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Homework 6

September 26, 2013

(Do not hand in this assignment)

1. A store stocks a particular item. The demand for the product each day is 1 item with probability 1/6th, 2 items with probability 3/6th, and 3 items with probability 2/6th. Assume that the daily demands are independent and identically distributed. Each evening if the remaining stock is less than 3 items, the store orders enough to bring the total stock up to 6 items. These items reach the store before the beginning of the following day. Assume that any demand is lost when the item is out of stock.

   (a) Let X_n be the amount in stock at the *beginning* of day n; assume that $X_0 = 5$. If the process is a Markov chain, give the state space, initial distribution, and transition matrix. If the process is not, explain why it's not.

   (b) Let Y_n be the amount in stock at the *end* of day n; assume that $Y_0 = 2$. If the process is a Markov chain, give the state space, initial distribution, and transition matrix. If the process is not, explain why it's not.

2. Suppose each morning a factory post the number of days worked in a row without any injuries. Assume that each day there is injury free with probability 99/100. Let $X_0 = 0$ be the morning the factory first opened. Let X_n be the number posted on the morning after n full days of work. Is $X_0, X_1, \ldots$ a Markov chain? If so, give its state space, initial distribution, and transition matrix P. If not, show that it is not a Markov chain.

3. A six-sided die is rolled repeatedly. After each roll $n = 1, 2, ...$, let X_n be the largest number rolled in the first n rolls. Is $\{X_n, n \geq 1\}$ a discrete-time Markov chain? If it's not, show that it is not. If it is, what is the state space and the transition probabilities of the Markov chain?

4. Redo the previous problem except replace X_n with Y_n where Y_n is the number of sixes among the first n rolls. (So the first question will be, is $\{Y_n, n \geq 1\}$ a discrete-time Markov chain?)