Math 2603-F **Quiz 5**

Date: February 24, 2016

Last Name (Print): ____________________ **First Name (Print):** ____________________

Instructions Please print your name at the top of this page. To obtain maximum marks show all your work, carefully justifying your answers.

1. (4 points) Let $a \in \mathbb{Z}$ such that $a \not\equiv 0 \pmod 5$. Show that $a \equiv 3^k \pmod 5$ for some $k \in \mathbb{N}$.

Solution: We have four possible cases, depending on the value of a. If $a \equiv 1 \pmod 5$, then taking $k = 4$ yields $3^4 \equiv 81 \equiv 1 \pmod 5$. Now, if $a \equiv 2 \pmod 5$, then taking $k = 3$ yields $3^3 \equiv 27 \equiv 2 \pmod 5$. For the case $a \equiv 3 \pmod 5$ taking $k = 1$ yields $3^1 \equiv 3 \pmod 5$. Finally, for the case $a \equiv 4 \pmod 5$ taking $k = 2$ yields $3^2 \equiv 9 \equiv 4 \pmod 5$.

2. (6 points) Find all integers x, y with $0 \le x, y < 14$ that satisfy the following congruences

$$x + 2y \equiv 3 \pmod{14} \tag{1}$$
$$2x - 5y \equiv 0 \pmod{14}. \tag{2}$$

Solution: Multiplying (1) by 2 yields

$$2x + 4y \equiv 6 \pmod{14}.$$

Subtracting (2) from the previous congruence results in

$$9y \equiv 6 \pmod{14}.$$

Let us compute the multiplicative inverse of $9 \pmod{14}$.

	a	b
14	1	0
9	0	1
5	1	-1
4	-1	2
1	2	-3

Thus $9^{-1} \equiv -3 \pmod{14}$. Multiplying the previous congruence by -3 yields

$$y \equiv -18 \equiv 10 \pmod{14}.$$

Using this in congruence (1) gives us

$$x \equiv -3 \equiv 11 \pmod{14}.$$

Thus $x = 11$ and $y = 10$.

3. (6 points) Use the Chinese remainder theorem to find all integers x with $70 \leq x < 105$ that satisfy the following congruences

$$x \equiv 2 \pmod{5} \tag{3}$$
$$x \equiv 5 \pmod{7}. \tag{4}$$

Solution: From (3) we deduce that $x = 5k + 2$ for some $k \in \mathbb{Z}$. Using this in (4) yields

$$5k \equiv 3 \pmod{7}.$$

We now compute the multiplicative inverse of $5 \pmod{7}$.

	a	b
7	1	0
5	0	1
2	1	-1
1	-2	3

Thus $5^{-1} \equiv 3 \pmod{7}$. Multiplying the previous congruence by 3 results in

$$k \equiv 2 \pmod{7}.$$

Therefore $k = 7p + 2$ for some $p \in \mathbb{Z}$. Thus $x = 5(7p + 2) + 2 = 35p + 12$ for some $p \in \mathbb{Z}$. We conclude by observing taking $p = 2$ results in the desired value for x, that is, $x = 82$.