Name: ....

ISyE 3103 Introduction to Supply Chain Modeling: Logistics

Summer 2016
Exam 1
June 28, 2016

Instructions

1. There are 10 pages and 100 points.
2. No books, notes, computers, calculators, cell phones, or any other electronic equipment allowed within your reach.
3. No bathroom breaks during the exam allowed. Complete your bathroom visits before starting the exam.
4. Do your own work.
5. Show all calculations.

Question 1 (34 points)

1. You use a dataset that includes observations $Y_1, Y_2, \ldots, Y_n$ to develop a forecasting model. You use your model to produce forecasts $\hat{Y}_{n+1}, \hat{Y}_{n+2}, \ldots, \hat{Y}_{n+m}$, and then you compare these forecasts with the corresponding actual observed values $Y_{n+1}, Y_{n+2}, \ldots, Y_{n+m}$. Write down the expressions for 3 measures of forecasting error based on these forecasts and observed values. (6)

   Answer:

$$
\begin{aligned}
\text{Mean Absolute Error (MAE)} &= \frac{\sum_{i=1}^{m} \left|Y_{n+i} - \hat{Y}_{n+i}\right|}{m} \\
\text{Mean Absolute Percentage Error (MAPE)} &= \frac{\sum_{i=1}^{m} \left|\frac{Y_{n+i} - \hat{Y}_{n+i}}{Y_{n+i}}\right|}{m} \\
\text{Percent Mean Absolute Deviation (PMAD)} &= \frac{\sum_{i=1}^{m} \left|Y_{n+i} - \hat{Y}_{n+i}\right|}{\sum_{i=1}^{m} \left|Y_{n+i}\right|} \\
\text{Mean Squared Error (MSE)} &= \frac{\sum_{i=1}^{m} \left(Y_{n+i} - \hat{Y}_{n+i}\right)^2}{m} \\
\text{Root Mean Squared Error (RMSE)} &= \sqrt{\frac{\sum_{i=1}^{m} \left(Y_{n+i} - \hat{Y}_{n+i}\right)^2}{m}}
\end{aligned}
$$

2. Overconfidence in the accuracy of one's forecasts is a widespread phenomenon. List 3 factors discussed in class that are positively correlated with overconfidence in the accuracy of one's forecasts. (6)

   Answer:

(a) Level of expertise.

(b) Prior success.

(c) Males tend to be more overconfident that females.

(d) Organizational pressure to produce desirable forecasts.

3. In the article "Delusions of Success: How Optimism Undermines Executives' Decisions", the authors Dan Lovallo and Daniel Kahneman suggest that people should incorporate "the outside view" when generating forecasts. What do they mean with "the outside view"? (4)
Answer: The "outside view" means that you take into account data from many other similar settings when producing a forecast, and not just (wishful thinking about) your own setting. For example, when you want to create a new start-up company, and you estimate the probability of success (for example, the probability that the company will be solvent in 5 years or that you were able to sell it profitably). Most people who create a new start-up company do it because they think that they have a great business idea (if they did not think so, they probably would not create a start-up company to begin with), and therefore they forecast that the probability of success is high, say more than 50%. That is the inside view. However, when one looks at the data of a large number of start-up companies, then one observes that fewer than 20% of start-ups survive more than a few years (see eg., forbes.com), in spite of the fact that the vast majority of people who start them think that they have great business ideas. Thus, a forecast based on the outside view would state that the probability of success of a new start-up company is no more than 20%, even though one thinks that the business idea is great.

4. In a classical article, "A Contribution to the Study of Actuarial and Individual Methods of Prediction", the author Theodore R. Sarbin compares two methods ("actuarial and individual methods") of forecasting something.

   (a) Explain the two methods of prediction that were compared. (4)
   Answer: The "actuarial method" was a simple regression model with 2 explanatory variables. The "individual method" was a judgmental method involving highly qualified experts (clinical psychologists).

   (b) What was predicted in the study? (2)
   Answer: The academic success (as measured by honor-point ratio) of students.

