MGT 2251 Management Science

Exam 1

Instructor: Tatiana Rudchenko

Duration: 80 minutes

Last Name (Print):____

First Name (Print):____

ID #:____

Read each question carefully before you answer. Work at a steady pace.
Good Luck!

My signature *certifies* that I have taken this exam in accordance with the Georgia Tech honor Code.

Signature____

Part I (100 points)

Multiple-choice Questions

Please, use scantron form and circle only one answer for each question
Don't forget to PRINT your last, first names and section (A, B or D).

1. A solution that satisfies all the constraints in an LP minimization problem and gives lowest value of the objective function is called
   a. the unbounded solution
   b. the optimal solution
   c. the infeasible solution
   d. the feasible solution

Answer: B

2. At the optimal solution for an LP problem, if a constraint has the left-hand-side value **not equal** to the right-hand-side values, then which of the following is **false?**
   a. the constraint is nonbinding
   b. the constraint has positive slack or surplus
   c. the constraint has zero shadow price
   d. an infinitesimal change in the right-hand side of the constraint changes optimal objective function value.

Answer: D

3. Which of the following does **not** change the feasible region?
   a. increasing an objective function coefficient in a maximization problem
   b. adding a new constraint
   c. increasing the right-hand side of a constraint
   d. changing a coefficient of a constraint

Answer: A

4. When more than one optimal solutions exist in an LP problem, then
   a. one of the constraints will be redundant
   b. the objective function will be parallel to one of the constraints
   c. two constraints will be parallel
   d. the problem will also be unbounded

Answer: B

5. What is limitation of graphical method over "excel solver" method
   a. graphical method is not suitable when the number of constraints is more than 2
   b. graphical method is not suitable when the number of constraints is more than 4
   c. graphical method is not suitable when the number of decision variables is more than 2
   d. graphical method is not suitable when either the number of constraints is more than 2 or the number of decision variables is more than 2

Answer: C

Use the following data to answer questions Q6-Q16.
"Personal Mini Warehouses" is planning to expand its successful Orlando business into Tampa. In doing so, the company must determine how many storage rooms of large and small size to build. Its objective and constraints are as follows:

Maximize monthly earnings = $50X + 20Y$
Subject to $2X + 4Y \leq 400$ (monthly advertising budget available)
$100X + 50Y \leq 8{,}000$ (storage space)
$X \leq 60$ (rental limit expected)
$X, Y \geq 0$
Where X =number of large rooms developed
Y=number of small rooms developed

Variable Cells							
			Final	**Reduced**	**Objective**	**Allowable**	**Allowable**
	Cell	**Name**	**Value**	**Cost**	**Coefficient**	**Increase**	**Decrease**
	B3	Large Rooms	60	0	50	1E+30	10
	C3	Small Rooms	40	0	20	5	20
Constraints							
			Final	**Shadow**	**Constraint**	**Allowable**	**Allowable**
	Cell	**Name**	**Value**	**Price**	**R.H. Side**	**Increase**	**Decrease**
	D6	Advertising Budget	280	0	400	1E+30	120
	D7	Storage Space	8000	0.4	8000	1500	2000
	D8	Rental Limit	60	10	60	20	20

6. Based on the optimal solution, what is the optimal monthly earnings?
a. $3000
b. $3800
c. $2000
d. $800

Answer: B

Solution:
60*50+40*20=3,800

7. Which of the constraints is nonbinding?
a. Advertising Budget
b. Storage Space
c. Rental Limit
d. None

Answer: A

Solution:
Final value 280 (LHS of constraint).Constraint RHS value is 400. Constraint has a positive slack 120.

8. Assuming all other parameters remain unchanged, if the objective function coefficient associated with "**Small Rooms**" decreases by $10, what will be the change in the objective function?

a. increases by $200
b. decreases by $200
c. increases by $400
d. decreases by $400

Answer: D

Solution:

Allowable decrease is 20.$10 is within the allowable decrease of $20.Thus the optimal solution (60,40) will remain optimal. Let's compute new maximum of earnings 50X+ 10Y=50(60)+10(40)=3000+400=3400.
3800-3400=400. Or 10*40=400.

