**ISyE 3232 Exam # 2**
**Spring 2013**

**Name**

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK.

**Question 1**
**Question 2**
**Question 3**
**Question 4**
**Total**

(25) **1.** Three students arrive at the beginning of a professor's office hours. The amount of time they will stay is exponentially distributed with means 1, 1/2, and 1/3 hours, respectively.

**a.** (10) What is the expected time until only one student remains?

**b.** (15) What is the expected time until all three students are gone?

(25) A six-sided die is rolled repeatedly. After each roll $n = 1, 2, ...$, let $X_n$ be the largest number rolled in the first $n$ rolls. Can you model $\{X_n, n \geq 1\}$ using a discrete-time Markov chain? If so, answer the following questions

**a.** (10) What is the state space and the transition probability matrix?

**b.** (15) Is the Markov chain irreducible? Classify the states. Do the long-run probabilities exist? .

(25) **3.** Suppose a person's emotional mood on a particular day is dependent only on his mood the previous two days. If he was happy both yesterday and today, he will be happy tomorrow with probability 0.4. If he was happy today but not yesterday, he will be happy tomorrow with probability 0.7. If he was happy yesterday but not today, he will be happy tomorrow with probability 0.35. Finally, if he was sad both the previous days, he will be happy tomorrow with probability 0.8. What is the long-run probability that he is happy?

(25) **4.** Traffic on Lavista Road follows a Poisson process with rate 2/3 vehicles per minute. Assume that 10% of the vehicles are trucks, the other 90% are cars.

**a.** (5) What is the probability that at least one truck passes in an hour?

**b.** (10) Given that 10 trucks passed by in an hour, what is the expected number of vehicles that have passed in this one hour?

**c.** (10) Given that 40 vehicles passed by in an hour, what is the probability that there were exactly 5 trucks and 35 cars?