Quiz 9 (25 mins), MA1501 Calculus I, 10/24/2013 PRINTED NAME: ________________GT ID:____________

This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

---

1. (a) (15 pts) Using ϵ and N, give the definition of the statement: the sequence $\{a_n\}$ converges to the L, or simply $\lim_{n\to\infty} a_n = L$.

(b) (20 pts) Find the limit of the sequence

$$a_n = \frac{1}{2^n}\sin^2 n.$$

Answer: (a) For any $\epsilon > 0$, there exsits an integer N such that

$$\text{for all } n > N \implies |a_n - L| < \epsilon.$$

(b) The range of $\sin^2 n$ is $[0,1]$. Observe that

$$0 \le \frac{1}{2^n}\sin^2 n \le \frac{1}{2^n}.$$

Since $\lim_{n\to\infty} \frac{1}{2^n} = 0$, by the Sandwich Theorem for sequences, the limit of the sequence is 0.

2. Right, or wrong? Say which for each formula and give a reason for each answer.

(a) (15 pts) $\int x\sin x\, dx = -x\cos x + \sin x + C$

$\to$RIGHT.

$\int f(x)dx = F(x)$ is right if $\frac{d}{dx}F(x) = f(x)$. Otherwise it is wrong.

$$\begin{aligned}(-x\cos x + \sin x + C)' &= -\cos x + x\sin x + \cos x \\ &= x\sin x.\end{aligned}$$

(b) (20 pts) $\int -\frac{15(x+3)^2}{(x-2)^4}dx = \left(\frac{x+3}{x-2}\right)^3 + C$

$\to$RIGHT.

$$\begin{aligned}\frac{d}{dx}\left(\left(\frac{x+3}{x-2}\right)^3 + C\right) &= 3\left(\frac{x+3}{x-2}\right)^2 \frac{d}{dx}\left(\frac{x+3}{x-2}\right) \ (\textit{chain rule}) \\ &= 3\left(\frac{x+3}{x-2}\right)^2 \frac{(x-2)-(x+3)}{(x-2)^2} \ (\textit{quotient rule}) \\ &= 3\left(\frac{x+3}{x-2}\right)^2 \frac{-5}{(x-2)^2} \\ &= -\frac{15(x+3)^2}{(x-2)^4}\end{aligned}$$

3. (30 pts) We use finite approximations to estimate the area under the graph of the function

$$f(x) = \frac{1}{x} + 1, \quad x \in [1, 5].$$

Evaluate the following finite sums: (a) a lower sum with four rectangles of equal width and (b) an upper sum with four rectangles of equal width. (Hint: draw the graph of $f(x)$ first)

Answer: width $\Delta x = \frac{5-1}{4} = 1$ and $[1, 5]$ is divided into

$$[1, 2], [2, 3], [3, 4], [4, 5].$$

Since f is decreasing on $[1, 5]$, we use left endpoints to obtain upper sums and right endpoints to obtain lower sums.
(a) a lower sum is

$$\begin{aligned}
(\frac{1}{2}+1)\cdot 1 + (\frac{1}{3}+1)\cdot 1 + (\frac{1}{4}+1)\cdot 1 + (\frac{1}{5}+1)\cdot 1 &= \frac{30}{60} + \frac{20}{60} + \frac{15}{60} + \frac{12}{60} + 4 \\
&= \frac{77}{60} + \frac{240}{60} \\
&= \frac{317}{60}
\end{aligned}$$

(b) an upper sum is

$$\begin{aligned}
(1+1)\cdot 1 + (\frac{1}{2}+1)\cdot 1 + (\frac{1}{3}+1)\cdot 1 + (\frac{1}{4}+1)\cdot 1 &= 1 + \frac{6}{12} + \frac{4}{12} + \frac{3}{12} + 4 \\
&= \frac{13}{12} + \frac{60}{12} \\
&= \frac{73}{12}
\end{aligned}$$