PHYS 2212 Test 4
Spring 2014

Name(print) Key **Lab Section** ∞

Lab section by day and time: Curtis(H), Ballantyne(Q), Kim(P)						
Monday	12:05-2:55pm	H01 or Q01	3:05-5:55pm	H02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	Q03 or P03	3:05-5:55pm	Q04 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	H03 or Q05	3:05-5:55pm	P05 or Q06	6:05-8:55pm	H04 or P06
Thursday	12:05-2:55pm	H05 or Q07	3:05-5:55pm	Q08 or H06	6:05-8:55pm	H07 or P07

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Sign your name on the line above

The final exam for this class is scheduled for:
Period 13, May 2 (Fri) at 8:00am - 10:50am.

The conflict final exam for this class is

ADAPTS Student will need to schedule their final exam with the **ADAPTS office as soon as possible.**

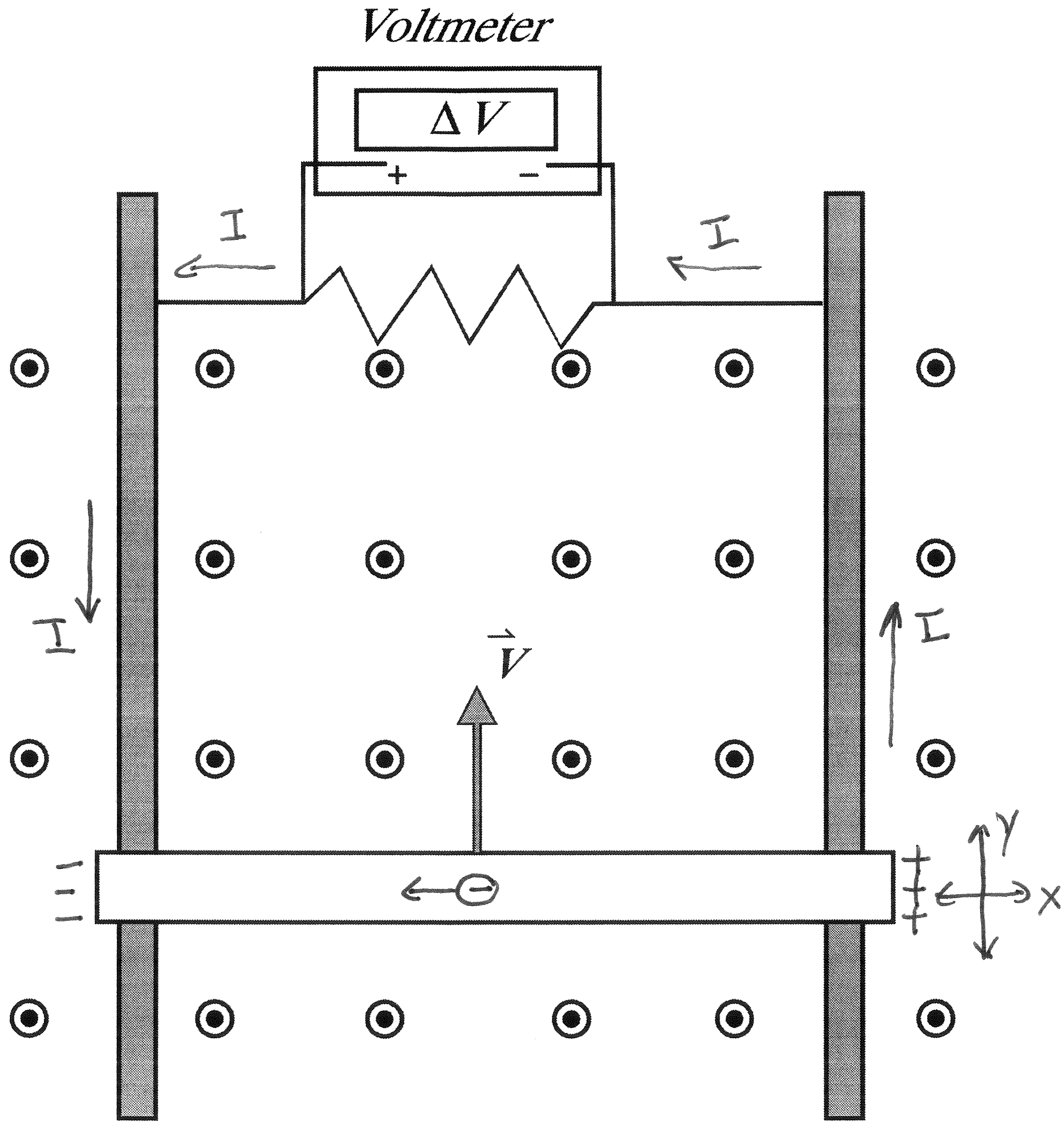
PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

