PHYS 2211 Test 2
Spring 2015

Name(print) ~~~ Test ~~~ Key ~~~~ Lab Section L

Greco (K or M), Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Tuesday	M01 N01	M02 N02	M03 N03
Tuesday	K03 K05	K04 K07	K06 K08
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Ryuzaki

Sign your name on the line above

PHYS 2211

Please do not write on this page

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

A polarized molecule (e.g. water) can be treated as two charges $+Q$ and $-Q$ permanently separated by a small distance s. An electron with charge $-e$ and mass m_e is placed initially at rest far from the dipole a distance R. Fill in the missing VPython statements below to update the position of the electron. During the ensuing motion of the electron, you can assume that the dipole remains motionless. You may also safely ignore the gravitational interaction of these particles.

```
from visual import *
# Constants and Mass
k = 9e9 # coulombs constant
m_e = 9.109382e-31 #mass of an electron
e = -1.9e-16 #charge of an electron
Q = 10*e #molecular charge
s = 3.9e-12 #charge Q separation distance in meters
R = 10*s #initial distance of electron from dipole in meters
# Initialization
electron = sphere(pos=vector(R,0,0), radius=1e-13, color=color.cyan)
Qpos = sphere(pos=vector(0,s/2,0), radius= 3e-13,color=color.blue) #+Q
Qneg = sphere(pos=vector(0,s/2,0), radius= 3e-13,color=color.blue) #-Q
velectron = vector(0,-4e3,0) $electron velocity m/s
pelectron = m_e*velectron #initial momentum of the electron
t = 0
while t < 3058992:
```

(a 15 pts) Add the necessary statements **here** to update the electron's momentum and position.

```
rplus = electron.pos - Qpos.pos
rplusmag = mag(rplus)
rplushat = norm(rplus)
rminus = electron.pos - Qneg.pos
rminusmag = mag(rminus)
rminushat = norm(rminus)
Fplus = (k * Q * e/rplusmag ** 2) * rplushat
Fminus = (k * (-Q) * e/rminusmag ** 2) * rminushat
Fnet = Fplus + Fminus
pelectron = pelectron + Fnet * deltat
electron.pos = electron.pos + (pelectron/m_e) * deltat
t = t + deltat
```

3 pts each

(b 5pts) What is the electron's initial position? (Answer should be a vector with units.)

$\vec{r}_i = \langle (3.9e{-12})(10), 0, 0 \rangle = \boxed{\langle 3.9e{-11}, 0, 0 \rangle \text{ m}}$ All //

(c 5pts) What is the electron's initial momentum? (Answer should be a vector with units.)   All //

$\vec{P}_i = \langle 0, (9.109382e{-31})(-4e3), 0 \rangle = \boxed{\langle 0, -3.6437528e{-27}, 0 \rangle \text{ Kg}\cdot\text{m/s}}$

Problem 2 (25 Points)

At $t = 0$, a star of mass M is located at $< x_s, y_s, 0 >$. At the same instant, a planet of mass m is located at $< x_p, y_p, 0 >$ and is moving with a velocity of $< v_{xi}, v_{yi}, 0 >$. When answering the following questions you can assume the star is so massive that it effectively remains at rest.

(a 10pts) At $t = 0$, what is the vector gravitational force exerted by the star on the planet. Please show your work to earn full credit.

✓ Position vector $\vec{r}$:

$$\vec{r} = \vec{r}_p - \vec{r}_s = \langle x_p, y_p, 0 \rangle - \langle x_s, y_s, 0 \rangle = \langle x_p - x_s, y_p - y_s, 0 \rangle$$

✓ Magnitude of $\vec{r}$:

$$|\vec{r}| = \sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}$$

−0.5
−1.5
−3.0
−8.0

✓ Unit vector:

$$\hat{r} = \frac{\langle x_p - x_s, y_p - y_s, 0 \rangle}{\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}}$$

✓ Magnitude of gravitational force:

$$F_{mag} = \frac{GMm}{|\vec{r}|^2} = \frac{GMm}{(x_p - x_s)^2 + (y_p - y_s)^2}$$

✓ Vector gravitational force:

$$\Rightarrow \vec{F} = -F_{mag}\,\hat{r} = \boxed{\frac{-GMm}{(x_p - x_s)^2 + (y_p - y_s)^2} \frac{\langle x_p - x_s, y_p - y_s, 0 \rangle}{\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}}}$$

(b 10pts) At $t = T$, what is the position of the planet? Assume that T is small enough so that you can perform your iterative calculations using a single time step. Please show your work to earn full credit.

