GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 – BIOTRANSPORT

EXAM 2 (SPRING 2014) – KEMP

STUDENT NAME: Key

GTID NUMBER: ______________________

RECITATION SECTION: ______________________

(Section A is Wednesdays at 12 pm; Section B is Wednesdays at 10 am)

Open Book
All non-communicating calculator types allowed
Time allotted: 50 minutes
Do all work in this booklet

Reminder: for questions that require numerical answers, units are required and worth 50%

Question	Maximum Mark	Actual Mark
1	50	
2	50	
Total	100	

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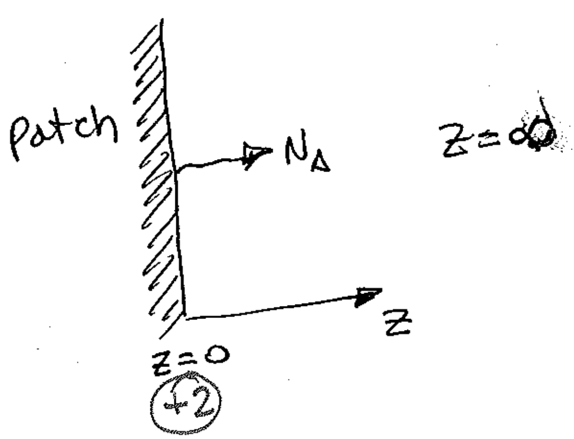
1) (50 points total) Dr. Prausnitz has led a team here at Georgia Tech and at the CDC towards the development of microneedle patches that can be applied on the skin for delivery of vaccines as easily as putting on a band-aid. The ability of these patches to pass clinical trials is dependent on demonstrating that they can effectively pass viral particles across the dermal layer. His group characterizes the dynamic immune response of primate models to different formulations of vaccines. He would like to know what the effective loading concentration in his patches is, given knowledge obtained from the animals 12 hours later. In this problem, consider a 1 cm^2 square patch that is 1 mm thick containing heat-inactivated virus with a diffusion coefficient in tissue of 4×10^{-8} cm^2/s. The patch is applies to the skin at time t=0.

a) (4 points) Draw a physical representation of the system with your defined coordinate system (for reference in the remainder of your analysis). State any additional assumptions needed for completing part c.



① Assume semi-infinite approximation (+1)

② Virus diffusion into body results in negligible concentration far from surface (+1)

b) (6 points) What are the boundary conditions you will use for this problem?

① at $t=0 \rightarrow C(z,0)=0$ for all z (+2)

② at $z=0 \rightarrow C(0,t)=C_s$ for all $t>0$ (+2)

③ at $z=\infty \rightarrow C(\infty,t)=0$ for all t (+2)

where C_s is unknown and we are solving for

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c) (40 points) Solve for the initial concentration within the patch if the Prausnitz lab measures the total mass transfer rate from the patch 12 hours post-application to be 1x10^{-11} mole/min.

$N(z=0) * \text{Area} = \text{rate}$

$N(z=0) \cdot 1\ cm^2 = 10^{-11}\ mole/min$

$$\boxed{N = 10^{-11}\ \frac{mole}{cm^2 \cdot min}}$$ +10

$$\boxed{N(z=0) = \sqrt{\frac{D}{\pi t}}\,(c_s - \cancel{c_0})}$$ +10

$$10^{-11}\ \frac{mol}{cm^2 \cdot min} = \sqrt{\frac{4\times10^{-8}\ cm^2/s}{\pi\left(12\ hr \times \frac{60\ min}{1\ hr} \times \frac{60\ s}{1\ min}\right)}} \cdot c_s$$

$$10^{-11}\ \frac{mol}{cm^2 \cdot min} = 5.43\times10^{-7}\ cm/s \cdot c_s$$

$$c_s = 3.07\times10^{-7}\ \frac{mol}{cm^2 \cdot \cancel{min}} \times \frac{\cancel{min}}{cm}$$

$$c_s = 3.07\times10^{-7}\ \frac{mol}{cm^3} = 3.07\times10^{-7}\ \frac{mol}{10^{-3}\ L}$$

+20

$= 0.307\ mM$

incorrect units -5

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2) (50 points total) Asthma is defined by pathological changes in the bronchial wall leading to a smaller cross-sectional area that air can flow through; this leaves the patient gasping for air. In this problem, we will ignore the changes in mechanical properties of the bronchial tube under these circumstances and just consider the effects of cross section on air velocity.

