Quiz #1 (Total points: 100) COE 2001 Section H

September 28, 2011

Printed Name: ANSWER KEY

Please read and sign the Honor Pledge below

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community.

Signature:______________________ **Date:**________________

1. A plane bent bar is subjected to a system of three coplanar forces, as shown in Figure 1.

(a). Find the sum of the three forces; (15 point)

(b). Find the sum of the moments of the three forces about the origin O; (15 points)

(c). Reduce the system of forces to a force and a couple at an arbitrary point $A = (x, y)$; (15 points)

(d). Locate a point about which the system of forces can be reduced to a single force. (10 points)

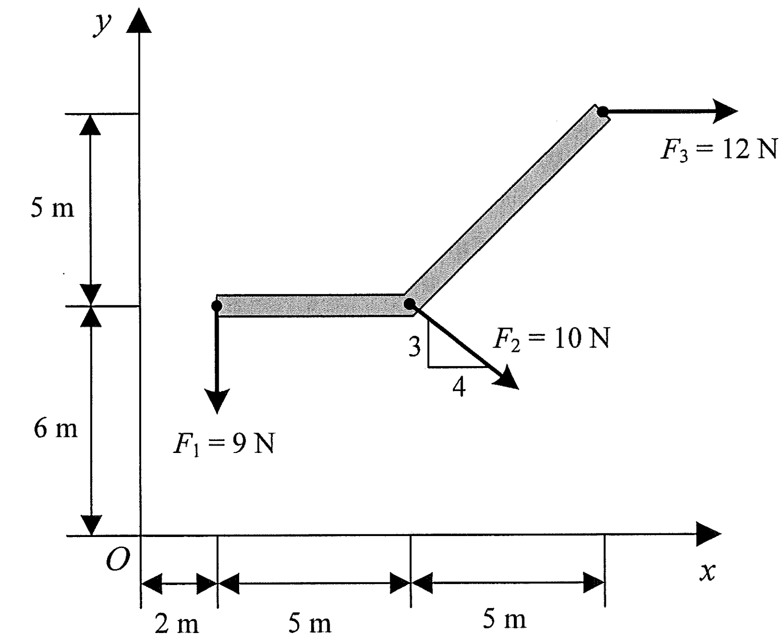


Figure 1

Solution

(a) $\vec{F}_1 = -9\vec{j}\ N,\ \vec{F}_2 = 8\vec{i} - 6\vec{j}\ N,\ \vec{F}_3 = 12\vec{i}\ N$

$$\vec{F}_{sum} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$
$$= -9\vec{j} + (8\vec{i} - 6\vec{j}) + 12\vec{i}$$
$$= 20\vec{i} - 15\vec{j}\ N$$

(b) $\vec{r}_1 = 2\vec{i} + 6\vec{j}\ m,\ \vec{r}_2 = 7\vec{i} + 6\vec{j}\ m,\ \vec{r}_3 = 12\vec{i} + 11\vec{j}\ m$

$$\vec{M}_{sum} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3$$

$$= (2\vec{i}+6\vec{j})\times(-9\vec{j}) + (7\vec{i}+6\vec{j})\times(8\vec{i}-6\vec{j})$$

$$+ (12\vec{i}+11\vec{j})\times(12\vec{i})$$

$$= -18\vec{k} - 90\vec{k} - 132\vec{k}$$

$$= \underline{-240\vec{k} \quad N\cdot m}$$

(c) We first reduce the system to a force & a couple at O.

$$\vec{R} = \vec{F}_{sum} = 20\vec{i} - 15\vec{j} \quad N$$

$$\vec{M}_O^R = \vec{M}_{sum} = -240\vec{k} \quad N.m$$

Then, at point $A = (x, y)$, we have

$$\underline{\vec{R} = 20\vec{i} - 15\vec{j} \ N}$$

$$\vec{M}_A^R = \vec{M}_O^R + \vec{r}_{O/A}\times\vec{R}$$

$$= (-240\vec{k}) + (-x\vec{i} - y\vec{j})\times(20\vec{i} - 15\vec{j})$$

$$= \underline{(-240 + 15x + 20y)\vec{k} \quad N.m}$$

(d) Set $\vec{M}_A^R = 0$

$$\Rightarrow -240 + 15x + 20y = 0$$

$\Rightarrow$ The system can be reduced to a single fore at any point lying on the line $3x + 4y - 48 = 0$.

