Full name: Solutions Math 2551, Section B___

Quiz 6 — §15.2-5

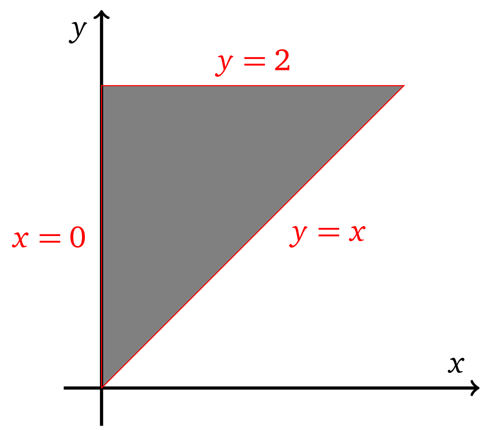
Please ***clearly*** show all work. Scientific calculators are allowed, but no graphing calculators!

(1) [10 points] Consider the iterated integral

$$\int_0^2 \int_x^2 x\, dy\, dx.$$

Rewrite the integral as an iterated integral in the following two ways (don't evaluate!):
(a) In the order $dx\, dy$ (b) In polar coordinates

The region of integration is drawn below.



(a) In the order $dx\, dy$ the integral is

$$\int_0^2 \int_0^y x\, dx\, dy$$

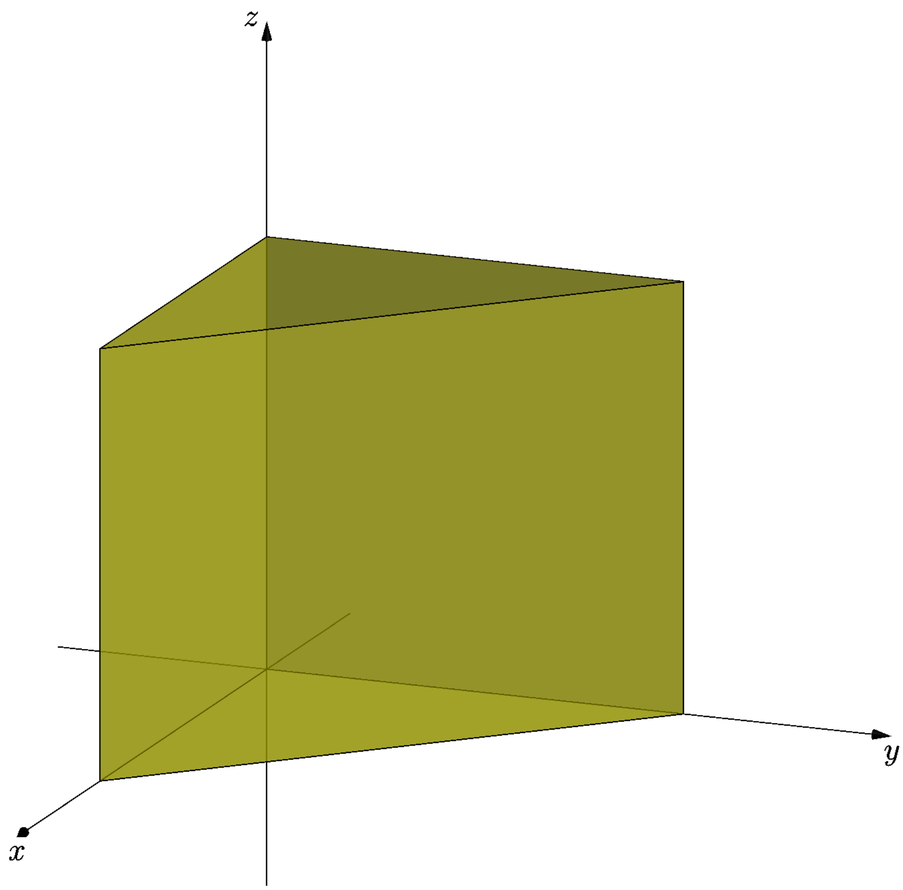
(b) In polar coordinates the integral is

$$\int_{\pi/4}^{\pi/2} \int_0^{2\csc\theta} r^2 \cos\theta\, dr\, d\theta$$

(2) [10 points] Let R be the region cut from the first octant by the surfaces $z = 1$ and $y = 1 - x$. Sketch the region R, and then evaluate the integral

$$\iiint_R 4xyz\,dV$$

The region is drawn below.



We will evaluate in the order $dy\,dz\,dx$, though any other order would work equally well.

$$\iiint_R 4xyz\,dV = \int_0^1 \int_0^1 \int_0^{1-x} 4xyz\,dy\,dz\,dx = \int_0^1 \int_0^1 2x(1-x)^2 z\,dz\,dx$$
$$= \int_0^1 x(1-x)^2\,dx = \boxed{\frac{1}{12}}$$

The last integral can be evaluated by expanding out the integrand first, or by making the u-substitution $u = 1 - x$, or even using integration by parts with $u = x$ and $dv = (1-x)^2\,dx$.