

**MATH 2602 K1-K3, Midterm 3 practice exercises, April 3**

**Problem 1 :** Let  $a_1 = 4, a_2 = 7$  and  $a_n = 10a_{n-1} - 21a_{n-2} + 7 + n$  for  $n \geq 3$ . Find  $a_n$  for all  $n$ .

**Problem 2:** How many integers between 1 and 200 (inclusive) are

- (a) divisible by at least one of 3, 5, 7?
- (b) divisible by neither 5 nor 11?
- (c) divisible by exactly one of 6 or 8?
- (d) divisible by 3 and by 5 but not by 10?
- (e) divisible by 5 but neither 3 nor 11?

**Problem 3:** Find the number of arrangements of the letters  $A, B, C, D, E, F, G, H$  which contain.

- (a) at least one of the patterns  $ABC$  or  $CED$ .
- (b) exactly one of the patterns  $ABC$  or  $CED$ .
- (c) contain neither  $ABC$  nor  $CED$ .

**Problem 4:** How many five-digit numbers can be formed

- (a) So that all the digits are distinct?
- (b) They have one or more repeated digits?

**Problem 5:** In a room where there are 51 people with ages between 1 and 100, show the following:

- (a) Either two people have the same age or there are two people whose ages are consecutive integers.
- (b) Either two people have the same age or one person's age is a multiple of another's.
- (c) Some people shake hands. Show that among those who shook at least one hand, two people shook the same number of hands.

**Problem 6:** An urn contains 10 red numbered balls and 6 white numbered balls. A sample of 8 balls is selected.

- (a) How many samples contain exactly 3 red balls?
- (b) How many samples contain at most 2 red balls?

**Problem 7:** In how many ways can 10 identical stones be distributed among 14 (labelled) boxes. So that

- (a) Each box contains no more than one stone.
- (b) The first box contains at least 3 stones and all the rest can contain any number of stones.

**Problem 8:** Find the coefficient of  $x^{10}$  in the expansion of  $(2x - \frac{1}{x})^{20}$ .