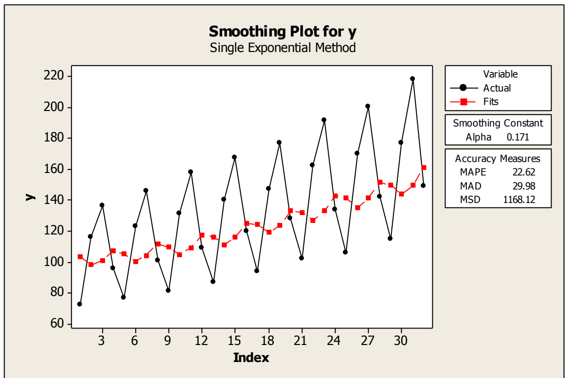
# ISyE 4031 Regression and Forecasting
# Homework 10 Solutions
# Spring 2016

1. Tiger Sports Drink data.

– Minitab solutions

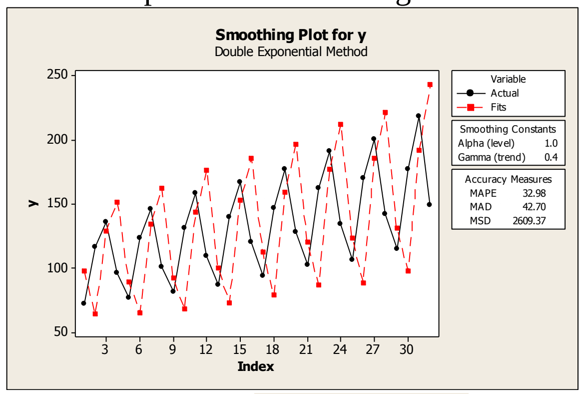
## Simple Exponential Smoothing




| Smoothing Constant | |
|---|---|
| Alpha | 0.171 |

| Accuracy Measures | |
|---|---|
| MAPE | 22.62 |
| MAD | 29.98 |
| MSD | 1168.12 |

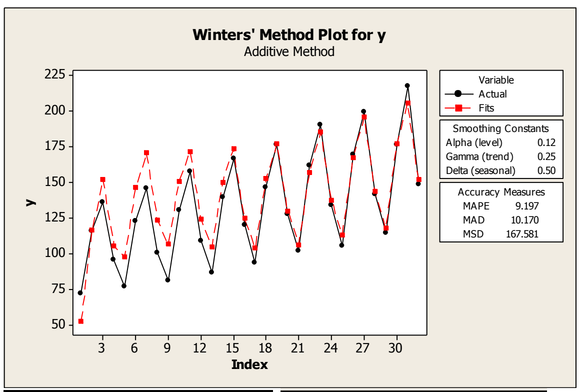
## Double Exponential Smoothing




| Smoothing Constants | |
|---|---|
| Alpha (level) | 1.0 |
| Gamma (trend) | 0.4 |

| Accuracy Measures | |
|---|---|
| MAPE | 32.98 |
| MAD | 42.70 |
| MSD | 2609.37 |

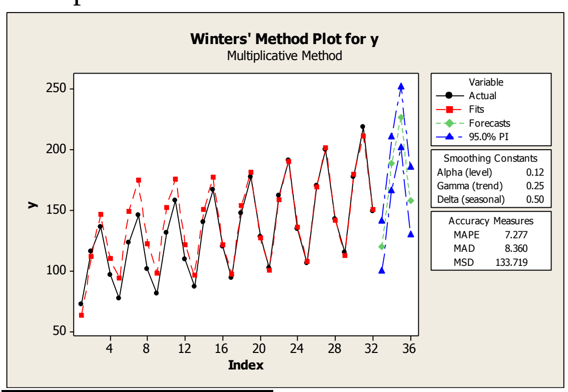
## Additive Holt-Winters'




| Smoothing Constants | |
|---|---|
| Alpha (level) | 0.12 |
| Gamma (trend) | 0.25 |
| Delta (seasonal) | 0.50 |

| Accuracy Measures | |
|---|---|
| MAPE | 9.197 |
| MAD | 10.170 |
| MSD | 167.581 |

## Multiplicative Holt-Winters'




| Smoothing Constants | |
|---|---|
| Alpha (level) | 0.12 |
| Gamma (trend) | 0.25 |
| Delta (seasonal) | 0.50 |

