8-47 a) 95% upper CI and df = 24 $\chi^2_{1-\alpha,df} = \chi^2_{0.95,24} = 13.85$

b) 99% lower CI and df = 9 $\chi^2_{\alpha,df} = \chi^2_{0.01,9} = 21.67$

c) 90% CI and df = 19

$\chi^2_{\alpha/2,df} = \chi^2_{0.05,19} = 30.14$ and $\chi^2_{1-\alpha/2,df} = \chi^2_{0.95,19} = 10.12$

8-56 a) 99% two-sided confidence interval on σ^2

$n = 10$ $s = 1.913$ $\chi^2_{0.005,9} = 23.59$ and $\chi^2_{0.995,9} = 1.73$

$$\frac{9(1.913)^2}{23.59} \le \sigma^2 \le \frac{9(1.913)^2}{1.73}$$
$$1.396 \le \sigma^2 \le 19.038$$

b) 99% lower confidence bound for σ^2

For $\alpha = 0.01$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.01,9} = 21.67$

$$\frac{9(1.913)^2}{21.67} \le \sigma^2$$
$$1.5199 \le \sigma^2$$

c) 90% lower confidence bound for σ^2

For $\alpha = 0.1$ and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.1,9} = 14.68$

$$\frac{9(1.913)^2}{14.68} \le \sigma^2$$
$$2.2436 \le \sigma^2$$
$$1.498 \le \sigma$$

d) The lower confidence bound of the 99% two-sided interval is less than the one-sided interval. The lower confidence bound for σ^2 is in part (c) is greater because the confidence is lower.

8-64 a) 95% Confidence Interval on the true proportion of helmets showing damage

$$\hat{p} = \frac{18}{50} = 0.36 \qquad n = 50 \qquad z_{\alpha/2} = 1.96$$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.36 - 1.96\sqrt{\frac{0.36(0.64)}{50}} \le p \le 0.36 + 1.96\sqrt{\frac{0.36(0.64)}{50}}$$

$$0.227 \le p \le 0.493$$

b) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$

$n \cong 2213$

c) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$

8-82 90% prediction interval on wall thickness on the next bottle tested.

Given $\bar{x} = 4.05$ s = 0.08 n = 25 for $t_{\alpha/2,n-1} = t_{0.05,24} = 1.711$

$$\bar{x} - t_{0.05,24}s\sqrt{1+\frac{1}{n}} \le x_{n+1} \le \bar{x} + t_{0.05,24}s\sqrt{1+\frac{1}{n}}$$

$$4.05 - 1.711(0.08)\sqrt{1+\frac{1}{25}} \le x_{n+1} \le 4.05 - 1.711(0.08)\sqrt{1+\frac{1}{25}}$$

$$3.91 \le x_{n+1} \le 4.19$$

8-94 a) 90% tolerance interval on wall thickness measurements that have a 90% CL

Given $\bar{x} = 4.05$ s = 0.08 n = 25 we find *k*=2.077

$$\bar{x} - ks,\ \bar{x} + ks$$

$$4.05 - 2.077(0.08),\ 4.05 + 2.077(0.08)$$

$$(3.88,\ 4.22)$$

The lower bound of the 90% tolerance interval is much lower than the lower bound on the 95% confidence interval on the population mean ($4.023 \le \mu \le \infty$)

b) 90% lower tolerance bound on bottle wall thickness that has confidence level 90%.

given $\bar{x} = 4.05$ s = 0.08 n = 25 and k = 1.702

$$\bar{x} - ks = 4.05 - 1.702(0.08) = 3.91$$

The lower tolerance bound is of interest if we want the wall thickness to be greater than a certain value so that a bottle will not break.

9-1 a) $H_0: \mu = 25,\ H_1: \mu \neq 25$ Yes, because the hypothesis is stated in terms of the parameter of interest, inequality is in the alternative hypothesis, and the value in the null and alternative hypotheses matches.

b) $H_0: \sigma > 10,\ H_1: \sigma = 10$ No, because the inequality is in the null hypothesis.

c) $H_0: \bar{x} = 50,\ H_1: \bar{x} \neq 50$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

d) $H_0: p = 0.1,\ H_1: p = 0.3$ No, the values in the null and alternative hypotheses do not match and both of the hypotheses are equality statements.

e) $H_0: s = 30,\ H_1: s > 30$ No, because the hypothesis is stated in terms of the statistic rather than the parameter.

