QUIZ 6 Name Key

Math 2551 D Steinbart Section _____ March 2, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(5) 1. We wish to find the minimum value of $F(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$. We will use Lagrange Multiplier techniques.

a. Set up the appropriate equations that one would solve. (Do not give vector equations as your final answer.)

b. Does the function $F(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$ have a maximum value? Why or why not?

Let $g(x,y,z) = 2x-3y-4z-49$. Then the constraint is $g(x,y,z)=0$

(a) Solve $\nabla F = \lambda \nabla g$, $g=0$ for $F(x,y,z)=2x^2+y^2+3z^2$, $g(x,y,z)=2x-3y-4z-49$ $\rightarrow$

$$\left.\begin{aligned} 4x &= \lambda 2 \\ 2y &= \lambda(-3) \\ 6z &= \lambda(-4) \\ 2x-3y-4z-49 &= 0 \end{aligned}\right\} (*)$$

We would solve these equations $(*)$. If there is a minimum value of F subject to the constraint $g=0$, the point(s) where F has the minimum will be one of the solutions to $(*)$.

(b). F subject to the constraint $g=0$ does not have a maximum value.

$g=0$ mean $2x-3y-4z-49=0$

For example we can take $2x-4z=0$ and $-3y-49=0$

So $2x=4z$, $x=2z$

So $y=\frac{-49}{3}$

We can make z as big as we want and take $x=2z$ and $y=\frac{-49}{3}$

Then $g(x,y,z) = g(2z, \frac{-49}{3}, z) = 2(2z) - 3(\frac{-49}{3}) - 4z = 0$

And $F(x,y,z) = F(2z, \frac{-49}{3}, z) = 2(2z)^2 + (\frac{-49}{3})^2 + 3z^2$ which can be made large as we want by taking z sufficiently large.

That's $\lim_{z\to\infty} F(2z, \frac{-49}{3}, z) = \infty$.

(5) 2. Consider the iterated integral $\int_1^4 \int_{3\sqrt{x}}^6 (5xy^2)\, dy\, dx$.

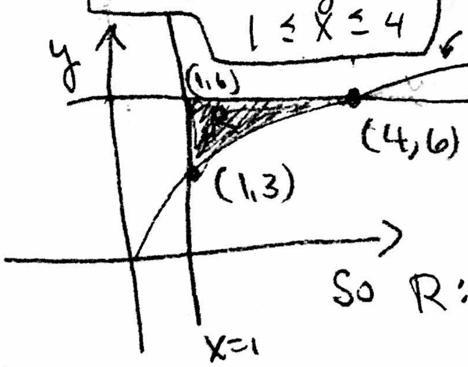
a. Sketch the region of integration for the integral. R

b. Write an equivalent integral with the order of integration reversed.

$\boxed{R: 3\sqrt{x} \le y \le 6,\ 1 \le x \le 4}$ $\rightarrow$ $3\sqrt{x} = y$, $y = 6$. These intersect when $3\sqrt{x} = 6$, $\sqrt{x} = 2$, $x = 4$

$x = y^2/9$, $y = 3\sqrt{x}$, $y = 6$

$x=1$, $y=3\sqrt{x}$ intersect when $y = 3\sqrt{1}$, $y=3$



Solve for x: $3\sqrt{x} = y$, $\sqrt{x} = \frac{y}{3}$, $x = \frac{y^2}{9}$

So R: $1 \le x \le y^2/9$, $3 \le y \le 6$

$$I = \int_3^6 \int_1^{y^2/9} (5xy^2)\, dy\, dx$$