PHYS 2212 Test 1
Fall 2014

Name(print) Jason Bourne **Lab Section**________________

Lab section by day and time: Greco(N,P), Zangwill(Q) FIX						
Monday	12:05-2:55pm	N01 or Q01	3:05-5:55pm	N02 or P01	6:05-8:55pm	Q02 or P02
Tuesday	12:05-2:55pm	N03 or P03	3:05-5:55pm	Q03 or P04	6:05-8:55pm	
Wednesday	12:05-2:55pm	N05 or P05	3:05-5:55pm	Q05 or P06	6:05-8:55pm	N04 or Q04
Thursday	12:05-2:55pm	P07 or N06	3:05-5:55pm	N07 or Q06	6:05-8:55pm	

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^{4}$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

____

Sign your name on the line above

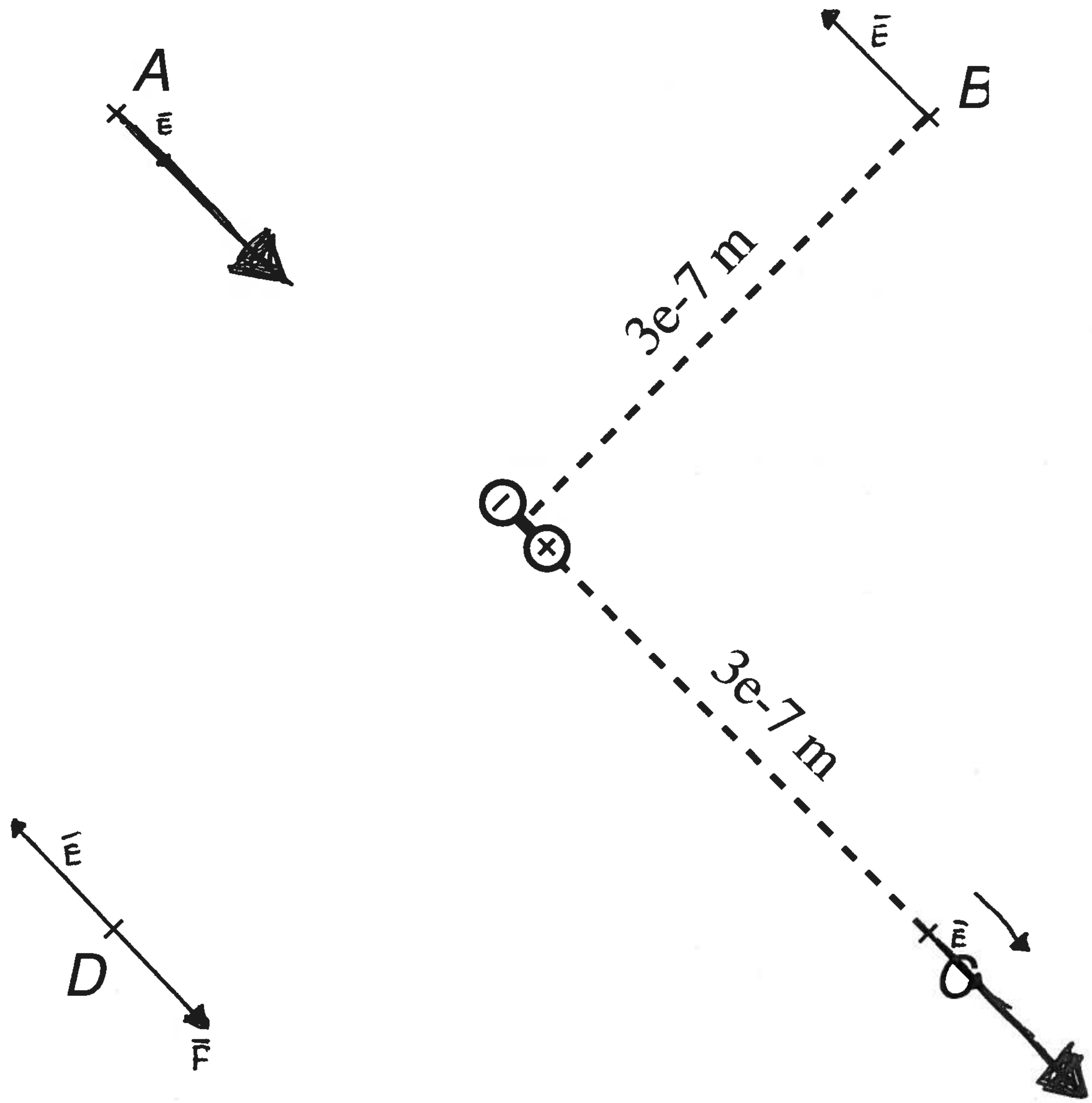
PHYS 2212

Please do not write on this page.