9. Assuming all other parameters remain unchanged, if the objective function coefficient associated with "**Large Rooms**" increases by $20, what will be the change in the objective function?

a. increase by $1000
b. decrease by $1000
c. increases by $1200
d. decrease by $1200

Answer: C

Solution:

Allowable increase is $\mathbf{10^{30}}$ $(infinit\ number)$.20 is within the allowable increase. The optimal solution (60,40) will not change. Let's compute new maximum of earnings 70X+ 20Y=70(60)+20(40)=4200+800=5000.5000-1800=1200.Or 20*60=1200.

10. Assuming all other parameters remain unchanged, if the objective function coefficient associated with "**Large Rooms**" increases by $20, what will be the change in the **optimal solution**?

a. The optimal number of Large Rooms decreases
b. The optimal number of Large Rooms remains the same.
c. The optimal number of Large Rooms increases
d. The optimal number of Large Rooms can either decrease or increases

Answer: B

Solution:

Allowable increase is $\mathbf{10^{30}}$ $(infinit\ number)$.20 is within the allowable increase. The optimal solution (60,40) will not change. The optimal number of large room will remain the same-60 rooms.

11. Assuming all other parameters remain unchanged, what is the range of the objective function coefficient associated with **Small Rooms** for which the current optimal solution still remains optimal?

a. Between 20 and 25
b. Between 15 and 40
c. Between 0 and 25
d. Between 40 and plus infinity

Answer: C

Solution:

Let's compute sensitivity range for objective function coefficient associated with **Small Rooms**: 20-current objective function coefficient, 5-allowable increase, 20-allowable decrease (table "Variable Cells)
Upper limit 20+5=25
Low limit: 20-20=0

12. If the "**Rental Limit**" constraint's right-hand side were 75, what would have been optimal profit of the Personal Mini Warehouses?

a. $3800
b. $3950
c. $4100
d. $3650

Answer: B

Solution:

Maximize monthly earnings = 50X + 20Y
Subject to $2X + 4Y \le 400$ (monthly advertising budget available)
$100X + 50Y \le 8{,}000$ (storage space)
$X \le 60$ (rental limit expected)
$X, Y \ge 0$
$X \le 75$ RHS value of "Rental Limit" is 75 now. That represents increase by 15 units in the value of RHS. Allowable increase is 20 (table "Constraints"). 15 is within the allowable increase of 20.The shadow price 10 is operative. Let's compute the increase in the value of optimal profit: 15*10=150.New value of the optimal profit is 3800+150=3950.

13. If the "**Advertising Budget**" available were only $300, what would have been optimal profit of the Personal Mini Warehouses?

a. $3800
b. $3700
c. $3500
d. $3600

Answer: A

Solution:

Maximize monthly earnings = 50X + 20Y
Subject to $2X + 4Y \le 400$ (monthly advertising budget available)
$100X + 50Y \le 8{,}000$ (storage space)
$X \le 60$ (rental limit expected)
$X, Y \ge 0$

RHS value of "monthly advertising budget available" constraint is 300 now. That represents decrease by 100 units in the value of RHS. Allowable decrease is 120 (table "Constraints"). 100 is within the allowable increase of 120.The shadow price 0 is operative. The optimal profit is $3800.

14. Assuming all other parameters remain unchanged, if another firm is ready to provide loan for **Advertising Budget** to Personal Mini Warehouses at interest rate of **1% per month**, should Personal Mini Warehouses accept the offer?

a. Yes
b. No
c. Can't say

Answer: B

Solution:

No because shadow price is 0. If we add some units of resources it will have no impact on the optimal value of the objective function.

Please, note:

A non-binding constraint will always have a shadow price of 0 and changing the RHS of a non-binding constraint by any amount within its allowable increase or decrease will have no impact on the optimal solution and no impact on the optimal value of the objective function

15. Assuming all other parameters remain unchanged, if another firm is ready to lease out its **Storage Space** to Personal Mini Warehouses, what is the maximum monthly rental per unit of storage space below which Personal Mini Warehouses should accept the offer?

a. $0
b. $0.4
c. $9500
d. $10

Answer: B

Solution:

$0.4 because shadow price is 0.4. The **shadow prices are the maximum amounts the manager would pay for additional units of resource**.

