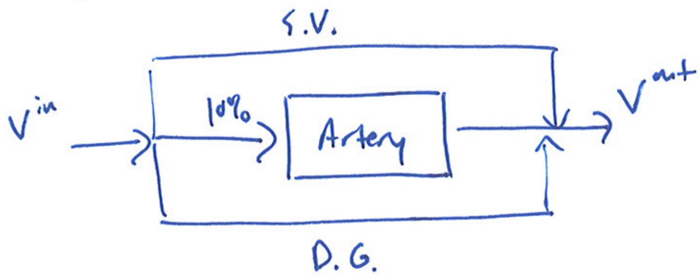
Name: Solutions

Problem #1 (25 points)

A double bypass procedure is performed using a saphenous vein (S.V.) and an artificial Dacron graft (D.G.) of comparable lengths to treat an occluded artery that only permits passage of 10% of the original blood flow. If total blood flow is completely restored following the surgery and the saphenous vein is 50% wider than the Dacron graft, what percentage of the total volumetric blood flow passes through the Dacron graft?



Basis: some volume or time if volumetric rate used (i.e. 100 mL or 100 mL/min → 1 min)

Assumptions:

- $l_{S.G.} = l_{D.G.}$ (given)
- cylindrical bypass grafts ∴ cross-sect area $= \pi r^2$

$$\text{diameter}_{S.V.} = 1.5 \text{ diameter}_{D.G.}$$

$$\therefore \quad A_{S.V.} = (1.5)^2 \pi = 2.25\pi$$

$$A_{D.G.} = (1)^2 \pi = \pi$$

$$V^{in} = V^{out} = V_{S.V.} + V_{artery} + V_{D.G.}$$

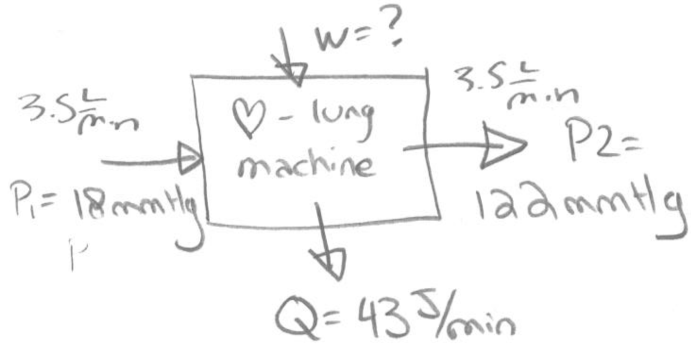
$$1 = 2.25 \cancel{\pi} x + 0.1 + \cancel{\pi} x$$

$$\Rightarrow \quad 0.9 = 3.25 \cancel{\pi} x \qquad x = 0.277$$

(relative flow thru D.G.)

convert to % & sig figs

$$= \boxed{30\%}$$



Basis

1min

Assumptions

- steady state
- $\Delta H = \emptyset$ (no potential energy)
- $\Delta V = \emptyset$ (no kinetic energy)

Energy balance

$$W = m(\cancel{\Delta \hat{KE}}^{\emptyset} + \cancel{\Delta \hat{PE}}^{\emptyset}) + m\int_{P1}^{P2} \hat{V}\,dP + Q$$

$\star$ Power $= \frac{J}{S} = \frac{Pa \cdot m^3}{S} = \frac{kg \cdot m^2}{S^3}$

$$W_{pump,\ ideal} = m\int_{P1}^{P2} \hat{V}\,dP + Q$$

$P_2 = \frac{122mmHg \mid 133.3Pa}{\mid 1mmHg} = 16262.6Pa$ $\quad P_1 = \frac{18mmHg \mid 133.3Pa}{\mid 1mmHg} = 2399.4Pa$

$\hat{V} = \frac{1}{\rho} = \frac{mL}{1.06g} \rightarrow \frac{L \mid .001m^3}{1.06kg \mid 1L} = .000943 \frac{m^3}{kg}$

$m = 3.5L\left(\frac{1.06kg}{1L}\right) = 3.71kg$

(could also use $\int_{P1}^{P2} V\,dP$ where $V = .0035m^3$)

$$W_{pump,\ ideal} = 3.71\frac{kg}{min}\left(.000943\frac{m^3}{kg}\right)(2399.4Pa - 16262.6Pa) + 43\frac{J}{min})$$

$= 48.5 + 43$ Joules

$= 91.521$ Joules/min

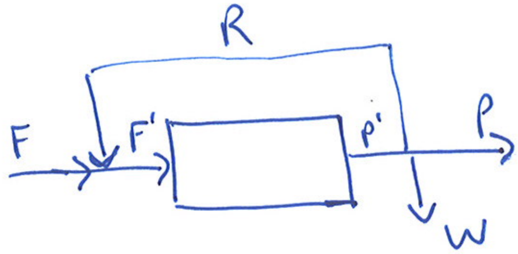
$W_{pump\ ideal} = W_{pump,\ actual}(.74)$

$W_{pump,\ actual} = 123.67 \frac{Joules}{min} = \boxed{2.1 \text{ watts}}$

Name: Solutions

Problem #3 (25 points)

The overall conversion of component into product by a bioreactor system with an integrated recycle stream is 90%, whereas the single-pass conversion for the bioreactor itself is only 60%. If the concentration of the entering component stream is 10 mg/ml and the volume output from the bioreactor is divided equally among the product, waste and recycle streams, what is the concentration of component in the recycle stream?



