PHYS 2211 Test 2
Spring 2014

Name(print) ~~~Test Key~~~~ **Lab Section** Key

Greco(K)			
Day	12-3pm	3-6pm	6-9pm
Monday		K01 K02	
Wednesday	K03 K05	K04 K07	K06 K08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

Phileas Fogg

Sign your name on the line above

PHYS 2211

Do not write on this page!

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (25 pts)		
Problem 3 (25 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

In a recent lab, you studied the motion of a spacecraft orbiting the Earth; you wrote a computer model (VPython) to predict the spacecraft's motion.
The spacecraft starts at an initial location $< -5.824e7, 5.44e6, 0 >$ m and with an initial velocity of $< 604, 2795, 0 >$ m/s. The code given below, which is nearly identical to your computer model from lab, is missing a few lines of code. In the space provided in the body of the program, add the statements necessary to complete the code.

```
from __future__ import division
from visual import *

G = 6.7e-11
mEarth = 6e24 #mass of the Earth in kg
mcraft = 15000 $mass of the craft in kg
deltat = 60 #time step in seconds

#Initial conditions
Earth = sphere(pos=vector(1e7,-2e6,0), radius=6.4e6, color=color.cyan)

#(a 10pts) Add the necessary initial conditions for the craft.

5  craft = sphere(pos=vector(-5.824e7, 5.44e6, 0), radius = 3e6,
                  color = color.blue)
5  craft.p = mcraft * vector(604, 2795,0)

t = 0
while t < 3058992:

#(b 15pts) Add the necessary statements to update the craft's position.
       r = craft.pos - Earth.pos
       rmag = mag(r)
10     rhat = norm(r)
       Fgravmag = G * mEarth * mcraft /rmag **2
       Fnet = -Fgravmag * rhat
5pt    craft.p = craft.p + Fnet * deltat
       craft.pos = craft.pos + (craft.p/mcraft) * deltat



    t = t + deltat
```

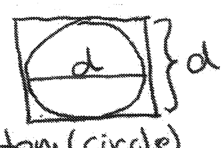
Problem 2 (25 Points)

One mole of platinum (6×10^{23} atoms) has a mass of 195 grams (0.195 kg). The density of platinum metal is $21.4\ \frac{\text{grams}}{\text{cm}^3}$ ($21.4 \times 10^3\ \frac{\text{kg}}{\text{m}^3}$).

(a 5pts) What is the diameter of a platinum atom inside a block of platinum metal? Show your work.

$$\rho = \frac{M_{total}}{V_{total}} \Rightarrow V_{total} = \frac{M_{total}}{\rho} = \frac{0.195\ \cancel{kg}}{21.4e3\ \cancel{kg}/m^3} = 9.11e{-6}\ m^3 \Big\}\ 1$$

$$V_{total} = N_{atoms} V_{atom} \Rightarrow V_{atom} = \frac{V_{total}}{N_{atoms}} = \frac{9.11e{-6}\ m^3}{6e23} = 1.52e{-29}\ m^3 \Big\}\ 1$$



atom (circle) in a cubic volume (cross-section)

$$V_{atom} = d^3 \quad 2\ pts$$

$$\Rightarrow d = (V_{atom})^{1/3} = (1.52e{-29}\ m^3)^{1/3} =$$

$$\boxed{d = 2.48e{-10}\ m} \quad 1\ pt$$

A rod of platinum hangs vertically. The rod is 2 m long with a square cross section 3 mm by 3 mm, so the cross-sectional area is $9 \times 10^{-6}\ \text{m}^2$. When a 90 kg mass is attached to the bottom of the rod, the rod stretches 1.2 mm (1.2×10^{-3} m).

(b 5pts) As measured by this experiment, what is Young's modulus for platinum? Remember that the gravitational force on a mass m near the Earth's surface is mg, where $g = 9.8$ N/kg. Show your work.

$$Y = \frac{F/A}{\Delta L/L_o} = \frac{F}{A} \cdot \frac{L_o}{\Delta L} = \frac{m g L_o}{A\, \Delta L} = \quad 3\ pts$$

$$= \frac{(90\ kg)(9.8\ m/s^2)(2\ \cancel{m})}{(9e{-6}\ m^2)(1.2e{-3}\ \cancel{m})} = 1.63e11\ kg\,m/s^2/m^2$$

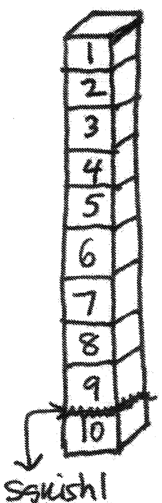
$$\boxed{Y = 1.63e11\ N/m^2} \quad 2\ pts$$

(c 5pts) Determine the effective stiffness of the spring-like interatomic force that acts between neighboring atoms in platinum. Show your work.

