MIDTERM I

ISyE 2028 A
Instructor Yao Xie
Monday, Sept 23th (8:05-9:25pm) 2013.
100 points total.

Name _____

1. **Home run record** (30 points)

   The current major league single-season home run record is held by Mark McGwire of the St. Louise Cardinals. Here are McGwire's home run count for 1987 to 2000:

   ```
   49 32 33 39 22 42 9 9 39 52 58 70 65 32
   ```

   (a) [**10pts**] Find the 5-numerical summary for the number of McGwire's home-runs.

   (b) [**10pts**] McGwire was injured in 1993 and there was a baseball strike in 1994. These two observations are in practice outliers (there was a special reason why these observations are so low). We generally identify an outlier if it is smaller that $Q_1 - 1.5 \times \text{IQR}$. Are the two values outliers?

   (c) [**10pts**] Using (a) and (b) create a box plot for the number of McGwire's home-runs.

2. **Performance of estimator** (40 points)

   Consider the probability density function

   $$f(x|\theta) = c(1 + \theta x), \quad -1 \leq x \leq 1.$$

   Assume that we observe $X_1, \ldots, X_n$ random variables with the density function specified above. Define the sample mean $\bar{X} = (X_1 + \ldots + X_n)/n$.

   (a) [**10pts**] Find the value of c.

   Hint: You will have to use one of the properties of the probability density function.

   (b) [**10pts**] Show that $\widehat{\theta} = 3\bar{X}$ is an unbiased estimator for θ.

   Hint: You will have to first compute the expectation of $\bar{X}$.

   (c) [**10pts**] Find the variance of $\widehat{\theta} = 3\bar{X}$.

   (d) [**5pts**] Based on the above calculation, is $\widehat{\theta}$ a consistent estimator?

   (e) [**5pts**] Find the mean square error (MSE) of $\widehat{\theta} = 3\bar{X}$.

3. **Finding point estimators** (30 points)

   Consider the probability density function

   $$f(x|\theta) = \theta x^{\theta - 1}, \quad 0 \leq x \leq 1, \quad 0 < \theta < \infty$$

   (a) [**15pts**] Find an estimator $\widehat{\theta}_{\text{MOM}}$ for θ using the method of moments.

   (b) [**15pts**] Find an estimator $\widehat{\theta}_{\text{ML}}$ for θ using maximum likelihood.