16. Assuming all other parameters remain unchanged, if another firm leases out 400 units its **Storage Space** to Personal Mini Warehouses at monthly rental **$20 per 100 units** of storage space, what is total profit to Personal Mini Warehouses in this new setting?

a. $3800
b. $3880
c. $3720
d. $3960

Answer: B

Solution:

$100X + 50Y \leq 8{,}000+400$

$100X + 50Y \leq 8{,}400$

Let's find an improve of the optimal solution value: 400*0.4(shadow price)=160

160-80=80

3,800+80=$3,880

II. Problems (20 points)

1. MSA Computer Corporation manufactures three models of computers: Netbook, Notebook and Desktop. The firm employs five technicians, working 160 hours each per month, on its assembly line. It requires 10 labor hours to assemble each Netbook, 15 labor hours to assemble each Notebook and 8 labor hours to assemble each Desktop. Each computer model requires data storage space. However, the supply of data storage space available is only 10000 GB per month. Each Netbook requires 200 GB, each Notebook requires 300 GB and each Desktop requires 100 GB data storage space. MSA wants at least 6 Netbooks and at most 10 Desktops to be produced each month. Furthermore, the number of Notebooks must be at least twice as the number of netbooks.

   Netbooks generate $800 profit per unit, Notebooks generate $1000 profit per unit and Desktops generate $600 profit per unit. MSA aims to maximize its profit by choosing how many computers of each model to manufacture. Formulate this as linear programming problem. (30 points)

(a). Define the decision variables for MSA Computer Corporation. Write the symbol or notation and its plain description for each decision variable. (1 point)

X- Number of Netbooks manufactured.
Y- Number of Notebooks manufactured.

Z- Number of Desktops manufactured.

(b). If MSA Computer Corporation's goal is to maximize its profit, formulate its objective function. (1 points)

Max: 800X+1000Y+600Z

(c). Formulate the constraints including non-negativity for MSA Computer Corporation. Label each constraint in parenthesis. (3 points)

10X+15Y+ 8Z =< 800 (Assembly Hours)
200X+300Y+100Z=< 10000 (Data Storage Space)
X>= 6 (MSA wants at least 6 Netbooks)
Y>= 2X (The number of Notebooks must be at least twice as the number of netbooks)
Z=< 10 (MSA wants at most 10 Desktops to be produced each month.)
X, Y, Z >= 0

2. Consider the following linear programming problem: (10 points)

Maximize	$30x + 40y$	(OBJ)
Subject to	$2x + y \le 8$	(1)
	$2x - y \geq 2$	(2)
	$y \le 2$	(3)
	$x, y \geq 0$	

Solve the problem graphically and answer the following:
1. Clearly plot and label the constraints. (1 points)
2. Identify and shade the feasible region. (1 points)
3. Identify all the corner points or extreme points and their coordinates (i.e. the values of x and y). (1 points)
4. Compute the objective function values on all the corner points. (2 points)
5. Determine the optimal solution (i.e. the values of x and y), and also compute the value of the objective function at the optimal solution. (2 points)
   (3,2), 170
6. Identify the non-binding constraint(s).Why? (2 points)
   Constraint 2
7. Is coordinate (4, 1) feasible solution? Is (1, 3) feasible? Why? (1 point)
   No, No

NOTE: Graph paper follows on the next page. You may answer all the parts of the question using the graph provided on the next page.

Solution 1-5:
Step 1: graph constraints equations. Graph must be located in the first quadrant of XY plain because x, y $\geq$ 0.
<u>Constraint 1:</u> $2x+y\le 8$ (1)
$2x+y=8$
$x=0$, $y=8$
$x=4$, $y=0$.
To plot the line, please connect two points (0, 8) and (4, 0)
<u>Constraint 2:</u> $2x-y\geq 2$ (2)

EXAM 1 Review of Exam 1 (Solutions)

2x-y=2

x=2, y=2

x=1, y=0

To plot the line, please connect two points (2,2) and (1,0)

<u>Constraint 3:</u> $y \geq 2$ (3).

