PHYS 2212 Test 2
Spring 2014

Name(print) KEY **Lab Section** ____________

Lab section by day and time: Curtis(H), Ballantyne(Q), Kim(P)						
Monday	12:05-2:55pm	H01 or Q01	3:05-5:55pm	H02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	Q03 or P03	3:05-5:55pm	Q04 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	H03 or Q05	3:05-5:55pm	P05 or Q06	6:05-8:55pm	H04 or P06
Thursday	12:05-2:55pm	H05 or Q07	3:05-5:55pm	Q08 or H06	6:05-8:55pm	H07 or P07

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^{4}$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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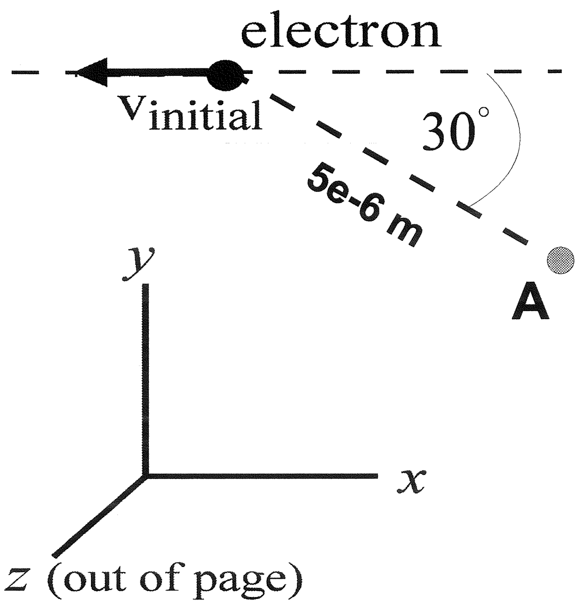
Sign your name on the line above

PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		BATMAN
Problem 2 (25 pts)		SUPERMAN
Problem 3 (25 pts)		WONDERWOMAN
Problem 4 (25 pts)		GREEN LANTERN

Problem 1 (25 Points)



(a 5pts) A electron is traveling with speed 3.2×10^5 m/s in the -x direction. What are the magnitude and the direction of the magnetic field due to this particle at location A? As shown in the diagram, the x axis runs to the right, the y axis runs up, and the z axis comes toward you out of the page.

$$\vec{v} = \langle -3.2 \times 10^5, 0, 0 \rangle \ \tfrac{m}{s}$$

$$q = -1.6 \times 10^{-19} \ C$$

1 $\vec{r} = \langle 5 \times 10^{-6} \cos 30, -5 \times 10^{-6} \sin 30, 0 \rangle$ m

$$|\vec{r}| = 5 \times 10^{-6} \ m$$

1 $$\vec{B} = \frac{\mu_0}{4\pi} \frac{q \vec{v} \times \hat{r}}{|\vec{r}|^2}$$

$$\hat{r} = \langle \cos\theta, -\sin\theta, 0 \rangle$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q}{|\vec{r}|^2} \langle V_x, 0, 0 \rangle \times \langle \hat{r}_x, \hat{r}_y, 0 \rangle$$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q}{|\vec{r}|^2} (+3.2 \times 10^5 \sin 30) \langle 0, 0, 1 \rangle$$

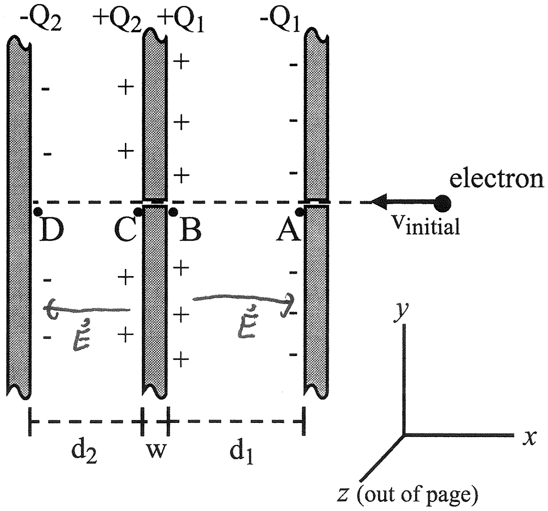
1pt

$$\vec{B} = +1.02 \times 10^{-10} \langle 0, 0, -1 \rangle \ T$$

Into the page

2pts

Three charged metal plates are arranged as shown in the diagram (with nothing in the gaps between the plates, only vacuum). A small hole in the middle of the plates allows an electron to pass through. Each plate is held apart by insulating supports (not shown), and each has an area of 1.8 m^2. Each plate is $w = 0.02$ mm thick. Distance $d_1 = 13$ mm, and distance $d_2 = 7$ mm. Charge $Q_1 = 2\times10^{-7}$ C, and charge $Q_2 = 4\times10^{-8}$ C. The charges are uniformly distributed over the surfaces of the plates as shown.



