ISYE 3232A Spring 2016 Quiz 3

February 1, 2016

David buys fruits and vegetables wholesale and retails them at David's Produce on La Vista Road. One of the difficult decisions is the amount of bananas to buy. Let us make some simplifying assumptions, and assume that David purchases bananas once a week at 20 cents per pound and retails them at 90 cents per pound during the week. Bananas that are more than a week old are too ripe to sell and David will pay workers to take them away. It costs 10 cent to get rid of each pound of unsold bananas. Suppose that the weekly demand for bananas is exponentially distributed with mean 1000 pounds.

Note that the pdf of exponential is $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ where mean is $1/\lambda$.
Its CDF is given as $F(x) = 1 - e^{-\lambda x}$ for $x \geq 0$

1. When 800 pounds are ordered, what is the expected weekly profit in dollars? Leave your answer with integrals.

   Answer:

$$\begin{aligned}
\mathsf{E}[\text{weeklyprofit}] &= (.9-.2)\mathsf{E}[D \wedge 800] - (.2+.1)\mathsf{E}[(800-D)^+] \\
&= .7\left\{\int_0^{800} x \times \frac{1}{1000} e^{-\frac{x}{1000}}\, dx + \int_{800}^{\infty} 800 \times \frac{1}{1000} e^{-\frac{x}{1000}}\, dx\right\} \\
&\quad -.3\int_0^{800} (800-x) \times \frac{1}{1000} e^{-\frac{x}{1000}}\, dx.
\end{aligned}$$

2. What is the optimal ordering quantity?

   Answer: We need to solve the following equation for y:

$$F(y) = 1 - e^{-\frac{y}{1000}} = \frac{90-20}{90+10} = 0.7.$$

   Then $y = -1000\ln(0.3) \approx 1203.97$ pounds.

3. Assuming that the weekly demand for bananas is normally distributed with mean 1000 pounds and variance 200^2 pounds^2, what is the optimal ordering quantity?

   Answer: Again we need to find y such that $F(y) = \frac{90-20}{90+10} = 0.7$. Note

$$F(y) = \Pr(D \leq y) = \Pr\left(\frac{D-1000}{200} \leq \frac{y-1000}{200}\right) = \Pr(Z \leq \frac{y-1000}{200}) = 0.7.$$

   From the standard normal table, $z_{0.7} \approx 0.52$ and $\frac{y-1000}{200} = 0.52$. Finally $y^* = 1104$ pounds.