Problem	Score	Grader
Problem 1 (25 pts)		
Problem 2 (30 pts)		
Problem 3 (20 pts)		
Problem 4 (25 pts)		

Problem 1 (25 Points)

Locations A, B, C, and D are each 3×10^{-7}m from the center of the dipole shown in the diagram below. The dipole consists of ions with charges $+e$ and $-e$, separated by a distance of 2×10^{-10} m (note that the diagram is not drawn to scale).



(a 8pts) At each of these four locations, draw an arrow representing the electric field due to the dipole. The relative magnitudes of the arrows should be correct (that is, a longer arrow represents a larger magnitude). Clearly label each of the four arrows $\vec{E}$.

for each Arrow 1pt direction, 1pt mAg

(b 2pts) At a particular instant an electron is at location D. Draw an arrow representing the force on the electron due to the dipole. Clearly label the arrow $\vec{F}$.

2pts direction

(c 8pts) Which of these (circle one) is the magnitude of the force on the electron due to the dipole?

A. 0 N

B. 8.5×10^{-9} N

C. 1.7×10^{-18} N   All

D. 3.4×10^{-18} N $\longrightarrow$ −2 pts

E. 1.0×10^{-24} N

F. 5.1×10^{-25} N

(d 7pts) Which of these statements about a dipole are correct? ***Write "T" (true) or "F" (false) beside each statement.***

1pt each

T (A) The net charge of a dipole is zero.

F (B) The net electric field due to a dipole is zero at all points in space, since the contribution of the negative charge cancels out the contribution of the positive charge.

F (C) At the center of the dipole, halfway between the two charges, the net electric field is zero.

T (D) The electric field at any location in space, due to a dipole, is the vector sum of the electric field due to the positive charge and the electric field due to the negative charge.

F (E) The electric field at any location in space, due to a dipole, always points radially away from the center of the dipole.

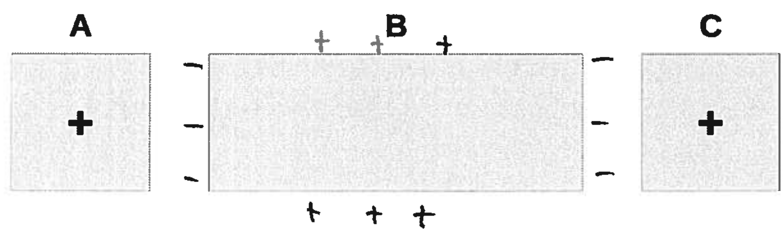
F (F) At a small distance d from a dipole, where $d << s$ (the separation between the charges), the magnitude of the electric field due to the dipole is proportional to $\frac{1}{d^3}$.

T (G) At a large distance d from a dipole, where $d >> s$ (the separation between the charges), the magnitude of the electric field due to the dipole is proportional to $\frac{1}{d^3}$.

Problem 2 (30 Points)   TA Discuss

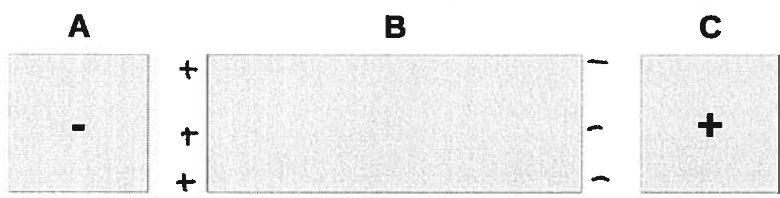
(a 5pts) A plastic cube (A), with a net positive charge $+Q$ is placed to the left of a neutral copper block (B). On the right of the copper block is a plastic cube (C) with a net positive charge $+Q$. Both cubes, A and C, are equidistant from the copper block as indicated in the figure. Using the diagrammatic conventions discussed in the textbook and in class, draw the polarization of charge on the copper block B. If there is no polarization, say so explicitly.

