BME 3400 Midterm Exam # 2 October 20, 2009 Name: SOLUTION

This is a closed-book exam. Calculators are allowed, but integrals should be solved before numbers are plugged in.

Show all your work! Free-body diagrams must be present and correct for full credit. Plug in numbers only at the end of a problem.

HONOR CODE

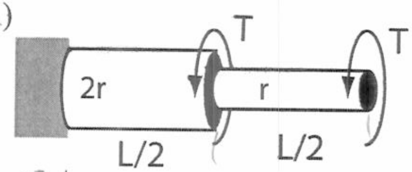
The conditions of this examination are subject to the Georgia Institute of Technology Academic Honor Code.

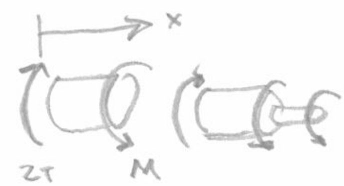
I pledge that the work in this exam represents my own, original work. I have not communicated with anyone about the contents of this exam, nor participated in or observed any conduct prohibited by the Honor Code.

Signature ______________________________

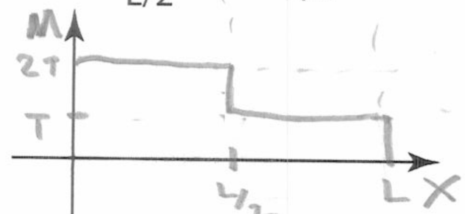
Problem 1A-C (40 points)
Sketch the internal load, polar moment of inertia, maximum shear stress, and shear strain in each cylinder as a function of x. Shear modulus G is constant, cross-sectional dimensions are given as a radius.
Be sure to label all axes and key values.

A)

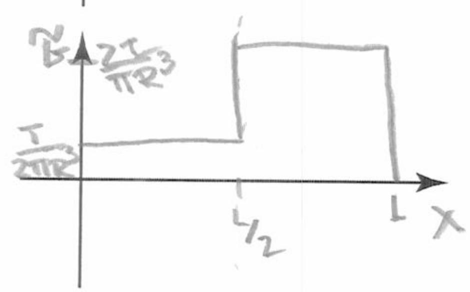




$\Sigma M_x = -2T + M \rightarrow M = 2T$



$J = \frac{\pi R^4}{2}$



$\tau = \frac{TR}{J}$

$\gamma = \tau / G$

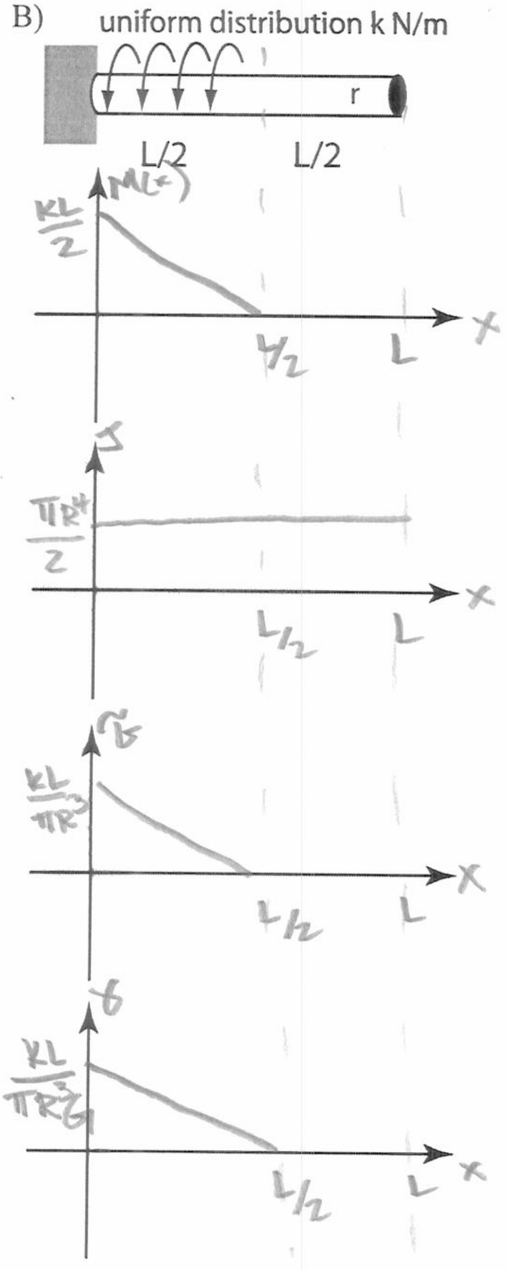
Give an equation for the total deformation of the bar.

