Quiz 5 (25 mins), MA1501 Calculus I, 09/24/2013a PRINTED NAME: ________________GT ID:____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

Answers must appear in the box provided! No cheat!

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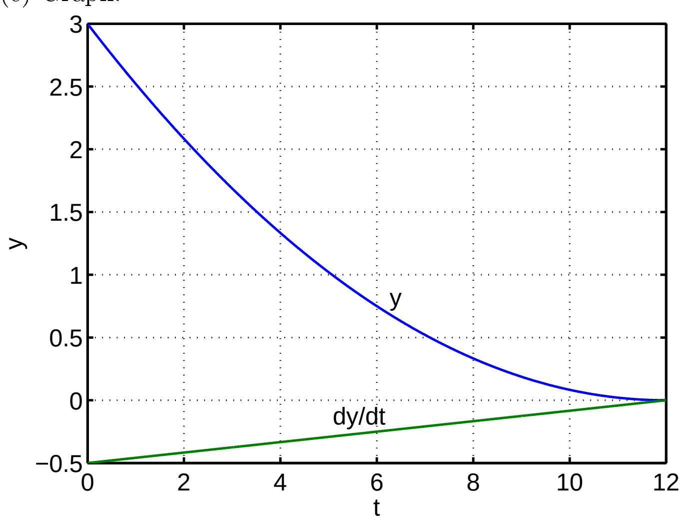
1. It takes 12 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank t hours after the valve is opened is given by the formula

$$y = 3\left(1 - \frac{t}{12}\right)^2 m.$$

(a) (10 pts) Find the rate $\frac{dy}{dt}$ (m/h) at which the tank is draining at time t.
(b) (20 pts) When is the fluid level in the tank falling fastest? Slowest? What are the values of $\frac{dy}{dt}$ at these times?
(c) (10 pts) Graph y and $\frac{dy}{dt}$ together for $t \in [0, 12]$ and discuss the behavior of y in relation to the signs.

Answer: (a) $\frac{dy}{dt} = 3\frac{d}{dt}\left(1 - \frac{t}{12}\right)^2 = 3\frac{d}{dt}\left(1 - \frac{t}{6} + \frac{t^2}{144}\right) = 3\left(-\frac{1}{6} + \frac{t}{72}\right) = -\frac{1}{2} + \frac{t}{24}$ (m/h)
or using the chain rule: $\frac{dy}{dt} = 3 \cdot 2\left(1 - \frac{t}{12}\right) \cdot \left(-\frac{1}{12}\right) = -\frac{1}{2}\left(1 - \frac{t}{12}\right) = -\frac{1}{2} + \frac{t}{24}$ (m/h)
(b) The largest value of $\frac{dy}{dt}$ is 0 (m/h) when $t = 12$ and the fluid level is falling the slowest at that time.
The smallest value of $\frac{dy}{dt}$ is $-\frac{1}{2}$ (m/h) when $t = 0$ and the fluid level is falling the fastest at that time.
(c) Graph:



Please circle your answer: Since $\frac{dy}{dt} \leq 0$, the graph of y is always **decreasing**.

2. (30pts) Find $\frac{dp}{dq}$

$$p = \frac{3q + \tan\ q}{q \sec\ q}$$

Answer: Method 1)

$$p = \frac{3q + \tan\ q}{q \sec\ q} = \frac{3q + \frac{\sin\ q}{\cos\ q}}{q\frac{1}{\cos\ q}} = \frac{3q \cos\ q + \sin\ q}{q}.$$

$$\begin{aligned}
\frac{dp}{dq} &= \frac{(3q\cos q + \sin q)'q - (3q\cos q + \sin q)q'}{q^2} \\
&= \frac{(3q'\cos q + 3q(\cos q)' + \cos q)q - (3q\cos q + \sin q)}{q^2} \\
&= \frac{(3q'\cos q + 3q(\cos q)' + \cos q)q - (3q\cos q + \sin q)}{q^2} \\
&= \frac{(4\cos q - 3q\sin q)q - 3q\cos q - \sin q}{q^2} \\
&= \frac{q\cos q - 3q^2\sin q - \sin q}{q^2}
\end{aligned}$$

Method 2) using $(\tan q)' = \sec^2 q$ and $(\sec q)' = \sec q \tan q$,

$$\begin{aligned}
\frac{dp}{dq} &= \frac{(3q + \tan q)'(q\sec q) - (3q + \tan q)(q\sec q)'}{q^2\sec^2 q} \\
&= \frac{(3 + \sec^2 q)(q\sec q) - (3q + \tan q)(\sec q + q\sec q\tan q)}{q^2\sec^2 q} \\
&= \frac{(3 + \sec^2 q)q - (3q + \tan q)(1 + q\tan q)}{q^2\sec q} \\
&= \frac{3q + q\sec^2 q - 3q - 3q^2\tan q - \tan q - q\tan^2 q}{q^2\sec q} \\
&= \frac{q(\sec^2 q - \tan^2 q) - 3q^2\tan q - \tan q}{q^2\sec q} \\
&= \frac{q - 3q^2\tan q - \tan q}{q^2\sec q} \\
&= \frac{q\cos q - 3q^2\sin q - \sin q}{q^2}
\end{aligned}$$

3. (30 pts) Find the second derivative of the function

$$y = \sin(x^2e^x) + (2x+1)^7.$$

Answer: The first derivative of y is

$$\begin{aligned}
y' &= \cos(x^2e^x)(x^2e^x)' + 7(2x+1)^6(2x+1)' \\
&= \cos(x^2e^x)(2xe^x + x^2e^x) + 14(2x+1)^6.
\end{aligned}$$

The second derivative of y is

$$\begin{aligned}
y'' &= -\sin(x^2e^x)(x^2e^x)'(2xe^x + x^2e^x) + \cos(x^2e^x)(2xe^x + x^2e^x)' + 84(2x+1)^5(2x+1)' \\
&= -\sin(x^2e^x)(2xe^x + x^2e^x)^2 + \cos(x^2e^x)(2e^x + 4xe^x + x^2e^x) + 168(2x+1)^5.
\end{aligned}$$