note: only looking for qualitative agreement

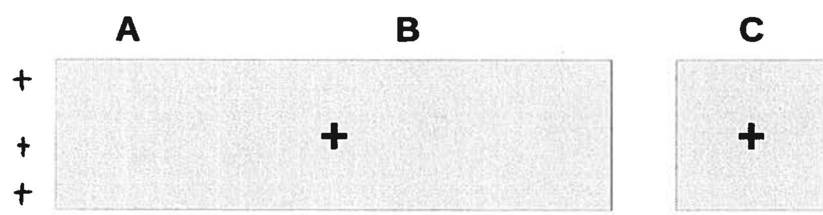


neutral
↓
equal + & −
−2

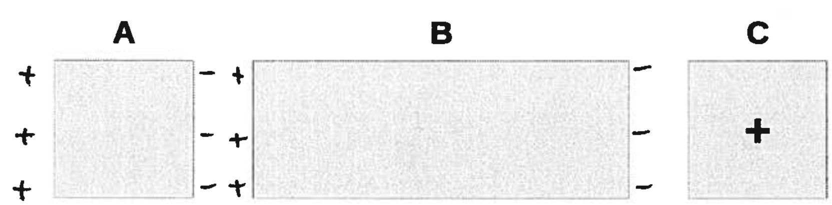
(b 5pts) A plastic cube (A), with a net negative charge $-Q$ is placed to the left of a neutral copper block (B). On the right of the copper block is a plastic cube (C) with a net positive charge $+Q$. Both cubes, A and C, are equidistant from the copper block as indicated in the figure. Using the diagrammatic conventions discussed in the textbook and in class, draw the polarization of charge on the copper block B. If there is no polarization, say so explicitly.



(c 5pts) A copper cube (A), with a net positive charge $+Q$ is placed to the left of a neutral copper block (B). On the right of the copper block is a plastic cube (C) with a net positive charge $+Q$. The metal cube A is brought into contact with block B and quickly reaches equilibrium. Using the diagrammatic conventions discussed in the textbook and in class, draw the polarization of charge on the combine copper blocks A+B. If there is no polarization, say so explicitly.



(d 5pts) A neutral copper cube (A) is placed to the left of a neutral copper block (B). On the right of the copper block is a plastic cube (C) with a net positive charge $+Q$. Both cubes, A and C, are equidistant from the copper block as indicated in the figure. Using the diagrammatic conventions discussed in the textbook and in class, draw the polarization of charge on the copper block B and cube A. If there is no polarization, say so explicitly.

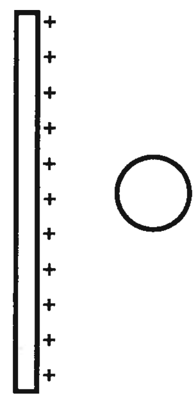


(e 5pts) You bring a positively charged piece of invisible tape near a piece of paper and find that the paper is attracted to the tape. Which of the following statements do you know must be FALSE? ***Write "F" (false) beside each statement you know CANNOT be true.***

- F The paper has a net positive charge. (2pts)
- ___ The paper has a net negative charge.
- ___ The paper is neutral.
- ___ The electric field of the tape causes the electron clouds that are bound to individual atoms in the paper to shift, creating an array of induced dipoles.
- F The electric field of the tape causes the mobile electron sea in the paper to shift. (2pts)

