# ISyE 3044 — Practice Problems for Exam #1

1. Consider the integral $\mu = \int_0^{1/2} \sin(\pi t)\, dt$.

   (a) Use the following 10 pseudo-random numbers to compute an estimate of $\mu$.

   0.60 0.73 0.35 0.08 0.99 0.47 0.22 0.16 0.54 0.87

   (b) Compute an approximate 90% confidence interval for $\mu$.

2. The random variable $X$ has density function

$$f(x) = |x| = \begin{cases} -x & \text{if } -1 \le x < 0 \\ x & \text{if } 0 \le x \le 1. \end{cases}$$

   (a) Find the mean $E(X)$.

   (b) Find the c.d.f. $F(x)$.

3. Short questions.

   (a) Suppose that $U$ is a Uniform$(0,1)$ random variable. What is the distribution of $X = -\frac{1}{5}\ln U$? Find the mean of $X$.

   (b) The discrete random variable $X$ has the following probability function:

| $k$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = k)$ | 0.30 | 0.35 | 0.20 | 0.10 | 0.05 |

   Use the uniform random number $U = 0.79$ to generate an observation for $X$.

4. Customers arrive at a post office branch according to a Poisson process with a rate of 2 per minute.

   (a) What is the expected number of arrivals between 10:30 and 10:40 a.m.?

   (b) Assume that the post office opens at 9 a.m. with no customers present. What is the probability that the third customer will arrive after 9:02 a.m.?

   (c) Suppose that no customer has arrived between 10 and 10:05 a.m. What is the probability that the next customer will arrive within the next minute?

5. A job shop operates continuously. The interarrival times for jobs are i.i.d. from the following distribution:

| Interarrival Time (Hours) | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Probability | .25 | .35 | .28 | .12 |

The processing times are i.i.d. normally distributed with mean 50 minutes and standard deviation 8 minutes.

Construct a simulation table, and perform a simulation for 10 customers. Assume that when the simulation begins there is one job being processed (scheduled to be completed in 25 minutes) and there is one job in queue with a 50-minute processing time.

Use the following uniform(0, 1) observations to generate the interarrival times

$$.53 \quad .37 \quad .08 \quad .62 \quad .53 \quad .56 \quad .14 \quad .67 \quad .82 \quad .68$$

and the following $N(0,1)$ numbers to generate the respective processing times

$$0.23 \quad -0.17 \quad 0.43 \quad 0.02 \quad 2.13 \quad -0.04 \quad -0.18 \quad 0.42 \quad 0.24 \quad -1.17$$

Round each service time to the nearest integer. In both cases, read from left to right.

(a) What is the average time in queue for the 10 new jobs?

(b) What is the (average) utilization of the machine?

6. Short questions.

   (a) The discrete random variable $X$ has probability (mass) function $\Pr(X = -1) = 0.45$, and $\Pr(X = 2) = 0.55$. Give the Simio expression that generates realizations of $X$.

   (b) Which distribution does the Simio expression Random.Exponential(2) generate data from? Give its parameter(s).

   (c) Suppose $X$ is geometric(0.8). Find $P(X > 5 \mid X > 3)$.

   (d) Suppose $X_1, X_2, \dots, X_8$ are i.i.d. uniform$(0, 2)$. Use the central limit theorem to approximate $P(5 < \sum_{i=1}^{8} X_i < 11)$.

   (e) Suppose $X \sim$ binomial$(3, 0.75)$ and $Y \sim$ binomial$(2, 0.75)$ are independent. Find $P(X + Y \geq 3)$.

7. Consider a single-server queuing system with FIFO service discipline and let $X(t)$ be the number of customers in the *system* (including the customer in service). A simulation run for 180 minutes produced the following sample path (output)

| $t$-interval | $[0, 40)$ | $[40, 72)$ | $[72, 83)$ | $[83, 96)$ | $[96, 129)$ | $[129, 136)$ |
|---|---|---|---|---|---|---|
| $\hat{X}(t)$ | 0 | 1 | 2 | 1 | 2 | 1 |
| $t$-interval | $[136, 148)$ | $[148, 165)$ | $[165, 175)$ | $[175, 180)$ | $[180, \infty)$ | |
| $\hat{X}(t)$ | 2 | 3 | 2 | 1 | 0 | |

   (a) Compute an estimate of the mean number of customers in the system during $[0, 180]$.

(b) Compute an estimate of the mean server utilization during [0, 180].

(c) Compute an estimate of the mean customer delay in queue during [0, 180].

8. Parts at a machine (with an infinite-capacity buffer) according to a Poisson process at the rate of 2 parts per minute. They are processed one-at-a-time by the machine. The processing times are i.i.d. uniform between 20 and 30 seconds.

   (a) Give the notation for this queueing system.

   (b) Find the long-run fraction of time the machine is starving for parts.

   (c) Find the long-run mean waiting time in queue per part.

   (d) Identify a service time distribution with mean 25 seconds that will minimize the mean waiting time in part (c).

