ISyE 4232 Spring 2014
H. Ayhan

Homework 3
February 6, 2014
(due February 13)

1. Consider a production system consisting of three single-server stations in series. Customer orders arrive at the system according to a Poisson process with rate 1 per hour. Each customer order immediate triggers a job that is released to the production system to be processed at station 1 first, and then at station 2. After being processed at station 2, a job has $p_1 = 10\%$ probability going back to station 1 for rework and $1 - p_1$ probability continuing onto station 3. After being processed at station 3, a job has $p_2 = 15\%$ probability going back to station 1, $p_3 = 10\%$ probability going back to station 2, and $1-p_2-p_3$ probability leaving the production system as a finished product. Assume that the processing times of jobs at each station are iid, having exponential distribution, regardless of the history of the jobs. The average processing times at stations 1, 2 and 3 are $m_1 = 0.8$, $m_2 = 0.70$ and $m_3 = 0.8$ hours, respectively.

   (a) Find the long-run fraction of time that there are 2 jobs at station 1, 1 job at station 2 and 4 jobs at station 3.

   (b) Find the long-run average (system) size at station 3.

   (c) Find the long-run average time in system for each job.

2. Consider the production system in Problem 1 with following modification: $p_1 = 10\%$, $p_2 = p_3 = 0$ (station 3 makes 100% reliable operations.) Furthermore, the Management decides to adopt the make-to-stock policy, using the CONWIP job release policy. Recall that when the system is operating CONWIP policy only a job leaving station 3 triggers a new job to be released to station 1. Let N be the CONWIP level.

   (a) For $N = 2$, compute the throughput of the production system. What is the average time in system per job?

   (b) Is it possible to double the throughput? If so, what N is needed to achieve it? What is the corresponding average time in system per job?