

ISyE 2027 Exam # 1  
Fall 2014

Name KEY

Please be neat and show all your work so that I can give you partial credit.  
GOOD LUCK.

Question 1  
Question 2  
Question 3  
Question 4  
Total

(25) 1. Nine children are seated at random in three rows of three desks. Let  $A$ =Al and Bobby sit in the same row and  $B$ =Al and Bobby both sit at one of the corner desks. Are  $A$  and  $B$  independent? Justify your answer.

$$P(B) = \frac{\binom{4}{2} 2! 7!}{9!} = \frac{12 \times 7!}{7! \times 8 \times 9} = \frac{12}{72} = \frac{1}{6}$$

$$P(A) = \frac{\binom{3}{1} \binom{3}{2} 2! 7!}{9!} = \frac{18 \times 7!}{7! \times 8 \times 9} = \frac{18}{72} = \frac{1}{4}$$

$$P(A \cap B) = \frac{2 \times 2 \times 7!}{9!} = \frac{4 \times 7!}{7! \times 8 \times 9} = \frac{4}{72} = \frac{1}{18}$$

Since  $P(A \cap B) \neq P(A)P(B)$ ,  $A$  and  $B$  are not independent.

(25) 2. (a) (15) Suppose  $X$  has the probability density function

$$f(x) = 4x^3 \quad 0 < x < 1$$

$$f(x) = 0 \text{ otherwise}$$

Compute the corresponding cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ x^4 & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

(b) (10) Compute  $P\{X > \frac{1}{3}\}$

$$\begin{aligned} P\left\{X > \frac{1}{3}\right\} &= 1 - P\left\{X \leq \frac{1}{3}\right\} = 1 - \left(\frac{1}{3}\right)^4 = 1 - \frac{1}{81} \\ &= \frac{80}{81} \end{aligned}$$

(25) 3. A company gives a test to 100 salesmen, 80 with good sales records and 20 with poor records. 60% of the good salesmen pass the test, but only 30% of the poor salesmen do. A new applicant takes the test and passes. What is the probability that he is a good salesman?

G: good salesman

P: poor salesman

T: pass the test

$$\begin{aligned}P(G|T) &= \frac{P(T|G)P(G)}{P(T)} = \frac{P(T|G)P(G)}{P(T|G)P(G) + P(T|P)P(P)} \\&= \frac{0.6 \times 0.8}{0.6 \times 0.8 + 0.3 \times 0.2} \\&= \frac{0.48}{0.54} \\&= \frac{8}{9}\end{aligned}$$

(25) 4. Two events have  $P(A) = 1/4$ ,  $P(B|A) = 1/2$ , and  $P(A|B) = 1/3$ . Compute

(a) (10)  $P(A \cap B)$

$$P(A \cap B) = P(B|A) P(A) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

(b) (10)  $P(B)$

$$\frac{1}{3} = P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{8}}{P(B)} \Rightarrow P(B) = \frac{3}{8}$$

(c) (5)  $P(A \cup B)$ .

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= \frac{1}{4} + \frac{3}{8} - \frac{1}{8} \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$