There was a mistake in the data of question 1. The corrected question is

1 a. Consider the sequence of cash flows given in the table. Find the net future value (NFV) at the end of period 22, using MARR = 10%.

End of period	0	1	2	3	4	5	6	7	8	...	22
Cash flow	-22,222	220	240	260	280	300	320	340	360	...	640

b. Consider the sequence of cash flows given in the table. Find the net future value (NFV) at the end of period 22, using MARR = 10%.

End of period	0	1	2	3	4	5	6	7	8	...	22
Cash flow	-22,222	220	210	200	190	180	320	340	360	...	640

c. What is the NPV and the NAV for parts (a) and (b) above?

a. NFV = -22,222*[F/P,10%,22]+220*[F/A,10%,22]+20*[F/G,10%,22]
= -22,222* 8.14 + 220* 71.4 + 20* 494
= -155,299

b. NFV = solution.to.a – 30*[P/G,10%,5]*[F/P,10%,22]
= -155,299 – 30 * 6.86 * 8.14
= -156,974

c. A. NPV = solution.to.a * [P/F,10%,22]
= -155,299 * 0.123
= -19,102

NAV = solution.to.a * [A/F,10%,22]
= -155,299 * 0.0140
= -2,174

B. NPV = solution.to.b * [P/F,10%,22]
= -156,974 * 0.123
= -19,308

NAV = solution.to.b * [A/F,10%,22]
= -156,974 * 0.0140
= -2198

3025 HW2

2. a) $NPV = -9900 + 600(P/A, 15\%, 25) - 10(P/G, 15\%, 25)$
$+1000(P/F, 15\%, 26)$
$= -9900 + 600 \cdot (1.15^{25}-1)/(0.15 \cdot 1.15^{25}) - 10 \cdot \frac{1.15^{25} - 0.15 \cdot 25 - 1}{0.15^2 \cdot 1.15^{25}}$
$+1000 \cdot 1.15^{-26}$
$= -9900 + 600 \cdot (6.464) - 10 \cdot 38.0514 + 1000 \cdot 0.0264$
$= -6375.5$

b) $NPV = -9900 + 200(P/F, 15\% 1) + 300(P/F, 15\%, 2)$
$+ 400 \cdot (P/F, 15\%, 3) + 500 \cdot (P/A, 15\%, 22)(P/F, 15\%, 3)$
$- 2000 \cdot (P/F, 15\%, 26)$
$= -9900 + 200 \cdot 1.15^{-1} + 300 \cdot 1.15^{-2} + 400 \cdot 1.15^{-3} + 500 \cdot \frac{1.15^{22}-1}{0.15 \cdot 1.15^{22}} \cdot 1.15$
$-2000 \cdot 1.15^{-26}$
$= -7198.6$

c) For part a, $NFV = NPV \cdot (F/P, 15\%, 26)$
$= -6375.5 \cdot 1.15^{26} = -241356$
$NAV = NPV(A/P, 15\%, 26)$
$= -6375.5 \cdot \frac{1.15^{26} \cdot 0.15}{1.15^{26}-1} = -982.3$

For part b, $NFV = -7198.6(F/P, 15\%, 26) = -272515.93$
$NAV = -7198.6 \cdot (A/P, 15\%, 26)$
$= -7198.6 \cdot \frac{0.15 \cdot 1.15^{26}}{1.15^{-26} - 1} = -1109.09$

3. You are presented with two mutually exclusive investment projects, A and B, as shown below. A choice of either one would require the corresponding Initial Investment now.

   If you accept a project, you will receive the Annual cash inflow for the Project life.

   You cannot accept fractional investments, and you cannot duplicate investments.

   In other words, you can select A, B, or neither one. However, you can always invest any extra cash in a fund that pays MARR per year. You currently have Initial Amount available to invest.

a. Compute the net future value (NFV) at the end of the Planning period for each of the two projects, A and B, using MARR. [easy]

   1) moving the outflow for the investment forward to the end of the planning period using (F/P, i%, N),

   2) obtaining the future value of the annual cash inflow during the lifetime of each project using (F/A, i, $N_{project}$), and

   3) If $N_{project}$ is smaller than the planning period, moving that lump sum forward the remaining time using (F/P, i, N-$N_{project}$) or (F/P, i, N_{rem}).

   The leftover funds need not be considered, because the FW or NFV of such funds at MARR is zero.