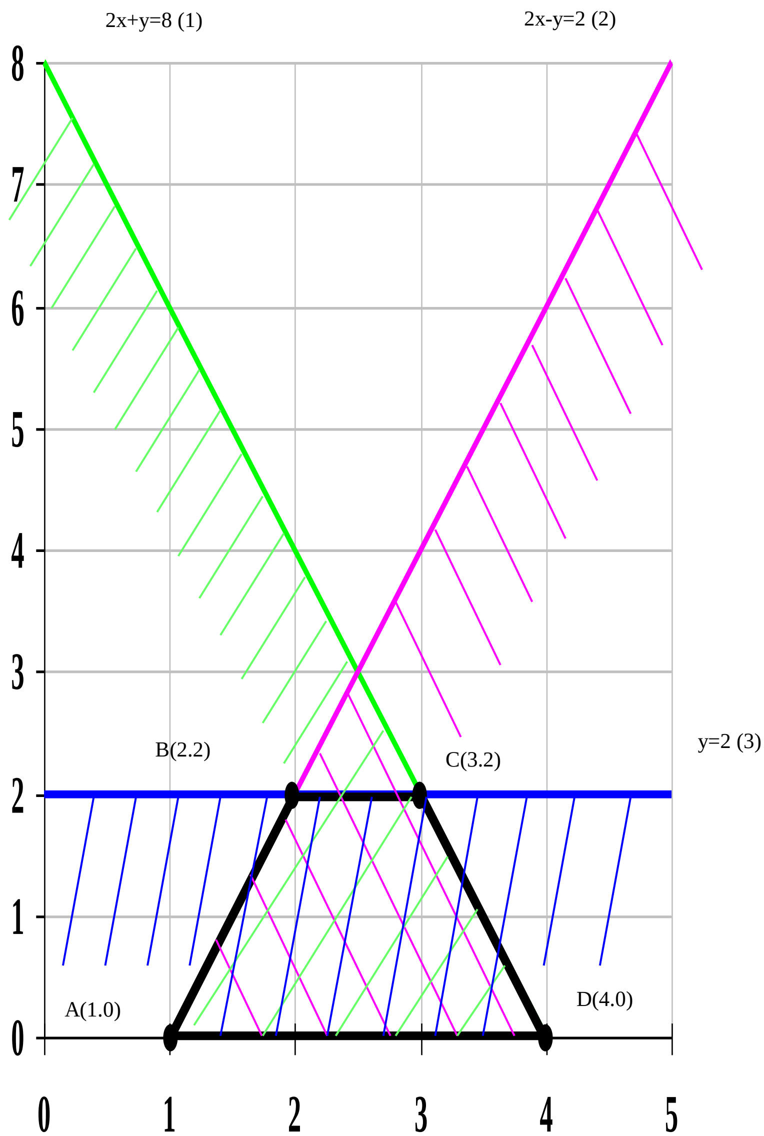
y=2. The line y=2 is parallel to X-axis.

