PHYS 2211 Test 3
Spring 2015

Name(print) ~~~ Test ~~~ Key ~~~~ **Lab Section** 0

Greco (K or M), Schatz(N)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Tuesday	M01 N01	M02 N02	M03 N03
Tuesday	K03 K05	K04 K07	K06 K08
Thursday	M04 N04	M05 N05	M06 N06

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Sakura Kinomoto

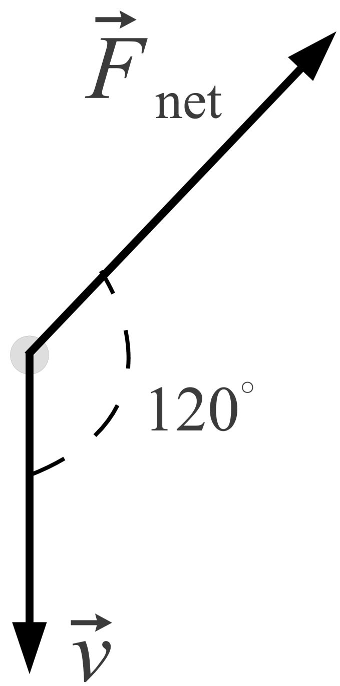
Sign your name on the line above

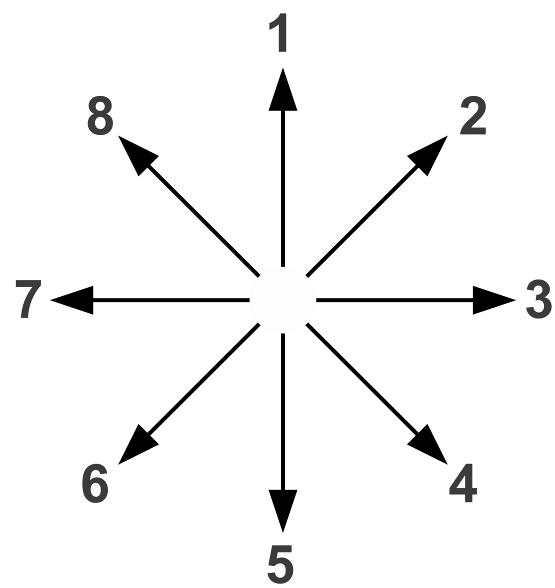
Please do not write on this page

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

An object of mass m and velocity $\vec{v}$ is acted upon by a net force $\vec{F}_{net}$ at time t, as indicated in figure below.





(a 6pts) Using the numbered direction arrows shown, indicate (by number) which arrow *best represents* the direction of the quantities listed below. If the quantity has zero magnitude or if more information is needed to determine the direction, indicate using the corresponding number listed below.

