MATH 2602, Midterm 1

June 6th, 2012

Name: ____________________________ **GTID:**____________________________

Section: ___________________________

Problem	*Points*
1	
2	
3	
4	
5	

TOTAL:________________

Please do show all your work including intermediate steps and also explain in words how you solve each problem. Partial credit is available.

Problem 1 (20 points).

Use mathematical induction to prove that for every integer $n \geq 1$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

Problem 2 (20 points).

Jump induction is another induction scheme, which works in the following way: Given a statement $P(n)$, if

(a) $P(1)$ and $P(2)$ are both true and

(b) For any $k \geq 1$, $P(k)$ is true implies $P(k+2)$ is true

then $P(n)$ is true for all $n \geq 1$.

Use jump induction to show that for every integer $n \geq 1$

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1}n^2 = (-1)^{n-1}(1 + 2 + \cdots + n).$$

Problem 3 (20 points).

Consider the following recurrence relation:

$$a_n = 3a_{n-1} - 2a_{n-2} + 3^n, \quad a_0 = 1, \quad a_1 = 1.$$

1. Find a particular solution to the recurrence relation.
2. Find the general solution to the corresponding homogeneous recurrence.

Problem 4 (20 points).

How many integers between 1 and 300 (inclusive) are

1. divisible by both 3 and 5?
2. divisible by both 3 and 5 but not divisible by 11?
3. divisible by exactly two of 3, 5 and 11?

Problem 5 (20 points).

Suppose 51 numbers are chosen from the set $\{1, 2, 3, ..., 100\}$. Show that among those chosen numbers there are two numbers such that one is a multiple of the other.

Hint: Any natural number n can be written in the form $n = 2^k a$ with $k \geq 0$ and a odd.