PHYS 2211 Test 3
Spring 2012

Name(print)____ **Lab Section**____________________

Lab section by day and time: Greco(M), Schatz(K,N)			
Day	12:05pm-2:55pm	3:05pm-5:55pm	6:05pm-8:55pm
Monday	M01 K01	M02 N01	
Tuesday	M03 N03	M04 K03	K02 N02
Wednesday	K05 N05	M05 N06	M06 K06
Thursday	K07 N07	M07 K08	M08 N08

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- **Correct answers without adequate explanation will be counted wrong.**
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you do not want us to grade
- Make explanations correct but brief. You do not need to write a lot of prose.
- Include diagrams!
- **Show what goes into a calculation, not just the final number, e.g.:** $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can not calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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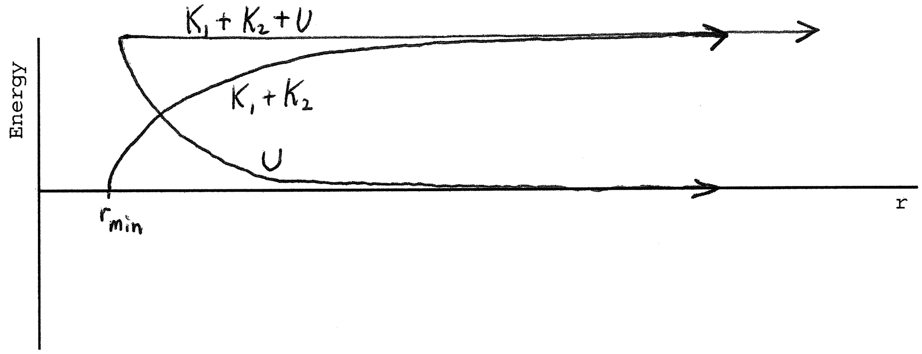
Sign your name on the line above

Problem 1 (25 Points)

An alpha particle (two protons and two neutrons) moving with momentum p is shot toward a carbon-12 nucleus (containing six protons and six neutrons) that is moving with the same momentum p toward the alpha particle. The speeds are non-relativistic.

(a 6pts) For this process, as a function of the separation r between the alpha particle and the carbon-12 nucleus, plot $K_1 + K_2$ (the sum of the kinetic energies of the two nuclei), plot the potential energy, and plot the sum of the kinetic energies and the potential energy. Label each of the three plots clearly.



(b 9pts) What is the minimum momentum p necessary, so that the alpha particle and carbon-12 nucleus come in contact? The radius of the alpha particle is R_α and the radius of the carbon-12 nucleus is R_c. Your answer should be symbolic.

$$\Delta E = 0 \Rightarrow \Delta K + \Delta U = 0 \Rightarrow \cancel{K_f} - K_i + U_f - \cancel{U_i} = 0$$

$$\Rightarrow U_f = K_i$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_c}{|r|} = \frac{p_\alpha^2}{2m_\alpha} + \frac{p_c^2}{2m_c}$$ where $q_\alpha = 2e$, $q_c = 6e$, $|r| = R_\alpha + R_c$, and $p_\alpha = p_c = p$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{12e^2}{(R_\alpha+R_c)} = \frac{p^2}{2}\left(\frac{1}{m_\alpha}+\frac{1}{m_c}\right)$$

$$\Rightarrow \boxed{|p| = \sqrt{\frac{6e^2}{\pi\varepsilon_0(R_\alpha+R_c)}\left(\frac{m_\alpha m_c}{m_\alpha+m_c}\right)}}$$

(c 10pts) After coming into contact, the two particles fuse creating an oxygen-16 nucleus (containing eight protons and eight neutrons) and a photon. Determine the sum of the kinetic energy of the oxygen nucleus and the energy of the photon. Your answer should be symbolic.

$$\Delta E = 0 \Rightarrow \Delta K + \Delta U + \Delta E_{rest} + \Delta K_{photon} = 0$$

$$\Rightarrow K_f - \cancel{K_i} + \cancel{U_f} - U_i + E_{rest,f} - E_{rest,i} + K_{photon,f} - \cancel{K_{photon,i}} = 0$$

