**Name:** Soln

# Transport Processes I, ChBE3200 — Exam #2 — Spring 2015

The exam consists of 3 problems worth the points indicated (for a total of 38 points). Please box your final answers in the space below each question in the specified units. Show all work and state any assumptions made to receive full credit. Unit conversions and physical property data are given below. You may use 1 personally made note sheet on this exam. Please sign the honor code.

Honor Code:
I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

______________________________
Signature

**Useful Information**

Densities: air (25°C) 1.2 kg/m$^3$
Water (4°C) 1000 kg/m$^3$ or 62.4 lb$_m$/ft$^3$

Acceleration due to gravity: $g = 9.8$ m/s$^2$ or 32 ft/s$^2$

Conversion factor: $g_c = 32(\text{lb}_m\text{ft}/\text{lb}_f\text{s}^2)$

**FACTORS FOR UNIT CONVERSIONS**

| Quantity | Equivalent Values |
|---|---|
| Mass | 1 kg = 1000 g = 0.001 metric ton = 2.20462 lb$_m$ = 35.27392 oz<br>1 lb$_m$ = 16 oz = $5 \times 10^{-4}$ ton = 453.593 g = 0.453593 kg |
| Length | 1 m = 100 cm = 1000 mm = $10^6$ microns (μm) = $10^{10}$ angstroms (Å)<br>= 39.37 in. = 3.2808 ft = 1.0936 yd = 0.0006214 mile<br>1 ft = 12 in. = 1/3 yd = 0.3048 m = 30.48 cm |
| Volume | 1 m$^3$ = 1000 L = $10^6$ cm$^3$ = $10^6$ mL<br>= 35.3145 ft$^3$ = 219.97 imperial gallons = 264.17 gal<br>= 1056.68 qt<br>1 ft$^3$ = 1728 in.$^3$ = 7.4805 gal = 0.028317 m$^3$ = 28.317 L<br>= 28,317 cm$^3$ |
| Force | 1 N = 1 kg·m/s$^2$ = $10^5$ dynes = $10^5$ g·cm/s$^2$ = 0.22481 lb$_f$<br>1 lb$_f$ = 32.174 lb$_m$·ft/s$^2$ = 4.4482 N = $4.4482 \times 10^5$ dynes |
| Pressure | 1 atm = $1.01325 \times 10^5$ N/m$^2$ (Pa) = 101.325 kPa = 1.01325 bar<br>= $1.01325 \times 10^6$ dynes/cm$^2$<br>= 760 mm Hg at 0°C (torr) = 10.333 m $H_2O$ at 4°C<br>= 14.696 lb$_f$/in.$^2$ (psi) = 33.9 ft $H_2O$ at 4°C<br>= 29.921 in. Hg at 0°C |
| Energy | 1 J = 1 N·m = $10^7$ ergs = $10^7$ dyne·cm<br>= $2.778 \times 10^{-7}$ kW·h = 0.23901 cal<br>= 0.7376 ft-lb$_f$ = $9.486 \times 10^{-4}$ Btu |
| Power | 1 W = 1 J/s = 0.23901 cal/s = 0.7376 ft·lb$_f$/s = $9.486 \times 10^{-4}$ Btu/s<br>= $1.341 \times 10^{-3}$ hp |

Example: The factor to convert grams to lb$_m$ is $\left(\frac{2.20462 \text{ lb}_m}{1000 \text{ g}}\right)$.

check pg 1
check -1 for Prems

Copied from Felder & Rousseau, 3rd. ed. Wiley. 2005.

**#1 (10 points)**

(a) Explain why a laminar boundary layer forms on the walls of a pipe when the bulk flow is turbulent.

There is shear stress on fluid by wall → slows flow so $v=0$ at wall. Turbulent flow is high Re (high v) so laminar boundary layer serves as transition zone between stagnant fluid at wall due to viscous forces & high velocity fluid in center pipe.

3

(b) Spin coating is a technique often used during photolithography in the manufacture of computer chips. Consider a circular silicon wafer with radius R, rotating about its axis with angular speed Ω (units of inverse time). A Newtonian fluid of viscosity μ and density ρ put on the surface of the wafer flows radially outward under the action of the centrifugal force. It quickly achieves a uniform thickness h, which depends on the spinning time t.

