**GEORGIA INSTITUTE OF TECHNOLOGY**

**COLLEGE OF ENGINEERING**

**BMED3300 - BIOTRANSPORT**

**SECOND TERM TEST FALL 2013 - ETHIER**

STUDENT NAME: SOLUTION

GTID NUMBER: ______________________

RECITATION SECTION: ______________________

(Section A is 10-11 am Monday; Section B is 3-4 pm on Monday)

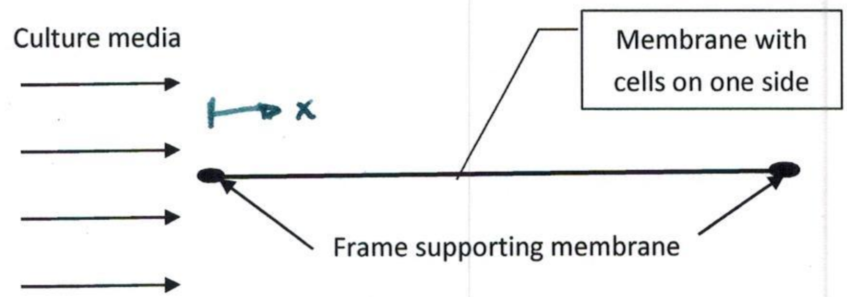
*Open book*
*All non-communicating calculator types allowed*
*Time allotted: 80 minutes*
*Do all work in this booklet*

**Reminder: for questions requiring numerical answers, units are required and worth 50%**

| Question | Maximum Mark | Actual Mark |
|---|---|---|
| 1 | 20 | |
| 2 | 35 | |
| 3 | 45 | |
| **Total** | **100** | |

5 min
8 min
10 min

1. Endothelial cells are being cultured on a very thin flexible membrane that is 4 cm long and 4 cm wide. The membrane is supported on its edges by a thin frame, and is placed in flowing culture media. The velocity of the media is adjusted so that cells 1 cm from the leading edge of the membrane experience a shear stress of 24 dynes/cm$^2$. There is a concern that the membrane may tear, which will occur if the total frictional drag force on the membrane (both sides) due to fluid flow exceeds 750 dynes. Assuming laminar flow, and neglecting any effects of the frame itself on the flow, determine whether the membrane will tear.




Do your GIM analysis here

This is a laminar b-layer problem.

We know $\tau = \tau_o$ @ $x = x_o = 1$ cm.

Use this to compute $F_D$ → use $C_{f,L}$

$\tau = \frac{1}{2}\rho V^2 C_f \qquad C_{f_x} = 0.664\, Re_x^{-1/2}$

At $x = x_o$ $\qquad \tau_o = \frac{1}{2}\rho V^2 \left(0.664\, Re_{x_o}^{-1/2}\right) \qquad$ —①

$\tau = \tau_o$

2 sides $\qquad$ w = width

$F_D = wL\bar{\tau}\cdot 2 \qquad$ L = length

$= wL\cdot\frac{1}{2}\rho V^2 (2)(0.664)\, Re_L^{-1/2}\cdot 2 \qquad$ —②

Combine ① & ②

$$\frac{F_D}{\tau_o} = \frac{wL\, 2\, Re_L^{-1/2}\cdot 2}{Re_{x_o}^{-1/2}} = 4wL\left(\frac{Re_L}{Re_{x_o}}\right)^{-1/2}$$

$$F_D = 4wL\tau_o\left(\frac{x_o}{L}\right)^{1/2}$$

$$= 4w\,(Lx_o)^{1/2}\,\tau_o$$

Blug in numbers

$$F_D = 4(4)(4\cdot 1)^{1/2}\,24 \quad \frac{\text{dyne}}{\text{cm}^2} \quad \text{cm}^2$$

$$= 8(96)\ \text{dyne}$$

$$= 768\ \text{dyne}$$

$\therefore$ membrane will tear.

2. A porous polymeric sphere of diameter 500 μm is to be loaded with a drug. Initially the sphere has no drug in it. It is loaded by suspending it in a very large bath, in which the drug is present at a uniform concentration of 2 mg/ml. Assuming a diffusivity of the drug in the sphere of $D_s = 1 \times 10^{-10}\ cm^2/sec$ and a diffusivity of the drug in the bath of $D_b = 1 \times 10^{-6}\ cm^2/sec$, estimate when the average concentration of drug in the sphere will have reached 90% of the value in the bath. You may treat the transport of drug in the bath as slow (quasi-steady).

Do your GIM analysis here

This is unsteady diffusion, sphere $R_s = 250\ \mu m$
Need to compute Bi, which requires $k_f$.
Solve steady diffusion problem outside sphere to get $k_f$.

Outside sphere: $\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{0 \text{ steady}} + \underbrace{\cancel{\underline{v} \cdot \nabla \rho}}_{\text{no convection}} = \mathcal{D} \nabla^2 \rho + \underbrace{\cancel{r}}_{\text{no rxn.}}$

$\therefore \nabla^2 \rho = 0$ use spherical co-ords.