-1
-2
-4.5
-12

(b 15pts) Calculate the potential difference $V_D - V_A$. Show all work explicitly. Make sure you account for all parts of the path you choose.

$d_1 = 13\times10^{-2}$ m
$d_2 = 7\times10^{-3}$ m
$w = 2\times10^{-5}$ m
$R = \sqrt{\frac{A}{\pi}} = 0.76$ m
$R \gg d_2, w, d_1$

$$V_D - V_A = (V_D - V_C) + (V_C - V_B) + (V_B - V_A)$$

$$(V_D - V_C) = -\int_C^D \vec{E}_{DC} \cdot d\vec{l} \qquad \vec{E}_{DC} = \frac{Q_2/A}{\varepsilon_0}\langle -1,0,0\rangle \text{ since } R \gg d_2$$

$$d\vec{l} = dl\,\langle 1,0,0\rangle$$

$$(V_D - V_C) = \frac{Q_2/A}{\varepsilon_0}\, l\,\Big|_0^{-d_2} = -\frac{Q_2/A}{\varepsilon_0} d_2 = -17.6\ \frac{Nm}{C}$$

$$(V_C - V_B) = -\int_B^C \vec{E}_{CB}\cdot d\vec{l} \qquad \vec{E}_{CB} = -\frac{Q_1/A}{2\varepsilon_0}\langle 1,0,0\rangle + \frac{Q_2/A}{2\varepsilon_0}\langle 1,0,0\rangle + \frac{Q_1/A}{2\varepsilon_0}\langle -1,0,0\rangle - \frac{Q_2/A}{2\varepsilon_0}\langle 1,0,0\rangle$$

since $w \ll R$

$$(V_C - V_B) = 0 \qquad \vec{E}_{CB} = 0$$

$$(V_B - V_A) = -\int_A^B \vec{E}_{BA}\cdot d\vec{l} \qquad \vec{E}_{BA} = \frac{Q_1/A}{\varepsilon_0}\langle 1,0,0\rangle \quad \text{since } R \gg d_1, w, d_2$$

$$(V_B - V_A) = -\frac{Q_1/A}{\varepsilon_0}\, l\,\Big|_0^{-d_1} = \frac{Q_1/A}{\varepsilon_0} d_1 = 163.2\ \frac{Nm}{C}$$

$$V_D - V_A = 163.2 + 0 - 17.6 = 145.6\ \frac{Nm}{C}$$

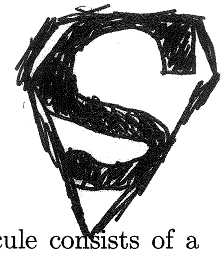
(c 5pts) Determine the change in kinetic energy for the electron as it moves from A to D.

$q = -1.6\times10^{-19}$

$$\Delta K + \Delta U = 0 \quad \Delta K = -q\,(V_D - V_A)$$

$$\underbrace{\Delta K + q\Delta V = 0}_{3pts} \quad \Delta K = +2.33\times10^{-17}\ Nm$$