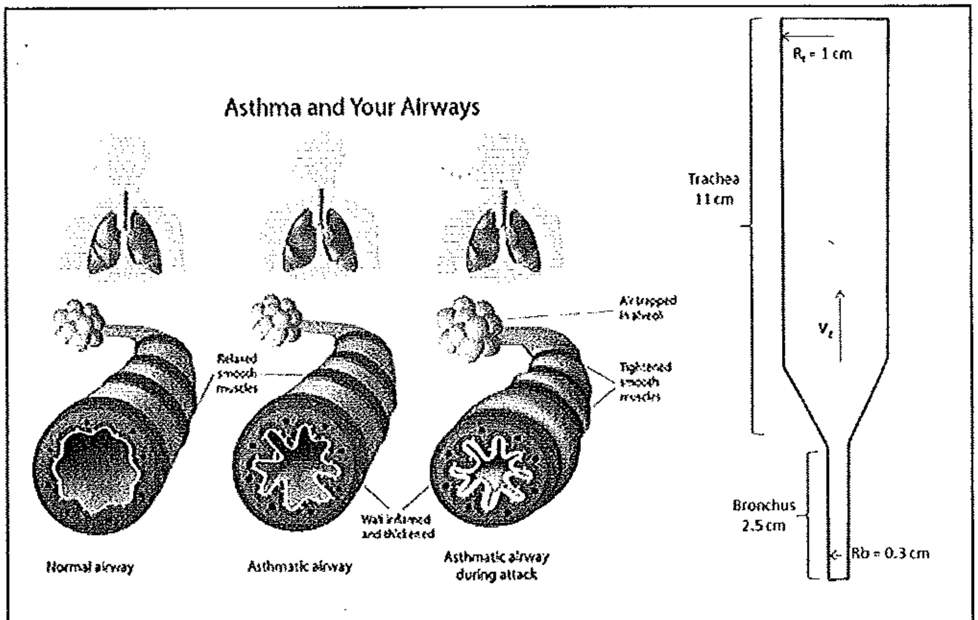


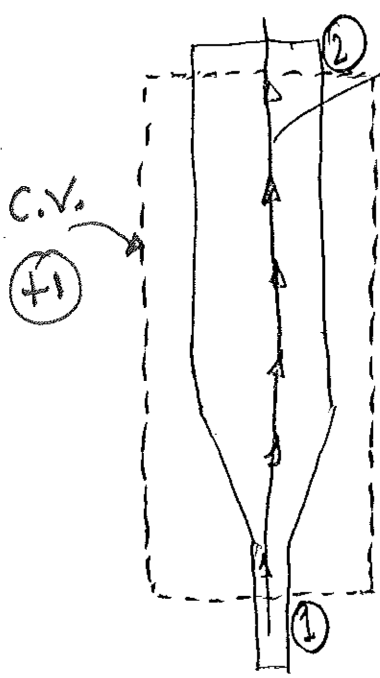
Image courtesy of NIH

A normal person breathing will expel air from the lung through the bronchi at 33 cm/s and a normal bronchus will have a radius of 0.3 cm and a length of 2.5 cm. A normal trachea has a length of 11 cm and a radius of 1 cm. The degree of airway obstruction is unknown but occurs at the bronchial level. You are interested in evaluating the change in cross-sectional area associated with the condition.

Assume for the purposes of solving this problem that the air flow is steady, inviscid, and incompressible. You may ignore any branching that exists in the lung structure. The density of air is approximately 1×10^{-3} g/cm^3.

a) (5 points) Perform your GIM analysis here

streamlined from ① to ② (define ① → ② line) (+2)



① A_1 and A_2 are given in problem statement
V_1 is given in problem statement

(defining origin/coordinates) (+2)

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b) (20 points) Determine the pressure change between the bronchus and trachea associated with a normal patient's exhaled air flow.

Need V_2 from $\begin{cases} A_1 = 0.3\ cm \\ A_2 = 1\ cm \end{cases}$ $V_1 = 33\ \frac{cm}{s}$

$$V_1 A_1 = V_2 A_2$$

$$33\ \frac{cm}{s} \times \pi (0.3\ cm)^2 = V_2 \times \pi (1\ cm)^2$$

$$\boxed{V_2 = 2.97\ cm/s}$$ +5

Applying Bernoulli's:

$$\boxed{\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}}$$ +5

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1)$$ ← exclusion of "z" terms -5

$$\Delta P = \rho \cancel{g}\left(\frac{V_2^2 - V_1^2}{2\cancel{g}}\right) + \rho g (z_2 - z_1)$$

If wrong -2

$$\Delta P = 10^{-3}\ \frac{g}{cm^3}\left[\frac{(2.97\ cm/s)^2 - (33\ cm/s)^2}{2}\right] + 10^{-3}\ \frac{g}{cm^3}\left(980\ \frac{cm}{s^2}\right)(13.5\ cm)$$

$$\Delta P = 12.689\ \frac{g}{cm.s^2} = 1.2689\ Pa$$ +10

incorrect units -5

(25 points)

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c) (~~30 points~~) The air velocity exiting the trachea of an asthmatic patient is measured to be 50 cm/s. Anatomically, the tracheal dimensions are identical to the healthy person but the bronchial radius is halved. How much larger is the pressure change in this individual compared to part b? Does this value make sense considering that airway collapse during asthma attacks is attributed to extreme negative pressure gradients? Same as (b)

we need new "V_1" from { new "V_2" and new "A_1" } $\rightarrow V_2 = 50\ \text{cm/s}$

$A_1 = \frac{(0.3)}{2}\text{cm} \rightarrow A_1 = 0.15\ \text{cm}$

$$V_1 \times \pi (0.15\ \text{cm})^2 = (50\ \text{cm/s}) \times \pi (1\ \text{cm})^2$$

$$\boxed{V_1 = 2222\ \text{cm/s}} \quad (+5)$$

$$\boxed{\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}} \quad (+5)$$

$$\text{new } \Delta P = \rho g \left[\frac{V_2^2 - V_1^2}{2g} + (z_2 - z_1) \right] \rightarrow \text{exclusion of "z" terms } (-5)$$

$$\text{new } \Delta P = 10^{-3}\frac{\text{g}}{\text{cm}^3}\left(\frac{(50\ \text{cm/s})^2 - (2222\ \text{cm/s})^2}{2}\right) + 10^{-3}\frac{\text{g}}{\text{cm}^3} \times \left(980\frac{\text{cm}}{\text{s}^2}\right) \times (13.5\ \text{cm})$$

→ if wrong (-2)

$$\text{new } \Delta P = -2454\ \frac{\text{g}}{\text{cm s}^2} = -245.4\ \text{Pa} \quad \} (+5)$$

(+10) { this is a 193-fold change in negative pressure drop which is consistent with the information above suggesting that asthma induces a larger negative pressure drop to cause airway collapse