ChBE 3200
Transport Phenomena I
Fall 2013

Exam II
Oct 25, 2013

This exam is closed-book, closed-notes. Some equations and other relevant information are provided. The use of wireless devices (e.g. cell phones, IR transmitters/receivers) is not permitted. The use of programmable calculators is only allowed if all relevant content has been erased from the calculator memory.

To receive full credit on each problem, it is advised to start with the appropriate full form of the balance equation(s) needed to solve the problem. Label all variables and equations. Include a brief word description to explain each step in your problem if appropriate. State <u>all</u> your assumptions clearly. Present your solution clearly. Numerical answers without units or explanations will not receive credit.

Name: SOLUTION
(PLEASE WRITE YOUR NAME ALSO ON THE BACK OF THE EXAM.)

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

Signed: ____________________

Problem I ________/20

Problem II ________/50

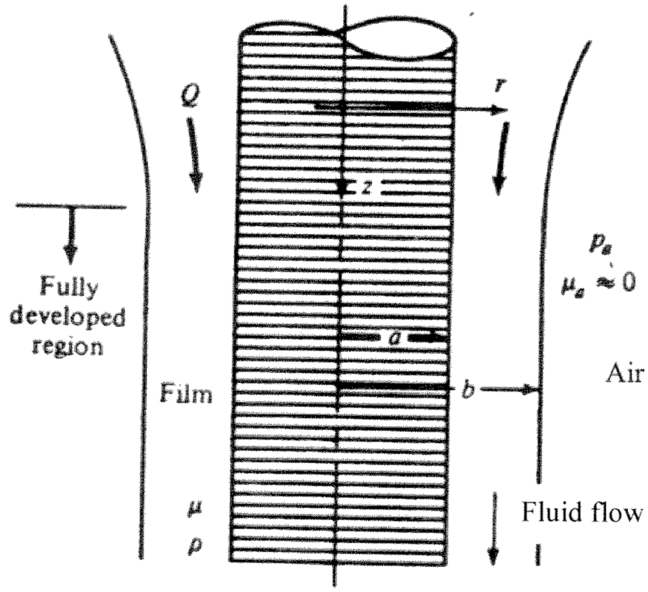
Problem III ________/30

Total ________/100

PLEASE SCAN THROUGH THE ENTIRE EXAM BEFORE WORKING ON IT.

Problem 1. (20 pts)

1.1. A Newtonian fluid flows down the side of a vertical rod of radius a as shown in the figure below. At a certain distance down the rod, the film will approach a fully developed flow of constant radius b. Assume that the atmosphere offers no shear resistance to the film motion. Assume that the thickness of the flowing fluid is small (i.e. (b-a) is a small value). (10 pts)



a. Which velocity component(s) is (are) non-zero?

$v_z(r)$ $\quad$ $v_\theta = 0$ $\quad$ $v_r = 0$

b. What are the boundary conditions to solve for the velocity components?

at $r = a$, $v_z = 0$ (no slip)

at $r = b$, $\tau_{rz} = 0$ (free surface)

c. Write out the Navier Stokes equations only in the direction of the non-zero velocity components. Cross out all terms in the Navier Stokes equations that you can neglect in this situation. Next to each term that you have crossed out, write a clear explanation why this term can be neglected (note: no credit will be given if the explanation is unclear or illegible). DO NOT SOLVE THE EQUATIONS.

fully developed $\qquad$ symmetry

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\cancel{\frac{\partial v_z}{\partial \theta}} + v_z\cancel{\frac{\partial v_z}{\partial z}}\right) = -\cancel{\frac{\partial P}{\partial z}} + \rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right]$$

S.S. $\qquad$ symmetry $\qquad$ open system $\qquad$ fully developed

no flow in r direction
(also because of continuity)

