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# Homework 1

August 16, 2013

Due: at the start of class on Monday, August 26/ Tuesday, August 27

1. Suppose a beagle is working in customs inspecting passengers' luggage for banned substances, and the beagle alerts the handler by sitting next to the location of the banned substance. What would be a reasonable guess for the distribution (e.g., Bernoulli, binomial, geometric, Poisson, exponential, uniform, normal) of each the following: (a) the number of alerts by the beagle during the next 3 hours, (b) the length of time until the next alert, (c) the number of bags sniffed before alerting, (d) whether or not the beagle alerts when sniffing the next bag, (e) the number of alerts out of the next 25 bags sniffed, and (f) the combined weight of the next 25 bags that the beagle indicates are carrying banned substances?

2. Let $X$ have mean 3 and variance 25. What is the mean and variance of $6 - 4X$? What is the mean and variance of $(X - 3)/5$?

3. Let $X$ be a discrete random variable with $\Pr\{X = i\} = ci$ for positive, odd integers $i < 10$; otherwise, the probability is zero. (a) Compute the value of $c$. (b) What is the mean of $X$? (c) What is the second moment of $X$? (d) What is the variance of $X$? (e) Compute $\mathrm{E}[(X - 2)^+]$ where $x^+$ is defined to be $\max(x, 0)$.

4. Let $X$ be a Poisson random variable with parameter 5, and let $Y = \min(X, 2)$. (a) What is the p.m.f. of $X$? (b) What is the mean of $X$? (c) What is the variance of $X$? (d) What is the p.m.f. of $Y$ (e) Compute $\mathrm{E}[Y]$.

5. Let $Y$ be a random variable with p.d.f. $ce^{-4s}$ for $s \geq 0$. (a) Determine $c$. (b) What is the mean, variance, and squared coefficient of variation of $Y$ where the squared coefficient of variation of $Y$ is defined to $\mathrm{Var}[Y]/(\mathrm{E}[Y]^2)$? (c) Compute $\Pr\{Y > 4\}$. (d) Compute $\Pr\{Y > 6 \mid Y > 2\}$. (e) What is the point $x^*$ such that $\Pr\{Y > x^*\} = 2/3$?

6. Let $X$ and $Y$ have joint probability density function $f_{(X,Y)}(s, t) = 8e^{-(3s+6t)}$ for $0 \leq s$, and $0 \leq t$. Find (a) $\Pr\{X = Y\}$ (b) $\Pr\{\min(X, Y) > 1/3\}$, (c) $\Pr\{X \leq Y\}$, (d) the marginal probability density function of $X$, and (e) $\mathrm{E}[XY]$.

7. Suppose a worker needs to process 100 items. The time to process each item is exponentially distributed with a mean of 2 minutes, and the processing times are independent. Approximately, what is the probability that the worker finishes in less than 6.25 hours?