ISyE 4031 Regression and Forecasting
Homework 4 Solutions
Spring 2016

1. The solution:

Regression Equation
Minutes = 11.46 + 24.602 Copiers

Coefficients

Term	Coef	SE Coef	T-Value	P-Value
Constant	11.46	3.44	3.33	0.009
Copiers	24.602	0.805	30.58	0.000

Model Summary

S	R-sq	R-sq(adj)
4.61521	99.05%	98.94%

Analysis of Variance

Source	DF	SS	MS	F-Value	P-Value
Regression	1	19918.8	19918.8	935.15	0.000
Error	9	191.7	21.3		
Total	10	20110.5			

R solution:

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	11.4641	3.4390	3.334	0.00875	**
Copiers	24.6022	0.8045	30.580	2.09e-10	***

Residual standard error: 4.615 on 9 degrees of freedom
Multiple R-squared: 0.9905, Adjusted R-squared: 0.9894
F-statistic: 935.1 on 1 and 9 DF, p-value: 2.094e-10

Analysis of Variance Table
Response: Minutes

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
Copiers	1	19918.8	19918.8	935.15	2.094e-10	***
Residuals	9	191.7	21.3			

2. Exercise 3.25.

a. Total variation = 20,110.5; Unexplained variation = 191.7; Explained variation = 19,918.8; r^2 = 0.9905; r = 0.9952.
Interpretation: 99.05% the total variation in service time can be explained by the linear relationship between service time and the number of copiers serviced.

b. $t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \dfrac{.9952\sqrt{11-2}}{\sqrt{1-.9905}} = 30.63$ (Difference from 30.58 is round-off error)

Since the test statistic is less than both $t^{(9)}_{[.025]} = 2.262,$ and $t^{(9)}_{[.005]} = 3.250$, we reject $H_0:\ \rho = 0$ at $\alpha = 0.05$ and $\alpha = 0.01$.

3. Exercise 3.29.

a. F =19919 / (191.70166/9) = 935.16 (approximately 935.15, round-off error).

b. Since 935.15 > $F_{[.05,1,9]} = 5.12$, reject $H_0: \beta_1 = 0$ with strong evidence of a linear relationship between x and y.

c. Since 935.15 > $F_{[.01,1,9]} = 10.56$, reject $H_0: \beta_1 = 0$ with strong evidence of a linear relationship between x and y.

d. p-value = 0 which is less than .001; Reject H_0 at all levels of α , extremely strong evidence of a linear relationship between x and y.

e. $t^2 = (30.58)^2 = 935.14$ (approximately equals F = 935.15)

$\left(t^{(9)}_{[.025]}\right)^2 = (2.262)^2 = 5.12 = F^{(1,9)}_{[.05]}$.

4. Exercise 3.36.

Minitab Solution:

```
Analysis of Variance
Source          DF   Adj SS  Adj MS  F-Value  P-Value
Regression       1   470.74  470.74    18.21    0.000
Error           52  1344.33   25.85
Total           53  1815.07

Model Summary
      S    R-sq  R-sq(adj)  R-sq(pred)
5.08454  25.93%     24.51%      19.02%

Coefficients
Term        Coef  SE Coef  T-Value  P-Value
Constant    0.85     1.98     0.43    0.670
AcctRt     0.610    0.143     4.27    0.000

Regression Equation
MarketRt = 0.85 + 0.610 AcctRt

Variable  Setting
AcctRt         15
    Fit    SE Fit        95% CI               95% PI
10.0042  0.752591  (8.49400, 11.5144)  (-0.309854, 20.3182)
```

R Solution:

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   0.8468     1.9751   0.429     0.67
AcctRt        0.6105     0.1431   4.267  8.4e-05 ***
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Residual standard error: 5.085 on 52 degrees of freedom
Multiple R-squared:  0.2593,   Adjusted R-squared:  0.2451
F-statistic: 18.21 on 1 and 52 DF,  p-value: 8.4e-05
```

```
> predict(lmod,newdata,level = 0.95, interval="confidence")
       fit      lwr      upr
1 10.00419 8.494002 11.51437
> predict(lmod,newdata, level = 0.95, interval="prediction")
       fit        lwr      upr
1 10.00419 -0.3098535 20.31823
```

a. When x = 15, $\hat{y} = 10.004$ and a 95% C. I. for mean market return rate is [8.494, 11.514].

b. When x = 15, $\hat{y} = 10.004$ and a 95% P. I. for market return rate of this individual stock is [−0.310, 20.318].

5. a. The error assumption, $E[\varepsilon_i] = 0$ is justified, because the mean of residuals is basically zero. From the normality test results "Mean" of errors= $-3.69*10^{-13}$.

b. Each ε_i has normal distribution. From the A−D test results: p−value of the test = 0.571 is greater than any reasonable significance level, α, e.g. 0.01, 0.05, 0.10, 0.15, etc. We do not reject H_0: Random errors are normal.

c. The assumption, each ε_i has identical distribution (identical variance) is violated. Residual vs. fitted value plot displays a non−random pattern. It seems there is a parabolic pattern.

6. Exercise B.1.

a. $\mathbf{A}' = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

b. $\mathbf{A}'\mathbf{A} = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 9 \\ 9 & 9 \end{bmatrix}$

7. Exercise B.2.

a. $\mathbf{A} + \mathbf{B} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 3 \end{bmatrix} + \begin{bmatrix} 0 & .6 & -.2 \\ .5 & -.2 & -.1 \\ -.5 & 0 & .5 \end{bmatrix} = \begin{bmatrix} 1 & 3.6 & .8 \\ 2.5 & .8 & .9 \\ .5 & 3 & 3.5 \end{bmatrix}$

b. $\mathbf{AB} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} 0 & .6 & -.2 \\ .5 & -.2 & -.1 \\ -.5 & 0 & .5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\mathbf{BA} = \begin{bmatrix} 0 & .6 & -.2 \\ .5 & -.2 & -.1 \\ -.5 & 0 & .5 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. $\mathbf{B} = \mathbf{A}^{-1}$