Math 1712 - Spring 2012
Test 3 - Show your work

Name: ______________________________ TA: ______________________________

1. (10 points) Find all anti-derivatives for each function:

a. $2x^5 - 4e^{3x}$ **ANS:** $\displaystyle\int (2x^5 - 4e^{3x})\,dx = \frac{x^6}{3} - \frac{4}{3}e^{3x} + C$

b. $\dfrac{1}{\sqrt{x}} - \dfrac{1}{x}$ **ANS:** $\displaystyle\int \left(\frac{1}{\sqrt{x}} - \frac{1}{x}\right) dx = \int \left(x^{-\frac{1}{2}} - \frac{1}{x}\right) dx = 2\sqrt{x} - \ln(x) + C$

2. (10 points) The ABC Company has determined that it's marginal cost funtion is given by: $x^3 - x$. Find the total cost function $C(x)$ if the fixed costs are \$6, 500.

$$C'(x) = MCF = x^3 - x \Rightarrow C(x) = \int (x^3 - x)\,dx = \frac{x^4}{4} - \frac{x^2}{2} + K$$

$$C(0) = 6500 \Rightarrow 6500 = 0 - 0 + K \Rightarrow C(x) = \frac{x^4}{4} - \frac{x^2}{2} + 6500$$

3. (10 points) Find the function $f(x)$, given that $f'(x) = 6x^2 - 4x + 2$ and $f(1) = 9$.

$$f'(x) = 6x^2 - 4x + 2 \Rightarrow f(x) = \int (6x^2 - 4x + 2)\,dx = 2x^3 - 2x^2 + 2x + C$$

$$f(1) = 9 \Rightarrow 9 = 2 \text{ -}2 + 2 + C \Rightarrow C = 7 \Rightarrow f(x) = 2x^3 - 2x^2 + 2x + 7$$

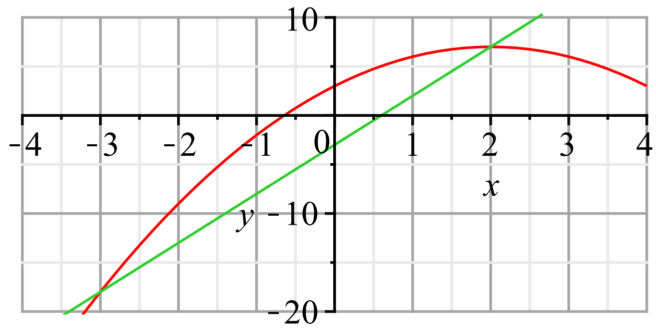
4. (20 points) Evaluate the following integrals. You do not need IBS nor IBP for these integrals.

a. $\displaystyle\int \left[\sqrt[3]{x^2} - \frac{1}{\sqrt[3]{x^2}}\right] dx = \int \left[x^{\frac{2}{3}} - x^{-\frac{2}{3}}\right] dx = \frac{3}{5}x^{\frac{5}{3}} + 3x^{\frac{1}{3}} + C$

b. $\displaystyle\int \left[e^{-3x} - \frac{5}{x}\right] dx = -\frac{1}{3}e^{-3x} - 5\ln(x) + C$

c. $\displaystyle\int \left[t^{-5} + \frac{1}{t^{-5}}\right] dt = \int (t^{-5} + t^5)\,dt = -\frac{t^{-4}}{4} + \frac{t^6}{6} + C = -\frac{1}{4t^4} + \frac{t^6}{6} + C$

5. (20 points) The graphs of $g(x) = -x^2 + 4x + 3$ and $f(x) = 5x - 3$ are shown below. a. Shade in the region between the two graphs between the two intersection points. b. Find the shaded area.



$$\boldsymbol{Area} = \int_{-3}^{2} \left[(-x^2 + 4x + 3) - (5x - 3) \right] dx = \int_{-3}^{2} \left[(-x^2 - x + 6) \right. dx$$