   (c) How did the two methods of prediction compare in terms of accuracy of forecasts and variance of forecasts? (2)
   Answer: The regression model was slightly more accurate and had less variance than the experts' judgmental method.

5. Name the 5 requirements for a successful implementation of Vendor Managed Inventory discussed in class, and briefly describe how each of these requirements were addressed in the Barilla case. (10)
Answer:

(a) Convince own management. Vitali had convinced the senior management at Barilla, who put pressure on Barilla's VP of sales, Rossini, to support the VMI effort.

(b) Convince other party's management. A successful trial implementation of VMI at two Barilla warehouses (Milan and Florence) helped to convince the distributors' management.

(c) Timely and accurate data of inventory levels and demand. EDI connections between Barilla and distributors were established, and each day data were transmitted from distributors to Barilla headquarters.

(d) Accurate demand forecasting. Barilla used a simple weighted average of demand over the last 30 days.

(e) Make efficient shipping decisions. Battistini, together with consultants Ferrozi and Tellarini, developed an algorithm for determining daily shipment quantities.

Question 2

(34 points)

Suppose that we want to estimate a model to forecast the cost to transport shipments from Guadalajara to Atlanta by various modes. Let

$$
\begin{aligned}
Y_i &= \text{cost for shipment } i \text{ in the data set, in USD} \\
X_{1i} &= \begin{cases} 1 & \text{if only truck transport is used} \\ 0 & \text{otherwise} \end{cases} \\
X_{2i} &= \begin{cases} 1 & \text{if truck and ocean transport is used} \\ 0 & \text{otherwise} \end{cases} \\
X_{3i} &= \begin{cases} 1 & \text{if truck and air transport is used} \\ 0 & \text{otherwise} \end{cases} \\
X_{4i} &= \text{the weight of shipment } i \text{ in kg} \\
X_{5i} &= \text{the volume of shipment } i \text{ in } m^3 \\
X_{6i} &= \text{the value of shipment } i \text{ in USD}
\end{aligned}
$$

Suppose that we estimate the following model:

$$\hat{Y}_i = \hat{\beta}_0+\hat{\beta}_1 X_{2i}+\hat{\beta}_2 X_{3i}+\hat{\beta}_3 X_{4i}+\hat{\beta}_4 X_{1i}X_{4i}+\hat{\beta}_5 X_{2i}X_{4i}+\hat{\beta}_6 X_{5i}+\hat{\beta}_7 X_{2i}X_{5i}+\hat{\beta}_8 X_{3i}X_{5i}+\hat{\beta}_9 X_{6i}$$

1. The cost model above implies a cost model for each of the modes "truck only", "truck and ocean", and "truck and air". Write down the cost model for the mode "truck only" implied by the cost model above (that is, plug in the appropriate values for X_{1i}, X_{2i}, and X_{3i}, and simplify). (2)
   Answer:

   $$\text{truck only:} \quad \hat{Y}_i = \hat{\beta}_0 + (\hat{\beta}_3 + \hat{\beta}_4)X_{4i} + \hat{\beta}_6 X_{5i} + \hat{\beta}_9 X_{6i}$$

2. Write down the cost model for the mode "truck and ocean" implied by the cost model above. (2)
   Answer:

   $$\text{truck and ocean:} \quad \hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_1) + (\hat{\beta}_3 + \hat{\beta}_5)X_{4i} + (\hat{\beta}_6 + \hat{\beta}_7)X_{5i} + \hat{\beta}_9 X_{6i}$$

3. Write down the cost model for the mode "truck and air" implied by the cost model above. (2)
   Answer:

   $$\text{truck and air:} \quad \hat{Y}_i = (\hat{\beta}_0 + \hat{\beta}_2) + \hat{\beta}_3 X_{4i} + (\hat{\beta}_6 + \hat{\beta}_8)X_{5i} + \hat{\beta}_9 X_{6i}$$

4. In terms of the $\hat{\beta}$s, how would you estimate the following quantities?

   (a) The average fixed cost for a shipment if only truck transport is used, in USD. (2)

