GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 - BIOTRANSPORT

FIRST TERM TEST FALL 2013 - ETHIER

STUDENT NAME: Solution

GTID NUMBER: ______

RECITATION SECTION: ______

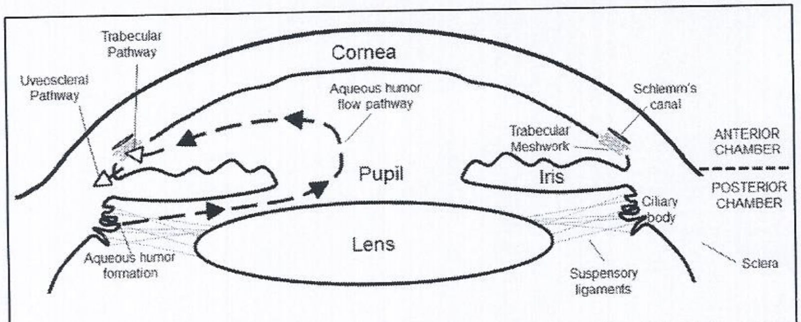
Open book
All non-communicating calculator types allowed
Time allotted: 80 minutes
Do all work in this booklet

Reminder: for questions requiring numerical answers, units are required and worth 50%

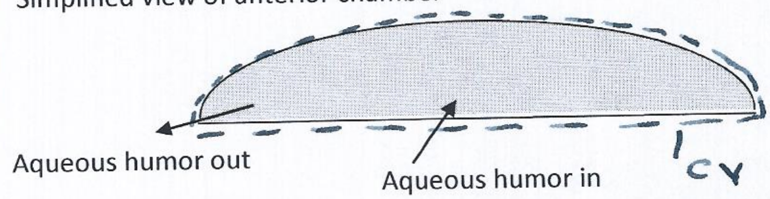
Question	Maximum Mark	Actual Mark	
1	20		5 min
2	35		6 min
3	45		8 min
Total	**100**		

58

1. The front part of your eye (the anterior chamber) is filled by a fluid called the aqueous humor. This fluid is produced just behind the anterior chamber and drains out of the eye at the edge of the anterior chamber (see arrows on figure[1]). The flow rate of aqueous humor can be measured in vivo by using a fluorescent tracer. At time t=0, tracer is delivered into the anterior chamber and rapidly mixed with the aqueous humor. (No further tracer is delivered to the eye after this initial dose.) The fluorescence in the anterior chamber is then measured over time. You may treat the anterior chamber as a well-mixed reservoir of fixed volume V = 125 μL. The aqueous humor inflow is steady. At time t=27 minutes, the concentration of tracer in the anterior chamber is measured to be half of the concentration present at time t=0. What is the flow rate of aqueous humor?



Simplified view of anterior chamber



Do your GIM analysis here

Conserve mass of aqueous humor 2 & tracer 2

Problem is steady for AH 1

— " — unsteady for tracer 1

Use cv as shown 2

Since AH is steady, we have $\mathcal{Q}_{in} = \mathcal{Q}_{out} = \mathcal{Q}$ 1

Call c the concentration of tracer in the anterior chamber. Mass balance on tracer:

rate of accum = rate of inflow − rate of outflow 2

$$V \frac{dc}{dt} = 0 - \mathcal{Q} c \quad 2$$

$$\therefore \boxed{\frac{dc}{dt} = -\frac{\mathcal{Q}}{V} c}$$

4

[1] Image from Ito, Y. A., & Walter, M. A. (2013). Genetics and Environmental Stress Factor Contributions to Anterior Segment Malformations and Glaucoma.

Integrate :