(i) If h is the main variable of interest, what core variables would you select to create dimensionless groups? (You do not need to actually make the groups.)

$R=[L]$ $\Omega=[T^{-1}]$ $\mu=[M/LT]$ $\rho=[M/L^3]$ $t=[T]$

best core involve fewest variables, but need all units

2

$R\,\Omega\,\rho$ or $R\,t\,\rho$ or $R\,\Omega\,\mu$ or $R\,t\,\mu$

(ii) List one reason why it would be useful to identify dimensionless groups for this process.

so that you could understand how changing the process variables ($R, \Omega, t, \mu, \rho$) affect h ~~[illegible]~~ with a minimal # of experiments

2

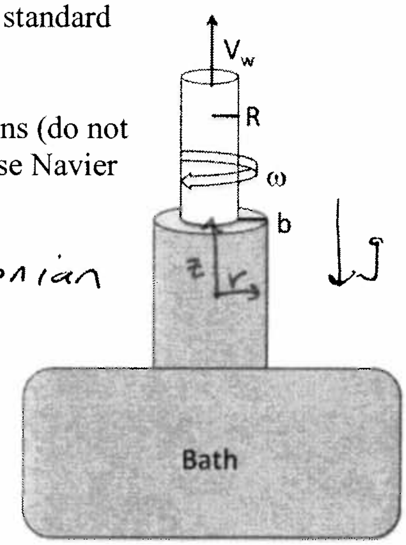
also for scaling for chips of different sizes

(c) Explain what it means when a falling object has reached its terminal velocity.

it is the velocity when the drag forces are balanced by the gravitational forces (weight) of the object

3

**#2 (14 points)**
A fluid is drawn up from a bath on a cylindrical wire of radius R. The wire is being drawn up at a velocity of $v_w$ and is also rotating at speed $\omega$. The fluid coats the wire with a thin coating of thickness b, as shown in the figure. The surrounding environment is air at standard pressure and temperature.




(a) What assumptions must be made in order to use Navier Stokes equations (do not list assumptions you are making to get rid of terms, just those needed to use Navier Stokes in the first place)?

3 Laminar flow, incompressible Newtonian fluid

(b) What are the boundary conditions for the fluid flow?

at $r = R$ $\quad V_z = V_w$, $\quad V_\theta = R\omega$

4 (1 each) at $r = R+b$ $\quad \tau_z = 0$, $\quad \tau_\theta = 0$

$\left(\mu \frac{\partial V_z}{\partial r} = 0\right), \left(\mu \frac{\partial V_\theta}{\partial r} = 0\right)$

(c) Simplify the Navier Stokes equations for this flow in the directions with non-zero velocity components, as if you were going to solve for the velocity profile of the fluid. Using the equations below cross out all terms you are neglecting. Do NOT solve (integrate). For each term you cross out, you must list a reason to receive credit. If it helps to save space, you can put a number for the reason next to the term in the equation and then write a list at the bottom of the actual reasons that correspond to each number since some reasons may be repeated.

$V_r = 0$, only simplify $V_\theta$ & $V_z$ 1

n cylindrical coordinates:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \rho g_r + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\rho\left(\underset{1}{\cancel{\frac{\partial v_\theta}{\partial t}}} + \underset{2}{\cancel{v_r}}\frac{\partial v_\theta}{\partial r} + \underset{3}{\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}} + \underset{2}{\cancel{\frac{v_r v_\theta}{r}}} + \underset{4}{\cancel{v_z\frac{\partial v_\theta}{\partial z}}}\right) = \underset{3}{\cancel{-\frac{1}{r}\frac{\partial P}{\partial \theta}}} + \underset{5}{\cancel{\rho g_\theta}} + \boxed{\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)\right.} + \underset{3}{\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}} + \underset{2}{\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}} + \left.\underset{4}{\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}}\right]$$

$$\rho\left(\underset{1}{\cancel{\frac{\partial v_z}{\partial t}}} + \underset{2}{\cancel{v_r}}\frac{\partial v_z}{\partial r} + \underset{3}{\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}}} + \underset{4}{\cancel{v_z\frac{\partial v_z}{\partial z}}}\right) = \underset{6}{\cancel{-\frac{\partial P}{\partial z}}} + \boxed{\rho g_z + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right.} + \underset{3}{\cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}}} + \left.\underset{4}{\cancel{\frac{\partial^2 v_z}{\partial z^2}}}\right]$$

