MATH 2401 Name: ______________

Quiz 2 (Jan. 22, 2015) GT Id: ______________

This quiz contains 2 questions. Write neatly and show all your work.

1. A straight line passes through $P(3,-2,0)$ and is perpendicular to the plane $3x-4z=8$.

   (a) Find parametric equations for the line. [3]

The line is parallel to $\vec{v}=\langle 3,0,-4\rangle$, a vector normal to the given plane. So the parametric equations are: (1)

$$\begin{cases} x = 3+3t \\ y = -2+0t \\ z = 0+(-4)t \end{cases}$$

i.e. $\underbrace{x=3+3t}_{0.5}$, $\underbrace{y=-2}_{0.5}$, $\underbrace{z=-4t}_{0.5}$, $\underbrace{-\infty<t<\infty}_{0.5}$.

   (b) Find the distance to the line from the point $S(2,-2,1)$. [3]

0.5 $\vec{PS}=\vec{OS}-\vec{OP}=\langle 2,-2,1\rangle-\langle 3,-2,0\rangle=\langle -1,0,1\rangle$.

With $\vec{v}=\langle 3,0,-4\rangle$ as above,

(1) $$\vec{PS}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1 & 0 & 1\\ 3 & 0 & -4\end{vmatrix}=\begin{vmatrix}0 & 1\\ 0 & -4\end{vmatrix}\vec{i}-\begin{vmatrix}-1 & 1\\ 3 & -4\end{vmatrix}\vec{j}+\begin{vmatrix}-1 & 0\\ 3 & 0\end{vmatrix}\vec{k}$$
$$=(0)\vec{i}-(1)\vec{j}+(0)\vec{k}$$
$$=-\vec{j}$$

Now, $|\vec{PS}\times\vec{v}|=1$

$|\vec{v}|=\sqrt{3^2+0^2+(-4)^2}=\sqrt{25}=5$ 0.5

$\therefore$ The distance to the line from $(2,-2,1)$ $=\dfrac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}$

$=\dfrac{1}{5}$. (1)

(c) Find the point where the line intersects the $yz-$ plane. [2]

Equation of the yz-plane is $x = 0$.

Solving the equations of the line & $x=0$, we get

$3+3t = 0$ or, $3t = -3$ i.e. $t = -1$. 1

For $t=-1$, $y = -2$ & $z = -4(-1) = 4$.

$\therefore$ The intersection point is $(0,-2,4)$ 1

2. Identify by type (ellipsoid, elliptical paraboloid etc.) the surfaces defined by each of the following equations.

(a) $x^2 + 4y^2 = z$ elliptical paraboloid. [1]

(b) $x^2 + 4y^2 = z^2$ elliptical cone. [1]