareto improving over (20, 20)