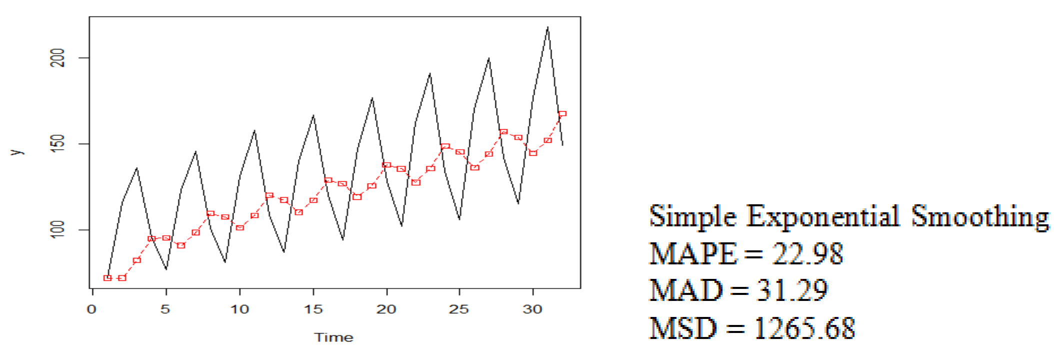
| Accuracy Measures | |
|---|---|
| MAPE | 7.277 |
| MAD | 8.360 |
| MSD | 133.719 |

– R solutions:

Simple Exponential Smoothing (Optimum $\alpha$)

```
> y.ts = ts(Mydata, frequency =4)
>plot(y.ts)
# By using R's optimal alpha:
> ses.exp = hw(y.ts, initial="simple", beta=FALSE, gamma=FALSE)
# Alternatively use function hw(y.ts, alpha = 0.171, beta=FALSE, gamma=FALSE)
# You can also use HoltWinters( ) function as below.
> ses = ses.exp$fitted
> lines(ses, type="o", pch=22, lty=2, col="red")
> mad.ses = sum(abs(y-ses)/length(y))
> mape.ses = 100*sum(abs((y-ses)/y)/length(y))
> msd.ses = sum((y-ses)^2/length(y))
```

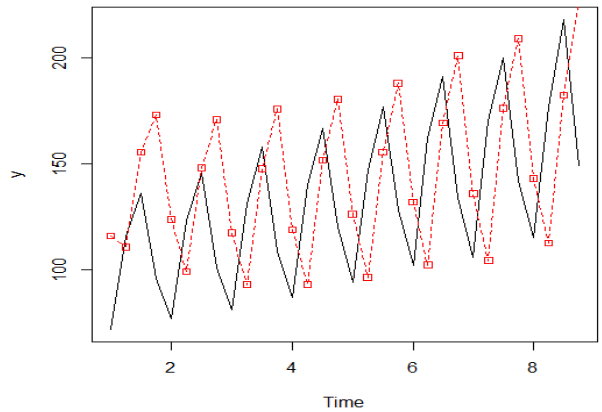
```
HoltWinters(x = y.ts, beta = FALSE, gamma = FALSE)
Smoothing parameters:
 alpha: 0.2363389
 beta : FALSE
 gamma: FALSE
```




Double Exponential Smoothing (Optimum $\alpha$, $\gamma$)

```
> des.exp <- hw(y.ts,initial = 'simple', gamma=FALSE)
> des = des.exp$fitted
# Alternatively use function hw(y.ts, alpha = 1, beta=0.4, gamma=FALSE)
```

```
HoltWinters(y.ts, gamma=FALSE)
Smoothing parameters:
 alpha: 1
 beta : 1
 gamma: FALSE
```

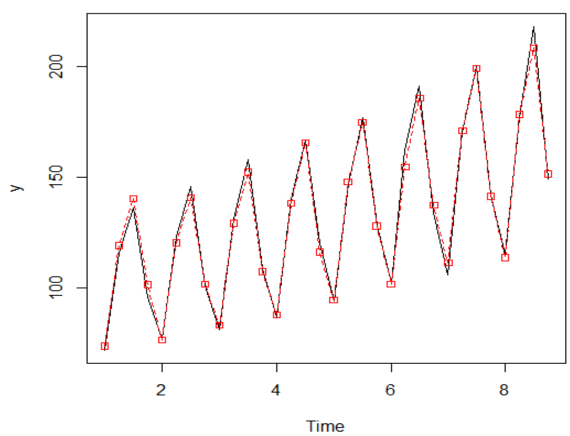



Double Exponential Smoothing
MAPE = 34.17
MAD = 41.56
MSD = 2190.66

Additive Holt-Winters’ Exponential Smoothing (Optimum α, γ, δ)
> hwa.exp <- hw(y.ts, seasonal="additive", initial="simple")
> hwa <- hwa.exp

