BMED 2210: Conservation Principles in BME
Fall 2013,

Exam 3
November 15, 2013
12:05 – 12:55 pm

Instructor: Edward Botchwey

Instructions: This is a closed book exam. The use of wireless devices is not permitted. The use of programmable calculators is only permitted if all relevant content has been erased from the calculator memory.

To receive full credit show ALL work. If appropriate draw the system (indicating the boundary and system). Write the conservation equations needed to solve the problem. Label all variables and equations, and present your solution clearly. Numerical answers without units will not receive full credit.

If unable to finish due to time write how you would solve the problem with the appropriate equations to receive partial credit for the process.

Name: Answer Key

GT ID: ____________________

The work presented here is solely my own. I did not receive any assistance nor did I assist other students during the exam. I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

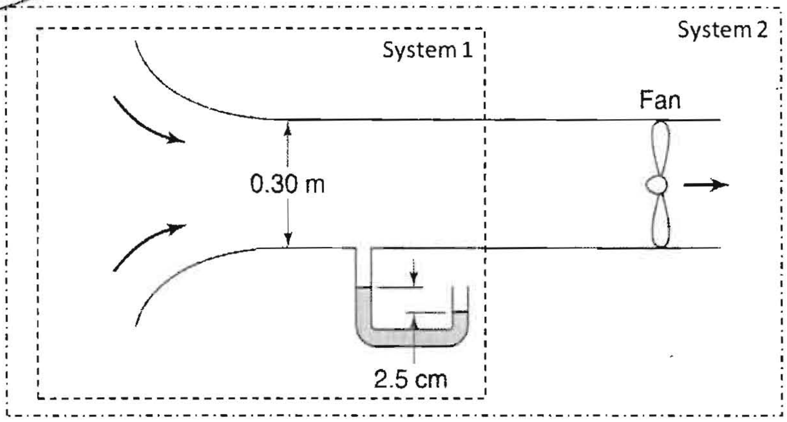
Signature: ____________________

Problem 1: _____/ 20
Problem 2: _____/ 25
Problem 3: _____/ 25
Problem 4: _____/ 30

Total: _____/ 100 points

Problem 1: A fan draws air from the atmosphere through a 0.30 m diameter round duct that has a smoothly rounded entrance. A differential manometer connected to an opening in the wall of duct shows a vacuum pressure ΔP = 2.5 cm of water. The density of air is 1.22 kg/m^3. Using Bernoulli's Equation, determine the output power of the fan?

$\Delta P = P_{in} - P_{out}$ b/c water is being pulled into the pipe



a) List four assumptions needed to apply Bernoulli (6 possible answers).

1) Steady State
2) Incompressibility
3) Inviscid fluid
4) No Rxns
5.) only mechanical + thermal energy
6.) Along streamlines

b) Examine System 1 and determine the velocity of the air duct.

$$\frac{\cancel{V_{in}^2} - V_{pipe}^2}{2} + \frac{P_{in} - P_{pipe}}{\rho} + g\cancel{(h_{in} - h_{pipe})} + \frac{\cancel{\dot{W}}}{\dot{m}} = 0$$

$$V_{pipe} = \sqrt{\frac{2(P_{in} - P_{pipe})}{\rho}} = \sqrt{\frac{2(2.5\,cm\,H_2O)\left(\frac{1.01325\times10^5\,Pa}{10.333\,m\,H_2O}\right)\left(\frac{1\,m}{100\,cm}\right)}{1.22\,kg/m^3}}$$

$$\boxed{V_{pipe} = 20.05\ \frac{m}{s}}$$

c) Examine System 2 and determine the power generated by the fan in watts.

