BME 3400 Midterm Exam # 3 November 24, 2009 Name: SOLUTION

This is a closed-book exam. 1 8.5x11" page of handwritten notes allowed.
Calculators are allowed, but integrals should be solved before numbers are plugged in.

Show all your work! Free-body diagrams must be present and correct for full credit. Plug in numbers only at the end of a problem.

HONOR CODE
The conditions of this examination are subject to the Georgia Institute of Technology Academic Honor Code.

I pledge that the work in this exam represents my own, original work. I have not communicated with anyone about the contents of this exam, nor participated in or observed any conduct prohibited by the Honor Code.

Signature ______________________________

Problem 2 (50 points)

You are contracted by Burton to build a new snowboard. You build the board with 140 bald-cypress strips of 2 mm width and 6 mm height (Figure B). Bald-cypress has an elastic modulus of 9.7 GPa and a maximum allowable stress of 42 MPa.

- How much does the board deflect under the rider's feet in this maneuver where the rider is balanced on a rail midway between the feet. *Hint*: place x=0 at the rail.

- (Extra credit) How much does the board deflect at the end of the board?

Figure A: Snowboarder riding a rail

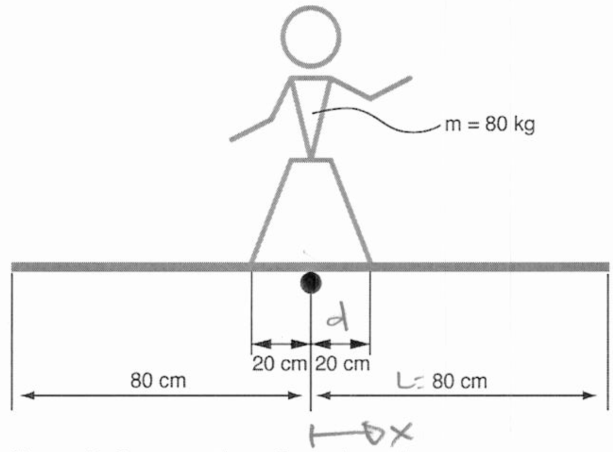
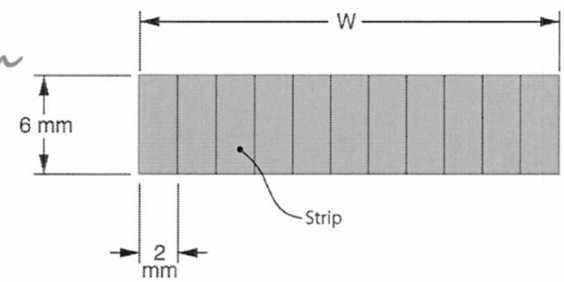


Figure B: Cross section of snowboard



Find $M(x)$ over $0 < x < 20\,cm$

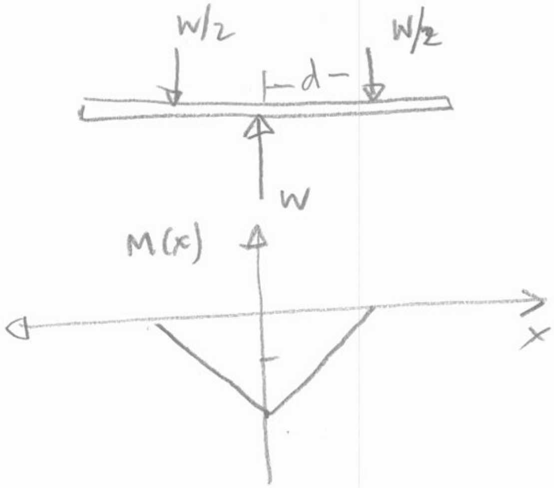
$EI\,v(x) = \iint M(x)\,dx$

boundary conditions are

$v(0) = 0 \quad , \quad v'(0) = 0$

deflection at foot is $v(20\,cm)$

$I = \frac{1}{2} b h^3 \qquad b = 140 \times 2\,mm \qquad h = 6\,mm$



$0 < x < 20\,cm$

$M(x) = -\frac{W}{2} \cdot d + \frac{W}{2} x = -78.4 + 392x$

$EI\,v(x) = \iint M(x)$

$EI\,v'(x) = -\frac{Wd}{2} x + \frac{W}{4} x^2 + C_1$

$EI\,v(x) = -\frac{Wdx^2}{4} + \frac{Wx^3}{12} + C_1 x + C_2$

$v'(0) = 0 \Rightarrow C_1 = 0$

$v(0) = 0 \Rightarrow C_2 = 0$

$$v(x) = \frac{1}{EI}\left(-\frac{wdx^2}{4} + \frac{wx^3}{12}\right)$$

$E = 9.7\ \text{GPa}$

$I = \frac{bh^3}{12} = \frac{(0.280\,m)(0.006\,m)^3}{12} = 5.04 \times 10^{-9}\ m^4$