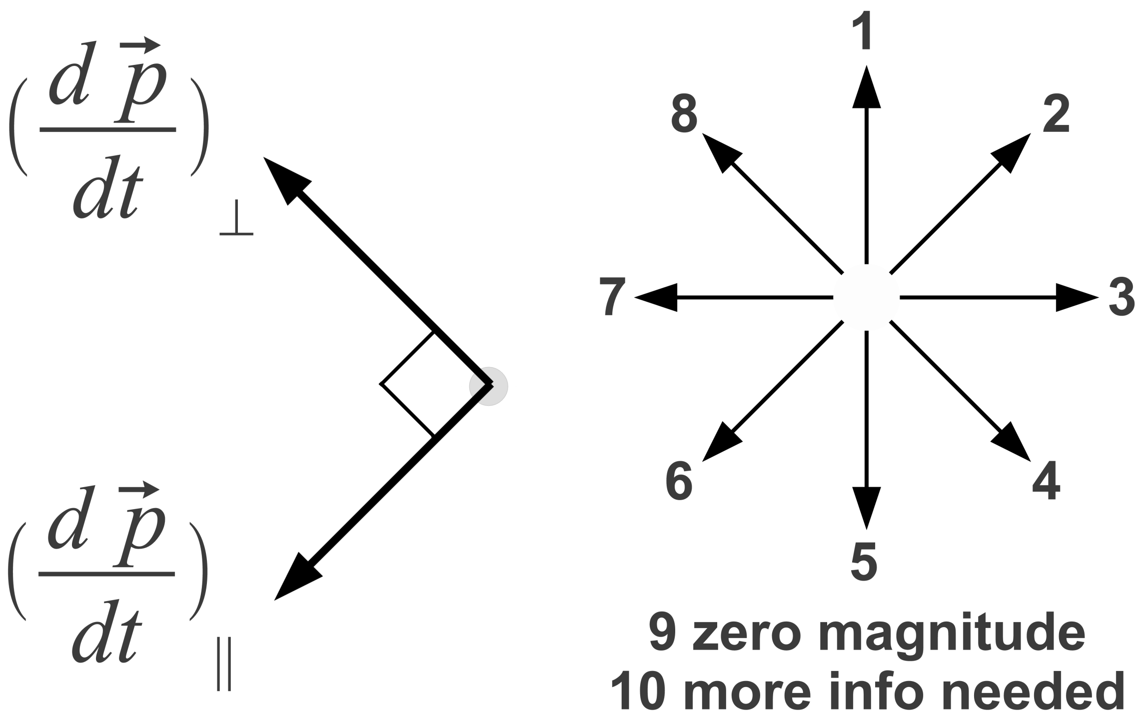
- $\vec{p}$, the object's momentum. __5__
- $(\vec{F}_{net})_{\parallel}$, the component of $\vec{F}_{net}$ that is parallel to the object's velocity. __1__
- $(\vec{F}_{net})_{\perp}$, the component of $\vec{F}_{net}$ that is perpendicular to the object's velocity. __3__
- $\frac{d\vec{p}}{dt}$, the time rate of change of the object's momentum. __2__
- $(\frac{d\vec{p}}{dt})_{\parallel}$, the component of $\frac{d\vec{p}}{dt}$ that is parallel to the object's velocity. __1__
- $(\frac{d\vec{p}}{dt})_{\perp}$, the component of $\frac{d\vec{p}}{dt}$ that is perpendicular to the object's velocity. __3__

1pt each

(b 4pts)At time t, the object's speed is (circle one):

- increasing.
- $\boxed{\text{decreasing.}}$ All
- constant.
- unable to be determined with the given information.

$(\frac{d\vec{p}}{dt})_{\parallel}$ and $(\frac{d\vec{p}}{dt})_{\perp}$ for an object of mass m are shown at time t in the figure below.



(c 10pts) Using the numbered direction arrows shown, indicate (by number) which arrow *best represents* the direction of the quantities listed below. If the quantity has zero magnitude or if more information is needed to determine the direction, indicate using the corresponding number listed below.

2pts each

- $\vec{p}$, the object's momentum. 10 -or- (6 or 2)
- $\vec{F}_{net}$, the net force acting on the object 7
- $(\vec{F}_{net})_{\parallel}$, the component of $\vec{F}_{net}$ that is parallel to the object's velocity. 6
- $(\vec{F}_{net})_{\perp}$, the component of $\vec{F}_{net}$ that is perpendicular to the object's velocity. 8
- $\frac{d\vec{p}}{dt}$, the time rate of change of the object's momentum. 7

(d 5pts)At time t, the object's speed is (circle one):

All

- increasing.
- decreasing.
- constant.
- $\boxed{\text{unable to be determined with the given information.}}$

Problem 2 (25 Points)

A person rides in an elevator above the surface of the Earth. The elevator descends a distance h towards the Earth at a constant speed.

All

(a 5pts) The work done on the person by the Earth is (circle one) $\boxed{\textbf{Positive}}$ **Negative** **Zero**

Insufficient Information to Answer

$$W = \vec{F} \cdot \Delta \vec{r} = (-mg)(-h) \Rightarrow W > 0$$

All

(b 5pts) The change in gravitational potential energy of the person+Earth system is (circle one) **Positive**

