

Solutions to Homework 3

$$1) P(X=1)=c \quad P(X=3)=3c \quad P(X=5)=5c \quad P(X=7)=7c \quad P(X=9)=9c \quad P(X=11)=11c$$

$$\text{We need } c+3c+5c+7c+9c+11c=1 \Rightarrow 36c=1 \Rightarrow c=\frac{1}{36}$$

$$2) a) \text{ Probability of getting a 6 in 6 throws} \\ = \binom{6}{1} \left(\frac{1}{6}\right) \left(\frac{5}{6}\right)^5$$

$$= 6 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^5 = 0.4019$$

$$b) \text{ Probability of getting 2 6's in 12 throws}$$

$$= \binom{12}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{10} = \frac{12!}{2!10!} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{10}$$

$$= \frac{10!11\cancel{12}}{2!10!} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{10} = 2 \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{10}$$

$$= 0.2961$$

So, (a) is more likely

$$3) a) \text{ Al wins the first two sets: } (0.7)^2$$

$$\text{Al wins 1st and 3rd or 2nd and 3rd in 3 sets: } 2 \times 0.3 \times (0.7)^2$$

$$(0.7)^2 + 0.6 \times (0.7)^2 = 1.6 \times (0.7)^2 = 0.784$$

$$b) \text{ Al wins the first 3 sets: } (0.7)^3$$

$$\text{Al wins 1st, 2nd, 4th or 1st, 3rd, 4th or 2nd, 3rd, 4th in 4 sets: } 3 \times 0.3 \times (0.7)^3$$

$$\text{Al wins 1st, 2nd, 5th or 1st, 4th, 5th or 3rd, 4th, 5th or 2nd, 4th, 5th or 1st, 3rd, 5th or 2nd, 3rd, 5th in 5 sets} \\ = 6 \times (0.3)^2 \times (0.7)^3$$

$$\begin{aligned}
 &= (0.7)^3 + 0.9 \times (0.7)^3 + 6 \times (0.3)^2 \times (0.7)^3 \\
 &= (0.7)^3 (1 + 0.9 + 0.54) \\
 &= (0.7)^3 (2.24) \\
 &= 0.8369
 \end{aligned}$$

4. $P(X=0) = \frac{1}{2}$ $P(X=1) = \frac{1}{10}$ $P(X=2) = \frac{1}{5}$

$P(X=3) = \frac{1}{10}$ $P(X=3.5) = \frac{1}{10}$