## ISyE 3232 Final Spring 2013

## Name

Please be neat and show all your work so that I can give you partial credit. HAVE A WONDERFUL SUMMER.

- Question 1
- Question 2
- Question 3
- Question 4
- Question 5

Total

(20) 1. There are two servers Robin and Pat. Service times are exponentially distributed with mean $1/20$ hours for Robin and $1/10$ hours for Pat. The two service times are independent. Both start servicing customers at time 0.
(a) (5) What is the probability that Robin finishes before Pat?
(b) (5) What is the probability that the first customer to finish, finishes before $t$ hours have elapsed?
(c) (10) What is the probability that both customers complete service within $t$ hours?

(20) **2.** We have an electronics store that sells a video game system, selling 0, 1, 2, or 3 of these units each day with probabilities 0.3, 0.4, 0.2, and 0.1, respectively. Each night at the close of business new units can be ordered which will be available when the store opens in the morning. Suppose that sales produce a profit of \$12 but it costs \$2 a day to keep unsold units in the store overnight. Assume we use the (1,3) inventory policy which means that if we have less than or equal to 1 unit we order up to 3 units otherwise we do not order any new units. What is the long run expected daily profit under this policy?

(20) 3. Consider a call center in which calls arrive according to a Poisson pro-
cess with rate $\lambda = 30$ per hour. Assume that there are two operators handling
three phone lines. However, the second operator only works when all three lines
are busy. Assume that the service times for either operator are independent
identically distributed exponential random variables with a mean of 4 minutes.
Also, assume that the arrival process and the service process are independent
(a) (10) What is the long run probability that both operators are busy?

(c) (5) What is the expected amount of time that a customer spends in this call center in the long run?

(20) 4. Customers arrive at the Smartshop convenience store with respect to a Poisson process of rate 20 per hour. When 2 or fewer customers are present in the check out line, a single clerk works and the service time is 3 minutes. However, when there are three or more customers are present, an assistant comes over to bag up the groceries and reduces the service time to 2 minutes. Assume that the store has a large enough capacity that it can accept all incoming customers and service times are exponentially distributed. What is the expected number of customers in the store in the long-run?

(20) 5. Arrivals come to a rock concert with respect to a Poisson process of
rate $\lambda=50$ per hour. Assume that %60 of these arrivals are females and %40
percent are males.
(a) (5) What is the probability that first arrival is a male?

(b) (10) If exactly two customers arrived in the first five minutes, what is the probability that both arrived in the first 3 minutes?

(c) (5) Suppose that regardless of gender customers buy 1 ticket with probability 1/2, 2 tickets with probability 2/5, and 3 tickets with probability 1/10. What is the expected number of tickets sold in one hour?