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Solutions to Homework 9

1. (a) The state space is $\{\$10, \$20\}$. The transition matrix is $\begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.9 \end{bmatrix}$. It is irreducible.

   (b) The state space is $\{\$10, \$25\}$. The transition matrix is $\begin{bmatrix} 0.9 & 0.1 \\ 0.15 & 0.85 \end{bmatrix}$. It is irreducible.

   (c) The stationary distribution is $(\pi^X_{\$10}, \pi^X_{\$20}) = (1/3, 2/3)$.

   (d) The stationary distribution is $(\pi^Y_{\$10}, \pi^Y_{\$25}) = (3/5, 2/5)$.

   (e) What you need look at is $\mathbb{E}(\sum_{i=1}^{300} X_i)$ and $\mathbb{E}(\sum_{i=1}^{300} Y_i)$. And choose the one with larger expectation. By the stationary distribution obtained in (b) and (c), we have

$$\mathbb{E}(\sum_{i=1}^{300} X_i) = 300(10 \times \frac{1}{3} + 20 \times \frac{2}{3}) = 5000,$$

$$\mathbb{E}(\sum_{i=1}^{300} Y_i) = 300(10 \times \frac{3}{5} + 25 \times \frac{2}{5}) = 4800,$$

   so you should pick stock 1.

2. (a) Yes, it is a Markov Chain. The state space is $\{0, 1, 2, 3, \cdots\}$.

$$a = (1, 0, 0, \cdots), \quad P_{i,j} = \begin{cases} 98/100 & j = i+1; \\ 2/100 & j = 0. \end{cases}$$

   (b) Yes, it is irreducible. Starting from any state i, the probability of visiting state 0 in one step is $2/100$. And starting from state 0, the probability of visiting state i in i step is $(98/100)^i$. Therefore, state 0 communicates with any other state. Thus any state commutes with each other.

   (c) Starting from state 0, we have positive probability to visit state 0 again in one step. So state 0 is aperiodic. And periodicity is a class property, so the Markov chain is aperiodic.

   (d) Let $p = 98/100$. You can write down the following balance equations.

$$\pi_i = p\pi_{i-1} \quad (1)$$
$$\pi_0 = (1-p)\sum_i \pi_i. \quad (2)$$

   From (2), you can write $\pi_0 = 1 - p$ since $\sum_i \pi_i = 1$. From (1), you will have $\pi_i = p^i\pi_0 = (1-p)p^i$.

   (e) Yes, because it is irreducible, and the stationary distribution exists.