

MATH 1711, Midterm 3

10/30/2013

Name: key 1 GTID: _____

Circle your section below

D1 TA: Katie Stocker

D2 TA: Maggie Ginn

D3 TA: Kayla McKenzie

Problem No.	Points
1	
2	
3	
4	
5	
6	

TOTAL: _____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (20 points).

A fair die is rolled 180 times. Find the probability of observing at most 25 fives. Use the normal approximation to the binomial distribution to approximate this probability. Simplify your answer as far as possible.

$$p = \frac{1}{6}, \quad q = 1 - p = \frac{5}{6}, \quad n = 180$$

$$\mu = np = 30, \quad \sigma = \sqrt{npq} = \sqrt{25} = 5$$

$$\begin{aligned} \Pr(X \leq 25) &= \Pr(X \leq 25.5) \\ &= \Pr\left(Z \leq \frac{25.5 - 30}{5}\right) \\ &= \Pr(Z \leq -0.9) \\ &= 0.1841 \end{aligned}$$

Problem 2 (10 points).

The RREF matrix for a system of equations is given below. Write out the solution to the system of equations. You may label the columns with any variable names you wish.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 0 & 2 & 0 & 2 & 5 \\ 0 & 1 & 3 & 0 & -3 & 6 \\ 0 & 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$x_1 = 5 - 2x_3 - 2x_5$$

$$x_2 = 6 - 3x_3 + 3x_5$$

$$x_3 = \text{any } \#$$

$$x_4 = 3 - 5x_5$$

$$x_5 = \text{any } \#$$

Turn over for more problems

Problem 3 (20 points).

A pair of dice is rolled, and the sum of the numbers on the two uppermost faces is recorded. Find the mean and variance for this experiment. You do *not* need to simplify your final answer.

k	$P_r(X=k)$
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36

$$\mu = E(X) = 2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + \dots + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} = 7$$

$$\sigma^2 = E(X^2) - \mu^2 = (4 \cdot \frac{1}{36} + 9 \cdot \frac{2}{36} + \dots + 144 \cdot \frac{1}{36}) - 49$$

or

$$\sigma^2 = (2-7)^2 \cdot \frac{1}{36} + (3-7)^2 \cdot \frac{2}{36} + \dots + (12-7)^2 \cdot \frac{1}{36}$$

Problem 4 (10 points).

Prove or disprove that two different random variables can have the same mean and variance.

The statement is true.

pf: consider two random variables X and Y .

$$Pr(X = -1) = \frac{1}{2}$$

$$Pr(Y = -2) = \frac{1}{8}$$

$$Pr(X = 1) = \frac{1}{2}$$

$$Pr(Y = 2) = \frac{1}{8}$$

$$Pr(Y = 0) = \frac{3}{4}$$

then $\mu_X = \mu_Y = 0$

$$\sigma_X^2 = \sigma_Y^2 = 1$$

Turn over for more problems

Problem 5 (20 points).

Use the method of inverses to solve the following system of equations: given $AX = B$, first, find A^{-1} , then solve the equation $X = A^{-1}B$. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\begin{aligned}x + y + z &= 5 \\x + y + 2z &= 4 \\3x + 4y + 3z &= 6\end{aligned}$$

To find A^{-1} :

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 2 & 0 & 1 & 0 \\ 3 & 4 & 3 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\substack{R_2 = R_2 - R_1 \\ R_3 = R_3 - 3R_1}} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\text{swap } R_2, R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{R_1 = R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 4 & 0 & -1 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right]$$

$$\xrightarrow{R_1 = R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -1 & -1 \\ 0 & 1 & 0 & -3 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{array} \right]$$

\nwarrow
 A^{-1}

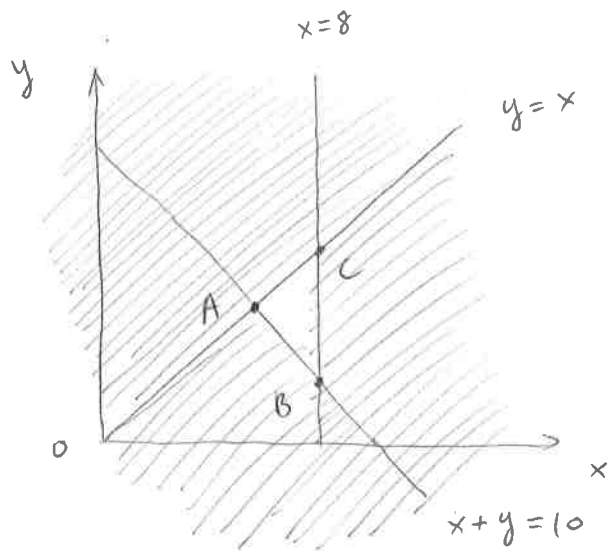
$$X = A^{-1}B = \begin{bmatrix} 5 & -1 & -1 \\ -3 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 15 \\ -9 \\ -1 \end{bmatrix}$$

Turn over for more problems

Problem 6 (20 points).

