MATH 1711 TEST 3, FALL 2009, PAGE I

Print Your Name: Key-1

T.A. or Section Number: ____________

WORK ALL OF PROBLEMS 1-3.

1. (30 points) The probability distribution of a random variable X is given below.

k	$Pr(X=k)$
-1	$\frac{1}{8}$
0	$\frac{1}{2}$
1	$\frac{1}{8}$
2	??

(a) Complete the table to find $Pr(X=2)$.

$\frac{1}{8}+\frac{1}{2}+\frac{1}{8}=\frac{6}{8}=\frac{3}{4}$, so $Pr(X=2)=1-3/4$
$=\boxed{1/4}$

(b) Find a probability distribution for the variable X^2+1.

K	$Pr(X^2+1)$
1	$1/2$
2	$1/8+1/8=1/4$
5	$1/4$

(c) Use your answer to part (b) to find $Pr[(X^2+1)\geq 2]$.

$Pr((X^2+1)\geq 2)=\frac{1}{4}+\frac{1}{4}$
$=\boxed{\frac{1}{2}}$

WORK ALL OF PROBLEMS 1-3.

(a) Complete the table to

(30 points) The probability distribution of a random variable X is given below.

(b) Find a. probability distribution for the variable X 2 + 1.

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2. (20 points) A coin is biased so that heads are twice as likely as tails. The coin is tossed three times and the number of heads appearing is recorded.

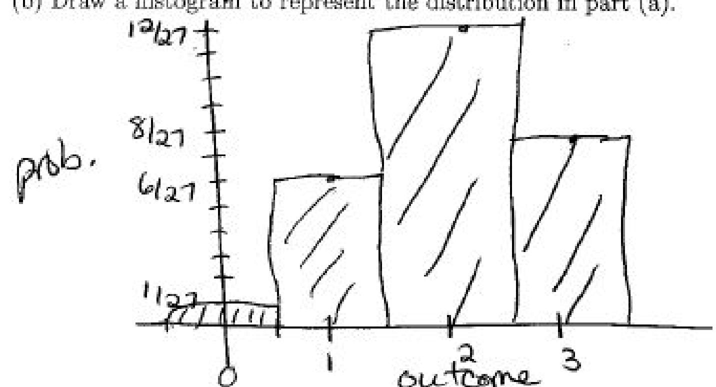
(a) Find a probability distribution for this experiment.

$Pr(H) = 2Pr(T) \Rightarrow Pr(H) = 2/3, \quad Pr(T) = 1/3$

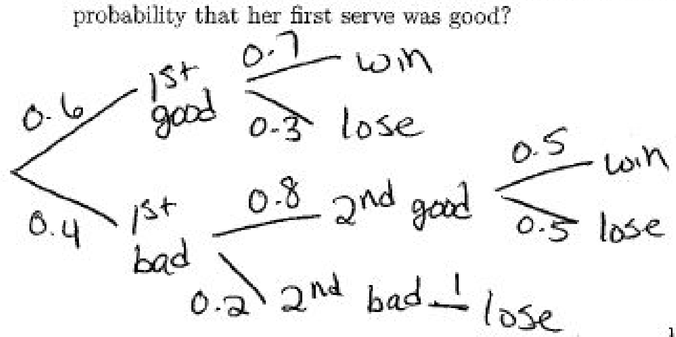
X = # of heads

k	$Pr(X=k)$
0	$(1/3)^3 = 1/27$
1	$\binom{3}{1}(2/3)(1/3)^2 = 6/27$
2	$\binom{3}{2}(2/3)^2(1/3) = 12/27$
3	$(2/3)^3 = 8/27$

(b) Draw a histogram to represent the distribution in part (a).



3. (16 points) Kim is a tennis player who has a very good first serve. If her first serve is good (which means "in"), then she wins the point 70% of the time. If she misses her first serve and has to have a second serve, then when her second serve is good, she wins the point 50% of the time. Her first serve is good 60% of the time, and her second serve is good 80% of the time. In a game, if it is known that Kim wins the point, what is the probability that her first serve was good?



$$Pr(1^{st}\text{ good} \mid \text{win}) = \frac{Pr(1^{st}\text{ good and win})}{Pr(\text{win})}$$

$$= \frac{(0.6)(0.7)}{(0.6)(0.7) + (0.4)(0.8)(0.5)} = \frac{42}{58} = \boxed{\frac{21}{29}}$$

2. (20 points) A coin is biased so that heads are twice as likely as tails. The coin is tossed three times and the number of heads

appearing is recorded. (a) Find a probability distribution for this experiment. A U

(b) Draw a histografn E0 represent the distfibution in part (3.).

