QUIZ 9 Name Key

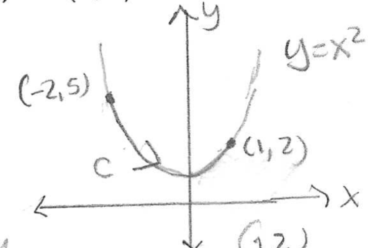
Math 2551 D Steinbart Section ____ April 7, 2016

Work neatly. Justify your answers and use proper notation. SHOW YOUR WORK TO RECEIVE CREDIT! No calculators or electronic devices are allowed (so no phones). Use exact values.

(6) 1. C is the portion of the parabolic curve $y = x^2 + 1$ from $(-2, 5)$ to $(1, 2)$. Sketch C.

a. Evaluate $\int_C (2y^2 - 5)\, dx + 4xy\, dy$

b. Evaluate $\int_C y\, dx + 2\, dy$



(a) Let $M\mathbf{i}+N\mathbf{j}=(2y^2-5)\mathbf{i}+4xy\mathbf{j}$

$\frac{\partial N}{\partial x}=4y \qquad \frac{\partial M}{\partial y}=4y$

So $\frac{\partial N}{\partial x}=\frac{\partial M}{\partial y}$ which means the differential form $M\,dx+N\,dy$ is exact and $\mathbf{F}=M\mathbf{i}+N\mathbf{j}$ is conservative. We'll find a potential fn for $\mathbf{F}$. $\frac{\partial f}{\partial x}=M=2y^2-5$

So $f=2xy^2-5x+g(t)$.

$N=4xy=\frac{\partial f}{\partial y}=4xy-0+g'(t)$, ← from above

So $4xy=4xy+g'(t)$ and $g'(t)=0$. Take $g(t)=0$

Thus we can take $f(x,y)=2xy^2-5x$

f is a potential fn for $\mathbf{F}$ and

$\int_C (2y^2-5)dx+4xy\,dy$

$= f(1,2)-f(-2,5) = 2xy^2-5x\Big|_{(-2,5)}^{(1,2)}$

$=2(1)4-5-(2(-2)25-5(-2)) = 3-(-90) = \boxed{93}$

(b) $\mathbf{F}=y\mathbf{i}+2\mathbf{j}$ is not conservative since

$\frac{\partial}{\partial y}(y)=1 \neq \frac{\partial}{\partial x}(2)=0$

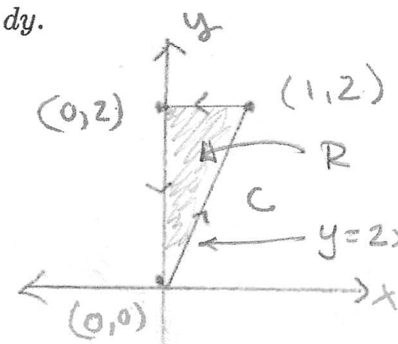
$C: \mathbf{r}(t)=t\mathbf{i}+(1+t^2)\mathbf{j}$. So $I=\int_C y\,dx+2\,dy$

$x=t \quad dx=dt \quad y=1+t^2 \quad dy=2t\,dt, \quad -2\le t\le 1$

$I=\int_{-2}^{1}(1+t^2)\,dt+2(2t)\,dt = t+\frac{t^3}{3}+4\frac{t^2}{2}\Big|_{-2}^{1}$

$=1+\frac{1}{3}+2-\left[-2-\frac{8}{3}+8\right]=-3+\frac{1}{3}+\frac{8}{3}=\boxed{0}$

(4) 2. Use Green's Theorem to evaluate $\int_C \underbrace{(x^2 + 4xy)}_{P}\, dx + \underbrace{(5y^2 + 2x)}_{Q}\, dy$.



$\int_C (x^2+4xy)\,dx+(5y^2+2x)\,dy$

R: $2x\le y\le 2$, $0\le x\le 1$

$=\iint_R \left[\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right]dA=\int_0^1\int_{2x}^{2}\left[2-4x\right]dy\,dx$

$=\int_0^1\left(2y-4xy\Big|_{2x}^{2}\right)dx=\int_0^1\left[4-8x-(4x-8x^2)\right]dx$

$=\int_0^1\left[4-12x+8x^2\right]dx = 4x-6x^2+\frac{8}{3}x^3\Big|_0^1 = 4-6+\frac{8}{3} = \boxed{\frac{2}{3}}$