9-10 a) $\alpha = P(\bar{X} \leq 98.5) + P(\bar{X} > 101.5)$

$$= P\left(\frac{\bar{X}-100}{2/\sqrt{9}} \leq \frac{98.5-100}{2/\sqrt{9}}\right) + P\left(\frac{\bar{X}-100}{2/\sqrt{9}} > \frac{101.5-100}{2/\sqrt{9}}\right)$$

$= P(Z \leq -2.25) + P(Z > 2.25) = (P(Z \leq -2.25)) + (1 - P(Z \leq 2.25))$

$= 0.01222 + 1 - 0.98778 = 0.01222 + 0.01222 = 0.02444$

b) $\beta = P(98.5 \leq \bar{X} \leq 101.5 \text{ when } \mu = 103)$

$$= P\left(\frac{98.5-103}{2/\sqrt{9}} \leq \frac{\bar{X}-103}{2/\sqrt{9}} \leq \frac{101.5-103}{2/\sqrt{9}}\right)$$

$= P(-6.75 \leq Z \leq -2.25) = P(Z \leq -2.25) - P(Z \leq -6.75) = 0.01222 - 0 = 0.01222$

c) $\beta = P(98.5 \leq \bar{X} \leq 101.5 \mid \mu = 105)$

$$= P\left(\frac{98.5-105}{2/\sqrt{9}} \leq \frac{\bar{X}-105}{2/\sqrt{9}} \leq \frac{101.5-105}{2/\sqrt{9}}\right)$$

$= P(-9.75 \leq Z \leq -5.25) = P(Z \leq -5.25) - P(Z \leq -9.75) = 0 - 0 = 0$

The probability of failing to reject the null hypothesis when it is actually false is smaller in part (c) because the true mean, μ = 105, is further from the acceptance region. That is, there is a greater difference between the true mean and the hypothesized mean.

9-12 $\mu_0 - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \overline{X} \le \mu_0 + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$, where σ =2

a) α = 0.01, n = 9, then $z_{\alpha/2}$ =2,57, then $98.29, 101.71$

b) α = 0.05, n = 9, then $z_{\alpha/2}$ =1.96, then $98.69, 101.31$

c) α = 0.01, n = 5, then $z_{\alpha/2}$ = 2.57, then $97.70, 102.30$

d) α = 0.05, n = 5, then $z_{\alpha/2}$ =1.96, then $98.25, 101.75$

9-13 δ = 103 - 100 = 3

δ > 0 then $\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$, where σ = 2

a) β = P(98.69< $\overline{X}$ <101.31|μ=103) = P(-6.47<Z<-2.54) = 0.0055

b) β = P(98.25< $\overline{X}$ <101.75|μ=103) = P(-5.31<Z<-1.40) = 0.0808

c) As n increases, β decreases

9-14 a) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{98-100}{2/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(3)$) = 2(1 - 0.99865) = 0.0027

b) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{101-100}{2/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(1.5)$) = 2(1 - 0.93319) = 0.13362

c) P-value = 2(1 - $\Phi(|Z_0|)$) = 2(1 - $\Phi\left(\left|\frac{102-100}{2/\sqrt{9}}\right|\right)$) = 2(1 - $\Phi(3)$) = 2(1 - 0.99865) = 0.0027

9-40 a) SE Mean from the sample standard deviation $= \frac{s}{\sqrt{N}} = \frac{1.015}{\sqrt{16}} = 0.2538$

$$z_0 = \frac{15.016 - 14.5}{1.1/\sqrt{16}} = 1.8764$$

P-value= $1 - \Phi(Z_0) = 1 - \Phi(1.8764) = 1 - 0.9697 = 0.0303$

Because the P-value < $\alpha = 0.05$, reject the null hypothesis that $\mu = 14.5$ at the 0.05 level of significance.

b) A one-sided test because the alternative hypothesis is $\mu > 14.5$

c) 95% lower CI of the mean is $\bar{x} - z_{0.05}\frac{\sigma}{\sqrt{n}} \le \mu$

$$15.016 - (1.645)\frac{1.1}{\sqrt{16}} \le \mu$$

$$14.5636 \le \mu$$

d) P-value = $2[1 - \Phi(Z_0)] = 2[1 - \Phi(1.8764)] = 2[1 - 0.9697] = 0.0606$

9-47 a)

1) The parameter of interest is the true mean speed, μ.

