PHYS 2212 Test 1
Spring 2014

KEY ____________________ **Lab Section**____________

Lab section by day and time: Curtis(H), Ballantyne(Q), Kim(P)						
Monday	12:05-2:55pm	H01 or Q01	3:05-5:55pm	H02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	Q03 or P03	3:05-5:55pm	Q04 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	H03 or Q05	3:05-5:55pm	P05 or Q06	6:05-8:55pm	H04 or P06
Thursday	12:05-2:55pm	H05 or Q07	3:05-5:55pm	Q08 or H06	6:05-8:55pm	H07 or P07

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a\cdot b}{c\cdot d}=\frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})}=5\times10^{4}$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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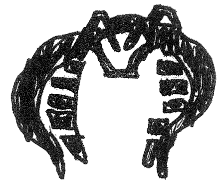
Sign your name on the line above

PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		G.I. JOE
Problem 2 (25 pts)		THUNDERCATS
Problem 3 (25 pts)		TEENAGE MUTANT NINJA TURTLES
Problem 4 (25 pts)		TRANSFORMERS

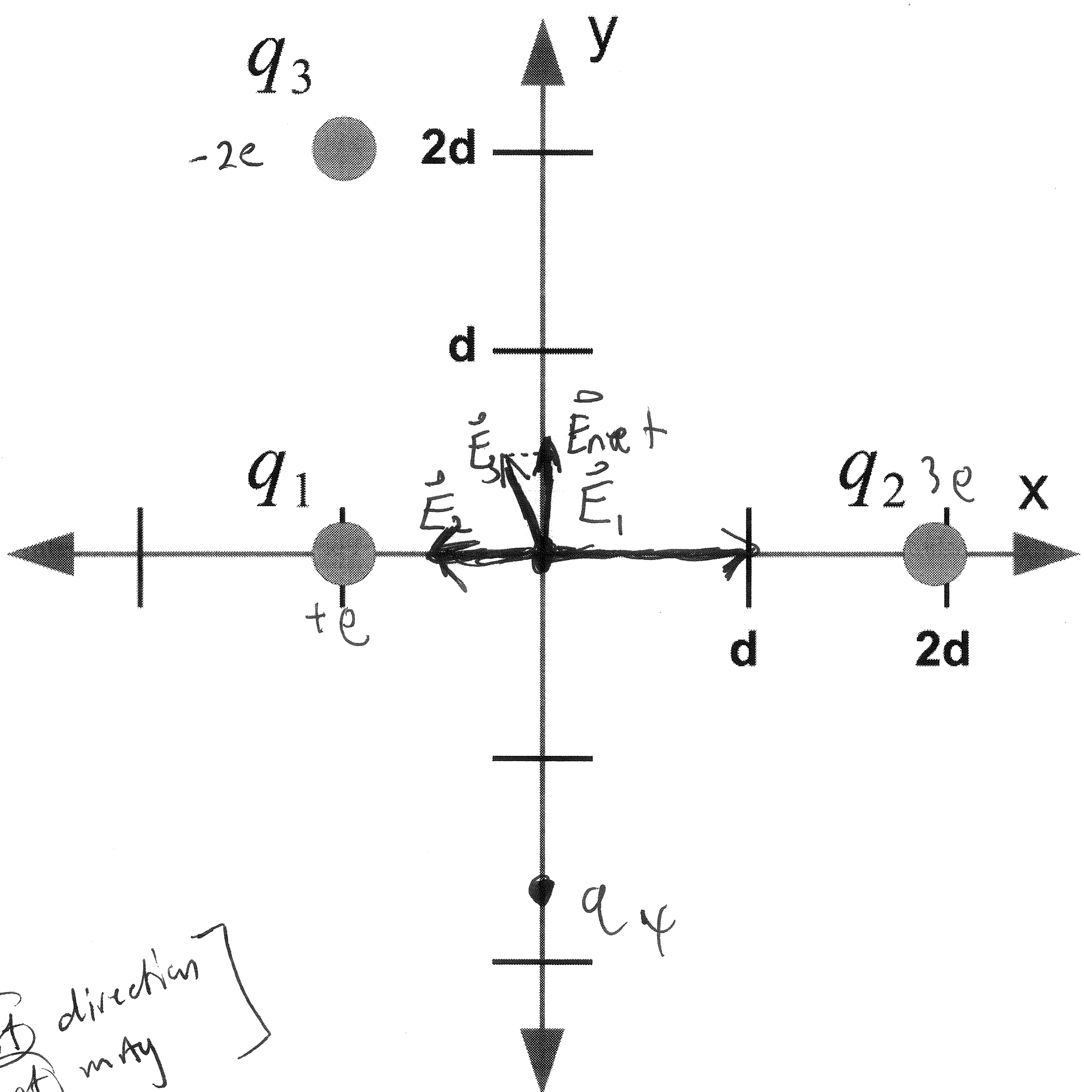


C O B R A

Problem 1 (25 Points)

Three particles are located on the x-y plane as indicated in the diagram. The first particle has charge $q_1 = e$ and is located at $< -d; 0; 0 >$. The second particle has charge $q_2 = 3e$ and is located at $< 1:91d; 0; 0 >$. The third particle has charge $q_3 = -2e$ and is located at $< -d; 2d; 0 >$.

Note that e is the charge of the proton and d is a positive distance.



1pt direction
1pt mag