A copper bar of length L and zero resistance slides at constant speed v along metal rails. The bar is moving through a region in which there is a uniform magnetic field B directed out of the page. A voltmeter is connected across a resistor of resistance R and reads ΔV. The resistor is connected to the metal rails as indicated in the diagram.



(a 5pts) On the diagram show the charge distribution in and/or on the copper bar. → All

(b 5pts) On the diagram indicate the direction of the conventional current. → All but cons/(a)

→ the force on a mobile electron in the bar is $(-e)\vec{v} \times \vec{B}_{ext}$

→ using the right hand rule $-e\vec{v} \times \vec{B}_{ext} = evB(-\hat{x})$

→ negative charge builds on the left end of the bar

→ the polarization of the bar cause a current I to flow counter-clockwise (c 10pts) Determine the magnitude of the magnetic force acting on the bar and indicate the direction of this force on the diagram. Your answer should be in terms of the given variables and known constants.

- The force on the bar is given by

$$\vec{F}_{BAR} = I\vec{L} \times \vec{B} \rightarrow |\vec{F}_{BAR}| = ILB$$

$$\boxed{\begin{array}{c} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{array}}$$

- the current running in the circuit is found by A loop rule $\Delta V_{resistor} = IR$ which is given as ΔV

$$\rightarrow I = \frac{\Delta V}{R}$$

- the force on the bar is therefore $\boxed{|\vec{F}_{BAR}| = \frac{\Delta V L B}{R}}$

note: $\left\{ E_{BAR}(-e) = (-e)vB \right\}$ since $\vec{F}_{net} = 0$ $\quad |\vec{F}_{BAR}| = \frac{v}{R}(LB)^2$

$EL = IR$

(d 5pts) Determine the reading on the voltmeter if the velocity of the copper bar was $2v$. Your answer should be in terms of the given variables and known constants.

3pt

- the force on an electron in the bar $\vec{F}_e = E_{BAR}(-e)\hat{x} + (-e)vB\hat{x}$
- But the bar is in steady state $\vec{F}_{net} = 0$

$$\rightarrow E_{BAR} = vB_{ext}$$

2pt

- $\Delta V_{loop} = E_{BAR} L + \Delta V_{resistor} = 0 \Rightarrow |\Delta V_{rest}| = vB_{ext}L$
- if the velocity doubles the reading on the voltmeter doubles $\boxed{|\Delta V_{new}| = 2vBL = 2\Delta V_{old}}$
- Conventional current I flows into the negative terminal so the sign of $\boxed{\Delta V < 0}$

Problem 2 (25 Points)

A sphere of radius R has a charge density $\rho(r) = \rho_0(r/R)$ where ρ_0 is a constant and r is the distance from the center of the sphere. At a point inside the sphere $r = r_{in}$ the total charge enclosed within that point can be found by integration:

$Q(r_{in}) = \int_0^{r_{in}} \rho(r) 4\pi r^2 dr$.

(a 5pts) Determine the total charge inside the sphere.

$$q(R) = \int_0^R \rho(r) 4\pi r^2 dr = \underbrace{\int_0^R \rho_0 \frac{r}{R} 4\pi r^2 dr}_{3pt}$$

$$= \frac{4\pi \rho_0}{R}\left[\frac{1}{4} r^4\right]\Bigg|_0^R = \boxed{\pi \rho_0 R^3} \quad 2pts$$

(b 5pts) Determine the electric field at a point outside of the sphere a distance r_{out} from the center of the sphere.