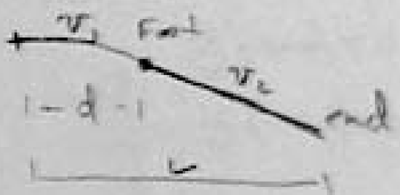
$w = 80\ kg \times 9.8\ m/s^2 = 784\ N$

$$v(0.2m) = \frac{\left[-\frac{(784\,N)(0.2\,m)(0.2\,m)^2}{4} + \frac{(784\,N)(0.2)^3}{12}\right]}{9.7\times10^{9}\ Pa \times 5.04\times10^{-9}\ m^4}$$

$= -0.021\ m$

$\boxed{v_{foot} = -2.1\ cm}$

EXTRA CREDIT



Board continues to deflect downward w/o any curvature since $M = 0$ for $d < x < L$

Compute a second curve. $v_2(L) =$ defl @ end

$v_1(0.2) = v_2(0.2) = -0.021\,m$

$v_1'(0.2) = v_2'(0.2) =$

$d < x < L$

$M = 0$

$EI v_2(x) = \iint M\,dx$

$EI v_2'(x) = C_3$

$EI v_2(x) = C_3 x + C_4$

$v_1'(0.2) = \frac{1}{EI}\left(-\frac{wd}{2}x + \frac{w}{4}x^2\right) = -0.16$

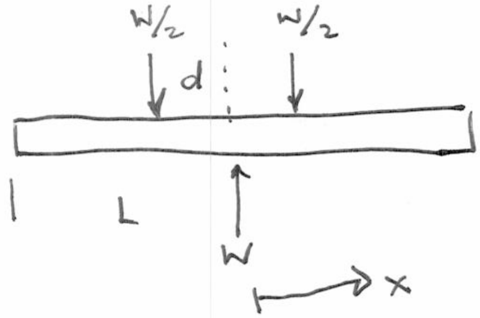
$EI v_1'(0.2) = EI v_2'(0.2) = C_3 = EI(-0.16)$

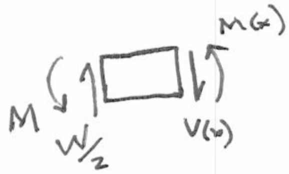
$EI v_1(0.2) = EI v_2(0.2) = C_3 x + C_4 = EI(-0.021)$

$C_4 = EI(-0.021) - EI(-0.16)(0.20\,m)$
$= 0.011$

$v_{end} = v_2(0.8\,m) = \frac{C_3(0.8) + C_4}{EI} = \boxed{-0.118\,m}$

ALTERNATE METHOD FOR E.C.



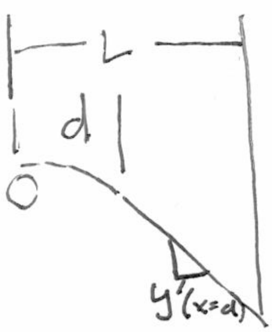


$$M = \frac{W}{2}\cdot d \quad \rightarrow \quad M(x) + \frac{W}{2}\cdot d - \frac{W}{2}\cdot x = 0$$

$$M(x) = \frac{W}{2}(x - d) \qquad 0 < x < d$$

$$M(x) = 0 \qquad d < x < L$$

$$y(x) = \iint \frac{M(x)}{EI}\,dx\,dx = \frac{1}{EI}\frac{W}{2}\left[\frac{x^3}{6} - \frac{dx^2}{2} + C_1 x + C_2\right]$$



$$y'(x=0) = 0 \rightarrow C_1 = 0 \qquad y(x) = \frac{W}{2EI}\left[\frac{x^3}{6} - \frac{dx^2}{2}\right]$$

$$y(x=0) = 0 \rightarrow C_2 = 0 \qquad y'(x) = \frac{W}{2EI}\left[\frac{x^2}{2} - dx\right]$$

$$y(x=d) = \frac{Wd^3}{2EI}\left[\frac{1}{6} - \frac{3}{6}\right] = -\frac{Wd^3}{6EI} = -\frac{784\,N(0.2m)^3}{6(9.7e9\,Pa)(5.04e^{-9}m^4)} \rightarrow -0.021\,m$$

$$y'(x=d) = \frac{Wd^2}{2EI}\left[\frac{1}{2} - 1\right] = -\frac{Wd^2}{4EI} \rightarrow 0.16$$

$$y^*(x=L) = y(x=d) + (L-d)\cdot y'(x=d)$$

$$y^*(x=L) = \frac{-Wd^2}{EI}\left[\frac{d}{6} + \frac{L-d}{4}\right] = \frac{-Wd^2}{EI}\left[\frac{L}{4} - \frac{d}{12}\right] = \frac{-Wd^2}{12EI}(3L - d)$$

$$= -\frac{\left(784\,N(0.2m)^2(3\cdot 0.8m - 0.2m)\right)}{12(9.7e9\,Pa)(5.04e^{-9})}$$

