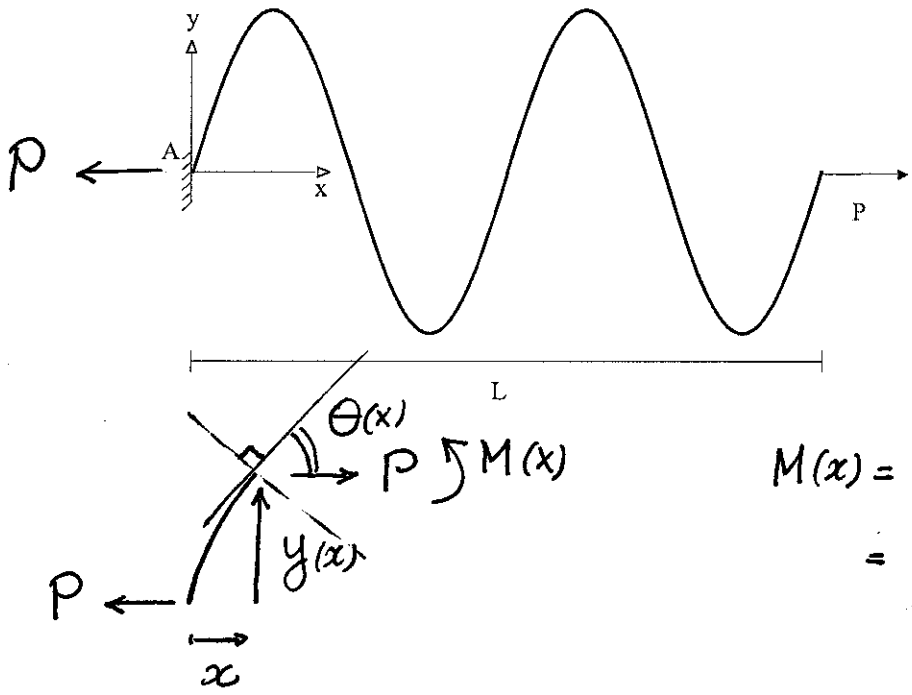
Structural Analysis Test 2 Solution

Problem 1

Find the shear force and bending moment as a function of x for the sinusoidal arch shown below. Assume that $a = L/4$.

$$y(x) = a\sin(4\pi x/L)$$



$M(x) = Py(x)$

$= \frac{PL}{4}\sin\left(\frac{4\pi x}{L}\right)$

$V(x) = P\sin\theta(x)$

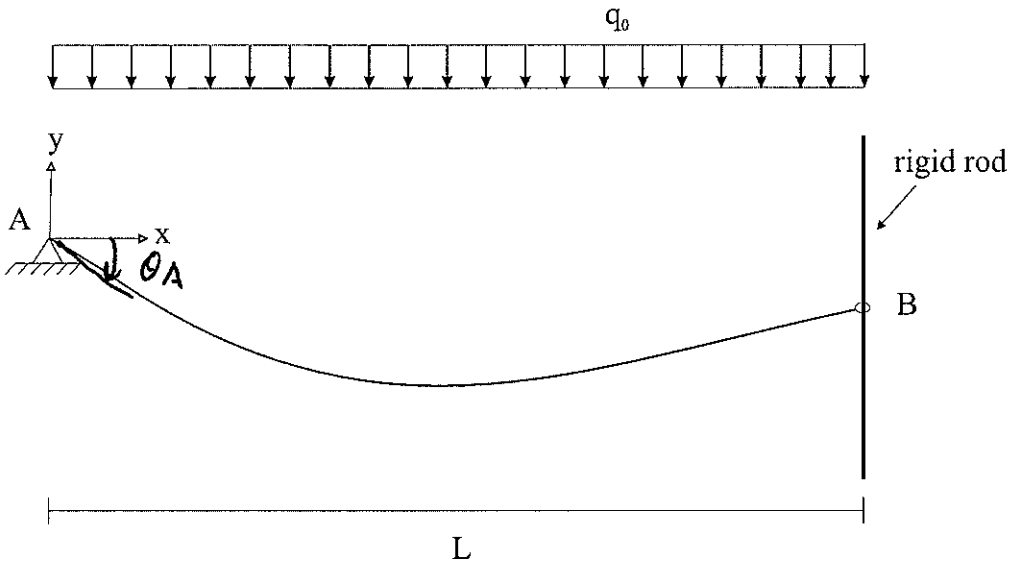
$y'(x) = \frac{4\pi}{L}a\cos\frac{4\pi x}{L} = \pi\cos\frac{4\pi x}{L} = \tan\theta(x)$

$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \Rightarrow V(x) = \frac{P\pi\cos 4\pi x/L}{\sqrt{1+\pi^2\cos^2\frac{4\pi x}{L}}}$

Problem 2

A cable of length ℓ is under a uniform distributed force q_0. The cable is free to move vertically at point B. Assuming that tension at point A is known to be T_A, find the location of point B in the equilibrium configuration. Ignore friction.

