Math 1712 - Spring 2013
Test 2 Show your work!

Name: ______________________________ TA: ____

1. (20 points) For each of the following functions, find all **relative maximums and minimums,** both the x & y values. Round to 4 decimal places. **SHOW YOUR WORK!**

a. $g(x) = (x-2)^{\frac{2}{3}} - 4$

* $\mathbf{g'(x) = \dfrac{2}{3(x-2)^{\frac{1}{3}}} = 0}$ **has no solutions.** $\mathbf{g'(x)}$ **DNE has the solution** $\mathbf{x = 2}$

$$\begin{array}{ccc} g' < 0 & & g' > 0 \\ \text{\_\_\_\_\_\_\_\_\_\_\_\_} & 2 & \text{\_\_\_\_\_\_\_\_\_\_\_\_} \end{array}$$

$\Rightarrow \mathbf{x = 2}$ **&** $\mathbf{y = g(2) = -4}$ **is a relative minimum, no relative maximum.**

b. $h(x) = x^2 e^{\frac{1}{3}x}$

* $\mathbf{h'(x) = 2xe^{\frac{1}{3}x} + \frac{1}{3}x^2 e^{\frac{1}{3}x} = xe^{\frac{1}{3}x}\left(2 + \frac{1}{3}x\right) = 0} \Rightarrow \mathbf{x = 0}$ **and** $\mathbf{x = -6; g'(x)}$ **DNE has no solutions**

$$\begin{array}{ccccc} h' > 0 & & h' < 0 & & h' > 0 \\ \text{\_\_\_\_\_\_\_\_\_\_\_\_} & -6 & \text{\_\_\_\_\_\_\_\_\_\_\_\_} & 0 & \text{\_\_\_\_\_\_\_\_\_\_\_\_} \end{array}$$

$\Rightarrow \mathbf{x = -6}$ **&** $\mathbf{y = h(-6) = 36e^{-2} \approx 4.8720}$ **is a relative maximum and**
$\mathbf{x = 0}$ **&** $\mathbf{y = h(0) = 0}$ **is a relative minimum**

2. (10 points) For the function $f(x) = x^2 + \dfrac{128}{x}$ defined on the interval $(0, \infty)$, find all absolute maximums and minimums if they exist and justify your answer. Find both the x & y values and round to 4 decimals places. Use the maximum-minimum principles from your book.

* $\mathbf{f'(x) = 2x - 128x^{-2} = 2x - \dfrac{128}{x^2} = 0 \Rightarrow 2x^3 - 128 = 0 \Rightarrow x^3 = \dfrac{128}{2} = 64 \Rightarrow x = 4}$

* $\mathbf{f''(x) = 2 + 256x^{-3} = 2 + \dfrac{256}{x^3} \Rightarrow f''(x) = 2 + \dfrac{256}{64} > 0.}$

By the maximum-minimum principal, $\mathbf{x = 4}$ **&** $\mathbf{y = f(4) = 4^2 + \dfrac{128}{4} = 48}$ **is the absolute minimum; no absolute maximum.**

3. (15 points) Let $E(t) = -28t^3 + 382t^2 - 1162t + 16905$ be the number of employees of the XYZ Company, where t is the number of years since the year 2000 ($t = 0$). This function is valid for the years 2000 to 2009. a. Find the year and the **absolute** extreme values of E, rounding E to 3 decimal places and rounding t to the nearest whole number. Use the correct year and the correct units on E. Use the

maximum-minimum principles from your book. You will need to use the QF: $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

*** $E'(t) = -84t^2 + 764t - 1162 = 0$. Solve using the QF**

*** $t = \dfrac{\left(-764 \pm \sqrt{764^2 - 4(-84)(-1162)}\right)}{2(-84)}$ $\Rightarrow t = 2$ and $t = 7$ (rounded to the nearest whole number).**

*** To find the absolute max and min, complete the table below:**

*** t:**	**0**	**2**	**7**	**9**
*** E:**	**16905**	**15885**	**17885**	**16977**

So the maximum number of employees is 17, 885 which occurs in the year 2007
and the minimum number of employees is 15, 885 which occurs in the year 2002

4. (10 points) At a small college, a flu epidemic has broken out with 4 cases initally reported. As the flu spreads about the campus, the number of flu cases N can be modeled by the restricted growth model $N(t) = \dfrac{5000}{1 + 1249e^{-0.33t}}$, where t is time in days since the initial 4 cases were reported. How long will it take for the number of flu cases to reach 100? Round to the nearest day and use the correct units in your answer.

$100 = \dfrac{5000}{1 + 1249e^{-0.33t}}$ (divide by 5000) $\Rightarrow$

$\dfrac{1}{50} = \dfrac{1}{1 + 1249e^{-0.33t}}$ (take the reciprocal of both sides)

$50 = 1 + 1249e^{-0.33t}$ (subtract 1 and then divide by 1249)

$\dfrac{49}{1249} = e^{-0.33t}$ (convert to logarithmic form)

$\ln\left(\dfrac{49}{1249}\right) = -0.33t \Rightarrow t = \dfrac{\ln\left(\dfrac{49}{1249}\right)}{-0.33} \approx 9.81$. Round to nearest day, $t \approx 10$ days.