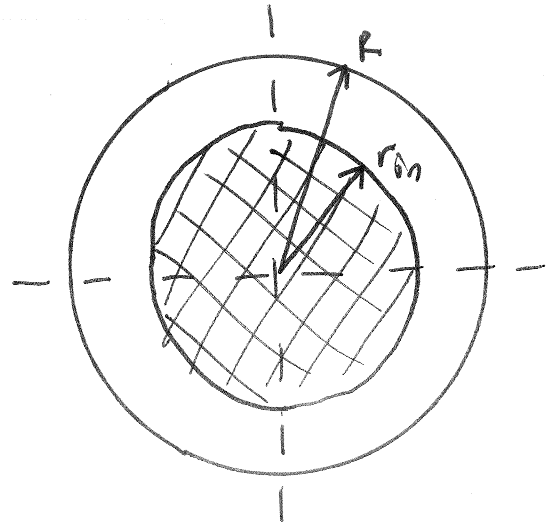
→ outside a charged sphere the field looks like a point charge @ the center. This comes from Gauss' Law $\vec{E} \cdot \vec{A}_{sphere} = \frac{q_{enc}}{\epsilon_0}$ 3pt

→ $\boxed{\vec{E}(r > R) = \frac{1}{4\pi\epsilon_0} \frac{\pi \rho_0 R^3}{r^2} \hat{r} \;\; *}$ 2pts

* ρ_0 is assumed positive, if $\rho_0 < 0$ then $(-\hat{r})$ direction

(c 15pts) Determine the electric field at a point inside of the sphere a distance r_{in} from the center.

Gauss' Law says $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$



→ pick a spherical surface inside the sphere of radius r_{in}

→ the charge enclosed inside r_{in} is $q(r_{in}) = \int_0^{r_{in}} \rho_0 \frac{r}{R} 4\pi r^2 dr = \frac{4\pi\rho_0}{R}\left[\frac{1}{4}r^4\right]_0^{r_{in}} =$

$= \frac{\pi \rho_0}{R} r_{in}^4$

→ By symmetry the electric field looks the same everywhere on the surface of the sphere r_{in} ($\vec{E}$ points out)

$\oint \vec{E} \cdot dA = E \oint dA = \frac{q_{in}}{\varepsilon_0}$

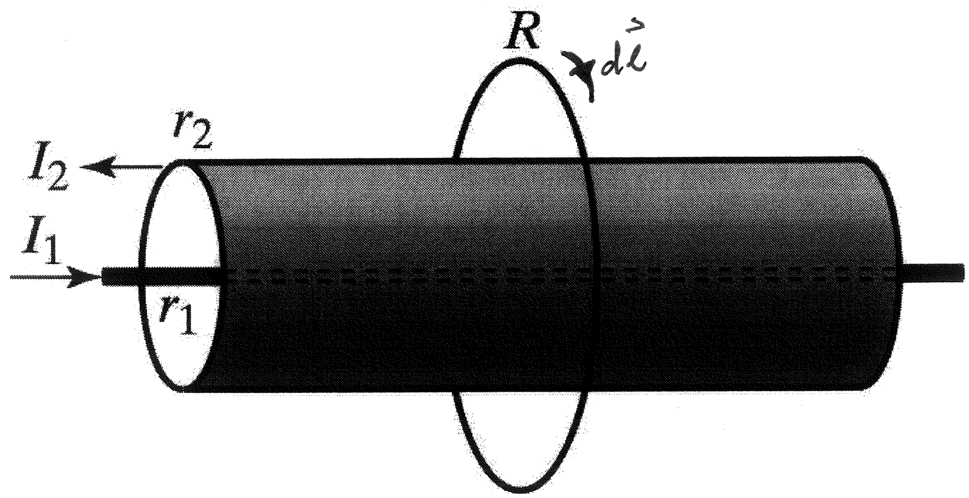
$E\, 4\pi r_{in}^2 = \frac{\pi \rho_0 r_{in}^4}{\varepsilon_0 R}$

$$\boxed{\vec{E}(r_{in} < R) = \left(\frac{\rho_0}{R\varepsilon_0}\right) \frac{r_{in}^2}{4} \hat{r}}$$

−1.0
−2.0
−4.5
−12

Problem 3 (25 Points)

A coaxial cable consists of an inner metal wire of radius r_1 and an outer (hollow) metal cylinder of radius r_2 as seen in the diagram. There is current I_1 to the right in the wire and current $I_2 < I_1$ to the left in the cylinder. Consider a circular Amperian path of radius R centered on the wire and perpendicular to the wire. Determine the magnitude of the magnetic field at a location R above the wire, far from the ends of the coaxial cable.



