Name:	KEY.	(
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GT username:

Circle your TA/section:

- (F1) Qiqin
- (F2) Changong
- (F3) George
- (L1) Scott
- (L2) Vaidehi

- 1. No books or notes are allowed.
- 2. You will not need a calculator for this exam. All electronic devices are not allowed.
- 3. Show all work and fully justify your answer to receive full credit.
- 4. Good luck!

Page	Max. Possible	Points
1	24	
2	24	
3	24	
4	28	
Total	100	

- 1. Let $A = \{1, 2, 3\}$.
 - (a) Give an example of a relation \mathcal{R} on A which is reflexive, not symmetric, and not transitive. (8 pts.)

For example.

$$R = \{(1,1),(2,2),(3,3),(1,2)\}$$

(b) Explain why your relation \mathcal{R} is not transitive.

(4 pts.)

Not transitive since

- **2.** Let $f: A \to B$ and $g: B \to C$ be functions.
 - (a) Prove that if $g \circ f$ is one-to-one, then f is one-to-one.

(8 pts.)

Proof: suppose got is one-to-one. (Proof by contradiction) Suppose, sections

(Prove the contrapositive)

Suppose of is not one-to-one.

Then 3x, , x = EA s.t. x, + x = but f(x,) = f(xe).

Thus g(f(x1)) = q(f(x2)), but x1+x2, so gof(x1) = gof(x2) but

MITHZ, so got is not one-to-one.

131

(b) Write down the converse of the statement you proved above and give an example of functions f, g where it is false. (4 pts.)

converse: if f is one-to-one then so is god.

false consider $f: 31, 2, 33 \rightarrow 3a, b, c3$ and $g: 3a, b, c3 \rightarrow 323$

Then f is one-to-one but god(x)= Z txef1.2735 so god is not 1-1.

(12 pts. each)

(a)

$$\begin{array}{ccc} \hat{\mathbf{O}} & p \vee q \\ & & \neg p \\ & \mathbf{G} & q \rightarrow r \\ & & \mathbf{G} & s \rightarrow \neg r \\ & & & \neg s \\ \end{array}$$

Proof. Suppose all promises are true.

- 1) is true so either por 7 is tom.
- @ is true so P is false, hence by O of is true
- 3) is true and 9 is true, so is true
- (4): If s were true then since (1) is true r would be false, but ris true so 5 most be false.

Hence (5) is true: S is fals.

8

4.

$$\frac{p \lor (q \land \neg r)}{\neg q \to \neg r}$$

Proof. Soppose The conclusion is false.

Then p is false. If the first premise is true then (qn 7r) is true since p is false.

But $(q_{17}) \Leftrightarrow \neg (r \rightarrow q)$, and $(\neg q \rightarrow r) \Leftrightarrow r \rightarrow q$.

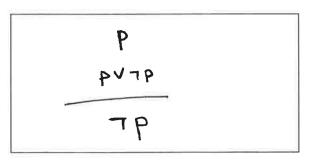
Therefore, if the conclusion is false and premise 1 is true, Then premise 2 is false.

in particular, the argument is valid



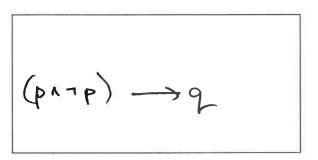
- 5. Short answer section: put a number, statement, or argument in each box. Please show your work for potential partial credit. (6 pts. each)
 - (i) Give an example of an invalid argument with at least two premises using only one atomic variable p.

For example:



(ii) Give an example of an implication which is always true using the atomic variables p, q.

for example



(iii) Simplify the negation of the statement $p \lor (q \to r)$.

7 (pv (q→r)) ⇔ 2p ∧ 7 (q→r)

←) 7p ∧ q ∧7r.

(iv) Let $A = \{a, b\}$. Then, the number of subsets of A equals



Subsets of A are

\$1, \$a3, \$63, A. 3

- 5. True and false questions. Instructions: for each statement below, circle TRUE if the statement is always true and circle FALSE otherwise. Your work will NOT be graded. (4 pts. each)
 - (i) The relation $\mathcal{R} = \{(a, b) \in \mathbb{Z}^2 \mid a b \ge 0\}$ is reflexive, anti-symmetric, and transitive.



(ii) The statement $q \to (\neg p \to q)$ \iff \uparrow is a neither a contradiction nor a tautology.

TRUE FALSE

(iii) The statement $\neg p \land (q \rightarrow r)$ is true when p,q, and r are all true.

TRUE (FALSE)

- false? op is false

 so the "and"

 is false
- (iv) For any $a \in \mathbb{Z}$, the number a is odd if and only if the number a^2 is odd.

TRUE FALSE

(v) $\forall r \in \mathbb{R} (\exists n \in \mathbb{Z} (n < r < n+1)).$

TRUE FALSE

(vi) There are n equivalence classes of integers under the equivalence relation remainder modulo n.

TRUE FALSE

(vii) If A, B are sets and both $A \subseteq B$ and $A \cap B = \emptyset$, then $B = \emptyset$.

TRUE (FALSE

 $A = \emptyset \quad B = \{1, 2\}$

Then ASB and AAB=\$
but B\$\$\omega\$.