$\boxed{\textbf{Negative}}$ **Zero** **Insufficient Information to Answer**

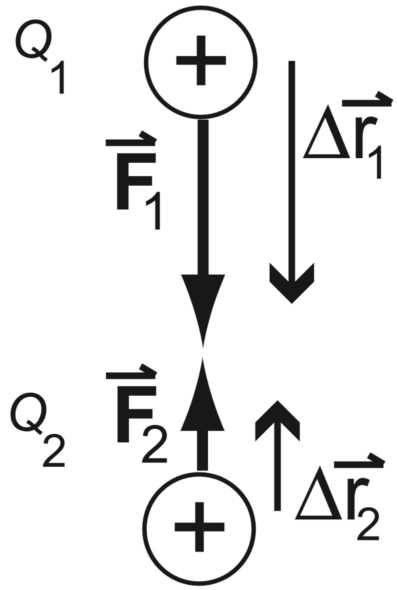
$$\Delta U = mg\,\Delta h = mg(h_f - h_i) \Rightarrow h_f < h_i \Rightarrow \Delta U < 0$$

All

(c 5pts) The elevator cable breaks and the person+elevator fall an additional distance h. During this second phase, the work done on the person by the Earth is compared to that done in part (a). The work done now is (circle one) **More Than** **Less Than** $\boxed{\textbf{The Same As}}$ **Insufficient Information to Answer**

$$W = (-mg)(-h) = \text{same as in Part (a)}$$

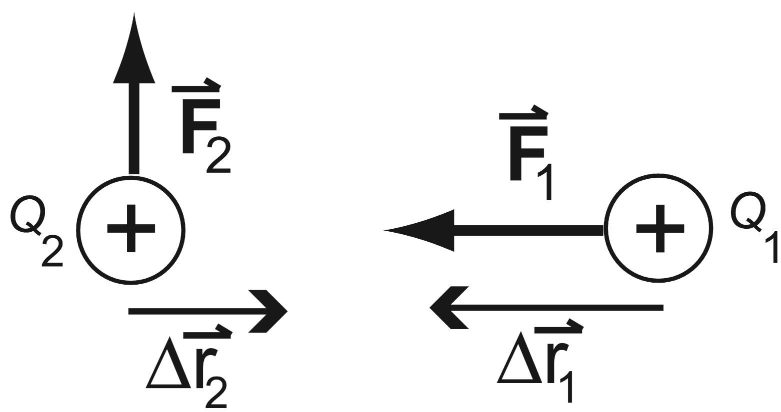
Note: $F = mg$ close to the surface of the Earth. If the elevator moved at much greater heights, then you need to consider $F = \frac{GMm}{r^2}$, so the force is larger at smaller heights, and in that case, the correct answer here would be "More than".



All (d 5pts) You push down on a positive charge Q_1 and your friend pushes up on a positive charge Q_2. Charge Q_1 moves down through a distance Δr_1 and Q_2 moves up through a distance Δr_2, as shown in the diagram. For the system of the two charges, the net work done by the external forces F_1 and F_2 is (circle one):

Positive **Negative** **Zero** **Insufficient Information to Answer**

$\vec{F}_1 \cdot \Delta\vec{r}_1 > 0$ and $\vec{F}_2 \cdot \Delta\vec{r}_2 > 0$, so $W_{net} > 0$



All (e 5pts) You push to the left on a positive charge Q_1 and your friend pushes up on a positive charge Q_2. Charge Q_1 moves through a distance Δr_1 to the left and Q_2 moves through a distance Δr_2 to the right, as shown in the diagram. For the system of the two charges, the net work done by the external forces F_1 and F_2 is (circle one):

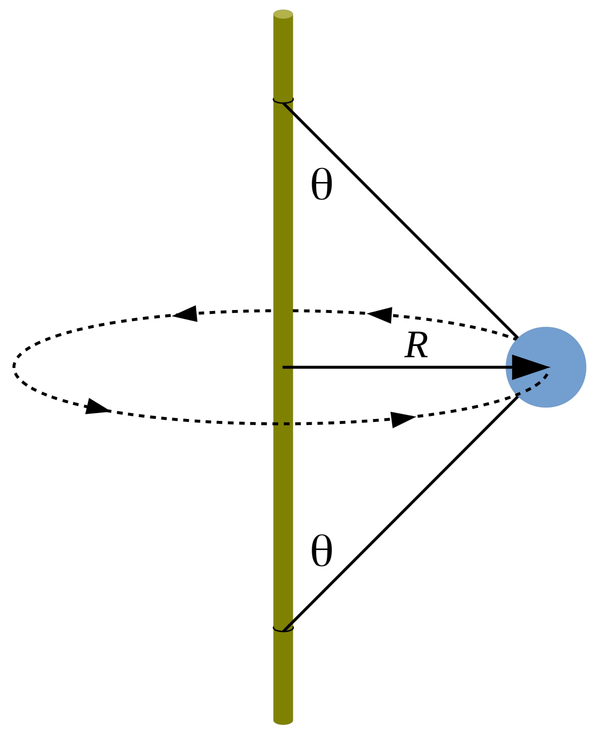
Positive **Negative** **Zero** **Insufficient Information to Answer**