Basis: 1 mL of F

Assumptions:

- 1 product for every converted component

Overall balance:

$$F = P + W \qquad P = W = R$$

$$1.0\ \text{mL} = 2P \quad \therefore \quad P = 0.5\ \text{mL} = W = R$$

Component/product:

$$F_c = P_c + W_c \qquad P_c = 0.9 F_c$$

$$10\ \text{mg} = 0.9\,(10\ \text{mg}) + W_c \qquad W_c = 1\ \text{mg}$$

Mixer:

$$F + R = F' \quad \therefore \quad F' = 1.5\ \text{mL}$$

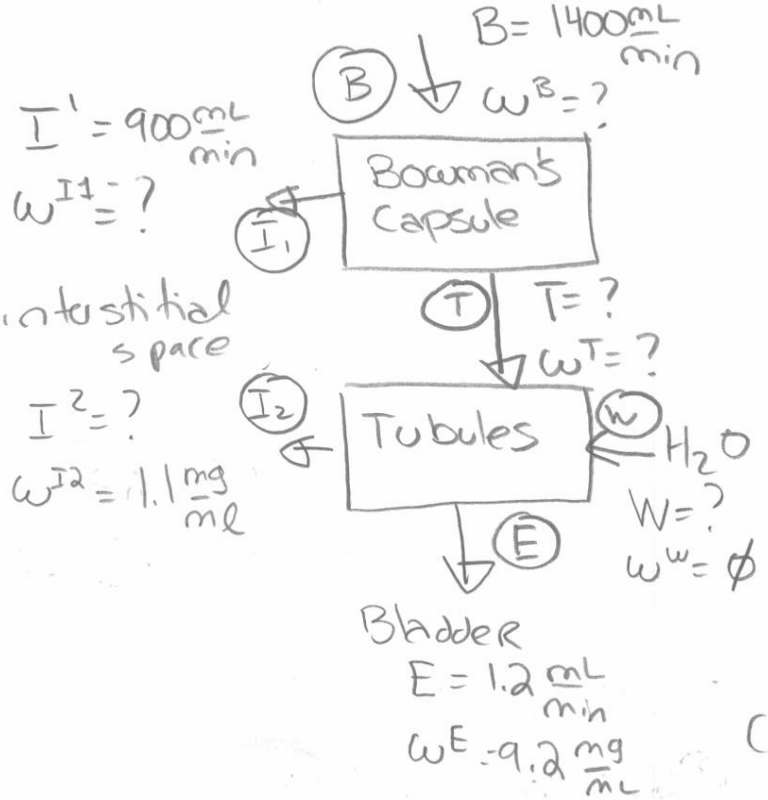
$$0.6 F_c' = P_c \quad \therefore \quad F' = 15\ \text{mg}$$

Component/product:

$$F_c + R_c = F_c'$$

$$R_c = 15 - 10 = 5\ \text{mg}$$

$$\Rightarrow [R_c] = \frac{5\ \text{mg}}{0.5\ \text{mL}} = \boxed{10\ \text{mg/mL}}$$



Legend

- ω^i = concentration of urea $\left(\frac{mg}{mL}\right)$ in stream i

Basis

1 min

Assumptions

- steady state
- no reaction

Given Relations

(1) $I^1 \omega^{I1} = T\omega^T(.5)$

$\Rightarrow 900 \frac{mL}{min}(\omega^{I1}) = T\omega^T(.5)$

(2) $I^2 \omega^{I2} = T\omega^T(.4)$

$\Rightarrow I^2 \left(1.1 \frac{mg}{mL}\right) = T\omega^T(.4)$

mass balance on urea; Bowman's Capsule

(3) $B\omega^B = I^1(\omega^{I1}) + T\omega^T$

$\Rightarrow 1400 \frac{mL}{min}(\omega^B) = 900 \frac{mL}{min}(\omega^{I1}) + T\omega^T$

mass balance on urea; Tubules

(4) $T\omega^T = I^2 \omega^{I2} + E\omega^E$

$\Rightarrow T\omega^T = I^2\left(1.1 \frac{mg}{mL}\right) + 1.2 \frac{mL}{min}\left(9.2 \frac{mg}{mL}\right)$

substitute (1) into (3)

(5) $B\omega^B = T\omega^T(.5) + T\omega^T = 1.5\,T\omega^T$

substitute (2) into (4)

$T\omega^T = T\omega^T(.4) + 11.04 \frac{mg}{min}$

(6) $\Rightarrow T\omega^T = 18.4\,mg/min$

Plug (6) into (5)

$1400 \frac{mL}{min} \omega^B = 1.5\left(18.4 \frac{mg}{min}\right)$

$\Rightarrow \boxed{\omega^B = .02 \frac{mg}{mL}}$