MATH 2401 WORKSHEET #3
SECTION K

Name:

(1) Find the limit of the function or show that the limit does not exist.

(a) $f(x,y) = \dfrac{xy^2 - 1}{y - 1}$, as $(x,y) \to (1,1)$.

Sol: Path 1: $x=1$. $\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(x,y)\to(1,1)} \dfrac{y^2-1}{y-1} = \lim_{(x,y)\to(1,1)} (y+1) = 2$.

3 points
1 point per path
1 point for the limit not existing

Path 2: $x=y$. $\lim_{(x,y)\to(1,1)} f(x,y) = \lim_{(x,y)\to(1,1)} \dfrac{y^3-1}{y-1} = \lim_{y\to 1} (y^2+y+1) = 3$.

Therefore the limit DNE.

(b) $g(x,y) = cos(\dfrac{x^3 - y^3}{x^2 + y^2})$, as $(x,y) \to (0,0)$.

3 points
1 point to make the change of variable
1 point for algebra
1 point to get the limit

Sol: Let $\begin{cases} x = r\cos\theta \\ y = r\sin\theta. \end{cases}$

Then $g(x,y) = \cos\left(\dfrac{r^3\cos^3\theta - r^3\sin^3\theta}{r^2}\right) = \cos(r(\cos^3\theta - \sin^3\theta))$

Therefore $\lim_{(x,y)\to(0,0)} g(x,y) = \lim_{r\to 0} \cos(r(\cos^3\theta - \sin^3\theta)) = \cos(0) = 1$ (Since $\cos^3\theta - \sin^3\theta$ is bounded).

(2) Find an equation for and sketch the graph of the level curve of the function $f(x, y) = \sqrt{x + y^2 - 3}$, that passes through the point $(3, -1)$.

Level curve: $f(x,y) = c$.

$\sqrt{x+y^2-3} = c$

$x+y^2-3 = c^2$.

4 points
1 point for graph
1 point for determining c
1 point for setting f(x,y)=c
1 point for correct equation

Plugging $(3,-1) \Rightarrow 3+1-3=c \Rightarrow c=1$.

$\Rightarrow$ Level curve: $y^2=4-x$.