NAME: SOLUTIONS

GRADE:

ISyE 3044 — Final Exam

Summer 2012

The exam is 90-minutes long. No books and notes are allowed. All communications devices, such as cellular phones and PDAs, must be turned off. You can use only the supplied formula sheet and tables, as well as a scientific calculator.

1. **[The first 3 questions count for 3 points; the last counts for 3 points]** Short questions.

   (a) A Simio Model Property can be changed during a simulation run.

   True **False**

   (b) The typical definition of the negative binomial distribution involves an integer parameter r (number of trials until the rth success). As discussed in the Solutions to Homework 6, the negative binomial distribution can be defined with a noninteger r. Can ExpertFit fit negative binomial models with noninteger parameter r?

   Yes **No**

   (c) The Subset Selection Analysis add-in implemented in Simio uses (Tukey's) simultaneous confidence intervals to identify potential scenarios with the best mean.

   True **False**

   (d) Use the uniform(0, 1) numbers below

   .10 .23 .68 .37 .72 .94 .47 .28 .04 .59

   and Monte Carlo integration to compute a point estimate of the integral

   $$\mu = \int_0^4 \frac{1}{1+e^{-t}}\, dt.$$

   ANSWER: $\hat{\mu} = 3.23$

Using the transformation $t = 4u$, we write

$$\mu = \int_0^1 \frac{4}{1+e^{-4u}}\, du$$

2. **[The first four questions count for 4 points; the fifth counts for 2 points]**

(a) The random number generator in the latest version of Excel is the Mersenne Twister.

True False

(b) Use the uniform random number 0.63 and an approximate method to generate a realization from the standard normal distribution.

ANSWER: 0.33 $= \frac{(.63)^{.135} - (.37)^{.135}}{.1975}$

(c) Use the normal realization from part (b) to generate a realization from the Poisson distribution with mean 25.

ANSWER: 27 $= \lceil 25 + .33\sqrt{25} - .5 \rceil$

(d) The random variable U is uniformly distributed on $(0, 1)$. What is the distribution of

$$X = \left\lceil \frac{\ln(1-U)}{\ln(0.4)} \right\rceil ?$$

Specify the parameter(s).

ANSWER: geometric $(p = 0.6)$

(e) The random variable X has density function $f(x) = e^{-x}/(1 - e^{-1})$, $0 \leq x \leq 1$. Use the inverse-transform method to derive a formula for generating realizations of X.

$F(x) = (1 - e^{-x})/(1 - e^{-1})$

$(1 - e^{-X})/(1 - e^{-1}) = U \Rightarrow 1 - e^{-X} = (1 - e^{-1})U \Rightarrow e^{-X} = 1 - (1 - e^{-1})U$

ANSWER: $X = -\ln[1 - (1 - e^{-1})U] = -\ln(1 - 0.63U)$

(f) **[Bonus question (2 points)]** Use the acceptance-rejection method with majoring function $t(x) = 1/(1 - e^{-1})$ to derive an algorithm for generating realizations of X. Use as many of the following uniform random numbers as needed to generate a realization of X:
~~.25~~, ~~.33~~, .67, .83, .54, .48, .28, .04, .17, .72

$c = \int_0^1 t(x)\,dx = \frac{1}{1 - e^{-1}} = 1.58$

$h(x) = 1,\ 0 \leq x \leq 1$ (uniform)

$g(x) = \frac{f(x)}{t(x)} = e^{-x}$

ANSWER: $X = 0.33$

1. $U = 0.25$, $Y = 0.33$; $.25 \leq e^{-.33} = 0.72$

Deliver $X = 0.33$

3. **[8 points]** The following data are repair times (in hours) for sensors on a conveyor: .25, .53, .98, .88, .72.

(a) Assuming that the data come from the uniform distribution on the interval $(a, 1)$, use the method of moments to compute an estimate of a.

1.5

$\frac{a+1}{2} = \bar{X} \Rightarrow \tilde{a} = 2\bar{X}-1$

ANSWER: $\tilde{a} = 0.344$

0.5 Does the estimate make sense? Give a short explanation.

ANSWER: No, because $\tilde{a}$ is larger than .25

(b) Assuming that the data come from the uniform distribution on (a, b), compute the MLEs of the endpoints a and b.