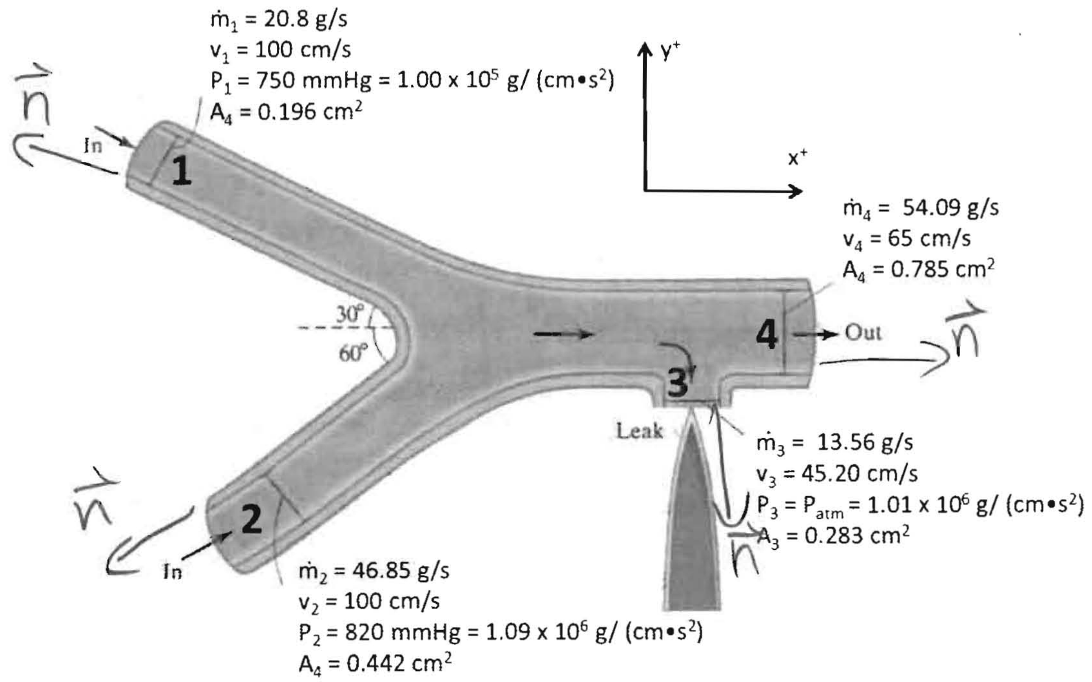
$P_{in} = P_{out} = P_{atm}$

$$\frac{\cancel{V_{in}^2} - V_{out}^2}{2} + \frac{P_{in} - \cancel{P_{out}}}{\rho} + g\cancel{(h_{in} - h_{out})} + \frac{\dot{W}}{\dot{m}} = 0$$

$$\dot{W} = \frac{\dot{m}\,V_{out}^2}{2} \qquad V_{out} = V_{pipe} \Rightarrow \dot{W} = \frac{\rho A_{pipe} V_{pipe}^3}{2}$$

$$\dot{W} = \frac{(1.22\,kg/m^3)(0.3\,m)^2\left(\frac{\pi}{4}\right)(20.05\,\frac{m}{s})^3}{2} \Rightarrow \boxed{\dot{W} = 347.54\,W}$$

Problem 2: Two blood vessels join to form a larger vessel as shown below. During a surgical procedure a needle accidentally pokes a hole with an area of 0.283cm^2, exposing the vessel to air. The pressures of the inlet vessels, 1 and 2, are measured to be 750 mmHg and 820 mmHg respectively. Assume the system is at steady state and blood has a density of 1.06 g/cm^3. Determine the resultant force in the y-direction required to keep the vessel stationary.



a) Write and simplify the conservation equation for momentum. Then solve for the resultant force in the y-direction <u>in terms of variables only</u>, accounting for all streams and forces.