(a 6pts) Draw the direction and approximate magnitude of the electric fields from the 3 charged particles at the origin. Label each field (e.g. E_1 for the electric field due to the first particle).

(b 4pts) Sketch an arrow that indicates the net electric field (from the 3 charged particles) at the origin.

2 pts direction } must match (a)
2 pts mag

(c 15pts) Where should a fourth particle with charge $q_4 = -e$ be positioned such that the net electric field at the origin $< 0; 0; 0 >$ is zero? Please be sure to show your work.

$$\vec{E}_{net} = 0 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1}{d^2}\langle +\hat{x}\rangle = \frac{1}{4\pi\epsilon_0}\frac{e}{d^2}\langle +1, 0, 0\rangle$$

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q_2}{(1.91d)^2}\langle -1, 0, 0\rangle = \frac{1}{4\pi\epsilon_0}\frac{0.82e}{d^2}\langle -1, 0, 0\rangle$$

$$\vec{E}_3 = \frac{1}{4\pi\epsilon_0}\frac{q_3}{((1d)^2 + (2d)^2)^{\frac{3}{2}}}\langle +1d, -2d, 0\rangle = \frac{1}{4\pi\epsilon_0}\frac{-2e}{(5)^{\frac{3}{2}}d^2}\langle +1, -2, 0\rangle$$

$$= \frac{1}{4\pi\epsilon_0}\frac{e}{d^2}\langle -0.18, +0.36, 0\rangle$$

$$\boxed{\begin{array}{c} -1.0 \\ -2.0 \\ -4.5 \\ -12.0 \end{array}}$$

$$\vec{E}_4 = \frac{1}{4\pi\epsilon_0}\frac{q_4}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{-e}{r^2}\hat{r}$$

$$-\vec{E}_4 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \frac{1}{4\pi\epsilon_0}\frac{e}{d^2}\langle \cancel{1 - 0.82 - 0.18}\ 0, 0.36, 0\rangle$$

$$+\frac{1}{4\pi\epsilon_0}\frac{+e}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{e}{2.8d^2}\langle 0, 1, 0\rangle$$

