MATH 2401 QUIZ #2 SECTION K

Name:

4 points total
1 point for evaluating each component
1 point for correct answer

(1) Evaluate the integral

$$\int_1^{\ln 3} [te^t\mathbf{i} + e^t\mathbf{j} + \ln t\mathbf{k}]dt.$$

$$\int_1^{\ln 3} [te^t i + e^t j + \ln t\, k]\, dt$$

$$= \int_1^{\ln 3} (te^t + e^t - e^t)\,dt\, i + e^t\Big|_1^{\ln 3} j + \int_1^{\ln 3} \ln t\, dt\, k$$

$$= (te^t - e^t)\Big|_1^{\ln 3} i + (3-e)\, j + \left(t\ln t\Big|_1^{\ln 3} - \int_1^{\ln 3} dt\right) k$$

$$= (3\ln 3 - 3)\, i + (3-e)\, j + [\ln 3 \ln(\ln 3) - \ln 3 + 1]\, k$$

$$= 3(\ln 3 - 1)\, i + (3-e)\, j + [\ln 3(\ln(\ln 3) - 1) + 1]\, k$$

(2) The motion of a particle is described by $\mathbf{r}(t) = \cos t^2\mathbf{i} + \sin t^2\mathbf{j}, t \geq 0$. Find the acceleration vector of the particle at time t.

3 points total
1 point for computing v(t)
1 point for computing a(t)
1 point for complete correctness

$$v(t) = \dot{r}(t) = -2t\sin t^2\, i + 2t\cos t^2\, j$$

$$a(t) = \dot{v}(t) = [-2\sin t^2 - 4t^2\cos t^2]\, i + [2\cos t^2 - 4t^2\sin t^2]\, j$$

(3) A particle is located at the $(0, 0, 0)$ and has 0 speed at time $t = 0$. If the acceleration of the particle is given by $\mathbf{a}(t) = (2e^{t^2} + 4t^2e^{t^2})\mathbf{i} + e^{-t}\mathbf{j} + \mathbf{k}$, find the position vector for the particle at each time t.

3 points
1 point for v
1 point for r
1 point for correctness

$$v(s) = \int_0^s a(t)\,dt = \int_0^s \left[(2e^{t^2} + 4t^2e^{t^2})i + e^{-t}j + k\right] dt$$

$$= 2te^{t^2}\Big|_0^s i - e^{-t}\Big|_0^s j + sk$$

$$= 2se^{s^2} i + (1 - e^{-s}) j + sk$$

$$r(t) = \int_0^t v(s)\,ds = \int_0^t \left[2se^{s^2} i + (1 - e^{-s}) j + sk\right] ds$$

$$= e^{s^2}\Big|_0^t i + \left[t + e^{-s}\Big|_0^t\right] j + \frac{t^2}{2} k$$

$$= e^{t^2} i + (t + e^{-t} - 1) j + \frac{t^2}{2} k$$