ISyE 4803 Exam # 1
Spring 2011

Name

Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Total

(35) **1.** Suppose that in a production line there are 5 stations. Assume that jobs arrive at station 1 with respect to a Poisson process of rate 20/hr. The output from station i is used as input into station $i+1$, for $i = 1, \ldots, 4$. However, since there are defective items at each station only 90% of the items move from station i to station $i + 1$, for $i = 1, \ldots, 4$. Similarly, only 90% of the items leaving station 5 are non-defective. The remaining 10% at each station is discarded. Assume that there is a single server at each station with exponentially distributed service times with rate 30 per hour.

a. (15) Does the joint distribution (in the long-run) for the number of jobs at five stations exist? If it does, compute it.

b. (10) What is the expected number of non-defective items in the production line in the long-run?

c. (10) What is the expected time that a job spends in the line if it is processed at all five stations?

(30) **2.** Consider a production system with two stations in series (i.e. all the jobs leaving station 1 go to station 2) operating under CONWIP (Constant Work In Process) policy. Recall that when the system is operating under CONWIP policy, only a job leaving station 2 triggers a new job to be released to station 1. Suppose there are two servers at station 1 and one server at station 2. Assume that the service times at both stations are exponential with rates 4/hr (for each server) and 6/hr, respectively. Let $N = 2$ be the CONWIP level. Compute the expected number of jobs at station 1 in the long run.

(35) **3.** Two people who prepare tax forms are working in a store at a local mall. Each has a chair next to his desk where customers can sit and be served. In addition there is one chair where customers can sit and wait. Customers arrive with respect to a Poisson process of rate λ but will go away if there is already someone sitting in the chair waiting. Suppose that server i requires an exponential amount of time with rate μ_i and that when both servers are free and arriving customer is equally likely to choose either one. Assume that $\lambda = 2$, $\mu_1 = 3$, and $\mu_2 = 3$ and compute the long run probability of having 3 customers in the store?