$$\hat{r} = \langle 0, 1, 0\rangle$$

$$|\vec{r}| = 1.67d$$

$$\vec{r}_{obs} = \langle 0, 0, 0\rangle$$

$$\vec{r} = \vec{r}_{obs} - \vec{r}_{source}$$

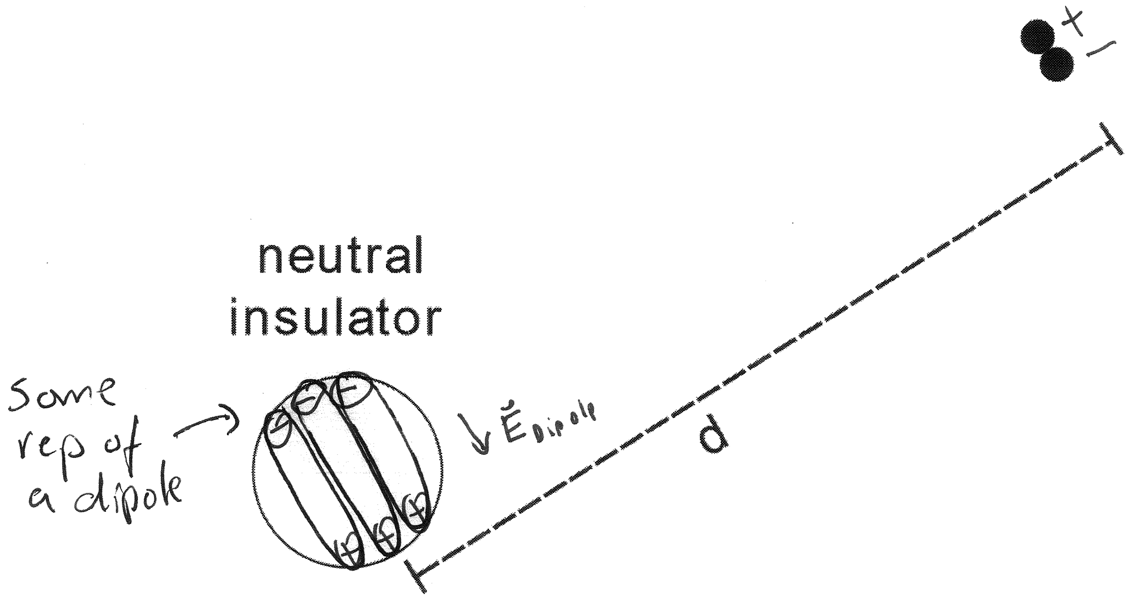
Thus q_4 is placed at

$$\boxed{\langle 0, -1.67d, 0\rangle}$$

Problem 2 (25 Points)

In the question below you will be asked to draw arrows and indicate how objects are polarized. The relative lengths of the arrows in your diagram below must be correct. Label your arrows clearly. Arrows without labels will be counted wrong. **If a quantity is zero, you must state that explicitly.** Note that the diagram is not to scale; the dipole is shown larger than its actual size.

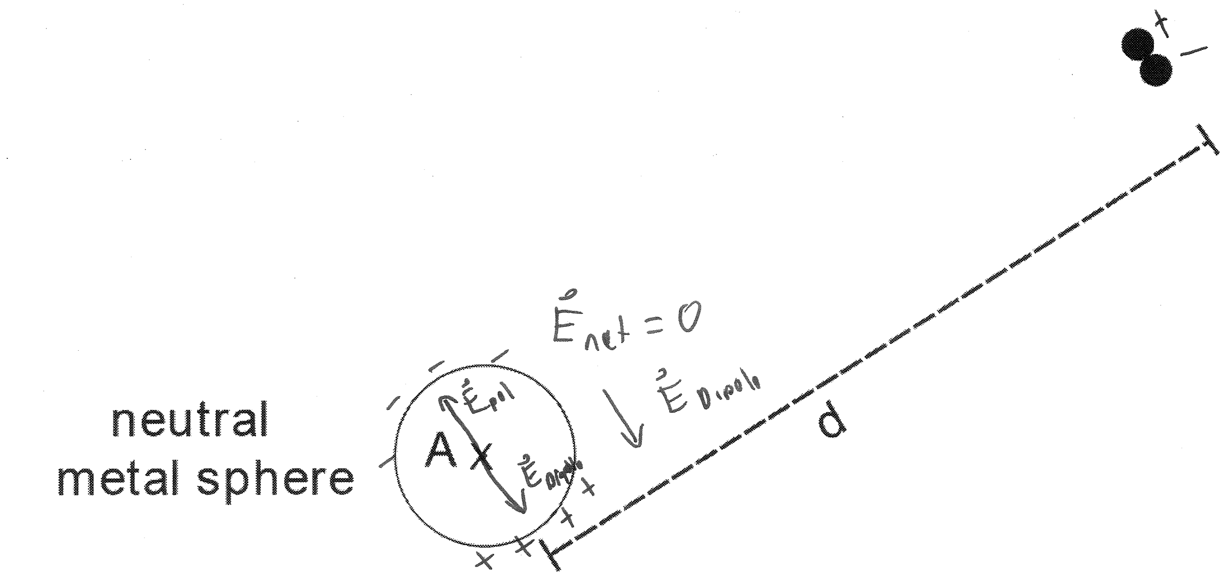
A permanent dipole is at a distance d from the center of a **neutral** spherical insulator.