1

ANSWER: $\hat{a} = .25, \hat{b} = .98$

(c) Assuming that the data come from the uniform distribution on (a, b), compute the MLE of the mean of this distribution

1

ANSWER: $\frac{\hat{a}+\hat{b}}{2} = 0.615$

and the standard deviation.

1

ANSWER: $\frac{(\hat{a}-\hat{b})^2}{12} = .044$ Answer is $\sqrt{.044} = 0.21$

(d) Use the Kolmogorov-Smirnov test with level of significance 0.10 to assess the fit of the uniform distribution on (0, 1).

2

ANSWER: We fail to reject the claim.

$X_{(i)}$	.25	.53	.72	.88	.98
$\hat{F}(X_{(i)})$	.25	.53	.72	.88	.98
$\frac{i}{5} - \hat{F}(X_{(i)})$	–	–	–	–	.02
$\hat{F}(X_{(i)}) - \frac{i-1}{5}$	.25	.33	.32	.28	.18

Adjusted test statistic $= \left(\sqrt{5} + 0.12 + \frac{0.11}{\sqrt{5}}\right) 0.33 = 0.79 \leq 1.224$

4. **[5 points]** The following simulated data are times-in-system (in minutes) of 25 consecutive customers at a post office:

8.7	13.6	12.5	8.5	5.2	2.8	5.0	6.1	5.1	8.4	3.4	7.9	14.1
8.7	13.5	15.9	5.2	5.2	9.0	11.8	9.7	9.9	5.5	13.8	15.5	

(a) Assume that the data were collected in steady state. Use the method of batch means with batch size equal to 5 to compute an approximate 95% confidence interval for the steady-state mean time in system. (Assume that the batch means are approximately independent.)

ANSWER: $9.0 \pm 2.54 = (6.46, 11.54)$

The batch means are 9.7, 5.48, 9.52, 9.42, 10.88

$\bar{Y} = 9.0$

$S = 2.05$

$t_{4,.025} = 2.776$

The CI is $9.0 \pm \frac{2.05}{\sqrt{5}}(2.776) = 9.0 \pm 2.54$.

(b) We wish to obtain a 95% confidence interval for the mean time in system with a half-length ≤ 1 minute. Do we need to collect additional data? If yes, compute an estimate of the additional number of observations that need to be collected. Use the quantile of the normal distribution.

ANSWER: 12 additional batches $\equiv$ 60 additional observations

Solve $1.96 \frac{2.05}{\sqrt{k}} \leq 1 \Rightarrow k \geq \lceil 16.14 \rceil = 17$

5. **[5 points]** A simulation model was used to evaluate two alternative designs for a baggage-handling system. The measure of effectiveness was the mean travel time for a bag from origin to destination (in minutes) over an 8-hour peak period. Ten independent replications produced the following averages:

Replication	1	2	3	4	5	6	7	8	9	10
Design 1 (Y_{1i})	15.5	12.4	7.6	10.0	8.1	8.9	8.2	6.0	14.3	14.8
Design 2 (Y_{2i})	15.7	13.6	10.4	12.0	10.7	11.3	10.8	9.3	14.9	15.2

(a) Conduct a paired-t test with level of significance 0.10 to assess the hypothesis that the mean travel times under the two designs are equal: $\mathrm{E}(Y_{1i}) = \mathrm{E}(Y_{2i})$.

ANSWER: The means are not equal

$D_i = Y_{1i} - Y_{2i}$

$\bar{D} = -1.81$

$S = 1.118$

The half-length of the CI for $\mu_1 - \mu_2$ is $1.83 \frac{1.18}{\sqrt{10}} = 0.648$

The CI is $-1.81 \pm 0.65 = (-2.46, -1.16)$

(b) The experiments used common random numbers to generate the arrival times, origins, and destinations of the bags. Did this strategy affect the effectiveness of the test in part (a)? An estimate of the covariance between the components of the vectors (Y_{1i}, Y_{2i}) is

$$\hat{C} = \frac{1}{10}\sum_{i=1}^{10}(Y_{1i} - \bar{Y}_1)(Y_{2i} - \bar{Y}_2).$$

ANSWER: $\hat{C} = 6.98 > 0$; hence the CRN strategy reduced the variance of the D_i. In fact, the estimate of the correlation is 0.90.