$\begin{array}{c} \text{ISyE 4232 Exam } \# \ 1 \\ \text{Spring 2013} \end{array}$

Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK.

Question 1

Question 2

Question 3

 \mathbf{Total}

(35) 1. A company reviews the state of one of its important products on an annual basis and decides whether it is successful (state 1) or unsuccessful (state 2). The company must then decide whether or not to advertise the product to further promote the sales. The matrices P_1 and P_2 given below provide the transition probabilities with and without advertisement during any year. The associated returns are given by the matrices R_1 and R_2 . Find the optimal decisions over the next two years assuming that the value associated with both states at the beginning of the third year is 0.

$$P_1 = \begin{bmatrix} 0.9 & 0.1 \\ 0.6 & 0.4 \end{bmatrix} P_2 = \begin{bmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{bmatrix} R_1 = \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix} R_2 = \begin{bmatrix} 4 & 1 \\ 2 & -1 \end{bmatrix}$$

(30) **2.** Consider a production system consisting of a machine center followed by an inspection station. Arrivals from outside the system occur only at the machine center and follow a Poisson process with rate λ . The machine center and inspection station are each singe-server operations with rates μ_1 and μ_2 . Suppose that each item independently passes inspection with probability p. When an object fails inspection it is sent to the machine center for reworking. Find the conditions on the parameters that are necessary for the system to have a stationary distribution. Provide the stationary distribution when it exists. (**Hint:** Recall that for an M/M/1 queue with $\lambda < \mu$, $\pi_n = (1 - \rho)\rho^n$ where $\rho = \frac{\lambda}{\mu}$).

(35) 3. The exercise room at the Wonderland motel has three pieces of equipment. Four businessmen who are trapped there because of a snowstorm use machines 1.2,3 for an exponential amount of time with means 15, 10, 5 minutes. When a person is done with one piece of equipment, he picks one of the other two with equal probability. If that equipment is occupied, he stands in line to use it. What is the long-run probability that all three equipments are occupied?