ISyE 2027C Quiz 1 Solutions and some class notes January 13 Spring 2015

You draw three cards from a standard 52-card deck.

1. (30 points) Let A denote the event that all three cards are the same suit. Find $P(A)$, the probability that A occurs.

$$4\frac{\binom{13}{3}}{\binom{52}{3}} \text{ or } \frac{12}{51}\frac{11}{50}$$

2. (30 points) Let B denote the event that the three cards form a sequence (e.g. 2,3,4 or 10,J,Q). Find $P(B)$, the probability that B occurs.

$$\frac{11 \cdot 4^3}{\binom{52}{3}}$$

3. (20 points) Find the probability $P(A \cap B)$ that both A and B occur.

$$\frac{11 \cdot 4}{\binom{52}{3}}$$

4. (20 points) Find $P(A \cup B)$, the probability that A or B or both occur. Use whatever notation you need to make it easy to write your answer. For example, you've calculated $P(A)$ already, so you can write $P(A)$ instead of copying your formula.

   Full credit for $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

For the problem of finding $P(C^C \cap D)$ where $P(C) = .3; P(D) = .4; P(C \cap D) = .2$, the quickest way is to use the law of total probability, which we have not learned yet. A slower way is as follows: $1 = P(C \cap D) + P(C^C \cap D) + P(C \cap D^C) + P(C^C \cap D^C)$ because exactly one of these events must happen. This uses the property that if two events are disjoint the probability of their union equals the sum of their probabilities. We are given $P(C \cap D) = .2$. Apply the rule $P(A) + P(A^C) = 1$[1] and deMoivre's rule to the fourth term to get

$$P(C^C \cap D^C) = 1 - P((C^C \cap D^C)^C) = 1 - P(C \cup D) = 1 - (P(C) + P(D) - P(D \cap C)).$$

This implies

$$P(C^C \cap D^C) = 1 - (.3 + .4 - .2) = .5 \Rightarrow P(C^C \cap D) + P(C \cap D^C) = 1 - .5 - .2 = .3$$

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Similarly

$$P(C^C \cap D) = 1 - P(C \cup D^C) = 1 - (.3 + (1 - .4) - P(C \cap D^C)) \Rightarrow P(C^C \cap D) = 0.1 + P(C \cap D^C).$$

Then

$$P(C^C \cap D) = 0.1 + .3 - P(C^C \cap D) \Rightarrow P(C^C \cap D) = 0.2$$

[1] Here A is $C^C \cap D^C$

Keno: you pick 15 (different) numbers from 1 to 80. The house picks 20, independent of what you pick. You bet that they will match at least 11 of your 15. The chance that you win is the sum of 5 terms, one for exactly 11 matches, one for exactly 12 matches, ..., one for exactly 15 matches. The sum is

$$\frac{\binom{15}{11}\binom{65}{9}+\binom{15}{12}\binom{65}{8}+\binom{15}{13}\binom{65}{7}+\binom{15}{14}\binom{65}{6}+\binom{15}{15}\binom{65}{5}}{\binom{80}{20}}$$

Explanation of the first term: there are $\binom{15}{11}$ choices of which 11 of your numbers get matched. For each such choice, the house can pick those 11 plus 9 more *that are not among your 15* in $\binom{65}{9}$ ways. If instead you write

$$\binom{15}{11}\binom{69}{9}$$

you are counting ways that the house could match 12 or more of your numbers, too.

You might think that that is a good idea, since you would have one term instead of 5. But you would not be counting eaach way once. You would count matching 15 items $\binom{15}{11}$ times!