3 $\left\{ Y = \frac{k_{Si}}{d} \Rightarrow k_{Si} = Yd = (1.63e11\ N/m^2)(2.48e-10\ m) = \right.$

$$\boxed{k_{Si} = 40.424\ N/m}$$ 2pts

(d 5pts) A cube of platinum has dimensions 0.3 meters on a side. Ten of these blocks are stacked, single file, on top of each other. Calculate the compression of the bottom block due to the other nine blocks.



$\rho = M/V \Rightarrow M = \rho V = (21.4e3\ kg/m^3)(0.3^3\ m^3) = 577.8\ kg$

Weight of the top nine cubes: $F = 9Mg = (9)(577.8\ kg)(9.8\ m/s^2) = 50961.96\ N$ 2pt

$Y = \frac{F/A}{\Delta L/L_o} \Rightarrow \Delta L = \frac{F L_o}{AY} = \frac{(50961.96\ N)(0.3\ m)}{(0.3^2\ m^2)(1.63e11\ N/m^2)} =$ 2pts

$$\Rightarrow \boxed{\Delta L = 1.04e-6\ m}$$ 1pt

(e 5pts) Determine the macroscopic spring stiffness of the ten block stack?

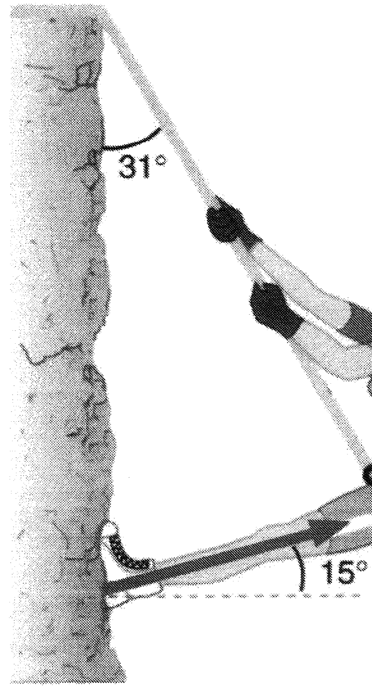
$F = k_s \Delta L \Rightarrow k_s = \frac{F}{\Delta L} = \frac{50.961.96\ N}{1.04e-6\ m} =$ 1pt

3pt

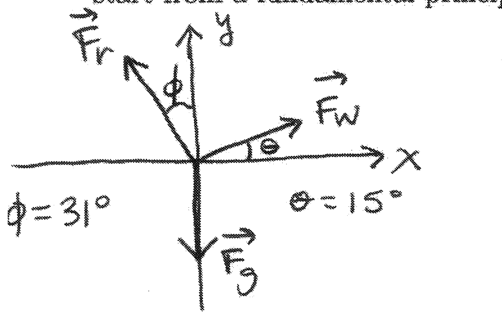
$$\Rightarrow \boxed{k_s = 4.9e10\ N/m}$$ 1pt

Problem 3 (25 Points)

Consider a rock climber of mass $m = 60$ kg who is ascending a vertical wall and stops for a rest by leaning back on her rope as seen in the diagram. The rope has some unknown tension and makes and angle of 31 degrees with the vertical. As the climber rest, the wall exerts an unknown force F on her foot. This force is parallel to her legs and makes and angle of 15 degrees with the horizontal.



(a 15pts) Determine the tension in the rope required for the mountain climber to remain stationary. Please start from a fundamental principle and show your work.



$\phi = 31°$ $\theta = 15°$

$$\vec{F}_{net} = \vec{F_r} + \vec{F_W} + \vec{F_g} = 0$$

components:

$$F_W \cos\theta - F_r \sin\phi = 0$$

$$F_W \cos\theta = F_r \sin\phi$$

$$\boxed{F_W = F_r \frac{\sin\phi}{\cos\theta}} \quad (\text{Eq } 1)$$

-components:

$$F_r \cos\phi + F_W \sin\theta - mg = 0$$

$$F_r \cos\phi + F_W \sin\theta = mg$$

Plug in (Eq 1) into above equation:

$$F_r \cos\phi + \left(F_r \frac{\sin\phi}{\cos\theta}\right) \sin\theta = mg$$

$$F_r \left(\cos\phi + \frac{\sin\phi \sin\theta}{\cos\theta}\right) = mg$$

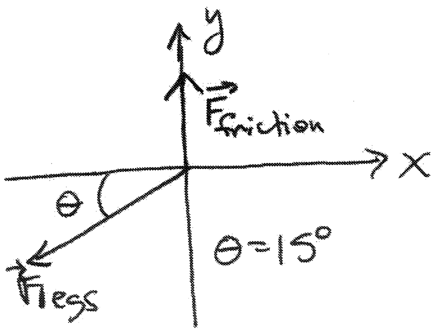
$$F_r = \frac{mg}{\cos\phi + \sin\phi \tan\theta} =$$