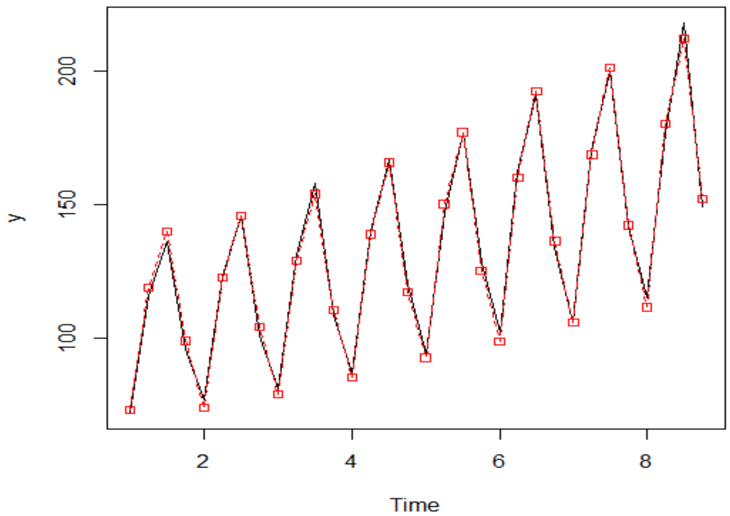
#Alternatively, use
#hw(y.ts, seasonal="additive", initial="simple", alpha=0.12, beta=0.25, gamma=0.5)

HoltWinters(x = y.ts)
Smoothing parameters:
 alpha: 0.0168574
 beta : 1
 gamma: 1




Additive Holt-Winters
MAPE = 2.001
MAD = 2.69
MSD = 12.36

Multiplicative Holt-Winters’ Exponential Smoothing (Optimum α, γ, δ)
> hwm.exp <- hw(y.ts, seasonal="multiplicative", initial="simple")
> hwm <- hwm.exp$fitted




Multiplicative Holt-Winters
MAPE = 1.74
MAD = 2.17
MSD = 6.47

Among all methods multiplicative Holt-Winter's performs the best. All accuracy measures: MAPE, MAD, and MSD are the smallest.

When we look at the time-series, we see a trend and increasing seasonality (multiplicative seasonality). Multiplicative Holt-Winter's method is the most appropriate for this type of time-series.

2. Exercise 8.14.

a. $\hat{y}_{20}(16) = \ell_{16} + 4b_{16} + sn_{20-4}$

$$= \ell_{16} + 4b_{16} + sn_{16} = 36.3426 + 4(.9809) + (\text{-}10.9088) = 29.3574$$

A 95% prediction interval for bike sales in period 20.

$$c_\tau = c_4 = 1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2 + \alpha^2(1+3\gamma)^2$$

$$= 1 + (.561)^2(1+0)^2 + (.561)^2(1+2(0))^2 + (.561)^2(1+3(0))^2 = 1 + 3(.561)^2 = 1.9442$$

$$[\hat{y}_{20}(16) \pm z_{[.025]} s\sqrt{c_4}]$$

$$= [29.3574 \pm 1.96(1.2025)\sqrt{1.9442}] = [29.3574 \pm 3.2863] = [26.0711, 32.6437]$$

b. $\hat{y}_{21}(16) = \ell_{16} + 5b_{16} + sn_{21-4} = \ell_{16} + 5b_{16} + sn_{17}$

$$= \ell_{16} + 5b_{16} + sn_{13} \quad (sn_{13} \text{ is last estimate for seasonal factor in quarter 1})$$

$$= 36.3426 + 5(.9809) \text{-} 14.2162 = 27.0309$$

A 95% prediction interval for bike sales in period 21

$$c_\tau = c_5 = 1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2 + \alpha^2(1+3\gamma)^2 + [\alpha(1+4\gamma) + (1-\alpha)\delta]^2$$

$$= c_4 + [\alpha(1+4\gamma) + (1-\alpha)\delta]^2 = 1.9442 + [.561(1+4(0) + .439(0)]^2 = 2.2589$$

$$[\hat{y}_{21}(16) \pm z_{[.025]} s\sqrt{c_5}]$$

$$= [27.0309 \pm 1.96(1.2025)\sqrt{2.2589}] = [27.0309 \pm 3.5423] = [23.4886,\ 30.5732].$$

3. Exercise 9.3.

a. The SAC dies down very slowly. Therefore, the original values are nonstationary.

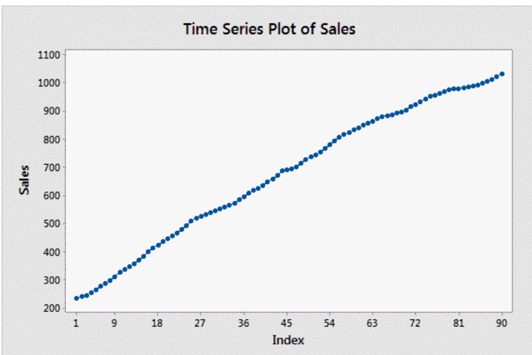
d. The SAC in Figure 9.16 dies down quickly. Therefore, the first differences are stationary.

