Homework 8
SOLUTIONS

1. Consider a Transportation problem with 2 supply points and 3 demand points. The supply at points 1 and 2 is $\{40, 60\}$ while the demand points request $\{20, 30, 40\}$. The costs from Supply point 1 to the demand points is $\{2, 3, 6\}$ respectively, while the costs from supply point 2 to the demands is $\{5, 18, 20\}$. Model this as a balanced transportation problem, and then solve for a basic feasible solution using Vogel's Method.

	2		3		6		0	40	p=2
	5		18		20		0	60	p=5
20	p=3	30	p=15	40	p=14	10	p=0		

	2	30	3		6		0	10	p=2
	5	x	18		20		0	60	p=5
20	p=3	0	p=0	40	p=14	10	p=0		

x	2	30	3	10	6	x	0	0	p=2
	5	x	18		20		0	60	p=5
20	p=3	0	p=0	30	p=14	10	p=0		

x	2	30	3	10	6	x	0	0	p=2
20	5	x	18	30	20	10	0	0	p=5
0	p=3	0	p=0	0	p=14	0	p=0		

Figure 1: Solution

2. You are in charge of handing out project assignments to five groups. The projects are labeled A,B,C,D,E. Each group hands in a list of their preferred project. Their rankings are in the following table (top project is first choice).

1	2	3	4	5
A	B	C	A	A
B	A	B	C	E
C	C	A	B	C
D	D	E	D	D
E	E	D	E	B

Model this problem and solve for an optimal pairing of group to project trying to minimize the sum of slot values assigned to the groups.

Solution: To start we want to minimize the cost of assigning projects to groups, so we have an assignment problem. In order to do this, we also want to assign values to giving group i their j^{th} choice. These values should be smaller for higher priority choices to give us a minimum. We can start the Hungarian method with the following matrix:

	1	2	3	4	5
A	1	2	3	1	1
B	2	1	2	3	5
C	3	3	1	2	3
D	4	4	5	4	4
E	5	5	4	5	2

Subtracting the smallest value in each row gives:

	1	2	3	4	5
A	0	1	2	0	0
B	1	0	1	2	4
C	2	2	0	1	2
D	0	0	1	0	0
E	3	3	2	3	0

There is a 0 in every column, so subtracting the smallest value in each column yields the same. It also takes 5 lines to cover all the 0's, so this is an optimal table. By choosing cells with 0 value, one per column and one per row, you get the following solution: $x_{1,1} = x_{2,2} = x_{3,3} = x4, 4 = x_{5,5} = 1$. The other possible solution is: $x_{1,4} = x_{2,2} = x_{3,3} = x4, 1 = x_{5,5} = 1$

3. Consider the following edge costs for a network (blank cells have no edge connecting)

-	1	2	3	4	5	6	7
1	0	2	3	-	-	-	-
2	-	0	-	-	3	-	-
3	-	-	0	1	3	-	-
4	-	-	-	0	-	3	4
5	-	-	-	2	0	-	2
6	-	-	-	-	-	0	-
7	-	-	-	-	-	-	0

(a) Draw the graph of this network

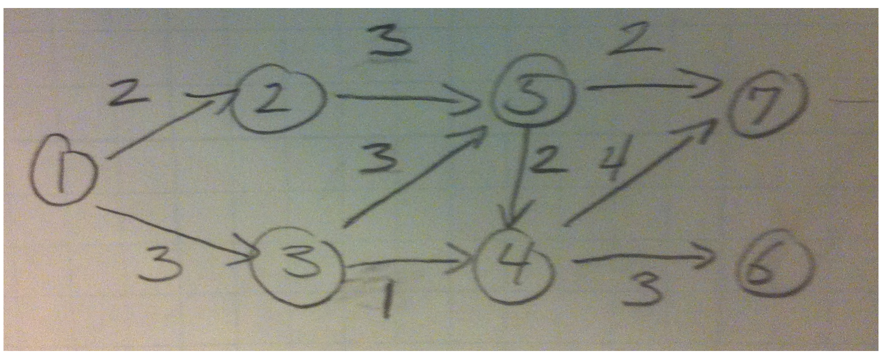

Figure 2: Solution

(b) Find the shortest path from node 1 to all other nodes in the network using Dijkstra's algorithm

(1) $L = \{1\}, d_1 = 0, d_2 = 2, d_3 = 3$

(2) $L = \{1,2\}, d_1 = 0, d_2 = 2, d_3 = 3, d_5 = 5$

(3) $L = \{1,2,3\}, d_1 = 0, d_2 = 2, d_3 = 3, d_4 = 4, d_5 = 5$

(4) $L = \{1,2,3,4\}, d_1 = 0, d_2 = 2, d_3 = 3, d_4 = 4, d_5 = 5, d_6 = 7, d_7 = 8$

(5) $L = \{1,2,3,4,5\}, d_1 = 0, d_2 = 2, d_3 = 3, d_4 = 4, d_5 = 5, d_6 = 7, d_7 = 7$

(6) $L = \{1,2,3,4,5,6\}, d_1 = 0, d_2 = 2, d_3 = 3, d_4 = 4, d_5 = 5, d_6 = 7, d_7 = 7$

(7) $L = \{1,2,3,4,5,6,7\}, d_1 = 0, d_2 = 2, d_3 = 3, d_4 = 4, d_5 = 5, d_6 = 7, d_7 = 7$

(c) What path do you follow to get from 1 to 7 in the shortest distance?

Solution: From backtracking, or keeping the last node visited during the algorithm, we get the path: 1-2-5-7

(d) What is the length of the shortest path from 2 to 1?

Solution: There is no path from 2 to 1. Therefore we represent the distance as ∞ or not possible.

4. Consider a 6 node graph with the following edge capacities.

-	1	2	3	4	5	6
1	0	3	4	-	-	-
2	-	0	-	2	-	-
3	-	-	0	2	3	-
4	-	-	-	0	-	3
5	-	-	-	-	0	4
6	-	-	-	-	-	0

Solve for the maximum flow that can be sent from 1 to 6.

Solution: In order to find the optimal max flow, we use the Ford-Fulkerson algorithm. We can start with all flow being 0, and all edges in set I. The source is node 1, and the sink is 6. We start by labeling node 1. Then we can label node 2, and (1,2) is a forward edge. From there we can label 4, then 5, with (2,4),(4,6) as forward arcs. Solving we find k=2, and let $x_{1,2} = x_{2,4} = x_{4,6} = 2$. Relabeling the nodes we start with node 1 again. We can once again label 2, and (1,2) as a forward edge, but cannot create a path from there so we look at labeling 3, and (1,3) as a forward edge. Next we can look at node 4 and (3,4) then 6 and edge (4,6). This gives us a path of all forward edges with k=1. Now $x_{1,2} = 2, x_{2,4} = 2, x_{4,6} = 3, x_{1,3} = 1, x_{3,4} = 1$. Now we see if there is another path by updating the sets I and R. Labeling 1, we can still go to 3, and (1,3) forward. From there we can go to 5 and 6 following (3,5) and (5,6) as forward edges. Here k=3. Now $x_{1,2} = 2, x_{2,4} = 2, x_{4,6} = 3, x_{1,3} = 4, x_{3,4} = 1, x_{3,5} = 3, x_{5,6} = 3$. Labeling again we cannot move past the source node, so we have the optimal flow, where the total flow is 6.