ISYE 3232A Spring 2016 Quiz 4

February 9, 2016

Next month's production at a manufacturing company will use a certain solvent for part of its production process. Assume that there is an ordering cost of \$1,500 incurred whenever an order for solvent is placed and the solvent costs \$25 per liter. Due to short product life cycle, unused solvent cannot be used in following months. There will be a \$15 disposal charge for each liter of solvent left over at the end of the month. If there is a shortage of solvent, the production process is seriously disrupted at a cost of \$85 per liter short. (Hint: The pdf of an exponential random variable with mean $1/\lambda$ is $f(x) = \lambda e^{-\lambda x}$ and its CDF is $F(x) = 1 - e^{-\lambda x}$.)

Remark: When you write your answer with integrals, be careful with notation. Wrong notation can lead to a completely different answer and thus a 0 point. For example, $\int_2^3 x dx$ and $\int_2^3 D dx$ are different because the former is equal to $5/2$ while the latter is equal to $\int_2^3 D dx = D\int_2^3 1 dx = D$. We will be very strict with notation when grading.

1. What is the optimal ordering quantity when the demand is governed by the exponential distribution with mean 600 liters (assuming there is no initial inventory)?

   Answer: First note that $1/\lambda = 600$ and thus $\lambda = 1/600$. Then

   $$F(q) = 1 - e^{-q/600} = \frac{85-25}{85+15} = 0.6,$$

   and $q^* = -600\ln(0.4)$ liters.

2. Assume that the initial inventory is 400. What is the total expected cost when no additional quantity is ordered. Leave your answer with integrals.

   Answer:

   $$\begin{aligned}\mathsf{E}[\text{Cost}] &= 85\mathsf{E}[\text{shortage}] + 15\mathsf{E}[\text{leftover}] \\ &= 85\mathsf{E}[(D-400)^+] + 15\mathsf{E}[(400-D)^+] \\ &= 85\int_{400}^{\infty}(d-400)\cdot\frac{1}{600}e^{-\frac{d}{600}}dd + 15\int_0^{400}(400-d)\cdot\frac{1}{600}e^{-\frac{d}{600}}dd\end{aligned}$$

   Remark: $\int_{400}^{\infty}(d-400)\cdot\frac{1}{600}e^{-\frac{d}{600}}dd$ and $\int_0^{400}(400-d)\cdot\frac{1}{600}e^{-\frac{d}{600}}dd$ could be replaced with

   $$\int_{400}^{\infty}(x-400)\cdot\frac{1}{600}e^{-\frac{x}{600}}dx \quad \text{and} \quad \int_0^{400}(400-x)\cdot\frac{1}{600}e^{-\frac{x}{600}}dx, \text{ respectively.}$$

   However, $\int_{400}^{\infty}(d-400)\cdot\frac{1}{600}e^{-\frac{d}{600}}dx$ and $\int_0^{400}(400-d)\cdot\frac{1}{600}e^{-\frac{d}{600}}dx$ or integrals with mismatched notation are wrong and will receive a 0 point.

3. Assume that the initial inventory is 400 and additional quantity is ordered. How many liters should be ordered if the answer for (a) is assumed to be 550 liters? What is the total expected cost when the additional quantity is ordered? Leave your answer with integrals.

<u>Answer:</u> We need to order 150 $(= 550 - 400)$ liters.

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\begin{aligned}
\mathsf{E}[\text{Cost}] &= 1500 + 25 \cdot 150 + 85\mathsf{E}[\text{shortage}] + 15\mathsf{E}[\text{leftover}] \\
&= 1500 + 25 \cdot 150 + 85\mathsf{E}[(D - 550)^+] + 15\mathsf{E}[(550 - D)^+] \\
&= 1500 + 25 \cdot 150 + 85 \int_{550}^{\infty} (d - 550) \cdot \frac{1}{600} e^{-\frac{d}{600}}\, dd + 15 \int_{0}^{550} (550 - d) \cdot \frac{1}{600} e^{-\frac{d}{600}}\, dd
\end{aligned}
$$