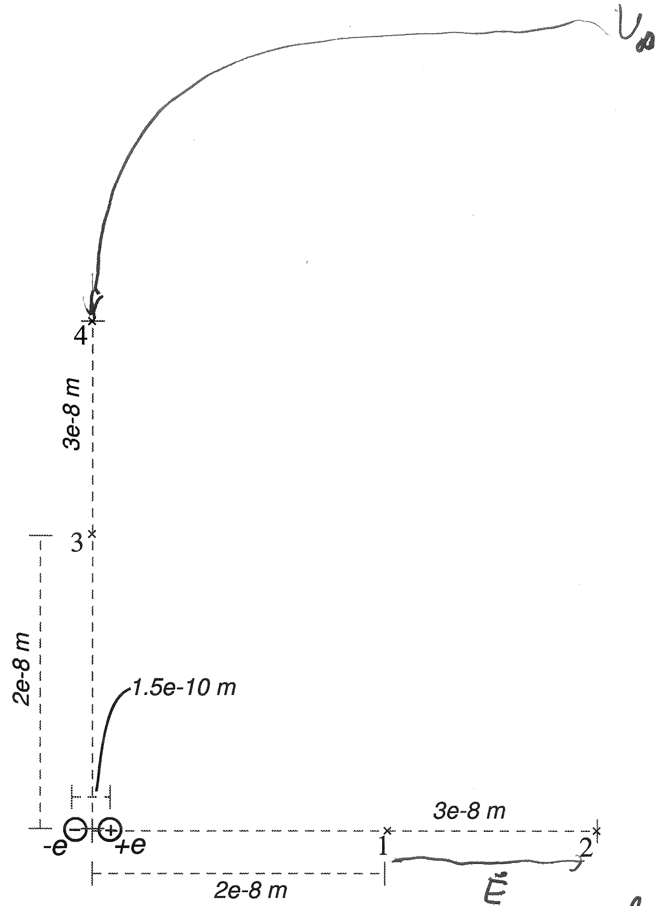
2pts



Problem 2 (25 Points)

A particular diatomic molecule consists of a positive ion with charge $+e$ and a negative ion with charge $-e$, separated by a distance of 1.5×10^{-10} m, as shown in the diagram. Locations 1, 2, 3, and 4 are shown in the diagram.

(a 15pts) Location 1 is 2×10^{-8} m from the center of the molecule, and location 2 is 3×10^{-8} from the center of the of the molecule. Calculate the potential difference $V_2 - V_1$. Show all steps in your work.



$l \gg s$

$$(V_2 - V_1) = -\int_{P_1}^{P_2} \vec{E}_{21} \cdot d\vec{l}_{21}$$

$$\vec{E}_{21} = \frac{1}{4\pi\epsilon_0} \frac{2qs}{l^3} \langle 1, 0, 0 \rangle$$

$$d\vec{l}_{21} = dl \langle 1, 0, 0 \rangle$$

$P_1 = 2 \times 10^{-8}$ m

$P_2 = 5 \times 10^{-8}$ m

$$(V_2 - V_1) = -\frac{1}{4\pi\epsilon_0} 2qs \int_{P_1}^{P_2} \frac{dl}{l^3}$$

$$(V_2 - V_1) = -\frac{1}{4\pi\epsilon_0} 2qs \left(\frac{-2}{l^2} \Big|_{P_1}^{P_2} \right) = +\frac{1}{4\pi\epsilon_0} 4qs \left(\frac{1}{P_2^2} - \frac{1}{P_1^2} \right)$$

$$(V_2 - V_1) = -1.81 \times 10^{-3} \frac{Nm}{C}$$

$$\boxed{\begin{array}{l} -1 \\ -2 \\ -4.5 \\ -12 \end{array}}$$

(b 5pts) Location 3 is 2×10^{-8} m from the center of the molecule, and location 4 is 3×10^{-8} from the center of the of the molecule. Calculate the potential difference $V_4 - V_3$. Show all steps in your work.

$$(V_4 - V_3) = -\int_{P_3}^{P_4} \vec{E}_{43} \cdot d\vec{l}_{43}$$

$$\vec{E}_{43} = \frac{1}{4\pi\epsilon_0} \frac{qs}{l^3} \langle 1,0,0 \rangle$$

$$d\vec{l}_{43} = dl \langle 0,1,0 \rangle$$

$$\langle 1,0,0 \rangle \cdot \langle 0,1,0 \rangle = 0$$

3pts

$$V_4 - V_3 = 0$$

2pts

(c 5pts) Calculate the potential difference $V_4 - V_\infty$. Show all steps in your work.

3

$V_\infty = 0$

Path indepence so can choose a path such that $(V_{\infty,y} - V_\infty) = 0$

1

$$\text{So } (V_4 - V_\infty) = (V_4 - V_{\infty,y}) + \cancel{(V_{\infty,y} - V_\infty)}^{0}$$

$$(V_4 - V_{\infty y}) = -\int_{P_{\infty,y}}^{P_4} \vec{E} \cdot d\vec{l}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qs}{l^3} \langle 1,0,0 \rangle$$

