NAME: SOLUTIONS

GRADE:

ISyE 3044-B — Test #2

Fall 2010

No books and notes are allowed. You can use only the supplied formula sheet and the tables at the end.

1. [**10 points**] Short questions.

   (a) Use as many random numbers as needed from the following list to generate two independent realizations from the Poisson distribution with mean 1.5. Go from left to right.

   0.85 0.10 | 0.54 0.10 | 0.12 0.08 0.35 0.49

   $e^{-1.5} = 0.223$

   ANSWER: $X_1 = 1, X_2 = 1$

   (b) What was the original name of ExpertFit?

   ANSWER: UniFit

   How was it misspelled in the mid 1990s?

   ANSWER: UnFit

   (c) An entity uses the Seize and Release blocks to capture and free units of a Resource. Name the blocks that are used to capture and free a unit of a Transporter.

   ANSWER: Request / Free

   (d) Arena allows the definition of a Transporter with capacity 2. **True** False

   (e) Tee Kolmogorov-Smirnov test can be used with discrete distributions. **True** False

   (f) The value of an Arena Expression can be altered by an entity. **True** False

ExpertFit uses only the chi-square test!

2. **[8 points]** The following data are times between arrivals of orders for an SKU at a warehouse (in days):

0.75 2.34 0.83 4.35 0.32

(a) Assuming that the data follow the exponential distribution with a rate λ, compute the maximum likelihood estimate (m.l.e.) of λ.

ANSWER: $1/\bar{X} = 0.582$

(b) Find the m.l.e. of the mean interarrival time.

ANSWER: $\bar{X} = 1.72$

(c) Find the m.l.e. of the standard deviation of the interarrival time.

ANSWER: $\bar{X} = 1.72$

(d) Use the Kolmogorov-Smirnov test with type I error $\alpha = 0.10$ to assess the goodness-of-fit of the exponential distribution. Since the parameter λ is unknown, use the m.l.e. from part (a) and the appropriate adjusted test statistic.

ANSWER: We fail to reject

(e) Now assume that the above data are from a gamma distribution. Use the method of moments to estimate the shape parameter α and the scale parameter λ.

ANSWER: $\hat{\lambda} = 0.79$, $\hat{\alpha} = 1.36$

(e) $$\left.\begin{array}{l} \dfrac{\alpha}{\lambda} = 1.72 \\ \left(\dfrac{\alpha}{\lambda}\right)^2 + \dfrac{\alpha}{\lambda^2} = 5.15 = \dfrac{1}{5}\sum_{i=1}^{5} X_i^2 \end{array}\right\} \Rightarrow$$

(d) Sort the data!

$X_{(i)}$	.32	.75	.83	2.34	4.35
$\hat{F}(X_{(i)}) = 1 - e^{-\hat{\lambda} X_{(i)}}$	.169	.354	.383	.744	.921
$\frac{i}{5} - \hat{F}(X_{(i)})$	.030	.046	.217	.056	.079
$\hat{F}(X_{(i)}) - \frac{i-1}{5}$	.170	.154	-	.144	.121

Adjusted test statistic: $\left(.217 - \frac{0.1}{\sqrt{5}}\right)\left(\sqrt{5} - 0.01 + \frac{0.85}{\sqrt{5}}\right) = 0.332 < 0.990.$

3. **[6 points]** The random variable X has density function $f(x) = 1 - x/2$, $0 \le x \le 2$.

(a) Find the c.d.f. $F(x) = \Pr(X \le x)$.

ANSWER: $x - x^2/4$

(b) Use the inverse-transform method to derive a formula for generating realizations of X.

ANSWER: $X = 2 - 2\sqrt{1-U}$

(b) Solve $X - \frac{X^2}{4} = U$

$$\frac{X^2}{4} - X + 1 = 1 - U$$

$$\left(\frac{X}{2} - 1\right)^2 = 1 - U$$

$$\frac{X}{2} = 1 \pm \sqrt{1-U}$$

$$X = 2 \pm 2\sqrt{1-U}$$

The solution that lies in $[0,2]$ is $2 - 2\sqrt{1-U}$.

4. **[6 points]** The scope of a simulation model was to estimate the mean cost per month (say μ) for an inventory management system. The simulationist used 10 independent replications, each for one month, and the model delivered the following 95% confidence interval for μ: $(12500, 16000)$.

   (a) What is the point estimate for μ?

   ANSWER: midpoint = 14250

   (b) Compute a 90% confidence interval for μ.

   ANSWER: $14250 \pm 1417 = [12833, 15667]$

   (c) The simulationist would like to derive a 90% confidence interval for μ with a half-width ≤ 500 dollars. How many additional replications should s/he conduct?

   ANSWER: 55 additional replications

(b) Half-width $= 1750 = 2.26 \dfrac{S_{10}}{\sqrt{10}} \Rightarrow S_{10} = 2448.7$

The half-width of the 90% CI is

$$1.83 \cdot \frac{S_{10}}{\sqrt{10}} = 1417$$

(c) Solve

$$1.645 \frac{2448.7}{\sqrt{K}} \leq 500$$

$$K \geq \left\lceil \frac{1.645^2 (2448.7)^2}{500^2} \right\rceil = 65$$

Standard Normal Quantiles

α	0.10	0.05	0.025	0.005	0.01
$z_{1-\alpha} = \Phi^{-1}(1-\alpha)$	1.28	1.645	1.96	2.58	3.09

Critical Values $c_{1-\alpha}$ for Adjusted K-S Statistics

		α				
Case	Adjusted Test Statistic	0.15	0.10	0.05	0.025	0.01
All parameters known	$\left(\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}}\right) D_n$	1.138	1.224	1.358	1.480	1.628
$\text{Nor}(\bar{X}_n, S_n^2)$	$\left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}}\right) D_n$	0.775	0.819	0.895	0.995	1.035
$\text{Expo}(1/\bar{X}_n)$	$\left(D_n - \frac{0.2}{\sqrt{n}}\right)\left(\sqrt{n} - 0.01 + \frac{0.85}{\sqrt{n}}\right)$	0.926	0.990	1.094	1.190	1.308