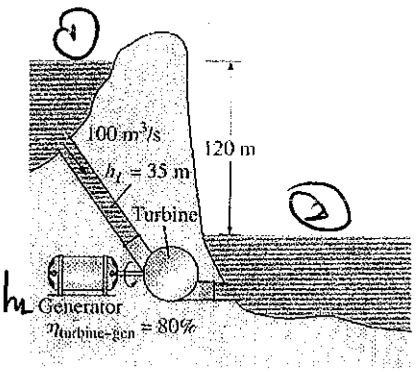
CEE 3040 Spring 2010 Exam #3 Name:____________________

(Use the extra pages. Do not write solutions on the back of the papers)

Problem 1 (5pts)

In a hydroelectric power plant, 100 m^3/s of water ($\rho = 1000$ kg/m^3, $\mu = 1\times10^{-3}$ Ns/m^2, $g = 9.81$ m/s^2) flows from an elevation of 120 m to a turbine, where electric power is generated. The total irreversible head loss in the piping system is 35 m. If the overall efficiency of the turbine-generator is 80 percent, find the electric power output.

Cons. of Energy



1' $$\cancel{\frac{\dot{Q}}{\dot{m}g}} + \cancel{h_p} + \cancel{\frac{P_1}{\gamma}} + z_1 + \cancel{\alpha_1\frac{V_1^2}{2g}} + \cancel{\frac{u_1}{g}}$$

$$= h_t + \cancel{\frac{P_2}{\gamma}} + z_2 + \cancel{\alpha_2\frac{V_2^2}{2g}} + \cancel{\frac{u_2}{g}} + h_L$$

$$\Rightarrow h_t = z_1 - z_2 - h_L$$

1' $$= 120m - 35m$$

$$= 85m$$

1' $$\Rightarrow \dot{W}_t = \eta \dot{m} g h_t = 0.8 \cdot 1000\,kg/m^3 \cdot 100\,m^3/s$$

1' $$\cdot 9.81\,m/s^2 \cdot 85m$$

1' $$= 66.7\,MW$$

Problem 2 (7pts)

Another good approximation of the laminar boundary layer velocity profile over a flat plate is:

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Find the boundary layer thickness $\delta(x)$ and skin friction coefficient $c_f(x)$.

① $\Theta = \int_0^\delta \frac{u}{U}(1-\frac{u}{U})dy$ 1'

$= \int_0^\delta [\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}(\frac{y}{\delta})^3][1-\frac{3}{2}(\frac{y}{\delta}) + \frac{1}{2}(\frac{y}{\delta})^3]dy$

$= 0.1393\,\delta$ 1'

② Momontum Integral Eqn.

$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} \Rightarrow \frac{\tau_w}{\rho U^2} = 0.1393\frac{d\delta}{dx}$ 1'

③ $\tau_w = \mu \frac{du}{dy}\Big|_{y=0} = \mu \cdot \frac{3}{2}\frac{U}{\delta}$ 1'

④ $\frac{\mu \cdot \frac{3}{2}\frac{U}{\delta}}{\rho U^2} = \frac{3}{2}\frac{\mu}{\rho U \delta} = 0.1393\frac{d\delta}{dx}$

$\Rightarrow \frac{\delta^2}{2} = \frac{3}{2}\frac{1}{0.1393}\frac{\nu x}{U}$

$\Rightarrow \delta = 4.64\sqrt{\frac{\nu x}{U}}$ 1'

⑤ $\tau_w = \mu \cdot \frac{3}{2}\frac{U}{\delta} = \mu \cdot \frac{3}{2}\frac{U}{4.64\sqrt{\frac{\nu}{Ux}}}$ 1'

$\Rightarrow C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = 0.646\sqrt{\frac{\nu}{Ux}}$ 1'

Problem 3 (8 pts)

A cylinder 0.16m in diameter is to be mounted in a stream of water in order to estimate the force on a tall chimney of 1m diameter which is subject to wind of 33m/s. The table summarizes the significant variables and known information about the prototype in air and the model in water.

Variables	Water	Air
Velocity, u	u_{water}	33m/s
Force, F	F_{water}	F_{air}
Dynamic viscotiy, μ	8×10^{-4}kg/ms	16×10^{-6}kg/ms
Density, ρ	1000kg/m^3	1.12kg/m^3
Diameter, d	0.16m	1m

(a) Find dimensionless groups for this system.

(b) Find the speed of the stream necessary to give dynamic similarity between the model and chimney;

(c) Find the ratio of forces.

(a) 1. $\phi(u, d, \rho, \mu, F) = 0$

1'

2. $\phi\left(u[LT^{-1}], d[L], \rho[ML^{-3}], \mu[ML^{-1}T^{-1}], F[MLT^{-2}]\right) = 0$

3. $5 - 3 = 2\ \Pi_s$ 1'

4. Repeating variables: u, d, ρ 1'

5. $\Pi_1 = \mu u^a d^b \rho^c$ 1'

$= [ML^{-1}T^{-1}][LT^{-1}]^a[L]^b[ML^{-3}]^c = L^0T^0M^0$

$\Rightarrow \begin{cases} \text{for } M: & 1 + c = 0 \Rightarrow c = -1 \\ T: & -1 - a = 0 \Rightarrow a = -1 \\ L: & -1 + a + b - 3c = 0 \Rightarrow b = -1 \end{cases}$

$\Rightarrow \Pi_1 = \dfrac{\mu}{ud\rho}$

6. $\Pi_2 = Fu^a d^b \rho^c \Rightarrow \Pi_2 = \dfrac{F}{\rho u^2 d^2}$ 1'

7. check dimensions: $\Pi_1 = \dfrac{\mu}{u\rho d} = [\cancel{M}\cancel{L^{-1}}\cancel{T^{-1}}][\cancel{L^{-1}}\cancel{T}][\cancel{L^{-1}}][\cancel{M^{-1}}\cancel{L^{3}}] = 1$

8. $\phi(\Pi_1, \Pi_2) = 0$

$\Pi_2 = \dfrac{F}{\rho u^2 d^2} = [\cancel{M}\cancel{L}\cancel{T^{-2}}][\cancel{M^{-1}}\cancel{L^{3}}][\cancel{L^{-2}}\cancel{T^{2}}][\cancel{L^{-2}}]$
$= 1$

(b) $(\Pi_1)_{water} = (\Pi_1)_{air}$

$$\Rightarrow \frac{\mu_w}{U_w d_w \rho_w} = \frac{\mu_a}{U_a d_a \rho_a}$$

$$\Rightarrow \frac{8\times10^{-4}\,kg/m\cdot s}{U_w \cdot 0.16 \cdot 1000\,kg/m^3} = \frac{16\times10^{-6}\,kg/m\cdot s}{33\,m/s \cdot 1\,m \cdot 1.12\,kg/m^3}$$

$$\Rightarrow U_w = 11.55\,m/s \qquad 1'$$

(c) $(\Pi_2)_w = (\Pi_2)_a$ $\qquad 1'$

$$\Rightarrow \frac{F_w}{\rho_w U_w^2 d_w^2} = \frac{F_a}{\rho_a U_a^2 d_a^2}$$

$$\Rightarrow \frac{F_w}{F_a} = \frac{\rho_w U_w^2 d_w^2}{\rho_a U_a^2 d_a^2}$$

$$= \frac{1000\,kg/m^3 \cdot (11.55\,m/s)^2 (0.16\,m)^2}{1.12\,kg/m^3 \cdot (33\,m/s)^2 (1\,m)^2}$$

$$= 2.8 \qquad 1'$$

or $\frac{F_a}{F_w} = 0.3571$