$$\sum_{in} \dot{\vec{P}} - \sum_{out} \dot{\vec{P}} + \sum \vec{F} = \cancel{\frac{d\vec{P}}{dt}}^{\,0}$$

$$y: \dot{m}_1 \vec{V}_{1,y} + \dot{m}_2 \vec{V}_{2,y} - \dot{m}_3 \vec{V}_{3,y} + \cancel{\dot{m}_4 \vec{V}_{4,y}}^{\,0} + \vec{F}_{P1,y} + \vec{F}_{P2,y} + \vec{F}_{P3,y} + \cancel{\vec{F}_{P4,y}}^{\,0} + \vec{F}_y = 0$$

$$F_y = -(\dot{m}_1 \vec{V}_{1,y} + \dot{m}_2 \vec{V}_{2,y} - \dot{m}_3 \vec{V}_{3,y} + \vec{F}_{P1,y} + \vec{F}_{P2,y} + \vec{F}_{P3,y})$$

b) If blood has a viscosity of 3.4 mPa•s, what is the Reynolds number at the leak? What kind of velocity profile is this?

$$Re = \frac{\rho V D}{\mu} \qquad \frac{\pi}{4} D^2 = A \Rightarrow D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(0.283\,cm^2)}{\pi}}$$

$$D = 0.6 \text{ cm}$$

$$= \frac{(1.06\ g/cm^3)(45.20\ cm/s)(0.6\ cm)}{(3.4\times10^{-3}\ \frac{kg}{m\cdot s^2}\ s)\left(\frac{1000g}{1kg}\right)\left(\frac{1m}{100cm}\right)} \Rightarrow \boxed{Re = 846}$$

$\Rightarrow$ Laminar Flow

c) Calculate the force in the y-direction due to pressure ($F_{P_{total,y}}$) in [g•cm/s^2].

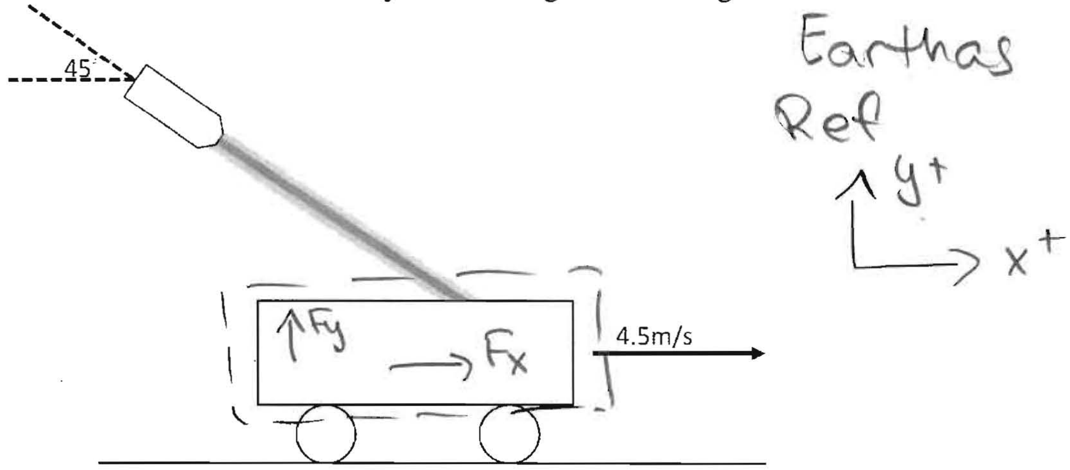
$$F_{P_{total,y}} = \sum F_{P_{i,y}} = \sum (-P\vec{n}A)_{i,y}$$

$$= -\left(1\times10^{5}\,\frac{g}{cm\cdot s^{2}}\right)(+1\sin 30)(0.196\,cm^{2}) - \left(1.09\times10^{6}\,\frac{g}{cm\cdot s^{2}}\right)(-1\sin 60)(0.442\,cm^{2})$$

$$-\left(1.01\times10^{5}\,\frac{g}{cm\cdot s^{2}}\right)(-1)(6.283\,cm^{2})$$

$$\boxed{F_{P_{total,y}} = 693264\,\frac{g\cdot cm}{s^{2}}}$$

Problem 3: An open tank cart as shown travels to the right at a uniform velocity of 4.5 m/s. At the instant shown the car passes under a jet of water issuing from a stationary pipe with a mass flow rate of 157.08 kg/s relative to the cart. The velocity of the jet is measured to be 20 m/s relative to the Earth. What is the force exerted by the cart? Ignore the weight of the cart.