   Answer: $\hat{\beta}_0$

(b) The average fixed cost for a shipment if truck and air transport is used, in USD. (2)
Answer: $\hat{\beta}_0 + \hat{\beta}_2$

(c) The incremental fixed cost for a shipment if truck and ocean transport is used instead of truck transport only, in USD. (2)
Answer: $\hat{\beta}_1$

(d) The average variable cost per kg for a shipment if truck and ocean transport is used, in USD per kg. (2)
Answer: $\hat{\beta}_3 + \hat{\beta}_5$

(e) The incremental average variable cost per kg for a shipment if truck and ocean transport is used instead of truck transport only, in USD per kg. (2)
Answer: $\hat{\beta}_5 - \hat{\beta}_4$

(f) The incremental average variable cost per m^3 for a shipment if truck and air transport is used instead of truck transport only, in USD per m^3. (2)
Answer: $\hat{\beta}_8$

5. Use your common sense.

(a) What sign do you expect β_0 to have, and why? (2)
Answer: $\hat{\beta}_0 > 0$. Fixed cost for trucking only is positive.

(b) What sign do you expect β_1 to have, and why? (2)
Answer: $\hat{\beta}_1 > 0$. Fixed cost for a shipment if truck and ocean transport is used is greater than fixed cost for the same shipment if truck transport only is used, because of additional (at port) and more expensive handling operations.

(c) What sign do you expect β_3 to have, and why? (2)
Answer: $\hat{\beta}_3 > 0$. Cost if truck and air transport is used increases as the shipment weight increases.

(d) What sign do you expect β_5 to have, and why? (2)
Answer: $\hat{\beta}_5 < 0$. Average variable cost per kg for a shipment if truck and ocean transport is used is less than average variable cost per kg for the same shipment if truck and air transport is used.

6. Suppose that we obtained data with 25 observations and that we estimated the following from these data. We assumed that the error terms ε_i are independent $N(0, \sigma^2)$ distributed.

$$
\begin{array}{rcrlcl}
\hat{\beta}_0 & = & 253 & \hat{\text{Var}}(\hat{\beta}_0) & = & 1.65 \times 10^4 \\
\hat{\beta}_1 & = & -7.81 & \hat{\text{Var}}(\hat{\beta}_1) & = & 2.62 \times 10^2 \\
\hat{\beta}_2 & = & 92 & \hat{\text{Var}}(\hat{\beta}_2) & = & 3.85 \times 10^1 \\
\hat{\beta}_3 & = & 0.123 & \hat{\text{Var}}(\hat{\beta}_3) & = & 4.12 \times 10^{-4} \\
\hat{\beta}_4 & = & -0.048 & \hat{\text{Var}}(\hat{\beta}_4) & = & 4.14 \times 10^{-5} \\
\hat{\beta}_5 & = & 0.013 & \hat{\text{Var}}(\hat{\beta}_5) & = & 1.62 \times 10^{-4}
\end{array}
$$

$$\begin{aligned}
\hat{\beta}_6 &= 25.0 & \hat{\text{Var}}(\hat{\beta}_6) &= 3.72 \times 10^{-5} \\
\hat{\beta}_7 &= -20.8 & \hat{\text{Var}}(\hat{\beta}_7) &= 2.71 \times 10^{-5} \\
\hat{\beta}_8 &= 53.0 & \hat{\text{Var}}(\hat{\beta}_8) &= 3.85 \times 10^{-5} \\
\hat{\beta}_9 &= 0.027 & \hat{\text{Var}}(\hat{\beta}_9) &= 2.45 \times 10^{-6}
\end{aligned}$$

(a) Write down an expression for the 95% confidence interval for β_3 based on the results. You do not have to simplify your expressions. (2)
Answer: $(0.123 - 2.131 \times \sqrt{4.12 \times 10^{-4}}, 0.123 + 2.131 \times \sqrt{4.12 \times 10^{-4}})$