$$= \frac{(60 \text{ kg})(9.8 \text{ m/s}^2)}{\cos(31°) + \sin(31°)\tan(15°)} =$$

$$\boxed{|\vec{F}_{rope}| = 590.85 \text{ N}}$$

−1 cleric
−2 minor
−4.5 major
−12 BTN

(b 10pts) Calculate the minimum static coefficient of friction, between her shoes and the cliff, required to remain motionless in the position shown. Please start from a fundamental principle.



$\vec{F}_{net} = 0$

y-components:

$F_{friction} - F_{legs} \sin\theta = 0$

$F_{friction} = F_{legs} \sin\theta$

$\mu F_{normal} = F_{legs} \sin\theta$

$\mu \cancel{F_{legs}} \cos\theta = \cancel{F_{legs}} \sin\theta$

$\mu \cos\theta = \sin\theta$

$\mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$

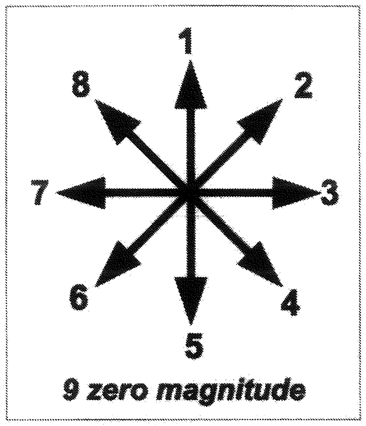
$\mu = \tan(15°)$

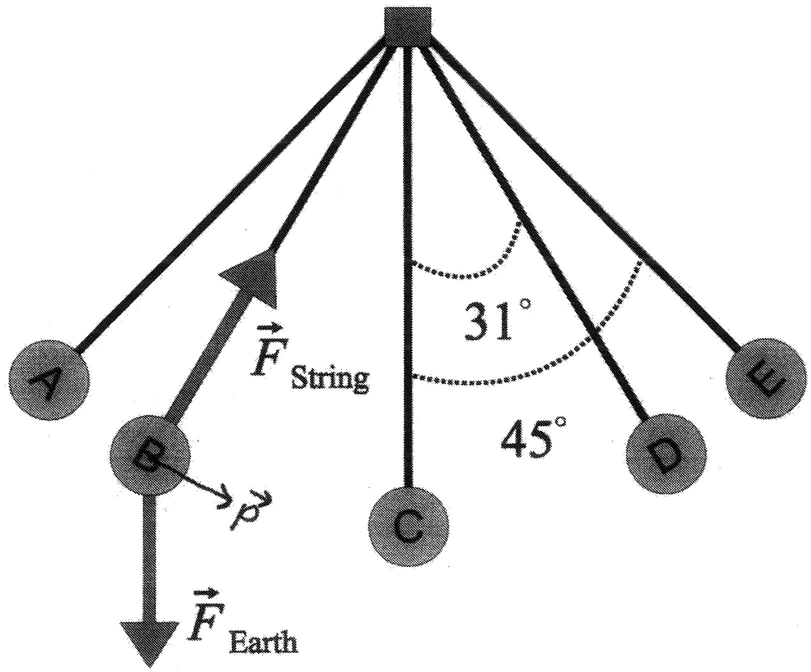
$\boxed{\mu = 0.27}$

−0.5
−1.5
−3.0
−8.0

Problem 4 (25 Points)

A 10 kg ball is attached to one end of a string that is 5 m in length; the other end of the string is attached to a fixed support. The ball swings freely in a circular arc. Figure 1 shows snapshots of the ball at different times. When the ball is at location C, the string is vertical. When the ball is at location A or E, the ball is momentarily at rest and the string makes an of 45 degree angle with the vertical. When the ball is at location B or D, the string make an angle of 31 degrees with the vertical and the ball's speed is 3.83 m/s (air resistance has been neglected).



