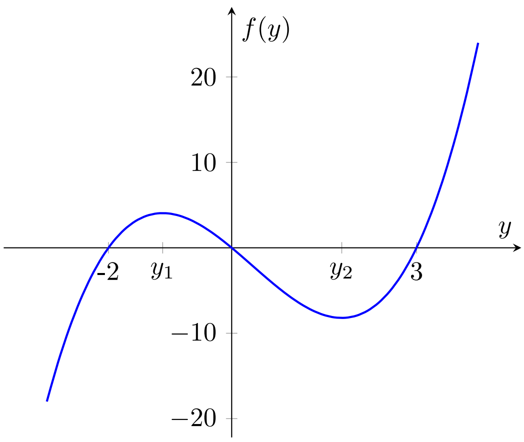
Quiz 1 **-** Name ____ Math 2403 - Section L ___

Good Luck!

This quiz has a back side! Don't forget about Question 2, 3 and Bonus Question!

1. (5 points) Consider the initial value problem $\frac{dy}{dt} = y(y^2 - y - 6)$, $y(0) = y_0$, $-\infty < y_0 < \infty$. Given the graph of the function $f(y) = y(y^2 - y - 6)$



   (a) Sketch the phase line.

   (b) List and classify any critical points.
   $y = 0$ asymptotically stable $y = -2, 3$ unstable;

   (c) Let y_1 and y_2 be the inflection points, analyze the concavity of solutions.
   y is concave up for $-2 < y < y_1, 0 < y < y_2$ and $y > 3$;
   y is concave down on the remaining intervals.

   (d) Sketch some of the solutions in the y versus t plane.

sorry, graphs are missing

2. (5 points) Solve the initial value problem

$$y' = -2x(y^2 - 3y + 2) \quad y(0) = 3$$

Solution:

$$\frac{y'}{(y-1)(y-2)} = -2x; \quad \left[\frac{1}{y-2} - \frac{1}{y-1}\right] y' = -2x; \quad \ln\left|\frac{y-2}{y-1}\right| = -x^2 + k; \quad \frac{y-2}{y-1} = ce^{-x^2}$$

$$y(0) = 3 \to c = \frac{1}{2}; \quad \frac{y-2}{y-1} = \frac{e^{-x^2}}{2}; \quad y = \frac{4 - e^{-x^2}}{2 - e^{-x^2}}$$

3. (5 points) Given the equation

$$3x^2 y dx + 2x^3 dy = 0$$

(a) Show it is not exact.
$M(x,y) = 3x^2 y$ and $N(x,y) = 2x^3$. $M_y(x,y) = 3x^2 \neq 6x^2 = N_x(x,y)$.

(b) Find an integrating factor to make the equation exact.

$$\frac{M_y - N_x}{N} = -\frac{3}{2x} \quad \mu(x) = e^{\int(-\frac{3}{2x})dx} = e^{-\frac{3}{2}\ln|x|} = x^{-\frac{3}{2}}$$

Therefore $3x^{1/2} y dx + 2x^{3/2} dy = 0$ is exact.

(c) Find an implicit solution.

$$F(x,y) = 2x^{3/2} y + \phi(y) \text{ and } F_y(x,y) = 2x^{3/2} + \phi'(y) = N(x,y), \text{ therefore } \phi(y) = k$$

The implicit solution is given by

$$2x^{3/2} y = c$$

[Bonus] (2 points) Given the equation

$$(x^2 + y^2)dx + 2xy dy = 0$$

(a) Show it is exact.
$M(x,y) = (x^2 + y^2)$ and $N(x,y) = 2xy$.
$M_y(x,y) = 2y = N_x(x,y)$.

(b) Find an implicit solution.

$$F(x,y) = \int M(x,y)dx = \frac{x^3}{3} + xy^2 + \phi(y)$$

Differentiating F with respect to y and comparing it to N gives us $\phi'(y) = 0$ and therefore $\phi(y) = k$, thus $F(x,y) = \frac{x^3}{3} + xy^2 + k$.
The implicit solution is given by

$$\frac{x^3}{3} + xy^2 = c$$