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Print Your Name: Key – 1

T.A. or Section Number: ____________

WORK ALL OF THE FIRST THREE PROBLEMS (NUMBERS 1-3).

1. (14 points) Given a binomial distribution with $n = 18$ and $p = \frac{1}{3}$, use the normal approximation to the binomial to estimate $Pr(X \leq 7)$.

$\mu = np = (18)(\frac{1}{3}) = 6 \qquad \sigma = \sqrt{npq} = \sqrt{18 \cdot \frac{1}{3} \cdot \frac{2}{3}} = \sqrt{4} = 2$

$Pr(X \leq 7) = Pr(X \leq 7.5) = Pr\left(Z \leq \frac{7.5-6}{2}\right)$

$= Pr(Z \leq 0.75)$

$= \boxed{0.7734}$

2. (18 points) Use the method of Gauss-Jordan elimination to solve the following system of equations. You should continue your row operations until your matrix is in RREF. SHOW ALL YOUR WORK AND CLEARLY LABEL EACH ROW OPERATION.

$$x + 2y + z = 1$$
$$y + 2z = 5$$
$$2x - y + 3z = -1$$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 2 & -1 & 3 & -1 \end{array}\right] \xrightarrow{R_3 = R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & -5 & 1 & -3 \end{array}\right]$

$\xrightarrow[R_3 = R_3 + 5R_2]{R_1 = R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -9 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 11 & 22 \end{array}\right] \xrightarrow{\frac{1}{11}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -9 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$\xrightarrow[R_2 = R_2 - 2R_3]{R_1 = R_1 + 3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$ so

$x = -3$
$y = 1$
$z = 2$

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T.A. or Section Number: ______________________

WORK ALL OF THE FIRST THREE PROBLEMS (NUMBERS 1-3).

- - 2. (18 points) Use the method of Gauss-Jordan elimination to solve the-following system

se the normal

of equations. You should continue your row operations until your matrix is in RREF. SHOW ALL YOUR WORK AND CLEARLY LABEL EACH ROW OPERATION.

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3. Let $A = \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.7 \end{bmatrix}$ represent the input-output matrix for an economy with two industries. Follow the steps below to find the solution to the input-output problem.

(a) (8 points) Evaluate $I - A$.

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{bmatrix}$$

(b) (12 points) Find the inverse of your answer in (a), $(I - A)^{-1}$.

$$(I - A)^{-1} = \frac{1}{\underbrace{(0.4)(0.3)}_{0.12} - \underbrace{(-0.2)(-0.1)}_{-0.02}} \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}$$

$$= \frac{1}{0.1} \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

(c) (10 points) If the consumers demand is $\begin{bmatrix} 100 \\ 200 \end{bmatrix}$, determine the amount that each industry should produce by calculating $X = (I - A)^{-1} D$.

$$X = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \end{bmatrix} = \begin{bmatrix} 500 \\ 1000 \end{bmatrix}$$

$X_1 = \$500$ worth

$X_2 = \$1000$ worth

3. Let A : represent the input-output matrix for an economy with two industries. Follow the steps below to find the solution to the

input~output problem. (a) (8 points) Evaluate $I - A$.

(c) (10 points) If the consumers demand is , determine the amount that each industry

should produce by calculating $X = (I - A)^{-1}D$.

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Print Your Name: Key-1

T.A. or Section Number: ________

WORK ONLY THREE OF THE LAST FOUR PROBLEMS (NUMBERS 4-7). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST THREE PROBLEMS WILL BE GRADED.

4. (14 points) A certain distribution has a mean of 10 and a standard deviation of 2. Use Chebychev's inequality to find a lower bound for the probability that a random outcome lies between 4 and 16. Simplify your final answer.

$\mu = 10,\ \sigma = 2$

$$Pr(4 \le X \le 16) = Pr(10-6 \le X \le 10+6)$$

$[\text{so } c = 6]$

$$\ge 1 - \frac{2^2}{6^2}$$

$$= 1 - \frac{1}{9} = \boxed{\frac{8}{9}}$$

5. (14 points) A normal distribution has a mean of 60 and a standard deviation of 8. Find the probability that a random score will be greater than 72. Simplify your final answer.

$\mu = 60,\ \sigma = 8$

$$Pr(X > 72) = Pr\left(Z > \frac{72-60}{8}\right)$$

$$= Pr\left(Z > \frac{12}{8} = 1.5\right)$$

$$= 1 - \text{area at } 1.5$$

$$= 1 - 0.9332$$

$$= \boxed{0.0668}$$

WORK ONLY THREE OF THE LAST FOUR PROBLEMS (NUMBERS 4-7 WRITE ”OMIT” OVER THE PROBLEM YOU DO NOT WANT GRADED.

IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY

THE FIRST THREE PROBLEMS WILL BE GRADED.

4. (14 points) A certain distribution has a mean of 10 and a standard deviation of 2. Use Chebychev's inequality to find a lower bound for the probability that a random outcome lies between 4 and 16. Simplify your final answer.

5. (14 points) A normal distribution has a mean of 60 and a standard deviation of 8. Find the probability that a random score will be greater than 72. Simplify your final

6. (14 points) Given the matrices A and B below, find AB and BA, or explain why the product is not possible.

$$A = \overset{2\times 3}{\begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \end{bmatrix}}, \quad B = \overset{3\times 3}{\begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 4 \\ 3 & -3 & -2 \end{bmatrix}}$$

Cannot multiply BA because # of colums in B $\neq$ # rows of A

$$AB = \begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 5 \\ 2 & 0 & 4 \\ 3 & -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4+3 & -3+0-3 & 15-8-2 \\ 0-2+12 & 0+0-12 & 0-4-8 \end{bmatrix} = \begin{bmatrix} 2 & -6 & 5 \\ 10 & -12 & -12 \end{bmatrix}$$

7. (14 points) SET UP the matrices A, A^T, X, and Y that you would use to find the least-squares line through the data points (1,1), (2,4), (3,6), and (4,7). DO NOT SOLVE THE LEAST-SQUARES PROBLEM.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} m \\ b \end{bmatrix}, \quad y = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 7 \end{bmatrix}$$

6. (14 points) Given the matrices A and B below, find AB and BA, or explain why the

product is not possible.

7. (14 points) SET UP the matrices A, AT, X, and Y that you would use to find the 1east—squares line through the data points (1,1), (2,4), (3,6), and (4,7). DO NOT SOLVE THE LEAST-SQUARES PROBLEM.

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Print Your Name: Key-2

T.A. or Section Number: ______

WORK ALL OF THE FIRST THREE PROBLEMS (NUMBERS 1-3).

1. Let $A = \begin{bmatrix} 0.7 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}$ represent the input-output matrix for an economy with two industries. Follow the steps below to find the solution to the input-output problem.

(a) (8 points) Evaluate $I - A$.

$$I-A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.7 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3 & -0.2 \\ -0.1 & 0.4 \end{bmatrix}$$

(b) (12 points) Find the inverse of your answer in (a), $(I - A)^{-1}$.

$$(I-A)^{-1} = \frac{1}{\underbrace{(0.3)(0.4)-(-0.1)(-0.2)}_{0.12-0.02}} \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}$$

$$= \frac{1}{0.1} \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$$

(c) (10 points) If the consumers demand is $\begin{bmatrix} 100 \\ 200 \end{bmatrix}$, determine the amount that each industry should produce by calculating $X = (I - A)^{-1}D$.

$$X = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 100 \\ 200 \end{bmatrix} = \begin{bmatrix} 800 \\ 700 \end{bmatrix}$$

$x_1 = \$800$ worth
$x_2 = \$700$ worth

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TA. or Section Number: ________._.._____________

WORK ALL OF THE FIRST THREE PROBLEMS (NUMBERS 1-3).

1. Let A = represent the input-output matrix for an economy with two industries. Follow the steps below to find the solution to the input-output problem. (a) (8 points) Evaluate I — A.

(0) points) If the consumers demand is , determine the amount that each industry

should produce by calculating X = (I A)_1D.

2. (18 points) Use the method of Gauss-Jordan elimination to solve the following system of equations. You should continue your row operations until your matrix is in RREF. SHOW ALL YOUR WORK AND CLEARLY LABEL EACH ROW OPERATION.

$$x - y + 3z = 10$$
$$2x + y - 4z = -3$$
$$y + 5z = 9$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 2 & 1 & -4 & -3 \\ 0 & 1 & 5 & 9 \end{array}\right] \xrightarrow{R_2 = R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & 3 & -10 & -23 \\ 0 & 1 & 5 & 9 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 3 & 10 \\ 0 & 1 & 5 & 9 \\ 0 & 3 & -10 & -23 \end{array}\right] \xrightarrow[R_3 = R_3 - 3R_2]{R_1 = R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 0 & 8 & 19 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & -25 & -50 \end{array}\right]$$

$$\xrightarrow{-\frac{1}{25}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 8 & 19 \\ 0 & 1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 = R_2 - 5R_3]{R_1 = R_1 - 8R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

