MATH 2401 QUIZ 1

Name:

Problem 1. (3 points) Find the parametric equations for the line through the point $(2, 3, 0)$ and perpendicular to the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

Solution 1. The vector $\mathbf{u} \times \mathbf{v}$ is perpendicular to $\mathbf{u}$ and $\mathbf{v}$ (1 point). We have (1 point)

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{pmatrix} = (-2, 4, -2).$$

Then the line through the point $(2, 3, 0)$ in the direction perpendicular to the vectors $\mathbf{u}$ and $\mathbf{v}$ is given by (1 point)

$$\ell(t) = (2, 3, 0) + t(-2, 4, -2) = (2 - 2t, 3 + 4t, -2t)$$

Problem 2. (4 points) Find the volume of the parallelepiped given by the vectors $\mathbf{u} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = -\mathbf{i} - \mathbf{k}$ and $\mathbf{w} = 2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

Solution 2. (1 point) The volume of the parallelepiped given by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is given by

$$\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}.$$

(2 points) Now

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -2 \\ -1 & 0 & -1 \end{pmatrix} = (-1, 3, 1).$$

Thus (1 point) we have

$$\mathbf{u} \times \mathbf{v} \cdot \mathbf{w} = (-1, 3, 1) \cdot (2, 4, -2) = -2 + 12 - 2 = 8.$$

Problem 3. (3 points) Let $\mathbf{u} = {}^3/{}_2\mathbf{i} - {}^1/{}_2\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$. Find the length and direction of $\mathbf{u} \times \mathbf{v}$.

Solution 3. We have by definition (1 pt):

$$\mathbf{u} \times \mathbf{v} = \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{3}{2} & -\frac{1}{2} & 1 \\ 1 & 1 & 2 \end{pmatrix} = (-2, -2, 2).$$

The magnitude of $\mathbf{u} \times \mathbf{v}$ is then

$$|\mathbf{u} \times \mathbf{v}| = \sqrt{4 + 4 + 4} = \sqrt{12} = 2\sqrt{3}.$$

Thus, the vector in the direction of $\mathbf{u} \times \mathbf{v}$ is (1 point)

$$\frac{\mathbf{u} \times \mathbf{v}}{|\mathbf{u} \times \mathbf{v}|} = \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right).$$