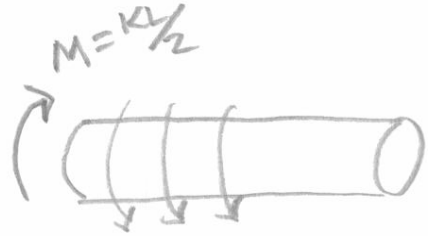
A)

$\varphi_T = \varphi_1 + \varphi_2 = \frac{2T L/2}{8\pi R^4 G} + \frac{T L/2}{\frac{\pi R^4}{2} G} = \frac{9}{8}\frac{TL}{\pi R^4 G}$

$\varphi = \frac{TL}{JG}$

B)





$$T(x)+kx-\frac{KL}{2}=0$$

$$J=\frac{\pi R^4}{2}$$

$$\tau=\frac{TR}{J}$$

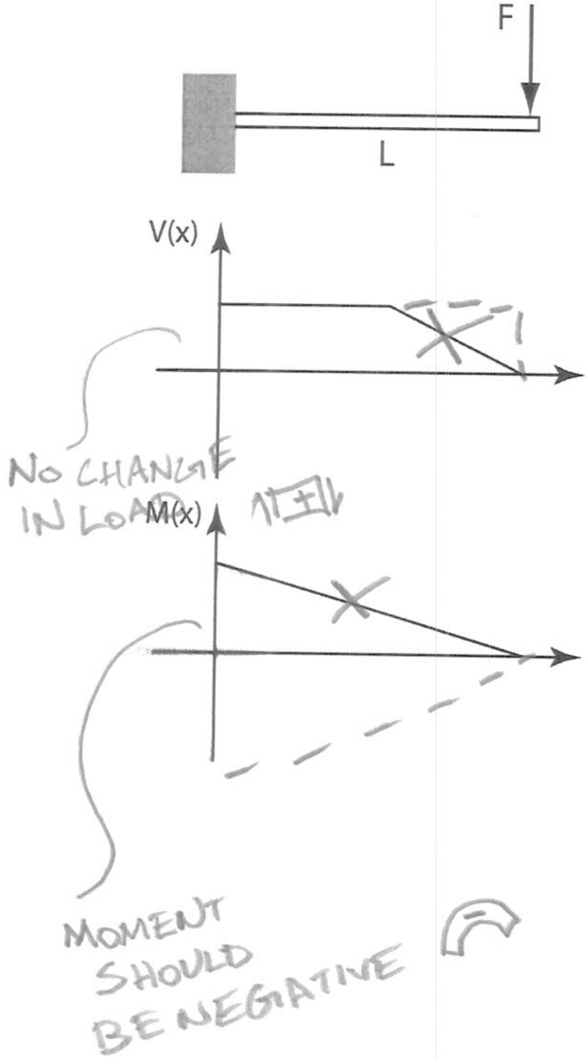
$$\gamma=\frac{\tau}{G}$$

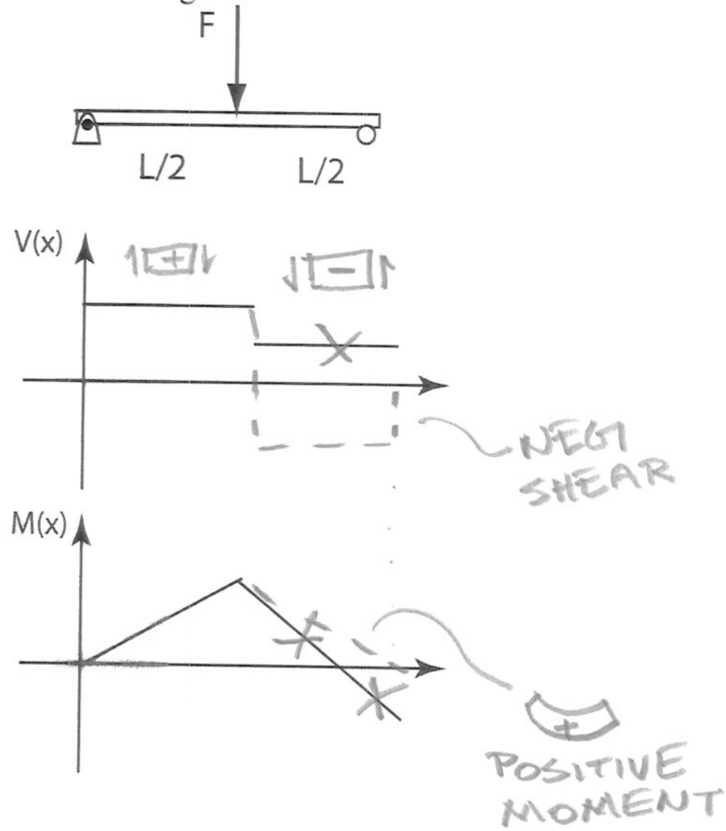
Give an equation for the total deformation of the bar.

B)

$$\varphi_{TOTAL}=\int_0^{L/2}\frac{T(x)}{J(x)G(x)}\,dx+\int_{L/2}^{L}\frac{\cancelto{0}{T(x)}}{J(x)G(x)}\,dx=\int_0^{L/2}\frac{k(L/2-x)}{\frac{\pi R^4}{2}G}\,dx=\frac{kL^2}{4\pi R^4 G}$$

C) **Find** and **explain** the mistakes in these shear and moment diagrams.





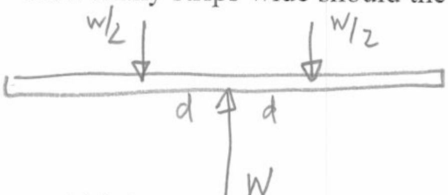