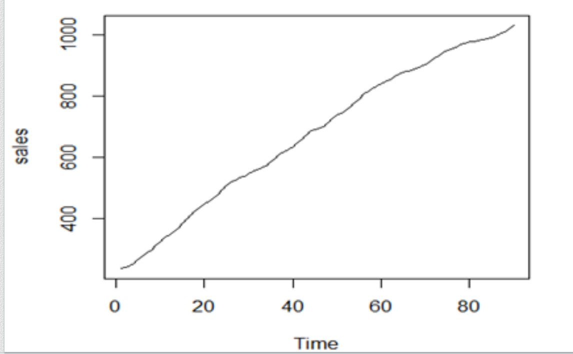
e. The SAC in Figure 9.16(a) dies down quickly. The SPAC in Figure 9.16(b) has a spike at lag 1 and cuts off after lag 1.

4. Toothpaste sales data.

a. Time−series plot.

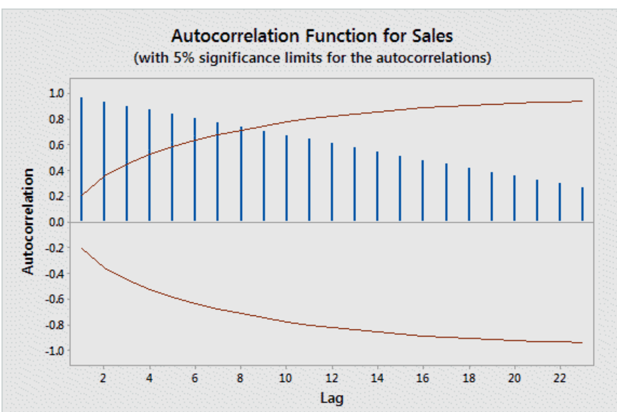
− Minitab solution − R solution

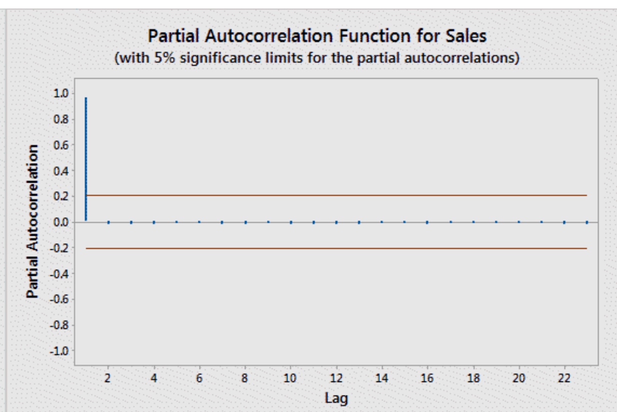


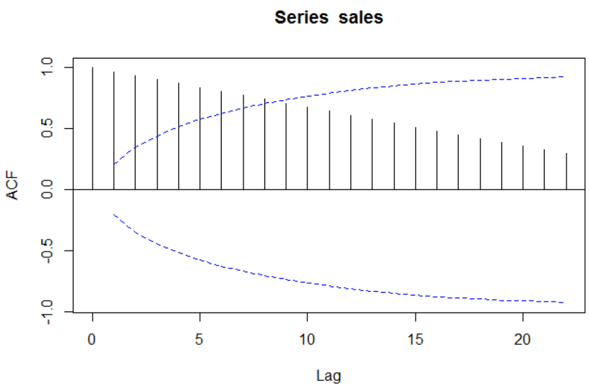

b. SAC and SPAC for the original data.

