ISyE 4031 Regression and Forecasting
Homework 7 Solutions
Spring 2016

1. Exercise 6.3.

a. The trend appears to be linear.

b. The seasonal variation appears to be constant. No transformation is required.

c. All of the independent variables in the model seem to be important. This is because the *p*-values associated with Time, *Q*2, *Q*3, and *Q*4 are all much less than $\alpha = .05$ or , i.e., we reject each H_0: $\beta_j = 0$, and conclude that each variable is significant.

d. $Q2 = \begin{cases} 1 \text{ if time period for sales quarter is 2} \\ 0 \text{ otherwise} \end{cases}$, $Q3 = \begin{cases} 1 \text{ if time period for sales quarter is 3} \\ 0 \text{ otherwise} \end{cases}$

$Q4 = \begin{cases} 1 \text{ if time period for sales quarter is 4} \\ 0 \text{ otherwise} \end{cases}$

e. $\hat{y}_{17} = 17.250$, $\hat{y}_{18} = 38.750$, $\hat{y}_{19} = 51.750$, $\hat{y}_{20} = 23.250$

f. $\hat{y}_t = 8.75 + .5t + 21Q_2 + 33.5Q_3 + 4.5Q_4$

$\hat{y}_{17} = 8.75 + .5(17) + 21(0) + 33.5(0) + 4.5(0) = 17.25$

$\hat{y}_{18} = 8.75 + .5(18) + 21(1) + 33.5(0) + 4.5(0) = 38.750$

g. A 95% *P.I.* for y_{17}: $[15.395, 19.105]$. We are 95% confident that sales of the TRK-50 Mountain Bike in the first quarter of year 5 will be between 15.395 and 19.105 (15 to 19 bikes).

95% *P.I.* for y_{18}: $[36.895, 40.605]$

95% *P.I.* for y_{19}: $[49.895, 53.605]$

95% *P.I.* for y_{20}: $[21.395, 25.105]$.

h. We test H_0: $\rho = 0$ vs H_a: $\rho > 0$ (for positive correlation, it's one-sided test, $\alpha = .05$). The number of independent variables = 4, n = 16. From the table: $d_{L,.05} = 0.74$ and $d_{U,.05} = 1.93$. Since $d = 2.20 > d_{U,.05} = 1.93$. Hence, there is no evidence of positive correlation at $\alpha = .05$.

2. Exercise 6.5.

a. Yes, all the associated *p*-values are less the $\alpha = .05$. We reject each H_0: $\beta_j = 0$, and conclude that each variable is significant.

b. $\hat{y}^*_{169} = 5.3489$ and $\hat{y}^*_{170} = 5.2641$.

c. $\hat{y}^*_t = 4.80732 + .00352t - .05247M_1 - .14079M_2 - .10710M_3 + .04988MM_4 + .02542M_5$
$+ .19017M_6 + .38245M_7 + .41377M_8 + .07142M_9 + .05064M_{10} - .14194M_{11}$

$\hat{y}^*_{169} = 4.80732 + .00352(169) - .05247 = 5.3497$

$\hat{y}^*_{170} = 4.80732 + .00352(170) - .14079 = 5.2649$

d. $\hat{y}_{169} = \left(\hat{y}^{*}_{169}\right)^4 = (5.3489)^4 = 818.5739$ (819 rooms)

95% *P.I.*: $[(5.2913)^4, (5.4065)^4] = [783.8799, 854.4071]$

e. $\hat{y}_{170} = \left(\hat{y}^{*}_{170}\right)^4 = (5.2641)^4 = 767.8856$ (768 rooms)

95% *P.I.* $[(5.2065)^4, (5.3217)^4]$ = [734.8243, 802.0502]

f. We test H_0: $\rho = 0$ vs H_a: $\rho > 0$ (for positive correlation, it's one-sided test, $\alpha = .05$). The number of independent variables = 12, n = 168. Table A5 does not provide the critical values for these parameters, we can use the critical values for the largest k and n. ($d_{L,.05}$ = 1.57 and $d_{U,.05}$ = 1.78). Since d =1.262, it appears that we reject H_0, hence, there is positive autocorrelation.

3. Exercise 6.6.

a. The use of a growth curve model seems appropriate since it appears that the data might be described by the model

$$y_t = \beta_0\left(\beta_1^t\right)\varepsilon_t \text{ (see Figure 6.22).}$$

b. Yes.

c. $y_t = \beta_0\left(\beta_1^t\right)\varepsilon_t$

$ln y_t = ln\ (\beta_0) + t ln(\beta_1) + ln\,\varepsilon_t$

$ln y_t = \alpha_0 + \alpha_1 t + u_t$ where $\alpha_0 = ln\ (\beta_0)$, $\alpha_1 = ln\ (\beta_1)$, and $u_t = ln\,\varepsilon_t$.

d. 1. $\hat{\alpha}_0 = -.54334$ and $\hat{\alpha}_1 = .38997$.

2. A point estimate of β_1 is $\hat{\beta}_1 = e^{\hat{\alpha}_1} = e^{.38997} = 1.4769$.

   Growth rate = 100 $(1 - \hat{\beta}_1) = 100(1 - 1.4769) = 47.69\%$

3. The point prediction of $ln\, y_{12}$ is $ln\, \hat{y}_{12} = 4.1363$. Thus, the point forecast of

   y_{12} is $\hat{y}_{12} = e^{4.1363} = 62.5709$

4. The 95% prediction interval for $ln\, y_{12}$ is [3.8303, 4.4423]. Thus, a 95% Prediction interval for y_{12} is

   $[e^{3.8303}, e^{4.4423}]$ = [46.0764, 84.9701].

We can be 95% confident that the number of reported cased of the disease in Month 12 will be at least 46.0764 (or roughly 46) cases and will be at most 84.9701 (or roughly 85) cases.