TEST 2 Name Key

Math 2551 D Section ____ March 9, 2016

No books, notes, calculators, cell phones, or other electronic devices are allowed. Show your work and justify your answer to receive credit. Work neatly. There is a total of 100 points. Put your name and section number on each page of the test.

(14 pts) 1. Locate and classify all critical points of $f(x,y) = 2x^2 + 4xy - \frac{2}{3}y^3 + 2$.

$f_x = 4x + 4y$

$f_y = 4x - 2y^2$

$f_x = 0$ $4x + 4y = 0$

$f_y = 0$ $4x = -4y$

$x = -y$

$4x - 2y^2 = 0$

$4(-y) - 2y^2 = 0$

$-4y - 2y^2 = 0$

$-2y(2+y) = 0$

$y = 0$ or $y = -2$

$\Downarrow$

$x = 0$ $x = -(-2) = 2$

Crit pts $(0,0)$ $(2,-2)$

$f_{xx} = 4$

$f_{yy} = -4y$

$f_{xy} = 4$

At $(0,0)$ $f_{xx} = 4$ $f_{yy} = 0$, $f_{xy} = 4$

so $f_{xx}f_{yy} - (f_{xy})^2 = 4(0) - 4^2 < 0$

f has a saddle pt at (0,0)

At $(2,-2)$: $f_{xx} = 4$, $f_{yy}(+8)$ $f_{xy} = 4$

so $f_{xx}f_{yy} - (f_{xy})^2 = 4(8) - 16 > 0$

$f_{xx} > 0$

so f has a local min at $(2,-2)$

2. Find the tangent plane to the surface $2x^2 - 4y^3z - zx = 4$ at the point $(3, 1, 2)$.

$18 - 8 - 6 = 4$ ✓

Pts

1 ~~15~~ 14

2 12

3 ~~15~~ ~~14~~ 13

68 4 ~~8+5+3~~ ~~8+7+3~~ ~~9~~+~~4~~+2 ~~12~~ ~~13~~ 14

32 5 ~~15~~ ~~14~~ 15

6 ~~15~~ 16

7 ~~15~~ 16

26

32

47

Pts	
1	14
2	12
3	13
4	~~15~~ 14
5	15
6	15
7	16

39

29

32

0

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Name and section Key

(13pts) 3. Suppose that $T(x,y,z) = 100 - x^2 - y^2 - 3z^2 - xyz$ gives the temperature T at the point (x,y,z) in space.

10pt → a. Find the linearization $L(x,y,z)$ of $T(x,y,z)$ at the point $(3,2,1)$.

3pt → b. Use this linearization to estimate the value of $T(3.1, 1.8, 1.4)$. You do not need to simplify your answer.

$T(3,2,1) = 100-9-4-3(1) = 84$

$T_x = -2x \quad T_x(3,2,1) = -6$

$T_y = -2y \quad T_y(3,2,1) = -4$

$T_z = -6z \quad T_z(3,2,1) = -6$

$L(x,y,z) = T(3,2,1) + T_x(3,2,1)(x-3) + T_y(3,2,1)(y-2) + T_z(3,2,1)(z-1)$

$= 84 - 6(x-3) - 4(y-2) - 6(z-$

(b) $T(3.1, 1.8, 1.4) \approx L(3.1, 1.8, 1.4) = 84 - 6(3.1-3) - 4(1.8-2) - 6(1.4-1)$

$= 84 - 6(.1) - 4(-.2) - 6(.4) = 84 - .6 + .8 - 2.4$

$= 81.8$ $\boxed{\text{So } T(3.1, 1.8, 1.4) \approx 81.8}$

15pts 4. Let $f(x,y,z) = xy^2 + z^3$. Let P be the point $(1,2,-1)$.

8 a. Find the derivative of $f(x,y,z)$ at $(1,2,-1)$ in the direction $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$.

2+2 b. Find the direction of maximum increase of f at the point $(1,2,-1)$. What is the rate of change of f in this direction?

3 c. Find a (non-zero) direction in which $f(x,y,z)$ at $(1,2,-1)$ is NOT changing. (There is more than one correct answer.)