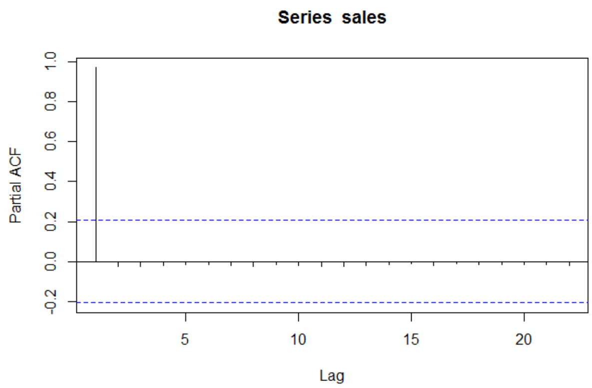
− Minitab solutions




− R solutions

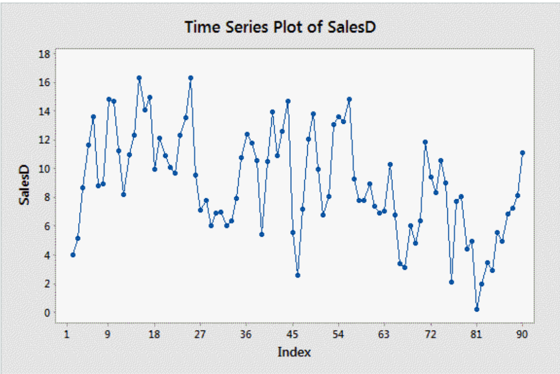


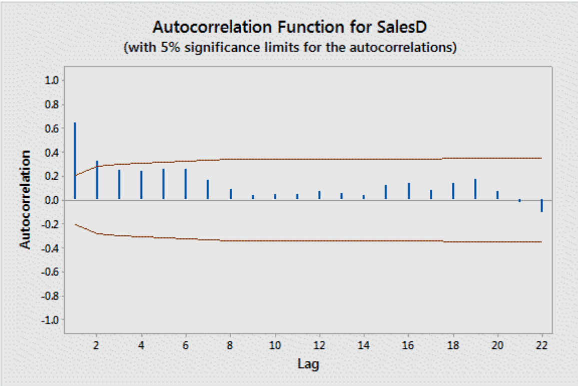



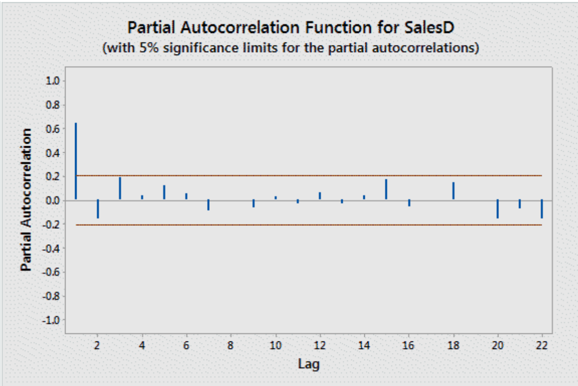

The SAC dies down very slowly. Therefore, the original values are nonstationary. Taking the differences is required.

c. SAC and SPAC for the first differences.

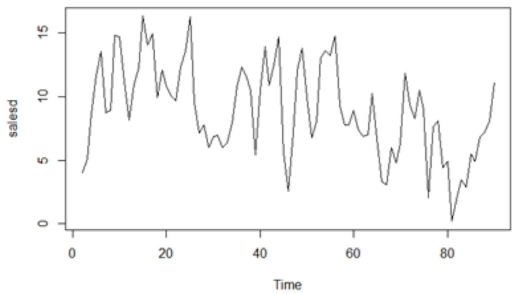
− Minitab solutions

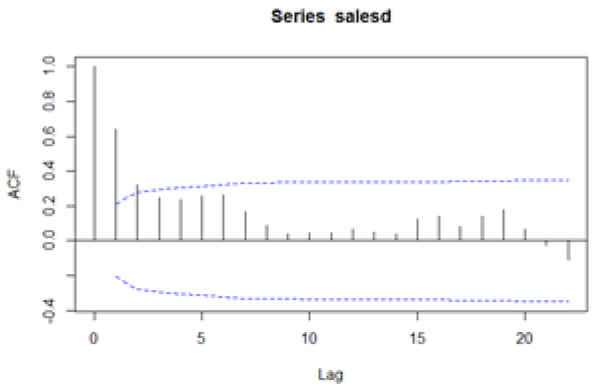


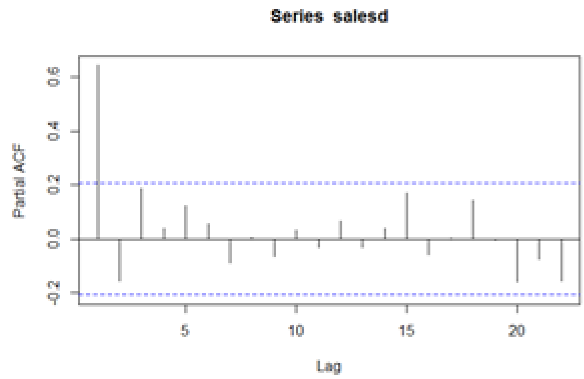




− R solutions




The SAC dies down quickly and the SPAC has a spike at lag 1 and cuts off after lag 1. Therefore, the first differences are stationary.