$$\ln c = -\frac{q t}{V} + \text{const}$$ ①

$c = c_0$ at time $t = 0$ $\therefore$ const $= \ln c_0$

$$\therefore \ln \frac{c_0}{c} = \frac{q t}{V}$$ ②

@ $t = 27$ min

$\frac{c_0}{c} = 2$

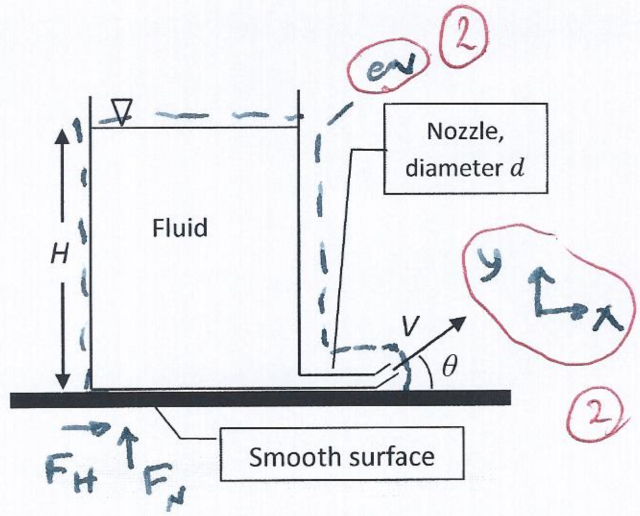
$V = 125\ \mu l.$

$$q = \frac{V \ln 2}{t} = \frac{(125) \ln 2}{27} \frac{\mu l}{\min}$$

$$= 3.21\ \mu l/\min.$$ ④

⑳

2. A large cylindrical tank of diameter D is filled to a depth H by fluid of density ρ. A small nozzle of diameter d is located at the bottom of the tank, as shown, with $d \ll H$. At the end of the nozzle there is an adjustable deflector that causes the fluid to leave the nozzle at an angle θ. The tank sits on a smooth surface with coefficient of static friction ζ, so that the maximum frictional force between the tank and the smooth surface is ζF_N, where F_N is the normal force exerted by the tank on the surface.



It is possible to show that fluid exits the nozzle with velocity $V = \sqrt{2gH}$. Given this information, and neglecting the weight of the tank, but not the weight of the fluid in it, derive a constraint for ζ to ensure that the tank will not slide sideways on the smooth surface as the fluid exits the nozzle. You may assume that the tank drains slowly enough that the process is steady. Your constraint for ζ should involve only θ, d and D.

Do your GIM analysis here

- steady (2)
- balance both x & y momentum (see def'n of x & y) (1) (2)
- note vertical component of fluid exiting affects F_N
- CV as shown
- normal & horizontal forces as shown (F_H & F_N)

y-momentum: $+\uparrow \sum F_y = \dot{m} V_y\big|_{out} - \dot{m} V_y\big|_{in}^{\,0}$ (6)

$$F_N - \frac{\pi}{4} D^2 H \rho g = \rho \frac{\pi}{4} d^2 V^2 \sin\theta$$

but $V = \sqrt{2gH}$

$$\therefore F_N = \rho \frac{\pi}{4}\left[d^2(2gH)\sin\theta + D^2 gH\right]$$

$$= \rho g H \frac{\pi}{4}\left[D^2 + 2d^2 \sin\theta\right]$$ (4)

x - mom $\quad \xrightarrow{+} \sum F_x = \dot{m} V_x|_{out} - \dot{m} \cancel{V_x}|_{in}^{0}$ (6)

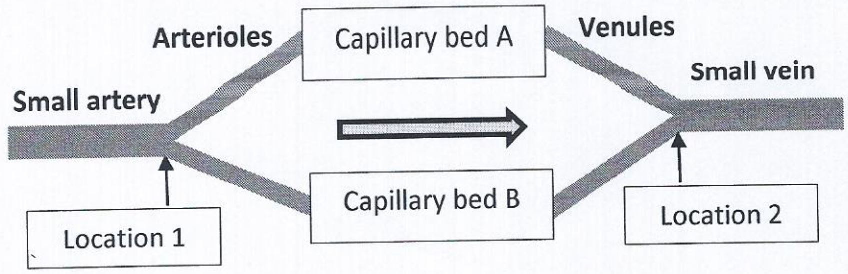
$$\left.\begin{aligned} F_H &= \rho \frac{\pi}{4} d^2 V^2 \cos\theta \\ &= \rho \frac{\pi}{4} d^2 \cdot 2gH \cdot \cos\theta \\ &= \rho g H \frac{\pi}{4} \left[2d^2 \cos\theta \right] \end{aligned}\right\} (4).$$

Need $\frac{F_H}{F_N} \leq \mu$ (2)

$$\therefore \mu \geq \frac{2d^2\cos\theta}{2d^2 \sin\theta + D^2} = \frac{2\cos\theta}{2\sin\theta + (D/d)^2} \Bigg\} (4).$$

