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ISyE 4232
H. Ayhan
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Homework 1

1. Customers arrive at a two-server system according to a Poisson process with rate $\lambda = 15$ per hour. An arrival finding server 1 free will begin his service with him. An arrival finding server 1 busy, server 2 free will join server 2. An arrival finding both servers busy goes away. Once a customer is served by either server, he departs the system. The service times at both servers are exponential random variables. Assume that the service rate of the first server is 6 per hour and the service rate of the second sever is 4 per hour.

   (a) Find the stationary distribution of the continuous time Markov chain.

   (b) Compute N.

   (c) What is the long-run average time spent in the system among all accepted customers?

2. Suppose we have a small call center staffed by three operators handling five telephone lines. Calls arrive according to a Poisson process with rate $\lambda = 150$ calls per hour. Service times are exponentially distributed with a mean of 4 minutes. Customers are willing to wait an exponentially distributed length of time with a mean of 8 minutes before reneging if service has not begun.

   (a) Find the stationary distribution of the continuous time Markov chain.

   (b) Find the probability that an arriving call immediately talks to an operator.

   (c) What is the long run proportion of calls that are blocked?

   (d) Compute N and N_q

   (e) What is the average time in system among all accepted calls (including those reneged)?

3. Consider two systems. The first system has two M/M/1 queues and the second system has an M/M/2 queue. Assume that the arrival process to both systems is Poisson with rate λ. In the first system, an arriving customer is equally likely to choose any one of the queues. All service times are exponentially distributed with rate μ. Assume that $\lambda < 2\mu$. Compare the four performance measures that we considered in class for these two systems.