Jan. 16, 2014

Math 2401 M - Quiz 1

First Name (Print): ___________ Last Name (Print): ________ Signature: ________________

- There are **2** questions on **2** pages. The quiz is worth 20 points in total.
- Answer the questions clearly and completely. You must provide work clearly justifying your solution.
- You can NOT write your work on the back of the page. Use it for scratch work if needed.
- You have 20 minutes to finish your work.

1. (10 points) Find parametrization for the line of intersection of the planes $x + y + 2z = 0$ and $2x - y - 5z = 3$.

Solution:

$\vec{n}_1 = \langle 1, 1, 2\rangle$ is normal to the plane $x + y + 2z = 0$.

$\vec{n}_2 = \langle 2, -1, -5\rangle$ is normal to the plane $2x - y - 5z = 3$.

The line of intersection of two planes is perpendicular to $\vec{n}_1 = \langle 1, 1, 2\rangle$ and $\vec{n}_2 = \langle 2, -1, -5\rangle$. Therefore, the line is parallel to $\vec{n}_1 \times \vec{n}_2$, and

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 2 \\ 2 & -1 & -5 \end{vmatrix} = -3\vec{i} + 9\vec{j} - 3\vec{k}.$$

Next, we find a point on the line. Let $z = 0$, [In fact, you can let x, y or z be any real number, then solve the equations of two planes for the other two variables], and solve the linear system

$$\begin{cases} x + y + 2z = 0, \\ 2x - y - 5z = 3, \\ z = 0 \end{cases} \Rightarrow (x, y, z) = (1, -1, 0)$$

The point $(4, 1, 0)$ is a point on the line. Hence, the standard parametrization of the line is

$$x = 1 - 3t, \ y = -1 + 9t, \ z = -3t, \qquad t \in \mathbb{R}.$$

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2. (10 points) Find the distance from the point $S = (2, -3, 4)$ to the plane $x + 2y + 2z = 13$.

Solution:

We need to find a point P in the plane $x + 2y + 2z = 13$, and calculate the length of the vector projection of $\overrightarrow{PS}$ onto a normal vector $\vec{n}$ of the plane.

Obviously, $P = (13, 0, 0)$ is a point in the plane, [You may choose any point $P = (x, y, z)$ satisfying the condition $x + 2y + 2z = 13$] then

$$\overrightarrow{PS} = \langle 2 - 13, -3 - 0, 4 - 0 \rangle = \langle -11, -3, 4 \rangle.$$

$\vec{n}_1 = \langle 1, 2, 2 \rangle$ is a normal vector of the plane $x + 2y + 2z = 13$, and

$$|\vec{n}| = \sqrt{1^2 + 2^2 + 2^2} = 3.$$

The distance from S to the plane is

$$\begin{aligned} d &= \left| proj_n \overrightarrow{PS} \right| = \frac{\left| \overrightarrow{PS} \cdot \vec{n} \right|}{|\vec{n}|} \\ &= \frac{\left| \langle -11, -3, 4 \rangle \cdot \langle 1, 2, 2 \rangle \right|}{3} = \frac{\left| (-11)(1) + (-3)(2) + (4)(2) \right|}{3} = 3. \end{aligned}$$

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