3. (16 points) Kim is a tennis player who has a very good first serve. If her first serve is good (which means ”in”), then she wins the point 70% of the time. If she misses her first serve and has to have a. second serve, then when her second serve is good, she wins the point 50% of the time. Her first serve is good 60% of the time, and her second serve is good 80% of the time. In a game, if it is known that Kim wins the point, what is the probability that her first serve was good?

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MATH 1711 TEST 3, FALL 2009, PAGE II

Print Your Name: Key-1

T.A. or Section Number: ____________

WORK ONLY THREE (3) OF THE NEXT FOUR PROBLEMS (NUMBERS 4-7). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST THREE WILL BE GRADED.

4. (12 points) A fair die is rolled 20 times. Find the probability that a "3" appears at least twice. You do not need to simplify your final answer.

$$Pr(3 \text{ at least twice}) = 1 - Pr(0 \text{ or } 1 \text{ "3"s})$$
$$= 1 - \left[\left(\frac{5}{6}\right)^{20} + \binom{20}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{19}\right]$$

OR count directly

$$Pr(\text{"3"} \geq 2 \text{ times}) = \binom{20}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{18} + \cdots + \left(\frac{1}{6}\right)^{20}$$

5. (12 points) A game at the fair costs $1 to play. You draw a card from a standard deck of 52. If the card is a king or ace, you win $4. You win $2 for a queen and $1 for a jack. Otherwise, you lose the game. Let X be a random variable representing your earnings. Find the mean and variance of X. Simplify the mean as far as possible, but you do not have to simplify the variance.

X = earnings

	k	Pr(X=k)
K or A	\$3	2/13
Q	\$1	1/13
J	\$0	1/13
any other card	−\$1	9/13

k^2	$Pr(X^2=k^2)$
9	2/13
1	10/13
0	1/13

$$\mu = E(X) = 3\cdot\frac{2}{13} + 1\cdot\frac{1}{13} + 0\cdot\frac{1}{13} + (-1)\left(\frac{9}{13}\right) = -\frac{2}{13} \text{ mean}$$

$$E(X^2) = 9\cdot\frac{2}{13} + 1\cdot\frac{10}{13} = \frac{28}{13}$$

$$\sigma^2 = E(X^2) - \mu^2 = \frac{28}{13} - \left(-\frac{2}{13}\right)^2 = \frac{360}{169}$$

variance

can also use other formula to compute σ^2

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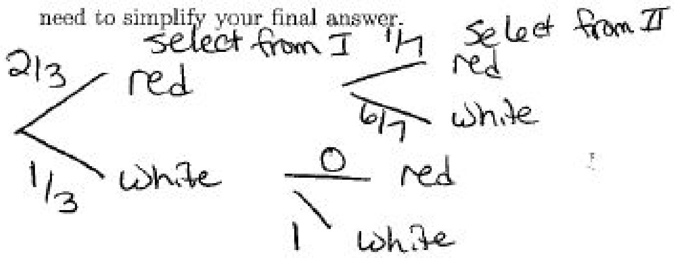
T.A. or Section Number: ____________________

WORK ONLY THREE (3) OF THE NEXT FOUR PROBLEMS (NUMBERS 4-7). WRITE OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU D0 NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST THREE WILL BE GRADED.

4. (12 points) A fair die is rolled 20 times. Find the probability that a appears at least twice. You do not need to simplify your final answer.

5. (12 points) A game at the fair costs $1 to play. You draw a card from a standard deck of 52. If the card is a king or ace, you win $4. You win $2 for a queen and $1 for a jack. Otherwise, you lose the game. Let X be a random variable representing your earnings. Find the mean and variance of X. Simplify the mean as far as possible, but you do not have to simplify the variance.

6. (12 points) Urn I contains 10 red balls and 5 white balls. Urn II contains 6 white balls. A ball is selected at random from urn I and placed in urn II. Then a ball is selected at random from urn II. What is the probability that the second ball is white? You do not need to simplify your final answer.



$$Pr(2^{nd} \text{ is white}) = \left(\frac{2}{3}\right)\left(\frac{6}{7}\right) + \left(\frac{1}{3}\right)(1)$$

$$= \frac{4}{7} + \frac{1}{3} = \boxed{\frac{19}{21}}$$

7. (12 points) Find the sample mean and sample variance for the data below. You should simplify the mean, but you do not need to simplify the variance.

