MATH 2401 WORKSHEET # 1 SECTION K

Name:

(1) Assume vector $\vec{u}$ is of length 5 and direction $\frac{-3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$. Find the angle between $\vec{u}$ and $\vec{v}$ where $\vec{v} = 4\mathbf{i} + 3\mathbf{j}$.

$$\vec{u} = 5 \times \left(-\frac{3}{5}\vec{i} + \frac{4}{5}\vec{j}\right) = -3\vec{i} + 4\vec{j}$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\| \times \|\vec{v}\|} = \frac{-3\times 4 + 3\times 4}{5\times 5} = \frac{0}{25} = 0$$

$$\therefore \theta = \arccos(0) = \frac{\pi}{2}$$

3 points total
1 point for formula for the angle
1 point for computations
1 point for correct answer.

(2) Calculate the volume of the parallelepiped box determined by $\vec{u}$, $\vec{v}$ and $\vec{w}$, where $\vec{u} = 2\mathbf{i} + \mathbf{j}$, $\vec{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\vec{w} = \mathbf{i} + 2\mathbf{k}$.

$$Vol = \left| \det\left(\begin{bmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{bmatrix}\right)\right|$$

$$= \left| 2\times\begin{vmatrix} -1 & 1 \\ 0 & 2 \end{vmatrix} - 1\times\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 0\times\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix}\right|$$

$$= |2\times(-2) - 1\times(4-1) + 0|$$

$$= |-4-3| = 7.$$

3 points total
1 point for formula
1 point for computation
1 point for correct answer

(3) Find parametrizations for the line in which plane $x+y+z=1$ and plane $x+y=2$ intersect.

Med 1:

$$y = 2-x$$

$$z = 1-x-y = 1-x-(2-x) = -1$$

$$\therefore \begin{cases} x = t \\ y = 2-t \\ z = -1 \end{cases}$$

4 points total
1 point for finding direction vector
1 point for a point on the line
2 points for correct answer

Mtd 2: $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = \vec{i}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$$= -\vec{i} + \vec{j}$$

And $(0, 2, -1)$ is on both plane $x+y+z=1$ and $x+y=2$

$$\therefore \begin{cases} x = 0 + 1\cdot t \\ y = 2 + 1\cdot t \\ z = -1 + 0\cdot t \end{cases} \iff \begin{cases} x = t \\ y = 2 + t \\ z = -1 \end{cases}$$