MATH 2603, Fall 2015, Quiz 7, Oct 28 2015: Closed book, no calculators.
Instructor: Esther Ezra.

You can answer all questions on this sheet, but may use extra sheets (from your personal notepad) if needed.

Solution

Name GT IDnumber

Problem 1. (50 points)

Let U be a set of 52 elements, with five subsets A_1, A_2, A_3, A_4, A_5 having the following properties: (i) $|A_i| = 23$, for each $i = 1, \ldots, 5$, (ii) the intersection of any two subsets contains 10 elements, (iii) the intersection of any three of the subsets contains four elements, (iv) the intersection of any four of the subsets contains one element, (v) the intersection of all five subsets is empty.

How many elements belong to none of the five subsets?

By Principle of Inclusion - Exclusion

elements belonging to none of the five subsets:

$$|U| - \left|\bigcup_{i=1}^{5} A_i\right| = 52 - \left[5 \times 23 - \binom{5}{2} \times 10 + \binom{5}{3} \times 4 - \binom{5}{4} + 0\right]$$

Problem 2. (50 points)

A coin is tossed 10 times and the sequence of head and tails is recorded. How many different sequences are possible? In how many of these sequences there are exactly 3 heads?

different sequences: 2^{10} (10 times tossing, 2 possibilities per toss, repetition allowed)

sequences of exactly 3 heads: $\binom{10}{3} = \binom{10}{7}$

(# of heads & tails already decided so just need to choose the spots for the heads)