✓ Initial momentum:

$$\vec{p}_i = m\vec{v}_i = \langle mv_{xi}, mv_{yi}, 0 \rangle$$

−0.5
−1.5
−3.0
−8.0

✓ Final momentum (apply momentum principle):

$$\vec{p}_f = \vec{p}_i + \vec{F}\Delta t = \langle mv_{xi}, mv_{yi}, 0 \rangle + \vec{F}T =$$

$$= \langle mv_{xi}, mv_{yi}, 0 \rangle - \frac{GMmT}{(x_p - x_s)^2 + (y_p - y_s)^2} \frac{\langle x_p - x_s, y_p - y_s, 0 \rangle}{\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}}$$

✓ Final position:

$$\vec{r}_f = \vec{r}_i + \vec{v}\Delta t = \langle x_p - x_s, y_p - y_s, 0 \rangle + (\vec{p}_f / m)T =$$

$$= \boxed{\langle x_p - x_s, y_p - y_s, 0 \rangle + \langle v_{xi}T, v_{yi}T, 0 \rangle - \frac{GMT^2}{(x_p - x_s)^2 + (y_p - y_s)^2} \frac{\langle x_p - x_s, y_p - y_s, 0 \rangle}{\sqrt{(x_p - x_s)^2 + (y_p - y_s)^2}}}$$

(c 5pts) In your own words, describe how you would continue to update the position of the planet as it orbits the star.

✓ Use the new position to calculate the new force

✓ Repeat the process (new $\vec{F}$, new $\vec{p}$, new $\vec{r}$, new $\vec{F}$, new $\vec{p}$, new $\vec{r}$) over and over again as needed.

All

Problem 3 (25 Points)

The US Penny is actually made of zinc. A typical penny has a diameter of 1.905 cm and an average thickness of 1.228 mm. The density of zinc is 7140 $\frac{\text{kg}}{\text{m}^3}$ and its atomic weight is 65.4 amu = 65.4 $\frac{\text{g}}{\text{mol}}$.

(a 5pts) Determine the mass of a typical penny.

$$\rho = \frac{m}{V} \Rightarrow m = \rho V$$

$$m = (7140)(1.228e-3)(\pi)\left(\frac{1.905e-2}{2}\right)^2 =$$

$$\Rightarrow \boxed{m = 0.002499\,\text{Kg} = 2.499\,\text{g}}$$

All

(b 5pts) What is the diameter of a single zinc atom?

✓ Number of atoms in a penny:

$$\frac{2.499\,\cancel{g}}{} \Big| \frac{\cancel{mol}}{65.4\,\cancel{g}} \Big| \frac{6.02e23\ \text{atoms}}{\cancel{mol}} = 2.3e22\ \text{atoms}$$

✓ Mass of one atom:

$$\frac{0.002499\ \text{kg}}{2.3e22\ \text{atoms}} = 1.09e-25\ \text{kg}$$

or just $\frac{65.4 \times 10^{-3}\ \text{Kg}}{6.02 \times 10^{23}\ \text{atom}}$

✓ Volume of one atom:

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} = \frac{1.09e-25\ \cancel{kg}\ m^3}{7140\ \cancel{kg}} = 1.53e-29\,m^3$$

✓ Going from volume to diameter:

$$V \sim d_a^3 \Rightarrow d_a \sim (V)^{1/3}$$

$$\Rightarrow d_a = (1.53e-29\,m^3)^{1/3} = \boxed{2.48e-10\,m}$$

All

(c 5pts) How many zinc atoms make up one side (i.e. heads or tails) of the penny?

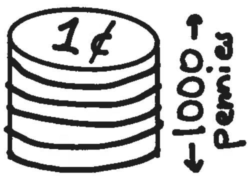
$$\frac{\text{area of penny face}}{\text{cross-sectional area of one atom}} = \frac{\cancel{\pi}\left(\frac{d}{\cancel{2}}\right)^2}{\cancel{\pi}\left(\frac{d_a}{\cancel{2}}\right)^2} = \frac{d^2}{d_a^2} =$$

All/

$$= \frac{(1.905e-2)^2}{(2.48e-10)^2} = \boxed{5.9e15 \text{ atoms}}$$

(d 5pts) You stack 1000 pennies, face to face, and apply a force of 25,000 N to the top penny. While the force is applied you find the thickness decreases by 1 mm. Calculate Young's modulus of zinc.