2) $H_0 : \mu = 100$

3) $H_1 : \mu < 100$

4) $z_0 = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

5) Reject H_0 if $z_0 < -z_\alpha$ where $\alpha = 0.05$ and $-z_{0.05} = -1.65$

6) $\bar{x} = 102.2$, $\sigma = 4$

$$z_0 = \frac{102.2 - 100}{4/\sqrt{8}} = 1.56$$

7) Because 1.56 > –1.65 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean speed is less than 100 at $\alpha = 0.05$.

b) $z_0 = 1.56$, then P-value= $\Phi(z_0) \cong 0.94$

c) $\beta = 1 - \Phi\left(-z_{0.05} - \frac{(95-100)\sqrt{8}}{4}\right)$ = 1-Φ(-1.65 - –3.54) = 1-Φ(1.89) = 0.02938

Power = 1-β = 1-0.0294 = 0.9706

d) n = $\dfrac{(z_\alpha + z_\beta)^2\sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.15})^2\sigma^2}{(95-100)^2} = \dfrac{(1.65+1.03)^2(4)^2}{(5)^2} = 4.60,$ n ≅ 5

e) 95% Confidence Interval

$$\mu \le \bar{x} + z_{0.05}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$\mu \le 102.2 + 1.65\left(\frac{4}{\sqrt{8}}\right)$$

$$\mu \le 104.53$$

Because 100 is included in the CI, there is not sufficient evidence to reject the null hypothesis.

9-61 a)

1) The parameter of interest is the true mean of body weight, μ.

2) H_0: μ = 300

3) H_1: μ ≠ 300

4) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

5) Reject H_0 if $|t_0| > t_{\alpha/2,n-1}$ where α = 0.05 and $t_{\alpha/2,n-1} = 2.056$ for n = 27

6) $\bar{x} = 325.496$, $s = 198.786$, n = 27

$$t_0 = \frac{325.496 - 300}{198.786/\sqrt{27}} = 0.6665$$

7) Because 0.6665 < 2.056 we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the true mean body weight differs from 300 at α = 0.05. We have 2(0.25) < P-value < 2(0.4). That is, 0.5 < P-value < 0.8

b) We reject the null hypothesis if P-value < α. The P-value = 2(0.2554) = 0.5108. Therefore, the smallest level of significance at which we can reject the null hypothesis is approximately 0.51.

c) 95% two sided confidence interval

$$\bar{x} - t_{0.025,26}\left(\frac{s}{\sqrt{n}}\right) \le \mu \le \bar{x} + t_{0.025,26}\left(\frac{s}{\sqrt{n}}\right)$$

$$325.496 - 2.056\left(\frac{198.786}{\sqrt{27}}\right) \le \mu \le 325.496 + 2.056\left(\frac{198.786}{\sqrt{27}}\right)$$

$$246.8409 \le \mu \le 404.1511$$

We fail to reject the null hypothesis because the hypothesized value of 300 is included within the confidence interval.

9-73 a) In order to use t statistics in hypothesis testing, we need to assume that the underlying distribution is normal.

1) The parameter of interest is the true mean distance, μ.

2) $H_0 : \mu = 280$

3) $H_1 : \quad \mu > 280$

4) $t_0 = \dfrac{\bar{x} - \mu}{s / \sqrt{n}}$

5) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $\alpha = 0.05$ and $t_{0.05,99} = 1.6604$ for n = 100

