solutions

Full Name: Solutions

Section B

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Signature: ______________________________

Math 2551 — Exam 3
October 28, 2015

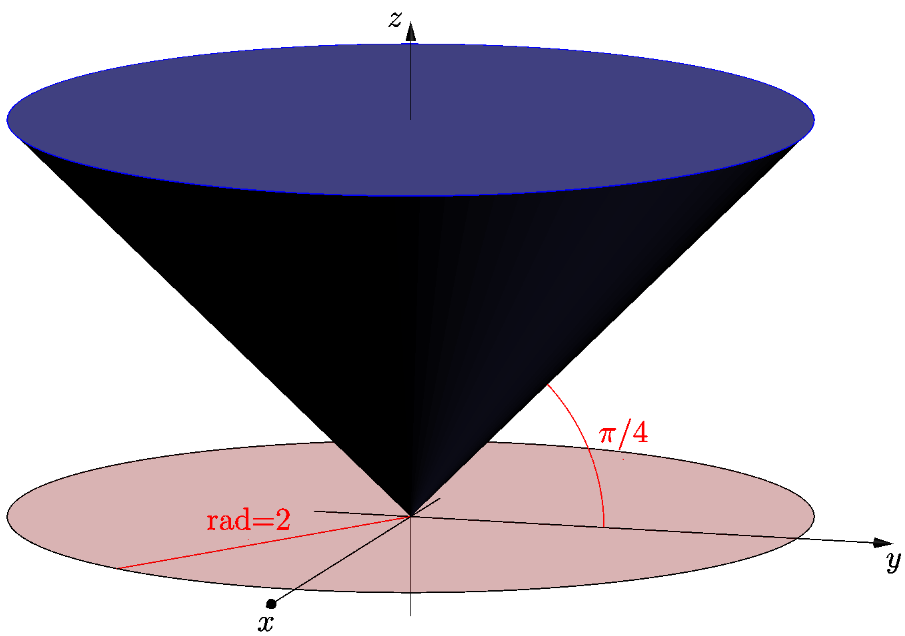
Write your solutions clearly and legibly, showing all work. Use of notes, cheat sheets, the textbook, or any outside materials is not permitted. Only non-graphing, non-programmable calculators are permitted.

Problem	Points Possible	Points Earned
1	16	16
2	13	13
3	15	15
4	16	16
Total	50	50

solutions

(1) Let R be the region in space which lies above $z = \sqrt{x^2 + y^2}$ and below $z = 2$. Set up (do not evaluate!) iterated integrals in both cylindrical and spherical coordinates which compute its volume. [16 points]

The region is drawn below. It is an upward opening cone with a flat top.



In cylindrical coordinates, the volume is given by

$$\int_0^{2\pi} \int_0^2 \int_r^2 r \, dz \, dr \, d\theta.$$

In spherical coordinates, the volume is given by

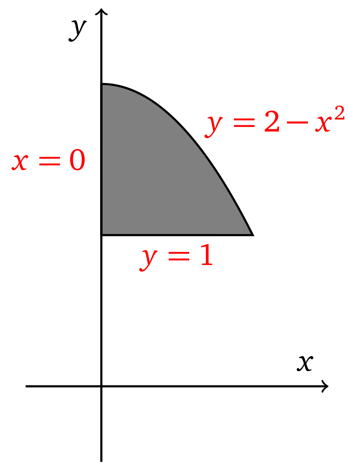
$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

8 points each, with partial credit possible.

