ChBE 3210 Spring 2015 Exam I Solutions

Problem I/ A) FALSE; $\delta/\delta_t \sim Pr^{1/3} < 1$ in this case $\Rightarrow \delta_t > \delta$

B) TRUE; apart from entry effects, h will be constant

C) TRUE

D) TRUE

E) TRUE; warm plate facing up has higher h than cold plate

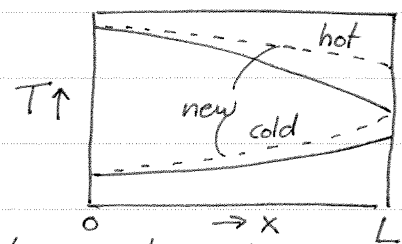
F) FALSE; F <u>never</u> greater than 1

G) TRUE; because $T_{h,in} = T_{h,out}$, mode of operation irrelevant for ΔT_{LM}

Problem II/ A) * $\dot{m}_H \uparrow \Rightarrow T_{H,out} \uparrow$

* larger ΔT (and probably h_{out})

$\Rightarrow$ greater $q \Rightarrow T_{C,out} \uparrow$



B) * Blasius: exact analysis without having to make many assumptions

* von Karman: can be used for non-laminar flow and more complex geometries

C) * 1) Hook up (or run water through) only one inlet and see which outlet produces flow

2) To test counter vs. co-flow: run hot and cold water at <u>low</u> flow rates: in counter-flow, $T_{H,out} < T_{C,out}$; in co-flow $T_{H,out} \approx T_{C,out}$

D) * Condensation on flat plate leads to build-up of condensate layer and -unlike tilted plates- lack of well-defined drainage prevents steady state that is predictable $\Rightarrow$ correlation not meaningful or useful

Problem III/ A) * Free convection from horizontal cylinder

$T_f = \frac{200+15}{2} = 107.5°C \approx 380K \Rightarrow Pr = 0.692$; $\frac{g\beta}{\nu^2} = 0.4742 \cdot 10^8 K^{-1} m^{-3}$

$k = 3.21 \cdot 10^{-2}\ W/m \cdot K$

$\Rightarrow Ra_D = \frac{g\beta}{\nu^2} \cdot \Delta T \cdot D^3 \cdot Pr$

$= 0.4742 \cdot 10^8 \cdot 185 \cdot (0.09)^3 \cdot 0.692 = 4.43 \cdot 10^6$

$\Rightarrow Nu_D = 0.480 \cdot Ra_D^{0.25} = 22.0 \Rightarrow h_D = \frac{Nu_D \cdot k}{D} = \frac{22.0 \cdot 0.0321}{0.09} = 7.85\ W/m^2 \cdot K$

$\Rightarrow q = h_D A \Delta T = h_D \pi D L \Delta T \Rightarrow \frac{q}{L} = 7.85 \cdot \pi \cdot 0.09 \cdot 185 = 411 \ W/m$

B) * Now forced convection (same $T_f \approx 380K$)

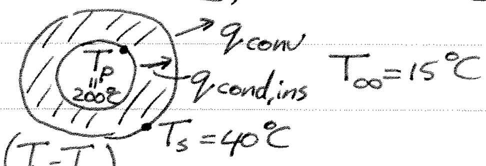
$Re_D = \frac{V \cdot D}{\nu} = \frac{8 \cdot 0.09}{2.37 \cdot 10^{-5}} = 3.04 \cdot 10^4 \Rightarrow Nu_D = 0.193 \cdot Re_D^{0.618} \cdot Pr^{1/3}$

$\nu \xrightarrow{@T_f} 2.37 \cdot 10^{-5}$

$= 0.193 \cdot (3.04 \cdot 10^4)^{0.618} \cdot (0.692)^{1/3} = 100.6$

$\Rightarrow$ Increase in Nu_D (and thus h_D and $\frac{q}{L}$) by factor $\frac{100.6}{22.0} = 4.57$