a) Draw the resultant forces, and label the coordinate according to the reference point of your choice. Simplify the conservation equation for momentum, and separate the equation into the appropriate Cartesian coordinates using the chosen reference ~~point~~. frame.

$$\sum_{in}\dot{\vec{p}} - \cancel{\sum_{out}\dot{\vec{p}}}^{\,0} + \sum\vec{F} = \frac{dp}{dt} = \dot{m}\vec{v} + \cancel{m\vec{a}}^{\,0}$$

$$x:\ \dot{m}_{jet}\vec{V}_{jet,x} + \vec{F}_x = \dot{m}_{jet}\vec{V}_{cart}$$

$$y:\ \dot{m}_{jet}\vec{V}_{jet,y} + \vec{F}_y = 0$$

b) Determine the resultant force in the y-direction.

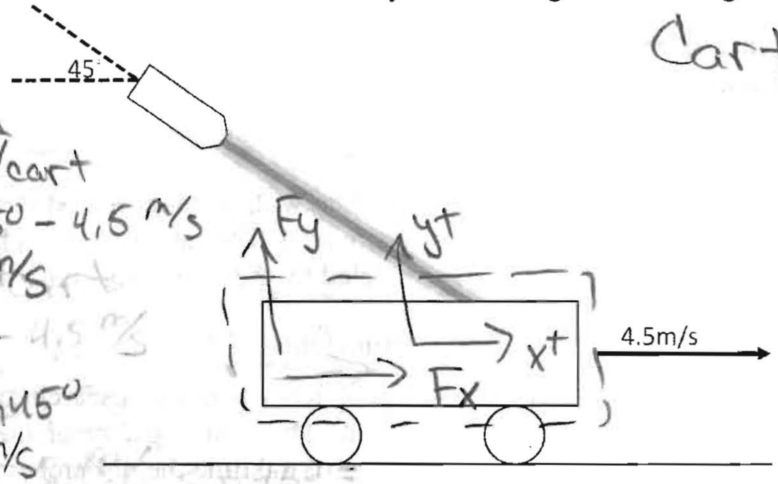
$$F_y = -\dot{m}_{jet}\vec{V}_{jet,y}$$
$$= -(157.08\ \tfrac{kg}{s})(-20\ \tfrac{m}{s}\sin 45^\circ)$$
$$\boxed{F_y = 2221.45\,N}$$

c) Determine the resultant force in the x-direction.

$$F_x = -\dot{m}_{jet}V_{jet,x} + \dot{m}_{jet}\vec{V}_{cart}$$
$$= \dot{m}_{jet}(\vec{V}_{cart} - V_{jet\,x})$$
$$F_x = (157.08\ \tfrac{kg}{s})(4.5\ \tfrac{m}{s} - 20\ \tfrac{m}{s}\cos 45^\circ)$$
$$\boxed{F_x = -1514.59\,N}$$

Problem 3: An open tank cart as shown travels to the right at a uniform velocity of 4.5 m/s. At the instant shown the car passes under a jet of water issuing from a stationary pipe with a mass flow rate of 157.08 kg/s relative to the cart. The velocity of the jet is measured to be 20 m/s relative to the Earth. What is the force exerted by the cart? Ignore the weight of the cart.