(2) Consider the following iterated integral:

$$\int_1^2 \int_0^{\sqrt{2-y}} dx\, dy$$

(a) Draw the region of integration, labeling each of its sides. [8 points]



(b) Write an equivalent iterated integral in the order $dy\, dx$. Don't evaluate! [5 points]

In the order $dy\, dx$ the integral is

$$\int_0^1 \int_1^{2-x^2} dy\, dx$$

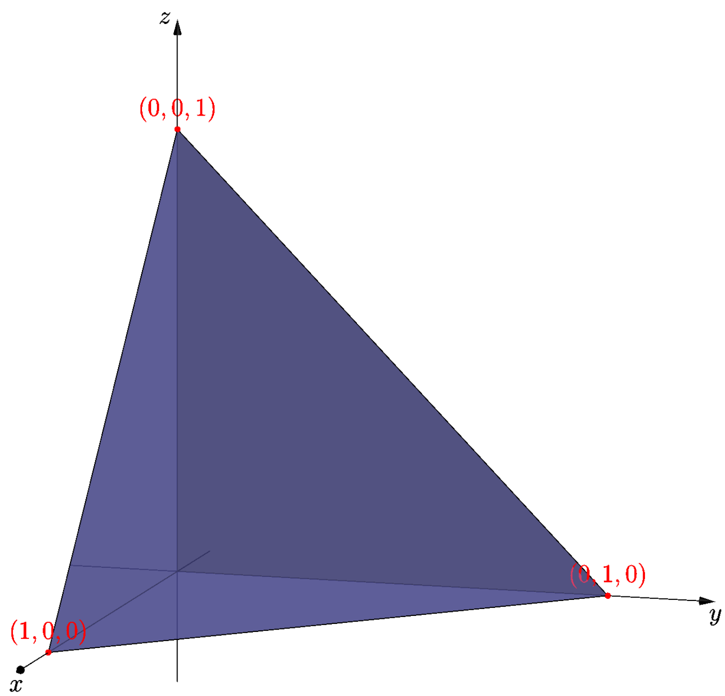
solutions

(3) Let R be the region in the first octant cut out by the plane $x+y+z=1$. Compute

$$\iiint_R x\,dV.$$

[15 points]

The region of integration is the pyramid drawn below.



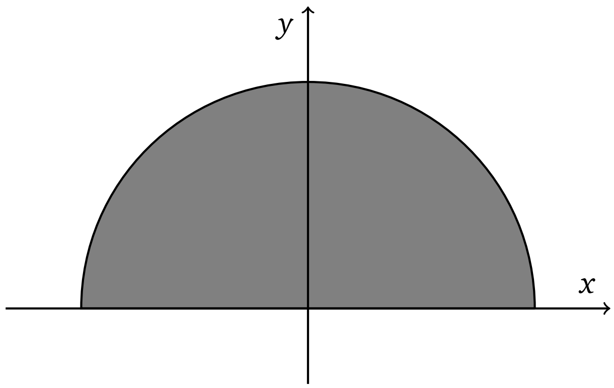
Any order of integration would apply here.

$$\iiint_R x\,dV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x\,dz\,dy\,dx = \int_0^1 \int_0^{1-x} x(1-x-y)\,dy\,dx$$
$$= \int_0^1 \frac{x(1-x)^2}{2}\,dx = \boxed{\frac{1}{24}}$$

10 points for setting up a correct iterated integral, 5 points for the evaluation of that integral. Partial credit possible for both.

(4) Let R be the region in the plane defined by the inequalities $x^2 + y^2 \leq 1$ and $y \geq 0$. What is the centroid of R? [16 points]

The region of integration is the half-disk of radius 1 drawn below.



The symmetry of the region across the y-axis implies immediately that $\overline{x} = 0$, so we need only compute $\overline{y}$. Naturally, we will do this with polar coordinates.

$$\overline{y} = \frac{1}{\text{Area}(R)} \iint_R y \, dA = \frac{1}{(\pi/2)} \int_0^\pi \int_0^1 r^2 \sin\theta \, dr \, d\theta = \frac{2}{\pi} \int_0^\pi \frac{\sin\theta}{3} \, d\theta = \frac{4}{3\pi}$$

Thus the centroid is $(0, 4/(3\pi))$.

4 points for recognizing or computing that $\overline{x} = 0$.
The remaining 12 points are for the computation of $\overline{y}$. The breakdown for these 12 points is the following: 2 points for the area of R (which you are not required to compute via an integral), 6 points for setting up the integral, and 4 points for evaluation.