(a 4pts) On the diagram draw the approximate charge distribution in and/or on the insulator.

All

Now the insulator is removed and replaced by a neutral metal sphere.



(b 4pts) At the center of the metal sphere, draw an arrow to represent the electric field due to just the dipole and label it $\mathbf{E}_d$. All

(c 4pts) Draw the charge distribution in and/ or on the metal sphere. Equal "+" & "-" All

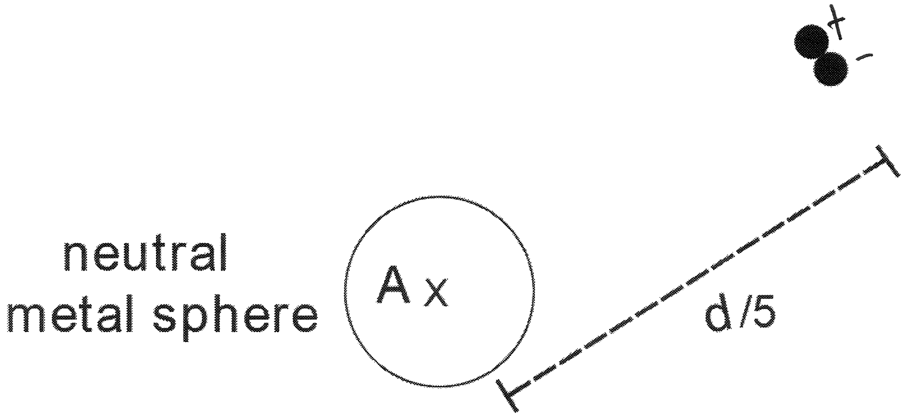
(d 4pts) At the center of the metal sphere, draw an arrow to represent the electric field due to the charges in and/or on the sphere and label it $\mathbf{E}_{pol}$. (2 pts) direction, (2pts) mag

(e 4pts) At the center of the metal sphere, draw an arrow to represent the net electric field and label it $\mathbf{E}_{net}$.

$\vec{E}_{net} = 0$

All & given from (b) & (d)

The metal sphere is now moved closer so that the distance from the dipole to the center of the sphere is d/5. The diagram is not to scale.