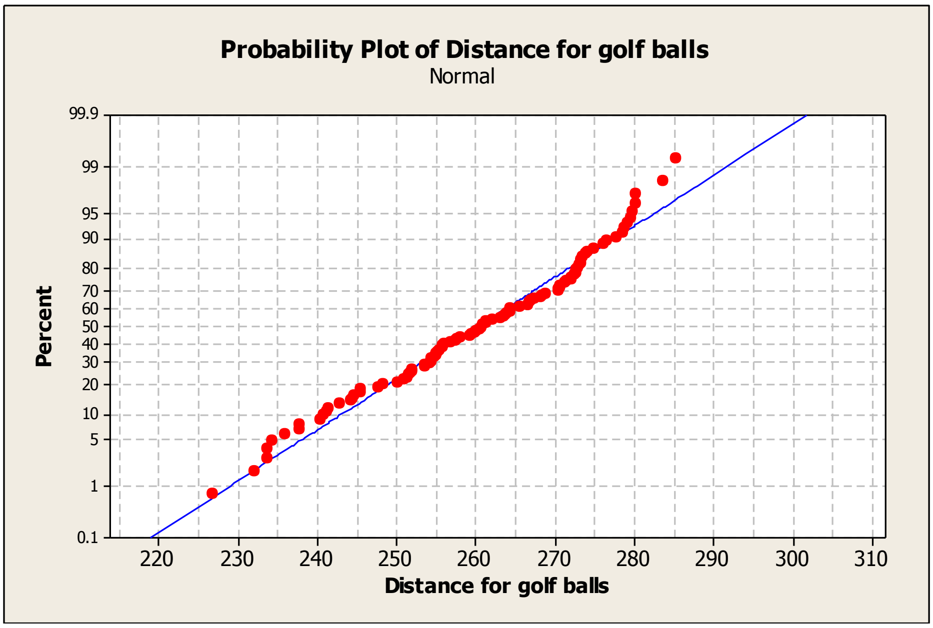
6) $\bar{x} = 260.3$ $\quad s = 13.41$ $\quad n = 100$

$$t_0 = \frac{260.3 - 280}{13.41 / \sqrt{100}} = -14.69$$

7) Because –14.69 < 1.6604 fail to reject the null hypothesis. There is insufficient evidence to conclude that the true mean distance is greater than 280 at $\alpha = 0.05$.

From Table V, the t_0 value in absolute value is greater than the value corresponding to 0.0005. Therefore, P-value > 0.9995.

b) From the normal probability plot, the normality assumption seems reasonable:



c) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|290 - 280|}{13.41} = 0.75$

Using the OC curve, Chart VII g) for $\alpha = 0.05$, $d = 0.75$, and $n = 100$, obtain $\beta \cong 0$ and power ≈ 1

d) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|290 - 280|}{13.41} = 0.75$

Using the OC curve, Chart VII g) for $\alpha = 0.05$, $d = 0.75$, and $\beta \cong 0.20$ (Power = 0.80), n =15

9-111

The estimated mean = 49.6741. Based on a Poisson distribution with $\lambda = 49.674$ the expected frequencies are shown in the following table. All expected frequencies are greater than 3.

The degrees of freedom are $k - p - 1 = 26 - 1 - 1 = 24$

Vechicles per minute	Frequency	Expected Frequency
40 or less	14	277.6847033
41	24	82.66977895
42	57	97.77492539
43	111	112.9507307
44	194	127.5164976
45	256	140.7614945
46	296	152.0043599
47	378	160.6527611
48	250	166.2558608
49	185	168.5430665
50	171	167.4445028
51	150	163.091274
52	110	155.7963895
53	102	146.0197251
54	96	134.3221931
55	90	121.3151646
56	81	107.6111003
57	73	93.78043085
58	64	80.31825
59	61	67.62265733
60	59	55.98491071
61	50	45.5901648
62	42	36.52661944

63	29	28.80042773
64	18	22.35367698
65 or more	15	62.60833394

a)

1) Interest is the form of the distribution for the number of cars passing through the intersection.

2) H_0: The form of the distribution is Poisson

3) H_1: The form of the distribution is not Poisson

4) The test statistic is

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

5) Reject H_0 if $\chi_o^2 > \chi_{0.05,24}^2 = 36.42$ for $\alpha = 0.05$

6) Estimated mean = 49.6741

$$\chi_0^2 = 1012.8044$$

7) Because 1012.804351 >> 36.42, reject H_0. We can conclude that the distribution is not a Poisson distribution at $\alpha = 0.05$.

b) P-value ≈ 0 (from computer software)

9-119 1) Interest is on the distribution of failures of an electronic component.

2) H_0: Type of failure is independent of mounting position.

3) H_1: Type of failure is not independent of mounting position.

4) The test statistic is:

$$\chi_0^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

5) The critical value is $\chi_{0.01,3}^2 = 11.344$ for $\alpha = 0.01$

6) The calculated test statistic is $\chi_0^2 = 10.71$

7) Because $\chi_0^2 \not> \chi_{0.01,3}^2$ fail to reject H_0. The evidence is not sufficient to claim that the type of failure is dependent on the mounting position at $\alpha = 0.01$. P-value = $P(\chi_0^2 > 10.71)$ = 0.013 (from computer software).