ISyE 4803 Final Exam
Spring 2011

Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK AND HAVE A WONDERFUL SUMMER.

Question 1
Question 2
Question 3
Question 4
Total

(25) **1.** Consider a model with $S = \{s_1, s_2\}$, $A_{s_1} = \{a_{11}, a_{12}\}$, $A_{s_2} = \{a_{21}, a_{22}, a_{23}\}$, $p\{s_1|s_1, a_{11}\} = 1$, $p\{s_1|s_1, a_{12}\} = 0.5$, $p\{s_1|s_2, a_{21}\} = 1$, $p\{s_1|s_2, a_{22}\} = 0$ and $p\{s_1|s_2, a_{23}\} = 0.75$.

a. (15) Provide all stationary deterministic policies.

b. (10) Compute the long run average gain under all the policies mentioned in part (a). Is the optimal gain constant?

(25) **2.** A decision maker observes a discrete time system which moves between states $\{s_1, s_2, s_3, s_4\}$ according to the following transition probability matrix:

$$P = \begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.5 & 0.0 \\ 0.1 & 0.0 & 0.8 & 0.1 \\ 0.4 & 0.0 & 0.0 & 0.6 \end{bmatrix}$$

At each point in time the decision maker may leave the system and receive a reward of $R = 20$ units or alternatively remain in the system and receive a reward of $r(s_i)$ units if the system occupies state s_i. If the decision maker decides to remain in the system, its state at the next decision epoch is determined by P. Assume that $r(s_i) = i$. The decision maker's objective is to maximize his discounted infinite horizon reward when the discount factor $\alpha = 0.8$.

a. (10) Formulate this as a Markov decision process problem. That is provide the state space, set of actions in each state, transition probabilities, rewards for each state action combination.

b. (15) Provide the primal and dual LP's to solve this problem.

(25) **3.** Consider a taxi station at an airport where taxis and customers arrive with respect to Poisson processes of rates 2/min and 3/min, respectively. Suppose that a taxi will wait no matter how many other taxis are present. However, if an arriving person does not find a taxi waiting he leaves to find an alternative transportation.

a. (15) What is the long run probability that an arriving customer gets a taxi?

b. (10) What is the average number of taxis waiting?

(25) **4.** Let $S = \{s_1, s_2, s_3\}$, $A_{s_1} = \{a_{11}, a_{12}\}$, $A_{s_2} = \{a_{21}\}$, and $A_{s_3} = \{a_{31}\}$, $r(s_1, a_{11}) = r(s_1, a_{12}) = 0$, $r(s_2, a_{21}) = 3$, and $r(s_3, a_{31}) = 4$, and $p(s_1|s_1, a_{11}) = p(s_2|s_1, a_{11}) = \frac{1}{2}$, $p(s_1|s_1, a_{12}) = \frac{2}{3}$, $p(s_3|s_1, a_{12}) = \frac{1}{3}$, $p(s_1|s_2, a_{21}) = 1$, and $p(s_1|s_3, a_{31}) = 1$. Use policy iteration to compute the long run average reward optimal policy.