$$d\vec{l} = dl \langle 0,1,0 \rangle$$

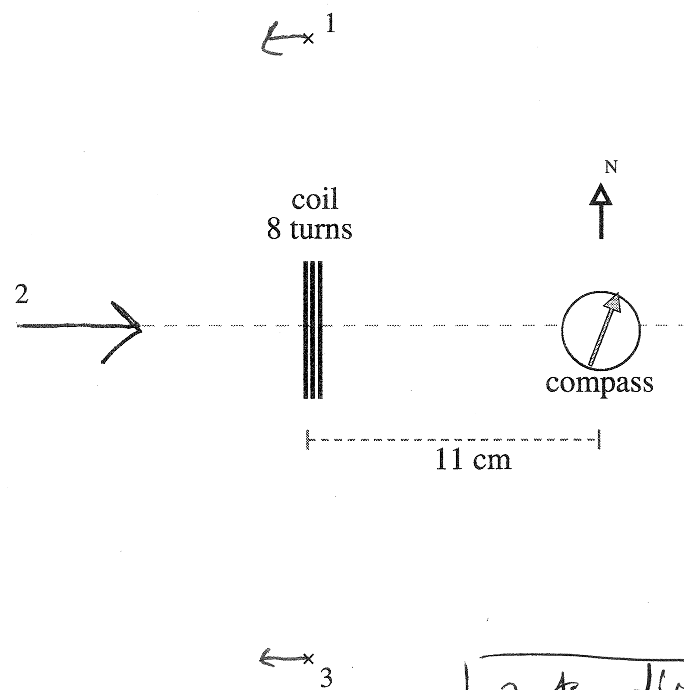
1pt

$$\langle 1,0,0 \rangle \cdot \langle 0,1,0 \rangle = 0$$

$$(V_4 - V_{\infty y}) = 0$$

Problem 3 (25 Points)

A compass, which originally points North, is placed 11 cm from a coil of wire, which is composed of 8 turns of wire, of radius 2 cm, as shown in the diagram. When the coil is connected to a battery (which is not shown in the diagram), the compass deflects 18 degrees to the East

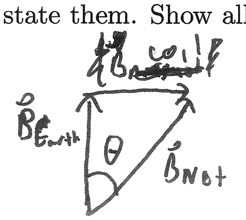


$\boxed{\text{2pts direction, 1pt mag}}$

(a 10pts) At each of the three locations marked 1, 2, and 3, draw and arrow representing the direction of the magnetic field due to the coil at that location. Draw the arrows to the same scale, in the sense that a longer arrow represents a larger magnitude of magnetic field.

(b 15pts) What is the magnitude of the conventional current running through the coil? If you make any approximations, state them. Show all of your work.

Compass



$|\vec{B}_{coil}| = |\vec{B}_{Earth}| \tan\theta$

$\theta = 18^\circ$

$|\vec{B}_{Earth}| \approx 2 \times 10^{-5}\,T$

$$|\vec{B}_{coil}| \approx \frac{\mu_0}{4\pi} \frac{2NI\pi R^2}{z^3}$$

assume ~~z>>R~~ $z \gg R$

$N = 8$

$R = 0.02\,m$

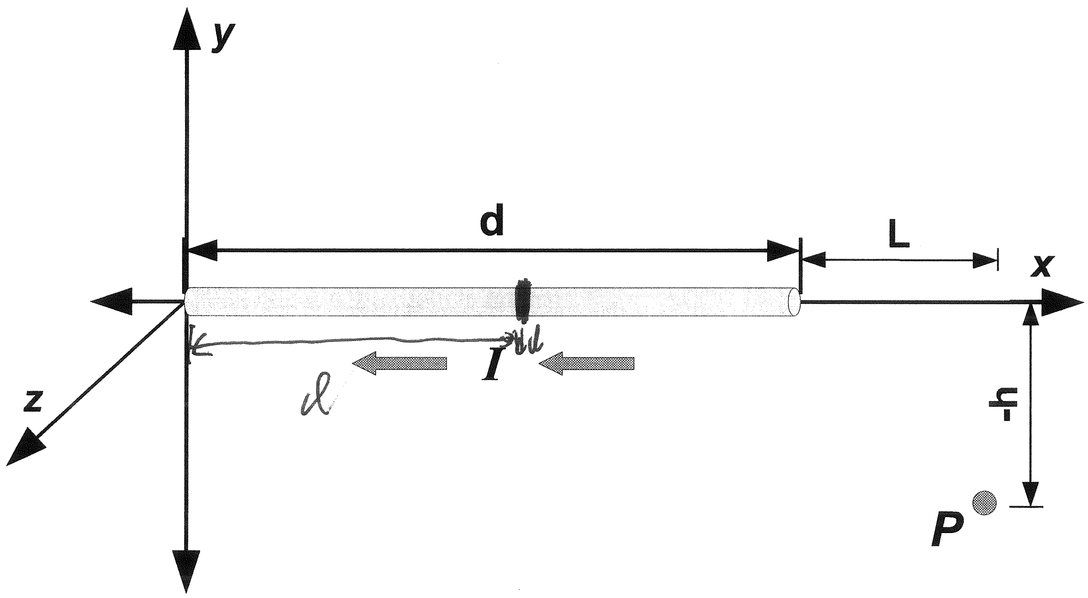
$z = 0.11\,m$

$$I = \frac{|\vec{B}_{Earth}| \tan\theta \, z^3}{(\frac{\mu_0}{4\pi}) 2N\pi R^2} = 4.3\ A$$

