ISyE 4031 Regression and Forecasting
Homework 8 Solution
Spring 2016

1. Exercise 6.4. (d).

(1) $\phi_1 = 0.59408$. Yes, p-value = 0.0003 < $\alpha = .001$ (Strong evidence).

(2) p-value = 0.1011 for β_1. That implies fail to reject H_0: $\beta_1 = 0$. However, because t^2 is significant (we reject reject H_0: $\beta_2 = 0$, since the corresponding p-value = 0.0121), we keep t in the model.

The p-value for β_4 is .0787 < 0.05. However if Q_1 and Q_3 are important, so "quarter" is important. The associated p-values for t^2, Q_1, and Q_3 are less than $\alpha = .05$.

(3) Point Forecasts: $\hat{y}_{41} = 605.33$ $\quad \hat{y}_{42} = 505.67$ $\quad \hat{y}_{43} = 426.94$ $\quad \hat{y}_{44} = 569.97$

95% $P.I.$ for y_{41}: [506.84, 703.82], 95% $P.I.$ for y_{42}: [391.11, 620.23]

95% $P.I.$ for y_{43}: [307.22, 546.66], 95% $P.I.$ for y_{44}: [448.49, 691.46]

(4) $$\hat{y}_{40+\tau} = 283.94906 - 9.21968(40+\tau) + .35348(40+\tau)^2 + 70.10688Q_1 - 35.42856Q_2 - 126.52509Q_3 + .59408\hat{\varepsilon}_{40+\tau-1}$$

where for $\tau = 1$: $\hat{\varepsilon}_{40} = y_{40} - [283.94906 - 9.21968(40) + .35348(40)^2]$

and for $\tau > 1$: $$\hat{\varepsilon}_{40+\tau-1} = \hat{y}_{40+\tau-1} - [283.94906 - 9.21968(40+\tau-1) + .35348(40+\tau-1)^2 + 70.10688Q_1 - 35.42856Q_2 - 126.52509Q_3]$$

(5) For period 41, $$\hat{y}_{41} = 283.94906 - 9.21968(41) + .35348(41)^2 + 70.10688(1) - 35.42856(0) - 126.52509(0) + .59408\hat{\varepsilon}_{40} = 570.24894 + .59408\hat{\varepsilon}_{40}$$

Since $y_{40} = 539.78$, we have

$$\hat{\varepsilon}_{40} = 539.78 - [283.94906 - 9.21968(40) + .35348(40)^2 + 70.10688(0) - 35.42856(0) - 126.52509(0)] = 539.78 - 480.72986 = 59.05014$$

Hence, $\hat{y}_{41}$ = 570.24894 + .59408 (59.05014) = 605.3285.

An approximate 95% prediction interval of y_{41} is

[$\hat{y}_{41} \pm z_{[.025]}s$] = [605.3285 $\pm$ 1.96 (50.25132)] = [506.84, 703.82]

For time period 42, $$\hat{y}_{42} = 283.94906 - 9.21968(42) + .35348(42)^2 + 70.10688(0) - 35.42856(1) - 126.52509(0) + .59408\hat{\varepsilon}_{41} = 484.83266 + .59408\hat{\varepsilon}_{41}$$

Here, $\hat{\varepsilon}_{41} = \hat{y}_{41} - [283.94906 - 9.21968(41) + .35348(41)^2$

$$+ 70.10688(1) - 35.42856(0) - 126.52509(0)]$$

$$= 605.3285 – 570.24894 = 35.07956$$

Hence, $\hat{y}_{42}$ = 484.83266 + .59408 (35.07956) = 505.6717

An approximate 95% prediction interval for y_{42} is

$$[\hat{y}_{42} \pm z_{[.025]} s\sqrt{1+\left(\hat{\phi}_1\right)^2}\ ] = [505.6717 \pm 1.96\ (50.25132)\ \sqrt{1+(.59408)^2}\ ]$$

$$= [505.6717 \pm 1.96\ (58.4501)] = [391.11, 620.23]$$

In a similar fashion we find that a point prediction of y_{43} is $\hat{y}_{43} = 426.9411$ and that an approximate 95% prediction interval for y_{43} is

$$[\ \hat{y}_{43} \pm 1.96 s\sqrt{1+\left(\hat{\phi}_1\right)^2 + \left(\hat{\phi}_1\right)^4}\ ]$$

$$= [426.9411 \pm 1.96\,(50.25132)\,\sqrt{1+(.59408)^2 + (.59408)^4}\ ] = [307.2235, 546.6591]$$

Finally, we find that a point prediction of y_{44} is $\hat{y}_{44} = 569.9732$ and that an approximate 95% prediction interval for y_{44} is

$$[\hat{y}_{44} \pm 1.96 s\sqrt{1+(\phi_1)^2 + (\phi_1)^4 + (\phi_1)^6}\ ]$$

$$= [569.9732 \pm 1.96(50.25132)\,(\sqrt{1+(.59408)^2 + (.59408)^4 + (.59408)^6})]$$

$$= [448.4872, 691.4592]$$

2. Exercise 8.2.

a. All values in spreadsheet should agree with the values in Figure 8.1.

b. When $\alpha = 0.4$, *SSE* = 35,688.

d. Resulting values should agree with the values in Figure 8.2.

3. Exercise 8.3.

a. The point forecast for the cod catch in time period 28 is

$$\hat{y}_{28}(24) = \hat{y}_{24+4}(24) = \ell_{24} = 354.5438$$

The 95% prediction interval is

$$\left[\ell_{24} \pm z_{[.025]} s\sqrt{1+3\alpha^2}\right] = 354.5438 \pm 1.96\ (34.95)\ \sqrt{1+3(.034)^2}$$

$$= 354.5438 \pm 68.6207 = [285.9231, 423.1645]$$

b. The point forecast for the cod catch in time period 29 is

$$\hat{y}_{29}(24) = \hat{y}_{24+5}(24) = \ell_{24} = 354.5438$$

The 95% prediction interval is

$$\ell_{24} \pm z_{[.025]} s\sqrt{1+4\alpha^2} = [354.5438 \pm 1.96\ (34.95)\,\sqrt{1+4(.034)^2}\ ]$$

$$= [354.5438 \pm 68.6602] = [285.8836, 423.2040].$$