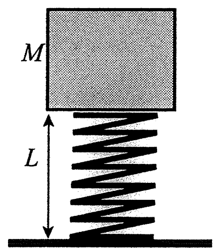
$$\Rightarrow K_f + K_{photon} = U_i + E_{rest,i} - E_{rest,f}$$

$$\Rightarrow K_f + K_{photon} = \frac{1}{4\pi\varepsilon_0} \frac{12e^2}{(R_\alpha + R_c)} + (m_\alpha c^2 + m_c c^2) - m_o c^2$$

$$\Rightarrow \boxed{K_f + K_{photon} = \frac{3e^2}{\pi\varepsilon_0 (R_\alpha + R_c)} + (m_\alpha + m_c - m_o) c^2}$$

Problem 2 (25 Points)

A spring with stiffness k and relaxed length L_0 stands vertically on a table. A mass M sits on the spring in static equilibrium.



(a 5pts) What is the compressed length of the spring L in terms of the variables given above and any known constants.

Since the system is in equilibrium,

$$|F_s| = |F_g|$$

$$\Rightarrow k|s| = Mg$$

$$\Rightarrow |s| = \frac{Mg}{k}$$

Thus, we must have:

$$L = L_0 - s = \boxed{L_0 - \frac{Mg}{k}}$$

(b 10pts) Using your hand, you compress the spring so that the spring now has a length of $L/3$ and you hold the spring motionless at this position. Calculate how much work your hand did? Explain if the sign you obtained for this work is reasonable using a different argument from the one you employed to calculate the work.

Choose the spring, mass, and Earth as your system, with your hand as the surroundings.

$$\Delta E = W \Rightarrow \cancel{\Delta K} + \Delta U_s + \Delta U_g = W$$

$$\Rightarrow \frac{1}{2}k(s_f^2 - s_i^2) + Mg\left(\frac{L}{3} - L\right) = W$$

$$\Rightarrow \frac{1}{2}k\left[\left(\frac{L}{3} - L_0\right)^2 - (L - L_0)^2\right] - \frac{2}{3}MgL = W$$

$$\Rightarrow \frac{1}{2}k\left[\left(\frac{L_0}{3} - \frac{Mg}{3k} - L_0\right)^2 - \left(L_0 - \frac{Mg}{k} - L_0\right)^2\right] - \frac{2}{3}MgL = W$$

$$\Rightarrow \frac{1}{2}k\left[\frac{4L_0^2}{9} + \frac{4MgL_0}{9k} + \frac{M^2g^2}{9k^2} - \frac{M^2g^2}{k^2}\right] - \frac{2}{3}MgL_0 + \frac{2}{3}\frac{M^2g^2}{k} = W$$

$$\Rightarrow \frac{2}{9}kL_0^2 + \frac{2}{9}MgL_0 - \frac{4}{9}\frac{M^2g^2}{k} - \frac{2}{3}MgL_0 + \frac{2}{3}\frac{M^2g^2}{k} = W$$

$$\Rightarrow \boxed{W = \frac{2}{9}\left(kL_0^2 + \frac{M^2g^2}{k}\right) - \frac{4}{9}MgL_0}$$ which factors to $$\boxed{W = \frac{2}{9}k\left(L_0 - \frac{Mg}{k}\right)^2}$$

The work your hand does on the system is positive, which makes sense because it adds more spring potential energy to the system than the gravitational potential energy it removes.

(c 10pts) You let go of the block and watch it shoot straight up into the air. How high does the block go before it starts to fall back towards the ground? Be sure to express your answers in terms of the variables given above.

This time, your hand does no work.

$$\Delta E = 0 \quad \Rightarrow \quad \Delta K + \Delta U_s + \Delta U_g = 0$$

$$\Rightarrow \cancel{K_f} - \cancel{K_i} + \cancel{U_{s,f}} - U_{s,i} + U_{g,f} - U_{g,i} = 0$$

$$\Rightarrow -\frac{1}{2}k\left(\frac{L}{3} - L_o\right)^2 + Mg y_f - Mg\left(\frac{L}{3}\right) = 0$$

$$\Rightarrow -\frac{1}{2}k\left(\frac{L_o}{3} - \frac{Mg}{3k} - L_o\right)^2 + Mg y_f - Mg\left(\frac{L_o}{3} - \frac{Mg}{3k}\right) = 0$$

