Quiz 7 (25 mins), MA1501 Calculus I, 10/17/2013a PRINTED NAME: ________________ GT ID:____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

Answers must appear in the box provided! No cheat!

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1. (a) (20 pts) State the Extreme Value Theorem. Be sure to include all hypotheses and conclusions.
(b) (30 pts) Find the critical points, and extreme values (global and local) for the following function.

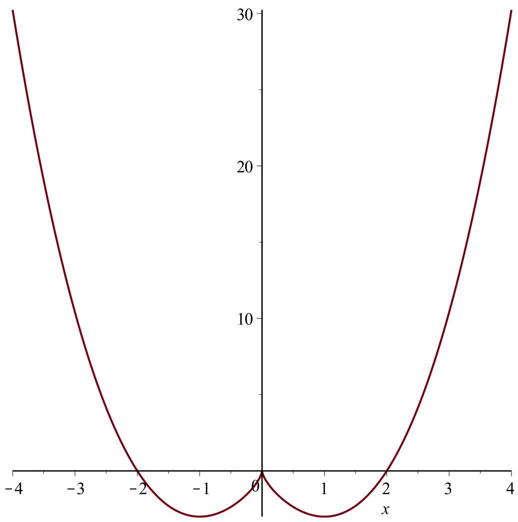
$$f(x) = x^{2/3}(x^2 - 4)$$

Answer: (a) If f is continuous on $[a, b]$, then f attains both a global(absolute) maximum and a global(absolute) minimum in $[a, b]$.

(b) $f'(x) = x^{2/3}(2x) + \frac{2}{3}x^{-1/3}(x^2 - 4) = \frac{8x^2-8}{3x^{1/3}} = \frac{8(x^2-1)}{3x^{1/3}}$
By solving $\frac{8(x^2-1)}{3x^{1/3}} = 0$, we have $f'(x) = 0$ at $x = -1$ and $x = 1$.
Moreover, $f'(x)$ is undefined at $x = 0$. Therefore, there are three critical points $x = -1$, $x = 0$ and $x = 1$.

critical point	value of derivative	extremum type	value of f
$x = -1$	$f'(-1) = 0$	local and global minimum	$f(-1) = -3$
$x = 0$	undefined	local maximum	$f(0) = 0$
$x = 1$	$f'(1) = 0$	local and global minimum	$f(1) = -3$

Since the domain of f is the whole real line, there f does not attain the global maximum.
As a remark, the graph of f is given below:



2. (a) (20 pts) The function

$$f(x) = \begin{cases} 1 + x^3, & 0 \le x < 1, \\ 1, & x = 1 \end{cases}$$

is 1 at $x = 0$ and $x = 1$ and differentiable on $(0, 1)$, but its derivative on $(0, 1)$ is never zero. Doesn't Rolle's Theorem says the derivative have to be zero somewhere in $(0, 1)$? Give reasons for your answer.

(b) (30 pts) Find the function with the given derivative whose graph passes through the point P.

$$f'(x) = e^{2x}, \qquad P(0, \frac{3}{2})$$

Answer: (a) Since $f(x)$ is not continuous on $0 \le x \le 1$, Rolle's Theorem does not apply.
(b) $f(x) = \frac{e^{2x}}{2} + C$.
$\frac{3}{2} = f(0) = \frac{1}{2} + C$ and $C = 1$.
Therefore $f(x) = 1 + \frac{e^{2x}}{2}$