C) * $q_{cond,ins} = q_{conv}$



$\frac{T_p - T_s}{\frac{\ln\left(\frac{D+2t}{D}\right)}{2 \cdot \cancel{\pi} \cdot \cancel{L} \cdot k_{ins}}} = h \cdot \cancel{\pi} (D+2t) \cancel{L} (T_s - T_\infty)$

$\Rightarrow k_{ins} = \frac{h}{2} \cdot \ln\left(\frac{D+2t}{D}\right) \cdot (D+2t) \frac{T_s - T_\infty}{T_p - T_s}$

$= \frac{h}{2} \cdot \ln\left(\frac{0.13}{0.09}\right) (0.13) \frac{25}{160} = h \cdot 3.73 \cdot 10^{-3}$

* New $T_f \Rightarrow$ new h !

$T_f = \frac{40+15}{2} = 27.5°C \approx 300K \Rightarrow Pr = 0.708$; $\frac{g\beta}{\nu^2} = 1.327 \cdot 10^8 \ K^{-1} m^{-3}$; $k = 0.0262 \frac{W}{m \cdot K}$

$\Rightarrow Ra_D = 1.327 \cdot 10^8 \cdot 25 \cdot (0.13)^3 \cdot 0.708 = 5.16 \cdot 10^6$

$\Rightarrow Nu_D = 0.480 \cdot (5.16 \cdot 10^6)^{0.25} = 22.9$ (not very different from A)!)

$\Rightarrow h_D = \frac{22.9 \cdot 0.0262}{0.13} = 4.61 \ W/m^2 \cdot K$ (quite different from A)!)

$\Rightarrow k_{ins} = 4.61 \cdot 3.73 \cdot 10^{-3} = 0.0172 \ W/m \cdot K$

D) * Because of insulation, relative importance of convective heat transfer is less. In C): $q = \frac{T_p - T_\infty}{R_{th,cond} + R_{th,conv}}$, while in A): $\frac{T_p - T_\infty}{R_{th,conv}} = q$

As a result, increasing h (and lowering $R_{th,conv}$) has less effect on q in C) (insulated pipe) than in A) (bare pipe)

E) * Inside pipe, heat transfer occurs through condensation of saturated steam, which has much greater h than convection of air (free and forced) $\Rightarrow T_p \approx T_{steam}$ is realistic

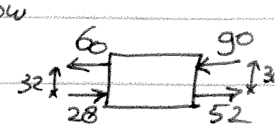
Problem IV/A) * $T_{s,in} = 90°C$ and $T_{s,ave} = 75°C \Rightarrow T_{s,out} = 60°C$

* $\Delta T_{water} = \frac{\dot{m}_{oil} \cdot c_{p,oil}}{\dot{m}_{water} \cdot c_{p,water}} \cdot \Delta T_{oil} = \frac{1.0}{0.6} \cdot \frac{2000}{4175} = 23.95°C$

$\Rightarrow T_{t,in} = 40 - \frac{23.95}{2} = 28\,°C$, $T_{t,out} = 40 + \frac{23.95}{2} = 52\,°C$

$\Rightarrow Y = \frac{T_{t,out} - T_{t,in}}{T_{s,in} - T_{t,in}} = \frac{52-28}{90-28} = 0.39$; $Z = \frac{30}{24} = 1.25 \Rightarrow F \approx 0.87$

B) $U \cdot A \cdot \Delta T_{LM} \cdot F = (\dot{m}\, c_p\, \Delta T)_{oil}$ with $\Delta T_{LM} \overset{\text{assume counter flow}}{=} \frac{38-32}{\ln(38/32)} = 34.9\,°C$



$\Rightarrow UA = \frac{(\dot{m}\, c_p\, \Delta T)_{oil}}{\Delta T_{LM} \cdot F} = \frac{1.0 \cdot 2000 \cdot 30}{34.9 \cdot 0.87} = 1975\ W/K$

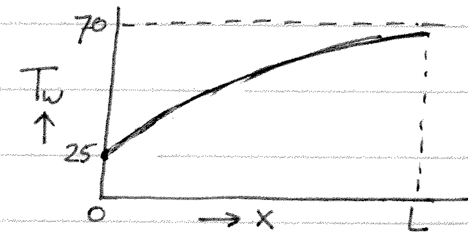
C) UA should increase if water flow rate increases; A will remain same, but h inside tube should increase, which enhances U

Problem V/A) * At inlet, $T_w = 25\,°C$.

