Quiz 1 - Name ______________________________ Math 2403 - Section L ___

Good Luck!

This quiz has a back side! Don't forget about Question 3 and Bonus Question!

1. (5 points) Solve the initial value problem

$$y' + \frac{1+x}{x} y = 0, \quad y(1) = 1$$

Solution:

$$y = ce^{-\int \frac{1+x}{x} dx} = ce^{-\ln|x| - x} = c\left(\frac{1}{x}e^{-x}\right).$$

Imposing the initial condition $y(1) = 1$, we have $c = e$.
Therefore the solution of the IVP is

$$y = \left(\frac{1}{x}e^{1-x}\right)$$

2. (5 points) Solve the initial value problem

$$(1 + x^2)y' + 4xy = \frac{2}{1 + x^2}, \quad y(0) = 1$$

Solution: Dividing by $(1 + x^2)$, the equation becomes

$$y' + \frac{4x}{1 + x^2} y = \frac{2}{(1 + x^2)^2}$$

The solution of the complementary equation is

$$y_1 = e^{\int \frac{4x}{1+x^2} dx} = e^{-\ln(1+x^2)^2} = \frac{1}{(1 + x^2)^2}$$

Look for the general solution of the form $y = y_1 u = \frac{u}{(1+x^2)^2}$ and therefore

$$u' = 2 \text{ and } u = 2x$$

The general solution of the problem is

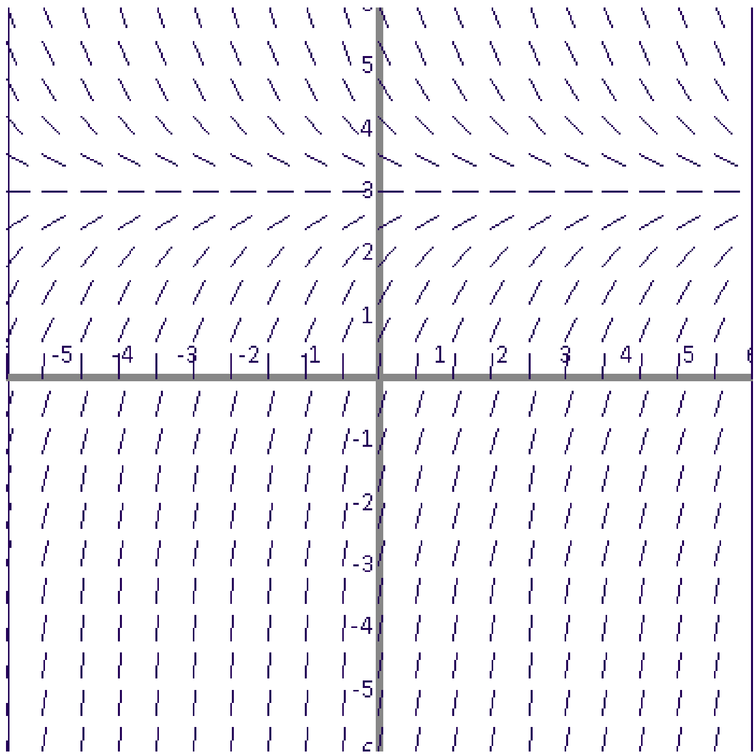
$$y = \frac{2x + c}{(1 + x^2)^2}$$

By imposing the initial condition we find $c = 1$.

3. (5 points) Draw the direction field for the following equation

$$y' = 3 - y$$

in the region $[0, 6] \times [0, 6]$.
Solution:



Bonus(2 points) Find a general solution of the equation.
Solution: The solution of the complementary equation is $y_1 = e^{-x}$.
The general solution is $y = 3 + ce^{-x}$.