ISYE 3133 Homework 1 Summer 2015

Due Wednesday, May 27 at noon

1. There are 3 factories on a lake (1, 2, and 3). Each emits 2 types of pollutants (1 and 2) into the lake. If the waste from each factory is processed, the amount of pollution in the lake can be reduced. It costs \$15 to process one ton of factory 1 waste, and each ton processed reduces the amount of pollutant 1 by .1 ton and the amount of pollutant 2 by .45 ton. It costs \$10 to process one ton of factory 2 waste, and each ton processed reduces the amount of pollutant 1 by .2 ton and the amount of pollutant 2 by .25 ton. It costs \$20 to process one ton of factory 1 waste, and each ton processed reduces the amount of pollutant 1 by .4 ton and the amount of pollutant 2 by .3 ton. The state wants to reduce the amount of pollutant 1 in the lake by at least 30 tons and the amount of pollutant 2 by at least 40 tons. Formulate an LP that will minimize the cost of reducing the pollution by the desired amounts.

For i = 1, 2, 3 let x_i = tons of processed factory i waste

$$\begin{aligned} \min z = \ & 15x_1 + 10x_2 + 20x_3 \\ \text{s.t.} \quad & .10x_1 + .20x_2 + .40x_3 \geq 30 \text{ (Pollutant 1)} \\ & .45x_1 + .25x_2 + .30x_3 \geq 40 \text{ (Pollutant 2)} \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{aligned}$$

2. CocoaCo manufactures two types of hot chocolate: premium and regular. Both products are made by combining two types of chocolates: grade 3 and grade 6. The chocolates in premium hot chocolate must have an average grade of at least 5, and the chocolates in regular hot chocolate must have an average grade of at least 4. During each of the next 2 months, CocoaCo can sell up to 1000 gallons of premium hot chocolate and up to 2000 gallons of regular hot chocolate. Premium hot chocolate sells for \$1 a gallon. Regular hot chocolate sells for 80 cents per gallon. At the beginning of month 1, CocoaCo has 3000 gallons of grade 6 chocolates and 2000 gallons of grade 3 chocolates. At the beginning of month 2, CocoaCo may buy additional grade 3 chocolates for 40 cents per gallon and additional grade 6 chocolates for 60 cents per gallon. Hot chocolate spoils at the end of the month, so CocoaCo cannot make hot chocolate in month 1 to be sold in month 2. At the end of month 1, a holding cost of 5 cents is assessed against each leftover gallon of grade 3 chocolates (10 cents for leftover gallons of grade 6 chocolates). In addition to the cost of the chocolates, it costs 10 cents to produce a gallon of hot chocolate (premium or regular). Formulate an LP that will help CocoaCo maximize profit over the next two months.

Let Pi = gallons of premium produced (and sold) during month i,
Ri = gallons of regular produced (and sold) during month i,
Bi = gallons of grade i chocolates bought for use during month 2,
Ij = gallons of grade j chocolates leftover at end of month 1,
Rij = gallons of grade j chocolates used to make regular hot chocolate during month i,
Pij = gallons of grade j chocolates used to make premium hot chocolate during month i.

max $0.9 P_1 + 0.9 P_2 + 0.7 R_1 + 0.7 R_2 - 0.4 B_3 - 0.6 B_6 - 0.05 I_3 - 0.1 I_6$

s.t. $R_1 = R_{16} + R_{13}$

$P_1 = P_{13} + P_{16}$

$R_2 = R_{26} + R_{23}$

$P_1 = P_{13} + P_{16}$

$P_2 = P_{26} + P_{23}$

$R_1 \leq 2000$

$R_2 \leq 2000$

$P_1 \leq 1000$

$P_2 \leq 1000$

$I_6 = 3000 - R_{16} - P_{16}$

$I_3 = 2000 - R_{13} - P_{13}$

$(6R_{16} + 3R_{13})/(R_{16} + R_{13}) \geq 4$

$(6R_{26} + 3R_{23})/(R_{26} + R_{23}) \geq 4$

$(6P_{16} + 3P_{13})/(P_{16} + P_{13}) \geq 5$

$(6P_{26} + 3P_{23})/(P_{26} + P_{23}) \geq 5$

$R_{26} + P_{26} = B_6 + I_6$

$R_{23} + P_{23} = B_3 + I_3$

all variables nonnegative

3. A company produces a cleaning solution which requires chemicals and labor. There are two production processes available: an hour of process 1 transforms 1 unit of labor and 2 units of chemicals into 3 oz of cleaning solution. An hour of process 2 transforms 2 units of labor and 3 units of chemicals into 5 oz of cleaning solution. It costs the company \$3 to purchase a unit of labor and \$2 to purchase a unit of chemicals. Each year up to 20,000 units of labor and 35,000 units of chemicals can be purchased. Without advertising, the company can sell 1000 oz of cleaning solution. However, to increase demand they can hire a model for \$100/hr. Each hour the model works increases the demand for the cleaning solution by 200 oz. Each ounce of cleaning solution sells for \$5. Formulate an LP to help the company maximize profits.