Cart as Ref



$\vec{V}_{jet,c,x} = \vec{V}_{jet,x} - \vec{V}_{cart}$

$= 20 \frac{m}{s} \cos 45^{\circ} - 4.5 \, m/s$

$V_{jet,c,x} = 9.64 \, m/s$

$V_{jet,c,y} = V_{jet,y}$

$= 20 \, m/s \sin 45^{\circ}$

$= 14.14 \, m/s$

a) Draw the resultant forces, and label the coordinate according to the reference point of your choice. Simplify the conservation equation for momentum, and separate the equation into the appropriate Cartesian coordinates using the chosen reference ~~point~~. frame

$$\sum_{in} \dot{\vec{p}} - \cancel{\sum_{out} \dot{\vec{p}}} + \sum \vec{F} = \frac{dp}{dt} = \cancel{\dot{m}\vec{V}} + \cancel{m\vec{a}}$$

$$0 \qquad\qquad 0 \quad 0$$

$x: \dot{m}_{jet} \vec{V}_{jet,c,x} + F_x = 0$

$y: \dot{m}_{jet} \vec{V}_{jet,y} + F_y = 0$

b) Determine the resultant force in the y-direction.

$F_y = - \dot{m}_{jet} \vec{v}_{jet,y}$

$= -(157.08 \, kg/s)(-14.14 \, m/s)$

$\boxed{F_y = 2221.11 \, N}$ ← off due to rounding

c) Determine the resultant force in the x-direction.

$F_x = - \dot{m}_{jet} V_{jet,c,x}$

$= -(157.08 \, kg/s)(9.64 \, m/s)$

$\boxed{F_x = -1514.25 \, N}$ ←

Problem 4: A heat sensitive sample stored in a capped test tube is taken from the freezer to be analyzed. Soon after the sample tube is immersed in an ice-water bath, the fire alarm goes off. The researcher leaves the room immediately leaving the sample still in the bath. Fortunately, the fire alarm is only an unscheduled fire drill.

The rate of heat exchange between the ice-water bath and its surrounding air can be modeled by the following equation:

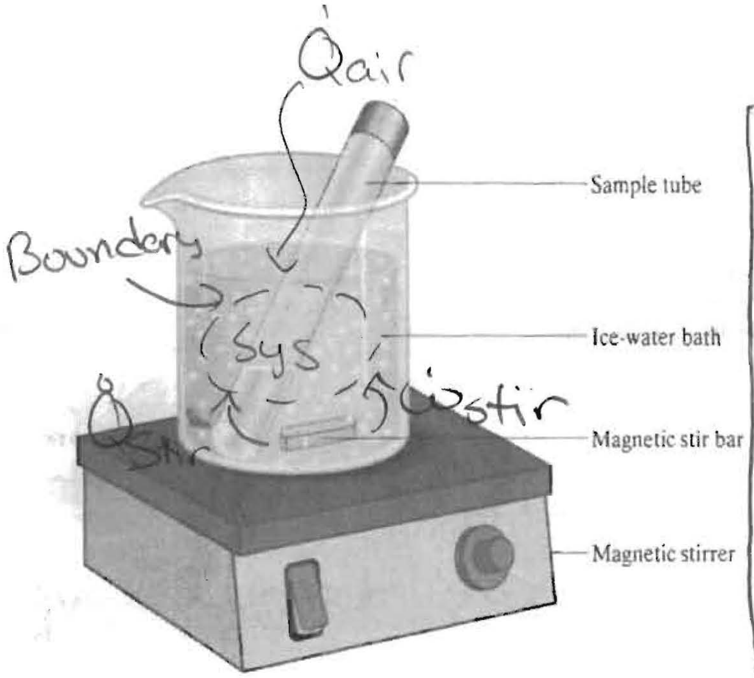
$$\dot{Q}_{Air} = h_{Air}A_{Air}(T_{Air} - T)$$

Additionally, since the ice-water bath is in contact with a magnetic stirrer, heat and work exchanges between the two systems need to be accounted. The rate of heat exchange between the ice-bath and magnetic stirrer can be similarly modeled by the following equation:

$$\dot{Q}_{Stir} = h_{Stir}A_{Stir}(T_{Stir} - T)$$

The ice bath contains 100g of water and 400 g of ice when the researcher leaves the room. Assume the work done by the stirrer is 70 cal/min. The heat capacity of the test tube can be ignore, and the ice will remain frozen for the entirety of the time the researcher is gone.

