Name (2 points): Dawson

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ChBE 3200
Quiz 4

BCs ① $v=0 \quad y=0$

$0 = c_2$

② $v = v_0 \quad y = h$

$v_0 = \frac{c_1 h}{\mu}$

$\boxed{c_1 = \frac{v_0 \mu}{h} = \tau_{yx}}$

$\boxed{v_x = v_0 \frac{y}{h}}$

<u>Question 1 (2 points):</u>

Define the substantial derivative of velocity $\frac{D\vec{v}}{Dt}$ as a function of time and position (x,y,z,t).

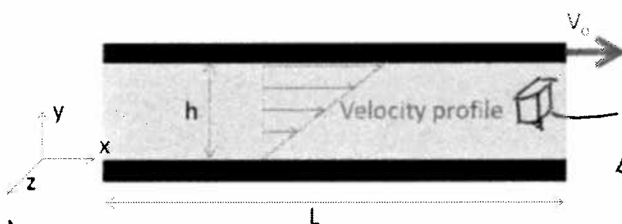
$$\rho \frac{Dv}{Dt} = \rho g - \nabla P + \mu \nabla^2 v$$

$$\boxed{\frac{\partial v}{\partial t} + v_x \frac{\partial v}{\partial x} + v_y \frac{\partial v}{\partial y} + v_z \frac{\partial v}{\partial z}}$$

<u>Question 2 (6 points):</u>

A fluid flows between two parallel horizontal plates (as shown below). The bottom plate is stationary and the top plate moves with constant velocity (v_o). Given assumptions required for the steady state velocity profile below, use **differential momentum balance** to derive an equation for the **velocity profile (v_x).**

(A) List your assumptions
(B) Write a differential balance equation
(C) Identify your BCs
(D) Provide an equation for v_x



CV
$\Delta x \Delta y \Delta z$

A. SS; FD flow; ρ, μ constant

$\frac{\partial}{\partial t} = \frac{\partial}{\partial z} = 0$; $v_y, v_z = 0$; $v_x = f(y)$

B. $P|_x \Delta y \Delta z - P|_{x+\Delta x} \Delta y \Delta z + \tau_{yx}|_{y+\Delta y} \Delta x \Delta z - \tau_{yx}|_y \Delta x \Delta z + \rho g \Delta x \Delta y \Delta z = 0$

$$-\frac{dP}{dx} + \frac{d\tau_{yx}}{dy} + \rho g = 0$$

$$\frac{d\tau_{yx}}{dy} = \frac{dP}{dx} - \rho g$$

v is linear
τ is constant
$\frac{dP}{dx} - \rho g = 0$ (flat, no pressure gradient)

$$\frac{d\tau_{yx}}{dy} = 0$$

$$\tau_{yx} = c_1$$

$$\mu \frac{dv_x}{dy} = c_1$$

$$\boxed{v_x = \frac{c_1}{\mu} y + c_2}$$