Let x_1 = Units of Process 1.

x_2 = Units of Process 2.

x_3 = Modeling hours hired.

max $z = 5(3x_1+5x_2)-3(x_1+2x_2)-2(2x_1+3x_2)-100x_3$

s.t. $x_1 + 2x_2 \leq 20{,}000$(Limited Labor)

$2x_1 + 3x_2 \leq 35{,}000$ (Limited Chemicals)

$3x_1 + 5x_2 = 1{,}000 + 200x_3$ (Cleaning solution Production = Cleaning solution Demand)

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$

4. Each year, a tire company faces demands (which must be met on time) for tires as follows: quarter 1 – 600, quarter 2 – 300, quarter 3 – 800, quarter 4 – 100. Workers work 3 consecutive quarters and then have 1 quarter off. For example, a worker may work

quarters 3 and 4 of 2015, quarter 1 of 2016, and then have quarter 2 of 2016 off. During a quarter, each worker can produce up to 50 tires. Each worker is paid \$500 per quarter. At the end of each quarter, a holding cost of \$50 per tire is assessed. Formulate an LP that can be used to minimize the cost per year (labor and holding) of meeting the demand. Assume at the end of each year the inventory is 0 and that each worker has the same schedule from year to year.

Let s_t = tires made during quarter t of each year, i_t = inventory of tires at the end of each year, x_t = workers getting quarter t off during each year

min z = 1500(x1 + x2 + x3 + x4) + 50i1 + 50i2 + 50i3 + 50i4
st s1 ≤ 50(x2 + x3 + x4)
s2 ≤5 0(x1 + x3 + x4)
s3 ≤ 50(x1 + x2 + x4)
s4 ≤ 50(x1 + x2 + x3)
i1 = 0 + s1 - 600
i2 = i1 + s2 - 300
i3 = i2 + s3 - 800
i4 = i3 + s4 - 100
i4 = 0
All variables ≥0

5. Describe all optimal solutions of the following LP:

$$\begin{aligned} &\max\ 4x_1 + x_2 \\ &s.t.\quad 8x_1 + 2x_2 \le 16 \\ &\qquad\ \ x_1 + x_2 \le 12 \\ &\qquad\ \ x_1, x_2 \ge 0 \end{aligned}$$

All optimal solutions have z = 8. All points on the line segment joining the points (0, 8) and (2, 0) are optimal.

6. An oil company produces three products: heating oil, gasoline and jet fuel. The average octane levels must be at least 4.5 for heating oil, 8.5 for gas, and 7.0 for jet fuel. To produces these products The company purchases two types of oil: crude 1 (at \$12 per barrel) and crude 2 (at \$10 per barrel). Each day, at most 10,000 barrels of each type of oil can be purchased. Before crude can be used to produce products for sale, it must be distilled. It costs 10 cents to distill a barrel of oil. The result of distillation is as follows: (1) Each barrel of crude 1 yields 0.6 barrel of naphtha, 0.3 barrel of distilled 1, and 0.1 barrel of distilled 2. (2) Each barrel of crude 2 yields 0.4 barrel of naphtha, 0.2 barrel of distilled 1, and 0.4 barrel of distilled 2. Distilled naphtha can be used only to produce gasoline or jet fuel. Distilled oil can be used to produce heating oil or it can be sent

through the catalytic cracker (at a cost of 15 cents per barrel). Each day, at most 5000 barrels of distilled oil can be sent through the cracker. Each barrel of distilled 1 sent through the cracker yields 0.8 barrel of cracked 1 and 0.2 barrel of cracked 2. Each barrel of distilled 2 sent through the cracker yields 0.7 barrel of cracked 1 and 0.3 barrel of cracked 2. Cracked oil can be used to produce gasoline and jet fuel but not to produce heating oil. The octane level of each type of oil is as follows: naphtha, 8; distilled 1, 4; distilled 2, 5; cracked 1, 9; cracked 2, 6. All heating oil produced can be sold at $14 per barrel; all gasoline produced, $16 per barrel; and all jet fuel produced, $16 per barrel. Marketing considerations dictate that at least 3000 barrels of each product must be produced daily. Formulate an LP to maximize the company's daily profit.

Let HO = barrels of heating oil sold
G = barrels of gasoline sold
J = barrels of jet fuel sold
C1 = barrels of crude 1 purchased
C2 = barrels of crude 2 purchased
HO1 = barrels of distilled crude 1 used for heating oil
HO2 = barrels of distilled crude 2 used for heating oil
NG = barrels of naptha used for jet fuel
NJ = barrels of naptha used for gasoline
DO1 = barrels of distilled crude 1 sent through cracker
DO2 = barrels of distilled crude 2 sent through cracker
CO1G = barrels of cracked oil 1 used for gasoline
CO2G = barrels of cracked oil 2 used for gasoline
CO1J = barrels of cracked oil 1 used for jet fuel
CO2J = barrels of cracked oil 2 used for jet fuel

max z = 14HO + 16G + 16J - 12.1C1 - 10.1C2 - .15DO1 - .15DO2

s.t. C1≤10000
C2≤10000
.3C1 + .2C2 = HO1 + DO1
.1C1 + .4C2 = HO2 + DO2
.6C1 + .4C2 = NG + NJ
.8DO1 + .7DO2 = CO1G + CO1J
.2DO1 + .3DO2 = CO2G + CO2J
HO = HO1 + HO2
G = NG + CO1G + CO2G
J = NJ + CO1J + CO2J
-8.5G + 8NG + 9CO1G + 6CO2G≥0
-7J + 8NJ + 9CO1J + 6CO2J≥0
-4.5HO + 4HO1 + 5HO2≥0
C1 + C2≤15,000
HO≥3000
G≥3000
J≥3000
All variables ≥0