

Instructions: *Print* your name, student ID number and recitation session in the spaces below.

Name: _____

Student ID: _____

Recitation session: _____

Practice Exam 2, Calculus III (Math 2551)

Question	Points
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2)	
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5)	
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Problem 1(20 points). Calculations.

(a) (5 pt) Find the directional derivative of

$$f(x, y, z) = xy + yz + zx$$

at $P(1, -1, 1)$ in the direction of $\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(b) (5 pt) Find the rate of change of $f(x, y) = xe^y + ye^{-x}$ along the curve $\vec{r}(t) = \ln t \mathbf{i} + t \ln t \mathbf{j}$.

(c)(5 pt) Find $\partial u / \partial s$ for $u = x^2 - xy$, $x = s \cos t$, $y = t \sin s$.

(d)(5 pt) Find dy/dx if $x \cos(xy) + y \cos x = 2$.

Problem 2(20 pt) Consider the function $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$.

- (a) (6 points) Find the equation for the tangent plane to the level surface $f = 4$ at the point $P(1, 4, 1)$.
- (b) (6 points) Find the equation for the normal line to $f = 4$ at $P(1, 4, 1)$.
- (c) (8 points) Use differentials to estimate $f(0.9, 4.1, 1.1)$.

Problem 3 (20 pt) Find the area of the largest rectangle with edges parallel to the coordinate axes that can be inscribed in the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

Problem 4 (20 points) Find the absolute extreme values taken on $f(x, y) = -\frac{2y}{x^2+y^2+1}$ on the set $D = \{(x, y) : x^2 + y^2 \leq 4\}$.

Problem 5 (20 points)

(a) (10 points) Find the area of the region enclosed by the parabolas $x = y^2$ and $x = 2y - y^2$.

(b) (10 points) Change the Cartesian integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x + 2y) \, dy dx$$

into an equivalent polar integral. Then evaluate the polar integral.