CEE 3040 – Fluid Mechanics – Fall 2014
Georgia Institute of Technology
Instructor: Dr. Hermann M. Fritz

Exam 3
Friday, 11/07/2014
11:05 to 11:55 p.m.

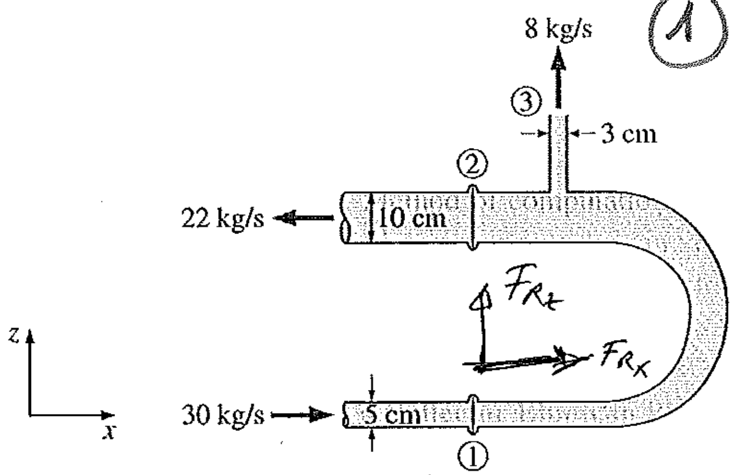
OVERALL: $\boxed{30 \text{ PTS}}$

NAME:____________________

This is a closed book exam. 1 additional sheet of US-Letter paper with personal notes/equations on both sides are allowed. Method of computation is up to you, but keep in mind that I cannot give partial credit if you used an equation solver in your calculator and come up with the wrong answer. Hence provide all necessary solving steps to follow through to the result. Show all work on attached pages and/or add additional pages if necessary.

1. (10 pts) **Force to Hold a Deflector Elbow in Place:**
Water is flowing into and discharging from a pipe U-section as shown in figure below. At flange (1), the total absolute pressure is 200 kPa, and 30 kg/s flows into the pipe. At flange (2), the total pressure is 150 kPa. At location (3), 8 kg/s of water discharges to the atmosphere, which is at 100 kPa. Determine the total x- and z-forces at the two flanges connecting the pipe?
Properties: We take the density of water to be 1000 kg/m^3.

10 TOTAL



1

$$V_1 = \frac{\dot m_1}{\rho A_1} = \frac{\dot m_1}{\rho\left(\frac{\pi}{4}D_1^2\right)} = \frac{30\ kg/s}{\left(1000\ \frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.05m)^2\right)}$$

$$V_1 = 15.3\ \frac{m}{s}$$

$$V_2 = \frac{\dot m_2}{\rho A_2} = \frac{\dot m_2}{\rho\left(\frac{\pi}{4}D_2^2\right)} = \frac{22\ kg/s}{\left(1000\ \frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.1m)^2\right)}$$

$$V_2 = 2.80\ \frac{m}{s}$$

1

$$V_3 = \frac{\dot m_3}{\rho A_3} = \frac{\dot m_3}{\rho\left(\frac{\pi}{4}D_3^2\right)} = \frac{8\ kg/s}{\left(1000\ \frac{kg}{m^3}\right)\left(\frac{\pi}{4}(0.03m)^2\right)} = 11.3\ \frac{m}{s}$$

1

CONSERVATION OF MOMENTUM X-DIR

1

$$\sum F_x = \cancel{\frac{\partial}{\partial t}\int_{CV} V_x \rho\, d\forall} + \int_{CS} V_x \rho\, \underset{\sim}{V} \cdot d\underset{\sim}{A}$$

STEADY

$$F_{Rx} + p_1 A_1 + p_2 A_2 = \dot m_2(-V_2) - \dot m_1 V_1$$ 1

$$\Rightarrow F_{Rx} = -p_1 A_1 - p_2 A_2 - \dot m_2 V_2 - \dot m_1 V_1$$ 1

$$F_{R_x} = -\left[(200-100)\,kN/m^2\right]\frac{\pi}{4}(0.05\,m)^2 - \left[(150-100)\,kN/m^2\right]\frac{\pi}{4}(0.10\,m)^2$$

$$-\left(22\,\frac{kg}{s}\right)\left(2.8\,\frac{m}{s}\right)\left(\frac{1\,kN}{1000\,kg\cdot m/s^2}\right) - \left(30\,\frac{kg}{s}\right)\left(15.3\,\frac{m}{s}\right)\left(\frac{1\,kN}{1000\,kg\cdot m/s^2}\right)$$

$$\underline{\underline{F_{R_x} = -1.110\ kN = -1{,}110\ N}}$$

1

$$\Sigma F_z = \cancel{\frac{\partial}{\partial t}\int_{CV} V_z\,\rho\,dV} + \int_{CS} V_z\,\rho\,\underline{V}\cdot d\underset{\sim}{A}$$