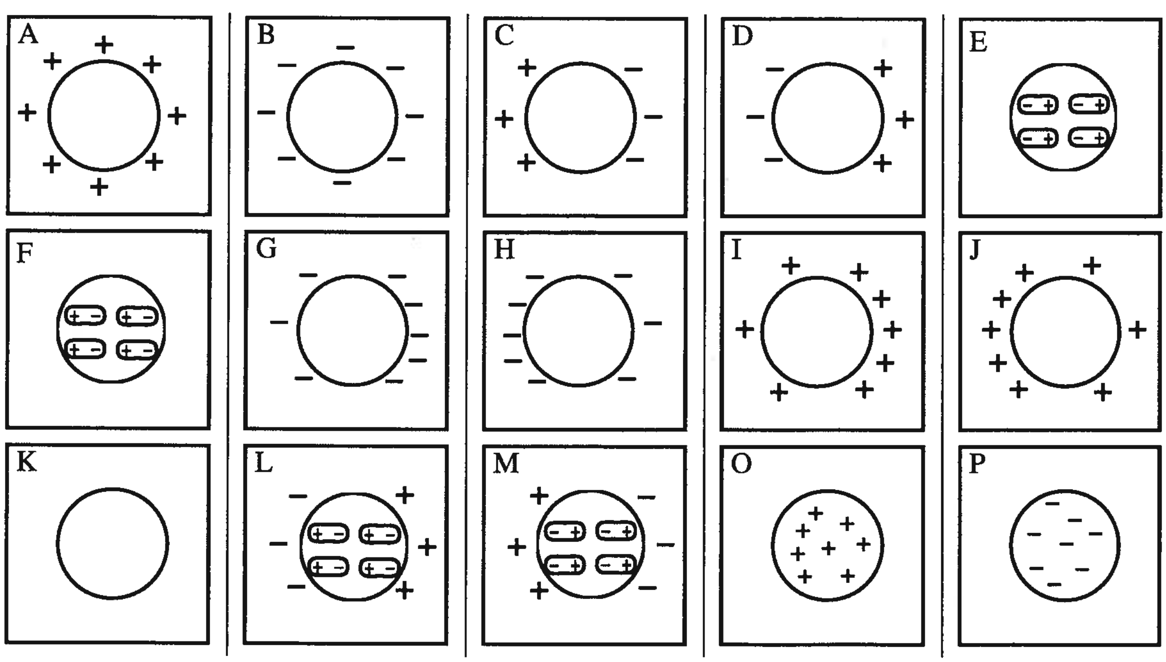
(−1) if selected but total score ≥ 0

(f 5pts) A small, neutral, solid plastic ball is brought near a rubber sheet with a positive charge uniformly distributed over one side, as shown in the diagram. Which of the diagrams below represents the best approximation to the charge distribution in or on the plastic ball?



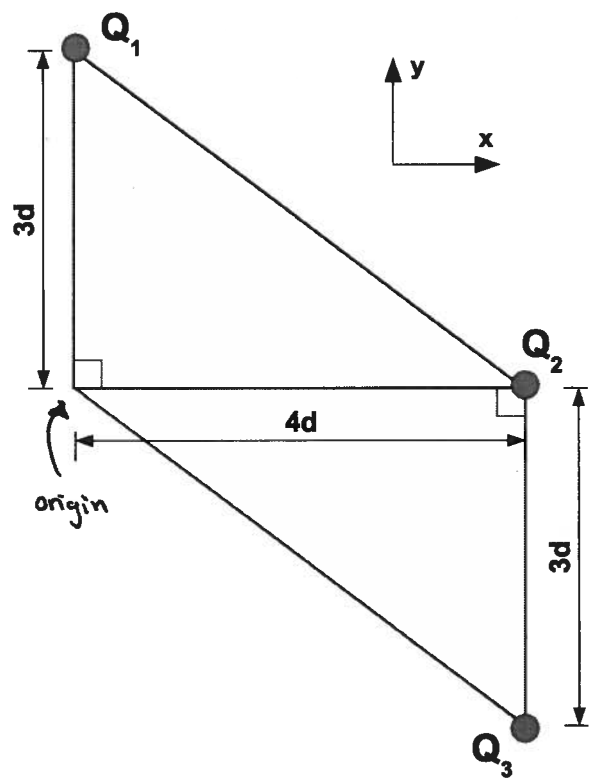
WRITE THE LETTER OF YOUR ANSWER HERE: E

All



Problem 3 (20 Points)

Three identical point charge Q_1, Q_2, and Q_3 all have positive charges $+Q$ and lie in the xy-plane. There relative locations are indicated directly on the diagram.



