MATH 2403, Midterm 2

10/12/2012

Name: _____ GTID:_____

Circle your section below

C1 TA: Ashley Bentley

C2 TA: Fahmid Sharkar

Problem No.	Points
1	
2	
3	
4	
5	

TOTAL:_____

Please do show all your work including intermediate steps. Partial credit is available. You may use a non-graphing calculator and a handwritten one-sided 11x8.5 note sheet.

Problem 1 (20 points).

Use Euler's method to find approximate value of the solution of the given initial value problem at $t = 0.2$ with $h = 0.1$.

$$y' = t^2 + y^2 + 1, \quad y(0) = 1.$$

Problem 2 (15+5 points).

Given the following linear system:

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 2 \\ 8 \end{pmatrix}.$$

1. Find the general solution of the given system.
2. Sketch the phase portrait.

Problem 3 (20 points).

Find the solution of the given initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

You may use the fact that the matrix $\begin{pmatrix} 1 & -5 \\ 1 & -3 \end{pmatrix}$ has

- eigenvalue $\lambda_1 = -1 + i$ with eigenvector $\mathbf{v}_1 = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$, and
- eigenvalue $\lambda_2 = -1 - i$ with eigenvector $\mathbf{v}_2 = \begin{pmatrix} 2-i \\ 1 \end{pmatrix}$.

Problem 4 (15+5 points).

Given the following linear system:

$$\mathbf{x}' = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix} \mathbf{x}.$$

1. Find the general solution of the given system. You may use the fact that the matrix $\begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}$ has repeated eigenvalues $\lambda_1 = \lambda_2 = -3$.

2. Describe the type of critical points and the stability property of the given system.

Problem 5 (15+5 points).

Consider the following two linear equations.

$$y'' + 4y' - 5y = 0. \tag{1}$$

$$y'' + 4y' - 5y + 5 = 0. \tag{2}$$

1. Find the general solution of equation (1).
2. let Y_1 and Y_2 be the general solutions of equation (1) and (2), resp.. What's the relation between Y_1 and Y_2?