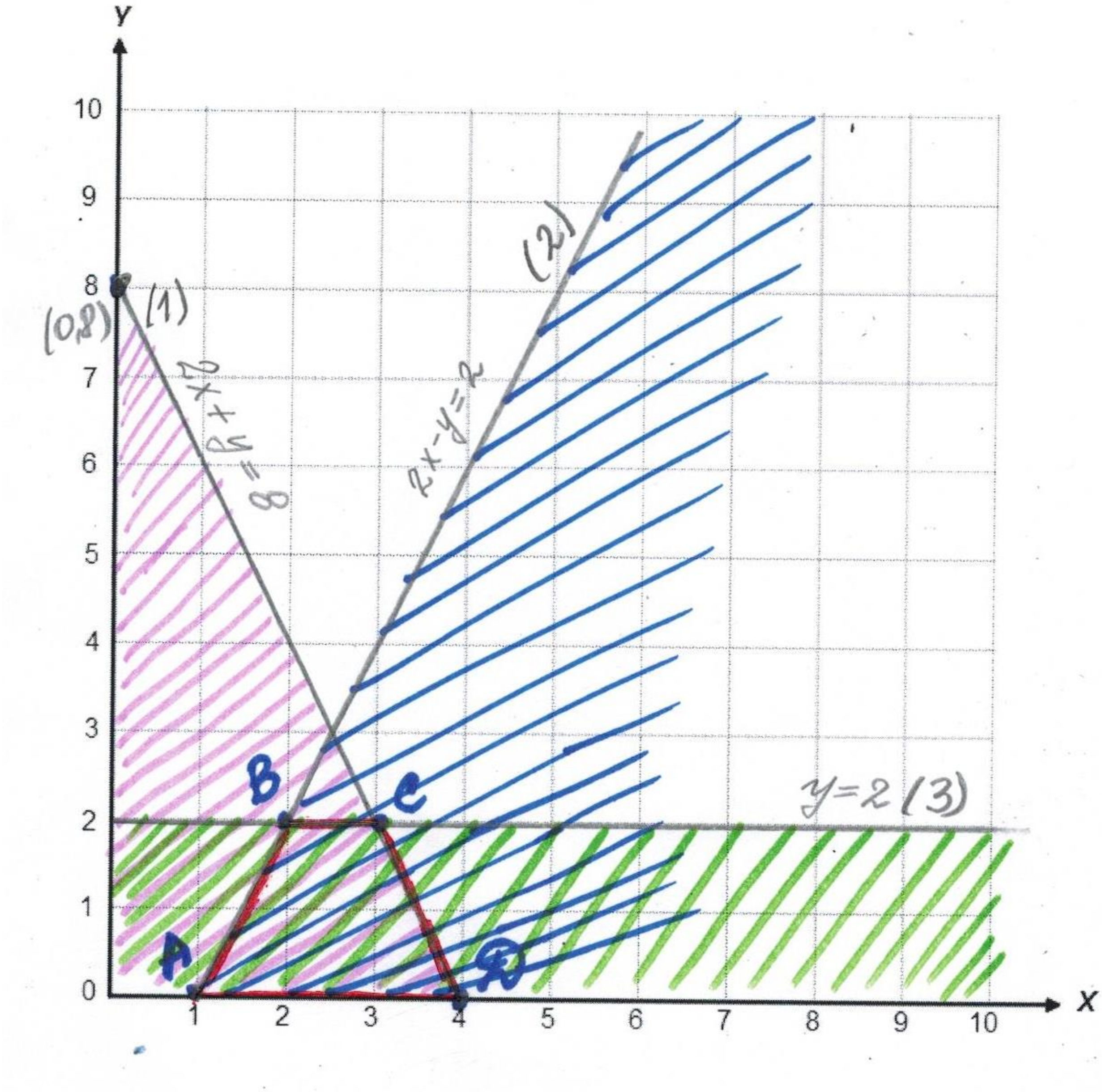
x=0, y=2

x=4, y=2

To plot the line, please connect two points (0,2) and (4,2)

Step 2: Graph feasible solution area or feasible region.





Step 4
Find all corner (extreme) points.
A (1,0), D(4,0)
Point B is an intersection of two constraint lines 2x-y=2 (2) and y=2(3).
2x-y=2
y=2.
B (2,2)
Point C is an intersection of two constraint lines 2x+y=8 (1) and y=2 (3)
2x+y=8
y=2.
C (3,2)

Step 3 Corner point Solution method. Compute objective function value at all corner points.
Table. 1 Corner Point Solution Method

point	Objective function value 30x + 40y	maximum
A(1,0)	30x + 40y=30*1+40*0=30	
B(2,2)	30x + 40y=30*2+40*2=140	
C(3,2)	30x + 40y=30*3+40*2=170	Maximum value 170 when x=3,y=2
D(4,0)	30x + 40y=30*4+40*0=120	

Step 4. Identify the optimal solution and make a conclusion about optimal solution and the optimal objective function value.
The optimal solution is x=3, y=2. The objective function value at point x=3, y=2 is 170.

Solution (6)
6. Identify the non-binding constraint(s).Why?

- When slack or surplus = 0, i.e., LHS = RHS, the constraint is called **binding**. That resource capacity (RHS) is used to the limit.
- When the constraint's slack or surplus ≠ 0, i.e., LHS ≠ RHS, it is a **non-binding** constraint. That resource has some slack left.

Optimal solution	Constraint	Usage (LHS)	Available (RHS)	Slack /surplus
x=3,y=2	2x + y ≤ 8	2x+y=2*3+2=8	8	8-8=0
x=3,y=2	2x - y ≥□ 2	2x-y=2*3-2=4	2	4-2=2
x=3,y=2	y≤2	y=2	2	2-2=0

