10-6 a)

1) The parameter of interest is the difference in mean burning rate, $\mu_1 - \mu_2$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

5) Reject H_0 if $z_0 < -z_{\alpha/2} = -1.96$ or $z_0 > z_{\alpha/2} = 1.96$ for $\alpha = 0.05$

6) $\bar{x}_1 = 18$ $\bar{x}_2 = 24$

$\sigma_1 = 3$ $\sigma_2 = 3$

$n_1 = 20$ $n_2 = 20$

$$z_0 = \frac{(18-24)}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}} = -6.32$$

7) Conclusion: Because $-6.32 < -1.96$ reject the null hypothesis and conclude the mean burning rates differ significantly at $\alpha = 0.05$.

P-value $= 2(1 - \Phi(6.32)) = 2(1-1) = 0$

b) $$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\left(18-24\right) - 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}} \leq \mu_1 - \mu_2 \leq \left(18-24\right) + 1.96\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}$$

$$-7.86 \leq \mu_1 - \mu_2 \leq -4.14$$

We are 95% confident that the mean burning rate for solid fuel propellant 2 exceeds that of propellant 1 by between 4.14 and 7.86 cm/s.

c) $$\beta = \Phi\left(z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) - \Phi\left(-z_{\alpha/2} - \frac{\Delta - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right)$$

$$= \Phi\left(1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right) - \Phi\left(-1.96 - \frac{2.5}{\sqrt{\frac{(3)^2}{20} + \frac{(3)^2}{20}}}\right)$$

$$= \Phi\left(1.96 - 2.64\right) - \Phi\left(-1.96 - 2.64\right) = \Phi\left(-0.68\right) - \Phi\left(-4.6\right) = 0.24825 - 0 = 0.24825$$

d) Assume the sample sizes are to be equal, use $\alpha = 0.05$, β = 1-power=0.1, and $\Delta = 4$

$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{\delta^2} = \frac{\left(1.96 + 1.28\right)^2\left(3^2 + 3^2\right)}{(4)^2} = 12$$

Use $n_1 = n_2 = 12$

10-9

Catalyst 1	Catalyst 2
$\bar{x}_1 = 65.22$	$\bar{x}_2 = 68.42$
$\sigma_1 = 3$	$\sigma_2 = 3$

$n_1 = 10$ $\qquad\qquad\qquad$ $n_2 = 10$

a) 95% confidence interval on $\mu_1 - \mu_2$, the difference in mean active concentration

$$\left(\bar{x}_1 - \bar{x}_2\right) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(65.22 - 68.42) - 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}} \le \mu_1 - \mu_2 \le \left(65.22 - 68.42\right) + 1.96\sqrt{\frac{(3)^2}{10} + \frac{(3)^2}{10}}$$

$$-5.83 \le \mu_1 - \mu_2 \le -0.57$$

We are 95% confident that the mean active concentration of catalyst 2 exceeds that of catalyst 1 by between 0.57 and 5.83 g/l.

P-value:

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(65.22 - 68.42)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}} = -2.38$$

Then P-value = 2(0.008656) = 0.0173

b) Yes, because the 95% confidence interval does not contain the value zero. We conclude that the mean active concentration depends on the choice of catalyst.

c)

$$\beta = \Phi\left(1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right) - \Phi\left(-1.96 - \frac{(5)}{\sqrt{\frac{3^2}{10} + \frac{3^2}{10}}}\right)$$

$$= \Phi(-1.77) - \Phi(-5.69) = 0.038364 - 0$$

$$= 0.038364$$

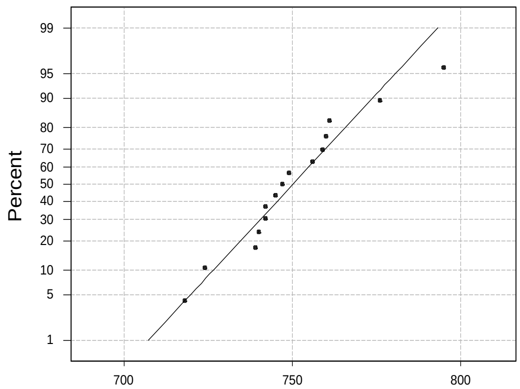
Power = 1 – β = 1– 0.038364 = 0.9616.

d) Calculate the value of n using α and β.

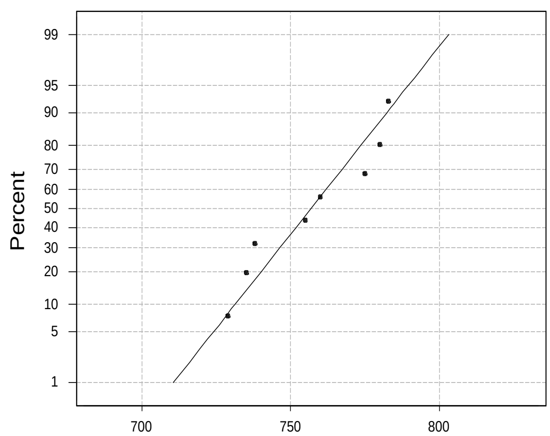
$$n \cong \frac{\left(z_{\alpha/2} + z_\beta\right)^2\left(\sigma_1^2 + \sigma_2^2\right)}{\left(\Delta - \Delta_0\right)^2} = \frac{\left(1.96 + 1.77\right)^2(9+9)}{(5)^2} = 10.02,$$

Therefore, 10 is only slightly too few samples. The sample sizes are adequate to detect the difference of 5.