Let $\underline{u} = \frac{\underline{w}}{|\underline{w}|} = \frac{1}{\sqrt{6}}(2\underline{i} - \underline{j} + \underline{k}) = \frac{2}{\sqrt{6}}\underline{i} - \frac{1}{\sqrt{6}}\underline{j} + \frac{1}{\sqrt{6}}\underline{k}$

$|\underline{w}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$ $\quad \nabla f|_{(1,2,-1)} = \langle y^2, x, 3z^2 \rangle|_{(1,2,-1)} = \langle 4, 2, 3 \rangle$

(a) So the derivative of f at $(1,2,-1)$ in the direction $\underline{w}$ is

$\nabla f|_{(1,2,-1)} \cdot \underline{u} = \langle 4,2,3 \rangle \cdot \langle \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \rangle = \frac{8}{\sqrt{6}} - \frac{2}{\sqrt{6}} + \frac{3}{\sqrt{6}} = \boxed{\frac{9}{\sqrt{6}}}$

(b) $\nabla f|_{(1,2,-1)} = \langle 4,2,3 \rangle$ is the direction of maximum increase.

The rate of change in this direction is $|\nabla f|_{(1,2,-1)}|$

$= \sqrt{4^2 + 2^2 + 3^2} = \sqrt{16+4+9} = \sqrt{29}$

(c) There are lots of correct answers. We need a direction $\langle a, b, c \rangle$ (non zero) with $\langle 4,2,3 \rangle \cdot \langle a,b,c \rangle = 0$

$4a + 2b + 3c = 0$ So ~~t~~ we can take $a = 1$ $b = -2$, $c = 0$

$\boxed{\langle 1, -2, 0 \rangle}$

Name and section Key

3. Suppose that $T(x,y,z) = 100 - x^2 - y^2 - 3z^2 - xyz$ gives the temperature T at the point (x,y,z) in space.
   a. Find the linearization $L(x,y,z)$ of $T(x,y,z)$ at the point $(3,2,1)$.
   b. Use this linearization to estimate the value of $T(3.1, 1.8, 1.4)$. You do not need to simplify your answer.

$T(3,2,1) = 100 - 9 - 4 - 3 - 3(2)(1) = 78$

$T_x(x,y,z) = -2x - yz$ $\quad T_y(x,y,z) = -2y - xz$ $\quad T_z(x,y,z) = -6z - xy$

$T_x(3,2,1) = -6 - 2 = -8$ $\quad T_y(3,2,1) = -4 - 3 = -7$ $\quad T_z(3,2,1) = -6 - 6 = -12$

$L(x,y,z) = T(3,2,1) + T_x(3,2,1)(x-3) + T_y(3,2,1)(y-2) + T_z(3,2,1)(z-1)$

$= 78 + (-8)(x-3) + (-7)(y-2) + (-12)(z-1) = 78 - 8x - 7y - 12z + 50$ $\quad (*)$

So $L(x,y,z) = -8x - 7y - 12z + 128$

(b) $T(3.1, 1.8, 1.4) \approx L(3.1, 1.8, 1.4) = -8(3.1) - 7(1.8) - 12(1.4) + 128 = 73.8$

(or use $*$: $L(3.1, 1.8, 1.4) = 78 - 8(3.1-3) - 7(1.8-2) - 12(1.4-1)$
$= 78 - .8 + 1.4 - 4.8 = 73.8$)

4. Let $f(x,y,z) = xy^2 + z^3$. Let P be the point $(1,2,-1)$.
   a. Find the derivative of $f(x,y,z)$ at $(1,2,-1)$ in the direction $\mathbf{w} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$.
   b. Find the direction of maximum increase of f at the point $(1,2,-1)$. What is the rate of change of f in this direction?
   c. Find a (non-zero) direction in which $f(x,y,z)$ at $(1,2,-1)$ is NOT changing. (There is more than one correct answer.)

Problem 3 Corrected ↑

Part (a) 10 pts
(b) 3 pts

Name and section Key

15pts 5. We wish to find the maximum value of $f(x, y, z) = 4x - 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 21$. There is a maximum value.

13 pts → a. Using the method of Lagrange multipliers, set up the appropriate equations that one would need to solve. (Do not give vector equations as your final answer.)

2 pt → b. Does $f(x, y, z)$ subject to the constraint $x^2 + y^2 + z^2 = 21$ have a minimum value? Why or why not?

Let $g(x,y,z) = x^2 + y^2 + z^2 - 21$

(a) $\nabla f = \langle 4, -2, 1 \rangle$ we need $\nabla f = \lambda \nabla g$

$\nabla g = \langle 2x, 2y, 2z \rangle$ $g = 0$

So solve

$$\boxed{\begin{aligned} 4 &= \lambda(2x) \\ -2 &= \lambda(2y) \\ 1 &= \lambda(2z) \\ x^2+y^2+z^2-21 &= 0 \end{aligned}}$$

(b) Yes it does have a minimum:

The surface $g(x,y,z) = 0$ is bdd (and closed). f is a continuous function. So f has a minimum (and maximum) value subject to the constraint.

