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Transport Processes I, ChBE 3200 Exam #3
Spring 2015

The exam consists of 2 problems worth the points indicated (for a total of 38 points). Please box your final answers in the space below each question in the specified units. Show all work and state any assumptions made to receive full credit. Unit conversions and physical property data are given below. You may use 3 personally made note sheets on this exam. Please sign the honor code.

Honor Code:
I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Signature

Useful Information

Densities: Air (25°C) 1.2 kg/m^3
Water (4°C) 1000 kg/m^3 or 62.4 lb$_m$/ft^3

Acceleration due to gravity: $g = 9.8$ m/s^2 or 32 ft/s^2
Conversion factor: $g_c = 32(\text{lb}_m\text{ft}/\text{lb}_f\text{s}^2)$

FACTORS FOR UNIT CONVERSIONS

Quantity	Equivalent Values
Mass	1 kg = 1000 g = 0.001 metric ton = 2.20462 lb$_m$ = 35.27392 oz 1 lb$_m$ = 16 oz = 5×10^{-4} ton = 453.593 g = 0.453593 kg
Length	1 m = 100 cm = 1000 mm = 10^6 microns (μm) = 10^{10} angstroms (Å) = 39.37 in. = 3.2808 ft = 1.0936 yd = 0.0006214 mile 1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm
Volume	1 m^3 = 1000 L = 10^6 cm^3 = 10^6 mL = 35.3145 ft^3 = 219.97 imperial gallons = 264.17 gal = 1056.68 qt 1 ft^3 = 1728 in.^{3} = 7.4805 gal = 0.028317 m^3 = 28.317 L = 28,317 cm^3
Force	1 N = 1 kg·m/s^2 = 10^5 dynes = 10^5 g·cm/s^2 = 0.22481 lb$_f$ 1 lb$_f$ = 32.174 lb$_m$·ft/s^2 = 4.4482 N = 4.4482×10^5 dynes
Pressure	1 atm = 1.01325×10^5 N/m^2 (Pa) = 101.325 kPa = 1.01325 bar = 1.01325×10^6 dynes/cm^2 = 760 mm Hg at 0°C (torr) = 10.333 m H_2O at 4°C = 14.696 lb$_f$/in.^{2} (psi) = 33.9 ft H_2O at 4°C = 29.921 in. Hg at 0°C
Energy	1 J = 1 N·m = 10^7 ergs = 10^7 dyne·cm = 2.778×10^{-7} kW·h = 0.23901 cal = 0.7376 ft-lb$_f$ = 9.486×10^{-4} Btu
Power	1 W = 1 J/s = 0.23901 cal/s = 0.7376 ft·lb$_f$/s = 9.486×10^{-4} Btu/s = 1.341×10^{-3} hp

Example: The factor to convert grams to lb$_m$ is $\left(\frac{2.20462 \text{ lb}_m}{1000 \text{ g}}\right)$.

Copied from Felder & Rousseau, 3rd. ed. Wiley. 2005.

#1 (15 points)
(a) Why does a packed bed have a larger pressure drop than a pipe of the same size?

There is a lot more frictional energy loss because the surface area between fluid + solid is much much higher when a pipe is packed w/ small beads.

2

(b) Given the additional work required to pump fluid through a packed bed, why would you choose a packed bed over a pipe? In other words, what is the advantage of a packed bed that may make up for the increased energy required? An example may be helpful in your explanation.

There are many benefits of the increased surface area; packing material can be
- a catalyst to increase production of valuable product
- an adsorbant to remove unwanted components from a fluid

2

(c) If you have a pipe containing a hot liquid flowing inside running through a room (room contains air at ~STP), name one thing you would do to the pipe (cannot change the diameter or length of pipe) to:
(ci) Increase heat transfer (i.e. heat the air). Why?

add fins to the pipe - will increase the area for heat transfer $q = Ah\Delta T$

$A\uparrow \; q\uparrow$

3

(cii) Decrease heat transfer (i.e. keep the liquid as hot as possible). Why?

add a layer of insulation around the pipe. Insulation with low K value provides resistance to conductive heat transfer more than normal pipe

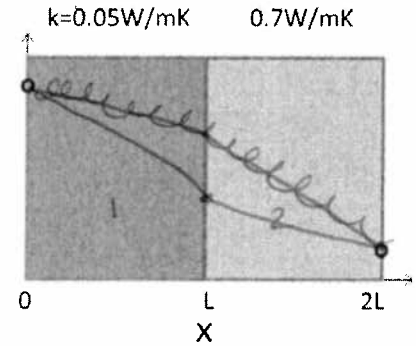
3

$$q = AK \frac{dT}{dX}$$

$K\downarrow \; q\downarrow$

5

(d) A wall with 2 different layers of materials separates hot air from cold air. The layers are of equal thickness (L), but their k values are 0.05 and 0.7 W/mK for layer 1 and layer 2 respectively. Layer 1 is in contact with hot air, while layer 2 is in contact with cold air. On the diagram at right, where x is the position in the wall, draw a rough temperature profile by connecting the starting and ending temperature points (o signs that were measured at the wall surfaces). Explain why you drew the plot the way that you did.



T decreases from left to right since given surface Temps.

