Math 1712 - Spring 2012
Test 1 - Show Your Work

Name: ____________________________________ TA: ____

Use the rules in Chapter 1 to find all derivatives; rules like the product rule, the chain rule, the sum/difference rule, ect. DO NOT USE THE DEFINITION OF THE DERIVATIVE AS A LIMIT!

1. (10 points) a. Find the **slope** of the line that goes thru the two points $P(-9, 2)$ & $Q(3, -4)$.

$$\boldsymbol{slope = m = \frac{-4-2}{3+9} = -\frac{1}{2}}$$

b. Find the **equation of the line** in slope-y intercept form $y = mx + b$ for the line with slope $\frac{1}{4}$ that goes thru the point $(-3, 7)$.

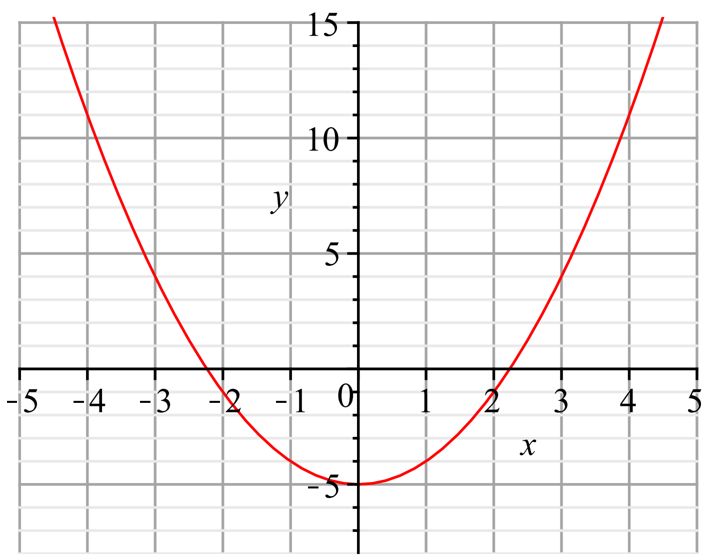
$$\boldsymbol{slope = m = \frac{1}{4} \Rightarrow y = \frac{1}{4}x + b.\ (-3, 7) \text{ is on the line } \Rightarrow 7 = \frac{1}{4}(-3) + b \Rightarrow b = \frac{31}{4}}$$

$$\boldsymbol{\Rightarrow y = \frac{1}{4}x + \frac{31}{4}}$$

c. Find the equation of the line **perpendicular** to the line $y = -\frac{1}{3}x + 4$ that goes thru the point $(2, 5)$. Put your answer in the $y = mx + b$ form.

Since the new line is perpendicular, $m = 3$. $(2, 5)$ is on the new line $\Rightarrow 5 = 3(2) + b \Rightarrow b = -1$ $\Rightarrow y = 3x - 1$

2. (10 points) A function $f(x)$ is graphed below. a. Find $f(1)$. b. Use the graph to find the range for $f(x)$. c. Put a dot on the graph for each x-intercept and for $f(1)$.



a. $f(1) = -4$

b. $range = [-5, \infty)$ or equivalent.

3. (10 points) Company XYZ has determined that the supply and demand functions for it's new product are given by:

Supply: $y = x^2 + x + 13$ Demand: $y = x^2 - 16x + 64$

where x is the unit price, in dollars, and y is the number of units (in thousands). Find the **equilibrium point**; that is, find the values of x & y for which $S = D$. Use the correct units in your answers.

$$\boldsymbol{S = D \Rightarrow x^2 + x + 13 = x^2 - 16x + 64 \Rightarrow x + 13 = -16x + 64 \Rightarrow 17x = 51 \Rightarrow x = 3}$$

$\boldsymbol{x = 3 \Rightarrow y = 3^2 + 3 + 13 = 25}$ **ANSWER: $\boldsymbol{x =\$ 3}$ & $\boldsymbol{y = 25}$ *thousand units***

4. (15 points) Let $f(x) = \dfrac{x^2 - 36}{x - 6}$. Use this function to answer the following questions.

a. Fill in the y values in the following tables (use your calculator)

x	5.9	5.99	5.999	5.9999
y	**11.9**	**11.99**	**11.999**	**11.9999**

x	6.1	6.01	6.001	6.0001
$\boldsymbol{y}$	**12.1**	**12.01**	**12.001**	**12.0001**

b. Based on your answer to a, find $\lim_{x \to 6} \dfrac{x^2 - 36}{x - 6} = \mathbf{12}$. Your table must justify your answer.

c. Use algebraic/analytic methods to evaluate $\lim_{x \to 6} \dfrac{x^2 - 36}{x - 6}$. Show your work.

$$\boldsymbol{\lim_{x \to 6} \frac{x^2 - 36}{x - 6} = \lim_{x \to 6} \frac{(x-6)\,(x+6)}{x - 6} = \lim_{x \to 6} x + 6 = 12}$$

5. (10 points) Find the following limits; show your computations to justify your limit. If the limit does not exist, state DNE and explain why it DNE.