Suppose the sample is the same temperature of the water bath and will be damaged if its temperature is above 5 ˚C. Estimate the maximum duration of the fire drill for the sample to remain intact. Assume the researcher returns to the laboratory immediately after the fire drill.



Parameters:

T_{Air} = 22 ˚C T_{Stir} = 32 ˚C

h_{Air} = 0.03 cal/(cm^2•min•˚C)

h_{Stir}= 0.1 cal/(cm^2•min•˚C)

A_{Air} = 500 cm^2 A_{Stir} = 200 cm^2

m_{Water} = 0.1 kg m_{Ice} = 0.4 kg

$C_{p,Water}$ = 75.4 J/(mol•˚C)

$C_{p,Ice}$ = 36.5 J/(mol•˚C)

W = 70 cal/min

T_i = -40 ˚C T_f = 5 ˚C

a) Label the system, boundary, surroundings, inputs and outputs. Then, write and simplify the conservation equation for energy in dynamic systems.

$$\cancel{\dot{m}\, C_p (T_{in} - T)} + \boxed{\sum \dot{Q} + \sum \dot{W}_{nonflow} = m\, C_v \frac{dT}{dt}}$$

$$C_p \approx C_v$$

$$\dot{Q}_{stir} + \dot{Q}_{air} + \dot{W}_{stir} = (m_{ice} C_{p,ice} + m_{water} C_{p,water}) \frac{dT}{dt}$$

b) Calculate the total rate of heat entering the system as a function of temperature. $\sum \dot{Q} = a + bT$

$$\sum \dot{Q} = \dot{Q}_{stir} + \dot{Q}_{air}$$

$$= h_{stir} A_{stir} (T_{stir} - T) + h_{air} A_{Air} (T_{Air} - T)$$

$$= \left(0.1 \frac{cal}{cm^2 \cdot min \cdot °C}\right)(200\,cm^2)(32°C - T) + \left(0.03 \frac{cal}{cm^2 \cdot min \cdot °C}\right)(500\,cm^2)(22°C - T)$$

$$= 640 \frac{cal}{min} - 20 \frac{cal}{min\,°C} T + 330 \frac{cal}{min} - 15 \frac{cal}{min\,°C} T$$

$$\boxed{\sum \dot{Q} = 970 \frac{cal}{min} - 35 \frac{cal}{min\,°C} T}$$

c) Calculate $m_{Total} \bullet C_{p,Total}$, the total heat capacitance for the system in [cal/°C].

$$m_T C_{p,T} = m_{ice} C_{p,ice} + m_{water} C_{p,water}$$

$$= (0.4\,Kg)\left(36.5 \frac{J}{mol\,°C}\right) + (0.1\,Kg)\left(75.4 \frac{J}{mol\,°C}\right)$$

$$= 22.14 \frac{\cancel{Kg}\,\cancel{J}}{\cancel{mol}\,°C} \left(\frac{1\,\cancel{mol}}{18\,\cancel{g}}\right)\left(\frac{1000\,\cancel{g}}{1\,\cancel{kg}}\right)\left(\frac{0.23901\,cal}{1\,J}\right)$$

$$\boxed{m_T C_{p,T} = 293.98 \frac{cal}{°C}}$$

d) Calculate the maximum time the sample can stay in the water-ice bath.

$$\sum \dot{Q} + \dot{W} = m_T C_{p,T} \frac{dT}{dt}$$

$$970 \frac{cal}{min} - 35 \frac{cal}{min\,°C} T + 70 \frac{cal}{min} = 293.98 \frac{cal}{°C} \frac{dT}{dt}$$

$$3.54 \frac{°C}{min} - 0.12 \frac{1}{min} T = \frac{dT}{dt}$$

$$\int_0^t dt = \int_{-40°C}^{5°C} \frac{dT}{3.54 - 0.12T}$$

$$t = \left(\frac{-1}{0.12\,min^{-1}}\right) \ln(3.54 - 0.12T) \Big|_{-40°C}^{5°C} \Rightarrow \boxed{t = 8.69\,min}$$