(a 10pts) Determine the electric force of Q_1 on Q_3. Make sure your answer is listed as a vector.

$$\bar{E} = \frac{Q_1}{4\pi\epsilon_0}\frac{\hat{r}}{r^2} \quad ; \quad \bar{F} = Q_3\bar{E}$$

$$\boxed{\begin{matrix} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{matrix}}$$

$$\bar{r} = \langle 4d, -3d, 0\rangle - \langle 0, 3d, 0\rangle$$
$$= \langle 4d, -6d, 0\rangle$$

$$\|\bar{r}\| = \sqrt{16d^2 + 36d^2}$$
$$= d\sqrt{52}$$
$$= 2d\sqrt{13}$$

$$\hat{r} = \frac{\bar{r}}{\|\bar{r}\|} = \frac{1}{2d\sqrt{13}}\langle 4d, -6d, 0\rangle = \langle \tfrac{2}{13}\sqrt{13}, -\tfrac{3}{13}\sqrt{13}, 0\rangle = \langle 0.555, -0.832, 0\rangle$$

$$\bar{F} = Q_3\bar{E} = \frac{Q\cdot Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$
$$= \frac{Q^2}{4\pi\epsilon_0}\frac{1}{52d^2}\langle \tfrac{2}{13}\sqrt{13}, -\tfrac{3}{13}\sqrt{13}, 0\rangle$$
$$= \frac{Q^2}{4\pi\epsilon_0}\frac{1}{d^2}\langle 0.01067, -0.01600, 0\rangle$$

(b 10pts) Determine the net electric force on Q_2. Make sure your answer is listed as a vector.

$$\bar{F}_{net, on\ 2} = \bar{F}_{12} + \bar{F}_{32} = Q\bar{E}_{12} + Q\bar{E}_{32}$$

$$\bar{E}_{12} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}_{12}}{r_{12}^2} \ ; \qquad \bar{r}_{12} = \langle 4d, 0, 0\rangle - \langle 0, 3d, 0\rangle$$

$$= \langle 4d, -3d, 0\rangle$$

$$\|\bar{r}_{12}\| = 5d$$

$$\hat{r}_{12} = \frac{\bar{r}_{12}}{\|\bar{r}_{12}\|} = \langle \tfrac{4}{5}, -\tfrac{3}{5}, 0\rangle$$

$$\boxed{\begin{matrix} -0.5 \\ -1.5 \\ -3.0 \\ -8.0 \end{matrix}}$$

$$\bar{E}_{32} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}_{32}}{r_{32}^2} \ ; \qquad \bar{r}_{32} = \langle 4d, 0, 0\rangle - \langle 4d, -3d, 0\rangle$$

$$= \langle 0, 3d, 0\rangle$$

$$\|\bar{r}_{32}\| = 3d$$

$$\hat{r}_{32} = \langle 0, 1, 0\rangle$$

$$\bar{F}_{net, on\ 2} = Q\bar{E}_{12} + Q\bar{E}_{32}$$

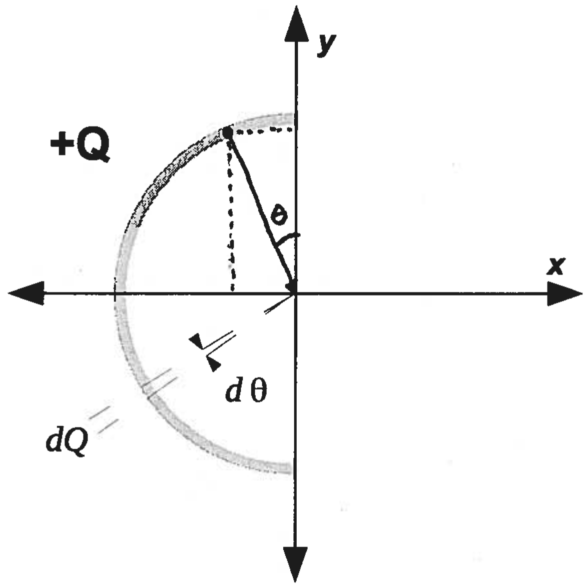
$$= \frac{Q^2}{4\pi\epsilon_0}\frac{1}{25d^2}\langle \tfrac{4}{5}, -\tfrac{3}{5}, 0\rangle + \frac{Q^2}{4\pi\epsilon_0}\frac{1}{9d^2}\langle 0, 1, 0\rangle$$

$$= \frac{Q^2}{4\pi\epsilon_0 d^2}\langle \frac{4}{125}, \frac{98}{1125}, 0\rangle$$

$$= \frac{Q^2}{4\pi\epsilon_0 d^2}\langle 0.032, 0.0871, 0\rangle$$

Problem 4 (25 Points)

Consider a thin plastic rod bent into a semicircular arc of radius R with center at the origin as shown in the diagram. The rod carries a uniformly distributed positive charge $+Q$.