1: steady state
2: $V_r = 0$, no flow in r direction
3: symmetrical about $\theta$ axis
4: fully developed flow
5: gravity only in z direction
6: fluid is completely exposed to atmosphere over whole length of wire

1 each

**#3 (14 points)**
A farmer has set up a piping system (shown below) to move water from one irrigation pond to another. The system includes two standard 90° elbows and a gate valve. When the valve is open the system should maintain a flowrate of 100 gallons/minute. The pipes are made of commercial steel and have a diameter of 4" (inches). The density and viscosity of the water (60°F) are 62.3 $lb_m/ft^3$ and $0.76\times10^{-3}$ $lb_m/ft\ s$. There is no change in height across the system. Figures 13.1 and 13.2 are on page 6 of the exam.

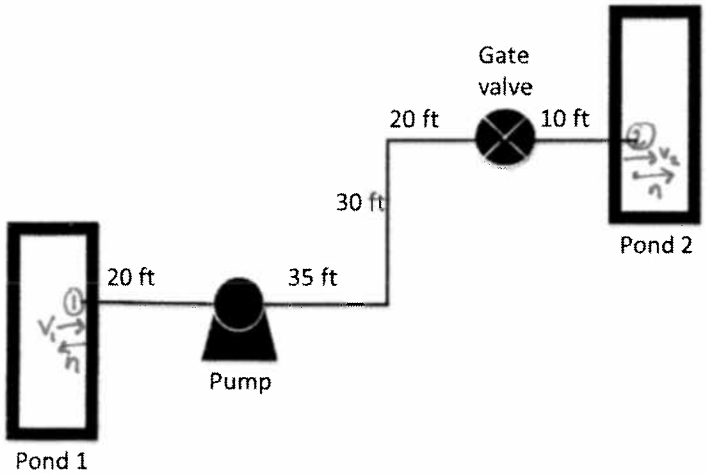



**Table 13.1** Friction loss factors for various pipe fittings

| Fitting | $K$ | $L_{eq}/D$ |
|---|---|---|
| Globe valve, wide open | 7.5 | 350 |
| Angle valve, wide open | 3.8 | 170 |
| Gate valve, wide open | 0.15 | 7 |
| Gate valve, $\frac{3}{4}$ open | 0.85 | 40 |
| Gate valve, $\frac{1}{2}$ open | 4.4 | 200 |
| Gate valve, $\frac{1}{4}$ open | 20 | 900 |
| Standard 90° elbow | 0.7 | 32 |

① $v = Q/A = 2.55\ ft/s$

(a) Is the flow laminar or turbulent? How do you know? Must answer both questions for credit.

① $Re = \frac{D v \rho}{\mu} = \frac{\left(\frac{4\ in \cdot 1\ ft}{12\ in}\right)\left[\left(\frac{100\ gal}{min}\cdot\frac{35.3\ ft^3}{264.17\ gal}\cdot\frac{1\ min}{60\ s}\right)/\pi\left(\frac{2\ in \cdot 1\ ft}{12\ in}\right)^2\right]\left(62.3\ \frac{lbm}{ft^3}\right)}{0.76\times10^{-3}\ lbm/ft\cdot s}$

$Re =$ 69,734

Turbulent ~~theoretically~~ since $Re > 2{,}300$ ① ~~it is laminar~~

~~but it is getting close to the transition region~~

(b) How much power in watts is required from the pump (assume 100% efficient)?

Energy balance ①

$$\cancel{\frac{\delta Q}{\delta t}} - \frac{\delta W_s}{\delta t} - \cancel{\frac{\delta W_\mu}{\delta t}} = \iint_{cs}\left(\cancel{\frac{V^2}{2}} + \cancel{gy} + u + \cancel{\frac{P}{\rho}}\right)\rho(\vec{v}\cdot\vec{n})dA + \cancel{\frac{\partial}{\partial t}\iiint_{cv}\left(\frac{v^2}{2}+gy+u\right)\rho dV}$$

no heat flow; viscous work in u term; diameter const. $v_1 = v_2$; no height diff.; both ponds open to atm ①; steady state

$$-\frac{\delta W_s}{\delta t} = u_1\rho(-v_1)A_1 + u_2\rho(+v_2)A_2 = \rho v A(u_2 - u_1) = \rho v g h_L A$$ ① ①