Note: $q_0 L = T_A \sin\theta_A \Rightarrow T_A > q_0 L$



$$\frac{d^2y}{dx^2} = \frac{q_0}{2H}$$

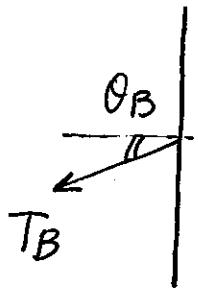
$$\frac{dy}{dx} = \frac{q_0 x}{H} + C_1$$

$y'(L) = 0 \Rightarrow C_1 = -\frac{q_0 L}{H}$

$$y = \frac{q_0 x^2}{2H} + C_1 x + C_2, \quad y(0) = 0 \Rightarrow C_2 = 0$$

$$y(L) = \frac{q_0 L^2}{2H} + C_1 L = -\frac{q_0 L^2}{2H} \Rightarrow y_B = -\frac{q_0 L^2}{2}\left(T_A^2 - q_0^2 L^2\right)^{-\frac{1}{2}}$$

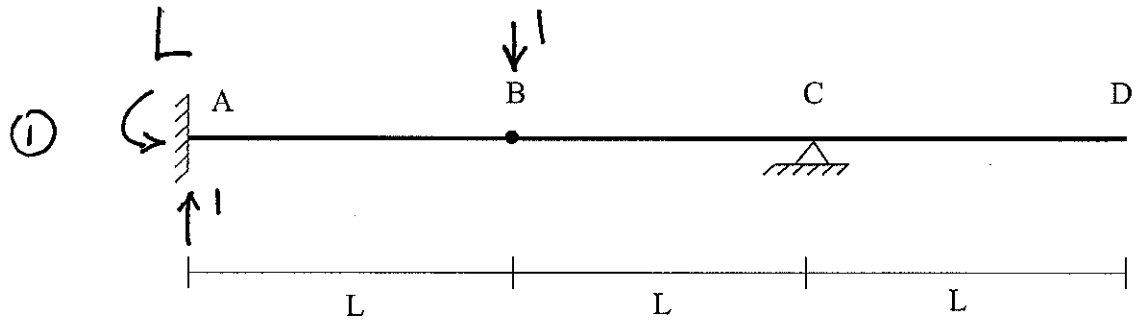
$\tan\theta_A = y'_A = -\frac{q_0 L}{H}$, $T_A \cos\theta_A = H^3 \Rightarrow H = \sqrt{T_A^2 - q_0^2 L^2}$