Neglecting effect of fluid jet on F_N, but everything else ok ~~(-12)~~ to (-16)

3. A simple vascular network is shown schematically. A small artery feeds two arterioles, which in turn feed 2 capillary beds. Blood drains from the capillary beds into two venules and then into a small vein. The capillary beds are essentially identical. Each bed may be considered to consist of 2×10^5 capillaries in parallel, with each capillary being 8 μm in diameter and 1000 μm long. For purposes of this question you need only consider the steady component of blood flow, and you can neglect the flow resistance of the venules. The effective viscosity of blood is 3.5 cP.



a. The blood pressures at Locations 1 and 2 are measured to be 75 mmHg and 35 mmHg, respectively. For the arteriole dimensions shown in the table, predict the total blood flow rate entering capillary bed A. State assumptions.

	Arteriole feeding bed A	Arteriole feeding bed B
Length L	0.3 cm	0.3 cm
Diameter D	0.015 cm	0.013 cm

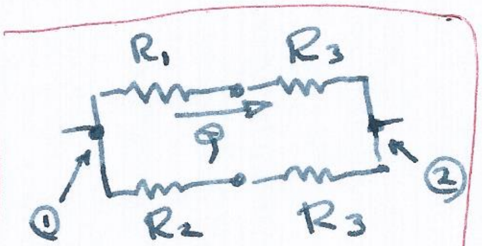
b. An agonist is administered that causes the arteriole feeding capillary bed A to decrease its diameter by 50% along its entire length. Assuming the blood pressures at Locations 1 and 2 do not change, what is the blood flow through capillary bed A after agonist administration?

Do your GIM analysis here

- treat as steady (2)
- system is blood in network (1)
- use the equivalent electrical circuit idea (2)

Assumptions: flow in all vessels obeys Poiseuille's law (steady, fully developed, Newtonian) (2)

(a)



(4)

q is flow in upper leg of circuit (6)

$$q = \frac{P_1 - P_2}{R_1 + R_3}$$

(resistance of lower leg is irrelevant)

$$R_1 = \frac{128 \mu L}{\pi D^4}$$ (2)

$\mu = 3.5 \times 10^{-2}$ g/(cm s)

$L = 0.3$ cm

$D = 0.015$ cm

$$= \frac{(128)(0.035)(0.3)}{\pi (0.015)^4} \quad \frac{g}{cm\ s\ cm^3}$$

$$= 8.451 \times 10^6 \ g/(cm^4 s)$$ (3)

$$R_3 = \frac{1}{N} \frac{128 \mu L}{\pi D^4}$$ (3)

$N = 2 \times 10^5$ $\quad$ $D = 8 \times 10^{-4}$ cm

$L = 0.1$ cm $\quad$ $\mu = 0.035$ g/(cm s)

$$= \frac{(128)(0.035)(0.1)}{(2 \times 10^5)\pi(8 \times 10^{-4})^4} \quad \frac{1}{cm^4 s}$$

$$= 1.741 \times 10^6 \ g/cm^4 s$$ (3)

R of one capillery = 3.482×10^{11} $g/cm^4 s$

$R_{tot} = 10.19 \times 10^6$ $g/cm^4 s$

$$Q = \frac{75-35}{(8.451 + 1.741)\times 10^6} \ mmHg \times \frac{1013250}{760} \quad \frac{g}{cm\ s^2\ mmHg} \frac{cm^4 s}{g}$$ (6)

$\Delta p = 5.33 \times 10^4 \ \frac{dyne}{cm^2}$

$$= 5.23 \times 10^{-3} \ cm^3/s$$

incorrect or no units (-10)

(35)

(b) R_1 changes, R_3 stays the same.

New $R_1 = \underbrace{16 \times \text{old } R_1}_{3} = \underbrace{1.352 \times 10^8}_{3} \ g/cm^4 s$ (6)

$$\text{new } Q = \frac{40}{(135.2 + 1.741)\times 10^6} \frac{1013250}{760} = 3.894 \times 10^{-4} \ cm^3/s$$ (4)

(10)

multiplying entire upper branch by 16 (-6)

~~Q changes~~

(45)