ISyE 2028, Fall 2015
Homework 2
100 points total. 10 points each question.

This homework is due Tuesday Sept 15 in class.

• Please remember to staple if you turn in more than one page.

• Please make sure to **SHOW ALL WORK** in order to receive full credit.

1. Suppose you a gambler at Las Vegas. The dealer has agreed to reveal you partial information about whether the card color is red or black. Given that partial information, you draw a card at random from the complete 52 deck of cards, and bet on cards. Being a smart GT student, you will calculate the following probabilities to win the bet:
   (a) $P(\text{card is heart 'A'}|\text{the card is red})$
   (b) $P(\text{card is heart}|\text{the card is red})$
   (c) $P(\text{card is King}|\text{the card is red})$
   (d) Calculate $P(\text{card is King})$. Compare this with your calculation in c., what can you say about whether or not knowing the color of the card is useful information? Is the event "card is King" independent of "card is red" or not?

2. A college professor never finishes his lecture before the end of the hour and always finishes his lecture within 2 min after the hour. Let X be the time that elapses between the end of the hour and the end of the lecture and suppose the pdf of X is $f(x) = kx^2$ for $0 \leq x \leq 2$ and $f(x) = 0$ elsewhere.
   (a) Find the value of k.
   (b) What is the probability that the lecture ends within 1 min of the end of the hour?
   (c) What is the probability that the lecture continues beyond the hour for between 60 and 90 sec?
   (d) What is the probability that the lecture continues for at least 90 sec beyond the end of the hour?
   (e) Find the cdf of X, $F(x)$ and repeat parts (b), (c) and (d) using $F(x)$ instead of $f(x)$.

3. For each of the following two situations discuss whether the Binomial assumptions would be reasonable:
   (a) Twenty animals are selected from a large population in which a particular trait occurs with probability .2. The number of animals that have this trait is recorded.
   (b) A rat is being trained to carry out a swimming task in which it must find and recover a small piece of food. The animal is put into the water on twenty successive occasions and the number of times it succeeds in getting the food within 10 seconds is recorded.

4. The firing rates of 25 neurons are recorded in a particular region of the brain of a monkey under each of two experimental conditions. The number of neurons in which the firing rate is lower in Condition 1 than in Condition 2 is counted. Suppose that in the population of neurons the probability of a higher firing rate for Condition 2 is .38.

   The R commands for calculating binomial probability, cumulative probability and quantile are as follows:

   - To find $P(X = x)$ for $X \sim Binomial(n, p)$, the command is

     `dbinom(x,n,p)`

     You will need to input all three values.
   - To find the cumulative probability $P(X \leq x)$ for $X \sim Binomial(n, p)$, the command is

     `pbinom(x,n,p)`
   - To find the quantile (inverse cumulative probability) x such that $P(X \leq x) = a$ for $X \sim Binomial(n, p)$, the command is

     `qbinom(a,n,p)`

     Note that for this, you will input the probability value a to find x.

   For $X \sim Binomial(n, p)$, n defines the number of trials and p denotes the probability of success. For a help menu about the binomial distribution functions type the command `help(qbinom)`.

   Using the R commands for the Binomial distribution, answer the following questions. To answer these questions, you will need to show the derivation and write down the R commands that you used for computations.

   (a) What is the probability that exactly 7 of the 25 neurons have higher firing rates for Condition 2?

   (b) What is the probability that 8 or more have higher firing rates for Condition 2?

   (c) What is the probability that 8 or fewer have higher firing rates for Condition 2?

   (d) What is the probability that more than 8 have higher firing rates for Condition 2?

   (e) What is the probability that from 8 to 10 have higher firing rates for Condition 2?

5. Astronomers treat the number of stars in a given volume of space as a Poisson random variable. The density in the Milky Way Galaxy in the vicinity of our solar system is one star per 16 cubic light-years.

   (a) What is the probability of two or more starts in 16 cubic light-years?

   (b) How many cubic light-years of space must be studied so that the probability of one or more stars exceeds 0.95?

6. Integration by parts is required. The probability density function for diameter of a drilled hole in mm is $10e^{-10(x-5)}$ for $x > 5$ mm. Although the target diameter is 5mm, variations, tool wear, and other nuisances produce diameters larger than 5mm.

(a) Determine the mean and variance of the diameter of the holes.
(b) Determine the probability that a diameter exceeds 5.1 mms.

7. The time between arrivals of small aircraft at a county airport is exponentially distributed with a mean of one hour.
   (a) What is the probability that more than three aircraft arrive within an hour?
   (b) If 30 separate one-hour intervals are chosen, what is the probability that no interval contains more than three arrivals?
   (c) Determine the length of an interval of time (in hours) such that the probability that no arrivals occur during the interval is 0.10.

8. The scores on an exam, represented by a random variable X, followed a normal distribution with mean 75 and standard deviation of 5. Since the exam was a bit hard, the instructor "corrected" the scores by applying the following linear combination: $0.8X + 20$ to all grades. Let Y be the random variable representing the corrected scores.
   (a) What is the distribution of Y?
   (b) find $P(Y > 75)$.
   (c) Find the first quartile $y_{.25}$ (the 25th percentile) of the corrected scores.

9. The yearly income of households in a certain city follows a distribution with mean $\mu = \$25,000$ and standard deviation $\sigma = \$10,000$.
   (**Comment:** Income is an example of a random variable that does *not* have a normal distribution. the distribution of income is skewed to the right)
   (a) A random sample of 400 households from the city is chosen. What is the probability that their average yearly income is between $24,000 and $25,500
   (b) Find the probability that the yearly income of a *single* randomly chosen household (from this city) is less than $26,000, or explain why you cannot find this probability with the given information.

10. Suppose that a certain compound melts at a temperature that may be considered a random variable with mean $150°C$ and standard deviation $2°C$. What are the mean and standard deviation of the melting temperature in degrees Fahrenheit? (Note that $°F = 1.8°C + 32$.)