ISyE 3833 - HW 5 - DO NOT SUBMIT - PRACTICE ONLY

1. *ISyE* is organizing a field trip for n of its students. There are m cabins available, and the jth cabin has capacity K_j, for each $j = 1, \ldots, m$. Assume that there is enough capacity for everyone, i.e., $\sum_{j=1}^{m} K_j \geq n$. If at least one person stays in cabin j, its cost is c_j, independent of how many people stay in the cabin, for each $j = 1, \ldots, m$. (There is no cost associated with a cabin in which no-one stays.) Each student must be assigned to exactly one cabin.

   (a) Formulate an IP model that minimizes the total cost of renting the necessary cabins.

   (b) Now, assume that each student has a car and that there is a cost for each student, $i \in \{1, \ldots, n\}$, to use his/her car to drive to any cabin: d_i. Each car can take up to 5 people. Each student must be assigned to exactly one car. If a student i doesn't drive then he/she doesn't have to pay the cost d_i, but does incur a cost for getting to the home of the student that is driving them. Suppose that the cost for student $i \in \{1, \ldots, n\}$ to get to the home of student $k \in \{1, \ldots, n\}$ is given by g_{ik}. Formulate an IP model that minimizes the total cost of renting the necessary cabins plus the total transportation cost (the cost to students who are not driving to get to the home of the student that is giving them a ride plus the cost to the students who are driving).

2. A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is \$400 for NY, \$500 for LA, \$300 for Chicago, and \$150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week, and region 3 requires 40 units per week. The costs (including production and shipping costs) of sending one unit from plant to a region are shown in the table.

From	To region 1	To region 2	To region 3
New York	20	40	50
Los Angeles	48	15	26
Chicago	26	35	18
Atlanta	24	50	35

   We want to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions:

   - If the New York warehouse is opened, then the Los Angeles warehouse must be opened.
   - At most two warehouses can be opened.
   - Either the Atlanta or the Los Angeles warehouse must be opened.

   (a) Formulate an IP that can be used to minimize the weekly costs of meeting demand.

   (b) Now suppose that if any units are shipped on a transport link (from a warehouse to a region), then at least 20 units must be shipped; anything less is not economic. Modify your integer linear programming model accordingly. [Hint: you may need to introduce additional variables.]

3. A local radio station is going to schedule commercials within 60 second blocks. There are m comercials to schedule. The duration of each commercial is t_i seconds, for $i = 1 \dots m$. You can assume $t_i \leq 60$ for any i. Each commercial must be played exactly once a day. Formulate an IP for which the optimal value corresponds to the minimum number of blocks necessary to run all ads in a day.

4. Suppose x and y are binary variables, w is an integer variable, and z is a continuous variable, with $0 \leq w \leq 6$ and $0 \leq z \leq 100$. Write linear constraints to model the following logical relationships. In same cases, you may need to define additional variables.

   (a) If $x = 1$ then $y = 1$.
   (b) x is not equal to y.
   (c) If $x = 1$ then $y = 0$.
   (d) If $y = 1$ then $z = 0$.
   (e) Either $x = 1$ or $y = 0$ or both.
   (f) If $x = 1$ and $y = 0$ then $z = 0$.
   (g) If $x = 1$ then $w \geq 3$.
   (h) Either $z = 0$ or $z \geq 20$.
   (i) If $x = 0$ then $z \geq 50$ and $w \leq 3$.
   (j) If $x = 1$ then $w = 3$.
   (k) If $z > 25$ then $x = 1$.
   (l) If $y = 1$ then $w + x \geq 2$.
   (m) If $w + 3x \leq 4$ then $y = 0$.
   (n) If $w + 3x \leq 4$ then $w + x \geq 2$.
   (o) Either $w + 3x \leq 4$ or $w + x \geq 2$, or both.

5. Answer Extra Exercise 3 from the Tutorial 14 handout. In addition, answer the following questions about the partial branch-and-bound tree given in that exercise.

   (a) To create the LP solved at Node 5, what constraint(s) were added to the (original) LP relaxation of the IP?
   (b) For each of Nodes 4, 5, 7, 8 and 9, indicate whether you need to branch on the node. In each case, briefly explain your answer. If branching is required, describe the branches.

   Important note: this is a tree for a **minimization** problem.

6. Consider an IP (maximization) problem P_{IP} and its linear relaxation P. Let Z_{IP} and Z_P denote their optimal values. Note that P has a dual problem: (D). Let Z_D denote the value of this dual. Assume that all problems have an optimal solution. Currently you have one feasible solution x^* to P_{IP} with a value equal to Z^*. You know that Z^* and the values of P and D lay in the set $\{50, 51, 52\}$:

   - What are all possible combination of values for Z^*, Z_P and Z_D?
   - In which cases can we be sure that x^* is optimal?
   - In which cases can we be sure that P has an optimal solution that is integer?