   This last fact is an important point: it makes project selection easy using NPV or NFV (see how much more work is involved in part b to select the better project. Values are in table below. In this example both alternatives are desirable.

b. If you wish to maximize your cash amount at the end of the Planning period, which of the two projects, A or B, is the better choice? Explain numerically. [somewhat involved]

   1) reinvestment of the annual cash inflow at MARR during the lifetime of each project using (F/A, i, $N_{project}$),

   2) If Nproject is smaller than the planning period, moving that lump sum forward the remaining time using

   (F/P, i, N-$N_{project}$) or (F/P, i, N_{rem}) and

3) investing the leftover funds for the entire planning period using (F/P, i, N). The money invested in a project is not considered, since that is spent on the project. Values are in table below. Note the difference in NFV amounts is the same as the difference in cash accumulation amounts.

Planning period, years	20
MARR	3.3%
Initial amount available	6000

Investment project	A	B
Initial investment amount needed	4,000	4,500
Annual cash inflow during life	587	671
Project life, years	12	10

Solution, part a

(F/P, MARR, 12)	1.476	1.476
F/P applied to investment	-5904	-6642
(F/A, MARR, N) for project	14.436	11.624
F/A applied to project inflow	8474	7799
Remaining years after project	0	2
(F/P, MARR, N) for remaining years	1	1.067
NFV of inflows	8474	8322
Total NFV	2570	1680

Solution, part b

Initial amount available	6,000	6,000
Investment project	A	B
Initial investment needed	4,000	4,500
Annual cash inflow	587	671
Project life, years	12	10
(F/A, MARR, $N_{project}$)	14.436	11.624
Intermediate amount from project inflows	8474	7799
Remaining years left, N_{rem}	0	2
(F/P, MARR, N_{rem})	1	1.067
Cash accumulation from project	8474	7799
Left-over funds at time 0	2000	1500
(F/P, MARR, 12)	1.476	1.476
Cash at time 12 from leftover funds	2952	2214
Total cash at end of planning period	**11426**	10013

4) a)

year	cash flow
0	$-250{,}000$
1-14	$-25000+100{,}000$
6	$+100{,}000$ (additional)
15	$22{,}000-25{,}000+100{,}000$

$$
\begin{aligned}
EUV &= -25{,}000+100000 \\
&\quad -250{,}000[A/P,10\%,15] \\
&\quad +22{,}000[A/F,10\%,15] \\
&\quad +100{,}000[P/F,10\%,6][A/P,10\%,15] \\
&= -25{,}000+100{,}000 \\
&\quad -250{,}000[0.131474] \\
&\quad +22{,}000[0.031474] \\
&\quad +100{,}000[0.564474][0.131474] \\
&= 50{,}245
\end{aligned}
$$

b) Answer to a $-\ 300[A/G,\ 10\%\ 15]$

$= 50{,}245 - 300[5.278933]$

$= 48{,}661$

Question 5

a $P = 22000 \qquad N = 22 \qquad i = \left(1 + \frac{0.11}{12}\right)^{12} - 1 = 11.57\%$

$A = 22000 \times (A/P, i, N)$

$= 22000 \times \frac{0.1157 \times 1.1157^{22}}{1.1157^{22} - 1}$

$= 2797.31$

b $P = 22000 - 1000 \qquad A = 2797.31$ find i^* using iterative meth

$A = P(A/P, i, N)$

$2797.31 = 21000\,(A/P, i^*, 22)$

Using excel: $i^* = 12.28\%$ effective interest rate

i_N is the nominal interest rate

$\left(1 + \frac{i_N}{12}\right)^{12} - 1 = 12.28\%$

$i_N = 11.64\%$

6.

	Group A	Group A	Group B	Group B	Group C	Group C
Investment name	A1	A2	B1	B2	C1	C2
Investment at time 0	50,000	65,000	70,000	80,000	85,000	65,000
Net inflow/year	2,020	9,865	15,435	17,090	35,830	10,705
Lifetime	10	10	10	10	10	10
(P/A, I, N)	6.1446	6.1446	6.1446	6.1446	6.1446	6.1446
Apply to inflows	12412.092	60616.479	94841.901	105011.214	220161.018	65777.943
NPV (MARR)	-37,588	-4,384	24,842	25,011	135,161	778

In part a, try to form feasible combinations, such as selecting one very profitable project from each letter group and check for feasibility with respect to the contingency constraint. The NPV of a combination is the sum of the NPVs of the included projects. In part b, budget = $180,000 so we need to delete any combinations that violate the budget constraint.

