MATH 1552 QUIZ 1, FALL 2015, GRODZINSKY

Print Your Name: Key-1

T.A.: (circle one) Miheer Brandon Stephen Kabir

1. (18 points) Solve the initial value problem:

$$\frac{dy}{dx} = xe^{x-3y}, \quad y(0) = 0.$$

$$\frac{dy}{dx} = xe^{x}e^{-3y}$$

$$\int e^{3y}\,dy = \int xe^{x}\,dx$$

$u = x \quad dv = e^{x}dx$

$du = dx \quad v = e^{x}$

$$\frac{1}{3}e^{3y} = xe^{x} - \int e^{x}\,dx$$

$$\frac{1}{3}e^{3y} = xe^{x} - e^{x} + C$$

$$y(0)=0 \Rightarrow \frac{1}{3}e^{0} = 0\cdot e^{0} - e^{0} + C$$

$$\frac{1}{3} = -1 + C, \quad C = 4/3$$

$$\text{So } \frac{1}{3}e^{3y} = xe^{x} - e^{x} + \frac{4}{3}$$

$$e^{3y} = 3xe^{x} - 3e^{x} + 4$$

$$3y = \ln(3xe^{x} - 3e^{x} + 4)$$

$$\boxed{y = \frac{1}{3}\ln(3xe^{x} - 3e^{x} + 4)}$$

2. (12 points) Evaluate the integral:

$$\int x^4 \ln(2x)dx.$$

By parts:

$u = \ln(2x) \qquad dv = x^4\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{5}x^5$

$$\int x^4 \ln(2x)\,dx = \frac{1}{5}x^5 \ln(2x) - \frac{1}{5}\int x^4\,dx$$

$$= \boxed{\frac{1}{5}x^5 \ln(2x) - \frac{1}{25}x^5 + C}$$

3. (20 points) Evaluate the integral: $\int \frac{x^2}{(9-x^2)^{3/2}}dx$.

Trig sub: Let $x = 3\sin\theta$, then $dx = 3\cos\theta\,d\theta$

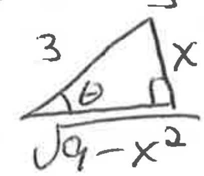
and $9 - x^2 = 9 - 9\sin^2\theta = 9\cos^2\theta$, so:

$$\int \frac{x^2}{(9-x^2)^{3/2}}dx = \int \frac{9\sin^2\theta}{(9\cos^2\theta)^{3/2}} \cdot 3\cos\theta\,d\theta$$

$$= \int \frac{9\sin^2\theta}{27\cos^3\theta} \cdot 3\cos\theta\,d\theta = \int \frac{\sin^2\theta}{\cos^2\theta}\,d\theta$$

$$= \int \tan^2\theta\,d\theta = \int (\sec^2\theta - 1)\,d\theta$$

$\sin\theta = \frac{x}{3}$



$$= \tan\theta - \theta + C$$

$$= \boxed{\frac{x}{\sqrt{9-x^2}} - \sin^{-1}\left(\frac{x}{3}\right) + C}$$

MATH 1552 QUIZ 1, FALL 2015, GRODZINSKY

Print Your Name: Key-2

T.A.: (circle one) Miheer Brandon Stephen Kabir

1. (20 points) Evaluate the integral: $\int \frac{x^2}{(16-x^2)^{3/2}}\,dx$.

Trig sub: let $x = 4\sin\theta$

$dx = 4\cos\theta\, d\theta$

and $x^2 = 16\sin^2\theta$

$$\Rightarrow \int \frac{x^2}{(16-x^2)^{3/2}}\,dx = \int \frac{16\sin^2\theta}{(16-16\sin^2\theta)^{3/2}} \cdot 4\cos\theta\, d\theta$$

$$= \int \frac{16\sin^2\theta}{(16\cos^2\theta)^{3/2}} \cdot 4\cos\theta\, d\theta$$

$$= \int \frac{16\sin^2\theta}{64\cos^3\theta} \cdot 4\cos\theta\, d\theta$$

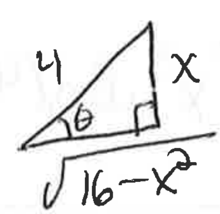
$$= \int \tan^2\theta\, d\theta$$

$$= \int (\sec^2\theta - 1)\, d\theta$$

$$= \tan\theta - \theta + C$$

$$= \boxed{\frac{x}{\sqrt{16-x^2}} - \sin^{-1}\left(\frac{x}{4}\right) + C}$$

$\sin\theta = \frac{x}{4}$



2. (18 points) Solve the initial value problem:

$$\frac{dy}{dx} = xe^{x-2y}, \quad y(0) = 0.$$

$$\frac{dy}{dx} = xe^{x}e^{-2y} \Rightarrow \int e^{2y}\,dy = \int xe^{x}\,dx$$

$u = x \quad dv = e^{x}dx$

$du = dx \quad v = e^{x}$

$$\frac{1}{2}e^{2y} = xe^{x} - \int e^{x}\,dx$$

$$\frac{1}{2}e^{2y} = xe^{x} - e^{x} + C$$

$$y(0)=0 \Rightarrow \frac{1}{2}e^{0} = 0 - e^{0} + C \Rightarrow \frac{1}{2} = -1 + C$$

$$C = 3/2$$

$$\text{So } \frac{1}{2}e^{2y} = xe^{x} - e^{x} + \frac{3}{2}$$

$$e^{2y} = 2xe^{x} - 2e^{x} + 3, \quad \boxed{y = \frac{1}{2}\ln(2xe^{x} - 2e^{x} + 3)}$$

3. (12 points) Evaluate the integral:

$$\int x^3 \ln(4x)dx.$$

By parts:

$u = \ln(4x) \qquad dv = x^3\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{4}x^4$

$$\int x^3 \ln(4x)\,dx = \frac{1}{4}x^4 \ln(4x) - \frac{1}{4}\int x^3\,dx$$

$$= \boxed{\frac{1}{4}x^4 \ln(4x) - \frac{1}{16}x^4 + C}$$