ISyE 2027C Probability with Applications

Homework 2 Hints **January 13, 2015**

be ready for a quiz January 20

An example: You are diving for treasure in a lake. The probability that the lake has treasure is 0.4. Each time you dive, you have a .1 chance of finding treasure if treasure is there, independent of previous dives. You dive twice but find no treasure. What is the probability that the lake has treasure? Answer: let T be the event that there is treasure in the lake. Let F be the event that you find treasure in two dives. We seek $P(T|F^C)$. $P(F|T^C) = 0$ and $P(T) = 0.4$. $P(F^C|T) = 0.9^2 = 0.81$, hence $P(F|T) = 1 - .81 = .19$. Next we need $P(F)$. From the law of total probabilty,

$$P(F) = P(T)P(F|T) + P(T^C)P(F|T^C) = .4 \cdot .19 + 0 = .076 = 1 - P(F^C)$$

Then $P(T|F^C) = P(F^C|T)P(T)/P(F^C) = .81 \cdot .4/.924$. Another way to look at it: there are two ways you could fail to find treasure in two dives. The first way is because there is no treasure, with probabilty .6. The second is that there is treasure but both dives fail, with probability $.4 \cdot .9^2$. The conditional probablity that there is treasure is the probability of the 2nd way divided by the sum of the probabilities, $.4 \cdot .9^2/(.6 + .4 \cdot .9^2)$.

Dekking *et al.*: Chapter 3 Section 6 Problems

3.1 Let T be the event of reaching B in two selections. Let U_C, U_D, U_E be the events of reaching C, D and E respectively in one selection. Events U_C, U_D, U_E are disjoint and each has probability 1/3. Therefore they form a partition of Ω. Now use the law of total probability. The answer is $\frac{7}{36}$.

3.2 $P(A|B) = 2/11$.

(b) They are not independent.

3.3 $P(S_1) = 1/4$. $P(S_2|S_1) = 4/17$. $P(S_2|S_1^C) = 13/51$. (b) Of course the answer will be 1/4. The point of the problem is to illustrate the law of total probability.

3.5 $\frac{3}{8}$.

3.8 $P(F|W) = \frac{11}{13}$.

3.11 A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy $P(A|B) = P(A^C|B^C) = p$, where A is the event breath analyzer indicates that legal limit is exceeded and B drivers blood alcohol percentage exceeds legal limit. On Saturday night about 5% of the drivers are known to exceed the limit. a. Describe in words the meaning of $P(B^C|A)$.

The chance of a "false positive." (Explain exactly what this means.)

b. Determine $P(B^C|A)$ if $p = 0.95$.

$1 - \frac{.05p}{.95-.9p} = \frac{.95(1-p)}{.95-.9p}$. Plugging in $p = 0.95$ gives the numerical value 0.5.

c. How big should p be so that $P(B|A) = 0.9$? Use the formula from (b). $p = .9 \cdot .95/.86 \approx 0.994186$.

3.16 You are diagnosed with an uncommon disease. You know that there only is a 1% chance of getting it. Use the letter D for the event you have the disease and T for the test says so. It is known that the test is imperfect: $P(T|D) = 0.98$ and $P(T^C|D^C) = 0.95$. a. Given that you test positive, what is the probability that you really have the disease?

0.16526138.

b. You obtain a second opinion: an independent repetition of the test. You test positive again. Given this, what is the probability that you really have the disease?

0.7950989.