ChBE 3210 Spring 2015 Final Exam Solutions

Problem I/ A) FALSE; $\vec{N}_A = 0$ only quantifies <u>average</u> motion

B) TRUE

C) FALSE; co-flow operation (negative OL slope)

D) TRUE; ethane is smaller and lighter than n-butane

E) FALSE; contact area much greater for small bubbles

F) TRUE; $\delta \ll D \Rightarrow$ plate correlation is OK

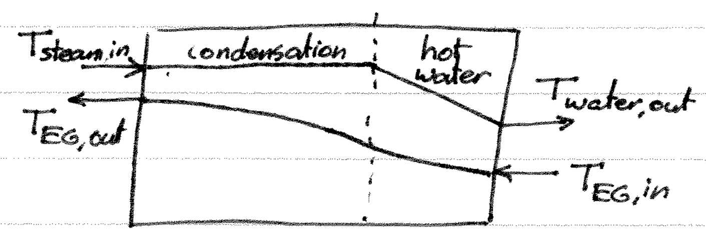
G) TRUE; $Bi = \frac{k_c L}{D_{AB}}$

H) FALSE; Hatta number is for <u>bulk</u> reactions

I) FALSE; total mass flow needs to be constant at different heights, but flux changes due to variable area

J) FALSE: $\vec{J}_A$ is molar, $\vec{n}_A$ is mass flux

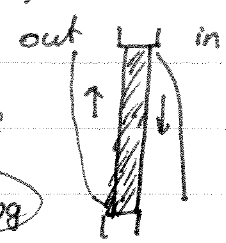
Problem II/ A)



- T_{steam} constant until condensation complete and then $T_{water} \downarrow$
- T_{EG} rises continuously

B) • Cooling effect occurs due to mass transfer of evaporating liquid, which requires/withdraws energy. Rubbing alcohol has higher vapor pressure and evaporates faster $\Rightarrow$ feels colder than water

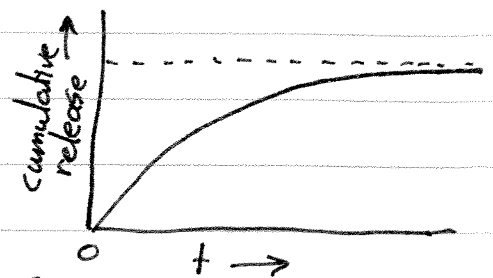
C) * Condensation happens when warm, moist air is cooled; this happens at inner surface of window.



* Natural convection on inside and outside surface are such that heat transfer window $\rightarrow$ outside is greatest at bottom (cooling air falling) (warming air rising)
(=> low window temp) and heat transfer room $\rightarrow$ window is lowest at bottom (thick BL) (=> also low window temp)

=> at location where window is coldest, frost will be observed first

D) * flux from slab → water greatest at $t=0$ ⇒ highest slope
* when $t\uparrow$, driving force and flux $\downarrow$ ⇒ lowest slope
* for $t \to \infty$, cumulative release plateaus



Problem III A) * minimum flow rate occurs if all refrigerant fully condensed between inlet and outlet: $q = \dot{m} \cdot h_{fg} \Rightarrow \dot{m} = \frac{q}{h_{fg}} = \frac{250\ J/s}{147000\ J/kg} = 1.70 \cdot 10^{-3}\ kg/s = 6.12\ kg/hr$

* $\dot{m} = \rho\, V_{ave} \cdot \frac{\pi}{4} D_{in}^2 \Rightarrow V_{ave} = \frac{4\dot{m}}{\pi \rho D_{in}^2} = \frac{4 \cdot 0.0070}{\pi (0.009)^2 \cdot \rho} = \frac{26.72}{\rho}$

⇒ 100% vapor: $V_{ave} = \frac{26.72}{7.38} = 3.62\ m/s$

100% liquid: $V_{ave} = \frac{26.72}{1511} = 0.0177\ m/s$

factor 200!

