Quiz 1 (20 mins), MA1501 Calculus I, 08/27/2013a PRINTED NAME: KEY

This quiz is worth a total of 100 points, and the value of each question is listed with each question. You must show your work; answers without substantiation do not count.

Answers must appear in the box provided! No cheat!

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1. (50 points) Write the formulas for $f \circ g$ and find the domain and the range.

$$f(x) = \sqrt{x+1}, \quad g(x) = \frac{1}{e^x - 1}$$

Answer: $(f \circ g)(x) = (f \circ g)(x) = \sqrt{\frac{1}{e^x-1} + 1}$, domain: $(0, \infty)$, range: $(1, \infty)$

For the domain of a function , we need to consider the following inequality

$$\frac{1}{e^x - 1} + 1 \geq 0$$

$$\Leftrightarrow \frac{1}{e^x - 1} \geq -1 \tag{1}$$

Note that x cannot be 0. We consider two cases for x:

case 1) $x > 0$

In this interval, every x satisfies (1).

case 2) $x < 0$

(1) $\Leftrightarrow e^x - 1 \leq -1 \Leftrightarrow e^x \leq 0$ which is invalid for any x.

Therefore, the domain of $f \circ g$ is $(0, \infty)$ and the range is $(1, \infty)$.

2. (50 points) Find a formula for the inverse function f^{-1} and verify that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$.

$$f(x) = \frac{100}{1 + 2^{-x}}$$

Answer: $f^{-1}(x) = \log_2 \left(\frac{x}{100-x}\right)$

Step 1) Solve for x:

$$1 + 2^{-x} = \frac{100}{y} \iff 2^{-x} = \frac{100}{y} - 1 \iff -x = \log_2 \left(\frac{100}{y} - 1\right) \iff x = \log_2 \left(\frac{y}{100 - y}\right)$$

Step 2) Interchange x and y

$$y = \log_2 \left(\frac{x}{100 - x}\right).$$

Therefore, $f^{-1}(x) = \log_2 \left(\frac{x}{100-x}\right)$.

Show your computation: $(f \circ f^{-1})(x) = \frac{100}{1+2^{-\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1+2^{\log_2\left(\frac{x}{100-x}\right)^{-1}}} = \frac{100}{1+\left(\frac{x}{100-x}\right)^{-1}} = \frac{100}{1+\frac{100-x}{x}} = \frac{100}{1+\frac{100}{x}-1} = \frac{100}{\frac{100}{x}} = x.$

$(f^{-1} \circ f)(x) = \log_2 \left(\frac{\frac{100}{1+2^{-x}}}{100-\frac{100}{1+2^{-x}}}\right) = \log_2 \left(\frac{100}{100\cdot(1+2^{-x})-100}\right) = \log_2 \left(\frac{100}{100\cdot 2^{-x}}\right) = \log_2 \left(\frac{1}{2^{-x}}\right) = \log_2 2^x = x.$