5. (10 points) Let $H(x) = \dfrac{e^x}{x}$. Find the equation of the tangent line to the graph of $H(x)$ when $x = 3$. **Round to 2 decimal places**.

$H'(x) = \dfrac{xe^x - e^x}{x^2} \Rightarrow m_{TL} = H'(3) = \dfrac{3e^2 - e^2}{9} \approx 4.46$

$x=3 \Rightarrow y=\frac{e^3}{3} \approx 6.70 \Rightarrow TL:\ y-6.70=4.46\,(x-3)$ **or** $y=4.46\,x-6.68$

6. (15 points) Suppose that the total daily cost C of producing x radios is $C(x)=0.002\,x^3+0.1\,x^2+42\,x+300$ and currently 40 radios are being produced daily. a. Find the current total daily cost. b. Use the marginal cost and the current cost to estimate the total daily cost of producing 41 radios. **SHOW YOUR WORK**.

a. Current total daily cost $= C(40) = \$\,2,268$

b. $MC(x)=C'(x)=0.006\,x^2+0.2\,x+42 \Rightarrow MC(40)=\$\,59.60$

$\Rightarrow C\,(x+1) \approx C(x)+MC(x) \Rightarrow C(41) \approx C(40)+MC(40)=2268+59.6=\$\,2,327.60$

7. (15 points) At a price of p dollars, the demand per month D (in thousands of units) for a new small tablet device is $\mathrm{D}(p)=240e^{-0.003p}$. (Recall that the demand is simply the number of tablet devices sold per month). a. Find the number of tablet devices sold per month if the price is \$200. b. Find the marginal demand if the price is \$200 and interprete what this number means in terms of the price and demand. Use the correct units in your answers and round to 3 decimal places.

a. $p=200 \Rightarrow \mathrm{D}(200)=240\,e^{(-0.003)\cdot(200)} \approx$ **131.715 thousand tablets or 131, 715 tablets**

b. Marginal demand $= MD(p)=\mathrm{D}'(p)=240\,e^{-0.003p}\,(-0.003)=-0.72\,e^{-0.003p}$

$\Rightarrow MD(200)=\mathrm{D}'(200) \approx -0.395;$ **If the price is \$ 200, then the demand is decreasing by approximately 395 tablets per dollar.**

8. (15 points) The concentration C of a drug in a person's blood t seconds after an injection is given by $C(t)=0.08+0.12\,e^{-0.02t}$, where the concentration C is measured in grams/cubic centimeter $\left(\frac{g}{cm^3}\right)$.

The concentration of the drug when initially injected $(t=0)$ is $0.20\frac{g}{cm^3}$ a. How long will it take for the concentration of the drug to reach **half** the initial amount? **Round to the nearest second**. b. Find $C'(60)$ and interptete this number in terms of C & t. Round to 4 decimal. Use the correct units in all answers

a. Half the initial concentration $= \frac{1}{2}(0.20)=0.10=0.08+0.12\ e^{-0.02t}$

$$\Rightarrow 0.02=0.12e^{-0.02t} \Rightarrow \frac{0.02}{0.12}=e^{-0.02t} \Rightarrow t=\frac{\ln\left(\frac{0.02}{0.12}\right)}{-0.02} \approx 90\ seconds$$

b. $C'(t)=\left(0.12\,e^{-0.02t}\right)\cdot(-0.02)=-0.0024\,e^{-0.02t} \Rightarrow C'(60) \approx -0.0007$

This means that after 60 seconds, the concentration of the drug is decreasing by approximately $0.0007\ \frac{g}{cm^3}$ **per second.**

9. (15 points) Find the indicated derivative.

a. $F(x) = e^{10x^2}$. Find the first derivative.

$\boldsymbol{F'(x) = e^{10x^2} \cdot (20x) = 20x\,e^{10x^2}}$ **(either answer or equivalent)**

b. $g(x) = 2^x e^x$. Find $\dfrac{dg}{dx}$ and simplify.

$\boldsymbol{g'(x) = (2^x)' e^x + (e^x)' 2^x = \ln(2) 2^x e^x + 2^x e^x = 2^x e^x (\ln(2) + 1)}$

c. $h(x) = (2x+1)\ln(2x+1)$. Find $h'(x)$ and simplify

$$\boldsymbol{h'(x) = 2\ln(2x+1) + \frac{2(2x+1)}{2x+1} = 2\ln(2x+1) + 2}$$

EXTRA CREDIT (5 points) The Government just released this statement:

"The high inflation that began in the year 2015 may be coming to an end. After 10 years, the rate of inflation is still increasing, but the rate at which it is increasing is decreasing".

If $I(t)$ = inflation with $t = 0$ corresponding to 2015, what does this statement say about $I'(10)$ & $I''(10)$?

Answer: $\boldsymbol{I'(10) > 0}$ **&** $\boldsymbol{I''(10) < 0}$