

Math 2602 K1-K3
Spring 2014
Midterm 1 practice
1/30/14
Time Limit: 80 Minutes

Name (Print): _____

Section _____

This exam contains 5 pages (including this cover page) and 8 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, but you *can* use non symbolic calculator on this exam.

You are required to show your work on each problem on this exam.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Total:	80	

1. (10 points) Show that $n(n+1)(2n+1)$ is divisible by 3 for all integers n .
2. (10 points) Prove that if a and $a+b$ are rational numbers then b is rational.

3. (10 points) Show that $\neg(p \rightarrow q) \leftrightarrow (p \wedge \neg q)$ is a tautology.

4. (10 points) Show the following logical equivalence $p \rightarrow (q \vee r) \Leftrightarrow (p \wedge \neg q) \rightarrow r$.

5. (10 points) The binary relation \mathcal{R} is defined by $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \leq y\}$. Is \mathcal{R} a) Reflexive, b) Symmetric, c) Antisymmetric, d) Transitive ?

Justify your answer.

6. (10 points) For integers a and b define $a \sim b$ if $a - b$ is divisible by 5.

a) Show that \sim defines an equivalence relation on \mathbb{Z} .

b) What are the equivalence classes for \sim ?

7. (10 points) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x|x|$. Check if f is one-to-one and onto function. If so find the inverse function of f .

8. (10 points) Check if the sets have the same cardinality and justify your answer.

- a) $\{\sqrt{n} \mid n \in \mathbb{N} \text{ and } n > 10\}$ and \mathbb{N} .
b) The intervals (a, b) and (c, ∞) . Assume $a < b$.