

GEORGIA INSTITUTE OF TECHNOLOGY

COLLEGE OF ENGINEERING

BMED3300 – BIOTRANSPORT

QUIZ 3 (SPRING 2014) – KEMP

STUDENT NAME: Key

GTID NUMBER: \_\_\_\_\_

RECITATION SECTION: \_\_\_\_\_

(Section A is Wednesdays at 12 pm; Section B is Wednesdays at 10 am)

*Closed Book*

*All non-communicating calculator types allowed*

*Time allotted: 15 minutes*

*Do all work in this booklet*

**Reminder: for questions that require numerical answers, units are required and worth 50%**

Question	Maximum Mark	Actual Mark
1	9	
2	3	
Total	12	

1. There is currently a great deal of interest in delivering drugs (e.g. insulin) by inhalation, thus avoiding injection (e.g. Henry et al., Diabetes Care, 26:764, 2003). Aerosol droplets of density  $\rho$  and diameter  $D$  containing the drug would be inhaled and reach the small airways, where they would be absorbed into the blood. One problem is that droplets can have a hard time following tortuous pathways in the lungs, and run into airway walls before reaching their target destination. This depends on the speed of the air,  $V_{air}$ , as well as the viscosity and density of the air,  $\mu_{air}$  ( $\text{kg}\cdot\text{s}^{-1}\cdot\text{m}^{-1}$ ), and  $\rho_{air}$ . Droplets can also sediment out of the air stream, which depends on gravitational acceleration,  $g$ .

a. Construct a  $\pi$ -matrix from the relevant parameters in this problem and confirm that 3  $\pi$ -groups can be formed from these parameters.

	$\rho$	$D$	$V_{air}$	$\mu_{air}$	$\rho_{air}$	$g$
$M$	1	0	0	1	1	0
$L$	-3	1	1	-1	-3	1
$t$	0	0	-1	-1	0	-2

$\# \text{ variables} + 2$

6 variables

rank = 3

$$6 - 3 = 3 \pi \text{ groups} + 1$$

b. Taking  $V_{air}$ ,  $D$  and  $\rho$  as the core group of variables, find three  $\pi$ -groups.

$$M^0 L^0 t^0 = (V_{air})^a D^b \rho^c g$$

$$= \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \cdot \left(\frac{L}{t^2}\right)$$

$$M^0 L^0 t^0 = (V_{air})^a D^b \rho^c \rho_{air}$$

$$= \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{L^3}\right)$$

$$M^0 L^0 t^0 = (V_{air})^a D^b \rho^c \mu$$

$$= \left(\frac{L}{t}\right)^a L^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{L \cdot t}\right)$$

$$M: 0 = c$$

$$M: 0 = c + 1$$

$$c = -1$$

$$M: 0 = c + 1$$

$$c = -1$$

$$L: 0 = a + b - 3c + 1$$

$$a + b = -1$$

$$L: 0 = a + b - 3c - 3$$

$$a = -b$$

$$L: 0 = a + b - 3c - 1$$

$$-2 = a + b$$

$$t: 0 = -a - 2$$

$$a = -2$$

$$t: 0 = -a$$

$$t: 0 = -a - 1$$

$$a = -1$$

$$b = 1$$

$$\pi_2 = \frac{\rho_{air}}{\rho}$$

$$b = -1$$

$$\pi_1 = V_{air}^{-2} D^1 \rho^0 g$$

$$= \frac{D \cdot g}{V_{air}^2}$$

$$= \left(\frac{t}{L}\right)^2 \cdot L \cdot \frac{L}{t^2} \checkmark$$

$$\pi_3 = V_{air}^{-1} D^{-1} \rho^{-1} \mu$$

$$= \frac{\mu}{V_{air} \rho D}$$

$$= \left(\frac{M}{L \cdot t}\right) \cdot \frac{1}{L} \cdot \frac{t}{L} \cdot \frac{L^2}{M}$$



For

$V_{air}, D, \mu, \rho;$

$V_{air}, D, \mu, \rho;$

$V_{air}, D, \mu, \rho;$

$$m^0 L^0 t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{m}{L \cdot t}\right)^c \left(\frac{m}{L^3}\right) + 1$$

$$m^0 L^0 t^0 = \left(\frac{L}{t}\right)^a L^b \left(\frac{m}{L \cdot t}\right)^c \cdot \frac{L}{t^2}$$

$$m: 0 = -c + 1$$

"

$$m: 0 = -c$$

$$c = -1 \quad +1$$

$$L: 0 = a + b - c - 3$$

$$\pi_2 = \frac{V_{air} \rho_{air} D}{\mu}$$

$$L: 0 = a + b - c + 1$$

$$0 = a + b - 1 + 1$$

$$-1 = a + b$$

$$t: 0 = -a - c$$

$$t: 0 = -a - c - 2$$

$$0 = -a + 1$$

$$a = 1$$

$$a = 1$$

$$b = 1$$

$$b = 1 \quad +1$$

$$\pi_1 = \frac{D \cdot \rho_{air}}{V_{air} \cdot \mu}$$

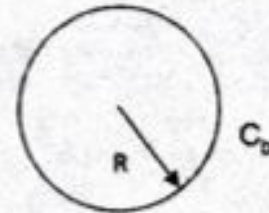
$$\pi_3 = \frac{D \cdot \rho_{air}}{V_{air} \cdot \mu}$$

$$= \frac{L}{t} \cdot \frac{m}{L^3} \cdot \frac{t^2}{m} \quad \checkmark$$

$$\pi_1 = \frac{V_{air} \rho_{air} D}{\mu} \quad \text{N} + 3$$

$$= \frac{L}{t} \cdot \frac{m}{L^3} \cdot L \cdot \frac{t^2}{m} \quad \checkmark$$

2. Medical implants are capable of releasing drugs at a constant rate into the systemic circulation, a convenient alternative to oral drug administration when a constant blood level of drug is desired in the patient for extended periods of time. Several slow-release corticosteroid intraocular implants are undergoing clinical trials for treating macular edema. Perhaps more familiar is the use of Norplant™, an implantable contraceptive device which releases the steroid hormone levonorgestrel into the blood when implanted under the skin in the arm. A new device is being considered which does not contain any surface coating, i.e. the implant of **spherical geometry** consists of a single bead of polymer gel material that the drug is imbedded in and can diffuse through. Consider the following definitions:



$c_b$  = concentration of drug in the body fluid at the surface of the device  
(assume constant)

$c_d$  = drug concentration in the implant

$R$  = radius of the implant

$D_{d,i}$  = effective diffusion coefficient of the drug in the implant

a) What are the boundary conditions you would use for determining a solution?

$$\text{at } r = R, \quad c_d = c_b \quad + 1 \frac{1}{2}$$

$$\text{at } r = 0, \quad \frac{\partial c_d}{\partial r} = 0 \quad + 1 \frac{1}{2}$$