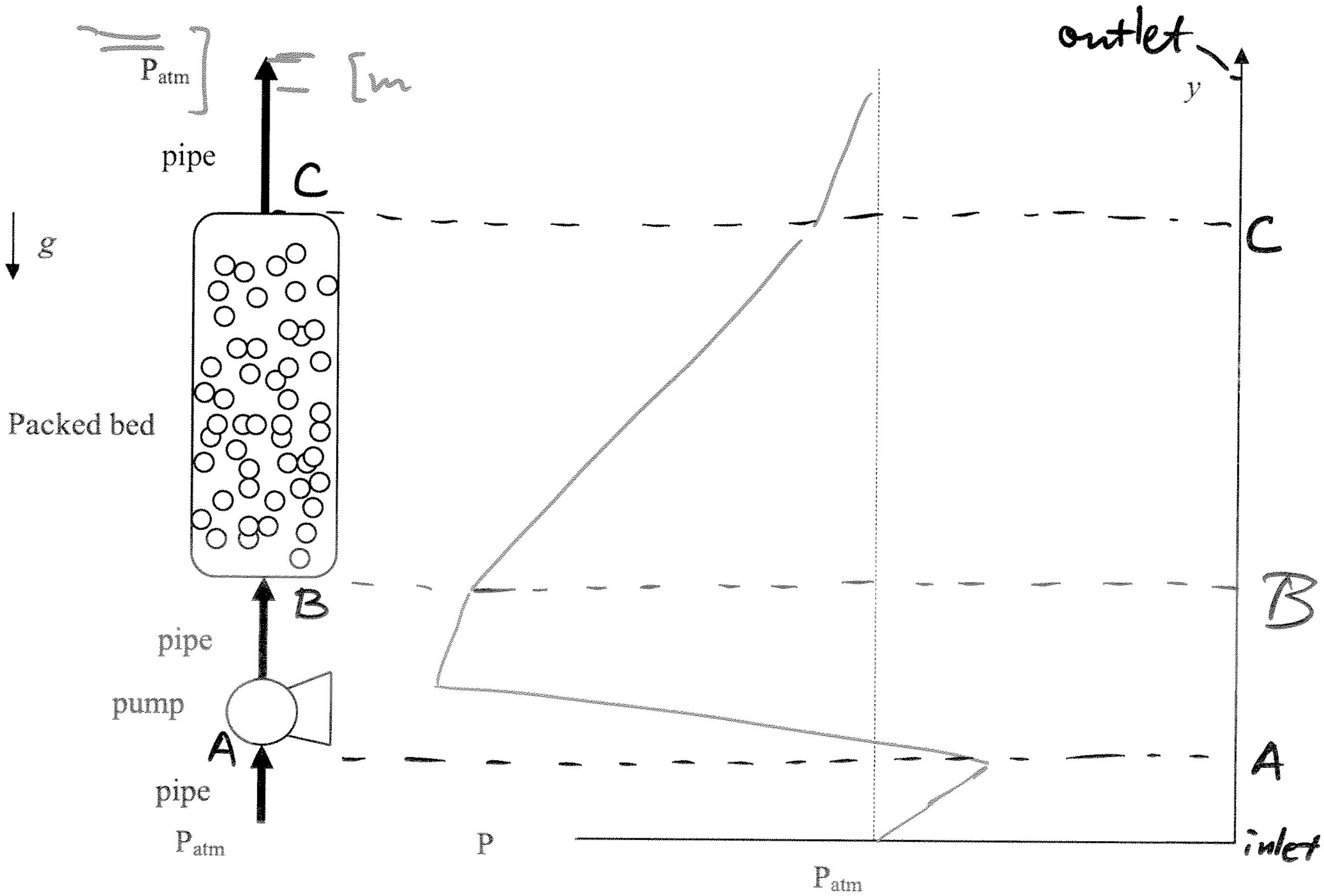
1.2. Boundary layers only exist in turbulent flow; more turbulent flow (higher Re) has thinner boundary layers. TRUE / FALSE (Correct or give reason if false.) (2pts)

1.3. In analyzing the boundary layer for flow over a semi-infinity plate (flow is in the x direction with v_∞), continuity equation for boundary layers is as follows: $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$ What does the continuity equation tell us about v_y? (Hint: do a scaling analysis of this equation.) (4 pts)

$$\frac{\partial v_x}{\partial x} \sim \frac{v_\infty}{x} \qquad \frac{\partial v_y}{\partial y} \sim \frac{\tilde{v}_y}{\delta}$$

since $\delta \ll x$, $\tilde{v}_y \ll v_\infty$

1.4. Draw pressure as a function of vertical distance for the following situation. (4 pts)



Problem II. (50 points)

A smooth water pipe slopes upward at an angle of 30 degrees. The pipe is 2 inches in diameter. The globe valve is fully open. The water is at a constant temperature of 20C. The density of water is 62.4 lb/cubic foot and the specific gravity of mercury is 13.6. The kinematic viscosity of water is 1.08 x 10^{-5} ft^2/s.

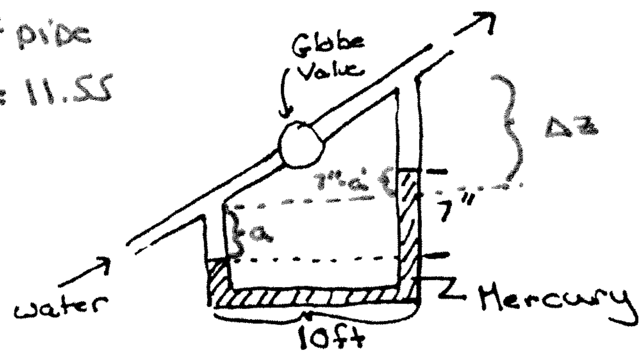
a. If the mercury manometer shows a 7 inch deflection, what is the flow rate in cubic feet per second.

b. Explain qualitatively what happens to the frictional losses as the temperature is increased.