So $x = 3$, $y = -1$, $z = 2$

3. (14 points) Given a binomial distribution with $n = 18$ and $p = \frac{1}{3}$, use the normal approximation to the binomial to estimate $Pr(X \leq 8)$.

$$\mu = 18 \cdot \frac{1}{3} = 6 \qquad \sigma = \sqrt{npq} = \sqrt{18 \cdot \frac{1}{3} \cdot \frac{2}{3}} = \sqrt{4} = 2$$

$$Pr(X \leq 8) = Pr(X \leq 8.5)$$
$$= Pr\left(Z \leq \frac{8.5 - 6}{2}\right)$$
$$= Pr(Z \leq 1.25)$$
$$= \boxed{0.8944}$$

(18 points) Use the method of Gauss—Jordan elimination to solve the following system of equations. You should continue your row

operations until your matrix is in REEF SHOW ALL YOUR WORK AND CLEARLY LABEL EACH ROW OPERATION

3. (14 points) Given a binomial distribution n = 18 and p approximation to the binomial to estimate 5 8

se the normal

Print Your Name: Key – 2

T.A. or Section Number: ____________

WORK ONLY THREE OF THE LAST FOUR PROBLEMS (NUMBERS 4-7). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST THREE PROBLEMS WILL BE GRADED.

4. (14 points) SET UP the matrices A, A^T, X, and Y that you would use to find the least-squares line through the data points (1,2), (2,5), (3,7), and (4,9). DO NOT SOLVE THE LEAST-SQUARES PROBLEM.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \\ 4 & 1 \end{bmatrix}, \quad A^T = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} m \\ b \end{bmatrix}, \quad Y = \begin{bmatrix} 2 \\ 5 \\ 7 \\ 9 \end{bmatrix}$$

5. (14 points) A certain distribution has a mean of 12 and a standard deviation of 3. Use Chebychev's inequality to find a lower bound for the probability that a random outcome lies between 6 and 18. Simplify your final answer.

$\mu = 12, \quad \sigma = 3$

$$\Pr(6 \le X \le 18) = \Pr(12-6 \le X \le 12+6)$$

(so $c = 6$)

$$\ge 1 - \frac{3^2}{6^2}$$

$$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

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T.A. or Section Number: ___“..w________________

WORK ONLY THREE OF THE LAST FOUR PROBLEMS (NUMBERS 4-7). WRITE OVER THE PROBLEM YOU DO NOT WANT GRADED.

IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY

THE FIRST THREE PROBLEMS WILL BE GRADED.

4. (14 points) SET UP the matrices A, AT, X, and Y that you would use to find the least-squares line through the data points (1,2), (2,5), (3,7), and (4,9). DO NOT SOLVE THE LEAST-SQUARES PROBLEM.

5. (14 points) A certain distribution has a mean of 12 and a standard deviation of 3. Use C11ebychev‘s inequality to find a lower bound for the probability that a random outcome

lies between 6 and 18. Simplify your final answer.

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6. (14 points) A normal distribution has a mean of 60 and a standard deviation of 8. Find the probability that a random score will be greater than 66. Simplify your final answer.

$\mu = 60, \ \sigma = 8$

$$Pr(X \geq 66) = Pr\left(Z > \frac{66-60}{8}\right)$$

$$= Pr\left(Z > \frac{6}{8} = 0.75\right)$$

$$= 1 - \text{area at } 0.75$$

$$= 1 - 0.7734$$

$$= \boxed{0.2266}$$

7. (14 points) Given the matrices A and B below, find AB and BA, or explain why the product is not possible.

$$A = \begin{bmatrix} 1 & 4 & -2 \\ -2 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 & 4 \\ 0 & 1 & 3 \\ -4 & 4 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 4 & -2 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -2 & 4 \\ 0 & 1 & 3 \\ -4 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+8 & -2+4-8 & 4+12-2 \\ -4+0-12 & 4+0+12 & -8+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 & 14 \\ -16 & 16 & -5 \end{bmatrix}$$

BA is not defined since # colums of B $\neq$ # rows of A

6. (14 points) A normal distribution has a mean of 60 and a standard deviation of 8.

Find the probability that a random score will be greater than 66. Simplify your final

answer.

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7. (14 points) Given the matrices A and B below, find AB and BA, or explain why the

product is not possible.