ISyE 4031 Regression and Forecasting
Spring 2016
Homework 1 Solutions

1. Exercise 2.7.

a. We wish to find $P(30.7 \le y \le 32.3)$,

$$z_{30.7} = \frac{30.7 - 31.5}{.8} = \frac{-.8}{.8} = -1 \text{ and } z_{32.3} = \frac{32.3 - 31.5}{.8} = \frac{.8}{.8} = 1$$

$$P(30.7 \le y \le 32.3) = P(-1 \le z \le 1) = P(-1 \le z \le 0) + P(0 \le z \le 1)$$

= 0.3413 + 0.3413 = 0.6826.

e. We wish to find $P(y \le 29.5)$,

$$z_{29.5} = \frac{29.5 - 31.5}{.8} = \frac{-2}{.8} = -2.5$$

$$P(y \le 29.5) = P(z \le -2.5) = 1 - P(-2.5 \le z \le 0) = 1 - 0.4938 - 0.5 = 0.0062$$

f. We wish to find $P(y \ge 29.5)$,

$$P(y \ge 29.5) = P(z \ge -2.5) = 0.5 + P(-2.5 \le z \le 0) = 0.5 + 0.4938 = 0.9938.$$

Or from part (e): $P(y \ge 29.5) = 1 - P(z \le 29.5) = 1 - 0.0062 = 0.9938$.

2. Exercise 2.8.

a. $z_{[.05]} = 1.645$.

b. $z_{[.02]} = 2.054$.

3. Exercise 2.9.

a. $t_{[.05]}^{(7)} = 1.895$.

b. $t_{[.01]}^{(7)} = 2.998$.

c. $t_{[.005]}^{(7)} = 3.499$.

4. Exercise 2.10.

a. $F_{[.05]}^{(2,5)} = 5.79$.

b. $F_{[.05]}^{(5,2)} = 19.30$.

5. Exercise 2.11.

a. $x^2{}_{[.05]}(3) = 7.81473$.

b. $x^2{}_{[.01]}(2) = 9.21034$.

6.a. The parameter of interest is the true mean interior temperature life, μ.

The hypothesis that we are testing: H_0: $\mu = 22.5$ vs H_1 : $\mu \ne 22.5$.

Test statistic: $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

We reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where $\alpha = 0.05$ and $t_{\alpha/2,n-1} = 2.776$ for $n = 5$.

$\bar{x} = 22.496$, $s = 0.378$, $n = 5$. So,

$$t_0 = \frac{22.496 - 22.5}{0.378/\sqrt{5}} = -0.0237$$

Because $-0.0237 > -2.776$, we cannot reject the null hypothesis. There is not sufficient evidence to conclude that the true mean interior temperature is not equal to 22.5 °C at $\alpha = 0.05$.

b. A 95% two sided confidence interval for μ,

$$\bar{x} - t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,4}\left(\frac{s}{\sqrt{n}}\right) =$$

$$22.496 - 2.776\left(\frac{0.378}{\sqrt{5}}\right) \le \mu \le 22.496 + 2.776\left(\frac{0.378}{\sqrt{5}}\right) = 22.027 \le \mu \le 22.965$$

Or a 95% two sided confidence interval for μ is (22.027, 22.965).

We cannot conclude that the mean interior temperature differs from 22.5 because the value is included in the confidence interval (i.e., the same conclusion as in part (a), do not reject H_0: $\mu = 22.5$).