Manometer Analysis:

L = length of pipe

$= \frac{10}{\cos 30} = 11.55$



$P_1 + \rho_w g a - \rho_{Hg} g \left(\frac{7}{12}\right) - \rho_w \left(\Delta z - \left(\frac{7}{12} - a\right)\right) = P_2$

$P_2 - P_1 = \frac{7}{12} g \left[\rho_w - \rho_{Hg}\right] - \rho_w g \Delta z$

~~$P_2 - P_1 = \frac{7}{12} g [-12.6]$~~

+8

Mechanical Energy Balance

$$\frac{\delta Q}{\partial t} - \frac{\delta W_s}{\partial t} - \frac{\delta W_\mu}{\partial t} = \iint_{C.S} \rho \left(e + \frac{P}{\rho}\right)(\vec{v} \cdot \vec{n})\, dA + \frac{\partial}{\partial t} \iiint_{C.V.} \rho e\, dV$$

No heat, no work, steady state

$$\frac{\cancel{V_2^2 - V_1^2}}{2g_c} + \frac{g}{g_c}(z_2 - z_1) + \frac{P_2 - P_1}{\rho} + h_L g = 0 \Rightarrow \frac{g}{g_c}(z_2 - z_1) + \frac{P_2 - P_1}{\rho} + h_L g = 0$$

+10

constant diameter pipe

$$g \cancel{\Delta z} + \frac{(7/12) g \left[\rho_w - \rho_{Hg}\right]}{\rho_w} - \frac{\rho_w g \cancel{\Delta z}}{\rho_w} + h_L g = 0$$

$$\frac{7}{12}[-12.6] + h_L = 0 \Rightarrow h_L = 7.35 ft = 2 f_F \frac{L}{D} \frac{V^2}{g} + K \frac{V^2}{2g}$$

+10

$$7.35 ft = V^2 \left[2 \frac{L}{Dg} f_F + \frac{K}{2g}\right] = V^2 \left[2 \frac{11.5 ft}{(\frac{1}{6} ft)(32.2 ft/s^2)} + \frac{7.5}{2(32.2)}\right]$$

$$236.67 = V^2 [138 f + 3.75] \quad \text{Equation } *$$

+10

Guess $f = .003$

From Equation * $V = 7.54\ ft/s$

$N_{Re} = \frac{VD}{\nu} = \frac{(7.54)(1/6)}{1.08 \times 10^{-5}} = 1.16 \times 10^5 \xrightarrow{\text{Moody}} f = .00425$

Guess $f = .00425$ $V = 7.38\ ft/s$

$N_{Re} = \frac{(7.38)(1/6)}{1.08 \times 10^{-5}} = 1.13888 \times 10^5 \xrightarrow{\text{Moody}} f = f = .0042$

Guess $f = .0042$ $\boxed{V = 7.39 \approx 7.4\ ft/s}$ $Q = VA = V\left(\frac{\pi}{4} D^2\right)$ +2

+10

b) As temperature increases the viscosity decreases while density remains relatively constant. As viscosity decreases, the Reynolds number decreases causing friction factor to decrease

Problem III. (30 points)

A string instrument-maker is to miniaturize an instrument for a child. The functional frequency *f* of a string is a function of the string length *L*, the diameter *D*, the density of the string material *ρ*, and the tensile force *T* applied to the string. Using the same material to make the mini instrument, what is fold increase (or decrease) in the tensile force applied to the string if the mini instrument is half the size of the real one?

variables L, D, ρ, T, f

pick core D, ρ, f (because L & D cannot both be core, and T is what we are interested)

so $\Pi = \dfrac{T}{D^a \rho^b f^c}$ and $\Pi_1 = \dfrac{L}{D}$

(you can do dimensional matrix to determine a, b, c or as I show here)

$$[T] = [m][a] = [m][L/t^2]$$

$$[\rho \cdot D^3] = [m] \qquad [L/t^2] = [D \cdot f^2]$$

$$\Rightarrow \Pi = \frac{T}{\rho D^3 \cdot D f^2} = \frac{T}{\rho D^4 f^2}$$

To make a mini instrument,

Π_1 doesn't change (also because of geometrical similarity)

$$\Rightarrow \frac{L_{mini}}{D_{mini}} = \frac{L_{real}}{D_{real}}$$

Also,

$$\Pi_{mini} = \Pi_{real}$$

$$\Rightarrow \frac{T_{mini}}{T_{real}} = \frac{\rho D_{mini}^4 f^2}{\rho D_{real}^4 f^2} = \left(\frac{D_{mini}}{D_{real}}\right)^4$$

$$= \left(\frac{L_{mini}}{L_{real}}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$