Homework 8

ISyE 4031 Regression and Forcasting
Due: Wednesday December 1st, 2010

1. Study example 6.10 on page 311. Solve the example the approximate method or the Cochrane-Orcutt method.

The approximate method is outlined in the book in detail on page 311-314. The majority of students did this. The Cochrane-Orcutt method involves similar analysis after the following key transformation:

$y_t - \Phi y_{t-1} = B_0(1-\Phi) + B_1(t-\Phi(t-1)) + B_2 M_1 + B_3 M_2 + B_4 M_3 + B_5 M_4 + B_6 M_5 + B_7 M_6 + B_8 M_7 + B_9 M_8 + B_{10} M_9 + B_{11} M_{10} + B_{12} M_{11} + a_t$

2. Do the following Exercises from the book.

Exercise 6.3 on page 318.

6.3 a. The trend appears to be linear.

b. The seasonal variation appears to be constant. No transformation is required.

c. All of the independent variables in the model seem to be important. This is because the *p*-values associated with Time, $Q2$, $Q3$, and $Q4$ are all much less than $\alpha = .05$ or $\alpha = .01$.

d. $Q2 = \begin{cases} 1 \text{ if time period for sales quarter is 2} \\ 0 \text{ otherwise} \end{cases}$

$Q3 = \begin{cases} 1 \text{ if time period for sales quarter is 3} \\ 0 \text{ otherwise} \end{cases}$

$Q4 = \begin{cases} 1 \text{ if time period for sales quarter is 4} \\ 0 \text{ otherwise} \end{cases}$

e. $\hat{y}_{17} = 17.250, \quad \hat{y}_{18} = 38.750, \quad \hat{y}_{19} = 51.750, \quad \hat{y}_{20} = 23.250$

f. $\hat{y}_t = 8.75 + .5t + 21Q_2 + 33.5Q_3 + 4.5Q_4$

$\hat{y}_{17} = 8.75 + .5(17) + 21(0) + 33.5(0) + 4.5(0) = 17.25$

$\hat{y}_{18} = 8.75 + .5(18) + 21(1) + 33.5(0) + 4.5(0) = 38.750$

g. 95% *P.I.* for y_{17} : $[15.395, 19.105]$

We are 95% confident that sales of the TRK-50 Mountain Bike in the first quarter of year 5 will be between 15.395 and 19.105 (15 to 19 bikes)

95% *P.I.* for y_{18} : $[36.895, 40.605]$

95% *P.I.* for y_{19} : $[49.895, 53.605]$

95% *P.I.* for y_{20} : $[21.395, 25.105]$

h. $n_p - 1$ = number of parameters –1

= number of independent variables = 4

$d = 2.20 > d_{U,.05} = 1.93$. Hence, there is no evidence of positive correlation at $\alpha = .05$.

Exercise 6.6 on page 321.

6.6 a. The use of a growth curve model seems appropriate since it appears that the data might be described by the model

$$y_t = \beta_0\left(\beta_1^t\right)\varepsilon_t \text{ (see Figure 6.22)}$$

b. Yes

c. $y_t = \beta_0\left(\beta_1^t\right)\varepsilon_t$

$lny_t = ln\left(\beta_0\right) + tln(\beta_1) + ln\varepsilon_t$

$lny_t = \alpha_0 + \alpha_1 t + u_t$

where

$\alpha_0 = ln\ (\beta_0)$, $\alpha_1 = ln\left(\beta_1\right)$, and $u_t = ln\varepsilon_t$

d. 1. $\hat{\alpha}_0 = -.54334$ and $\hat{\alpha}_1 = .38997$

2. A point estimate of

β_1 is $\hat{\beta}_1 = e^{\hat{\alpha}_1} = e^{.38997} = 1.4769$

Growth rate = $100\ (1 - \hat{\beta}_1) = 100(1 - 1.4769) = 47.69\%$

3. The point prediction of $ln\ y_{12}$ is $ln\ \hat{y}_{12} = 4.1363$

Thus, the point forecast of

y_{12} is $\hat{y}_{12} = e^{4.1363} = 62.5709$

4. The 95% prediction interval for $ln\ y_{12}$ is [3.8303, 4.4423]. Thus, a 95% Prediction interval for y_{12} is

$[e^{3.8303}, e^{4.4423}]$ = [46.0764, 84.9701].

We can be 95% confident that the number of reported cased of the disease in Month 12 will be at least 46.0764 (or roughly 46) cases and will be at most 84.9701 (or roughly 85) cases.