QUIZ 5 Name Key

Math 2551 D Steinbart Section ______ February 24, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(5) 1. Let $f(x, y) = x^2 \sin y - 4$.

a. Find the linearization $L(x, y)$ of f at $(6, \frac{\pi}{6})$.

b. Find the plane tangent to the surface $x^2 \sin y - 4 = z$ at $(6, \frac{\pi}{6}, 14)$.

$f_x = 2x \sin y$ $\quad f_x(6, \pi/6) = 2(6) \sin \pi/6 = 12(\frac{1}{2}) = 6$

$f_y = x^2 \cos y$ $\quad f_y(6, \pi/6) = 6^2 \cos \pi/6 = 36(\frac{\sqrt{3}}{2}) = 18\sqrt{3}$

$f(6, \pi/6) = 6^2 \sin \pi/6 - 4 = 36(\frac{1}{2}) - 4 = 14$.

(4) a) So $L(x,y) = 14 + 6(x-6) + 18\sqrt{3}(y - \pi/6)$.

(1) b) The plane is $z = 14 + 6(x-6) + 18\sqrt{3}(y - \pi/6)$.

(4) 2. Let $f(x, y) = 3y^2 - 2y^3 - 3x^2 - 6xy$. Find and classify the critical points of f. (Classify = at each critical point, does f have a local maximum, a local minimum, a saddle point, or there is not enough information to determine the nature of f at the critical point.)

$f_x = -6x - 6y$

$f_y = 6y - 6y^2 - 6x$

$f_x = 0$

$-6x - 6y = 0$

$-6x = 6y$

$-x = y$

$f_y = 0$

$6y - 6y^2 - 6x = 0$

Use ~~that~~ $-x = y$, so $x = -y$

$6y - 6y^2 + 6y = 0$

$12y - 6y^2 = 0$

$6y(2 - y) = 0$ So $y = 0, y = 2$

If $y = 0$ then $x = -0 = 0$. $(0,0)$ is a crit pt

If $y = 2$, then $x = -2$. So $(-2, 2)$ is a crit pt

$f_{xx} = -6$

D

$f_{yy} = 6 - 12y$

$f_{xy} = -6$

$(\cdot,\cdot)$	f_{xx}	f_{yy}	f_{xy}	$D = f_{xx}f_{yy} - (f_{xy})^2$
$(0,0)$	-6	6	-6	$-6(6) - (-6)^2 = -72$
$(-2,2)$	-6	-18	-6	$(-6)(-18) - (-6)^2 = 108 - 36 > 0$

The critical pts of f are $(0,0)$ and $(-2,2)$. At $(0,0)$, $D < 0$ so f has a saddle pt.

At $(-2,2)$, $D > 0$ and $f_{xx} < 0$; so f has a local maximum at $(-2,2)$.

(1) 3. Determine if the statement is true or false. If $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$ then the tangent plane to the surface given by $z = f(x, y)$ at $(x_0, y_0, f(x_0, y_0))$ must be parallel to the xy-plane. $\boxed{\text{True.}}$

Let P = tangent plane to $z = f(x,y)$ at $(x_0, y_0, f(x_0,y_0))$

why: A vector orthogonal to the tangent plane to $z = f(x,y)$ at $(x_0, y_0, f(x_0,y_0))$ is the vector $\underline{n} = \langle f_x(x_0,y_0), f_y(x_0,y_0), -1 \rangle = \langle 0, 0, -1 \rangle$

The vector $n_1 = \langle 0, 0, 1 \rangle$ is orthogonal to the xy plane (= surface $z = 0$)

$\underline{n} = -\underline{n}_1$; So $\underline{n}$ and $\underline{n}_1$ are parallel. So the plane P and the xy plane are parallel.