ISyE 4031 Regression and Forecasting
Homework 11 Solutions (Not collected)
Spring 2016

1. Exercise 9.4.

a. Since the SAC dies down quickly and the SPAC cuts off after lag 1, the first order autoregressive model $z_t = \delta + \phi_1 z_{t-1} + a_t$ describes $z_t = y_t - y_{t-1}$ (see Table 9.5).

b. $z_t = \delta + \phi_1 z_{t-1} + a_t$

$$y_t - y_{t-1} = \delta + \phi_1 (y_{t-1} - y_{t-2}) + a_t$$

$$y_t = \delta + y_{t-1} + \phi_1 (y_{t-1} - y_{t-2}) + a_t$$

c. If we include the constant term δ, we are assuming that the original time series values display a deterministic trend. From Figure 9.18 we see that this deterministic trend is in the upward direction.

d. 1. $y_3 = 244.09$

$$\hat{y}_3 = \hat{\delta} + y_2 + \hat{\phi}_1 (y_2 - y_1) = 3.06464 + 239 + .64774\,(239 - 235) = 244.6556$$

$$y_3 - \hat{y}_3 = 244.09 - 244.6556 = -.5656$$

2. $\hat{y}_{91}(90) = \hat{\delta} + y_{90} + \hat{\phi}_1 (y_{90} - y_{89})$

= 3.06464 + 1029.480 + .64774 (1029.480 – 1018.420) = 1039.7086

$\hat{y}_{92}(90) = \hat{\delta} + \hat{y}_{91}(90) + \hat{\phi}_1 (\hat{y}_{91}(90) - y_{90})$

$= 3.06464 + 1039.7086 + .64774(1039.7086 - 1029.480) = 1049.3987$

$\hat{y}_{93}(90) = \hat{\delta} + \hat{y}_{92}(90) + \hat{\phi}_1 (\hat{y}_{92}(90) - \hat{y}_{91}(90))$

= 3.06464 + 1049.3987 + .64774 (1049.3987 – 1039.7086) = 1058.7400.

3. [1039.7086 $\pm$ 5.4831] = [1034.2255, 1045.1917]

[1049.3987 $\pm$ 10.5683] = [1038.8304, 1059.9670]

[1058.7400 $\pm$ 15.4975] = [1043.2425, 1074.2375]

2. Exercise 9.5.

$\hat{y}_{123}(122) = y_{122} + \hat{a}_{123} - \theta_1 \hat{a}_{122}$

= 15.9265 + 0 $-(-.3534)(y_{122} - \hat{y}_{122}(121))$

= 15.9265 + .3534 (15.9265 – 16.1880) = 15.8341

$\hat{y}_{124}(122) = \hat{y}_{123}(122) + \hat{a}_{124} - \theta_1 \hat{a}_{123}$

$= 15.8341 + 0\ (-.3534)(0) = 15.8341$

3. Exercise 9.6. The SAC of the original viscosity values cuts off after lag 2. Therefore, the original viscosity values are stationary. Moreover, the SPAC dies down quickly. Thus, we identify the second-order moving average model

$$y_t = \delta + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2}$$

Here we include the constant term δ because $\bar{y}$ is not near zero.

4. Exercise 10.1. From the output: $\hat{\phi} = 0.6591$. The stationarity condition is $|\phi_1| < 1$. This condition is satisfied because $|\hat{\phi}_1| = 0.6591 < 1$.

5. Exercise 10.3. By using the Minitab output, we find that the *t*-statistic related to $\hat{\phi}_1$ is 8.15. Since $|t| = 8.15 > t_{[a/2]}^{(n-n_p)} = t_{[.025]}^{(90-2)} = 1.96$, we reject H_0: $\phi_1 = 0$. We conclude that ϕ_1 is significant and should be retained in the model.

The *t*-statistic related to $\hat{\delta}$ is 10.31. Since $|t| = 10.31 > t_{[.025]}^{(90-2)} = 1.96$, we conclude that δ should be retained in the model (reject H_0: $\delta = 0$).

6. Exercise 10.5. By using the Minitab output,

$$Q^* = 8.9 < x_{[.05]}^2(12-1) = 19.6751.$$

We do not reject H_0: The model is adequate.

7. Exercise 10.6.

$\hat{y}_{91}(90) = 1039.7086$ and $SE_{91}(90) = 2.7975$.

Since $t_{[a/2]}^{(n-n_p)} = 2.576$, the 99% prediction interval for y_{91} is

[1039.7086 $\pm$ 2.576 (2.7975) = [1039.7086 $\pm$ 7.2064] = [1032.5022, 1046.9150]

8. Exercise 10.7. The final point estimates of δ, θ_1, and θ_2 are

$$\hat{\delta} = 35.1822, \quad \hat{\theta}_1 = .5237, \text{ and } \hat{\theta}_2 = -.6569.$$

The invertibility conditions are $\theta_1 + \theta_2 < 1, \theta_2 - \theta_1 < 1$, and $|\theta_2| < 1$. These conditions are satisfied since

$$\hat{\theta}_1 + \hat{\theta}_2 = 0.5237 + (-0.6569) = -.1332 < 1$$

$$\hat{\theta}_2 - \hat{\theta}_1 = -0.6569 - 0.5237 = -1.1806 < 1$$

$$|\hat{\theta}_2| = 0.6569 < 1$$

9. Exercise 10.8.

For δ we have | *t* | = 158.23 > 1.96, we reject H_0: $\delta = 0$,
For θ_1 we have | *t* | = 8.29 > 1.96, we reject H_0: $\theta_1 = 0$,
For θ_2 we have | *t* | = 10.42 > 1.96, we reject H_0: $\theta_2 = 0$.

We conclude that each of δ, θ_1, and θ_2 should be retained in the model, i.e., all of them are significant.

10. Exercise 10.9. Since each of the *p*-values, i.e., 0.318 (*k* = 12), 0.369 (*k* = 24), 0.745 (*k* = 36), and 0.76 (*k* = 48) associated with Q^* is substantially greater than 0.05, we do not reject model adequacy at any *k* and conclude that the chemical product model is adequate.

11. Exercise 11.2. No, the SAC for the trade values obtained by using the transformation $z_t = y_t - y_{t-1}$ dies down extremely slowly at the seasonal level.

12. Exercise 11.3. No, the SAC for the trade values obtained by using the transformation $z_t = y_t - y_{t-12}$ dies down extremely slowly at the nonseasonal level.

13. Exercise 11.4. Yes, the trade values obtained by using the transformation $z_t = y_t - y_{t-1} - y_{t-12} + y_{t-13}$ should be considered stationary. At the seasonal level the SAC has a spike at lag 12 and cuts off after lag 12. At the nonseasonal level the SAC has a spike at lag 1 and cuts off after lag 1.

14. Exercise 11.5. At the nonseasonal level the SAC has a spike at lag 1 and cuts off after lag 1 and SPAC dies down (or possibly cuts off after lag 1).
This suggests using the nonseasonal moving average model of order 1

$$z_t = \delta + a_t - \theta_1 a_{t-1}$$

At the seasonal level the SAC has a spike at lag 12 and cuts off after lag 12 and the SPAC dies down. This suggests using the seasonal moving average model of order 1

$$z_t = \delta + a_t - \theta_{1,12} a_{t-12}.$$