H. Ayhan and S.H. Kimi

Homework 12

(optional – will be counted as extra points toward homework)

Due: at the start of class on Tuesday/Wednesday, December 3/4 November 21, 2013

1. Suppose we are producing wire which is extruded as a long, continuous strand. After each 100 meters, the wire is cut and placed on a spool. Defects in the wire appear at random and on the average there are 4 defects per kilometer of wire. Assume that the spacing between defects is exponentially distributed. For notation, let $N(t)$ be the number of defects in the first t meters of wire produced, $S_0 = 0$, S_n be the location of the nth defect, T_n be the time between the $(n-1)$st and nth defect, and λ be the rate of defects per meter of wire.
   (a) What is the rate of defects per meter of wire?
   (b) What is the expected number of defects in the first spool?
   (c) What is the mean number of defects in the 9 th spool?
   (d) What is the probability that the first spool contains no defects?
   (e) What is the probability that the tenth spool contains one defect?
   (f) What is the probability that the tenth spool contains one defect given that the first nine spools contained no defects?
   (g) What is the probability that there is exactly two defects among the first ten spools?
   (h) What is the average location of the first defect?
   (i) What is the expected location of the tenth defect?
   (j) What is the expected location of the first defect given that the first spool has zero defects?
   (k) What is the expected location of the first defect in the second kilometer of wire?
   (l) Suppose that a defect can either be minor or major, and each defect is minor with probability 3/4 and major with probability 1/4, independent of the location and status of the other defects. What is the expected number of major defects in the first 5 spools? For notation, let $L(t)$ be the number of minor and $M(t)$ the number of major defects in the first t meters. Let S_n^L be the location of the nth minor defect, and T_n^L be the time between the nth and $(n-1)$st minor defect. Similarly define, S_n^M and T_n^M for the major defects.
   (m) What is the probability of at least one major defect among the first 10 spools given that there were no minor defects?
   (n) What is the probability that the first three defects are minor?
   (o) Suppose each minor defect can be fixed at a cost of \$1, but a spool with any major defects must be recycled at a cost of \$10. What is the expected cost due to defects of the first spool? What is the expected cost due to defects of the first ten spools?

2. Siegbert runs a hot dog stand that opens at 8 am. From 8 until 11 am customers seem to arrive, on the average, at a steadily increasing rate that starts with an initial rate of 5 customers per hour at 8 am and reaches a maximum of 20 customers per hour at 11 am. From 11 am until 1 pm the (average) rate seems to remain constant at 20 customers per hour. However, the (average) arrival rate then drops steadily from 1 pm until closing time at 5 pm at which time it has the value of 12 customers per hour. We assume that the numbers of customers arriving at Siegbert's stand during disjoint time periods are independent.
   (a) What is a good probability model for the customer arrivals to the hot dog stand?
   (b) What is the rate function?
   (c) What is the probability that no customers arrive between 8:30 am and 9:30 am on Monday morning?

(d) What is the expected number of arrivals in this period?

(e) Consider the first customer that arrives after 1 pm and let T be the length of time from 1pm to the time when the customer arrives. What is the probability that this customer arrives after 1:10, after 1:20? Is T exponentially distributed?