									Part a.	Part b. with budget 180,000		
									Feasible	Budget	Budget	Feasible
Altern.	A1	A2	B1	B2	C1	C2	Invest.	NPV	NPV	Overrun	Feasible?	NPV
1	0	0	0	0	0	0	0	0.00	0.00	0	YES	0.00
2	0	1	0	1	1	0	230000	155788.71	155788.71	50,000	NO	0.00
3	0	1	1	0	1	0	220000	155619.40	155619.40	40,000	NO	0.00
4	0	1	0	0	1	0	150000	130777.50	130777.50	0	YES	130777.5
5	1	0	1	0	0	1	185000	-11968.06	-11968.06	5,000	NO	0.00
6	1	0	0	1	0	1	195000	-11798.75	-11798.75	15,000	NO	0.00
7	1	0	1	0	0	0	120000	-12746.01	-12746.01	0	YES	-12746.01
8	1	0	0	1	0	0	130000	-12576.69	-12576.69	0	YES	-12576.69
9	1	0	0	0	0	0	50000	-37587.91	-37587.91	0	YES	-37587.91
10	1	0	0	0	0	1	115000	-36809.97	-36809.97	0	YES	-36809.97
11	0	1	0	0	0	0	65000	-4383.52	-4383.52	0	YES	-4383.52
12	0	1	1	0	0	0	135000	20458.38	20458.38	0	YES	20458.38
13	0	1	0	1	0	0	145000	20627.69	20627.69	0	YES	20627.69
14	0	1	0	0	1	0	150000	130777.50	130777.50	0	YES	130777.50
15	0	1	0	0	0	1	130000	-3605.58	-3605.58	0	YES	-3605.58
16	0	1	1	0	1	0	220000	155619.40	155619.40	40,000	NO	0.00
17	0	1	0	1	1	0	230000	155788.71	155788.71	50,000	NO	0.00
18	0	1	1	0	0	1	200000	21236.32	21236.32	20,000	NO	0.00

19	0	1	0	1	0	1	210000	21405.64	21405.64	30,000	NO	0.00
20	0	0	1	0	0	1	135000	25619.84	25619.84	0	YES	25619.84
21	0	0	0	1	0	1	145000	25789.16	25789.16	0	YES	25789.16
22	0	0	1	0	0	0	70000	24841.90	24841.90	0	YES	24841.90
23	0	0	0	1	0	0	80000	25011.21	25011.21	0	YES	25011.21
24	0	0	0	0	0	1	65000	777.94	777.94	0	YES	777.94
Max								155788.71	155788.71			130777.50

7. Obtain the IRR (or ROR) for this cash flow, to the nearest percentage. What is a quick way to get an initial estimate of the IRR?

Time	0	1	2	3	4	5	6	7
Cash flow	-2,000	500	600	220	55	700	550	625

2000 = 500(P/F, i*, 1) + 600(P/F, i*, 2) + 220(P/F, i*, 3) + 55(P/F, i*, 4) + 700(P/F, i*, 5) + 550(P/F, i*, 6) + 625(P/F, i*, 7) The solving rate is i*= 13.8%

[Quick Way, obtain the sum of the inflows: 3250, then the average inflow: 3250/7 = 464,

then the (P/A,i,7) factor of 4.3103, which corresponds to ≈ 13.8%.]

(8) You can compute present value, future value or euv of each model and compare. Here, the EUV is easier to compute. (also faster).

Model X:

(cost) $EUV = 9000(A/P, 15\%, 3) - 800(A/F, 15\%, 3) + 1900$

$= 9000(0.402115) - 800(0.302115) + 1900$

$= 5519.033$

Model Y:

(cost) $EUV = 16000(A/P, 15\%, 8) - 5000(A/F, 15\%, 8) + 2200$

$= 16000(0.187444) - 5000(0.087444) + 2200$

$= 4761.884$

$\because$ EUV cost of Model Y < EUV cost of Model X

$\therefore$ Model Y is the preferred alternative.

If you choose to use NPV or NFV you would need to choose the same evaluate equal lengths of time (24 years)

Question 9

a. $NPV_A = -40000 + 15015\,(P/A, i_A^*, 7)$

$(P/A, i_A^*, 7) = 2.66$ By iterative me

$NPV_B = -30000 + 12012\,(P/A, i_B^*, 7)$

$(P/A, i_B^*, 7) = 2.50$ By iterative r

$NPV_C = -60000 + 22022\,(P/A, i_C^*$

$(P/A, i_C^*, 7) = 2.72$ By iterative

B has the highest rate of return:

B becomes the defender
consider investing instead in Project C

year	cash flow
0	-30,000
1-7	10,010

IRR ≈ 27.2% Accept C because IRR > MARR and this is an investment opportunity
C becomes the defender
consider investing instead in Project A

year	
0	20,000
1-7	-7,007

IRR ≈ 29.2% Reject A because IRR > MARR and this is a borrowing opportunity

Adopt project C

b.