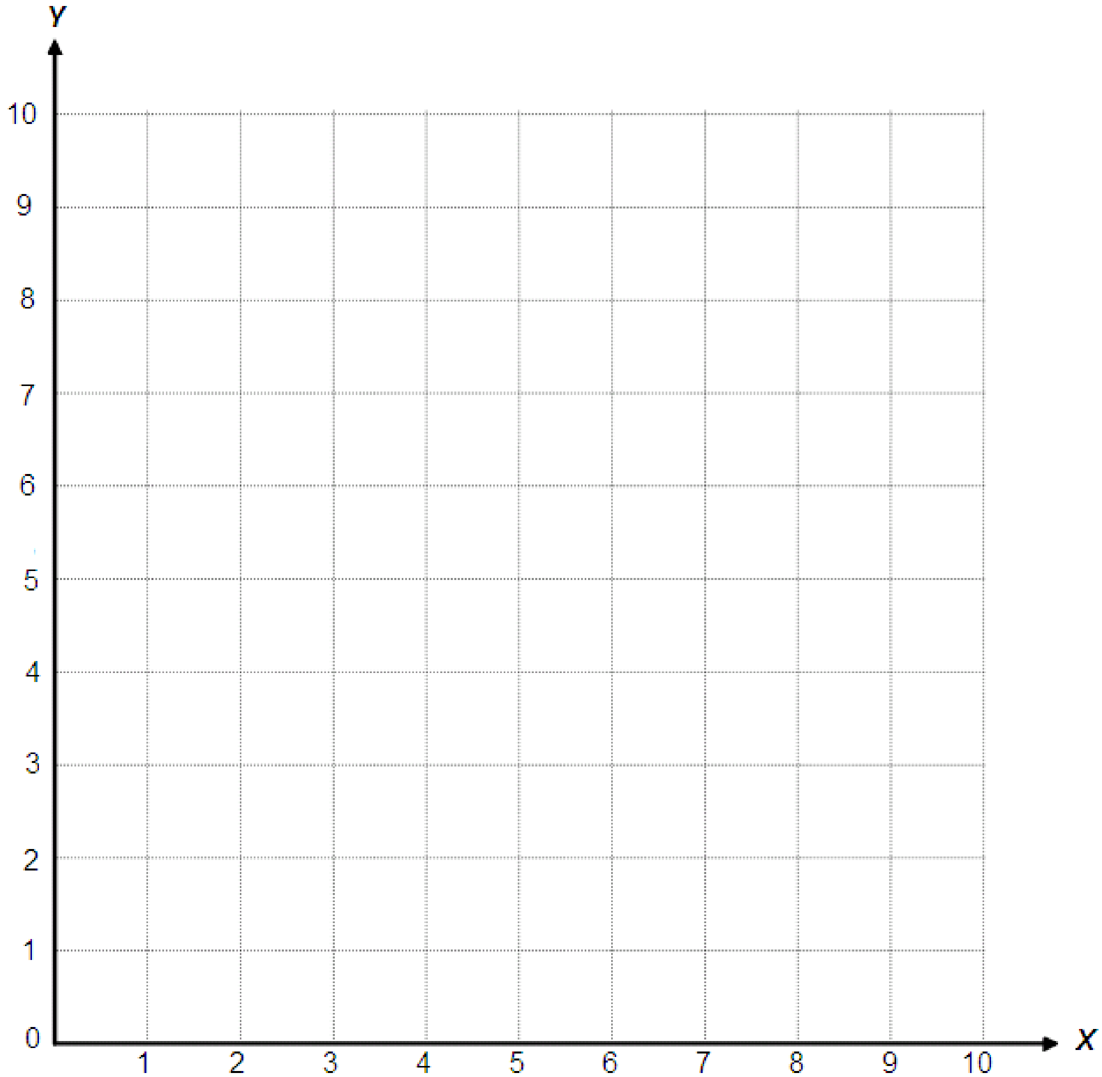
Conclusion: Constraint #2 is non-binding because RHS ≠LHS, surplus is 2.

Solution (7)
7. Is coordinate (4, 1) feasible solution? Is (1, 3) feasible? Why? (1 point)

A ***feasible solution*** does not violate ***any*** of the constraints.

Point (4,1)   2x + y ≤ 8
Constraint (1) check: 2*4+1≤ 8, 9≥8. *Thus, point (4,1) is infeasible solution.*
Answer: No

Point (1,3)
Constraint (1) check: 2x + y ≤ 8, 2*1+3≤ 8, 5≤8
Constraint (2) check: 2x - y ≥□ 2,2*1-3≥2, it is not a true because -2≤2.
Point (1,3) is infeasible solution.
Answer: No


Y
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