Full name: Solutions Math 2551, Section B___

Quiz 4 — §14.2-4

Please ***clearly*** show all work. Scientific calculators are allowed, but no graphing calculators!

(1) Does the following limit exist? If so, what is the limit; if not, explain why. [8 points]

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$$

This limit does not exist, which can be verified using the two-path test. For instance along the path $x=0$ the limit is 0, while along the path $y=0$ the limit is 1.

(2) Suppose that in an electric circuit two resistors are placed *in parallel*. Assume that the first has resistance R_1 and the second has resistance R_2. It is a fundamental fact that the so-called *effective resistance* R of such a circuit is given by the formula

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

(a) What are $\frac{\partial R}{\partial R_1}$ and $\frac{\partial R}{\partial R_2}$? [6 points]

$$\frac{\partial R}{\partial R_1} = \frac{R_2^2}{(R_1 + R_2)^2} \qquad \frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1 + R_2)^2}$$

(b) Assume that R_1 and R_2 are functions of time t, and that at time $t = 1$

$$R_1(1) = 4\,\Omega \qquad R_2(1) = 2\,\Omega \qquad \left.\frac{dR_1}{dt}\right|_{t=1} = 1\,\Omega/s \qquad \left.\frac{dR_2}{dt}\right|_{t=1} = -1\,\Omega/s$$

Using the multivariable chain rule, compute dR/dt at time $t = 1$. [6 points]

In this case, the multivariable chain rule says that

$$\frac{dR}{dt} = \frac{\partial R}{\partial R_1}\frac{dR_1}{dt} + \frac{\partial R}{\partial R_2}\frac{dR_2}{dt}.$$

Using our computations in part (a), it follows that at time $t = 1$:

$$\frac{dR}{dt} = \left(\frac{2^2}{(4+2)^2}\right)\cdot 1 + \left(\frac{4^2}{(4+2)^2}\right)\cdot(-1) = -\frac{12}{36} = \boxed{-\frac{1}{3}\,\Omega/s}$$