(a 15pts) Determine the electric field $d\vec{E}$ at the origin due to the representative piece of the rod dQ. Your answer should be a **vector** and given in terms of $d\theta$.

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{R^2}$$

$$dQ = \lambda R\,d\theta = \left(\frac{Q}{\pi R}\right)R\,d\theta = \left(\frac{Q}{\pi}\right)d\theta$$

$$\vec{r}_{obj} = \langle 0,0,0\rangle$$

$$\vec{r}_{src} = \langle -R\sin\theta, R\cos\theta, 0\rangle$$

$$\vec{r} = \vec{r}_{obj} - \vec{r}_{src} = \langle R\sin\theta, -R\cos\theta, 0\rangle$$

$$\|\vec{r}\| = R$$

$$\hat{r} = \frac{\vec{r}}{\|\vec{r}\|} = \langle \sin\theta, -\cos\theta, 0\rangle$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dQ}{R^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{(Q/\pi)\,d\theta}{R^2}\langle \sin\theta, -\cos\theta, 0\rangle = \frac{Q}{4\pi^2\epsilon_0}\frac{d\theta}{R^2}\langle \sin\theta, -\cos\theta, 0\rangle$$

−1.0
−2.0
−4.5
−12.0

(b 5pts) Calculate the electric field $\vec{E}$ at the origin due to the entire semicircular rod. Your answer should be a vector.

$$dE = \frac{Q}{4\pi^2\epsilon_0} \frac{d\theta}{R^2} \langle \sin\theta, -\cos\theta, 0 \rangle$$

limits match choice for θ (2pts)

x-comp: $dE_x = \frac{Q}{4\pi^2\epsilon_0} \frac{\sin\theta\, d\theta}{R^2} \Rightarrow E_x = \int_0^{\pi} \frac{Q}{4\pi^2\epsilon_0 R^2} \sin\theta\, d\theta$

$$= \frac{Q}{4\pi^2\epsilon_0 R^2} \int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{-Q}{4\pi^2\epsilon_0 R^2} \cos\theta \Big|_0^{\pi}$$

$$= \frac{2Q}{4\pi^2\epsilon_0 R^2}$$

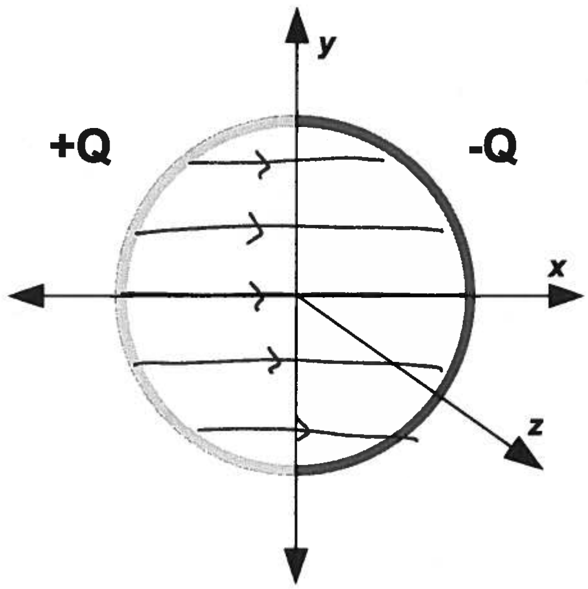
y-comp: $dE_y = \frac{-Q}{4\pi^2\epsilon_0} \frac{\cos\theta\, d\theta}{R^2} \Rightarrow E_y = \frac{-Q}{4\pi^2\epsilon_0 R^2} \int_0^{\pi} \cos\theta\, d\theta$

$$= -\frac{Q}{4\pi^2\epsilon_0 R^2} \sin\theta \Big|_0^{\pi}$$

$$= 0 \quad \text{(1pt)}$$

$\therefore \quad \vec{E} = \frac{Q}{2\pi^2\epsilon_0 R^2} \langle 1, 0, 0 \rangle$ (2pts)