$\boxed{\begin{array}{l} -1 \\ -2 \\ -4.5 \\ -8 \end{array}}$

Problem 4 (25 Points)

A thin wire is part of a complete electrical circuit which carries conventional current I. Consider only the piece of wire of length d as shown in the diagram. In this piece of the wire the current is flowing in the -x direction. Observation location P is in the xy plane at $\langle L + d, -h, 0\rangle$.



(a 5pts) Determine the direction of the magnetic field at location P due to the segment of wire. Briefly explain how you know this.

1 $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{|\vec{r}|^2}$

2 $\vec{r} = \langle L+d-l, -h, 0\rangle$

$d\vec{l} = dl\,\langle -1, 0, 0\rangle$

Current in $-\hat{x}$ direction

$\hat{r} = \frac{\langle L+d-l, -h, 0\rangle}{\sqrt{(L+d-l)^2 + h^2}}$

direction $d\vec{B}$

determined by

$\langle -1, 0, 0\rangle \times \hat{r} = \left\langle 0, 0, \frac{+h}{\sqrt{(L+d-l)^2 + h^2}} \right\rangle$ 2

unit vector $\Rightarrow \langle 0, 0, +1\rangle$

Or out of the page ✓

Also, right hand rule ✓

(b 15pts) Set up the integral for the vector magnetic field contributed by this piece of wire at the location P.

5 $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, h\, d\ell}{((L+d-\ell)^2+h^2)^{3/2}} \langle 0,0,1 \rangle$

$\vec{B} = \int_0^{l} \frac{\mu_0}{4\pi} \frac{I\, h\, d\ell}{((L+d-\ell)^2+h^2)^{3/2}} \langle 0,0,1 \rangle$

5 limits

5 integration variable & vector

(c 5pts) Determine the magnitude of the magnetic field at observation location at $\langle L+d, 0, 0\rangle$. Be sure to show your work for full credit.

$h=0$

2 $|\vec{B}| = \int_0^{l} \frac{\mu_0}{4\pi} \frac{I\,(0)\, d\ell}{((L+d-\ell)^2+(0)^2)^{3/2}} \cancel{\langle 0,0,1 \rangle} = 0$

3 $I\, d\vec{\ell} \times \hat{r} = \langle -I d\ell, 0, 0 \rangle \times \langle 1,0,0 \rangle = 0$

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge

Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \frac{d\vec{v}}{dt}$

$\Delta U_{el} = q\Delta V$

$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$

$\oint \vec{E} \bullet \hat{n} dA = \frac{\sum q_{inside}}{\epsilon_0}$

$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left|\frac{d\Phi_{mag}}{dt}\right|$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \frac{d}{dt} \int \vec{E} \bullet \hat{n} dA\right]$

$\frac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\frac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x \Delta x + E_y \Delta y + E_z \Delta z)$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$

$\oint \vec{B} \bullet \hat{n} dA = 0$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2 + (L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$, $\vec{p} = \alpha \vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2 + R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$ (z along axis)

$\left|\vec{E}_{disk}\right| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \frac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \frac{Q/A}{\epsilon_0}\left(\frac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{\ell} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$

$\left|\vec{B}_{loop}\right| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$) $\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \frac{1}{4\pi\epsilon_0}\frac{-q\vec{a}_\perp}{c^2 r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \frac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\, nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\, nu$

$J = \frac{I}{A} = \sigma E$

$R = \frac{L}{\sigma A}$

$E_{dielectric} = \frac{E_{applied}}{K}$

$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$ due to a point charge

$I = \dfrac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$ K $\approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\dfrac{d\vec{p}}{dt}_{\perp}\right| = \dfrac{|\vec{v}|}{R}|\vec{p}| \approx \dfrac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z\rangle \times \langle b_x, b_y, b_z\rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\,dx = ax + c \quad \int ax\,dx = \frac{a}{2}x^2 + c \quad \int ax^2\,dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\dfrac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N · m^2/C^2)$^{-1}$
Magnetic constant	$\dfrac{\mu_0}{4\pi}$	1×10^{-7} T · m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T · m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T