of the stack



✓ $F = 25{,}000 \text{ N}$

✓ $A = \pi\left(\frac{d}{2}\right)^2 = \pi(1.905e-2/2)^2 = 2.85e-4 \text{ m}^2$

✓ $\Delta L = 1\text{mm} = 1e-3 \text{ m}$

✓ $L_0 = (1000)(\text{thickness}) = (1000)(1.228e-3) = 1.228 \text{ m}$

(about 4 feet!)

$$\Rightarrow Y = \frac{F/A}{\Delta L/L_0} = \frac{F}{A}\frac{L_0}{\Delta L} = \frac{(25000\text{ N})(1.228\cancel{\text{m}})}{(2.85e-4\text{m}^2)(1e-3\cancel{\text{m}})} = \boxed{1.077e11 \text{ N/m}^2}$$

All

(e 5pts) Calculate the interatomic spring stiffness for zinc.

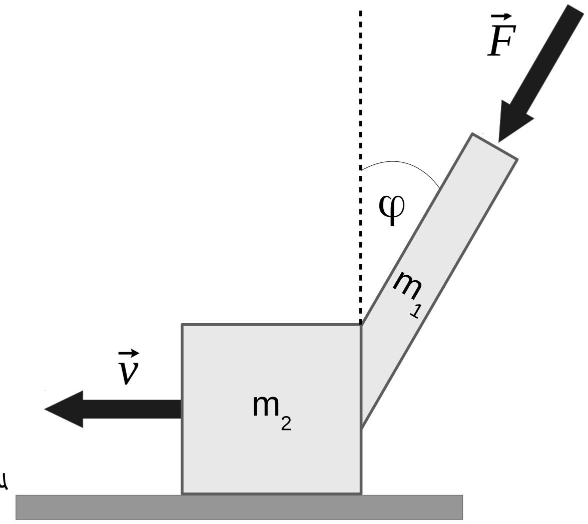
$$Y = \frac{K_{si}}{d_a} \Rightarrow K_{si} = Y d_a$$

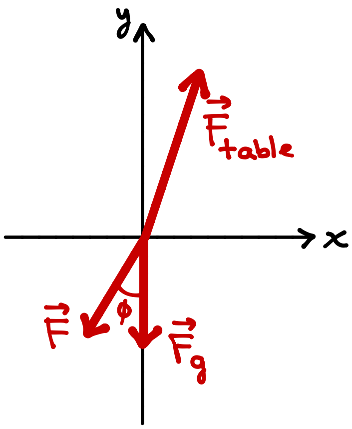
$$K_{si} = (1.077e11 \text{ N/m}^2)(2.48e-10\cancel{\text{m}}) = \boxed{26.7096 \text{ N/m}}$$

All

Problem 4 (25 Points)

(a 5pts) A constant force F is applied to a stick of mass m_1. The stick is connected to a block of mass m_2 and makes an angle ϕ with the vertical. The block slides across a table to the left at constant velocity. Draw and label a force body diagram for the system consisting of only the block.

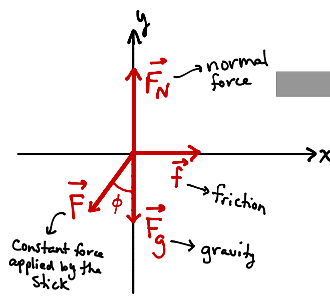




All

NOTE

This is also OK:



(b 10pts) Determine the coefficient of friction between the block and the table. Please show your work to earn full credit.

✓ System: block + stick → constant velocity, so $\frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{F}_{net} = 0$

x-components

$\vec{f} + \vec{F}_x = 0$ → constant velocity

$\vec{f} = -\vec{F}_x = -F\sin\phi\,\hat{x}$

$\vec{f} = F\sin\phi\,(-\hat{x})$

y-components

$\vec{F}_N + \vec{F}_g + \vec{F}_y = 0$ → equilibrium

$\vec{F}_N - (m_1+m_2)g\,\hat{y} - F\cos\phi\,\hat{y} = 0$

$\vec{F}_N = (m_1+m_2)g\,\hat{y} + F\cos\phi)\,\hat{y}$

✓ friction $= \mu F_{Normal}$, So:

$$F\sin\phi = \left[(m_1+m_2)g + F\cos\phi\right]\mu$$

$$\Rightarrow \boxed{\mu = \frac{F\sin\phi}{(m_1+m_2)g + F\cos\phi}}$$

−0.5
−1.5
−3.0
−8.0

(c 10pts) An unknown force is applied to the stick and both block and stick are observed to move to the left with constant acceleration a. Determine the magnitude of the contact force **on the block from the stick.**