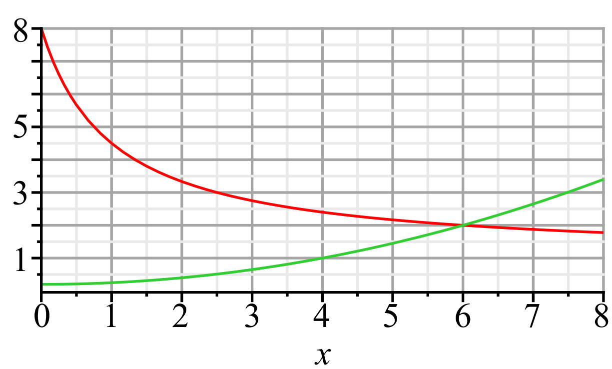
$$= -\frac{x^3}{3} - \frac{x^2}{2} + 6x \Big| = \frac{125}{6}$$

6. (15 points) The FGH Company has determined that their marginal profit function (\$) is given by: $75 - t$ where t is time in weeks. Find the total profit from $t = 0$ **to** $t = 10$ weeks assuming $P(0) = 0$.

$$P'(t) = MPF = 75 - t \Rightarrow P(10) - P(0) = \int_0^{10} (75 - t)\, dt = \left(75t - \frac{t^2}{2}\right) \Big| = 700 = P(10) \quad \textbf{OR}$$

$$P(t) = \int (75 - t)\, dt = 75t - \frac{t^2}{2} + C \ \ \& \ P(0) = 0 \Rightarrow C = 0 \quad P(t) = 75t - \frac{t^2}{2} \Rightarrow P(10) = 700$$

7. (20 points) The AMR Company has determined that it's supply and demand functions are given by: $\boldsymbol{S(x) = \frac{x^2 + 4}{20}}$ **&** $\mathbf{D}(\boldsymbol{x}) = \boldsymbol{\frac{x + 8}{x + 1}}$. The graphs of these two functions are shown below. a. Use the graph to find the equilibrium point; that is, find the equilibrium quantity (x_E) and the equilibrium price (p_E). b. Find the **producer's surplus** at the equilibrium point.



From the graph, $\boldsymbol{x_E = 6}$ **&** $\boldsymbol{p_E = 2 \Rightarrow PS = x_E p_E - \int_0^{x_E} S(x)\, dx = 12 - \frac{1}{20} \int_0^6 (x^2 + 4)\, dx = \frac{36}{5}}$

8. (20 points) Evaluate the following integrals using ***IBS*** **on the first two and** ***IBP*** **on the last one.**

a. $\displaystyle\int \frac{24x^5}{4x^6+3}\,dx = \int \frac{du}{u} = \ln(u) + C = \ln(4x^6+3) + C$

$u = 4x^6 + 3$
$du = 24x^5$

b. $\displaystyle\int_0^2 x\,e^{x^2+1}\,dx = \frac{1}{2}e^{x^2+1}\Big| = \frac{1}{2}(e^5 - e) \approx 72.85$

$u = x^2 + 1$
$\frac{1}{2}\,du = x$

c. $\displaystyle\int x^3 \ln(x)\,dx = \frac{x^4}{4}\ln(x) - \int \frac{x^4}{4}\,\frac{dx}{x} = \frac{x^4}{4}\ln(x) - \frac{1}{4}\int x^3\,dx = \frac{x^4}{4}\ln(x) - \frac{1}{16}x^4 + C$

$u = \ln(x)$ $\quad dv = x^3$

$du = \frac{dx}{x}$ $\quad v = \frac{x^4}{4}$

EXTRA CREDIT (5 points) Evaluate: $\displaystyle\int (x+4)\,e^x\,dx = (x+4)\,e^x - \int e^x\,dx = (x+4)\,e^x - e^x + C$
$= x\,e^x + 3e^x + C = (x+3)\,e^x + C$

$u = x + 4$ $\quad dv = e^x$

$du = dx$ $\quad v = e^x$