Math 2602 **Exam 3** Spring 2015

Instructor: Sal Barone

Name: KEY

GT username: _____

Circle your TA/section:

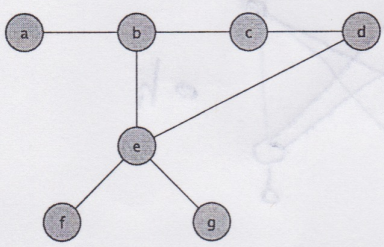
(F1) Qiqin (F2) Changong (F3) George (L1) Scott (L2) Vaidehi

1. No books or notes are allowed.
2. You will not need a calculator for this exam. All electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Good luck!

Page	Max. Possible	Points
1	28	
2	20	
3	28	
4	24	
Total	100	

1. Give an example of two graphs $\mathcal{G}_1, \mathcal{G}_2$ each with exactly 4 vertices and 4 edges such that $\mathcal{G}_1$ and $\mathcal{G}_2$ are not isomorphic. (8 pts.)

2. Consider the graph $\mathcal{G}$ whose model is pictured below. (5 pts. each)



(a) Is the graph bipartite? If so, give a bipartition of $\mathcal{G}$.

Yes. $V_1 = \{a, c, e\}$

$V_2 = \{b, d, f, g\}$

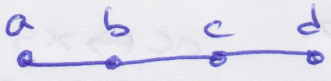
(b) Is the graph Eulerian? Justify your answer.

No. Some vertices have odd degree.

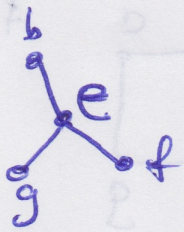
(c) Is there a subgraph of $\mathcal{G}$ that is isomorphic to K_3? If so, define it here.

No, but there is a subgraph homeomorphic to K_3.

(d) Find representatives for the isomorphism classes of connected subgraphs of $\mathcal{G}$ that have exactly 3 edges.



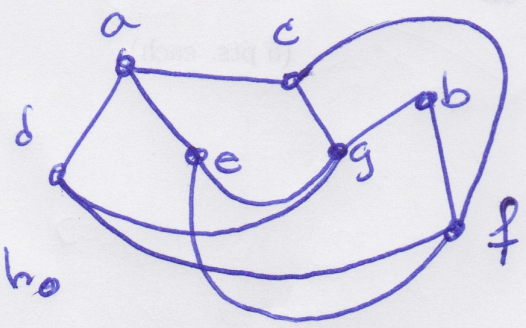
and



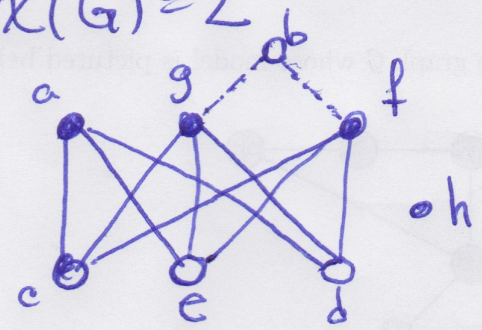
3. Consider the following graph (5 pts. each)

$$\mathcal{G} = (\{a, b, c, d, e, f, g, h\}, \{ac, ad, ae, bf, bg, cf, cg, df, dg, ef, eg\}).$$

(a) Draw a model for $\mathcal{G}$ and find the chromatic number of $\mathcal{G}$, and provide a $\chi(\mathcal{G})$-coloring of $\mathcal{G}$.



$\chi(G) = 2$



cycle?

(b) Is $\mathcal{G}$ Hamiltonian? If so, find a Hamiltonian ~~path~~. If not, justify your answer.

No. It is disconnected.

(c) Is $\mathcal{G}$ planar? Justify your answer.

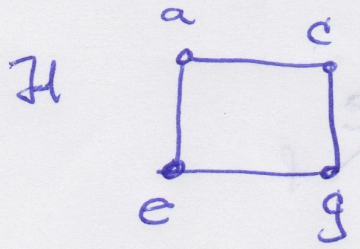
No. Has $K_{3,3}$ subgraph.

