Name: Key

ChBE 2120, Numerical Methods, Paravastu Section, Fall 2015
Quiz 3: 20 points possible

1) (5 points) For the function $f(x, y) = x\,y\,sin(x) + cos(y)$, show how the Bisection Method could be used to optimize $f(x, \pi/2)$ in terms of x. Perform one iteration of the algorithm with the initial bracket, $x_L = 1, x_U = \pi$.

$g(x) = \frac{\pi}{2} x \sin(x)$ $\quad g'(x) = \frac{\pi}{2}(x\cos(x) + \sin(x)) = h(x) = 0$ +2

$h(x_L) = \frac{\pi}{2}(\cos(1) + \sin(1)) = 2.17$ +1

$h(x_U) = \frac{\pi}{2}(\pi\cos(\pi) + \sin(\pi)) = 4.93$ +1

$x_r = \frac{1+\pi}{2} = 2.07$

$h(x_r) = \frac{\pi}{2}(2.07)\cos(2.07) + \sin(2.07) = -0.181 =$ new x_u +1

2) (7.5 points) For the function $f(x, y) = x\,y\,sin(x) + cos(y)$, and the initial guess $x = \pi/2, y = 0$, setup the numerical matrix equation that would need to be solved in order to calculate the next guess using Newton's method. You do not need to solve for the next guess or invert the coefficient matrix.

$$\nabla f = \begin{bmatrix} y(x\cos(x) + \sin(x)) \\ x\sin(x) - \sin(y) \end{bmatrix}$$ +2

$$H = \begin{bmatrix} y(2\cos(x) - x\sin(x)) & x\cos(x) + \sin(x) \\ x\cos x + \sin(x) & -\cos(y) \end{bmatrix}$$ +2

$$\underline{x}_0 = \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix} \quad \nabla f\big|_{x_0} = \begin{bmatrix} 0 \\ \pi/2 \end{bmatrix} \quad H\big|_{x_0} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$ +2

$$\underline{x}_1 = \underline{x}_0 - H^{-1}\nabla f = \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \pi/2 \end{bmatrix}$$ +1.5

3) (7.5 points) Perform one step of the Steepest Descent method using the function from (2) and the initial guess $x = 0, y = \pi/2$. Use the step size that optimizes the effectiveness of the step.

$$\underline{x}_0 = \begin{bmatrix} 0 \\ \pi/2 \end{bmatrix} \quad \underline{x}_1 = \underline{x}_0 - t\underbrace{\nabla f(\underline{x}_0)}_{\begin{bmatrix} 0 \\ -1 \end{bmatrix}} = \begin{bmatrix} 0 \\ \pi/2 + t \end{bmatrix}$$ +2 +1

$f(\underline{x}_1) = \cos(\frac{\pi}{2} + t) = h(t)$

$h'(t) = -\sin(t + \pi/2) \Rightarrow t = \pi/2$ +3

$$\underline{x}_1 = \begin{bmatrix} 0 \\ \pi \end{bmatrix}$$ +1.5