9. Short questions. For multiple-choice questions circle your answer.

   (a) In order to conduct independent replications in Simio, we must define an experiment and hit the Run (Fast Forward) button. **True False**

   (b) During the execution of a Simio experiment, we can view the animated model. **True False**

   (c) Simio always reports confidence intervals for quantiles of Responses, regardless of the number of replications in an experiment. **True False**

   (d) A Simio model traces a *user-defined* Model State (variable) over a period of time. An Experiment uses several independent replications. Statistical averages and confidence intervals for the expected value of the Model State are computed using

   **A StateStatistic and an Output Statistic**
   **By default (like the time an entity spends in the system)**

## Solutions

1. Using the transformation $t = u/2$, we write $\mu = \int_0^1 (1/2)\sin(\pi u/2)\,du$. The Monte Carlo estimate is
$$\hat{\mu} = \frac{1}{10}\sum_{i=1}^{10} X_i = 0.317; \quad X_i = (1/2)\sin(\pi U_i/2).$$
Using the variance estimate
$$S^2 = \frac{1}{9}\sum_{i=1}^{10}(X_i - \bar{X})^2 = 0.157^2$$
and the 95th percentile $t_{9,0.95} = 1.83$ of Student's $t$ distribution with 9 d.f., we get the confidence interval
$$0.317 \pm 1.83\frac{0.157}{\sqrt{10}} = (0.227, 0.408).$$

2. (a) $E(X) = 0$ by symmetry.
(b) We have
$$F(x) = \begin{cases} \int_{-1}^{x}(-t)\,dt = 1/2 - x^2/2 & \text{if } -1 \le x < 0 \\ F(0) + \int_0^x t\,dt = 1/2 + x^2/2 & \text{if } 0 \le x \le 1. \end{cases}$$

3. (a) Exponential with mean 1/5.
(b) $X = \min\{k : P(X \le k) \ge 0.79\} = 3$.

4. (a) $E[N(10)] = 2 \times 10 = 20$ customers.
(b) We want $P[N(2) < 3] = P[N(2) \le 2] = e^{-4}(1 + 4 + 4^2/2!) = 13e^{-4} = 0.238$.
(c) Let $X$ be an interarrival time. By the memoryless property, we need $P(X \le 1) = 1 - e^{-2} = 0.865$.

5. The arrival, delay, processing, and departure times of the 10 new jobs are:

| Job | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Arrival Time | 60 | 120 | 120 | 240 | 300 | 360 | 360 | 480 | 600 | 720 |
| Delay Time | 15 | 7 | 56 | 0 | 0 | 7 | 57 | 0 | 0 | 0 |
| Processing Time | 52 | 49 | 53 | 50 | 67 | 50 | 49 | 53 | 52 | 41 |
| Departure Time | 127 | 176 | 229 | 290 | 367 | 417 | 466 | 533 | 652 | 761 |

This is a short table. Learn how to generate the detailed table discussed in class.

(a) The mean delay is total delay$/10 = 14.2$.
(b) The answer is $591/761 = 0.776$.

6. (a) Random.Discrete(–1,0.45,2,1).
(b) Exponential with $\lambda = 1/2$.
(c) The geometric distribution is memoryless. Then $P(X > 5 \mid X > 3) = P(X > 2) = 0.2^2 = 0.04$.
(d) $E(X_i) = 1$, $\text{Var}(X_i) = 2^2/12 = 1/3$. Then

$$P\left(5 < \sum_{i=1}^{8} X_i < 11\right) \approx P\left(\frac{5-8}{\sqrt{8/3}} < Z < \frac{11-8}{\sqrt{8/3}}\right) = P(-1.83 < Z < 1.83) = 2\Phi(1.83){-}1 = 0.932.$$

(e) $X + Y \sim \text{binomial}(5, 0.75)$ and then

$$P(X + Y \geq 3) = \sum_{k=3}^{5} \binom{5}{k} (0.75)^k (0.25)^{5-k} = 0.896.$$

7. (a) The estimate for the mean number of customers in system is

$$\frac{1}{180} \int_0^{180} \hat{X}(t)\, dt = \frac{240}{180} = 1\frac{1}{3}.$$

(b) The estimate for the server utilization is

$$\frac{\text{time in } [0, 180] \text{ when } \hat{X}(t) \geq 1}{180} = \frac{140}{180} = 0.778.$$

(c) Five customers completed service during [0, 180]. The delays of these customers are

| customer $i$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| delay $D_i$ | 0 | 11 | 33 | 29 | 27 |

The estimate for the mean customer delay is

$$\frac{1}{5} \sum_{i=1}^{5} D_i = \frac{100}{5} = 20.$$

8. (a) We have an M/G/1 queue with $\lambda = 1/30$ per second, $\mu = 1/25$ per second, $\rho = 25/30 = 5/6$, and service-time variance $\sigma^2 = (30 - 20)^2/12 = 8.33$.
(b) $1 - \rho = 1/6$.
(c)

$$W_Q = \frac{(1/30)(25^2 + 8.33)}{2(1 - 5/6)} = 63.3 \text{ seconds.}$$

(d) Constant service time equal to 25 seconds.

9. (a) True.
(b) False.
(c) False.
(d) StateStatistic(s) and OutputStatistic(s).