Problem 2 (30 points)

You are contracted by Burton to build a new snowboard for young riders. It must be as light as possible and the board's width must be as small as possible. The company has given you the following design criteria:

-The board must not break when a snowboarder of mass 70 kg rides rails (Figure A). In this maneuver the rider is balanced on a rail midway between the feet.

-You must build the board with bald-cypress strips of 2 cm width and 6 mm height (Figure B). Bald-cypress has an elastic modulus of 9.7 GPa and a maximum allowable stress of 42 MPa.

How many strips wide should the snowboard be?



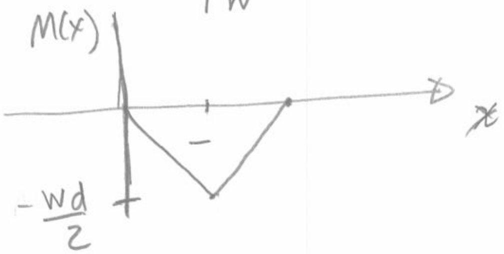


Figure A: Snowboarder riding a rail

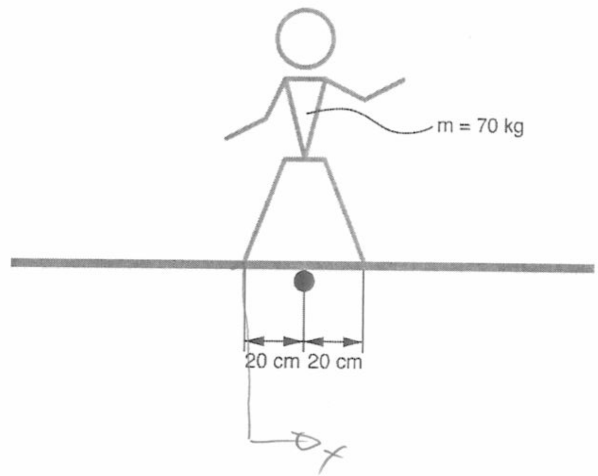
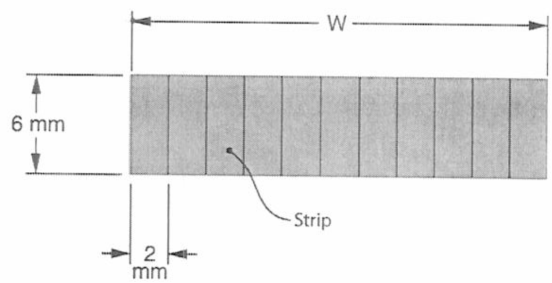


