7-4. $X_i \sim N(100, 10^2)$ $n = 25$

$$\mu_{\overline{X}} = 100 \quad \sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{25}} = 2$$

$$P[(100 - 1.8(2)) \le \overline{X} \le (100 + 2)] = P(96.4 \le \overline{X} \le 102) = P\left(\frac{96.4-100}{2} \le \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \le \frac{102-100}{2}\right)$$

$$= P(-1.8 \le Z \le 1) = P(Z \le 1) - P(Z \le -1.8) = 0.8413 - 0.0359 = 0.8054$$

7-11. $n = 36$

$$\mu_X = \frac{a+b}{2} = \frac{(3+1)}{2} = 2$$

$$\sigma_X = \sqrt{\frac{1^2 + 0^2 + 1^2}{3}} = \sqrt{\frac{2}{3}}$$

$$\mu_{\overline{X}} = 2, \sigma_{\overline{X}} = \frac{\sqrt{2/3}}{\sqrt{36}} = \frac{\sqrt{2/3}}{6}$$

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

Using the central limit theorem:

$$P(2.1 < \overline{X} < 2.5) = P\left(\frac{2.1-2}{\frac{\sqrt{2/3}}{6}} < Z < \frac{2.5-2}{\frac{\sqrt{2/3}}{6}}\right) = P(0.7348 < Z < 3.6742)$$

$$= P(Z < 3.6742) - P(Z < 0.7348) = 1 - 0.7688 = 0.2312$$

7-23. a) $\frac{S}{\sqrt{N}} = \text{SE Mean} \rightarrow \frac{10.25}{\sqrt{N}} = 2.05 \rightarrow N = 25$

Mean $= \frac{3761.70}{25} = 150.468$, Variance $= S^2 = 10.25^2 = 105.0625$

Variance $= \frac{\text{Sum of Squares}}{n-1} \rightarrow 105.0625 = \frac{SS}{25-1} \rightarrow SS = 2521.5$

b) Estimate of population mean = sample mean = 150.468

7-26. $$\mathrm{E}\left(\overline{X}_1\right) = E\left(\frac{\sum_{i=1}^{2n} X_i}{2n}\right) = \frac{1}{2n} E\left(\sum_{i=1}^{2n} X_i\right) = \frac{1}{2n}(2n\mu) = \mu$$

$$\mathrm{E}\left(\overline{X}_2\right) = E\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i\right) = \frac{1}{n}(n\mu) = \mu$$

$\overline{X}_1$ and $\overline{X}_2$ are unbiased estimators of μ.

The variances are $V\left(\overline{X}_1\right) = \frac{\sigma^2}{2\mathrm{n}}$ and $V\left(\overline{X}_2\right) = \frac{\sigma^2}{\mathrm{n}}$; compare the MSE (variance in this case),

$$\frac{MSE(\hat{\Theta}_1)}{MSE(\hat{\Theta}_2)} = \frac{\sigma^2 / 2n}{\sigma^2 / n} = \frac{n}{2n} = \frac{1}{2}$$

Because both estimators are unbiased, one concludes that $\overline{X}_1$ is the "better" estimator with the smaller variance.

7-35. **Descriptive Statistics**

Variable	N	Mean	Median	TrMean	StDev	SE Mean
Oxide Thickness	24	423.33	424.00	423.36	9.08	1.85

a) The mean oxide thickness, as estimated by Minitab from the sample, is 423.33 Angstroms.
b) The standard deviation for the population can be estimated by the sample standard deviation, or 9.08 Angstroms.
c) The standard error of the mean is 1.85 Angstroms.
d) Our estimate for the median is 424 Angstroms.
e) Seven of the measurements exceed 430 Angstroms, so our estimate of the proportion requested is 7/24 = 0.2917

7-46. $f(x) = (\theta+1)x^{\theta}$

$$L(\theta) = \prod_{i=1}^{n}(\theta+1)x_i^{\theta} = (\theta+1)x_1^{\theta} \times (\theta+1)x_2^{\theta} \times \ldots = (\theta+1)^n \prod_{i=1}^{n} x_i^{\theta}$$

$$\ln L(\theta) = n\ln(\theta+1) + \theta \ln x_1 + \theta \ln x_2 + \ldots = n\ln(\theta+1) + \theta\sum_{i=1}^{n} \ln x_i$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{n}{\theta+1} + \sum_{i=1}^{n} \ln x_i = 0$$

$$\frac{n}{\theta+1} = -\sum_{i=1}^{n} \ln x_i$$

$$\hat{\theta} = \frac{n}{-\sum_{i=1}^{n} \ln x_i} - 1$$

7-48.

$$L(\theta) = \prod_{i=1}^{n} \frac{x_i e^{-x_i/\theta}}{\theta^2} \qquad \ln L(\theta) = \sum \ln(x_i) - \sum \frac{x_i}{\theta} - 2n\ln\theta$$

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{1}{\theta^2}\sum x_i - \frac{2n}{\theta}$$

Setting the last equation equal to zero and solving for theta yields

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{2n}$$

7-52. a) $\hat{a}$ cannot be unbiased since it will always be less than a.

b) bias = $\frac{na}{n+1} - \frac{a(n+1)}{n+1} = -\frac{a}{n+1} \underset{n\to\infty}{\to} 0$.

c) $2\overline{X}$

d) $P(Y\le y)=P(X_1, \ldots, X_n \le y)=[P(X_1\le y)]^n=\left(\frac{y}{a}\right)^n$. Thus, f(y) is as given. Thus,

bias = E(Y) – a = $\frac{an}{n+1} - a = -\frac{a}{n+1}$.

e) For any $n > 1$, $n(n+2) > 3n$ so the variance of $\hat{a}_2$ is less than that of $\hat{a}_1$. It is in this sense that the second estimator is better than the first.