2. The system in Figure 2 is in equilibrium. The weight of block A is $W_A = 18$ lb. Neglect friction and the masses of the pulleys and the rope.

(a). Draw a free-body diagram of block A and pulley 1 together; (10 points)

(b). Draw a free-body diagram of block B and pulley 2 together; (10 points)

(c). Determine the weight of block B, W_B. (25 points)

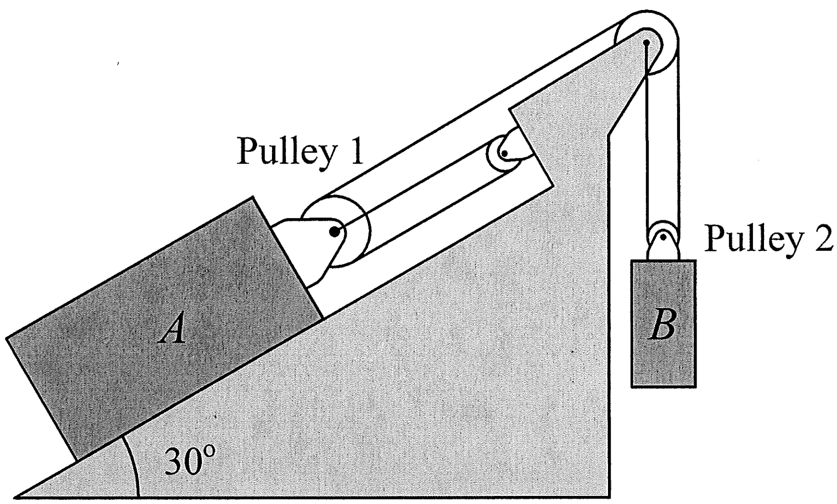
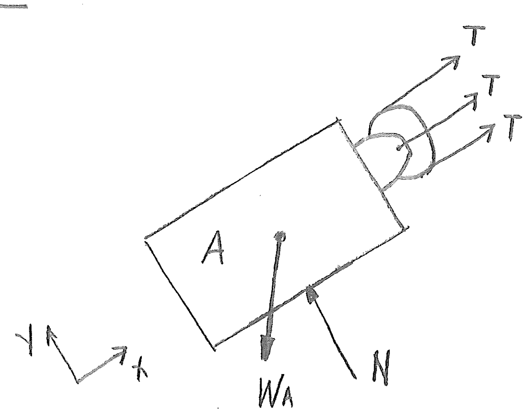


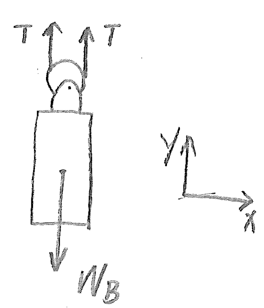
Figure 2

Solution

(a)



(b)



(c) Equilibrium condition of block A in the x-direction:

$$\Sigma F_x = 0$$

$$\Rightarrow 3T - W_A \sin 30^\circ = 0$$

$$\Rightarrow T = 18 \times \sin 30^\circ / 3$$

$$= 3 \text{ lb}$$

Equilibrium condition of block B in the y-direction:

$$\Sigma F_y = 0$$

$$\Rightarrow 2T - W_B = 0$$

$$\Rightarrow W_B = 2T$$

$$= 2 \times 3 = \underline{6 \text{ lb}}$$

(Note that two different x-y coordinate systems are used for block A & block B, in sake of simplicity of calculation.)