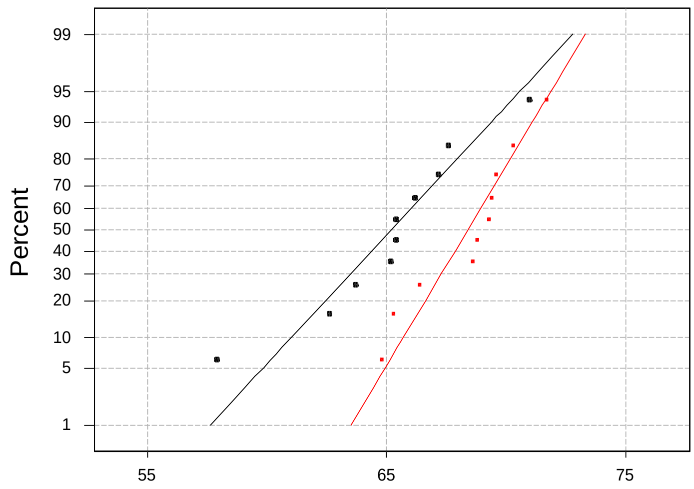
The data from the first sample n = 15 appear to be normally distributed.



The data from the second sample n = 8 appear to be normally distributed



Plots for both samples are shown in the following figure.



10-21 a) 1) The parameter of interest is the difference in mean catalyst yield, $\mu_1 - \mu_2$, with $\Delta_0 = 0$

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

5) Reject the null hypothesis if $t_0 < -t_{\alpha, n_1+n_2-2}$ where $-t_{0.01,25} = -2.485$ for $\alpha = 0.01$

6) $\bar{x}_1 = 86$ $\quad \bar{x}_2 = 89$

$s_1 = 3$ $\quad s_2 = 2$

$n_1 = 12$ $\quad n_2 = 15$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{11(3)^2 + 14(2)^2}{25}} = 2.4899$$

$$t_0 = \frac{(86-89)}{2.4899\sqrt{\frac{1}{12} + \frac{1}{15}}} = -3.11$$

7) Conclusion: Because $-3.11 < -2.485$, reject the null hypothesis and conclude that the mean yield of catalyst 2 exceeds that of catalyst 1 at $\alpha = 0.01$.

b) 99% upper confidence interval $\mu_1 - \mu_2$: $t_{0.01,25} = 2.485$

$$\mu_1 - \mu_2 \le (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p)\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\mu_1 - \mu_2 \le (86-89) + 2.485(2.4899)\sqrt{\frac{1}{12} + \frac{1}{15}}$$

$\mu_1 - \mu_2 \le -0.603$ or equivalently $\mu_1 + 0.603 \le \mu_2$

We are 99% confident that the mean yield of catalyst 2 exceeds that of catalyst 1 by at least 0.603 units.

10-28 a)

1) The parameter of interest is the difference in mean coating thickness, $\mu_1 - \mu_2$, with $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$

3) $H_1: \mu_1 - \mu_2 > 0$

4) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

5) Reject the null hypothesis if $t_0 > t_{0.01,18}$ where $t_{0.01,18} = 2.552$ for $\alpha = 0.01$ since

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)}{n_2 - 1}} 18.37$$

$$\nu \cong 18$$

(truncated)

6) $\bar{x}_1 = 103.5$ $\bar{x}_2 = 99.7$

$s_1 = 10.2$ $s_2 = 20.1$

$n_1 = 11$ $n_2 = 13$

$$t_0 = \frac{(103.5 - 99.7)}{\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}}} = 0.597$$

7) Conclusion: Because 0.597 < 2.552, fail to reject the null hypothesis. There is insufficient evidence to conclude that increasing the temperature reduces the mean coating thickness at $\alpha = 0.01$.

P-value = P(t > 0.597), 0.25 < *P*-value < 0.40

b) If $\alpha = 0.01$, construct a 99% two-sided confidence interval on the difference in means.
Here, $t_{0.005,19} = 2.878$

$$\left(\bar{x}_1 - \bar{x}_2\right) - t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \left(\bar{x}_1 - \bar{x}_2\right) + t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}} \le \mu_1 - \mu_2 \le (103.5 - 99.7) - 2.878\sqrt{\frac{(10.2)^2}{11} + \frac{(20.1)^2}{13}}$$

$$-14.52 \le \mu_1 - \mu_2 \le 22.12$$

Because the interval contains zero, there is not a significant difference in the mean coating thickness between the two temperatures.