B) • $R_{th,conv,out} = \frac{1}{h_o A_{out}} = \frac{1}{h_o \pi D_o L} = \frac{1}{20 \cdot \pi \cdot 0.01 \cdot L} = \frac{1.59}{L}\ [K/W]$

$R_{th,cond} = \frac{\ln(D_o/D_i)}{2\pi k L} = \frac{\ln(10/9)}{2\pi\, 300\, L} = \frac{5.6 \cdot 10^{-5}}{L} \ll R_{th,conv,out}$

C) • $R_{th,conv,in} = \frac{1}{h_i A_{in}} = \frac{1}{2000 \cdot \pi \cdot 0.009 \cdot L} = \frac{0.0177}{L}$

$q = \frac{\Delta T_{LM}}{\sum R_{th}} = \frac{\Delta T_{LM}}{\frac{1.59}{L} + \frac{0.018}{L}} = \frac{L}{1.61} \cdot \Delta T_{LM}$ with $\Delta T_{LM} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$

$\left.\begin{aligned} \Delta T_1 &= 321 - 293 = 28K \\ \Delta T_2 &= 321 - 313 = 8K \end{aligned}\right\} \Delta T_{LM} = 16.01$

$\Rightarrow L = \frac{1.61 \cdot q}{\Delta T_{LM}} = \frac{1.61 \cdot 250}{16.0} = 25.2\,m$

D) * Most of the coil is "horizontal cylinder in natural convection"; challenge is that T_∞ varies at subsequent cylinders (coil elements) in the flow. Use $T_{f,ave} = \frac{T_{f,top} + T_{f,bottom}}{2} = \frac{\frac{40+48}{2} + \frac{20+48}{2}}{2} = 39°C$ as representative average temperature. Can also argue "flat plate" (vertical).

E) * $R_{th,fouling} = 0.10 \cdot \sum R_{th} = \frac{0.161}{L} \Rightarrow \frac{R_{fouling}}{\pi D_o L} = \frac{0.161}{L}$

$\Rightarrow R_{fouling} = \pi \cdot D_o \cdot 0.161 = 0.00506\ m^2/K \cdot W$

$= 506 \cdot 10^{-5}\ m^2/K \cdot W$ (higher than all values in handout Table)

F) * If refrigerator back too close to wall, then natural convection is obstructed $\Rightarrow h_o \downarrow \Rightarrow q \downarrow$

⇒ performance degraded

Problem IV/ A) * convection: $q_{conv} = h \cdot A_{surf} \cdot \Delta T = h \cdot \pi \cdot D \cdot L \cdot \Delta T = 0.90 \cdot P_{heater}$ (10% not convection)

$$\Rightarrow h = \frac{0.9 \cdot P_{heater}}{\pi D L \, (T_s - T_\infty)} = \frac{0.9 \cdot 250}{\pi \cdot 0.015 \cdot 0.5 \cdot (130-25)} = 90.9 \ W/m^2 \cdot K$$

B) * external forced convection around cylinder $\Rightarrow Nu_D = B \cdot Re_D^n \cdot Pr^{1/3}$

$T_f = \frac{130+25}{2} = 77.5°C \approx 350K \Rightarrow Pr = 0.700, \ \nu = 2.06 \cdot 10^{-5} \ m^2/s$

$Re = \frac{V_\infty D}{\nu} = \frac{10 \cdot 0.015}{2.06 \cdot 10^{-5}} = 7282 \Rightarrow B = 0.193 \ \& \ n = 0.618 \Rightarrow Nu_D = 41.8$

$\Rightarrow h = \frac{k}{D} \cdot Nu_D = \frac{3.0 \cdot 10^{-2}}{0.015} \cdot 41.8 = 83.6 \ W/m^2 \cdot K$

(pretty close to experimental value)

C) * Heat needed for evaporation = Convection flux per area

$\Rightarrow \lambda \cdot M_{H_2O} \cdot N_{H_2O} = h (T_\infty - T_s)$ with $N_{H_2O} = k_c (c_{H_2O,sat} - c_{H_2O,\infty})$

$\Rightarrow T_s = T_\infty - \lambda M_{H_2O} \frac{k_c}{h} (c_{H_2O,sat} - c_{H_2O,\infty})$

* Chilton - Colburn:

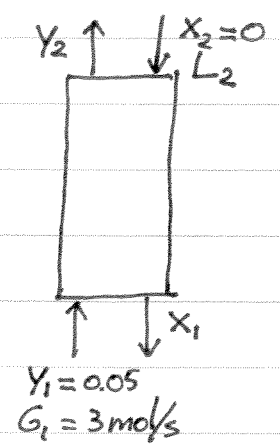
$$\frac{h}{\rho V_\infty C_p} Pr^{2/3} = \frac{k_c}{V_\infty} Sc^{2/3} \Rightarrow \frac{k_c}{h} = \frac{1}{\rho C_p} \left(\frac{Pr}{Sc}\right)^{2/3}$$

$$\Rightarrow T_s = T_\infty - \lambda M_{H_2O} \frac{1}{\rho C_p} \left(\frac{Pr}{Sc}\right)^{2/3} \left[\frac{P_{vap} - 0.3 \cdot P_{vap}}{RT}\right]$$