STEADY

1

$$F_{Rz} - \cancel{p_3} A_3 = \dot{m}_3 V_3 - 0$$

0 atm

$p_{3,gage} = 0$

1

$$\underline{\underline{F_{Rz}}} = \dot{m}_3 V_3 = \left(8\,\frac{kg}{s}\right)\left(11.3\,\frac{m}{s}\right)\left(\frac{1\,N}{1\,kg\cdot m/s^2}\right) = \underline{\underline{90.4\ N}}$$

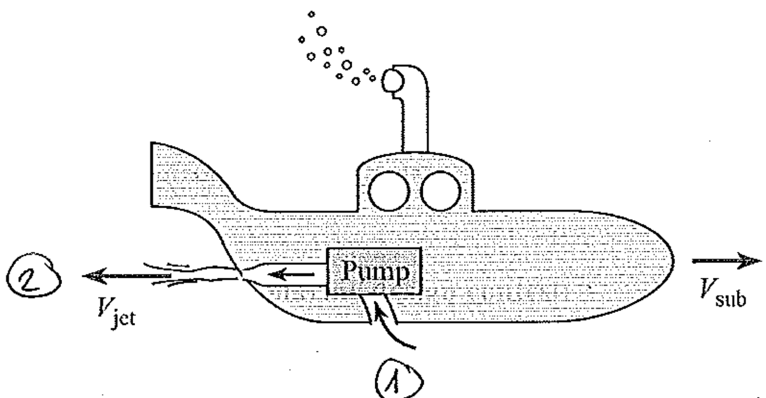
1

2. (10 pts) **Submarine design contest:** 10 PTS

A design contest features a submarine in water that will travel at a steady speed of v_{sub} = 1 m/s. The sub is powered by a water jet. This jet is created by drawing water from an inlet of diameter 25 mm, passing this water through a pump and then accelerating the water through a nozzle of diameter 5 mm to a speed of v_{jet}. The hydrodynamic drag force F_d acting against the moving submarine can be estimated by $F_d = C_d*[0.5*\rho*(v_{sub})\verb|^|2]*A_p$, where C_d = 0.3 and the projected area is A_p = 0.28 m^2. Specify the required value of v_{jet}?

Properties: We take the density of water to be ρ = 1000 kg/m^3.

Assumptions: The x-component of the inlet velocity is $v_{1x} = v_{sub}$.



SELECT A CONTROL VOLUME THAT SURROUNDS THE SUB. SELECT A REFERENCE FRAME LOCATED ON THE SUBMARINE.

CONSERVATION OF MOMENTUM IN X-DIRECTION:

$$\Sigma F_x = \cancel{\frac{\partial}{\partial t}\int_{CV} u\rho\, d\forall} + \int_{CS} u\rho\, \underset{\sim}{V}\cdot d\underset{\sim}{A}$$ 2