Ampere says $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$

→ the Amperian path has radius 'R' and the B-field will be the same along the path by symmetry

$$\oint \vec{B}\cdot d\vec{l} = B\oint dl = B\,2\pi R = \mu_0 I_{enc}$$

→ going around the path clockwise B from I_1 is positive and B from I_2 will be negative

· $I_{enc} = I_1 - I_2$

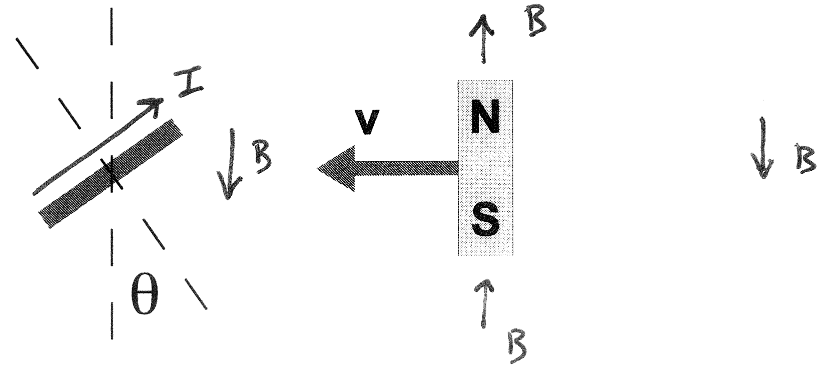
→ $\boxed{|\vec{B}(R)| = \left|\frac{\mu_0 (I_1 - I_2)}{2\pi R}\right|}$

-1.0
-3.75
-7.5
-20

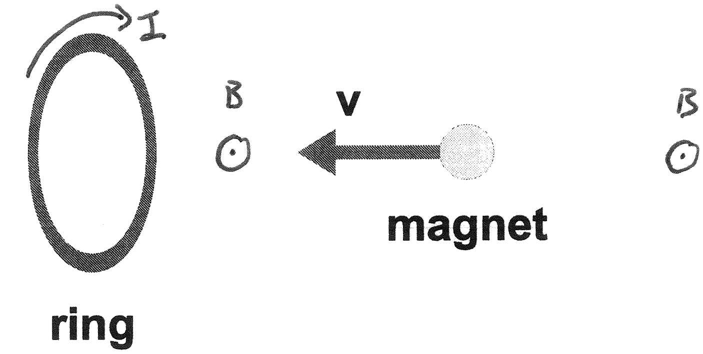
Problem 4 (25 Points)

A magnet with magnetic moment μ moves at speed v toward a metal ring of radius R as shown in the diagram. As shown, the ring is tilted so that it makes an angle of θ with a line parallel to the axis of the magnet.

Top View

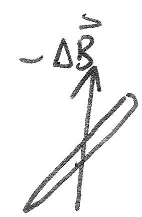


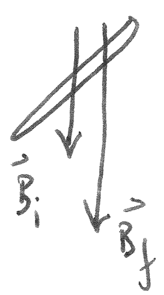
Front View



(a 5pts) On the diagram (Top View) indicate the direction of the induced current in the ring. Briefly state how you determined this.

from the top view we see that the B-field points down threw the ring and is increasing with time as the magnet moves closer



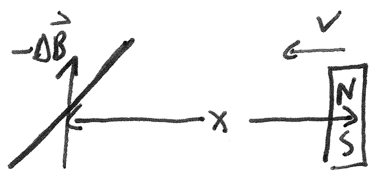


- Then $(-\Delta\vec{B})$ will point up

- putting our thumb in the direction of $(-\Delta\vec{B})$ our fingers curl clockwise

TA Discuss

(b 20pts) At the instant when the magnet is a distance x from the center of the coil, what is the magnitude of E_{nc}?



faraday says

$$\left| \oint \vec{E}_{nc} \cdot d\vec{l} \right| = \left| \frac{d\phi_{mag}}{dt} \right|$$

-1.0
-3.0
-6.0
-16.0

Φ_{mag} through the ring is equal to $\vec{B}_{mag} \cdot \vec{A}_{ring}$

$$\Phi_{mag} = B_{mag} \pi R^2 \cos\theta = \frac{\mu_0 \mu}{4\pi x^3} \pi R^2 \cos\theta$$

→ we assume B_{mag} is constant over the ring and that the magnet is far from the ring $(x > R)$

$$\frac{d\Phi_{mag}}{dt} = \frac{\mu_0 \mu R^2 \cos\theta}{4} \frac{d}{dt}\left(\frac{1}{x^3}\right) = \frac{\mu_0 \mu R^2 \cos\theta}{4} (-3) \frac{1}{x^4} \frac{dx}{dt}$$

