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Solutions to Homework 2

1. a) $\min(D,7)$ has the pmf given by

$$P(\min(D,7)=k)=\begin{cases} 1/10 & \text{if } k=5 \\ 3/10 & \text{if } k=6 \\ 6/10 & \text{if } k=7 \\ 0 & \text{otherwise} \end{cases}$$

Hence,

$$E[\min(D,7)]=\frac{1}{10}*5+\frac{3}{10}*6+\frac{6}{10}*7=6.5.$$

b) $(7-D)^+$ has the pmf given by

$$P((7-D)^+=k)=\begin{cases} 1/10 & \text{if } k=2 \\ 3/10 & \text{if } k=1 \\ 6/10 & \text{if } k=0 \\ 0 & \text{otherwise} \end{cases}$$

Hence,

$$E[(7-D)^+]=\frac{1}{10}*2+\frac{3}{10}*1=0.5.$$

2. a)

$$E[\max(D,8)]=8*P(D\le 8)+\int_8^{10}\frac{s}{5}ds=8*3/5+\frac{s^2}{10}\,|_8^{10}=\frac{24}{5}+\frac{36}{10}=8.4.$$

b)

$$E[(D-8)^+]=\int_8^{10}\frac{s-8}{5}ds=0.4.$$

3. $p=30$, $c_v=10$, $s_v=5$.

$$\text{Profit}(7)=(p-c_v)\min\{D,7\}-(c_v-s_v)(7-D)^+.$$

Hence

$$E[\text{Profit}(7)]=20E[\min\{D,7\}]-5E[(7-D)^+]=127.5$$

using the values found in question 1.

4. Similar to the previous question

$$E[\text{Profit}(7)]=20E[\min\{D,7\}]-5E[(7-D)^+]=130$$

the way to calculate is the same as in question 2.

5. If D has an exponential distribution with mean 7 then the pdf of D is given by

$$f_D(x) = \frac{1}{7}e^{-\frac{1}{7}x}.$$

As in the last 2 questions we need to find $E[\min\{D, 7\}]$ and $E[(7-D)^+]$.

$$E[\min(D, 7)] = 7 * P(D \geq 7) + \int_0^7 \frac{s}{7}e^{-s/7}ds = 7e^{-1} + (7 - 14e^{-1}) \approx 4.43$$

$$E[(7-D)^+] = \int_0^7 \frac{7-s}{7}e^{-s/7}ds = 7 - \int_0^7 \frac{s}{7}e^{-s/7}ds = 7 - (7 - 7e^{-1}) \approx 2.575$$

$$E[\text{Profit}(7)] = 20E[\min\{D, 7\}] - 5E[(7-D)^+] = 75.725.$$