10-56 a)

1) The parameter of interest is the mean difference in impurity level, μ_d, where d_i = Test 1 − Test 2.

2) $H_0: \mu_d = 0$

3) $H_1: \mu_d \ne 0$

4) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.005,7}$ or $t_0 > t_{0.005,7}$ where $t_{0.005,7} = 3.499$ for $\alpha = 0.01$

6) $\bar{d} = -0.2125$

$s_d = 0.1727$

$n = 8$

$$t_0 = \frac{-0.2125}{0.1727/\sqrt{8}} = -3.48$$

7) Conclusion: Because –3.499 < –3.48 < 3.499, fail to reject the null hypothesis. There is not sufficient evidence to conclude that the tests generate different mean impurity levels at $\alpha = 0.01$.

b)

1) The parameter of interest is the mean difference in impurity level, μ_d, where d_i = Test 1 – Test 2.

2) $H_0: \mu_d + 0.1 = 0$

3) $H_1: \mu_d + 0.1 < 0$

4) The test statistic is

$$t_0 = \frac{\bar{d} + 0.1}{s_d/\sqrt{n}}$$

5) Reject the null hypothesis if $t_0 < -t_{0.05,7}$ where $t_{0.05,7} = 1.895$ for $\alpha = 0.05$

6) $\bar{d} = -0.2125$

$s_d = 0.1727$

$n = 8$

$$t_0 = \frac{-0.2125 + 0.1}{0.1727/\sqrt{8}} = -1.8424$$

7) Conclusion: Because –1.895 < –1.8424, fail to reject the null hypothesis at the 0.05 level of significance.

c) $\beta = 1 - 0.9 = 0.1$

$$d = \frac{|0.1|}{0.1727} = 0.579$$

n = 8 is not an adequate sample size. From the chart VII*g*, n ≈ 30

10-69

a)

1) The parameters of interest are the time to assemble standard deviations, σ_1, σ_2 where Group 1 = men and Group 2 = women

2) $H_0: \sigma_1^2 = \sigma_2^2$

3) $H_1: \sigma_1^2 \neq \sigma_2^2$

4) The test statistic is

$$f_0 = \frac{s_1^2}{s_2^2}$$

5) Reject the null hypothesis if $f_0 < f_{1-\alpha/2, n_1-1, n_2-1} = 0.365$ or $f_0 > f_{\alpha/2, n_1-1, n_2-1} = 2.86$ for $\alpha = 0.02$

6) $n_1 = 25$ $n_2 = 21$ $s_1 = 0.98$ $s_2 = 1.02$

$$f_0 = \frac{(0.98)^2}{(1.02)^2} = 0.923$$

7) Conclusion: Because 0.365 < 0.923 < 2.86, fail to reject the null hypothesis. There is not sufficient evidence to support the claim that men and women differ in repeatability for this assembly task at the 0.02 level of significance.

ASSUMPTIONS: Assume random samples from two normal distributions.

b) 98% confidence interval:

$$\left(\frac{s_1^2}{s_2^2}\right) f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \left(\frac{s_1^2}{s_2^2}\right) f_{\alpha/2,n_2-1,n_1-1}$$

$$f_{1-\alpha/2,n_2-1,n_1-1} = \frac{1}{f_{\alpha/2,n_1-1,n_2-1}} = \frac{1}{f_{0.01,24,20}} = \frac{1}{2.86} = 0.350$$

$$(0.923)0.350 \le \frac{\sigma_1^2}{\sigma_2^2} \le (0.923)2.73$$

$$0.323 \le \frac{\sigma_1^2}{\sigma_2^2} \le 2.527$$

Because the value one is contained within this interval, there is no significant difference between the variance of the repeatability of men and women for the assembly task at a 2% significance level.

10-87 a)

1) The parameters of interest are the proportion of satisfactory lenses, p_1 and p_2

2) $H_0: p_1 = p_2$

3) $H_1: p_1 \neq p_2$

4) Test statistic is

$$z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \quad \text{where} \quad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

5) Reject the null hypothesis if $z_0 < -z_{0.005}$ or $z_0 > z_{0.005}$ where $z_{0.005} = 2.58$ for $\alpha = 0.01$

6) $n_1 = 300$ $\quad n_2 = 300$

$x_1 = 253$ $\quad x_2 = 196$

$\hat{p}_1 = 0.843$ $\quad \hat{p}_2 = 0.653$ $\quad \hat{p} = \frac{253+196}{300+300} = 0.748$

$$z_0 = \frac{0.843 - 0.653}{\sqrt{0.748(1-0.748)\left(\frac{1}{300} + \frac{1}{300}\right)}} = 5.36$$

7) Conclusion: Because 5.36 > 2.58, reject the null hypothesis and conclude that there is a difference in the fraction of polishing-induced defects produced by the two polishing solutions at the 0.01 level of significance.

P-value = $2[1 - P(Z < 5.36)] \approx 0$

b) By constructing a 99% confidence interval on the difference in proportions, the same question can be answered by whether or not zero is contained in the interval.