Test 2
Version A SOLUTIONS

1. **[3 pts] Yes or No:** Consider a maximization problem with objective function $z = 6x_1 + 3x_2 + 4x_3$ and feasible solution: $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$. You know that the dual objective function is $w = 10\pi_1 + 5\pi_2$. Given the informatin provided, is $\overline{\pi} = \begin{bmatrix} 3 & 4 \end{bmatrix}$ a possible feasible solution to the dual? Why or why not?

   SOLUTION: Yes. w=50, this means that not only is the solution feasible, it is optimal. $w \geq z$. **On White version: w=44, No.**

2. **[2 pts] Yes or No:** A feasible solution $\overline{x}$ to the primal problem is $x_1 = 5, x_2 = 1, s_1 = 0, s_2 = 4, s_3 = 0$. You also know that a feasible solution $\overline{\pi}$ to the dual problem is $\pi_1 = 0, \pi_2 = 2, \pi_3 = 1, e_1 = 0, e_2 = 0$. Based on these feasible solutions, is it possible that $\overline{x}$ is the optimal solution to the primal problem? Why or why not?

   SOLUTION: Yes. Even though $x_2e_2 \neq 0$ **that means that the two solutions are not BOTH optimal. One of them may still be optimal.**

3. **[2 pts] True or False:** A transportation problem will return an integer solution on flows if demand and supply are all integer values, but the costs of the edges contain fractional values.

   SOLUTION: TRUE. The cost only affects the final z value, not the flow. (White version: False, same reasoning, the statement was just the negative of this one.)

4. **[2 pts] True or False:** A transportation problem in which all of the demand is 1 is always an assignment problem.

   SOLUTION: FALSE. The supply also has to be 1 at every supply point.

5. **[2 pts] True or False:** It is not possible to represent a maximum benefit 1 to 1 pairing as an assignment problem.

   SOLUTION: FALSE. The negative of the maximum is a minimum, and we can use Hungarian. (White version: True, same reasoning, the statement was just the negative of this one.)

6. **[2 pts] True or False:** Dijkstra's algorithm works correctly if you have a negative cost edge if you add the same constant to all edge costs to make all of the costs non-negative.

   SOLUTION: FALSE. Consider the graph where you have 3 nodes. There is an edge from 1 to 2 with cost 2, (1,3) has cost 1, and (2,3) has cost -2. The true shortest path from 1 to 3 on this graph is to go 1-2-3 with cost of 0. Adding +2 to all edges and running Dijkstra gives the path 1-3 as the shortest with a new cost of 3, 1 before the constant addition.

7. **[2 pts] True or False:** Consider a graph with 5 nodes, labeled 1 through 5. A valid spanning tree of this graph is the set of edges $\{(1,2),(1,3),(1,4),(1,5)\}$.

   SOLUTION: TRUE. All edges are connected and there are no cycles.

8. **[2 pts] True or False:** In the Ford-Fulkerson Algorithm, it is possible to label a node more than once in the same iteration. (Where one iteration is the process of trying to find a path from the source to the sink).

   SOLUTION: FALSE. In the rules for labeling you can only label a node if it is unlabeled. (White version: When labeling an edge, one node in the edge needs to be labeled and the other unlabeled, then you label the edge and the node making both nodes in the edge labeled, preventing you from being able to label it again.)

9. **[5 pts]** You are trying to assign star athletes to recreational teams in such a way as to minimize the personal conflict on the team. You have 4 teams made and 4 more athletes to assign. The amount of conflict caused by adding a specific athlete to a team is given in the table below. Which athlete should you assign to which team? (Given that you need to assign one athlete and only one to each team).

	Team 1	Team 2	Team 3	Team 4
Mark	20	15	30	10
Greg	40	35	30	25
Ben	35	25	40	15
Andrew	35	30	50	10

SOLUTION: Using the Hungarian method you get 1-Mark, 2-Ben, 3-Greg, 4-Andrew. (White Version: 1-Andrew, 2-Greg, 3-Ben, 4-Mark. I just reversed the columns).

Extra Credit [2 pts]: The problem of finding the minimum number of lines it takes to cover all 0's is what type of problem? (one mentioned in class).

SOLUTION: Set Cover

10. **[3 pts]** Consider a transportation problem with 3 suppliers and 4 demand points. The amounts the suppliers have available is 10,50, and 20 respectively. The amount of demand at each demand point is 25,15,10, and 20 respectively. The cost to transport 1 unit from a supply point to a demand point is given in the following table. Given this information, formulate the problem as an LP, make sure you define your decision variables (you may also define some c_{ij} to condense the objective function, but I want all the constraints written out).

	Demand 1	Demand 2	Demand 3	Demand 4
Supply 1	5	6	8	10
Supply 2	6	7	5	9
Supply 3	8	9	6	8

SOLUTION: Let there be a dummy demand node 5 with d_5=10

$$\min \quad z = \sum c_{ij}x_{ij}$$

subject to

$$\begin{aligned}
x_{11} + x_{12} + x_{13} + x_{14} + x_{15} &= 10 \\
x_{21} + x_{22} + x_{23} + x_{24} + x_{25} &= 50 \\
x_{31} + x_{32} + x_{33} + x_{34} + x_{35} &= 20 \\
x_{11} + x_{21} + x_{31} &= 25 \\
x_{12} + x_{22} + x_{32} &= 15 \\
x_{13} + x_{23} + x_{33} &= 10 \\
x_{14} + x_{24} + x_{34} &= 20 \\
x_{15} + x_{25} + x_{35} &= 10 \\
x_{ij} &\geq 0 \qquad \forall i = 1,2,3; j = 1,2,3,4,5
\end{aligned}$$

SOLUTION: If you do not include the dummy demand and have constraints with =, your original problem is infeasible (plug it into GAMS or Excel to check), unless you change the supply constraints to be $\leq$.

11. [**5 pts**] Consider the problem:

$$\begin{aligned} \max \quad z = & 6x_1 + 3x_2 \\ \text{subject to} & \\ & 5x_1 + 3x_2 \leq 15 \\ & 2x_1 + 4x_2 \leq 8 \\ & x_1, x_2 \geq 0 \\ & x_1 \text{ integer} \end{aligned}$$

Graph the feasible region of the linear relaxation, and identify the feasible points of the original problem. Label x,y intercepts and axis.

SOLUTION: The feasible points for the original problem are all points in the feasible region for the relaxation where x_1 is an integer. These can be represented by vertical lines at 0,1,2 on the x_1 axis as well as the point (3,0). For the White Version, it is the same thing only x_2 must be an integer not x_1 so the lines are horizontal at 0 and 1, along with the point (0,2).