Use the graphical method from class to minimize the objective function $3x+2y$ subject to the constraints

$$\begin{cases} x+y \geq 10 \\ y \leq x \\ x \leq 8 \end{cases}$$



To find A:
$$\begin{cases} y = x \\ x + y = 10 \end{cases} \Rightarrow (x, y) = (5, 5)$$

To find B:
$$\begin{cases} x = 8 \\ x + y = 10 \end{cases} \Rightarrow (x, y) = (8, 2)$$

To find C:
$$\begin{cases} y = x \\ x = 8 \end{cases} \Rightarrow (x, y) = (8, 8)$$

At A, $3x + 2y = 3 \cdot 5 + 2 \cdot 5 = 25$

At B, $3x + 2y = 3 \cdot 8 + 2 \cdot 2 = 28$

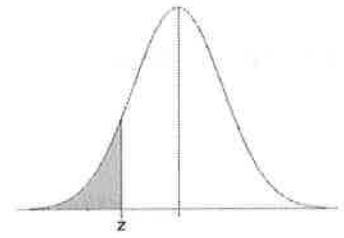
At C, $3x + 2y = 3 \cdot 8 + 2 \cdot 8 = 40$

So $3x + 2y$ is minimized at $(5, 5)$.
optimal value is 25.

The End.

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Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A normal distribution curve is shown. The horizontal axis is labeled with a point z . The area under the curve to the left of z is shaded gray.

[illegible]

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10/30/2013

Name: key 2 GTID: _____

Circle your section below

D1 TA: Katie Stocker

D2 TA: Maggie Ginn

D3 TA: Kayla McKenzie

Problem No.	Points
1	
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TOTAL: _____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (20 points).

A pair of dice is rolled, and the sum of the numbers on the two uppermost faces is recorded. Find the mean and variance for this experiment. You do *not* need to simplify your final answer.

see key 1.

Problem 2 (10 points).

Prove or disprove that two different random variables can have the same mean and variance.

see key 1

Turn over for more problems

Problem 3 (20 points).

A fair die is rolled 180 times. Find the probability of observing at most 22 fours. Use the normal approximation to the binomial distribution to approximate this probability. Simplify your answer as far as possible.

$$p = \frac{1}{6}, \quad q = 1 - \frac{1}{6} = \frac{5}{6}, \quad n = 180$$

$$\mu = np = 30, \quad \sigma = \sqrt{npq} = 5$$

$$\begin{aligned} \Pr(X \leq 22) &= \Pr(X \leq 22.5) \\ &= \Pr\left(Z \leq \frac{22.5 - 30}{5}\right) \\ &= \Pr(Z \leq -1.5) \\ &= 0.0668 \end{aligned}$$

Problem 4 (10 points).

The RREF matrix for a system of equations is given below. Write out the solution to the system of equations. You may label the columns with any variables names you wish.

$$\begin{array}{ccccc|c} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \hline 1 & 0 & 3 & 0 & -2 & 5 \\ 0 & 1 & 3 & 0 & 3 & 3 \\ 0 & 0 & 0 & 1 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$x_1 = 5 - 3x_3 + 2x_5$$

$$x_2 = 3 - 3x_3 - 3x_5$$

$$x_3 = \text{Any } \#$$

$$x_4 = 6 - 5x_5$$

$$x_5 = \text{Any } \#$$

Turn over for more problems

Problem 5 (20 points).

Use the method of inverses to solve the following system of equations: given $AX = B$, first, find A^{-1} , then solve the equation $X = A^{-1}B$. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$\begin{aligned}x + y + z &= 5 \\ 3x + 4y + 3z &= 6 \\ x + y + 2z &= 7\end{aligned}$$

To find A^{-1} :

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 4 & 3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array} \right] \xrightarrow[\substack{R_2 = R_2 - 3R_1 \\ R_3 = R_3 - R_1}]{} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{R_1 = R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 4 & -1 & 0 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \xrightarrow{R_1 = R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -1 & -1 \\ 0 & 1 & 0 & -3 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array} \right] \uparrow A^{-1}$$

$$X = A^{-1} \cdot B = \begin{bmatrix} 5 & -1 & -1 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 12 \\ -9 \\ 2 \end{bmatrix}$$

Turn over for more problems

Problem 6 (20 points).

Use the graphical method from class to minimize the objective function $2x+3y$ subject to the constraints

$$\begin{cases} x+y \geq 10 \\ y \leq x \\ x \leq 8 \end{cases}$$

see key 1 for the graph.

$$\text{At } A, \quad 2x+3y = 2 \cdot 5 + 3 \cdot 5 = 25$$

$$\text{At } B, \quad 2x+3y = 2 \cdot 8 + 3 \cdot 2 = 22$$

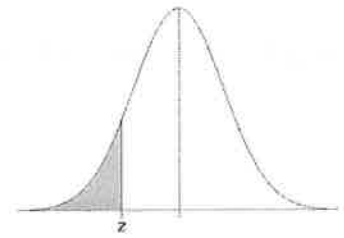
$$\text{At } C, \quad 2x+3y = 2 \cdot 8 + 3 \cdot 8 = 40.$$

So $2x+3y$ is minimized at $(8, 2)$
optimal value is 22.

The End.

[blank page 1]

Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

A normal distribution curve is shown. The area under the curve to the left of a point labeled z on the horizontal axis is shaded. The vertical axis passes through the peak of the curve.

A normal distribution curve is shown. The area under the curve to the left of a point labeled z on the horizontal axis is shaded. The vertical axis passes through the peak of the curve.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998