* When $x\uparrow$, $T_w \uparrow$ and $\Delta T = T_s - T_w \downarrow$

$\Rightarrow \frac{dT}{dx} \downarrow$

* If $L \to \infty$ (long pipe), $T_w \to 70\,°C$



B) * Internal forced convection

$Re_D = \frac{\rho V_{ave} D}{\mu}$, $\dot{m} = \rho V_{ave} \frac{\pi}{4} D^2 \Rightarrow \rho V_{ave} D = \frac{4\dot{m}}{\pi D} \Rightarrow Re_D = \frac{4\dot{m}}{\pi D \mu}$

* Problem: which $T_b = \frac{T_0 + T_L}{2}$ to use? $T_{L,max} = 70\,°C \Rightarrow T_{b,max} = \frac{25+70}{2} = 47.5\,°C \approx 320\,K$

* Using $T_b = 320\,K \Rightarrow Re_D = \frac{4 \cdot 0.01}{\pi \cdot 0.01 \cdot 577 \cdot 10^{-6}} = 2207 \Rightarrow$ laminar!

$\Rightarrow Nu_D = 1.86 \cdot \left(Pr \cdot Re \cdot \frac{D}{L}\right)^{1/3} \cdot \left(\frac{\mu_b}{\mu_w}\right)^{0.14} = 1.86 \cdot \left(3.77 \cdot 2207 \cdot \frac{0.01}{8}\right)^{1/3} \cdot \left(\frac{577}{\underset{\text{at } 70°C}{400}}\right)^{0.14} = 4.27$

$\Rightarrow h_D = \frac{Nu_D \cdot k}{D} = \frac{4.27 \cdot 0.64}{0.01} = 273\ W/m^2 K$

C) * $\underbrace{h}_{-\frac{\rho V_{ave} c_p D}{4L} \ln\left(\frac{T_s - T_L}{T_s - T_0}\right)} \cdot \overset{\pi D L}{A} \cdot \Delta T_{LM} = \underbrace{\dot{m}}_{\rho V_{ave} \frac{\pi}{4} D^2} \cdot c_p \cdot \underbrace{\Delta T}_{T_L - T_0}$

$\Rightarrow -\frac{\cancel{\rho} \cancel{V_{ave}} \cancel{c_p} \cancel{D}}{\cancel{4}\cancel{L}} \cancel{\pi} \cancel{D} \cancel{L}\, \Delta T_{LM} = \cancel{\rho} \cancel{V_{ave}} \frac{\cancel{\pi}}{\cancel{4}} \cancel{D^2} \cancel{c_p} \cdot \frac{T_L - T_0}{\ln\left(\frac{T_s - T_L}{T_s - T_0}\right)}$

$\Rightarrow \Delta T_{LM} = \frac{-(T_L - T_0)}{\ln\left(\frac{T_s - T_L}{T_s - T_0}\right)} = \frac{(T_s - T_L) - (T_s - T_0)}{\ln\left(\frac{T_s - T_L}{T_0 - T_L}\right)}$

D) * $\ln\left(\frac{T_s - T_L}{T_s - T_o}\right) = -\frac{h \pi D L}{\dot{m} c_p} = -\frac{273 \cdot \pi \cdot 0.01 \cdot 8}{0.01 \cdot 4180} = -1.64$

$\Rightarrow \frac{T_s - T_L}{T_s - T_o} = e^{-1.64} = 0.194 \Rightarrow T_L = T_s - (T_s - T_o) \cdot 0.194 = 70 - 45 \cdot 0.194$
$= 61.3°C$

* $q = \dot{m} c_p \Delta T = 0.01 \cdot 4180 \cdot (61.3 - 25) = 1517\ W$