(b) Suppose that we have to send a shipment that weighs $12300kg$ with a volume of $20m^3$ and a value of \$54000 by a combination of truck and ocean transport. Write an expression to forecast the cost for this shipment. You do not have to simplify your expression. (3)
Answer: $\hat{Y} = (253-7.81)+(0.123+0.013)12300+(25.0-20.8)20+0.027\times 54000$

(c) We estimate the variance of the forecast to be

$$\hat{\text{Var}}(\hat{Y}) = 3.84 \times 10^5$$

We want to make a conservative cost estimate, such that the probability that the cost will be more than the estimate is approximately 0.2. Write an expression for such a cost estimate. You do not have to simplify your expression. (3)
Answer: $\hat{Y} + 0.866 \times \sqrt{\hat{\text{Var}}(\hat{Y})}$

Question 3 (32 points)

Furniture is shipped from a manufacturing plant in High Point, NC to a retailer in Irvine, CA. We want to determine if it is more economical to ship by truckload carriers or by rail. Of course, there are many different types of furniture, but for the purpose of the calculations the quantities of furniture will be converted into *truckload quantities*, representing the amount of furniture that can be packed into a truck trailer of size $40 \times 8 \times 8ft^3$. The forecasted demand for the next year is for furniture that can be packed into 50 truck trailers. We know that 16 railroad boxcars of size $40 \times 10 \times 10ft^3$ each has the same volume as 25 truck trailers. Truckload transportation from High Point to Irvine costs \$2000. The journey by truck takes 5 days on average (door to door). Sending a rail car from High Point to Irvine costs \$2100. The journey by rail takes 14 days on average (door to door).

Each truckload quantity of furniture is valued at \$110,000 in High Point, \$112,000 in transit, and \$115,000 in Irvine. The inventory holding cost rate is estimated at 20% of value per year, including funds tied up in inventory, spoilage, and loss. In addition, the cost of storage in High Point is \$1800 per truckload quantity of furniture per year, and the cost of storage in Irvine is \$2400 per truckload quantity of furniture per year.

1. Order processing at the plant takes 2 days, that is, order lead time is equal to 2 days plus the transportation time. Assume that the *daily* demands are independent normally distributed with mean given above and standard deviation $\sigma_d = 0.1$ truckload quantities. Determine the safety stock required at the retailer for truck shipping and the safety stock required at the retailer for rail shipping so that the probability that

the retailer runs out of stock during a cycle between replenishments is 2%. You do not have to simplify your expressions, but remember to specify the units. (8)

Answer: Truck:
Variance of demand over $2+5=7$ days $=7\times 0.1^2$.
Standard deviation of demand over 7 days $=\sqrt{7\times 0.1^2}=\sqrt{7}\times 0.1$ truckload quantities.
Safety stock to cover demand over 7 days with probability $0.98 = 2.05\times 0.1\sqrt{7} = 0.205\times\sqrt{7}$ truckload quantities.
Rail:
Variance of demand over $2+14=16$ days $=16\times 0.1^2$.
Standard deviation of demand over 16 days $=\sqrt{16\times 0.1^2}=4\times 0.1=0.4$ truckload quantities.
Safety stock to cover demand over 16 days with probability $0.98=2.05\times 0.4$ truckload quantities.

2. As before, order processing at the plant takes 2 days, that is, order lead time is equal to 2 days plus the transportation time. Also, as before, the daily demands are normally distributed with mean given above and standard deviation $\sigma_d=0.1$ truckload quantities. However, daily demands are not independent. Instead, daily demands are positively correlated. Assume that the correlation coefficient between demand on any two days during the lead time is 0.2. Determine the safety stock required for truck shipping and the safety stock required for rail shipping so that the probability that the distribution center runs out of stock during a cycle between replenishments is 2%. You do not have to simplify your expressions, but remember to specify the units. (8)
Answer: Truck:
Variance of demand over $2+5=7$ days $=7\times 0.1^2+7\times 6\times 0.2\times 0.1^2=15.4\times 0.1^2$.
Standard deviation of demand over 7 days $=\sqrt{15.4\times 0.1^2}=\sqrt{15.4}\times 0.1$ truckload quantities.
Safety stock to cover demand over 7 days with probability $0.98=2.05\times\sqrt{15.4}\times 0.1 = 0.205\times\sqrt{15.4}$ truckload quantities.
Rail:
Variance of demand over $2+14=16$ days $=16\times 0.1^2+16\times 15\times 0.2\times 0.1^2=64\times 0.1^2$.
Standard deviation of demand over 16 days $=\sqrt{64\times 0.1^2}=0.8$ truckload quantities.
Safety stock to cover demand over 16 days with probability $0.98=2.05\times 0.8=1.64$ truckload quantities.