Estimate $T_f \approx 15°C$ (first guess) $= 290K$

$$\Rightarrow T_s = 25 - \underset{[kJ/kg]\,[kg/kmol]}{2500 \cdot 18} \cdot \frac{1}{\underset{[kg/m^3]\,[kJ/kg \cdot K]}{1.22 \cdot 1.006}} \cdot \left(\frac{0.70}{0.60}\right)^{2/3} \cdot \frac{\overset{[atm]}{0.7 \cdot 0.015}}{\underset{[atm \cdot m^3/kgmol \cdot K]}{0.08206} \cdot 288 \, [K]}$$

$$= 25 - 18.1 = \underbrace{6.9°C}_{\text{close to experiment!}} \quad \left[\Rightarrow T_f = \frac{25+6.9}{2} = 16°C \text{ close enough!}\right]$$



Problem V/ A) * $G_1 = 3$ mol/s $\Rightarrow G_s = 0.95 \cdot 3 = 2.85$ mol/s ($CO_2 + N_2$)

$\Rightarrow L_s = 3.0 \cdot G_s = 8.55$ mol/s (water flow rate)

* $Y_1 = 0.05 \Rightarrow 0.05 \cdot 3 = 0.15$ mol/s of EO enters tower

* 98% of this EO = 0.147 mol/s leaves in water

$\Rightarrow x_1 = \frac{0.147}{8.55 + 0.147} = 0.0169 = 1.69\%$

* 2% of EO = 0.003 mol/s leaves in gas

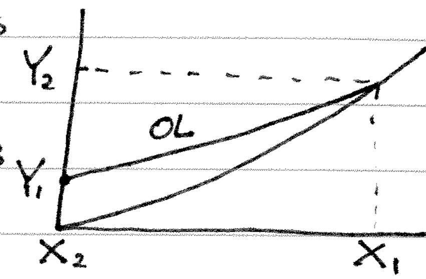
$\Rightarrow Y_2 = \frac{0.003}{2.85 + 0.003} = 0.00105 = 0.105\%$

B) * $x_1 \uparrow$: less water to dissolve EO

* $y_2 \uparrow$: less EO removed b/c lower driving force for transfer

C) * Now, operating line and equilibrium line intersect at (x_1, y_2)

* Need to use solute-free concentrations



$$Y_2 = \frac{0.05}{1-0.05} = 0.0526, \quad Y_1 = \frac{0.01}{1-0.01} = 0.0101$$

$$L_s/G_s = 1.5 = \frac{Y_2 - Y_1}{X_1 - \cancel{X_2}_{=0}} \Rightarrow X_1 = \frac{0.0425}{1.5} = 0.0283$$

$$\Rightarrow x_1 = \frac{X_1}{1+X_1} = 0.0276$$

$$\Rightarrow C = \frac{y_2}{x_1} = \frac{0.05}{0.0276} = 1.81$$

D) * Molar flux: $k_y(y_2 - y_{2,i}) = k_y(y_2 - Cx_{2,i})$

$$K_y(y_2 - y_2^*) = K_y(y_2 - \underbrace{\cancel{Cx_2}}_{x_2=0})$$

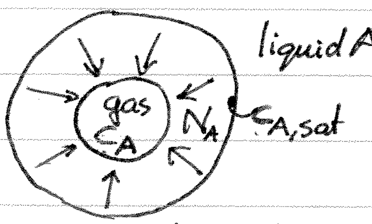
equal $\Rightarrow K_y \cdot y_2 = k_y(y_2 - Cx_{2,i})$

* 75% resistance in gas $\Rightarrow 1/k_y = 0.75 \cdot 1/K_y \Rightarrow K_y = 0.75 \cdot k_y$

$$\Rightarrow 0.75 \cdot \cancel{k_y} \cdot y_2 = \cancel{k_y}\, y_2 - \cancel{k_y}\, Cx_{2,i} \Rightarrow -0.25 \cdot y_2 = -C\, x_{2,i}$$

$$\Rightarrow x_{2,i} = \frac{0.25 \cdot 0.00105}{1.81} = 1.45 \cdot 10^{-4} = 0.0145\%$$