1, 1, 3, 4, 6

$$\bar{X} = \frac{1+1+3+4+6}{5} = \frac{15}{5} = 3$$

$$s^2 = \frac{(1-3)^2 \cdot 2 + (3-3)^2 + (4-3)^2 + (6-3)^2}{4}$$

$$= \frac{8+1+9}{4} = \frac{18}{4} \text{ or } \boxed{\frac{9}{2}}$$

6. (12 points) Urn I contains 10 red balls and 5 white balls. Urn II contains 6 white bails. A ball is selected at random from urn I

and placed in urn II. Then a ball is selected

at random from urn II. What is the probability that the second ball is white? You do not

7. (12 points) Find the sample mean and sample variance for the data below. You should

simplify the mean, but you do not need to simplify the variance. _

1, 1, 3, 4, 6

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MATH 1711 TEST 3, FALL 2009, PAGE I

Print Your Name: Key-2

T.A. or Section Number: ____________

WORK ALL OF PROBLEMS 1-3.

1. (16 points) Kim is a tennis player who has a very good first serve. If her first serve is good (which means "in"), then she wins the point 90% of the time. If she misses her first serve and has to have a second serve, then when her second serve is good, she wins the point 60% of the time. Her first serve is good 50% of the time, and her second serve is good 70% of the time. In a game, if it is known that Kim wins the point, what is the probability that her first serve was good?

0.5 1st good — 0.9 win; 0.1 lose

0.5 1st bad — 0.7 2nd good — 0.6 win; 0.4 lose

0.3 2nd bad — 1 lose

$$Pr(1^{st}\text{ good} \mid \text{win}) = \frac{Pr(1^{st}\text{ good and win})}{Pr(\text{win})}$$

$$= \frac{(0.5)(0.9)}{(0.5)(0.9)+(0.5)(0.7)(0.6)} = \frac{0.45}{0.66} = \boxed{\frac{15}{22}}$$

2. (20 points) A coin is biased so that tails are twice as likely as heads. The coin is tossed three times and the number of heads appearing is recorded.

(a) Find a probability distribution for this experiment.

X = # of heads

$Pr(T) = 2Pr(H)$

so

$Pr(H) = 1/3$

$Pr(T) = 2/3$

k	$Pr(X=k)$
0	$(2/3)^3 = 8/27$
1	$(3)(\frac{1}{3})(\frac{2}{3})^2 = 12/27$
2	$(3)(\frac{1}{3})^2(\frac{2}{3}) = 6/27$
3	$(\frac{1}{3})^3 = 1/27$

(b) Draw a histogram to represent the distribution in part (a).

MATH 1711 TEST 3, FALL 2009, PAGE I

Print Your Name:

T.A. or Section Number: _________,u____________

WORK ALL OF PROBLEMS 1-3.

1. (16 points) Kim is a tennis player who has a. very good first serve. If her first serve is good (which means ”in”), then she wins the point 90% of the time. If she misses her first serve and has to have a second serve, then when her second serve is good, she wins the point 60% of the time. Her first serve is good 50% of the time, and her second serve is good 70% of the time. -In a game, if it is known that Kim Wins the point, What is the

probability that her first serve was good"?

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2. (20 points) A coin is biased so that tails are twice as likely as heads. The coin is tossed three times and the number of heads appearing is recorded._ _ I (a) Find a probability distribution for this experiment. ‘Hi 39$ /U9-055

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(b) Draw a histogram to represent the distribution in pert (a.).

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3. (30 points) The probability distribution of a random variable X is given below.

k	$Pr(X = k)$
-1	$\frac{1}{2}$
0	$\frac{1}{4}$
1	$\frac{1}{8}$
2	??

(a) Complete the table to find $Pr(X = 2)$.

$1/2 + 1/4 + 1/8 = 7/8$

$1 - \frac{7}{8} = \frac{1}{8}$, so $\boxed{Pr(X=2) = \frac{1}{8}}$

(b) Find a probability distribution for the variable $X^2 + 1$.

K	$Pr((X^2+1) = k)$
1	$1/4$
2	$1/2 + 1/8 = 5/8$
5	$1/8$

(c) Use your answer to part (b) to find $Pr[(X^2 + 1) \geq 2]$.

$$Pr[(X^2+1) \geq 2] = \frac{5}{8} + \frac{1}{8}$$

$$= \boxed{\frac{3}{4}}$$

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3. (30 points) The probability distribution of a random variable X is given below.

(a) Complete the table to find 1 2).

(b) Find a probability distribution for the vazriable X 2 1.

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Print Your Name: Key-2

T.A. or Section Number: ____________

WORK ONLY THREE (3) OF THE NEXT FOUR PROBLEMS (NUMBERS 4-7). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST THREE WILL BE GRADED.

4. (12 points) Find the sample mean and sample variance for the data below. You should simplify the mean, but you do not need to simplify the variance.

