

Circle your TA's name: D1 - Lacy Hardcastle D2 - Kyla (Hewell) Adams D3 - Kelly Robinson

1. Let
- $y^6 + x^3 + (3y-x)^5 = 20$
- . Find
- $\frac{dy}{dx}$
- .

$$6y^5 \frac{dy}{dx} + 3x^2 + 5(3y-x)^4 (3 \frac{dy}{dx} - 1) = 0$$

$$6y^5 \frac{dy}{dx} + 3x^2 + 15(3y-x)^4 \frac{dy}{dx} - 5(3y-x)^4 = 0$$

$$6y^5 \frac{dy}{dx} + 15(3y-x)^4 \frac{dy}{dx} = -3x^2 + 5(3y-x)^4$$

$$(15 \text{ pts.}) \quad [6y^5 + 15(3y-x)^4] \frac{dy}{dx} = -3x^2 + 5(3y-x)^4$$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2 + 5(3y-x)^4}{6y^5 + 15(3y-x)^4}}$$

2. Use a differential (or the tangent line approximation method) to approximate
- $y = \frac{1}{\sqrt{7}}$
- .

$$\text{let } f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$$

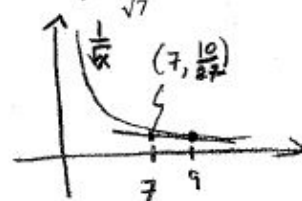
$$f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = -\frac{1}{2(\sqrt{x})^3}$$

$$\frac{1}{\sqrt{7}} \approx f(9) + f'(9)(7-9)$$

$$= \frac{1}{\sqrt{9}} + \left(-\frac{1}{2(\sqrt{9})^3} \right) (-2)$$

$$= \frac{1}{3} + \frac{1}{27} = \frac{1}{3} + \frac{1}{27} = \frac{9+1}{27}$$

$$= \boxed{\frac{10}{27}}$$



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2. Use a differential (or the tangent line approximation method) to approximate $y = \%$

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3. Let $f(x) = \frac{x^2 - 9}{x - 1}$.

- (a) Find the domain of
- f
- .

The denominator is not equal to zero.

(6 pts.)

$$x \neq 1$$

- (b) Find the x-intercepts and y-intercept of
- f
- .

x-int : Set $y = 0$

$$\frac{x^2 - 9}{x - 1} = 0$$

$$x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

(10 pts.)

y-int Set $x = 0$

$$y = \frac{0^2 - 9}{0 - 1}$$

$$y = 9$$

- (c) Find all asymptotes of
- f
- .

The vertical asymptote:

Set the denominator = 0

$$x = 1$$

(10 pts.)

check $\lim_{x \rightarrow 1} \frac{x^2 - 9}{x - 1}$ DNE

There is no horizontal asymptote

because $\lim_{x \rightarrow \infty} \frac{x^2 - 9}{x - 1} = \lim_{x \rightarrow \infty} \frac{x(x - \frac{9}{x})}{x(1 - \frac{1}{x})} = \infty$

$$\begin{array}{r} x+1 \\ x-1 \overline{) x^2 - 9} \\ \underline{\ominus x^2 \oplus x} \\ x - 9 \\ \underline{\ominus x \oplus 1} \\ -8 \end{array}$$

The oblique asymptote is $y = x + 1$.

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3.

(c) Find all asymptotes of f

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4. The management of Hotlanta Hot Sauce Company estimate that their profit (in dollars) from the daily production and sale of x boxes (each box consisting of a dozen bottles) of the hot sauce is given by $P(x) = -0.000001x^3 + 27x - 8000$. How many bottles of hot sauce does the company have to produce and sell to make the largest daily profit?

$$P'(x) = (-0.000001)(3x^2) + 27 = 0$$

$$(-0.000001) 3x^2 = -27$$

$$x^2 = \frac{-27}{-0.000001 \cdot 3} = 9000000$$

$$x = \sqrt{9000000} = 3000 \text{ boxes.}$$

(15 pts.)

$$P''(x) = (-0.000001)(6x)$$

$$P''(3000) = -0.000001(6 \cdot 3000) < 0$$

max
 $\hat{S} \ x=3000$
 concave
 down

\therefore Hotlanta Hot sauce company has to sell 36,000 bottles a day to make the largest daily profit.

5. Let $f(x) = \frac{x^3}{27}$. Find $f^{-1}(x)$.

$$y = \frac{x^3}{27}$$

$$27y = x^3$$

$$\sqrt[3]{27y} = x$$

$$x = 3\sqrt[3]{y}$$

$$y = 3\sqrt[3]{x}$$

(10 pts.)

$$\boxed{f^{-1}(x) = 3\sqrt[3]{x}}$$

MATH 1112 D1-D3 7 Test #2 Full Name

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4. The management of Hotlanta Hot Sauce Company estimate that their profit (in dollars) from the daily production and sale of x boxes (each box consisting of a dozen bottles) of the hot sauce is given by $P(x) = -0.000001x^3 + 27x - 8000$. How many bottles of hot sauce does the company have to produce and sell to make the largest daily profit?

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6. Determine whether each statement is true or false. Justify the answers.

- (a) If
- $f'(c) = 0$
- , then
- f
- has a relative maximum or a relative minimum at
- $x = c$
- .

(6 pts.)

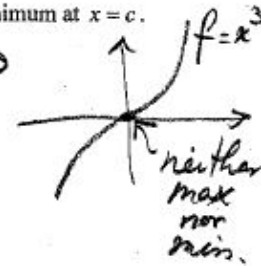
False

a counterexample

$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$f'(0) = 0$$



- (a) If
- $f'(c) = 0$
- and
- $f''(c) = 0$
- , the second derivative test fails.

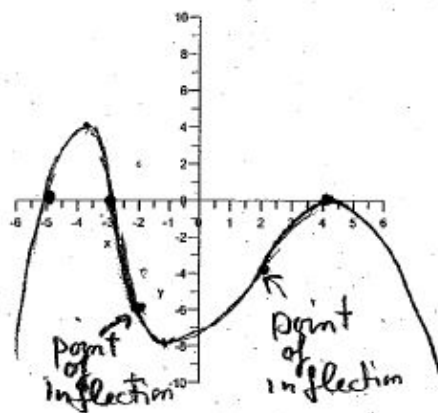
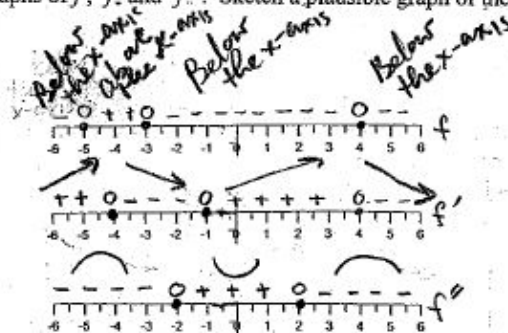
(6 pts.)

True

See the rule (3c) on p. 274.

7. The sign graphs of f , f' and f'' . Sketch a plausible graph of the function f .

(12 pts.)



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6. Determine whether each statement is true or false. Justifi the answers.

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