Mar. 13, 2014

Math 2401 M - Quiz 5

First Name (Print): _____ Last Name (Print): _____ Signature: _____

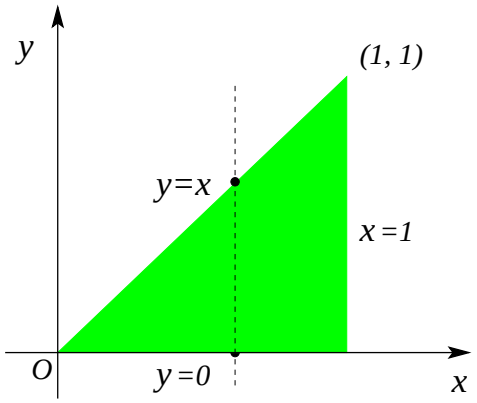
- There are **2** questions on **2** pages. The quiz is worth 20 points in total.
- Answer the questions clearly and completely. You must provide work clearly justifying your solution.
- You can NOT write your work on the back of the page. Use it for scratch work if needed.
- You have 20 minutes to finish your work.

1. (10 points) **Sketch** the region of integration for the integral

$$\int_0^1 \int_y^1 x^2 e^{xy} dxdy$$

and **evaluate** the integral with the **order of integration reversed**.

Solution.



$$\begin{aligned} & \int_0^1 \int_y^1 x^2 e^{xy} dxdy \\ = & \int_0^1 \int_0^x x^2 e^{xy} dydx = \int_0^1 xe^{xy}\Big|_0^x dx \\ = & \int_0^1 (xe^{x^2} - x)dx = \frac{1}{2}(e^{x^2} - x^2)\Big|_0^1 = \frac{e-2}{2}. \end{aligned}$$

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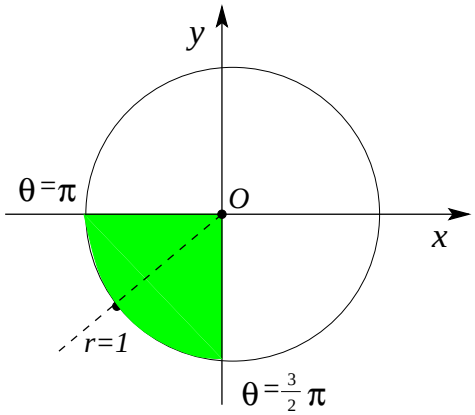
2. (10 points) Change the Cartesian integral

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0}\frac{2}{1+\sqrt{x^2+y^2}}dydx$$

into an equivalent polar integral, and evaluate the polar integral.

Solution.



$$\begin{aligned}
&\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0}\frac{2}{1+\sqrt{x^2+y^2}}dydx\\
=\ &\int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{1}\frac{2r}{1+r}drd\theta=2\int_{\pi}^{\frac{3\pi}{2}}\int_{0}^{1}(1-\frac{1}{1+r})drd\theta\\
=\ &2\int_{\pi}^{\frac{3\pi}{2}}\Big(r-\ln|1+r|\Big)\Big|_{0}^{1}d\theta=2\int_{\pi}^{\frac{3\pi}{2}}(1-\ln 2)d\theta=(1-\ln 2)\pi.
\end{aligned}$$

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