✓ System: block + stick

✓ $\frac{d\vec{p}}{dt} = \vec{F}_{net} = (m_1+m_2)\vec{a}$

✓ forces: gravity ($\vec{F}_g$), friction ($\vec{f}$), normal ($\vec{F}_N$), unknown ($\vec{F}_u$)

<u>x-components</u>

$\vec{F}_{net,x} = (m_1+m_2)(-a) = f - F_u \sin\phi$

$(m_1+m_2)a = F_u \sin\phi - \mu F_N$

<u>y-components</u>

$\vec{F}_{net,y} = 0 = F_N - F_u \cos\phi - (m_1+m_2)g$

$F_N = (m_1+m_2)g + F_u\cos\phi$

$$\Rightarrow F_u \sin\phi = (m_1+m_2)a + \mu\left[(m_1+m_2)g + F_u\cos\phi\right]$$

$$F_u \sin\phi - \mu F_u\cos\phi = (m_1+m_2)a + \mu(m_1+m_2)g$$

$$F_u(\sin\phi - \mu\cos\phi) = (m_1+m_2)(a+\mu g)$$

$$F_u = \frac{(m_1+m_2)(a+\mu g)}{\sin\phi - \mu\cos\phi}$$

→ the unknown force
("μ" comes from Part A)

−0.5
−1.5
−3.0
−8.0

→ force on the stick by the block

✓ System: <u>stick</u> only $\Rightarrow m_1\vec{a} = \vec{F}_u + \vec{F}_g + \vec{F}_{contact}$

<u>x-components</u>

$F_{contact,x} + F_{u,x} = -m_1 a$

$F_{contact,x} - F_u\sin\phi = -m_1 a$

$F_{contact,x} = F_u\sin\phi - m_1 a$

<u>y-components</u>

$F_{contact,y} + F_{u,y} + F_g = 0$

$F_{contact,y} - F_u\cos\phi - m_1 g = 0$

$F_{contact,y} = F_u\cos\phi + m_1 g$

$$\rightarrow \vec{F}_{contact} = \langle (F_u\sin\phi - m_1 a), (F_u\cos\phi + m_1 g), 0\rangle$$

✓ Finally, the force on the block by the stick is the negative of $\vec{F}_{contact}$:

$$\boxed{\vec{F}_b = \langle (m_1 a - F_u\sin\phi), -(m_1 g + F_u\cos\phi), 0\rangle}$$

(where "F_u" is the unknown force found above)

[Note — assumptions needed: $\vec{F}_u$ makes some angle ϕ, $\vec{a} = \langle -a, 0, 0\rangle$]

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\|} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

$$\Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}}$$

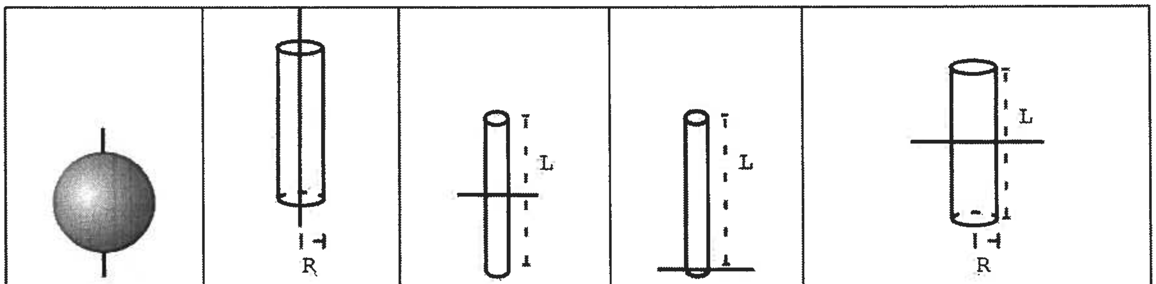
$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$



$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule $\cdot$ second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule $\cdot$ second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^3
micro	μ	1×10^{-6}	mega	M	1×10^6
nano	n	1×10^{-9}	giga	G	1×10^9
pico	p	1×10^{-12}	tera	T	1×10^{12}