Name (2 points): Dawson

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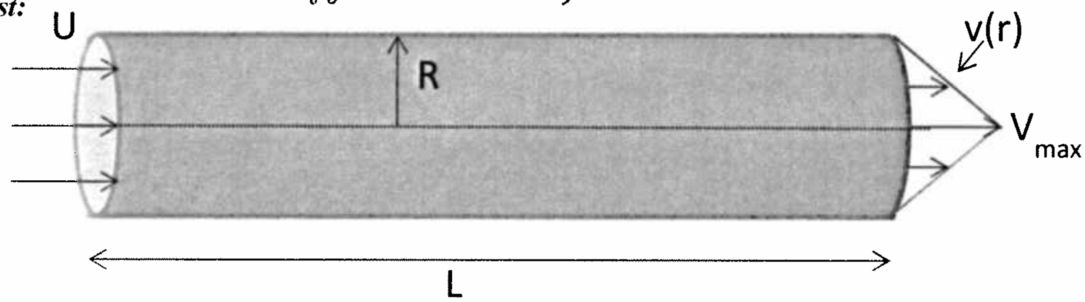
ChBE 3200
Quiz 2

Conservation of mass:

$$\int_0^{2\pi}\!\!\int_0^{R} \overset{\text{constant}}{\rho}(\boldsymbol{v}\cdot\boldsymbol{n})\,\overset{r\,dr\,d\theta}{dA} + \cancel{\frac{\partial}{\partial t}\iiint \rho\, dV}^{SS} = 0$$

System of interest:



Assume steady state incompressible flow in a circular pipe with uniform velocity profile at the entrance and linear velocity profile at the exit (as shown).

Question 1 (4 points):

Write **linear equation for the velocity profile at the exit** solving for the **constants** using the **boundary conditions**.

① $V = a + br$

BC 1 $\quad V = 0 \quad r = R$

BC 2 $\quad V = V_{max} \quad r = 0$

$0 = a + bR$

$V_{max} = a$

$b = -a/R = -\frac{V_{max}}{R}$

$V = V_{max}(1 - r/R)$

$v(r)$

$\boxed{v = V_{max}(1 - r/R)}$

② $V/V_{max} = a + b\,r/R$ dimensionless form

BC 1 $\quad V/V_{max} = 0 \quad r/R = 1$

BC 2 $\quad V/V_{max} = 1 \quad r/R = 0$

$0 = a + b$

$1 = a$

$b = -a$

$V/V_{max} = 1 - \frac{r}{R}$

Question 2 (4 points):

Use the integral equation for **conservation of mass** (continuity equation) to define the relationship between **U and V_{max}**

$$\int_0^{2\pi}\!\!\int_0^{R} \rho(-U)\,r\,dr\,d\theta + \int_0^{2\pi}\!\!\int_0^{R} \rho V_{max}(1 - r/R)\,r\,dr\,d\theta = 0$$

$$-\rho U \pi R^2 + 2\pi\rho V_{max}\int_0^R \left(r - \frac{r^2}{R}\right)dr = 0$$

$$-\rho U \pi R^2 + 2\pi\rho V_{max}\left[\frac{R^2}{2} - \frac{R^3}{3R}\right] = 0$$

$\boxed{U = V_{max}(\ 1/3\ )}$

$$\rho U \pi R^2 = 2\pi\rho V_{max}\frac{R^2}{6}$$

$$U = \frac{1}{3}V_{max}$$