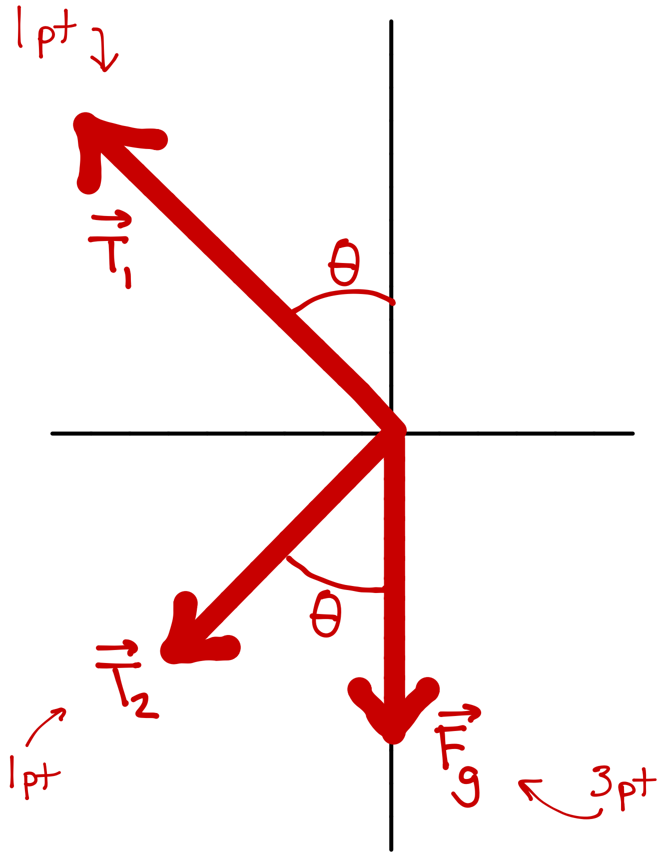
$\vec{F}_1 \cdot \Delta\vec{r}_1 > 0$ and $\vec{F}_2 \cdot \Delta\vec{r}_2 = 0$, so $W_{net} > 0$

Problem 3 (25 Points)

Consider the child's toy shown in the diagram. This toy is made by attaching a ball of mass m to a wooden stick by two identical strings. The stick is then rotated between the palms of your hand so that the ball travels around a circle of radius R at constant velocity v. In the diagram, gravity points down (i.e. parallel to the stick)

(a 5pts) In the space below draw a free body diagram for the ball.





(b 20pts) Determine the tension in each string if both strings make an angle of θ with the stick.

✓ Ball moves in a circle at constant speed, so $\vec{F}_{net//} = 0$

✓ Circular motion, so $|\vec{F}_{net\perp}| = \frac{mv^2}{R}$, pointing to the left

✓ Trajectory is parallel to the ground (no motion in y-direction)

$$\Rightarrow \left(\frac{d\vec{p}}{dt}\right)_{vertical} \Rightarrow T_1 \cos\theta - T_2 \cos\theta - mg = 0 \quad [Eq. 1]$$

$$\Rightarrow \left(\frac{d\vec{p}}{dt}\right)_{\perp} \Rightarrow T_1 \sin\theta\, \hat{n} + T_2 \sin\theta\, \hat{n} = \frac{mv^2}{R}\hat{n} \quad [Eq. 2]$$

