MATH 1552 - SPRING 2016
QUIZ 3 - SHOW YOUR WORK

NAME: _____ TA: _____

1. (13 points) Evaluate the integral: $\int \frac{(-x^2-7x-9)}{(x-3)(x^2+4)}\,dx = -3\int \frac{dx}{x-3} + \int \frac{2x}{x^2+4} - \int \frac{dx}{x^2+4}$

$$= -3\ln|x-3| + \ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

middle integral*, use** $u = x^2+4$ **&** ***last integral **use** $u = \frac{x}{2}$

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First do the partial fractions decomposition

$$\frac{(-x^2-7x-9)}{(x-3)(x^2+4)} = \frac{A}{x-3} + \frac{(Bx+C)}{x^2+4}$$ Clear fractions

$$\Rightarrow -x^2-7x-9 = (x^2+4)A + (x-3)(Bx+C)$$

$x=3 \Rightarrow -39 = 13A + 0 \Rightarrow \boldsymbol{A=-3}$

$x=0 \Rightarrow -9 = (4)(-3) + (-3)C \Rightarrow \boldsymbol{C=-1}$

$x=1 \Rightarrow -17 = (5)(-3) + (-2)(B-1) \Rightarrow \boldsymbol{B=2}$

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2. (12 points) Evaluate the integral: $\int \frac{x^2}{9+x^2}\,dx = \int \frac{(9\tan^2(u))\,(3\sec^2(u))}{9\sec^2(u)}\,du$ Remember that $\tan^2(x) + 1 = \sec^2(x)$

$$= \int 3\tan^2(u)\,du = 3\int \sec^2(u)\,du - 3\int du$$

$$= 3\,(\tan(u) - u) = \mathbf{3\left(\frac{x}{3} - \tan^{-1}\left(\frac{x}{3}\right)\right) + C}$$

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$x = 3\tan(u)$ **or** $u = \tan^{-1}\left(\frac{x}{3}\right)$

$dx = 3\sec^2(u)\,dx$

$\sqrt{9+x^2} = 3\sec(u) = \Rightarrow\ 9 + x^2 = 9\sec^2(u)$

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3. (5 points) Solve the IVP: $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ & $y(0) = 3$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \Rightarrow dy = \frac{dx}{\sqrt{1-x^2}} \Rightarrow \int dy = \int \frac{dx}{\sqrt{1-x^2}}$$

$y = \sin^{-1}(x) + C \Rightarrow 3 = \sin^{-1}(0) + C \Rightarrow C = 3$

$$\mathbf{y = \sin^{-1}(x) + 3}$$