Payback Period

MARR = 10%

	Undiscounted	Balance	Discounted	Balance
0	-40000	-40000	-40000	-40000
1	15015	-24985	13650	-26350
2	15015	-9970	12409.09	-13940.91
3	15015	5045	11280.99	-2659.92
4			10255.44	7595.53

Undiscounted Payback period for A = 3 years

Discounted Payback Period for A = 4 years

	Undiscounted	Balance	Discounted	Balance
0	-30000	-30000	-30000	-30000
1	12012	-17988	10920	-19080
2	12012	-5976	9927.27	-9152.73
3	12012	6036	9024.79	-127.94
4	12012		8204.35	8076.41

Undiscounted Payback Period for B = 3 years
Discounted Payback Period for B = 4 years

	Undiscounted	Balance	Discounted	Balance
0	-60000	-60000	-60000	-60000
1	22022	-37978	20020	-39980
2	22022	-15956	18200	-21780
3	22022	6066	16545.45	-5234.55
4	22022		15041.32	9806.77

Undiscounted Payback Period for C = 3 years
Discounted Payback Period for C = 4 years

10) a)

year	Project A B	Project A C	Project B B	Project B C
0		500,000		300,000
1-5	215,000		185,000	
5	415,000		335,000	
NPV	1,087,543	500,000	918,482	300,000

B/C = 2,1751 B/C = 3.0616

Both B/C > 1 so both are acceptable
Project B becomes defender and project A is the challenger

year	Ben.	Costs
0		200,000
1-5	30,000	
5	85,000	
NPV	172,978	200,000

B/C < 1 so reject challange

Adopt project B

10 b) Project A IRR ≈ 37%
Project B IRR ≈ 58%

Both are investments with IRR > MARR

Adopt B as defender and A is challenger. The data for the challenge can be found in part (a).

IRR of the challenge ≈ 0.7% < MARR

since the challenge is an investment opportunity, reject the challenge.

Adopt Project B

c) The NPV is the benefits − costs from part (a)

Project A NPV = 587543
Project B NPV = 616,482

choose project B, it has the greatest NPV.

11. Consider the cash flow given below. **MARR** = 10%

time	0	1	2	3	4	5	6	7	8	9	10	11
cash flow	-500	220	220	-140	220	-110	-110	220	220	220	220	220

a. Find the **undiscounted payback period.** What are some of the disadvantages of this method? [trivial]

b. Find the **discounted payback period. W**hat are some of the disadvantages of this method? [easy]

Solution: a) first table, b) second table. Method is to apply MARR to the most recent investment balance and update the balance with the current cash flow. Another way to obtain discounted payback period is to obtain contribution to PV of each cash flow element, and obtain cumulative sums of these contributions.

Time	0	1	2	3	4	5	6	7	8	9	10	11
Cash flow	-500	220	220	-140	220	-110	-110	220	220	220	220	220
Cumulative Cash Flow	-500	-280	-60	-200	20	-90	-200	20	240	460	680	900

So, the undiscounted payback period is time 7. The method ignores time value of money, and the cash flows that are realized after net flow becomes nonnegative and remains nonnegative, and the method possibly can trick users into identifying false early payback periods, such as time 4 in this example.

Time	0	1	2	3	4	5	6	7	8	9	10	11
Cash flow	-500	220	220	-140	220	-110	-110	220	220	220	220	220
Cumulative value	-500	-330	-143	-297	-107	-227	-361	-177	26	249	493	763

So, the discounted payback period is time 8. This method also ignores cash flows after the cumulative value becomes positive.

12. Consider the cash flow series below (all costs), which has a 3-year cycle that, once it begins, repeats forever. With MARR = 20%, how much would need to be deposited at time zero to all one to meet these costs?

time	0	1	2	3	4	5	6	7	8	9	10 and on
cost	-	-	-	-	100	180	220	100	180	220	repeat cycle

We are looking for the NPV. The 3-year cycle that repeats forever, but with a delayed start. First find the EUV of the 3-year cycle:
P_3 = 100 (P/F, 20%, 1) + 180 (P/F, 20%, 2) + 220 (P/F, 20%, 3) = 83.3 +125.0 +127.3 = \$335.6
A = 335.6(A/P, 20%, 3) = \$159.3. This uniform series continues forever starting from period 4.
The PV of this uniform series at time 3 is the limit of A(P/A, i, N) as N $\rightarrow \propto$, which is A/i.
Here we have 159.3/0.2 = 796.6. Shift this back to time 0, so obtain 796.6(1.2)^(-3) = 461