MATH 2403 H1-H3. Differential Equations. Quiz 1. Jan 14, 2014.
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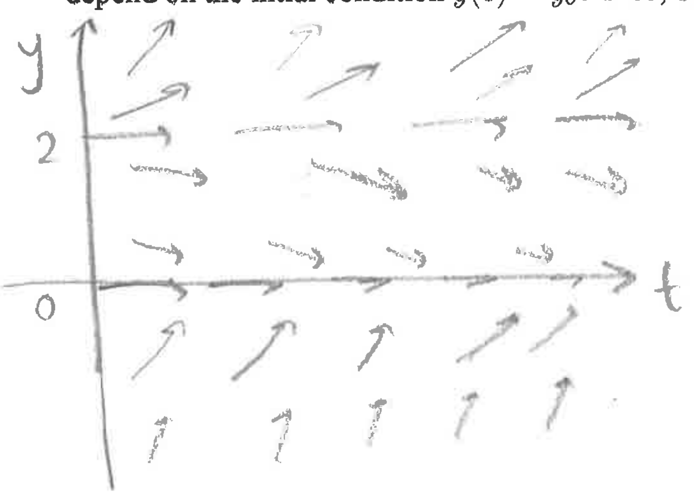
Student's Name: ______________________ Section ______

Show all work to receive credit

1. For the differential equation:

$$\frac{dy}{dt} = -y(-y+2),$$

draw the direction field and use it to describe the solutions as $t \to \infty$. Does the behavior at $t \to \infty$ depend on the intial condition $y(0) = y_0$? If so, describe this dependency.



If $y(0) = y_0$, then as $t \to \infty$:

If $y_0 = 0$, $y(t) \to 0$

If $y_0 = 2$, $y(t) \to 2$

If $y_0 > 2$, $y(t) \to \infty$

If $y_0 < 0$, $y(t) \to 0$

If $0 < y_0 < 2$, $y(t) \to 0$

2. Find the solution to the initial value problem

$$y' + 2ty = t + 2te^{-t^2}, \qquad y(0) = 2.$$

The equation is linear:

$$\mu(t) = e^{\int 2t} = e^{t^2}$$

$$\Rightarrow \quad e^{t^2}(y' + 2ty) = (t + 2te^{-t^2})e^{t^2}$$

$$\Leftrightarrow \quad \frac{d}{dt}(ye^{t^2}) = te^{t^2} + 2t$$

$$\Leftrightarrow \quad ye^{t^2} = \int (te^{t^2} + 2t)\,dt = \frac{1}{2}e^{t^2} + t^2 + C$$

$$\Leftrightarrow \quad y = \frac{1}{2} + t^2 e^{-t^2} + Ce^{-t^2} \quad ; \quad y(0) = 2 \Rightarrow C = \frac{3}{2}$$

$$\therefore \quad y = \frac{1}{2} + t^2 e^{-t^2} + \frac{3}{2}e^{-t^2}$$