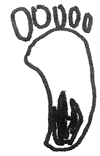


(f 5pts) Calculate the ratio $\frac{E_{pol,d/5}}{E_{pol,d}}$. Show your work.

$$|\vec{E}_{pol,d}| = \frac{1}{4\pi\epsilon_0}\frac{qs}{d^3}$$ 2pts

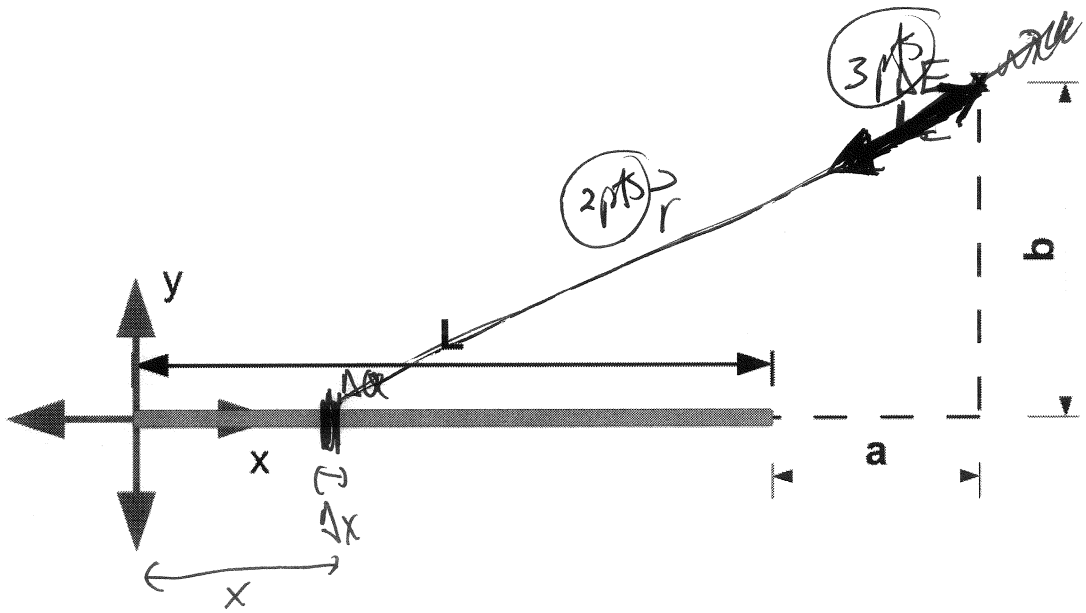
$$|\vec{E}_{pol,d/5}| = \frac{1}{4\pi\epsilon_0}\frac{qs}{\left(\frac{d}{5}\right)^3} = \frac{1}{4\pi\epsilon_0}\frac{125qs}{d^3}$$ 2pts

$$\frac{|\vec{E}_{pol,d/5}|}{|\vec{E}_{pol,d}|} = \frac{\cancel{\frac{1}{4\pi\epsilon_0}}\,\frac{125\cancel{qs}}{\cancel{d^3}}}{\cancel{\frac{1}{4\pi\epsilon_0}}\,\frac{\cancel{qs}}{\cancel{d^3}}} = 125$$ 1pt



Problem 3 (25 Points)

A very thin plastic rod of length L is rubbed with cloth and becomes uniformly charged negatively, with net charge $-Q$. In this problem you will work towards determining the Electric field of the rod at an observation location 'x" that is a distance a to the right of the rod and a distance b above the rod.



(a 5pts) Determine an expression for the charge on a slice of the rod, ΔQ, in terms of the infinitesimal length Δx of the slice.

Linear Charge Density $\lambda = \frac{-Q}{L}$

$$\Delta Q = \lambda \Delta x = \frac{-Q}{L} \Delta x$$

(b 5pts) On the diagram sketch the relative position vector $\vec{r}_{observation} - \vec{r}_{source}$ for an arbitrary slice of the rod ΔQ. Sketch the direction of the electric field $\Delta \vec{E}$ due to ΔQ at the observation location marked "x".

$$\vec{r}_{source} = \langle x, 0, 0 \rangle$$

$$\vec{r}_{observation} = \langle L+a, b, 0 \rangle$$

$$\vec{r}_{observation} - \vec{r}_{source} = \langle L+a-x, b, 0 \rangle$$

(c 10pts) Derive an expression for the vector electric field $\Delta \vec{E}$ of the slice of the rod at the observation location marked "x". Be sure to show your work to earn full credit.