→ the distance 'x' is changing at the rate 'v'

$$\left| \frac{d\Phi_{mag}}{dt} \right| = \frac{3}{4} \frac{\mu_0 \mu R^2 \cos\theta}{x^4} v$$

$$\oint E_{nc} \cdot dl = E_{nc} \int dl = E_{nc} 2\pi R = \frac{3}{4} \frac{\mu_0 \mu R^2 \cos\theta}{x^4} v$$

$$\boxed{|\vec{E}_{nc}| = \frac{3}{8\pi} \frac{\mu_0 \mu R \cos\theta}{x^4} v}$$

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net} \quad \text{and} \quad \frac{d\vec{p}}{dt} \approx m\vec{a} \text{ if } v << c$$

$$\Delta U_{el} = q\Delta V$$

$$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x\Delta x + E_y\Delta y + E_z\Delta z)$$

$$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$$

$$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$$

$$\oint \vec{E} \bullet \hat{n} dA = \frac{\sum q_{inside}}{\epsilon_0}$$

$$\oint \vec{B} \bullet \hat{n} dA = 0$$

$$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left|\frac{d\Phi_{mag}}{dt}\right|$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$$

$$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \frac{d}{dt} \int \vec{E} \bullet \hat{n} dA\right]$$

Specific Results

$$\left|\vec{E}_{dipole,axis}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3} \text{ (on axis, } r \gg s)$$

$$\left|\vec{E}_{dipole,\perp}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3} \text{ (on } \perp \text{ axis, } r \gg s)$$

$$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2 + (L/2)^2}} \text{ (} r \perp \text{ from center)}$$

electric dipole moment $p = qs$, $\vec{p} = \alpha \vec{E}_{applied}$

$$\left|\vec{E}_{rod}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r} \text{ (if } r \ll L)$$

$$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2 + R^2)^{3/2}} \text{ (} z \text{ along axis)}$$

$$\left|\vec{E}_{disk}\right| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right] \text{ (} z \text{ along axis)}$$

$$\left|\vec{E}_{disk}\right| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0} \text{ (if } z \ll R)$$

$$\left|\vec{E}_{capacitor}\right| \approx \frac{Q/A}{\epsilon_0} \text{ (}+Q \text{ and } -Q \text{ disks)}$$

$$\left|\vec{E}_{fringe}\right| \approx \frac{Q/A}{\epsilon_0}\left(\frac{s}{2R}\right) \text{ just outside capacitor}$$

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{\ell} \times \hat{r}}{r^2} \text{ (short wire)}$$

$$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$$

$$\left|\vec{B}_{wire}\right| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2 + (L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r} \text{ (} r \ll L)$$

$$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$$

$$\left|\vec{B}_{loop}\right| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2 + R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3} \text{ (on axis, } z \gg R) \qquad \mu = IA = I\pi R^2$$

$$\left|\vec{B}_{dipole,axis}\right| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3} \text{ (on axis, } r \gg s)$$

$$\left|\vec{B}_{dipole,\perp}\right| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3} \text{ (on } \perp \text{ axis, } r \gg s)$$

$$\vec{E}_{rad} = \frac{1}{4\pi\epsilon_0}\frac{-q\vec{a}_\perp}{c^2 r}$$

$$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$$

$$\left|\vec{B}_{rad}\right| = \frac{\left|\vec{E}_{rad}\right|}{c}$$

$$i = nA\bar{v} \qquad I = |q|\, nA\bar{v} \qquad \bar{v} = uE$$

$$\sigma = |q|\, nu \qquad J = \frac{I}{A} = \sigma E \qquad R = \frac{L}{\sigma A}$$

$$E_{dielectric} = \frac{E_{applied}}{K}$$

$$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right] \text{ due to a point charge}$$

$I = \dfrac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$ $\qquad$ $K \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\frac{d\vec{p}}{dt}_\perp\right| = \frac{|\vec{v}|}{R}\,|\vec{p}| \approx \frac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\,dx = ax + c \quad \int ax\,dx = \frac{a}{2}x^2 + c \quad \int ax^2\,dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N $\cdot$ m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N $\cdot$ m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N $\cdot$ m^2/C^2)$^{-1}$
Magnetic constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T $\cdot$ m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T $\cdot$ m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T