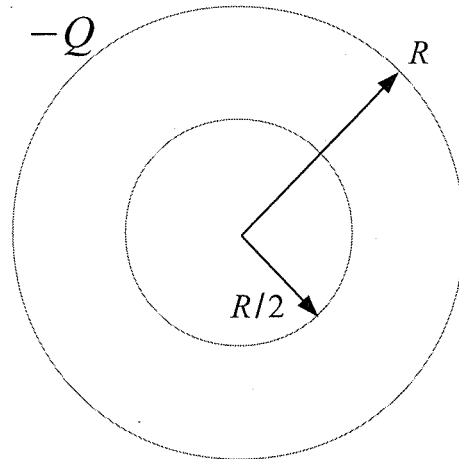


Name: Key Section: _____

Show all work clearly and in order, and box your final answers.

A plastic sphere of radius R has hollow center of radius $R/2$. This thick spherical shell has a charge $-Q$ distributed uniformly throughout the plastic. For the following three regions, determine the magnitude and direction of the electric field at an observation location a distance r from the center of the sphere. Be sure to provide a briefly explanation to earn full credit.



1. (20 points) $r < R/2$ (inside the hollow center)

$$r < \frac{R}{2}, \quad \vec{E}_{\text{net}} = 0$$

\vec{E}_{net} cancels everywhere
inside the hollow
center.

2. (20 points) $r > R$ (outside the plastic shell)

$$r > R$$

For $r > R$, the field will resemble the field of a point charge with charge $-Q$.

$$\vec{E}_{\text{net}} = -\frac{Q_{\text{tot}}}{4\pi\epsilon_0 r^2} \hat{r} \quad (\text{radially inward})$$

3. (40 points) $R/2 < r < R$ (in the plastic)

$$E = \frac{-\Delta Q}{4\pi\epsilon_0 r^2} \hat{r}$$

$$\begin{aligned} \Delta Q &= Q_{\text{tot}} \frac{\text{(Volume of charge)}}{\text{(Volume of total charge)}} \\ &= Q_{\text{tot}} \frac{\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi \frac{R^3}{8}\right)}{\left(\frac{4}{3}\pi R^3 - \frac{4}{3}\pi \frac{R^3}{8}\right)} \\ &= Q_{\text{tot}} \frac{(8r^3 - R^3)\pi/6}{\frac{7}{6}\pi R^3} \\ \Rightarrow \Delta Q &= Q_{\text{tot}} \frac{(8r^3 - R^3)}{7R^3} \end{aligned}$$

from $\frac{R}{2} \rightarrow r$

$$\begin{aligned} E &= \frac{-\Delta Q}{4\pi\epsilon_0 r^2} \hat{r} \\ &= \frac{-Q_{\text{tot}}}{4\pi\epsilon_0 r^2} \left(\frac{8r^3 - R^3}{7R^3}\right) \hat{r} \\ &= -\frac{Q_{\text{tot}}}{28\pi\epsilon_0 r^2} \left(\frac{8r^3 - R^3}{R^3}\right) \hat{r} \quad (\text{radially inward}) \end{aligned}$$

4. (20 points) Replace the plastic shell with an identical metal shell also with charge $-Q$. Briefly describe how your answer for the electric field outside of the plastic shell $r > R$ would change.

The answer will not change. the electric field for the metal solid shell will also be

$$\vec{E}_{\text{net}} = -\frac{Q_{\text{tot}}}{4\pi\epsilon_0 r^2} \hat{r} \quad \text{for } r > R$$