$$\frac{1}{r^2} \frac{d}{dr}\left(r^2 \frac{d\rho}{dr}\right) = 0.$$

$$r^2 \frac{d\rho}{dr} = k_1 \Rightarrow \frac{d\rho}{dr} = \frac{k_1}{r^2} \Rightarrow \rho(r) = k_2 - k_1/r$$

at $r = \infty$, $\rho = \rho_\infty$ $\qquad \therefore k_2 = \rho_\infty$

at $r = R_s$, $\rho = \rho_s$ $\qquad \therefore k_1 = -(\rho_s - \rho_\infty) R_s$

$$\therefore \rho(r) = \rho_\infty - (\rho_\infty - \rho_s)\frac{R_s}{r}$$

Compute flux at surface from Fick's law

$$n_s = \dot{j}_s = -\mathcal{D}_b \frac{d\rho}{dr}\Big|_{r=R_s} = -\frac{\mathcal{D}_b(\rho_\infty - \rho_s)R_s}{r^2}\Big|_{r=R_s}$$

$$= \frac{-\mathcal{D}_b(\rho_\infty - \rho_s)}{R_s}$$

Then $k_f = \dfrac{n_s}{\rho_s - \rho_\infty} = \dfrac{\mathcal{D}_b}{R_s}$

$$Bi = \frac{k_f R_s}{\mathcal{D}_s} = \frac{\mathcal{D}_b}{\mathcal{D}_s} = 10^4$$

$$Y \text{ (or } \theta) = \frac{\rho - \rho_\infty}{\rho_0 - \rho_\infty} = 0.1$$

$\rho_0 = 0$

$\rho_\infty = 2$ mg/ml

$\rho = 0.9\,\rho_\infty$

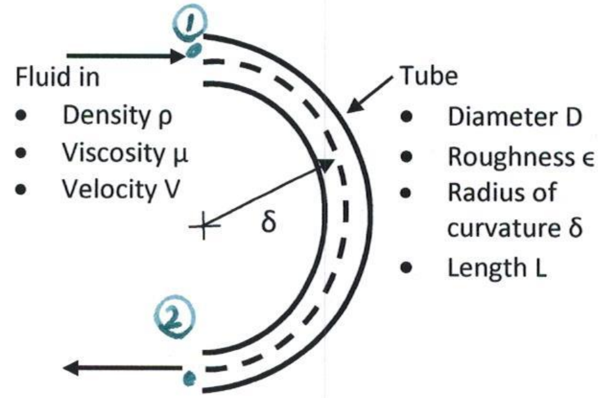
Use chart. $\Rightarrow F_o \doteq 0.18 = \dfrac{\mathcal{D}_s t}{R_s^2}$

$$\therefore t = \frac{(0.18)(R_s^2)}{\mathcal{D}_s} = \frac{(0.18)(250\times10^{-4})^2 \text{ cm}^2}{10^{-10} \text{ cm}^2/s}$$

$$= 1.125\times10^6 \text{ sec}.$$

3. In class, we considered the pressure drop due to flow in straight tubes. However, most "tubes" in the body (blood vessels and airways) are curved. Here we consider fully-developed flow in a curved tube, where the curvature is characterized by the tube's radius of curvature, $\delta$. (See the diagram for other relevant parameters.)

δ




a. We are interested in understanding the pressure drop due to air flow in a curved airway segment. It is decided to build a 5x scale model using a working fluid with density $\rho_{model} = 0.1\ g/cm^3$ and $\nu_{model} = 0.08\ cm^2/s$. Show that results from the 5x scale model will be valid if the working fluid is infused at a velocity which is 10 times smaller than that present in the real airway. The density and kinematic viscosity of air are $\rho_{air} = 1.2 \times 10^{-3} g/cm^3$ and $\nu_{air} = 0.16\ cm^2/s$. Hint: what are the relevant pi-groups? This is like flow in a tube but with curvature. **[25 marks]**

b. Using the parameters from part a, a pressure drop of 80 dynes/cm$^2$ is measured in the scale model (from inlet to outlet) for an inlet working fluid velocity of $V_{model} = 450\ cm/s$. What pressure drop does this correspond to in an actual airway? You can neglect effects of gravity. **[20 marks]**

Do your GIM analysis here

This is $\pi$-groups / similitude. The parameters are the same as in tube flow, plus one extra one, $\delta$. We need to match n-1 of the $\pi$-groups between the model & the real thing (airway).

(a) Flow in a straight tube: $\pi$-groups are $f$, $Re_D$ and $\epsilon/D$

Flow in a curved tube: all the same parameters, plus $\delta$. By inspection, $\pi$-groups are $f$, $Re_D$, $\epsilon/D$ and $\delta/D$.

Need to match 3 of these, then the 4th will match automatically. $\epsilon/D$ and $\delta/D$ are geometry-related & will match if the model is built correctly.

$\therefore$ Need $Re_{D\,air} = Re_{D\,model}$

$$\frac{V_{air} D_{air}}{\nu_{air}} = \frac{V_{model} D_{model}}{\nu_{model}}$$

$$V_{model} = V_{air}\left(\frac{D_{air}}{D_{mod}}\right)\left(\frac{\nu_{mod}}{\nu_{air}}\right) = V_{air}\left(\frac{1}{5}\right)\left(\frac{0.08}{0.16}\right) = \frac{V_{air}}{10}$$

QED.

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(b) Since $Re_D$, $\epsilon/D$ & $\delta/D$ match, so too does $f$. Use ME eq'n from ① → ②. Note $V_1 = V_2$. Neglect $h_1 - h_2$. Then

$$p_1 - p_2 = \Delta p = \rho g h_f = \rho g f \frac{L}{D}\frac{V^2}{2g} = \frac{\rho V^2}{2} f \frac{L}{D}$$

$$\therefore f = \frac{2D\Delta p}{\rho V^2 L}$$

$f_{air} = f_{mod} \Rightarrow$ $$\frac{\Delta p_{air}}{\rho_{air} V_{air}^2} = \frac{\Delta p_{mod}}{\rho_{mod} V_{mod}^2}$$

$$\Delta p_{air} = \Delta p_{mod}\left(\frac{\rho_{air}}{\rho_{mod}}\right)\left(\frac{V_{air}}{V_{mod}}\right)^2 = \left(\frac{1.2\times10^{-3}}{0.1}\right)(10)^2\, 80\ \frac{dyne}{cm^2}$$

$$= 96\ dyne/cm^2.$$