Homework 8

October 17, 2013

(Due: at the start of class on Tuesday/Wednesday, October 29/30)

1. Suppose that the weekly demand D of a non-perishable product is iid with the following distribution.

d	10	20	30
$\Pr(D = d)$	.2	.5	.3

Unused items from one week can be used in the following week. The management decides to use the following inventory policy: whenever the inventory level in Friday evening is less than or equal to 10, an order is made and it will arrive by Monday morning. The order-up-to quantity is set to be 30.

   (a) If this week (week 0) starts with inventory 20, what is the probability that week 2 has starting inventory level 10?

   (b) If this week (week 0) starts with inventory 20, what is the probability that week 100 has starting inventory level 10?

   (c) Draw a transition diagram.

   (d) Is the Markov chain irreducible?

   (e) Is the Markov chain periodic? Give the period of each state.

   (f) Suppose that each item sells at \$200. Each item costs \$100 to order, and each leftover item by Friday evening has a holding cost of \$20. Suppose that each order has a fixed cost \$500. Calculate the expected profit of the following week give that there are i item in stock this Friday evening for $i = 0, 10, 20, 30$.

   (g) Calculate the long-run expected profit per week.

2. Consider the following transition matrix:

$$P = \begin{pmatrix} 0 & 0.5 & 0 & 0.5 \\ 0.6 & 0 & 0.4 & 0 \\ 0 & 0.7 & 0 & 0.3 \\ 0.8 & 0 & 0.2 & 0 \end{pmatrix} \tag{1}$$

   (a) Draw a transition diagram.

   (b) Calculate the 100-step transition matrix.

   (c) Calculate the 101-step transition matrix.

   (d) Is the Markov chain irreducible?

   (e) Is the Markov chain periodic? Give the period of each state.

   (f) Is $(\pi_1, \pi_2, \pi_3, \pi_4) = (33/96, 27/96, 15/96, 21/96)$ the stationary distribution of the Markov Chain?

   (g) Is $P_{11}^{100} = \pi_1$? Is $P_{11}^{101} = \pi_1$? Give an expression for π_1 in terms of P_{11}^{100} and P_{11}^{101}.

3. For each of the following transition matrices, do the following: (1) Determine whether the Markov chain is irreducible; (2) Find a stationary distribution π; is the stationary distribution unique; (3) Determine whether the Markov chain is periodic; (4) Give the period of each state. (5) Without using any software package, find P^{100} approximately

   (a)

$$P = \begin{pmatrix} 0 & 1/3 & 2/3 \\ 2/3 & 0 & 1/3 \\ 1/3 & 2/3 & 0 \end{pmatrix}$$

   (b)

$$P = \begin{pmatrix} .2 & .8 \\ .5 & .5 \end{pmatrix}$$