Figure B: Cross section of snowboard



Maximum bending moment occurs at the center

$$M_{max} = -\frac{Wd}{2} = \frac{-(70\ kg)(9.8\ m/s^2)(0.2\ m)}{2}$$

$$M_{max} = -68.6\ N\text{-}m$$

$$\sigma_{max} = -\frac{Mc}{I} = \frac{-Mc}{\frac{1}{12}bh^3}$$ need to choose b

$$b = \frac{-12Mc}{\sigma_{max} h^3} = \frac{-12(-68.6\ N\text{-}m)(0.003\ m)}{(42\times10^6\ Pa)(0.006\ m)^3}$$

$$b = 0.2722$$

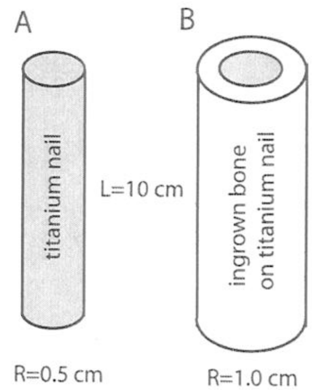
$$= 272.2\ mm$$

$$\#\ strips = \frac{272.2\ mm}{2\ mm/strip} = 136.+ strips$$

$\boxed{\text{So } 137 \text{ strips}}$

Problem 3 (30 points)

You are working at an orthopedic device company. You are testing the effects of stress-shielding due to an intermedullary nail on the shear modulus of bone. Healthy bone as a shear modulus of 3 GPa, Titanium has a shear modulus of 44 GPa, and stainless steel has a shear modulus of 77 GPa.



You have a A) sample of a new intermedullary nail with radius 0.5 cm, and B) a sample taken from a cadaver where the bone has grown on and around the same kind of intermedullary nail. The outer ~~diameter~~ of the bone is 1 cm.
radius

You have a device that applies 10 N-m of torque to each end of the sample.
- If A twists 1.33° degrees under the torsional load what material is it made of?
- If B twists 1.27° degrees under the same load, is the bone healthy?

$$\phi_A = 1.33^\circ \times \frac{\pi \text{ rad}}{180^\circ} = 0.0232 \text{ rad} \qquad J_N = \frac{\pi R^4}{2} = \frac{\pi (0.01 \text{ m})^4}{2}$$

$$= 9.8 \times 10^{-10} \text{ m}^4$$

$$\phi_A = \frac{TL}{GJ}$$

$$0.0232 = \frac{(10 \text{ N})(0.1 \text{ m})}{G_{nail}(9.8 \times 10^{-10} \text{ m}^4)} \qquad J_B = \frac{\pi (R_o^4 - R_i^4)}{2} = \frac{\pi (0.01^4 - 0.005^4)}{2}$$

$$= 1.4 \times 10^{-8} \text{ m}^4$$

$$\boxed{G_{nail} = 4.4 \times 10^{10} \text{ Pa}} \quad \text{titanium}$$

ϕ_B

$\phi_B = \phi_{Nail}\ \phi_{Bone}$ equal twist

$T = T_{\cdot} + T_{Bone}$ from statics

T

$\Sigma T = 0$

$$\phi = \frac{T_N L}{G_N J_N} = \frac{T_B L}{G_B J_B} = 1.27^\circ \times \frac{\pi}{180^\circ} \text{ rad}$$

$$T_N = \frac{\phi\, G_N J_N}{L} = \frac{0.202 \text{ rad} \cdot 44 \times 10^9 \text{ Pa} \times 9.8 \times 10^{-10} \text{ m}^4}{0.1 \text{ m}}$$

$$= 9.56 \text{ N}$$

$$T_B = T - T_N = 10 - 9.56 \text{ N} = 0.44 \text{ N}$$

$$G_B = \frac{T_B L}{\phi J_B} = \frac{(0.44 \text{ N})(0.1 \text{ m})}{(0.0222)(1.4 \times 10^{-8} \text{ m}^4)} = \boxed{1.25 \times 10^8 \text{ Pa}}$$

which is << 3GPa
so unhealthy