More space >



$h_L = \sum_i h_{Li}$ ①

$$h_L\,\text{pipe} = 2f_f \frac{L}{D}\frac{v^2}{g} = 2(0.0052)\left(\frac{20+35+30+20+10\,ft}{4\,in \mid \frac{1\,ft}{12\,in}}\right)\frac{(2.55\,ft/s)^2}{32\,ft/s^2}$$

comm. steel $e = 0.00015\ ft$ $\quad \frac{e}{D} = \frac{0.00015\ ft}{\left(4\,in \mid \frac{1\,ft}{12\,in}\right)} = 0.00045$

13.1 $f_f \approx 0.0052$

$$h_L\,\text{elbow} = K\frac{v^2}{2g} = \frac{0.7}{2}\,\frac{(2.55\ ft/s)^2}{32\ ft/s^2} = 0.07\ ft$$

$$h_L\,\text{gate} = K\frac{v^2}{2g} = \frac{0.15}{2}\,\frac{(2.55\ ft/s)^2}{32\ ft/s^2} = 0.015\ ft$$

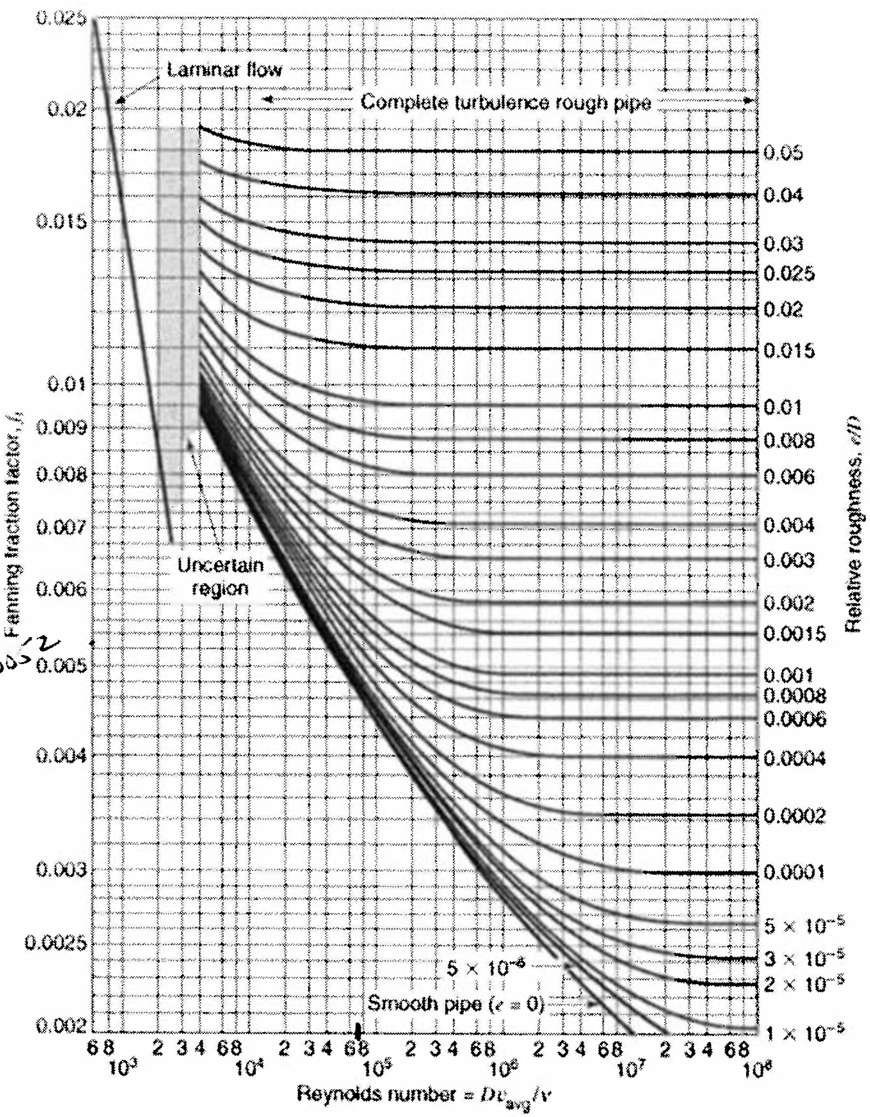
$$h_L\,\text{total} = h_L\,\text{pipe} + 2h_L\,\text{elbow} + h_L\,\text{gate valve} = 0.73\,ft + 2(0.07\,ft) + 0.01$$
$$= 0.885\ ft$$