$$\Delta E = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{|\vec{r}|^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{|\vec{r}|^3} \vec{r}$$

$$\Delta E = \frac{1}{4\pi\epsilon_0} \frac{(-Q/L)\,\Delta x}{((L-x+a)^2+b^2)^{\frac{3}{2}}} \langle L-x+a,\ b,\ 0\rangle$$

$$\boxed{\begin{array}{l} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$$

(d 5pts) Setup, but do not solve, the integral required to determine the electric field $\vec{E}$ at the observation location "x".

$$\vec{E} = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{-Q/L \, dx}{((L-x+a)^2+b^2)^{3/2}} \langle L-x+a, b, 0 \rangle$$

vector (1pt)

limits (3pts)

integration variable "dx" (1pt)

Extra Credit (5pts): Determine $\vec{E}$ at the observation location "x" if $b = 0$.

If $b = 0$

All

$$\vec{E} = \int_0^L \frac{1}{4\pi\epsilon_0} \frac{-Q/L \, dx}{(L-x+a)^{3}} \langle \cancel{L-x+a}\ 1, 0, 0 \rangle$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{-Q}{L} \langle 1,0,0 \rangle \int_0^L \frac{dx}{(L-x+a)^2}$$

$u = L - x + a$

$du = -dx$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{+Q}{L} \langle 1,0,0 \rangle \int \frac{du}{u^2}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{+Q}{L} \langle 1,0,0 \rangle \left[-\frac{1}{u} + C \right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \langle 1,0,0 \rangle \left[-\frac{1}{L-x+a} + C \right]_0^L$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{L} \langle 1,0,0 \rangle \left(-\frac{1}{a} + \frac{1}{L+a} \right)$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\cancel{L}} \left(\frac{\cancel{a} - \cancel{L} - \cancel{a}}{a(L+a)} \right) \langle 1,0,0 \rangle$$

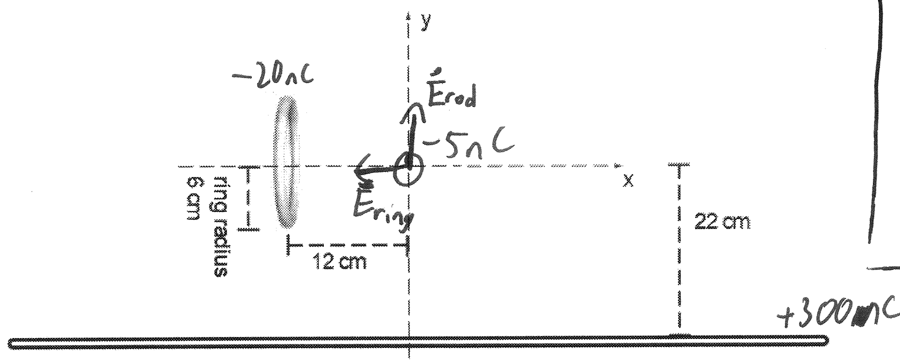
$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{a(L+a)} \langle -1, 0, 0 \rangle}$$

Problem 4 (25 Points)

A thin glass rod, of length 4 m, lies parallel to the x-axis and 22 cm below it. It has a charge of +300 nC uniformly distributed over its surface. A thin plastic ring, of radius 6 cm, is centered on the x-axis, 12 cm to the left of the origin. It has a uniform charge of −20 nC.

