ISYE 3133 Homework 2 Summer 2015

Due Thursday, June 4 at 10am.
Problems with asterisks (*) will be graded for correctness.

1. Use the simplex algorithm to solve the following LPs:

a. minimize $-x_1 - x_2$

s.t. $x_1 - x_2 \le 1$

$x_1 + x_2 \le 2$ $x_1 = 0,\ x_2 = 2,\ z = -2$

$x_1, x_2 \ge 0$

*b. maximize $2x_1 + x_2 - x_3$

s.t. $5x_1 + x_3 \ge 1$

$-2x_1 + x_3 \le 22$ $x_1 = x_2 = 0,\ x_3 = 6,\ z = -6$

$-4x_1 + x_2 - x_3 \le -6$

$x_1 \le 0;\ x_2, x_3 \ge 0$

c. minimize $-3x_1 + x_2$

s.t. $x_1 - 2x_2 \ge 2$ LP is infeasible

$-x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0$

2. Characterize all optimal solutions to the following LP:

maximize $-8x_5$

s.t. $x_1 + x_3 + 3x_4 + 2x_5 = 2$

$x_2 + 2x_3 + 4x_4 + 5x_5 = 5$

all $x \ge 0$

Any solution with $x_5 = 0$ and satisfying $x_1 + x_3 + 3x_4 = 2$, $x_2 + 2x_3 + 4x_4 = 5$, and all $x \ge 0$ is optimal.

*3. Solve the following LP:

maximize $-4x_1 + 5x_2$

s.t. $5x_1 + 7x_2 \le 35$ $x_1 = 1,\ x_2 = 0,\ z = 5$

$-x_1 + 2x_2 \le 2$

$x_1, x_2 \ge 0$

*4. Consider the following dictionary for a maximization problem:

$$
\begin{array}{rcl}
x_3 &=& b - 4x_1 + A_1x_2 + A_2x_5 \\
x_4 &=& 2 + x_1 + 5x_2 + x_5 \\
x_6 &=& 3 + A_3x_1 + 3x_2 + 4x_5 \\
\hline
z &=& 10 + C_1x_1 + C_2x_2
\end{array}
$$

Give all conditions on A_1, A_2, A_3, b, C_1, and C_2 necessary to make the following statements true:

a. The current basic solution is feasible but the LP is unbounded. $b, C_2, A_1 \ge 0$

b. The current basic solution is feasible but not optimal. $b \ge 0$, $C_1 \ge 0$ or $C_2 \ge 0$

c. The current basic solution is degenerate. $b = 0$

d. The current solution is optimal and alternative optimal solutions exist.

$b \ge 0;\ C_1, C_2 \le 0$ or $b \ge 0;\ C_1 = 0, C_2 \le 0$ or $b \ge 0;\ C_1 \le 0, C_2 = 0$

e. The current basic solution is not a basic feasible solution. $b < 0$