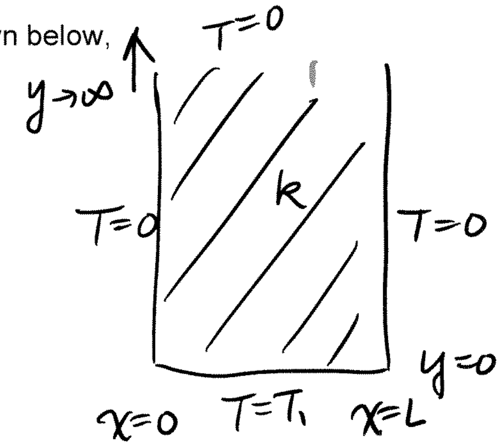
Problem I. Short answer questions. (20 points) For true/false questions, please correct the incorrect part(s) or explain why it is wrong. For multiple choice questions, circle the correct answer.

1. (3 pts) For a semi-infinite 2-D slab at steady state as shown below, simplify the heat transfer equation.



$$\vec{\nabla}\cdot k\vec{\nabla}T + \cancel{\dot{q}} = \rho C_v \cancel{\frac{DT}{Dt}}$$ S.S. solid

no gen.

$$k\nabla^2 T = 0 \Rightarrow \nabla^2 T = 0$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

2. (7 pts) The solution for problem 1 is

$$T(x,y) = (A\cos\lambda x + B\sin\lambda x)(Ce^{\lambda y} + De^{-\lambda y})$$

Use the boundary conditions and determine A and C.

B.C. ① $x=0$ $y>0$ $T=0$

② $x=L$ $y>0$ $T=0$

③ $0<x<L$ $y=0$ $T=T_1$

④ $0<x<L$ $y\to\infty$ $T=0$

use ④ $T=0$ for all x, so $Ce^{\lambda y}+De^{-\lambda y}=0$

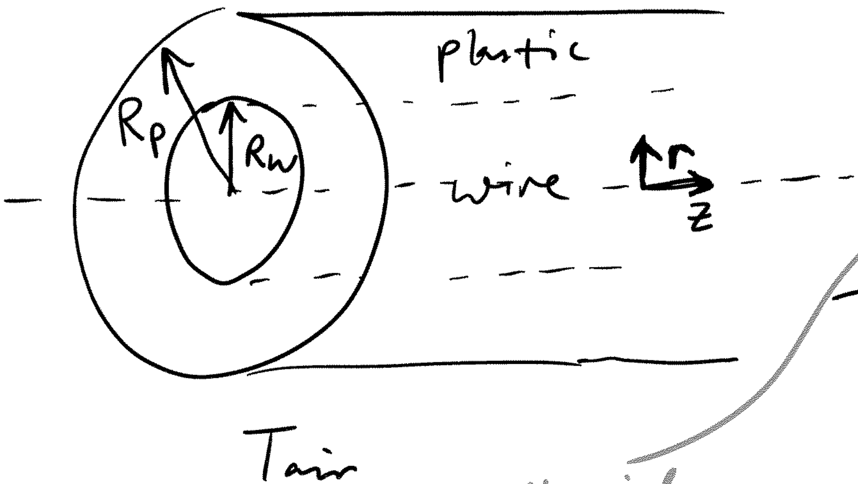
when $y\to\infty$, $Ce^{\lambda y}=0 \Rightarrow C=0$

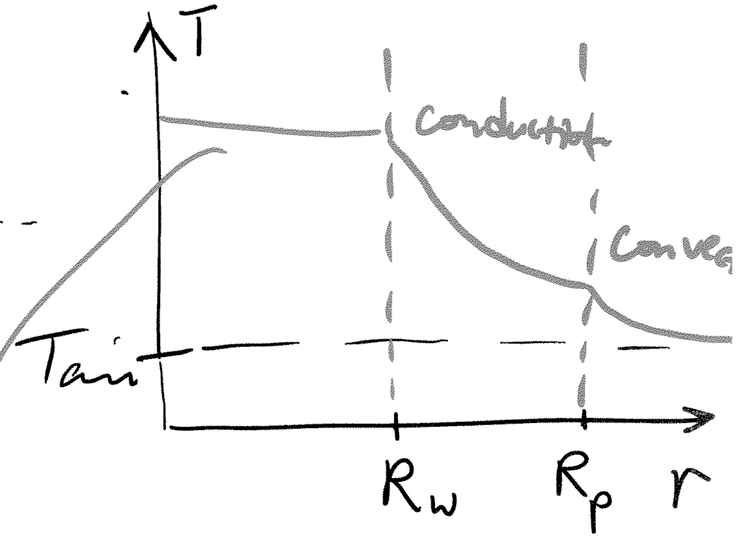
use ①, $T=0$ for all $y>0$, so $A\cos\lambda x + B\sin\lambda x = 0$

when $x=0$ $A\cos\lambda x = 0 \Rightarrow A=0$

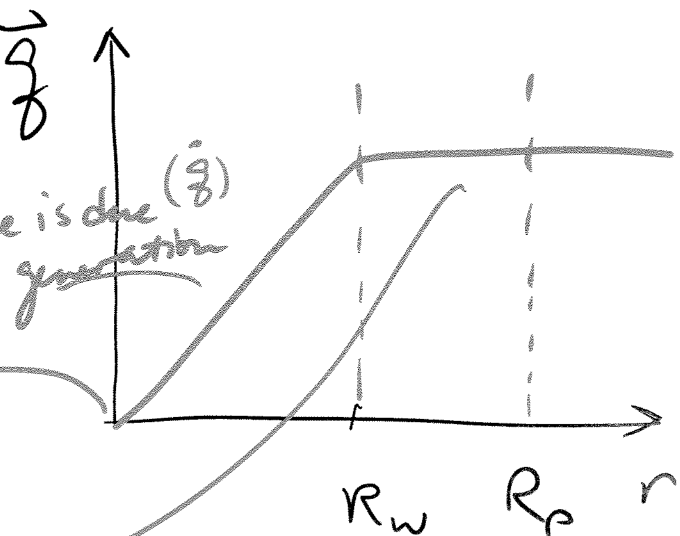
Name:

3. (10 pts) For a thin and long electrical wire that is resistively heated up (by applying a voltage across), assume the thermal conductivity of the wire to be much higher than that of the plastic. Draw the T profile and the heat flus profile as a function of radius, and give some reasons for the features of these curves.





flat'ish because of large k



slope is due to ($\dot{q}$) generation

zero at wall (symmetry)

S.S. same flux everywhere

2.

$$q = \frac{(T_{\infty,\text{inner}} - T_{\infty\,\text{out}})}{\Sigma R} = \frac{(90°C - 25°C)}{\Sigma R} = \frac{65}{\Sigma R}$$

$$q = \frac{65}{R_{\substack{\text{conv.}\\H_2O}} + R_{\text{cond}} + R_{\substack{\text{conv}\\\text{air}}}}$$

conduction thru inner pipe

$$R_{\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi r} = \frac{\ln(.016m/.013m)}{(2\pi)(20\,W/mK)} = 1.65\times10^{-3}\,mK/W$$

$$R_{\substack{\text{conv}\\H_2O}} = (hA)^{-1} = (h2\pi R)^{-1} = [(5000\,W/m^2K)(2\pi)(.013m)]^{-1}$$
$$= 2.45\times10^{-3}\,mK/W$$

$$R_{\substack{\text{conv.}\\\text{air}}} = (h_{\text{air}}[A_o + \eta_f A_f])^{-1}$$

$h_{air} = 200\,W/m^2K$

$A_o = 2\pi(.016m) - (4\,\text{fins})(.003m) = .0885\,m$

$A_f = \underset{\substack{\uparrow\\\text{top +}\\\text{bottom}}}{2}(.024m)(4\,\text{fins}) = .192\,m$

$$L\sqrt{\frac{h}{kt}} = (.024)\left[\frac{200}{20\times \underset{\substack{\uparrow\\\text{"t" not "2t"}}}{.0015m}}\right]^{1/2} = 1.95$$

From chart

$\eta_f \approx .50$

$$R_{\substack{\text{conv}\\\text{air}}} = \left[200\frac{W}{m^2K}\left(.0885 + (.5)(.192)\right)\right]^{-1} = 27.103\times10^{-3}\,\frac{mK}{W}$$

$$q_{\text{TOTAL}} = \frac{65}{(2.45\times10^{-3}\,mK/W) + (1.65\times10^{-3}\,mK/W) + (27.103\times10^{-3}\,mK/W)}$$

$$\boxed{q_{\text{TOTAL}} = 2083\,W/m}$$

Note Title 11/29/2013

3. Known: $D_p = 0.27\ mm = 2.7 \times 10^{-4}\ m$

$\rho = 2400\ kg/m^3$

$\varepsilon_{p.b.} = 0.4$

$\varepsilon_{mf} = 0.45$

$D = 0.2\ m$ $L_{p.b.} = 0.1\ m$

$\rho_a = 1.275\ kg/m^3$

$\mu = 1.9 \times 10^{-5}\ Pa.s$

(1) Min. fluidization

$$\frac{1.75}{\varepsilon_{mf}^3} Re_{mf}^2 + \frac{150(1-\varepsilon_{mf})}{\varepsilon_{mf}^3} Re_{mf} - \frac{D_p^3 \rho_f \rho_p g}{\mu} = 0 \quad \rho_p \gg \rho_f$$

Plug in #'s,

$$\frac{1.75}{0.45^3} Re_{mf}^2 + \frac{150(1-0.45)}{0.45^3} Re_{mf}^2 - \frac{(2.4 \times 10^{-4})^3 (1.275)(2400)(9.8)}{(1.9 \times 10^{-5})^2} = 0$$

$$19.2 Re_{mf}^2 + 905.3 Re_{mf} - 1630 = 0$$

solve quadratic eqn., take (+) root

$Re_{mf} = 1.74$

$$Re_{mf} = \frac{D_p \rho v'}{\mu} \Rightarrow v' = \frac{(1.74)(1.9 \times 10^{-5})}{(2.7 \times 10^{-4})(1.275)}$$

$= 0.096\ m/s$

$= \boxed{9.6\ cm/s}$

(2) $L_1(1-\varepsilon_1) = L_2(1-\varepsilon_2)$

packed min. fluidized

$$(0.1)(1-0.4) = L_{mf}(1-0.45)$$

$$\Rightarrow L_{mf} = 0.109\ m = 10.9\ cm$$

$$\Delta P = L_{mf}(1-\varepsilon_{mf})(\rho_p - \rho_f)\,g$$

$$= (0.109)(1-0.45)(2400-\cancel{1.275})(9.8)$$

$$= \boxed{1409\ Pa}$$

(3) Because $50\ Pa < 1409\ Pa$,

the bed is <u>not</u> fluidized

$$\Rightarrow L = 10\ cm, \quad \varepsilon = 0.4$$