Now consider an additional thin plastic rod bent into a semicircular arc of radius R with center at the origin as shown in the diagram. This rod carries a uniformly distributed negative charge $-Q$. Together these bent rods form a perfect circle



(c 5pts) What is the direction of the electric field at location $< 0, 0, L >$ where $L >> R$. Briefly explain how you determined this. Hint: consider how this object would appear from far away.

2pts

It is in the $(+\hat{x})$ direction. From far away the object's field would resemble a dipole. The location $\langle 0,0,L\rangle$ is $\perp$ to the dipoles axis.

3pts

This page is for extra work, if needed.

Things you must know

Relationship between electric field and electric force
Electric field of a point charge
Relationship between magnetic field and magnetic force
Magnetic field of a moving point charge
Conservation of charge
The Superposition Principle

Other Fundamental Concepts

$\vec{a} = \frac{d\vec{v}}{dt}$

$\frac{d\vec{p}}{dt} = \vec{F}_{net}$ and $\frac{d\vec{p}}{dt} \approx m\vec{a}$ if $v << c$

$\Delta U_{el} = q\Delta V$

$\Delta V = -\int_i^f \vec{E} \bullet d\vec{l} \approx -\sum (E_x\Delta x + E_y\Delta y + E_z\Delta z)$

$\Phi_{el} = \int \vec{E} \bullet \hat{n} dA$

$\Phi_{mag} = \int \vec{B} \bullet \hat{n} dA$

$\oint \vec{E} \bullet \hat{n} dA = \frac{\sum q_{inside}}{\epsilon_0}$

$\oint \vec{B} \bullet \hat{n} dA = 0$

$|\text{emf}| = \oint \vec{E}_{NC} \bullet d\vec{l} = \left|\frac{d\Phi_{mag}}{dt}\right|$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \sum I_{inside\ path}$

$\oint \vec{B} \bullet d\vec{l} = \mu_0 \left[\sum I_{inside\ path} + \epsilon_0 \frac{d}{dt} \int \vec{E} \bullet \hat{n} dA\right]$

Specific Results

$\left|\vec{E}_{dipole,axis}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2qs}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{E}_{dipole,\perp}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{qs}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\left|\vec{E}_{rod}\right| = \frac{1}{4\pi\epsilon_0}\frac{Q}{r\sqrt{r^2+(L/2)^2}}$ ($r \perp$ from center)

electric dipole moment $p = qs$, $\vec{p} = \alpha \vec{E}_{applied}$

$\left|\vec{E}_{rod}\right| \approx \frac{1}{4\pi\epsilon_0}\frac{2Q/L}{r}$ (if $r \ll L$)

$\left|\vec{E}_{ring}\right| = \frac{1}{4\pi\epsilon_0}\frac{qz}{(z^2+R^2)^{3/2}}$ (z along axis)

$\left|\vec{E}_{disk}\right| = \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{(z^2+R^2)^{1/2}}\right]$ (z along axis)

$\left|\vec{E}_{disk}\right| \approx \frac{Q/A}{2\epsilon_0}\left[1 - \frac{z}{R}\right] \approx \frac{Q/A}{2\epsilon_0}$ (if $z \ll R$)

$\left|\vec{E}_{capacitor}\right| \approx \frac{Q/A}{\epsilon_0}$ ($+Q$ and $-Q$ disks)

$\left|\vec{E}_{fringe}\right| \approx \frac{Q/A}{\epsilon_0}\left(\frac{s}{2R}\right)$ just outside capacitor

$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{\ell} \times \hat{r}}{r^2}$ (short wire)

$\Delta\vec{F} = I\Delta\vec{l} \times \vec{B}$

$\left|\vec{B}_{wire}\right| = \frac{\mu_0}{4\pi}\frac{LI}{r\sqrt{r^2+(L/2)^2}} \approx \frac{\mu_0}{4\pi}\frac{2I}{r}$ ($r \ll L$)