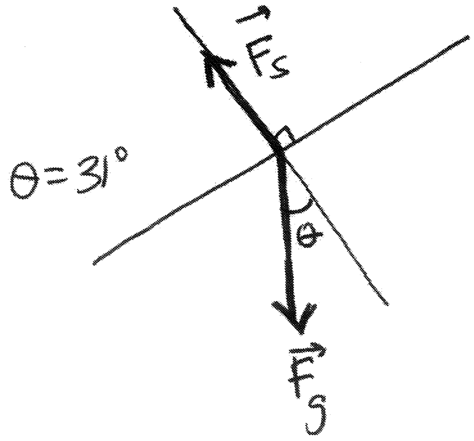


(a 15pts) Using the rosette of arrows shown above, indicate the number of the arrow that best represents the direction of the following quantities at locations A through E:

1 pt each

	A	B	C	D	E
$(\frac{d\vec{p}}{dt})_{\parallel}$	4	4	9	6	6
$(\frac{d\vec{p}}{dt})_{\perp}$	9	2	1	8	9
$\vec{F}_{net}$	4	3	1	7	6

(b 10pts) Draw a force diagram showing all of the forces acting in the ball at location D. What is the magnitude of the net force acting on the ball at this location? Please show all of your work in this calculation and put off substituting numerical values into you work until you have a final expression to evaluate.



Perpendicular:

$$\vec{F}_\perp = \left(\frac{d\vec{p}}{dt}\right)_\perp = \hat{n}\frac{mv^2}{R} =$$

$$= \frac{(10\text{ kg})(3.83^2\text{ m}^2/\text{s}^2)}{5\text{ m}} =$$

$$\vec{F}_\perp = 29.34\text{ N}\,\hat{n}$$

Parallel:

$$\vec{F}_{\parallel} = -F_g\sin\theta\hat{p} = -mg\sin\theta\hat{p} = (10\text{ kg})(9.8\text{ m/s}^2)\sin(31°) =$$

$$\vec{F}_{\parallel} = 50.47\text{ N}(-\hat{p})$$

−0.5
−1.5
−3.0
−8.0

Net force:

$$|\vec{F}_{net}|^2 = |\vec{F}_\perp|^2 + |\vec{F}_{\parallel}|^2$$

$$|\vec{F}_{net}| = \sqrt{(29.34\text{N})^2 + (50.47\text{N})^2}$$

$$\boxed{|\vec{F}_{net}| = 58.38\text{ N}}$$

This page is for extra work, if needed.

Things you must have memorized

The Momentum Principle Definition of Momentum	The Energy Principle Definition of Velocity	The Angular Momentum Principle Definition of Angular Momentum
Definitions of angular velocity, particle energy, kinetic energy, and work		

Other potentially useful relationships and quantities

$$\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|\vec{v}|}{c}\right)^2}}$$

$$E^2 - (pc)^2 = \left(mc^2\right)^2$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\vec{F}_{\parallel} = \frac{d|\vec{p}|}{dt}\hat{p} \text{ and } \vec{F}_{\perp} = |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n}$$

$$\vec{F}_{grav} = -G\frac{m_1 m_2}{|\vec{r}|^2}\hat{r}$$

$$U_{grav} = -G\frac{m_1 m_2}{|\vec{r}|}$$

$$\left|\vec{F}_{grav}\right| \approx mg \text{ near Earth's surface}$$

$$\Delta U_{grav} \approx mg\Delta y \text{ near Earth's surface}$$

$$\vec{F}_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|^2}\hat{r}$$

$$U_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}|}$$

$$\left|\vec{F}_{spring}\right| = k_s s$$

$$U_{spring} = \frac{1}{2}k_s s^2$$

$$U_i \approx \frac{1}{2}k_{si}s^2 - E_M$$

$$\Delta E_{thermal} = mC\Delta T$$

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$I = m_1 r_{1\perp}^2 + m_2 r_{2\perp}^2 + \ldots$$

$$K_{tot} = K_{trans} + K_{rel}$$

$$K_{rel} = K_{rot} + K_{vib}$$

$$K_{rot} = \frac{L_{rot}^2}{2I}$$

$$K_{rot} == \frac{1}{2}I\omega^2$$

$$\vec{L}_A = \vec{L}_{trans,A} + \vec{L}_{rot}$$

$$\vec{L}_{rot} = I\vec{\omega}$$

$$\omega = \sqrt{\frac{k_s}{m}}$$

$$v = d\sqrt{\frac{k_{si}}{m_a}}$$

$$Y = \frac{F/A}{\Delta L/L} \text{ (macro)}$$

$$Y = \frac{k_{si}}{d} \text{ (micro)}$$

$$\Omega = \frac{(q+N-1)!}{q!\,(N-1)!}$$

$$S \equiv k\ln\Omega$$

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

$$\Delta S = \frac{Q}{T} \text{ (small } Q\text{)}$$

$$\text{prob}(E) \propto \Omega(E)\, e^{-\frac{E}{kT}}$$

$$E_N = -\frac{13.6\text{eV}}{N^2} \text{ where } N = 1, 2, 3\ldots$$

$$E_N = N\hbar\omega_0 + E_0 \text{ where } N = 0, 1, 2\ldots \text{ and } \omega_0 = \sqrt{\frac{k_{si}}{m_a}} \text{ (Quantized oscillator energy levels)}$$