Problem VI/A)



A: species of interest, B: carrier gas, C: ceramic

System: tube wall ($D_i \leq r \leq D_o$)

Coord. system: cylindrical $\Rightarrow$ radial flux only

* $\vec{\nabla} \cdot \vec{N}_A + \underbrace{\cancel{\frac{\partial c_A}{\partial t}}}_{ss} - \underbrace{\cancel{R_A}}_{\text{no bulk reaction}} = 0 \Rightarrow \frac{1}{r}\frac{d}{dr}(r N_{A,r}) = 0 \Rightarrow r N_{A,r} = \text{constant}$

* $\vec{N}_A = -c\, D_{A,wall} \vec{\nabla} y_A + \cancel{y_A}(\vec{N}_A + \vec{N}_B + \vec{N}_C) \Rightarrow N_{A,r} = -c\, D_{A,wall} \frac{dy_A}{dr}$

$\hookrightarrow y_{A,max} < 0.02$ ($P_{A,sat} < 0.02$ atm) $\quad = -D_{A,wall} \frac{dc_A}{dr}$

B) Combine equations: $(r N_{A,r}) \frac{1}{r} = -D_{A,wall} \frac{dc_A}{dr}$

$$\Rightarrow \text{integrate: } (r N_{A,r}) \int_{D_i/2}^{D_o/2} \frac{dr}{r} = -D_{A,wall} \int_{C_A(z)}^{C_{A,sat}} dc_A$$

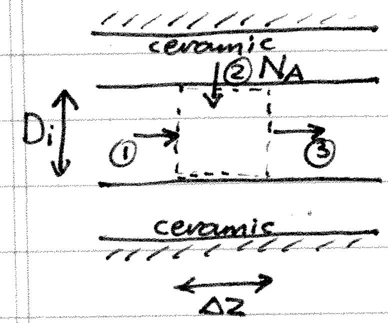
$$\Rightarrow (r N_{A,r}) \ln \frac{D_o}{D_i} = $$

$$= -D_{A,wall}\,(C_{A,sat} - C_A(z))$$

at inner wall ($r = D_i/2$): $N_{A,wall}(z) = \underbrace{\frac{-2\, D_{A,wall}}{D_i \ln(D_o/D_i)}}_{\alpha = 2.89\cdot 10^{-5}\ m/s} (C_{A,sat} - C_A(z))$

Note: N_A negative b/c in $-r$ direction!

C) Need local mass balance for section Δz of tube:



$$\left(c_A \cdot v \frac{\pi}{4} D_i^{\cancel{2}}\right)_{z+\Delta z} = \left(c_A \cdot v \cdot \frac{\cancel{\pi}}{4} \cdot D_i^{\cancel{2}}\right)_z + \cancel{\pi} \cancel{D_i} \Delta z |N_A|$$

$$\Rightarrow v \cdot \frac{D_i}{4} \Delta C_A = \Delta z \cdot \alpha \cdot (C_{A,sat} - C_A(z))$$

$$\xRightarrow{\Delta z \to 0} \frac{dc_A}{C_{A,sat} - C_A} = \frac{4\alpha}{v D_i} dz$$

$$\Rightarrow \text{integrate} \int_0^{C_{A,L}} \frac{dc_A}{C_{A,sat} - C_A} = \int_0^L \frac{4\alpha}{v D_i} dz$$

$$\Rightarrow -\ln\left(\frac{C_{A,L} - C_{A,sat}}{0 - C_{A,sat}}\right) = -\ln(0.5) = \frac{4\alpha}{v D_i} L \Rightarrow L = \frac{-v D_i \ln(0.5)}{4\alpha} = 0.60\ m$$

D) $N_{A,wall} = \underbrace{\alpha (C_{A,sat} - C_A(z))}_{\text{diffusion}} = \underbrace{k_c (C_A(z) - C_{A,\infty})}_{\text{convection inside tube}}$

$\Rightarrow$ assumption that $C_A(z) - C_{A,\infty}$ small is valid if $k_c \gg \alpha = 2.9 \cdot 10^{-5}$ m/s

- Need to find k_c for gas flow through tube:
  calculate Re and Sh-correlation to find k_c
  (Challenge is that D_{AB} is unknown, but can be estimated)