(d) Exhibit a subgraph $\mathcal{H}$ of $\mathcal{G}$ which is isomorphic to the graph

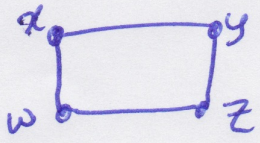
$$\mathcal{C} = (\{x, y, z, w\}, \{xy, yz, zw, wx\})$$

and provide an explicit isomorphism between $\mathcal{H}$ and $\mathcal{C}$. Check explicitly that your isomorphism is edge preserving.

$\mathcal{H}$



$\mathcal{C}$



$f: \{a, c, e, g\} \to \{x, y, z, w\}$

$a \mapsto x$ $\quad ac \mapsto xy$

$c \mapsto y$ $\quad cg \mapsto yz$

$e \mapsto w$ $\quad ge \mapsto zw$

$g \mapsto z$ $\quad ea \mapsto wx$

4. Short answer section: put a number, function, or graph in each box. Please show your work for potential partial credit, and *give the best possible answer.* (7 pts. each)

(i) Let $\mathcal{G}$ be the graph in Problem **2.**
The number of spanning trees of $\mathcal{G}$ is equal to

4

one 4-cycle.

(ii) How many colorings does K_5 have? That is, for how many values of n does K_5 have an n-coloring.

1

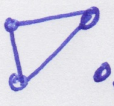
K_5 does not have a 1-coloring.
K_5 does not have a 2-coloring.
K_5 does not have a 3-coloring
K_5 does not have a 4-coloring.

K_5 has a 5-coloring ✓.

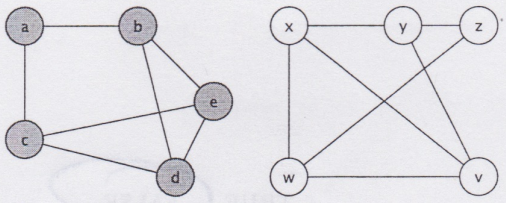
(iii) How many subgraphs of K_4 have exactly 3 edges?

24

$\binom{6}{3} = 20$

but  ← include or not.

(iv) Two graphs are shown. Give an explicit isomorphism between them.



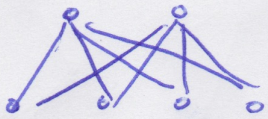
$f: \{a,b,c,d,e\} \to \{x,y,z,w,v\}$

$a \longmapsto z$
$b \longmapsto y$
$c \longmapsto w$
$d \longmapsto v$
$e \longmapsto x$

5. True and false questions. Instructions: **In the questions below assume that $\mathcal{G}$ is a graph and $\mathcal{H}$ is a subgraph of $\mathcal{G}$.** For each statement below, circle TRUE if the statement is true and circle FALSE otherwise. Your work will NOT be graded.

(4 pts. each)

(i) For every $n \geq 2$, $K_{n,n+2}$ contains a cycle of length $2n+2$. TRUE (FALSE)

$K_{2,4}$?  no.

(ii) If $\mathcal{G}$ has degree sequence 2,2,2,1,1 then the degree sequence of $\mathcal{H}$ consists entirely of 2's and/or 1's. TRUE (FALSE)

could be 0 (isolated vertex)

(iii) If $\mathcal{G}$ is a tree and $\mathcal{H}$ contains an edge then the chromatic number of $\mathcal{H}$ is equal to 2. (TRUE) FALSE

(iv) If $\mathcal{G}$ is a tree and $\mathcal{H}$ has no vertices of degree zero, then $\mathcal{H}$ is a tree. TRUE (FALSE)

trees are connected

(v) If $\mathcal{H}$ is isomorphic to K_n for some $n \geq 3$, then $\mathcal{G}$ is not bipartite. (TRUE) FALSE

(vi) There is a planar graph with exactly 6 vertices and 12 regions. TRUE (FALSE)

planar $\Rightarrow$ $V - E + R = 2$