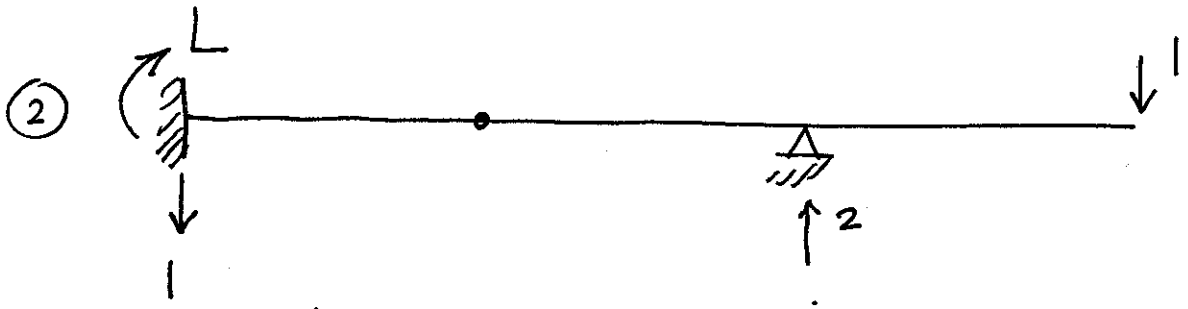
no friction ⇓ $\theta_B = 0$

Problem 3

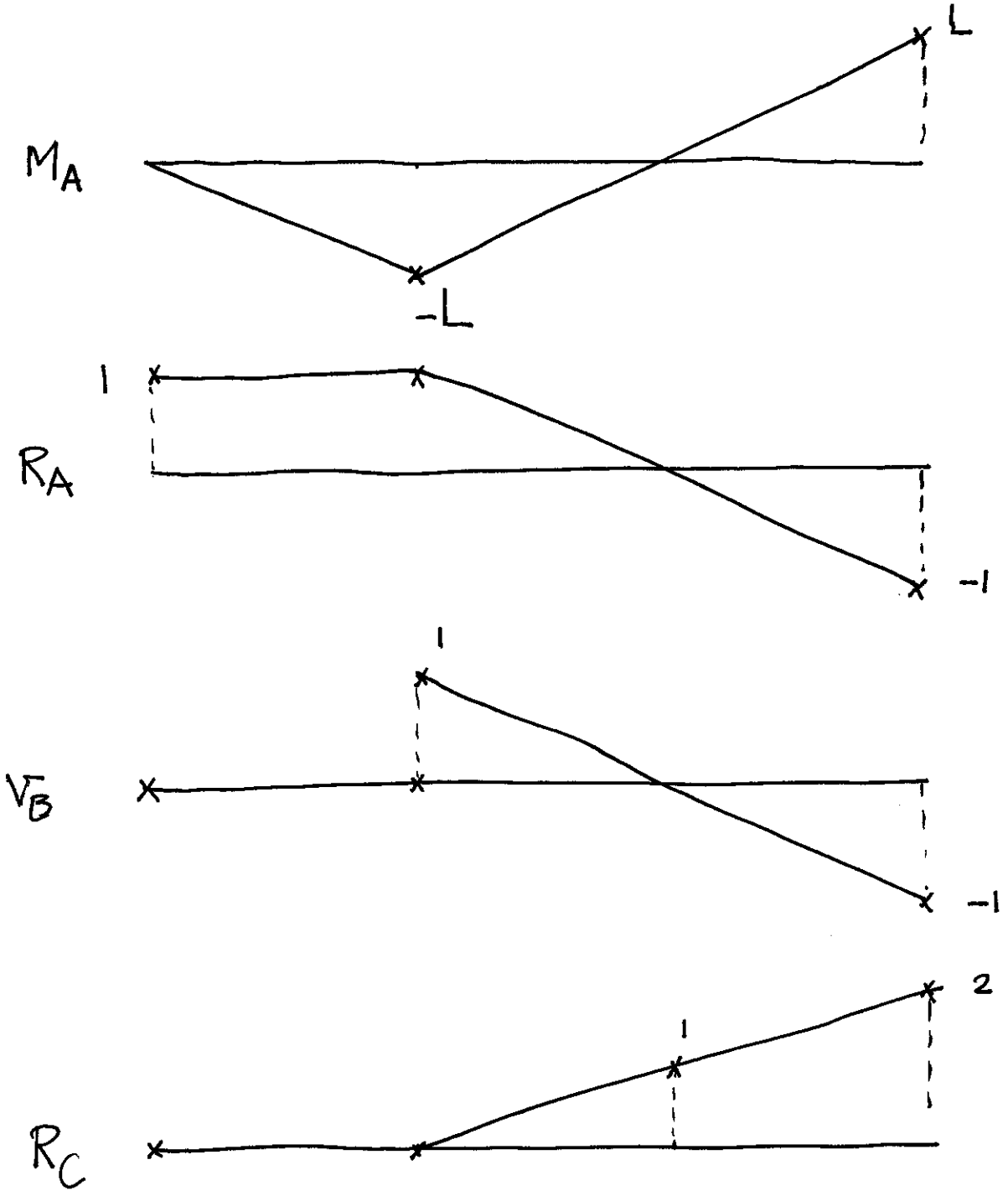
Draw the influence lines for M_A, R_A, V_B, and R_C.



$M_A = -L, R_A = 1, V_B^- = 0, V_B^+ = 1, R_C = 0$



$M_A = L, R_A = -1, V_B = -1 \qquad R_C = 2$


L
M_A
-L
1
R_A
-1
1
V_B
-1
2
1
R_C