two equations and two unknowns

✓ Solve Eq 1 for T_1

$$T_1 \cos\theta - T_2 \cos\theta - mg = 0$$

$$(T_1 - T_2)\cos\theta = mg$$

$$T_1 - T_2 = \frac{mg}{\cos\theta}$$

$$T_1 = T_2 + \frac{mg}{\cos\theta} \quad [Eq. 3]$$

✓ Plug Eq. 3 into Eq 2

$$T_1 \sin\theta + T_2 \sin\theta = \frac{mv^2}{R}$$

$$(T_1 + T_2)\sin\theta = \frac{mv^2}{R}$$

$$\left(T_2 + \frac{mg}{\cos\theta} + T_2\right)\sin\theta = \frac{mv^2}{R}$$

$$2T_2 + \frac{mg}{\cos\theta} = \frac{mv^2}{R\sin\theta}$$

$$2T_2 = \frac{mv^2}{R\sin\theta} - \frac{mg}{\cos\theta}$$

$$\boxed{T_2 = \frac{mv^2}{2R\sin\theta} - \frac{mg}{2\cos\theta}} \quad [Eq. 4]$$

✓ Use Eq. 4 in Eq. 3

$$T_1 = T_2 + \frac{mg}{\cos\theta} =$$

$$= \frac{mv^2}{2R\sin\theta} - \frac{mg}{2\cos\theta} + \frac{mg}{\cos\theta} =$$

$$\boxed{T_1 = \frac{mv^2}{2R\sin\theta} + \frac{mg}{2\cos\theta}}$$

[Eq. 5]

* The tensions in the two strings are given by Eq. 4 and Eq. 5.

−1 clerical
−3 minor
−6 major
−16 BTN

Problem 4 (25 Points)

A rock of mass m is released from rest very far from the Earth (i.e. $r = \infty$). The only force acting on the rock is the gravitational force of the Earth. In the following questions you can assume the mass of the rock is much less than the mass of the Earth and neglect air resistance.

(a 5pts) Determine the velocity of the rock the instant it reaches the surface of the Earth. Your answer should not be numeric.