$\left|\vec{B}_{wire}\right| = \left|\vec{B}_{earth}\right| \tan\theta$

$\left|\vec{B}_{loop}\right| = \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{(z^2+R^2)^{3/2}} \approx \frac{\mu_0}{4\pi}\frac{2I\pi R^2}{z^3}$ (on axis, $z \gg R$) $\mu = IA = I\pi R^2$

$\left|\vec{B}_{dipole,axis}\right| \approx \frac{\mu_0}{4\pi}\frac{2\mu}{r^3}$ (on axis, $r \gg s$)

$\left|\vec{B}_{dipole,\perp}\right| \approx \frac{\mu_0}{4\pi}\frac{\mu}{r^3}$ (on $\perp$ axis, $r \gg s$)

$\vec{E}_{rad} = \frac{1}{4\pi\epsilon_0}\frac{-q\vec{a}_\perp}{c^2 r}$

$\hat{v} = \hat{E}_{rad} \times \hat{B}_{rad}$

$\left|\vec{B}_{rad}\right| = \frac{\left|\vec{E}_{rad}\right|}{c}$

$i = nA\bar{v}$

$I = |q|\, nA\bar{v}$

$\bar{v} = uE$

$\sigma = |q|\, nu$

$J = \frac{I}{A} = \sigma E$

$R = \frac{L}{\sigma A}$

$E_{dielectric} = \frac{E_{applied}}{K}$

$\Delta V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$ due to a point charge

$I = \frac{|\Delta V|}{R}$ for an ohmic resistor (R independent of ΔV); power $= I\Delta V$

$Q = C\,|\Delta V|$

$\text{K} \approx \frac{1}{2}mv^2$ if $v \ll c$

circular motion: $\left|\frac{d\vec{p}}{dt}_{\perp}\right| = \frac{|\vec{v}|}{R}|\vec{p}| \approx \frac{mv^2}{R}$

Math Help

$$\vec{a} \times \vec{b} = \langle a_x, a_y, a_z \rangle \times \langle b_x, b_y, b_z \rangle$$

$$= (a_y\, b_z - a_z\, b_y)\hat{x} - (a_x\, b_z - a_z\, b_x)\hat{y} + (a_x\, b_y - a_y\, b_x)\hat{z}$$

$$\int \frac{dx}{x+a} = \ln(a+x) + c \quad \int \frac{dx}{(x+a)^2} = -\frac{1}{a+x} + c \quad \int \frac{dx}{(a+x)^3} = -\frac{1}{2(a+x)^2} + c$$

$$\int a\,dx = ax + c \quad \int ax\,dx = \frac{a}{2}x^2 + c \quad \int ax^2\,dx = \frac{a}{3}x^3 + c$$

Constant	Symbol	Approximate Value
Speed of light	c	3×10^8 m/s
Gravitational constant	G	6.7×10^{-11} N · m^2/kg^2
Approx. grav field near Earth's surface	g	9.8 N/kg
Electron mass	m_e	9×10^{-31} kg
Proton mass	m_p	1.7×10^{-27} kg
Neutron mass	m_n	1.7×10^{-27} kg
Electric constant	$\frac{1}{4\pi\epsilon_0}$	9×10^9 N · m^2/C^2
Epsilon-zero	ϵ_0	8.85×10^{-12} (N · m^2/C^2)$^{-1}$
Magnetic constant	$\frac{\mu_0}{4\pi}$	1×10^{-7} T · m/A
Mu-zero	μ_0	$4\pi \times 10^{-7}$ T · m/A
Proton charge	e	1.6×10^{-19} C
Electron volt	1 eV	1.6×10^{-19} J
Avogadro's number	N_A	6.02×10^{23} molecules/mole
Atomic radius	R_a	$\approx 1 \times 10^{-10}$ m
Proton radius	R_p	$\approx 1 \times 10^{-15}$ m
E to ionize air	E_{ionize}	$\approx 3 \times 10^6$ V/m
B_{Earth} (horizontal component)	B_{Earth}	$\approx 2 \times 10^{-5}$ T