

Quiz 4 (25 mins), MA1501 Calculus I, 09/17/2013a

PRINTED NAME: _____ GT ID: _____

This quiz is worth a total of 100 points, and the value of each question is listed with each question.

You must show your work; answers without substantiation do not count.

Answers must appear in the box provided! No cheat!

1. Consider the following function

$$f(x) = x + \frac{9}{x^2}.$$

(a) (20pts) Find the slope of the function's graph at the given point (1, 10) and (3, 4) .

(b) (20pts) Find the second derivative of $f(x)$.Answer: (a) We find the derivative of f w.r.t. x .

$$\begin{aligned} f(x) &= x + 9x^{-2}, \\ f'(x) &= 1 - 18x^{-3}. \end{aligned} \tag{1}$$

From (1), we have

$$f'(1) = -17, \quad f'(3) = 1 - \frac{18}{27} = \frac{1}{3}.$$

Therefore, slope at (1, 10) = -17, and slope at (3, 4) = $\frac{1}{3}$ (b) The first derivative of $f(x)$ is

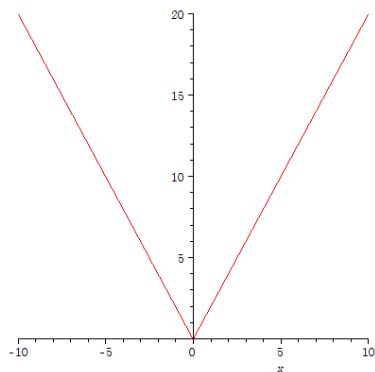
$$f'(x) = 1 - 18x^{-3}.$$

The second derivative is

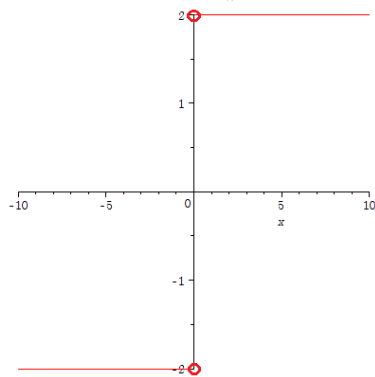
$$f''(x) = \frac{54}{x^4}.$$

2. (30pts) Graph the function of $f(x) = 2|x|$, $x = (-\infty, \infty)$. Then graph $y = \frac{2|x|-0}{x-0} = 2\frac{|x|}{x}$. What can you conclude about the differentiability of f at $x = 0$?

Answer: Graph of $f(x) = 2|x|$



Graph of $f(x) = 2\frac{|x|}{x}$



Both -2 and 2 should be marked, and since $f(x)$ is not defined at $x = 0$, circles should be specified as well.

f is not differentiable at $x = 0$ since the right-hand derivative $\lim_{h \rightarrow 0+} 2\frac{|h|}{h}$ and the left-hand derivative $\lim_{h \rightarrow 0-} 2\frac{|h|}{h}$ differ.

3. (30 pts) Find the first derivative of the function

$$y = \frac{x^2 - e^x}{x + 0.5}.$$

Answer: Using the derivative quotient rule,

$$\begin{aligned} y' &= \frac{(x^2 - e^x)'(x + 0.5) - (x^2 - e^x)(x + 0.5)'}{(x + 0.5)^2} \\ &= \frac{(2x - e^x)(x + 0.5) - (x^2 - e^x)}{(x + 0.5)^2} \end{aligned}$$

or one can use the derivative product rule,

$$\begin{aligned} y &= (x^2 - e^x)(x + 0.5)^{-1}, \\ y' &= (x^2 - e^x)'(x + 0.5)^{-1} - (x^2 - e^x)(x + 0.5)^{-2} \\ &= \frac{2x - e^x}{x + 0.5} - \frac{x^2 - e^x}{(x + 0.5)^2} \\ &= \frac{(2x - e^x)(x + 0.5) - (x^2 - e^x)}{(x + 0.5)^2} \end{aligned}$$