$$-\frac{\delta W_s}{\delta t} = \left(62.3\,\frac{lbm}{ft^3}\right)\left(2.55\,\frac{ft}{s}\right)\left(32\,\frac{ft}{s^2}\right)(0.885\,ft)\,\pi\left(2\,in \mid \frac{1\,ft}{12\,in}\right)^2$$

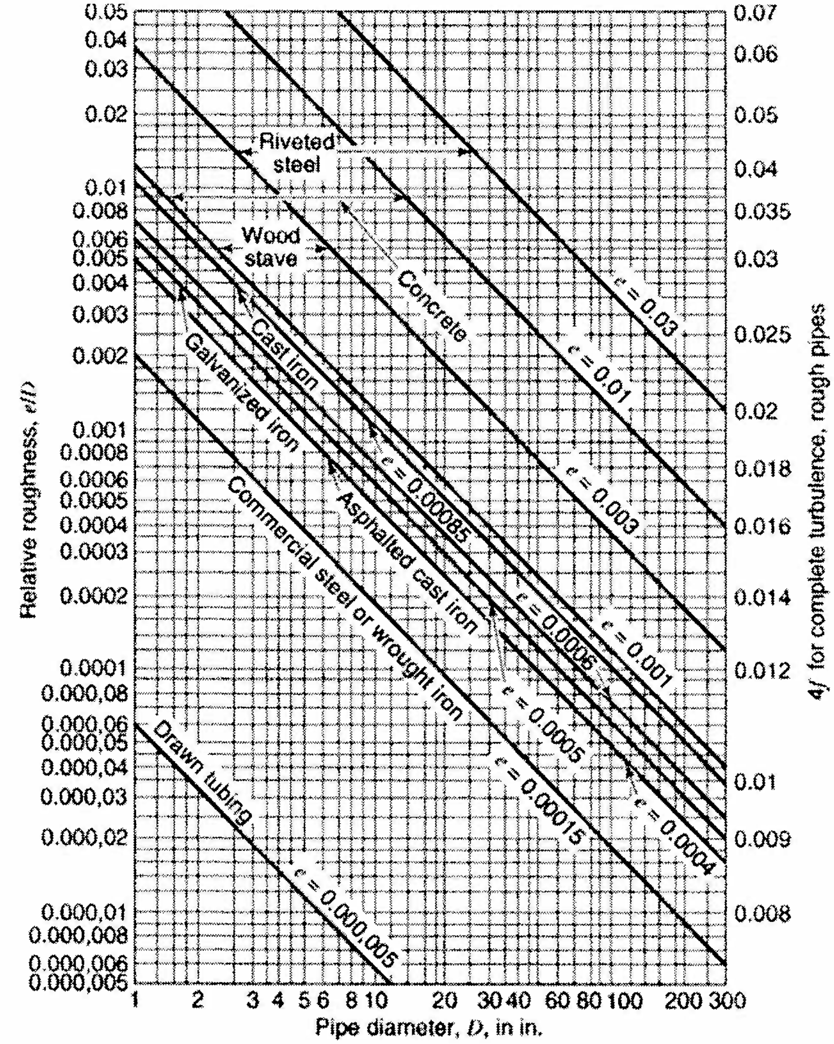
$$-\frac{\delta W_s}{\delta t} = 392.6\ \frac{\cancel{lbm}\cdot ft^{\cancel{2}}}{s^{\cancel{3}}}\left(\frac{lb_f\cdot \cancel{s^2}}{32\ \cancel{lbm}\cdot\cancel{ft}}\right) = 12.27\ \frac{lb_f\cdot ft}{s}$$

$\left(\frac{1}{g_c}\right)$

$$\text{power from pump} = 12.27\ \frac{lb_f\cdot ft}{s}\left(\frac{1\,W}{0.7376\ \frac{ft\cdot lb_f}{s}}\right) = 16.6\ W$$




$f = 0.0052$




Note: $e$ is in "ft"