Note that the diagram is not to scale. The rod is much longer than is shown here.

$$\boxed{\begin{array}{c} -1.0 \\ -4.0 \\ -7.5 \\ -20 \end{array}}$$



Calculate the force on a small ball with charge −5 nC placed at the origin. **Your answer must be a vector. Clearly show all steps in your work.**

$$|\vec{E}_{rod}| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q_{rod}/L_{rod}}{r} \qquad \text{since } L \gg r$$

$Q_{rod} = 3\times10^{-7}\,C$

$L_{rod} = 4\,m$

$r = 0.22\,m$

$$|\vec{E}_{rod}| \approx 6136\,\frac{N}{C}$$

$$\vec{E}_{rod} = 6136\langle 0, 1, 0\rangle\,\frac{N}{C}$$

$$|\vec{E}_{ring}| = \frac{1}{4\pi\epsilon_0}\frac{q_{ring}z}{(z^2+R_{ring}^2)^{3/2}}$$

$q_{ring} = -2\times10^{-8}\,C$

$z = 0.12\,m$

$R_{ring} = 0.06\,m$

$q_{ball} = -5\times10^{-9}\,C$

$$|\vec{E}_{ring}| = 8944\,\frac{N}{C}$$

$$\vec{E}_{ring} = 8944\langle -1, 0, 0\rangle\,\frac{N}{C}$$

$$\vec{E}_{net} = \vec{E}_{rod} + \vec{E}_{ring} = \langle -8944, 6136, 0\rangle\,\frac{N}{C}$$

$$\vec{F} = q_{ball}\vec{E}_{net} = \langle 4.47\times10^{-5}, -3.07\times10^{-5}, 0\rangle\,N$$

OR

$$\text{If } |\vec{E}_{rod}| = \frac{1}{4\pi\epsilon_0}\frac{Q_{rod}}{r\sqrt{r^2+\left(\frac{L_{rod}}{2}\right)^2}} = 6100\,\frac{N}{C}$$

$$\text{So } \boxed{\vec{F} = \langle 4.47, -3.05, 0\rangle\times10^{-5}\,N}$$

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \frac{d\vec{v}}{dt}$

$\frac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\frac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta U_{el} = q\Delta V$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x\Delta x + E_y\Delta y + E_z\Delta z)$

$\Phi_{el} = \int \vec{E} \bullet \hat{n}dA$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n}dA$

$\oint \vec{E} \bullet \hat{n}dA = \frac{\sum q_{inside}}{\epsilon_0}$

$\oint \vec{B} \bullet \hat{n}dA = 0$

$\text{emf} = \oint \vec{E}_{NC} \bullet d\vec{l} = \left[\frac{d\Phi_{mag}}{dt}\right]$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \frac{d}{dt}\int \vec{E} \bullet \hat{n}dA\right]$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2 + (L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$; $\vec{p} = \alpha\vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2 + R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$ (z along axis)

$\left|\vec{E}_{disk}\right| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \frac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \frac{Q/A}{\epsilon_0}\left(\frac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\Delta\vec{l} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\,\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right|\tan\theta$

$\left|\vec{B}_{loop}\right| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$)

$\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \frac{1}{4\pi\epsilon_0}\frac{-q\vec{a}_\perp}{c^2r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \frac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\,nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\,nu$

$J = \frac{I}{A} = \sigma E$

$R = \frac{L}{\sigma A}$

$E_{dielectric} = \frac{E_{applied}}{K}$

$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$ due to a point charge

$I = \dfrac{\Delta V}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\,\Delta V$

$Q = C\ \Delta V$ $\qquad\qquad$ $K \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\dfrac{d\vec{p}}{dt}\right|_{\perp} = \dfrac{v}{R}\,|\vec{p}| \approx \dfrac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y b_z - a_z b_y)\hat{x} - (a_x b_z - a_z b_x)\hat{y} + (a_x b_y - a_y b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \qquad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \qquad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\,dx = ax + c \qquad \int ax\,dx = \frac{a}{2}x^2 + c \qquad \int ax^2\,dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m²/kg²
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\dfrac{1}{4\pi\epsilon_0}$	9×10^9 N · m²/C²
Epsilon-zero	ϵ_0	8.85×10^{-12} (N · m²/C²)$^{-1}$
Magnetic constant	$\dfrac{\mu_0}{4\pi}$	1×10^{-7} T · m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T · m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T