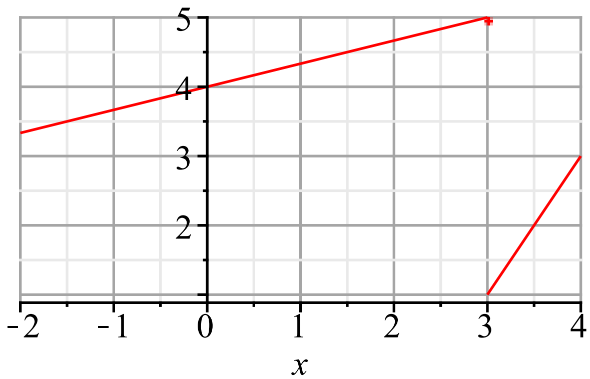
a. $\lim_{x \to 16} \dfrac{x - 16}{\sqrt{x} - 4} \, \dfrac{\boldsymbol{\sqrt{x} + 4}}{\boldsymbol{\sqrt{x} + 4}} = \boldsymbol{\lim_{x \to 4} \dfrac{(x - 16)\left(\sqrt{x} + 4\right)}{x - 16} = \lim_{x \to 4} \sqrt{x} + 4 = \sqrt{16} + 4 = 8}$

Rationalize.

b. $\lim_{x \to 0^+} \dfrac{1}{x}$ ***DNE* because if we divide 1 by a small positive number, we get a large number that increases without bound.**

6. (10 points) The definition and graph of $g(x)$ are given below:

$$\begin{cases} \frac{1}{3}x+4 & x \le 3 \\ 2x-5 & 3 < x \end{cases} \qquad (1)$$



Is $g(x)$ continuous at $x=3$? **(yes or no)** a. Explain your answer using the graph. b. Then explain your answer using limits.

NO, $g(x)$ is not continuous at $x=3$

a. The graph shows a jump or a gap at $x=3$. b. $\lim_{x \to 3} g(x)$ does not exist because of the gap OR
$\lim_{x \to 3^+} g(x) \neq \lim_{x \to 3} g(x)$

7. (10 points) Find the **equation of the tangent line** to the graph of the function $y=f(x) = \frac{x^2-4}{x}$, if $x=-4$. Put your answer in the form $y=mx+b$.

$$f'(x) = \frac{2x^2-x^2+4}{x^2} = \frac{x^2+4}{x^2} \Rightarrow slope = f'(-4) = \frac{16+4}{16} = \frac{5}{4}$$

$$x=-4 \Rightarrow y = \frac{16-4}{-4} = -3 \Rightarrow y+3 = \frac{5}{4}(x+4) \Rightarrow y = \frac{5}{4}x+2 \text{ (TL)}$$

8. (10 points) Let $g(x) = x^3 - 3x^2$. a. Find all **values of x** at which the tangent line to $g(x)$ is horizontal. b. Find all **values of x** at which the tangent line to $g(x)$ has slope 9.

a. tangent line horizontal
$\Rightarrow slope = 0 \Rightarrow g'(x) = 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0$ **and** $x = 2$
ANSWER: $x=0$ & $x=2$

b. slope = 9
$\Rightarrow g'(x) = 3x^2 - 6x = 9 \Rightarrow 3\left(x^2-2x-3\right) = 0 \Rightarrow 3(x-3)(x+1) = 0 \Rightarrow x=-1$ **and** $x=3$
ANSWER: $x=-1$ & $x=3$

9. (20 points) Find the derivatives of the following functions. Be sure to show your work.

a. $G(x) = 8\sqrt[3]{x} = 8x^{\frac{1}{3}} \Rightarrow G'(x) = \frac{8}{3}x^{-\frac{2}{3}} = \frac{8}{3x^{\frac{2}{3}}} = \frac{8}{3\sqrt[3]{x^2}}$ **any of these will do.**

b. $H(x) = \frac{-3}{x^5} = -3x^{-5} \Rightarrow H'(x) = 15x^{-6} = \frac{15}{x^6}$ **any of these will do**

c. $g(x) = (5 - x^2)(3x - 1)$ **<u>Simplify your answer</u>**..

$g'(x) = (-2x)(3x - 1) + (3)(5 - x^2) = -9x^2 + 2x + 15$

d. $h(x) = (3x^2 - 7x + 2)^3 \Rightarrow h'(x) = 3(3x^2 - 7x + 2)^2 (6x - 7)$

$h'(x) = 3(3x^2 - 7x + 2)^2 (6x - 7)$ **Or any equivalent**

10. (10 points) $C(x) = 4x^2 + 100$ is the cost function for the ABC Company, where x is the number of items produced and C is in dollars. The average cost function is $ACF(x) = \frac{C(x)}{x}$. Find the rate at which the average cost function is changing when 3 items are produced.

Rate of change $= ACF'(x) = \frac{4x^2 - 100}{x^2}$ **or equivalent**

The rate at which the average cost function is changing when 3 items are produced $= ACF'(3) = -7.11$

Answer: \$ -7.11 or decreasing (falling) at the rate of \$ 7.11

11. (10 points) An object moves on a hortzontal line so that it's position (in feet) at time t (in seconds) is given by: $s(t) = t + t^4$. a. Find $v(t)$ & $a(t)$. b. Find the velocity and acceleration of the object when $t = 2$ seconds. You do NOT need to put in the units.

a. $v(t) = s'(t) = 1 + 4t^3$ **&** $a(t) = v'(t) = 12t^2$

b. $velocity = v(2) = 1 + 4(2^3) = 33$ **&** $acceleration = a(2) = 48$

EXTRA CREDIT: 5 points The population of a city is given by: $P(t) = 50t^2 + 10000$, where P is the poulation and t is in years.

Find the rate at which the population is changing after 20 years. Use the correct units in your answer.

$P'(t) = 100t \Rightarrow P'(20) = 2000 \frac{people}{year}$