STEADY

$$F_{DRAG} = \dot{m}_2 V_2 - \dot{m}_1 V_{1X}$$ 3

CONTINUITY $\dot{m}_1 = \dot{m}_2 = \rho A_{jet}\cdot V_{jet}$ 1

$V_2 = V_{jet}$ 1

$V_{1X} = V_{sub}$ 1

$$F_{DRAG} = \rho A_{jet} V_{jet}\left(V_{jet} - V_{sub}\right)$$ 1

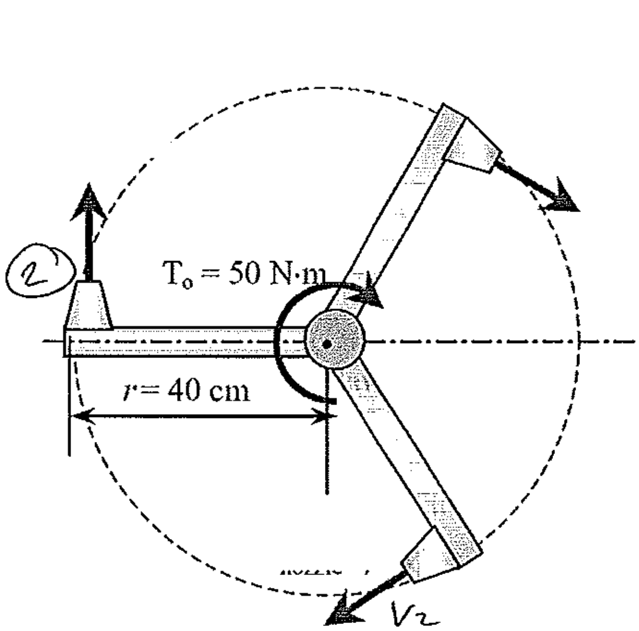
$$F_{DRAG} = C_D\left(\frac{\rho V_{sub}^2}{2}\right)A_p = 0.3\,\frac{(1000\,kg/m^3)(1.0\,\frac{m}{s})^2}{2}(0.28\,m^2) = 42\,N$$

$$42\,N = (1000\,kg/m^3)(1.96\times10^{-5}\,m^2)\,V_{jet}\left[V_{jet} - (1.0\,m/s)\right] \Rightarrow V_{jet} = 46.8\,\frac{m}{s}$$ 1

3. (10 pts) **Three armed lawn sprinkler:** 10 PTS

A lawn sprinkler with three identical arms is used to water a garden by rotating in a horizontal plane by the impulse caused by water flow. Water enters the sprinkler along the axis of rotation at a rate of 40 L/s and leaves the 1.2-cm-diameter nozzles in the tangential direction. The bearing applies a retarding torque of T_0 = 50 Nm due to friction at the anticipated operating speeds. For a normal distance of 40 cm between the axis of rotation and the center of the nozzles, determine the angular velocity of the sprinkler shaft?

Properties: We take the density of water to be ρ = 1000 kg/m³ = 1 kg/L.



$$\dot{m}_{nozzle} = \dot{m}/3 \quad (1)$$

$$Q_{nozzle} = Q/3 \quad (1)$$

$$V_{jet} = \frac{Q_{nozzle}}{A_{jet}} = \frac{40\ L/s}{3\left[\frac{\pi}{4}(0.012m)^2\right]}\left(\frac{1m^3}{1000L}\right)$$

$$V_{jet} = 117.9\ \frac{m}{s} \quad (1)$$

$$\dot{m}_{total} = \rho\, Q_{total} = \left(1\ \frac{kg}{L}\right)\left(\frac{40\ L}{s}\right) = 40\ \frac{kg}{s} \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM: (2)

$$\sum(\underset{\sim}{r} \times \underset{\sim}{F}) = \cancel{\frac{\partial}{\partial t}\int_{CV}(\underset{\sim}{r}\times\underset{\sim}{V})\rho\, d\forall} + \int_{CS}(\underset{\sim}{r}\times\underset{\sim}{V})\rho\, \underline{V}\cdot d\underline{A}$$

0 STEADY

ASSUMING COUNTERCLOCKWISE POSITIVE:

$$-T_0 = -3r\dot{m}_{nozzle}V_{2\,relative} \quad \text{or} \quad T_0 = r\,\dot{m}_{total}V_{2\,relative} \quad (1)$$

$$V_{2\,relative} = \frac{T_0}{r\,\dot{m}_{total}} = \frac{50\ Nm}{(0.4m)\left(40\ \frac{kg}{s}\right)}\left(\frac{1kg\cdot\frac{m}{s^2}}{1N}\right) = 3.1\ \frac{m}{s} \quad (1)$$

TANGENTIAL AND ANGULAR VELOCITIES OF THE NOZZLES BECOME:

$$V_{nozzle} = V_{jet} - V_{2\,relative} = 117.9\frac{m}{s} - 3.1\frac{m}{s} = 114.8\ \frac{m}{s} \quad (1)$$

$$\underline{\underline{\omega}} = \frac{V_{nozzle}}{r} = \frac{114.8\frac{m}{s}}{0.4m} = \underline{\underline{287\ rad/s}} \quad \text{OR} \quad \underline{\underline{\dot{n}}} = \frac{\omega}{2\pi} = \frac{287\ rad/s}{2\pi}\left(\frac{60s}{1min}\right) = \underline{\underline{2741\ rpm}} \quad (1)$$