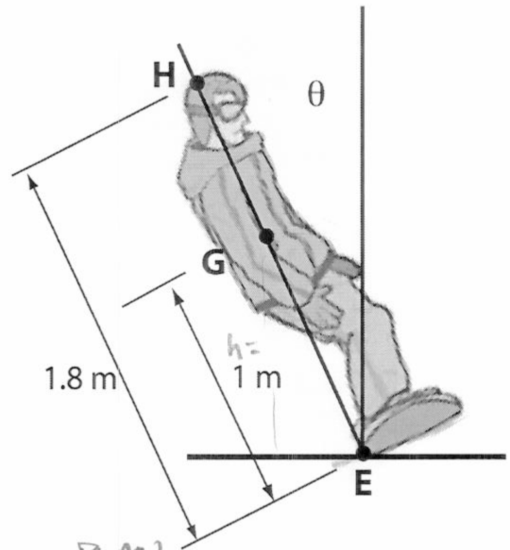
$$\boxed{\text{deflection at end is } = -0.118\,m}$$

Problem 1 (50 points)

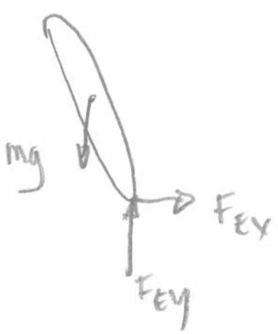
When snowboarding on a flat run, the edge of the board can get caught on the snow, causing the rider to rapidly fall backwards. Concussions are likely if the ~~velocity~~ Speed of the head exceeds 7 m/s on impact.

The rider is 1.8 m tall, 80 kg, and has a radius of gyration about the center of mass, G, of 0.2 m. He falls rigidly, and the edge of the board does not move relative to the ground.

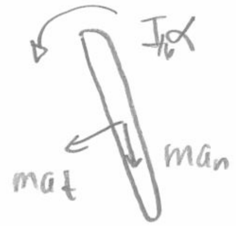
Will he likely get a concussion if his initial angular velocity in the upright position is ω = 2 rad/s **k**?



FBD



KIN



PLAN

$\Sigma M_{/E} = I_G\alpha + \vec{r} \times m\vec{a}_G$

solve $\alpha(\theta)$, integ. for ω_f

$\int_0^{\pi/2} \alpha(\theta)\, d\theta = \int_{\omega_i}^{\omega_f} \omega\, d\omega$

$\vec{V}_H = \vec{\omega}_f \times \vec{r}_{H/E}$ $\quad V_H > 7$ m/s

⇒ concussion

$h = 1$ m

$I_g = k^2 m \quad k = 0.2$ m

$a_t = h\alpha$

$$\Sigma M_{/E} = mgh\sin\theta = I_G\alpha + ma_t \cdot h$$

$$mgh\sin\theta = k^2 m\alpha + m \cdot h\alpha \cdot h$$

$$\alpha = \frac{\cancel{m}gh\sin\theta}{k^2\cancel{m} + mh^2}$$

$$\int_0^{\pi/2} \alpha\, d\theta = \int_{\omega_i}^{\omega_f} \omega\, d\omega$$

$$\frac{gh}{k^2+h^2}\int_0^{\pi/2} \sin\theta\, d\theta = \frac{1}{2}\omega^2\Big|_{\omega_i}^{\omega_f}$$

$$\frac{gh}{k^2+h^2}(-\cos\theta)\Big|_0^{\pi/2} = \frac{1}{2}\omega_f - \frac{1}{2}\omega_i$$

$$\frac{gh}{k^2+h^2} =$$

$$\frac{1}{2}\omega_f = \frac{gh}{k^2+h^2} + \frac{1}{2}\omega_i^2 = \frac{(9.8\ m/s^2)(1\ m)}{(0.2m)^2+(1m)^2} + \frac{1}{2}(2\ rad/s)^2$$

$$\frac{1}{2}\omega_f^2 = 11.42$$

$$\omega_f = \sqrt{2(11.42)} = 4.8\ rad/s$$

$$V_H = \omega_f \cdot l = (4.8\ rad/s)(1.8m)$$

$$\boxed{V_H = 8.6\ m/s} > 7\ m/s$$

concussion is likely