QUIZ 1 Name Solutions

Math 2551 D Steinbart Section ____ January 20, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values. That is, if the answer is $\sqrt{2}$ then write "$\sqrt{2}$" and not "1.7".

There is a total of 10 points.

1. (5 points) Consider the planes $x - 2y + 2z = -9$ and $6x + 2y - 3z = 19$.

   a. How do we know that these planes intersect?

   b. Find parametric equations for the line in which the planes intersect.

② The planes will intersect if the planes are not parallel. $\underline{n}_1 = \langle 1, -2, 2\rangle$ is orthogonal (normal) to the first plane. $\underline{n}_2 = \langle 6, 2, -3\rangle$ is orthogonal to the second plane. The planes are parallel when $\underline{n}_1$ and $\underline{n}_2$ are parallel. $\underline{n}_1$ and $\underline{n}_2$ are not parallel since $\underline{n}_2$ is not a scalar multiple of $\underline{n}_1$. [That is $\underline{n}_2 \neq c\underline{n}_1$ for any scalar c.] So the planes intersect, and the intersection is a line.

① $\underline{n}_1 \times \underline{n}_2$ is orthogonal to both $\underline{n}_1$ and $\underline{n}_2$ and so $\underline{n}_1 \times \underline{n}_2$ is parallel to the line of intersection. $\underline{n}_1 \times \underline{n}_2 = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -2 & 2 \\ 6 & 2 & -3 \end{vmatrix} = \underline{i}(6-4) - \underline{j}(-3-12) + \underline{k}(2-(-12)) = 2\underline{i} + 15\underline{j} + 14\underline{k}$ ← This vector is parallel to the line.

Now find a point on the line of intersection.

$x - 2y + 2z = -9$

$6x + 2y - 3z = 19$

$7x + 0y - z = 10$

We can take $x = 0$

Then $7(0) + 0 - z = 10$

$z = -10$

Then using first plane:

$x - 2y + 2z = -9$ with $x = 0$, $z = -10$

$0 - 2y + 2(-10) = -9$

$-2y = 11$

$y = -11/2$

So $(0, -11/2, -10)$ is a point on the line of intersection.

Then the line is

$\underline{r}(t) = \langle 0, -11/2, -10\rangle + t\langle 2, 15, 14\rangle$, $t \in \mathbb{R}$

$x = 2t$

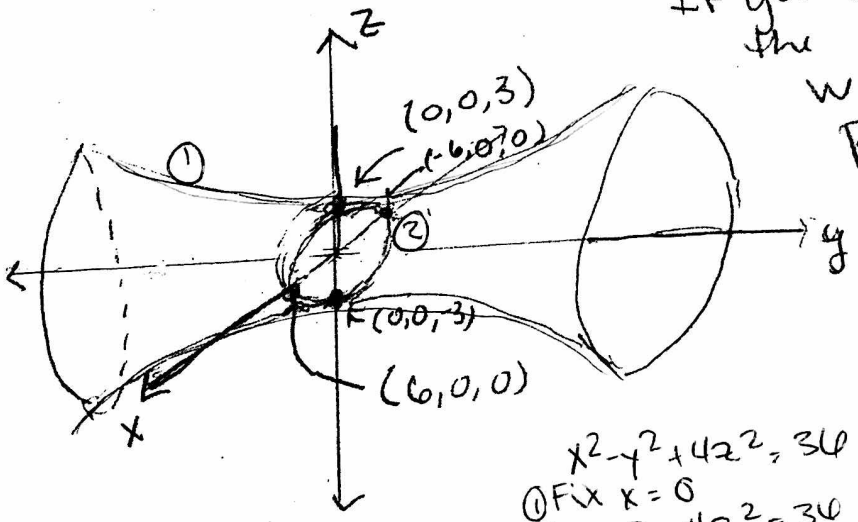
$y = -11/2 + 15t$, $t \in \mathbb{R}$

$z = -10 + 14t$

2. (5 points) Identify and sketch the quadric surface $x^2 - y^2 + 4z^2 = 36$.

$x^2 - y^2 + 4z^2 = 36$ is a hyperboloid of one sheet. Its axis is parallel to the y axis.

$x^2 + 4z^2 = y^2 + 36$



If you slice the surface with the plane $y = c$ we will see

$x^2 + 4y^2 = c^2 + 36$, an ellipse

$x^2 - y^2 + 4z^2 = 36$

① Fix $x = 0$

$-y^2 + 4z^2 = 36$

hyperbola: $z = \pm\frac{1}{2}\sqrt{y^2 + 36}$

$y = 0$, $z = \pm 3$

$y = \pm 3$, $z = \frac{1}{2}\sqrt{45}$

③ Fix $xy = 0$

$x^2 + 4z^2 = 36$

ellipse

③ Fix $z = 0$ hyperbola

$x^2 - y^2 = 36$