MATH 1712 - SPRING 2013
QUIZ 4 - SHOW YOUR WORK

Name: ______________________________ TA: _______________________________

1. (5 points) a. Find the absolute extrema (both the x & y $values$) for the function $f(x) = \frac{1}{3}x^3 - 4x + 6$ on the interval $[-3, 3]$.

* $\mathbf{f'(x) = x^2 - 4 = 0 \Rightarrow x = \pm 2}$ **are the critical points**

* **Since the domain is a closed interval, make a table**

x	$f(x)$	
-3	$f(-3) = 9$	
3	$f(3) = 3$	
-2	$f(-2) = 11.3$	**Absolute max**
2	$f(2) = 0.67$	**Absolute min**

(5 points) b. Find the absolute extrema (both the x & y $values$) for the function $g(x) = 2x + \frac{18}{x}$ on the interval $(0, \infty)$.

* $g'(x) = 2 - \frac{18}{x^2} = 0 \Rightarrow 2x^2 - 18 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm 3$. But $x = 3$ is the only one in the interval $(0, \infty)$.

* $g''(x) = \frac{36}{x^3} \Rightarrow g''(3) = \frac{36}{27} > 0 \Rightarrow x = 3$ **and** $g(3) = 9$ is the absolute minimum. There is no absolute maximum.

2. (10 points) Suppose that the total monthly cost (in \$) for producing x chairs is $C(x) = 0.001x^3 + 0.07x^2 + 19x + 700$ and that currently 25 chairs are produced monthly. a. Use the marginal cost function to **estimate the cost of producing the 26th chair**. b. Use the total cost function to find the **exact cost of producing the 26th chair**.

a. The MCF = $C'(x) = 0.003x^2 + 0.14x + 19 \Rightarrow C'(25) =$ **\$ 24.38** $\approx$ **cost of producing the 26th chair**

b. **The exact cost of producing the 26th chair** $= C(26) - C(25) = 1258.90 - 1234.38 =$ **\$ 24.52**

3. (10 points) The total costs, in dollars, of producing x units of a certain product is given by:

$$C(x) = 8x + 20 + \frac{x^3}{100}$$

a. Find the average cost function $A(x) = \frac{C(x)}{x}$.

$$\mathbf{A(x) = 8 + \frac{20}{x} + \frac{x^2}{100}}$$

b. Find the minimum average cost and the value x_0 at which it occurs.

* $\mathbf{A'(x) = -\frac{20}{x^2} + \frac{2x}{100} = 0 \Rightarrow -2000 + 2x^3 = 0 \Rightarrow x = 10}$

* $\mathbf{A''(x) = \frac{40}{x^3} + \frac{1}{50} \Rightarrow A''(10) = \frac{40}{1000} + \frac{1}{50} > 0 \Rightarrow x = 10}$ **and** $\mathbf{A(10) = 11}$ **is the minimum average cost**

c. Compute $C'(x_0)$ and compare it to $A(x_0)$.

$\mathbf{C'(x) = 8 + \frac{3x^2}{100} \Rightarrow C'(10) = 8 + 3 = 11.}$ **The marginal cost is equal to the minimum average cost or simply** $\mathbf{C'(10) = A(10)}$