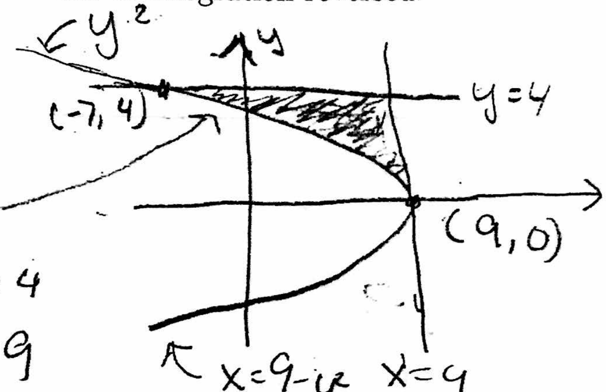
15 pts **TEST 2 - page 4** Name and section Key

4 → 6. a. Sketch the region of integration for the integral $I = \int_0^4 \int_{9-y^2}^{9} f(x,y)\,dx\,dy$.

11 → b. Write an equivalent iterated integral with the order of integration reversed.

$9 - y^2 \le x \le 9 \quad \rightarrow \quad x = 9,$

$0 \le y \le 4 \qquad\qquad x = 9 - y^2$



$y^2 = 9 - x$ and $y \ge$ here

so $y = \sqrt{9-x}$ $\qquad \sqrt{9-x} \le y \le 4$

$y = 4 \quad 4 = \sqrt{9-x} \qquad -7 \le x \le 9$

$16 = 9 - x$

$x = -7$

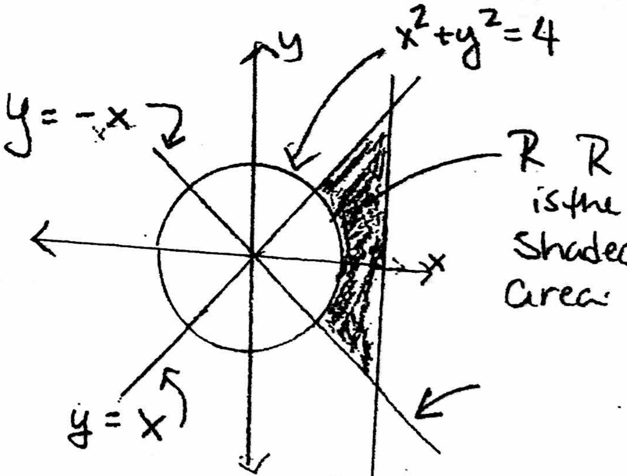
So $-7 \le x \le 9$

$y^2 = 9 - x$

$$\boxed{I = \int_{-7}^{9} \int_{\sqrt{9-x}}^{4} f(x,y)\,dy\,dx}$$

16 pts. 7. Convert the double integral to an iterated integral in polar coordinates. Include limits. Do not evaluate the integral.

$I = \int\int_R \frac{2x}{(3x^2+3y^2+8)}\,dA.$



$x = 3 \qquad x = r\cos\theta$

$r\cos\theta = 3$

$r = \frac{3}{\cos\theta}$

$x = -y$

$r\cos\theta = -r\sin\theta$

$\frac{r}{-r} = \frac{\sin\theta}{\cos}$

$-1 = \tan\theta$

$-\frac{\pi}{4} = \theta \leftarrow$ note: from picture $-\frac{\pi}{2} \le \theta \le 0$

$r\cos\theta = r\sin\theta$

$\frac{r}{r} = \frac{\sin\theta}{\cos\theta}$

$1 = \tan\theta$

$\theta = \pi/4 \leftarrow$ From picture, $\frac{\pi}{-4} < \theta$

$x = 3$

$r = \frac{3}{\cos\theta}$

$x^2 + y^2 = 4 \longrightarrow$ so $3x^2 + 3y^2 = 3r^2$

$r^2 = 4$; $r = 2$

So $2 \le r \le \frac{3}{\cos\theta}$

$-\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$$I = \int_{-\pi/4}^{\pi/4} \int_{2}^{\frac{3}{\cos\theta}} \frac{2r\cos\theta}{3r^2+8} \underbrace{r\,dr\,d\theta}_{dA}$$