ISyE 3833 - HW 2 Due: Feb 17 / 18

1. Write the following LP in canonical inequality form (i.e., formulate an equivalent LP that has canonical inequality form):

$$\begin{array}{lrcrcrcr}
\min & -2x_1 & + & x_2 & + & x_3 & & \\
\text{s.t.} & 4x_1 & + & 3x_2 & + & 2x_3 & \leq & 3 \\
 & 2x_1 & - & x_2 & - & 3x_3 & \geq & 1 \\
 & x_1 & + & x_2 & + & x_3 & = & 1 \\
 & x_1 & & & & & \geq & 0 \\
 & & & x_2 & & & \geq & -7 \\
 & & & & 0 \leq & x_3 & \leq & 3.
\end{array}$$

2. ("A Gentle Introduction to Optimization", by B. Guenin, J. Könemann and L. Tunçel, Cambridge University Press, 2014) A chemical plant produces a noxious byproduct, called Chemical X, that is highly toxic and needs to be disposed of properly. The chemical plant is connected by a pipe system to a recycling plant that can safely process Chemical X. The amount of Chemical X produced in each hour of the day, according to a standard day's production schedule, is shown in Table 1 (Chemical X is not produced in any hour omitted in the table).

Table 1: Amount of Chemical X produced during each of the day.

Time interval	9-10am	10-11am	11am-12pm	12-1pm	1-2pm	2-3pm
Chemical X (litres)	300	240	600	200	300	900

The chemical plant has a storage capacity of 1000 litres for Chemical X, and, for environmental safety reasons, no Chemical X can be kept, unprocessed, overnight at the chemical plant. The cost for the recycling plant to process Chemical X varies throughout the day, as given in Table 2.

Table 2: Price for processing Chemical X at the recycling plant, at each hour of the day.

Time	10am	11am	12pm	1pm	2pm	3pm
Price ($ per litre)	30	40	35	45	38	50

Formulate an LP model to assist the chemical plant manager to minimize the cost of safely disposing of Chemical X. Clearly define your variables, and briefly describe each constraint. Implement your model in the Xpress-IVe software, and solve it. What should the chemical plant manager do, and what will it cost?

3. A warehouse can store up to 100 units of a particular item. In each of the next N days we need to determine the number of units to sell and then the number of units to buy. Selling

in each day precedes buying so that the maximum amount that can be sold is the initial inventory at the beginning of the day. The maximum amount that can be bought is limited by the capacity minus the amount in inventory after selling takes place. Inventory can be held overnight in the warehouse, provided the total number of units held does not exceed the warehouse capacity. At the beginning of day 1, before selling takes place, there are 40 units of inventory in the warehouse. Suppose that p_n and c_n are the per unit selling and buying prices, respectively, for each day $n = 1, \ldots, N$. Formulate an LP to determine the amount to be sold and bought each day to maximize total profit. Make sure you have defined your variables clearly.

4. ("A Gentle Introduction to Optimization", by B. Guenin, J. Könemann and L. Tunçel, Cambridge University Press, 2014) You wish to build a house and you have divided the process into a number of tasks, namely:

   B. excavation and building the foundation,
   F. raising the wooden frame,
   E. electrical wiring,
   P. indoor plumbing,
   D. dry walls and flooring,
   L. landscaping.

   You estimate the following duration for each of the tasks (in weeks):

Tasks	B	F	E	P	D	L
Duration	3	2	3	4	1	2

   Some of the tasks can only be started when some other tasks are completed. For instance, you can only build the frame once the foundation has been completed, i.e. F can only start after B is completed. All the precedence constraints are summarized as follows:

   - F can start only after B is completed,
   - L can start only after B is completed,
   - E can start only after F is completed,
   - P can start only after F is completed,
   - D can start only after E is completed,
   - D can start only after P is completed.

   The goal is to schedule the starting time of each task such that the entire project is completed as soon as possible. As an example, here is a feasible schedule with a completion time of ten weeks.

Tasks	B	F	E	P	D	L
Starting time	0	3	6	5	9	6
End time	3	5	9	9	10	8

   Formulate this problem as an LP. Explain your formulation. Note that there is no limit on the number of tasks that can be done in parallel. (Hint: Introduce variables to indicate the times that the tasks start.)

5. *Read Chapter 3.8 of the Winston textbook, on blending problems, and then answer the following question.* Bullco blends silicon and nitrogen to produce two types of fertilizers. Fertilizer 1 must be at least 40% nitrogen and sells for $70 per pound. Fertilizer 2 must be at least 70% silicon and sells for $40 per pound. Bullco can purchase up to 80 pounds of nitrogen at $15 per pound and up to 100 pounds of silicon at $10 per pound. Assuming all fertilizer products can be sold, formulate an LP to help Bullco maximize profits.

6. (Taha) Three refineries with daily capacities of 6, 5, and 8 million gallons, respectively, supply three distribution areas with daily demands of 4, 8, and 7 million gallons, respectively. Gasoline is transported to the three distribution areas through a network of pipelines. The transportation cost is 10 cents per 1000 gallons per pipeline mile. Table 3 gives the mileage between the refineries and the distribution areas. Refinery 1 is not connected to distribution area 3. Construct the associated transportation problem model. Make sure you give the network, indicate the supply/demand at each node, and give the cost and capacity for each arc.

	Area 1	Area 2	Area 3
Refinery 1	120	180	-
Refinery 2	300	100	80
Refinery 3	200	250	120

Table 3: Distance between refineries and distribution areas (pipeline miles)