Slope $\left(\frac{dT}{dx}\right)$ is greater for (left) material #1 since $K_1 < K_2$ + q/A is same for both materials (see below)

$$q = -KA\frac{dT}{dx}$$

$$\left.\begin{aligned} q_1 &= q_2 \\ A_1 &= A_2 \\ K_1 &< K_2 \end{aligned}\right\} \quad \text{at steady state} \quad \underbrace{\frac{q_1}{A_1} = \frac{q_2}{A_2}}_{} = \frac{q}{A}$$

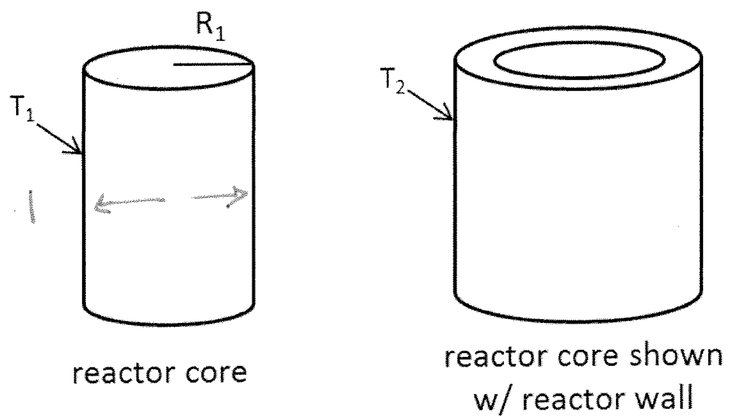
$$K_1\left(\frac{dT}{dx}\right)_1 = K_2\left(\frac{dT}{dx}\right)_2$$

$$\left(\frac{dT}{dx}\right)_1 > \left(\frac{dT}{dx}\right)_2$$

since $K_1 < K_2$

#2 (23 points)

A chemical reaction occurs in the solid core of a small cylindrical reactor. The heat of reaction provides an r-dependent heat source $\dot{q}=S*r$, where S is a constant and r is the radius. The core has a radius R_1 and conductivity k. The heat of reaction causes the temperature at $r = R_1$ reactor core surface to be T_1. The pictures are not drawn to scale, the reactor is much longer than it is wide, so heat loss from the top and bottom can be neglected. As indicated in the diagram, the reactor is surrounded by an insulating wall. The reactor wall surface is 430K and it sits in a room with an air temperature of 25°C. Radiant energy exchange may be neglected.



Information:
$R_1 = 0.08$ m
$k = 0.08$ W/m·K
$S = 20$ kW/m^4
Wall thickness = 0.02m
Wall k= 0.04 W/m K
$T_2 = 430$ K
Surrounding air T=25°C
Air h= 10 W/m^2 K

1 a) Draw an arrow on the reactor core picture to indicate in what direction heat will move.

7 b) What is T_1?

13 c) Find the temperature profile in the reactor core.

at steady state

2 $q_{\substack{reactor\\cond.}} = q_{\substack{wall\\cond.}} = q_{conv}$

$q_{conv} = Ah\Delta T = 2\pi(R_1 + t_w)L h_{air}(T_2 - T_{air})$

$q_{conv} = 2\pi(0.08m + 0.02m)(10\ W/m^2 \cdot K)(430\ K - \overset{298\ K}{\underset{+273\ K}{25°C}})\ L$

More space →

$q_{conv} = 829\ \frac{W}{m}\ (L)$

since all L same
just assume L = 1 m
for simplicity

$\rightarrow q_{conv}(\text{perimeter}) = 829\ W$

Extra space

or use resistance method

$q = \frac{\Delta T}{\Sigma R}$

$q = \frac{T_1 - T_{air}}{\frac{1}{h 2\pi (R_1 + t) L} + \frac{\ln(R_2/R_1)}{2\pi k}}$

$q_{conv} = q_{wall\ cond}$

$829 W = -kA \frac{dT}{dr} = -k_{wall} 2\pi r \cancel{L} \frac{dT}{dr}$ 1m

$\frac{829 W}{(0.04 W/mK) 2\pi r} dr = -dT$

$\frac{829 W}{(0.04 \frac{W}{m \cdot K}) 2\pi} [\ln R_2 - \ln R_1] = -(T_2 - T_1)$

$\frac{829 W}{(0.04 \frac{W}{m \cdot K}) 2\pi} [\ln(0.08m + 0.02m) - \ln(0.08m)] = T_1 - \cancel{T_2}\ 430K$

$\boxed{T_1 = 1166 K}$

To get T profile use diff T eqn at s.s.

$k\nabla^2 T + \dot{q} = 0$ for cylind. coord

T only function of r (symmetry in θ, very long in z so "fully developed T profile")

$\frac{k}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \dot{q} = 0$

$\frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) = -\frac{S r^2}{k}$

$r \frac{\partial T}{\partial r} = -\frac{S}{k} \int r^2 dr$

$$r\frac{\partial T}{\partial r} = -\frac{S}{k}\frac{r^3}{3} + C_1$$

B.C.

2 at $r = R_1$ ($0.08\,m$), $T = 1166\,K$

2 at $r = 0$, $T = T_{max}$ $\rightarrow \frac{dT}{dr} = 0$

1 $$\frac{dT}{dr} = -\frac{S}{k}\frac{r^2}{3} + \frac{C_1}{r}$$

at $r = 0$ ↗ goes to ∞ unless $C_1 = 0$

$\therefore C_1 = 0$, 1

$$\frac{dT}{dr} = -\frac{S}{k}\frac{r^2}{3}$$

$$\int dT = -\frac{S}{3k}\int r^2\,dr$$

1 $$T = -\frac{S}{9k}r^3 + C_2$$

$$1166\,K = \frac{-20{,}000\,W/m^4}{9\left(0.08\,\frac{W}{m\cdot K}\right)}(0.08\,m)^{\cancel{2}\,3} + C_2 \quad , 1$$

$C_2 = \cancel{1166}$
$1180\,K$

$$\boxed{T = \frac{-20{,}000\,W/m^4}{9\left(0.08\,\frac{W}{m\cdot K}\right)}r^3 + 1180\,K}$$

27,777 , 1