1, 3, 4, 6, 6

$$\bar{X} = \frac{1+3+4+6+6}{5} = \frac{20}{5} = \boxed{4}$$

$$s^2 = \frac{(1-4)^2 + (3-4)^2 + (4-4)^2 + (6-4)^2 \cdot 2}{4}$$

$$= \frac{9+1+8}{4} = \frac{18}{4} \text{ or } \frac{9}{2}$$

5. (12 points) A fair die is rolled 15 times. Find the probability that a "6" appears at least twice. You do not need to simplify your final answer.

$$Pr(\text{"6" appears} \geq 2 \text{ times}) = 1 - Pr(\text{"6" once or none})$$

$$= 1 - \left[\left(\frac{5}{6}\right)^{15} + \binom{15}{1}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{14}\right]$$

OR count directly:

$$Pr(\text{"6"} \geq 2 \text{ times}) = \binom{15}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{13} + \cdots + \left(\frac{1}{6}\right)^{15}$$

Print Your Name: __K<Q&jg:____

T.A. or Section Number: ______________________

WORK ONLY THREE (3) OF THE NEXT FOUR PROBLEMS (NUMBERS 4-7). WRITE ”OMIT” OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT,

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4. (12 points) Find the sample mean and sample variance for the data below. You should simplify the mean, but you do not need to simplify the variance.

1, 3, 4, 6, 6

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6. (12 points) A game at the fair costs \$1 to play. You draw a card from a standard deck of 52. If the card is an ace, you win \$5. You win \$3 for a king or a queen, and \$1 for a jack. Otherwise, you lose the game. Let X be a random variable representing your earnings. Find the mean and variance of X. Simplify the mean as far as possible, but you do not have to simplify the variance.

X = earnings

	K	Pr(X=k)
A	\$4	1/13
K or Q	\$2	2/13
J	\$0	1/13
o.w.	-\$1	9/13

K	$Pr(X^2=k)$
16	1/13
4	2/13
0	1/13
1	9/13

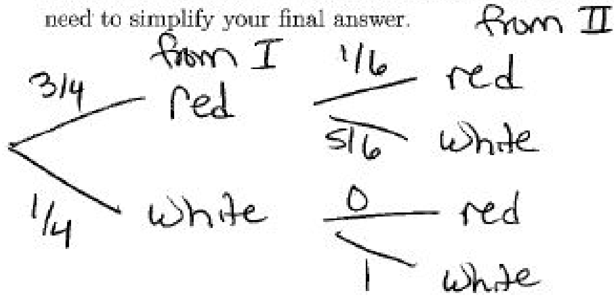
$$\mu = E(X) = 4\cdot\frac{1}{13} + 2\cdot\frac{2}{13} + 0\cdot\frac{1}{13} + (-1)\cdot\frac{9}{13} = -\frac{1}{13} \text{ mean}$$

$$E(X^2) = 16\cdot\frac{1}{13} + 4\cdot\frac{2}{13} + 0\cdot\frac{1}{13} + 1\cdot\frac{9}{13} = \frac{33}{13}$$

$$\sigma^2 = \frac{33}{13} - \left(-\frac{1}{13}\right)^2 = \frac{33}{13} - \frac{1}{169} = \frac{428}{169}$$

OR $\sigma^2 = \left(4+\frac{1}{13}\right)^2\cdot\frac{1}{13} + \left(2+\frac{1}{13}\right)^2\cdot\frac{2}{13} + \left(-\frac{1}{13}\right)^2\cdot\frac{1}{13} + \left(-1+\frac{1}{13}\right)^2\cdot\frac{9}{13}$

7. (12 points) Urn I contains 9 red balls and 3 white balls. Urn II contains 5 white balls. A ball is selected at random from urn I and placed in urn II. Then a ball is selected at random from urn II. What is the probability that the second ball is white? You do not need to simplify your final answer.



$$Pr(2^{nd} \text{ is } w) = \frac{3}{4}\cdot\frac{5}{6} + \frac{1}{4}\cdot 1 = \frac{5}{8} + \frac{1}{4} = \frac{7}{8}$$

6. (12 points) A game at the fair costs \$1 to play. You draw a card from a standard deck of 52. If the card is an ace, you Win \$5. You

win $3 for a king or a queen, and $1 for a jack. Otherwise, you lose the game. Let X be a random variable representing your earnings. Find the mean and variance of X. Simplify the mean as far as possible, but you

7. (12 points) Urn I contains 9 red balls and 3 white balls. Urn II contains 5 white balls.

A bail is selected at random from urn I and placed in urn II. Then a bail is selected at random from urn II. What is the probability that the second bail is white? You do not

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