MATH 3012 A, Midterm 2

06/19/2013

Name: ______________________ GTID: ______________________

key

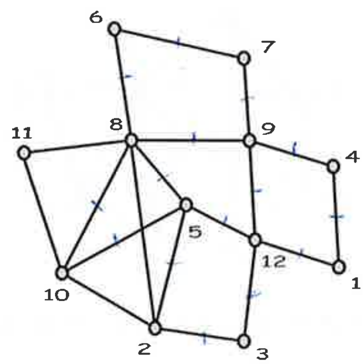
Problem No.	Points
1	10
2	20
3	10
4	25
5	5
6	10
7	20

TOTAL: 100

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (10 points).

Use the algorithm developed in class, with vertex 1 as root, to find an Euler circuit in the following graph:



(1, 4, 9, 7, 6, 8, 5, 2, 3, 12, 1)

add (9, 8, 10, 5, 12, 9)

⇒ (1, 4, 9, 8, 10, 5, 12, 9, 7, 6, 8, 5, 2, 3, 12, 1)

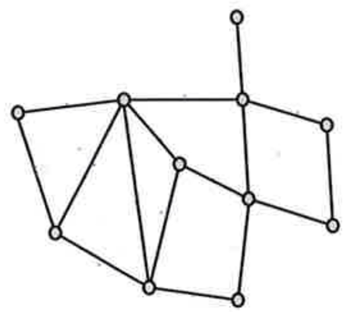
add (8, 2, 10, 11, 8)

⇒ (1, 4, 9, 8, 2, 10, 11, 8, 10, 5, 12, 9, 7, 6, 8, 5, 2, 3, 12, 1)

Turn over for more problems

Problem 3 (10 points).

Verify Euler's formula for the following planar graph.



$V = 11$

$E = 16$

$F = 7$

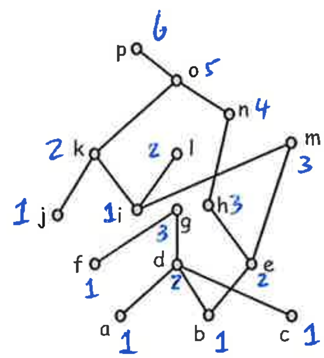
$V + F - E = 2$

$11 + 7 - 16 = 2$

Turn over for more problems

Problem 4 (25 points).

Consider the following poset:



3 pts 1. Find all points comparable to h.

p, o, n, e, b

3 pts 2. Find all points which cover h.

n

3 pts 3. Find a maximal chain of size 3.

{b, e, m} answer not unique

3 pts 4. Find a maximal antichain of any size.

{p, l, m, g} answer not unique

3 pts 5. Find the set of all minimal elements.

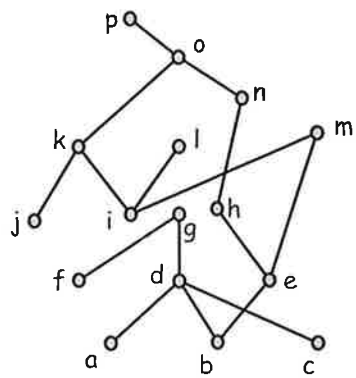
{a, b, c, f, i, j}

10 pts 6. Using the algorithm taught in class (recursively removing the set of minimal elements), find the height H of the poset and a partition of P into H antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam.

{b, e, h, n, o, p}

Turn over for more problems

Problem 5 (5 points).
Show that the following poset is **not** an interval order.



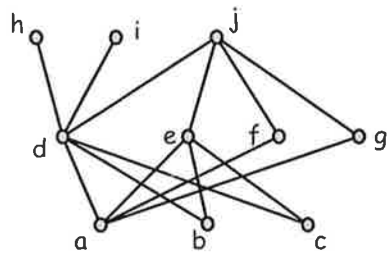
p contains $\underline{2} + \underline{2}$.

e.g. $\{j, k\}$ and $\{m, e\}$ form $\underline{2} + \underline{2}$.

Turn over for more problems

Problem 6 (10 points).

Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation.



$D(a) = \emptyset$ 1

$D(b) = \emptyset$ 1

$D(c) = \emptyset$ 1

$D(d) = \{a, b, c\}$ 3

$D(e) = \{a, b, c\}$ 3

$D(f) = \{a\}$ 2

$D(g) = \{a\}$ 2

$D(h) = \{d, a, b, c\}$ 4

$D(i) = \{d, a, b, c\}$ 4

$D(j) = \{d, e, f, g, a, b, c\}$ 5

$U(a) = \{d, e, f, g, h, i, j\}$ 1

$U(b) = \{d, e, h, i, j\}$ 2

$U(c) = \{d, e, h, i, j\}$ 2

$U(d) = \{h, i, j\}$ 3

$U(e) = \{j\}$ 4

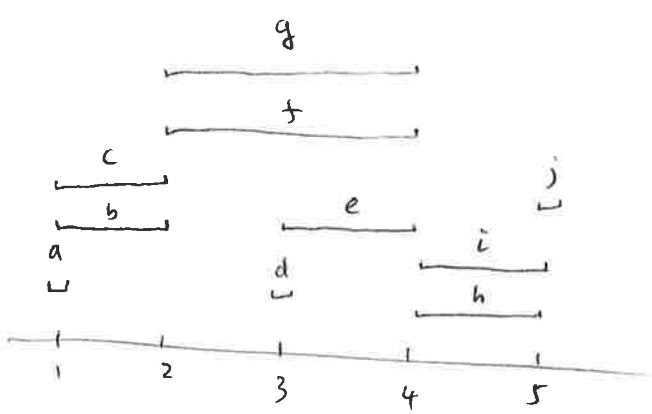
$U(f) = \{j\}$ 4

$U(g) = \{j\}$ 4

$U(h) = \emptyset$ 5

$U(i) = \emptyset$ 5

$U(j) = \emptyset$ 5



Turn over for more problems

Problem 7 (20 points).

Determine whether each of the following statements is true-or-false. If the statement is true, circle the "**T**"; if false, circle the "**F**".

[**T** \ **F**] A general graph is Eulerian if and only if every vertex of the graph is even.

[**T** \ **F**] An Eulerian graph is Hamiltonian, but a Hamiltonian graph is not necessarily Eulerian.

[**T** \ **F**] A graph that contains a proper cycle cannot be Hamiltonian.

[**T** \ **F**] Any edge added to a tree must produce a cycle.

[**T** \ **F**] The complete graph K_4 has four vertices and four edges.

[**T** \ **F**] A tree with more than one vertex has at most two leaves.

[**T** \ **F**] There is a planar graph with 100 vertices and 300 edges.

2pts each

[**T** \ **F**] There is a triangle-free graph with 30 vertices and 250 edges. — 15

[**T** \ **F**] Let P be the poset consisting of all subsets of $\{1, 2, 3, 4, 5, 6, 7\}$, ordered by inclusion. Then width$(P) = 7$.

[**T** \ **F**] There is a poset P with width(P) =height$(P) = 5$.

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