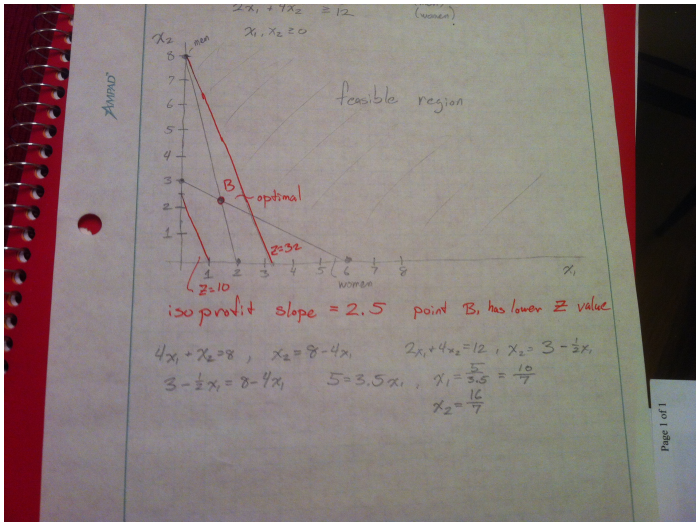
Homework 3
SOLUTIONS

1. Model the following, graph the feasible region, and find the optimal solution:

   A company wants to advertise its products, and has two options of ad types to run. A television commercial will cost them \$10 and a magazine ad will cost \$4. Suppose each television commercial reaches 4 new men and 2 new women, while each magazine ad is seen by 1 new man and 4 new women. The company wants to minimize their costs while running enough ads to reach 8 new men and 12 new women.

$$\begin{aligned} \min \quad & z = 10x_1 + 4x_2 && && (1)\\ & \text{subject to} \\ & 4x_1 + x_2 \geq 8 && \text{Men} && (2)\\ & 2x_1 + 4x_2 \leq 12 && \text{Women} && (3)\\ & x_i \geq 0 \qquad \forall i = 1, 2 && && (4) \end{aligned}$$



2. Convert the model of the previous problem into the standard form, using e for excess and all $e > 0$.

$$\begin{aligned}
\min \quad z =& 10x_1 + 4x_2 + 0e_1 + 0e_2 && && (5)\\
& \text{subject to} \\
& 4x_1 + x_2 - e_1 = 8 && \text{Men} && (6)\\
& 2x_1 + 4x_2 - e_2 = 12 && \text{Women} && (7)\\
& x_i \geq 0 \qquad \forall i = 1, 2 && && (8)\\
& e_i \geq 0 \qquad \forall i = 1, 2 && && (9)
\end{aligned}$$

3. Problem 1 on page 55 of the text book.

   Note: The variables are in terms of acres planted, but the price you receive is per bushel.

$$\begin{aligned}
\max \quad z =& (3)(10)x_1 + (4)(25)x_2 && && (10)\\
& \text{subject to} \\
& x_1 + x_2 \leq 7 && \text{Acres} && (11)\\
& 4x_1 + 10x_2 \leq 40 && \text{Labor} && (12)\\
& 10x_1 \geq 30 && \text{Government} && (13)\\
& x_i \geq 0 \qquad \forall i = 1, 2 && && (14)
\end{aligned}$$

4. Convert the model you just came up with for Farmer Jones into the standard form.

$$\begin{aligned}
\max \quad z =& (3)(10)x_1 + (4)(25)x_2 + 0x_3 + 0x_4 + 0x_5 && && (15)\\
& \text{subject to} \\
& x_1 + x_2 + x_3 = 7 && \text{Acres} && (16)\\
& 4x_1 + 10x_2 + x_4 = 40 && \text{Labor} && (17)\\
& 10x_1 - x_5 = 30 && \text{Government} && (18)\\
& x_i \geq 0 \qquad \forall i = 1, 2, 3, 4, 5 && && (19)
\end{aligned}$$

5. Provide at least 2 basic solutions to the standard form of the Farmer Jones problem. At least one of those 2 should be a basic feasible solution.

   There are 9 possible combinations/points you could have chosen (the x_1,x_5 combo is impossible). All that is required for a basic solution is 2 of the variables to be equal to 0.

   First look at the easiest solution: x_1,$x_2 = 0$. Then $x_3 = 7$,$x_4 = 40$,$x_5 = -30$. This is not feasible because x_5 is negative. Pick it to be non-basic and equal to 0 in our next point, which would require x_1 to be basic. Leave x_2 non-basic. $x_1 = 3$,$x_2 = 0$,$x_3 = 4$,$x_4 = 28$,$x_5 = 0$. This is a feasible solution and relates to planting just enough corn to satisfy the government constraint.

6. In the solutions to Homework 2 problem 3, I identified a number of points on the graph. For each point identify if it is a basic solution. If it is a basic solution identify if it is a basic feasible solution. If it is a basic feasible solution, identify the basic feasible solutions it is adjacent to.

	Basic?	Feasible?	Adjacent BFS?
A	Yes	No	n/a
B	Yes	Yes	D,C
C	Yes	Yes	E,B
D	Yes	Yes	E,B
E	Yes	Yes	D,C
F	Yes	No	n/a