3. Consider the possibility of shipping vehicles that are only partially full. Let q_T denote the shipment size in truckload quantities (fraction of a truckload) if truck shipping is used, and let q_R denote the shipment size in railcar quantities (fraction of a railcar) if rail shipping is used. For the case with correlated daily demands, and for each mode, write expressions for the supply chain's cost components (transportation costs and inventory costs) as a function of the shipment sizes q_T and q_R. The safety stock at the manufacturing plant is equal to 0.5 truckload quantities (verify that this is equal to 0.32 railcar quantities). You do not have to simplify your expressions, but remember to specify the units. (8)

Answer: Truck:
Transportation cost $= \$2000 \times 50/q_T = 100,000/q_T$ per year.
Holding cost in transit $= \$50 \times 0.2 \times 112,000 \times 5/365$ per year.
Storage and Holding cost at manufacturing plant $= \$1800(0.5+q_T)+(0.2\times 110,000)(0.5+q_T/2)$ per year.
Storage and Holding cost at retailer $= \$2400(0.205\times\sqrt{15.4}+q_T)+(0.2\times 115,000)(0.205\times\sqrt{15.4}+q_T/2)$ per year.
Rail:
Transportation cost $= \$2100 \times 32/q_R$ per year.
Holding cost in transit $= \$50 \times 0.2 \times 112,000 \times 14/365$ per year.
Storage and Holding cost at manufacturing plant $= \$1800(0.5+25q_R/16)+(0.2\times 110,000)(0.5+25q_R/32)$ per year. Storage and Holding cost at retailer $= \$2400(1.64+25q_R/16)+(0.2\times 115,000)(1.64+25q_R/32)$ per year.

4. For the case with correlated daily demands, and for each mode, write an expression for the optimal (least cost for supply chain) shipment size q_T and q_R. You do not have to simplify your expressions, but remember to specify the units. (8)
Answer: Truck:
Total cost

$$\begin{aligned}
f_T(q_T) &= \frac{100,000}{q_T} + \frac{50\times 0.2\times 112,000\times 5}{365} \\
&\quad +1800(0.5+q_T)+(0.2\times 110,000)(0.5+q_T/2)+2400(0.205\times\sqrt{15.4}+q_T) \\
&\quad +(0.2\times 115,000)(0.205\times\sqrt{15.4}+q_T/2) \\
f_T'(q_T) &= -\frac{100,000}{q_T^2} + 26,700 \\
\Rightarrow \quad q_T^* &= \min\left\{1, \sqrt{\frac{100,000}{26,700}}\right\} \text{truckload quantities}
\end{aligned}$$

Rail:
Total cost

$$\begin{aligned}
f_R(q_R) &= \frac{2100\times 32}{q_R} + \frac{50\times 0.2\times 112,000\times 14}{365} \\
&\quad +1800(0.5+25q_R/16)+(0.2\times 110,000)(0.5+25q_R/32)+2400(1.64+25q_R/16) \\
&\quad +(0.2\times 115,000)(1.64+25q_R/32) \\
f_R'(q_R) &= -\frac{2100\times 32}{q_R^2} + \frac{26,700\times 25}{16} \\
\Rightarrow \quad q_R^* &= \min\left\{1, 4\sqrt{\frac{2100\times 32}{26,700\times 25}}\right\} \text{railcar quantities}
\end{aligned}$$