MATH 1711, Midterm 3

11/5/2014

Name: Key GTID: _____

Circle your section below

D4 TA: Ethan Smith

D5 TA: Pranav Shenoy

Problem No.	Points
1	15
2	15
3	20
4	10
5	20
6	20

TOTAL: 100

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (15 points).

Alice received the following course grades during her first year at Tech: 4,4,4,4,3,3,2,2,2,0. Compute the sample mean and the variance of her grades. You do *not* need to simplify your final answer.

$$\bar{X} = \frac{1}{10}(4+4+4+4+3+3+2+2+2+0) = 2.8$$

$$s^2 = \frac{1}{9}\left[(4-\bar{x})^2 \cdot 4 + (3-\bar{x})^2 \cdot 2 + (2-\bar{x})^2 \cdot 3 + (0-\bar{x})^2\right]$$

Problem 2 (15 points).

Foxconn discovers that an average of 10 iPhone 6 Plus phones are defective in each batch, with a standard deviation of 8. Suppose that 10,000 batches are produced. Use the Chebychev inequality to estimate the number of batches that have 20 or fewer defective iPhone 6 Plus phones. Simplify your final answer as far as possible.

Let X be the # of phones in a batch.

defective

$$\begin{aligned} Pr(X \le 20) &= Pr(0 \le X \le 20) \\ &= Pr(|X-\mu| \le 10) \\ &\ge 1 - \frac{\sigma^2}{c^2} \\ &= 0.36 \end{aligned}$$

Hence the # of batches that have 20 or fewer defective phones is at least 3,600.

Turn over for more problems

Problem 3 (20 points).

Say that there is a 20% chance that a person chosen at random from the population has never heard of Leonhard Euler. Use the normal approximation to the binomial distribution to estimate the probability that in 625 people, we find exactly 100 people who have not heard of Euler. Simplify your final answer as far as possible.

Let X be the # of people who haven't heard of Euler.

$$Pr(X=100) = Pr(99.5 \le X \le 100.5)$$

$$\approx Pr\left(\frac{99.5-\mu}{\sigma} \le z \le \frac{100.5-\mu}{\sigma}\right) \qquad \mu = 125,\ \sigma = 10$$

$$= Pr(-2.55 \le z \le -2.45)$$

$$= A(-2.45) - A(-2.55)$$

$$= 0.0071 - 0.0054$$

$$= 0.0017$$

Problem 4 (10 points).

The RREF matrix for a system of equations is given below. Write out the solution to the system of equations. You may label the columns with any variables names you wish.

$$\begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{matrix}$$
$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 3 & -2 \\ 0 & 1 & 0 & 2 & 0 & -3 \\ 0 & 0 & 1 & 1 & -5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 = -2 - 3x_5$

$x_2 = -3 - 2x_4$

$x_3 = 5 - x_4 + 5x_5$

$x_4 =$ any #

$x_5 =$ any #

Turn over for more problems

Problem 5 (20 points).

Use the method of inverses to solve the following system of equations: given $AX = B$, first, find A^{-1}, then solve the equation $X = A^{-1}B$. ANY OTHER METHOD WILL RECEIVE NO CREDIT!

$$y + 2z = 1$$
$$2x + y + 3z = 2$$
$$x + y + 2z = 3$$

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 0 & 1 \\ 2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & -2 \\ 0 & 1 & 2 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{(-1)R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 - R_2]{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 & -1 & 2 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right]$$

$$\xrightarrow[R_2 - R_3]{R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & -1 & -2 & 4 \\ 0 & 0 & 1 & 1 & 1 & -2 \end{array}\right]$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -2 & 4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}B = \begin{bmatrix} -1 & 0 & 1 \\ -1 & -2 & 4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$$

Turn over for more problems

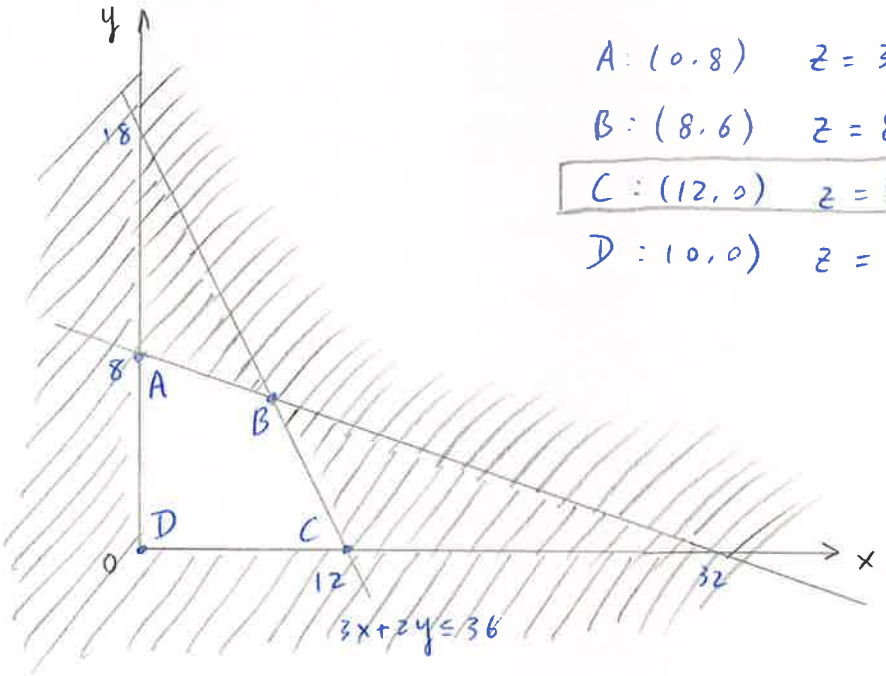
Problem 6 (20 points).

Tech Inc. makes electronic hockey and soccer games. A hockey game sells for \$7, and a soccer game sells for \$4. Each hockey game requires 3 labor-hours of assembly and 1 labor-hour of testing. Each soccer game requires 2 labor-hours of assembly and 4 labor-hours of testing. Each day, there are 36 labor-hours available for assembly and 32 labor-hours available for testing. How many of each game should Tech produce each day to maximize its daily revenue? Set up an LP problem, then solve it using graphic method.

Assume produce x hockey games, y soccer games.

$$\max: \quad 7x + 4y$$

$$\text{s.t.} \quad \begin{cases} 3x + 2y \le 36 \\ x + 4y \le 32 \\ x \ge 0, \; y \ge 0 \end{cases}$$



$x + 4y \le 32$

A: (0, 8) $z = 32$

B: (8, 6) $z = 80$

C: (12, 0) $z = 84$

D: (0, 0) $z = 0$

The End.

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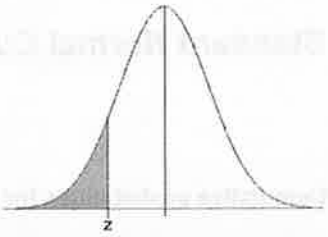
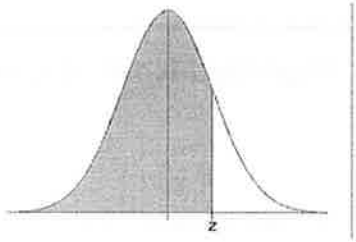
Standard Normal Cumulative Probability Table



Cumulative probabilities for NEGATIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998