Practice test 1, MA1501 Calculus I Seong Jun Kim

This sample test is to guide your review for the exam 1 (Thursday, September 12).
In the exam, *you must show your work. Answers without substantiation do not count.*

Keywords: properties of a function (graph, even/odd, shift, reflection, one-to-one, inverse, limit, continuity, a slope of a function), various functions (piecewise, trigonometric using π radians, exponential, logarithm, composite), concepts (domain, range, function, limits, one-sided limits, rate of change, tangent to curves, derivatives at a point), and theorems.

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1. Definitions:
- A function $f(x)$ is one-to-one on a domain D if

- The average rate of change of $y = f(x)$ with respect to x over the interval $[x_1, x_2]$ is

- $\lim_{x \to x_0} f(x) = L$ means that

- A function $f(x)$ is continuous at an interior point $x = c$ of its domain if and only if

- The slope of the graph of $y = f(x)$ at $x = x_0$ is defined by

2. Write the formula for $f \circ g$ and find the domain and the range.

$$f(x) = \sqrt{2x - 1}, \quad g(x) = \frac{\pi}{1 - e^x}$$

3. Draw the graph of the following function and check whether it is one-to-one over $x \in (-\infty, \pi]$.

$$f(x) = \begin{cases} -x + 1, & x < 0 \\ \cos(x), & 0 \le x \le \pi \end{cases}$$

4. Use ϵ and δ to show that

$$\lim_{x \to 4}(-5x + 17) = -3.$$

5. Evaluate (when the limit does not exist, give reasons for your answer)

$$\lim_{x\to 1} \frac{\frac{1}{x}-1}{x-1}$$

$$\lim_{x\to 0^-} \frac{\sqrt{6}-\sqrt{5h^2+11h+6}}{h}$$

$$\lim_{x\to 1} \sqrt{(x-1)^2}\frac{2x-1}{|2x-1|}$$

$$\lim_{x\to \infty} \frac{8x^4+x}{2x^4+4x^2-x+4}$$

$$\lim_{x\to 2} \frac{1}{x^2-4}$$

6. For what value of b is

$$g(x) = \begin{cases} \frac{x-b}{b+1}, & x<0 \\ x^2+b, & x>0 \end{cases}$$

continuous at $x=0$?

7. The Intermediate Value Theorem states that let f be continuous on $[a,b]$ and if y_0 is between $f(a)$ and $f(b)$, then there is some $c\in[a,b]$ for which $f(c)=y_0$. Use the Intermediate Value Theorem to show that there is a root of the equation $x^4-3x^3+1=0$ between 0 and 1.

8. Consider the following function

$$f(x)=x^3+x^2-12x.$$

(a) Using the difference quotient, find the slope of the function at the given point $(-1,12)$.
(b) Find an equation of the tangent line to the curve at $(-1,12)$.
(c) Using the difference quotient, find the derivative of the function at the given point $(-1,12)$.
(*Note that if you already know how to differentiate and if you do so in this problem, you will get no credit.*)