3pt → $\Delta E = \Delta K + \Delta U = 0$

$K_f - \cancel{K_i}_0 + U_f - \cancel{U_i}_0 = 0$

$K_f + U_f = 0$

$\frac{1}{2}mv^2 + \frac{-GMm}{r_f} = 0$

$\frac{1}{2}\cancel{m}v^2 = \frac{GM\cancel{m}}{1}$

$\frac{1}{2}v^2 = \frac{GM}{R}$

$v^2 = \frac{2GM}{R}$

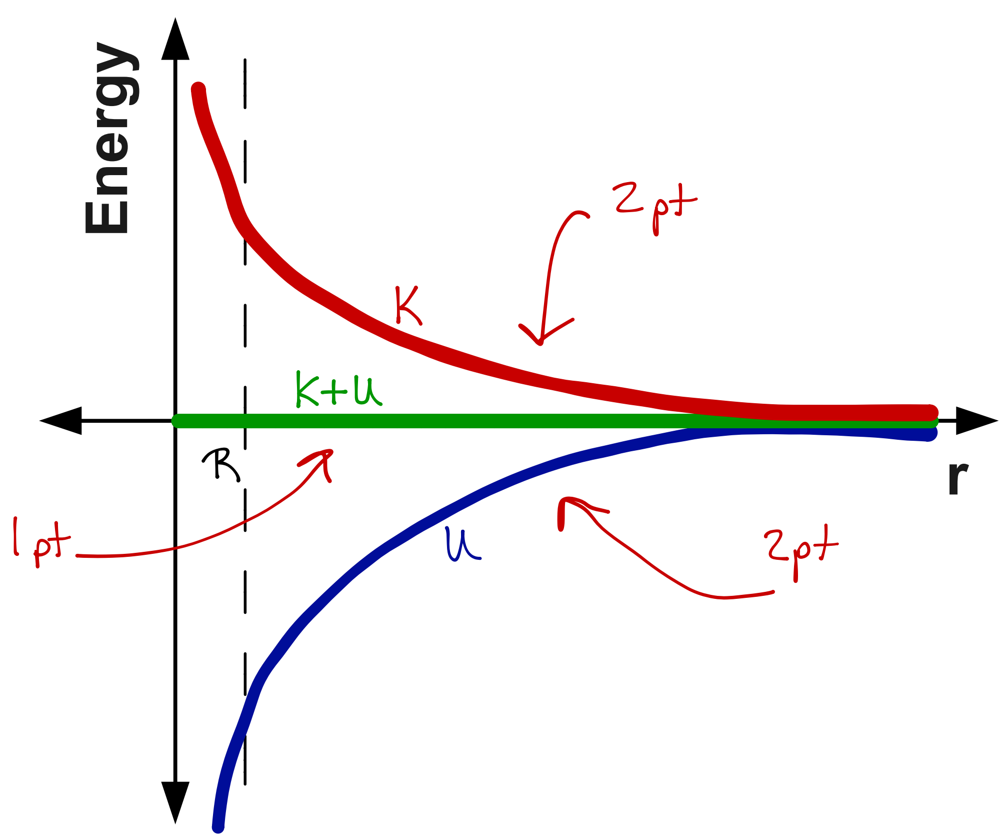
2pt

$$\boxed{v = \sqrt{\frac{2GM}{R}}}$$

direction: towards center of Earth

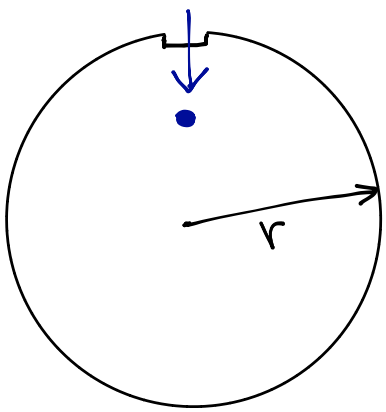
where: M = mass of Earth
R = radius of Earth

(b 5pts) For the case considered above, sketch the: Kinetic, Potential, and Total energy for the Earth+Rock system.



A rock with mass m is released from rest at the Earth's surface and drops through a hole all the way to the center of the Earth. The magnitude of the gravitational force on the rock is mgr/R, where r is the rock's distance from the center of the Earth and R is the Earth's radius. This force points towards the center of the Earth and is the only force acting on the rock. In the following questions you can assume the mass of the rock is much less than the mass of the Earth and neglect air resistance.

(c 10pts) Determine the velocity of the rock the instant it reaches the center of the Earth. Your answer should not be numeric.



✓ System: only the rock

✓ Initial: rock @ surface $(r = R)$

✓ Final: rock @ center $(r = 0)$

$$✓\ W = \int \vec{F} \cdot d\vec{r} = \int_R^0 \frac{-mgr}{R}\, dr = \frac{-mg}{R} \int_R^0 r\, dr =$$

$$= \frac{-mg}{R}\left[\frac{r^2}{2}\right]_R^0 = \frac{+mg}{\cancel{R}}\left(\frac{+R^{\cancel{2}}}{2}\right) = \frac{mgR}{2}$$

