Homework 5
SOLUTIONS

1. Using the simplex method, determine if the following LPs / tableaus are degenerate, unbounded, or have multiple solutions (or neither of these options). Use Bland's rules for choosing the entering variable.

   (a)

$$
\begin{aligned}
\max \quad z =& 10x_1 + 5x_2 \\
\text{subject to} & \\
& x_1 + 3x_2 \leq 6 \\
& -2x_1 + x_2 \geq 0 \\
& x_i \geq 0 \qquad \forall i = 1, 2
\end{aligned}
$$

Solution:

-10	-5	0	0	0
1	3	1	0	6
-2	1	0	-1	0

Need to get the -1 to be a +1 for the identity matrix. Note we have a feasible solution because -0 is still greater than or equal to 0.

-10	-5	0	0	0
1	3	1	0	6
[2]	-1	0	1	0

Degenerate because a basic variable x_4 is 0. Pivot on x_1, winner of ratio test is row 2 with value 0.

0	-10	0	5	0
0	[7/2]	1	-1/2	6
1	-1/2	0	1/2	0

Now enter x_2 into basis. Winner of ratio test is row 1, because the coefficient in row 2 is negative.

0	0	20/7	5/2-10/7	120/7
0	1	2/7	-1/7	12/7
1	0	1/7	3/7	6/7

Optimal solution found, no 0's in row 0, so we have just one optimal solution. Problem was Degenerate.

(b)

$$\begin{aligned} \max \quad z =& 10x_1 + 5x_2 \\ \text{subject to} & \\ & x_1 + 3x_2 \geq 6 \\ & -2x_1 + x_2 \leq 0 \\ & x_i \geq 0 \qquad \forall i = 1, 2 \end{aligned}$$

Solution:

-10	-5	0	0	0
[1]	3	-1	0	6
-2	1	0	1	0

This time to remove the -1 we have an infeasible starting basic solution. If we let $x_1 = 6$, we can remain feasible with x_4 in the basis and taking value 12.

0	25	-10	0	60
1	3	-1	0	6
0	7	-2	1	12

Now we have a negative coefficient with negative coefficients in all rows. This means that every ratio test is negative. Therefore the solution is unbounded. Note that the point in which we had the degeneracy was not feasible, therefore this problem IS NOT degenerate.

(c)

$$\begin{aligned} \max \quad z =& 5x_1 + 15x_2 \\ \text{subject to} & \\ & x_1 + 3x_2 \leq 6 \\ & -2x_1 + x_2 \geq 0 \\ & x_i \geq 0 \qquad \forall i = 1, 2 \end{aligned}$$

Solution:

-5	-15	0	0	0
1	3	1	0	6
[2]	-1	0	1	0

Here the constraints are the same as in problem a. So we can do the same thing we did there. We still have degeneracy.

0	-35/2	0	5/2	0
0	[7/2]	1	-1/2	6
1	-1/2	0	1/2	0

0	0	5	0	30
0	1	2/7	-1/7	12/7
1	0	1/7	3/7	6/7

We have an optimal solution, and a 0 coefficient in Row 0 for a non-basic variable. Therefore there are multiple optimal solutions. The problem is also degenerate.

(d)

$$\max \quad z = 6x_1 + 15x_2$$

subject to

$$\begin{aligned} & x_1 + 2x_2 \leq 6 \\ & 3x_1 + 2x_2 \leq 2 \\ & x_1 + x_2 \leq 6 \\ & x_1 \leq 3 \\ & x_i \geq 0 \qquad \forall i = 1, 2 \end{aligned}$$

Solution:

-6	-15	0	0	0	0	0
1	2	1	0	0	0	6
[3]	2	0	1	0	0	2
1	1	0	0	1	0	6
1	0	0	0	0	1	3

0	-11	0	2	0	0	4
0	4/3	1	-1/3	0	0	16/3
1	[2/3]	0	1/3	0	0	2/3
0	1/3	0	-1/3	1	0	16/3
0	-2/3	0	-1/3	0	1	7/3

16.5	0	0	15/2	0	0	15
-2	0	1	-1	0	0	4
3/2	1	0	1/2	0	0	1
-1/2	0	0	-1/2	1	0	5
1	0	0	0	0	1	3

2. **OPTIONAL Cycling Example.** Complete 6 iterations of Simplex using the following rules of selection: Choose most negative variable to enter basis. In case of ratio tie, pick the lowest numbered row with a positive coefficient in the entering column. (The first iteration you select column 1 row 1).

-3/4	150	-1/50	6	0	0	0	0
1/4	-60	-1/25	9	1	0	0	0
1/2	-90	-1/50	3	0	1	0	0
0	0	1	0	0	0	1	1