MATH 2603, Fall 2015, Quiz 6, Oct 21 2015: Closed book, no calculators.
Instructor: Esther Ezra.

You can answer all questions on this sheet, but may use extra sheets (from your personal notepad) if needed.

Name ______________ GT IDnumber ______________

Solution

Problem 1. (50 points)

Solve the recurrence relation

$$a_n = 5a_{n-1} - 2a_{n-2} + 3n^2,$$

where $a_0 = 0, a_1 = 3$.

$$a_n = \frac{6}{\sqrt{17}}\left(\frac{5+\sqrt{17}}{2}\right)^n - \frac{6}{\sqrt{17}}\left(\frac{5-\sqrt{17}}{2}\right)^n - \frac{3}{2}n^2 - \frac{3}{2}n - 3$$

+) Solve for homogeneous part

$\lambda^2 - 5\lambda + 2 = 0$

$\to \lambda = \dfrac{5 \pm \sqrt{25 - 4.2.1}}{2}$

$= (5 \pm \sqrt{17})/2$

$\to q_n = c_1\left(\frac{5+\sqrt{17}}{2}\right)^n + c_2\left(\frac{5-\sqrt{17}}{2}\right)^n$

+) Solve for particular solution

Guess $p_n = (An^2 + Bn + C)$

Solve A, B, C in

$An^2 + Bn + C = 5[A(n-1)^2 + B(n-1) + C] - 2[A(n-2)^2 + B(n-2) + C] + 3n^2$

$\to A = B = -3/2$; $C = -3$

$\to a_n = q_n + p_n$ and solve to get $c_1 = -c_2 = \frac{6}{\sqrt{17}}$

Problem 2. (50 points)

Write a recursive algorithm (use either a verbal description, or a pseudo-code) to search an element in a sorted array of n numbers. How many comparisons does your algorithm perform?

* Since the array is sorted we can do binary search
  - Let the element to be searched for be x.
  - Divide the arrays into 2 in the middle (each new set contains $\frac{n}{2}$ elements if n is even; if n is odd, one set will contain $\lfloor \frac{n}{2} \rfloor$ and the other $\lceil \frac{n}{2} \rceil$)
  - Call the new sets A and B and compare x with the last element ($\lfloor \frac{n}{2} \rfloor$th element) in A. If x is less than that then x must be in A, otherwise x in B.
  - Repeat this until we find x.
* The algorithm will perform, on the order of, $\log n$ comparisons
* If linear search is done then at most n comparisons