$$✓\ \Delta E = \Delta K = W$$

$$K_f - \cancel{K_i}^{\,0} = W$$

$$\frac{1}{\cancel{2}}\cancel{m}v^2 = \frac{\cancel{m}gR}{\cancel{2}}$$

$$v^2 = gR$$

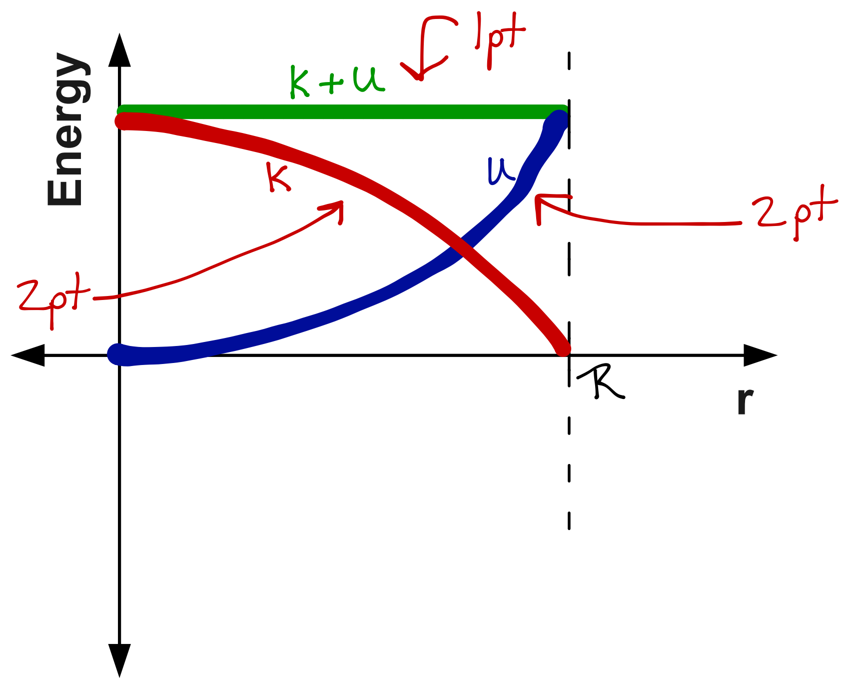
$$\boxed{v = \sqrt{gR}}$$

→ direction:

away from the center of the Earth

−0.5
−1.5
−3.0
−8.0

(d 5pts) For the case of the rock falling through the Earth, the potential energy of the Earth+Rock system is given by $mgr^2/(2R)$. On the graph below, sketch the Kinetic, Potential and Total energy for the Earth+Rock system.



All (Extra credit 5pts) How does your answer to part (c) compare to your answer in part (a)? Hint: the ratio of the two velocities should not depend on any of the parameters in the problem.

✓ What is g? $\Rightarrow \cancel{m}g = \frac{GM\cancel{m}}{R^2} \Rightarrow g = \frac{GM}{R^2}$

✓ Part A: $V_A = \sqrt{\frac{2GM}{R}}$

✓ Part C: $V_c = \sqrt{gR} = \sqrt{\frac{GM}{R^{\cancel{2}}}\cancel{R}} = \sqrt{\frac{GM}{R}}$

$\Rightarrow$ Ratio: $\frac{V_c}{V_A} = \frac{\sqrt{\cancel{GM}/\cancel{R}}}{\sqrt{2\cancel{GM}/\cancel{R}}} = \boxed{\frac{1}{\sqrt{2}}}$ or $\frac{V_A}{V_c} = \sqrt{2}$

* The speed at crashing (Part A) is $\sqrt{2}$ times larger than the speed at the center of Earth (Part C).

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}} \qquad E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt} \qquad \vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r} \qquad U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface} \qquad \Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r} \qquad U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s \qquad U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M \qquad \Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots}{m_1 + m_2 + \dots} \qquad I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \dots$$

$$K_{tot} = K_{trans} + K_{rel} \qquad K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I} \qquad K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot} \qquad \vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}} \qquad v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)} \qquad Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!} \qquad S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E} \qquad \Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

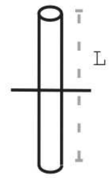
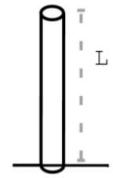
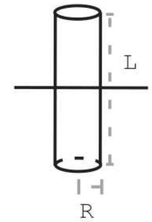
$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}} \qquad E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\dots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\dots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$

Moment of intertia for rotation about indicated axis

The cross product

$$\vec{A} \times \vec{B} = \langle A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x \rangle$$

				
$I = \frac{2}{5}MR^2$	$I = \frac{1}{2}MR^2$	$I = \frac{1}{12}ML^2$	$I = \frac{1}{3}ML^2$	$I = \frac{1}{12}ML^2 + \frac{1}{4}MR^2$

Constant	**Symbol**	**Approximate Value**
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} atoms/mol
Plank's constant	h	6.6×10^{-34} joule · second
hbar $= \frac{h}{2\pi}$	$\hbar$	1.05×10^{-34} joule · second
specific heat capacity of water	C	4.2 J/g/K
Boltzmann constant	k	1.38×10^{-23} J/K

milli	m	1×10^{-3}	kilo	K	1×10^{3}
micro	μ	1×10^{-6}	mega	M	1×10^{6}
nano	n	1×10^{-9}	giga	G	1×10^{9}
pico	p	1×10^{-12}	tera	T	1×10^{12}