$$\Rightarrow Mg y_f = \frac{1}{2}k\left(-\frac{2L_o}{3} - \frac{Mg}{3k}\right)^2 + \frac{1}{3}MgL_o - \frac{1}{3}\frac{M^2g^2}{k}$$

$$\Rightarrow Mg y_f = \frac{1}{2}k\left(\frac{4}{9}L_o^2 + \frac{4}{9}\frac{MgL_o}{k} + \frac{1}{9}\frac{M^2g^2}{k^2}\right) + \frac{1}{3}MgL_o - \frac{1}{3}\frac{M^2g^2}{k}$$

$$\Rightarrow Mg y_f = \frac{2}{9}kL_o^2 + \frac{2}{9}MgL_o + \frac{1}{18}\frac{M^2g^2}{k} + \frac{1}{3}MgL_o - \frac{1}{3}\frac{M^2g^2}{k}$$

$$\Rightarrow Mg y_f = \frac{2}{9}kL_o^2 + \frac{5}{9}MgL_o - \frac{5}{18}\frac{M^2g^2}{k}$$

$$\Rightarrow \boxed{y_f = \frac{2}{9}\frac{kL_o^2}{Mg} + \frac{5}{9}L_o - \frac{5}{18}\frac{Mg}{k}}$$

Problem 3 (25 Points)

The following two problems deal with the motion of a baseball near the surface of the Earth. For both of these problems, you will need to take into account the force of air resistance $\vec{F}_{air} = -\frac{1}{2}\rho C A v^2 \hat{v}$ acting on the baseball. A typical baseball has mass 0.155 kg and radius 0.035 m. Take the density of air to be $1.3\ \frac{\text{kg}}{\text{m}^3}$ and the bluntness coefficient for the baseball to be $C = 0.35$.

(a 15pts) At a certain instant in time the baseball's position is $\langle 25.00, 12.00, 3.00 \rangle$ m and its velocity is $\langle 40.00, 30.00, 0 \rangle$ m/s. Calculate the net force acting on the baseball. Your answer should be a vector.

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{air} = \langle 0, -mg, 0 \rangle + (-\tfrac{1}{2}\rho C A)|v|^2 \hat{v}$$

$$|v| = \sqrt{(40.00\ m/s)^2 + (30.00\ m/s)^2 + (0\ m/s)^2} = 50.00\ m/s$$

$$\hat{v} = \frac{\vec{v}}{|v|} = \frac{\langle 40.00, 30.00, 0 \rangle\ m/s}{50.00\ m/s} = \langle 0.8, 0.6, 0 \rangle$$

$$\Rightarrow \vec{F}_{net} = \langle 0, -(0.155\,kg)(9.8\ m/s^2), 0 \rangle - \tfrac{1}{2}(0.35)(1.3\ kg/m^3)[\pi(0.035\,m)^2](50.00\ m/s)^2\langle .8, .6, 0 \rangle$$

$$\Rightarrow \boxed{\vec{F}_{net} = \langle -1.75, -2.83, 0 \rangle\ N}$$

(b 10pts) A second baseball is released from rest at a very great height. Determine the terminal speed of the baseball as it falls to the Earth.

At terminal velocity, $|F_{air}| = |F_g|$

$$\Rightarrow \tfrac{1}{2}\rho C A v^2 = mg$$

$$\Rightarrow |v| = \sqrt{\frac{2mg}{\rho C A}}$$

$$\Rightarrow |v| = \sqrt{\frac{2(0.155\,kg)(9.8\ m/s^2)}{(1.3\ kg/m^3)(0.35)(\pi(0.035\,m)^2)}}$$

$$\Rightarrow \boxed{|v| = 41.65\ m/s}$$

Problem 4 (25 Points)

You put a thin metal pot containing 2 liters (2000 grams) of room-temperature (20°C) water on a hot electric stove. You also stir the water vigorously with a 100 watt electric beater, which does 100 J of work on the water every second. You observe that after 5 minutes (300 s) the water starts to boil (temperature 100°C).

(a 10pts) What was the change $\Delta E_{\text{thermal}}$ in the water?

$$\Delta E_{\text{thermal}} = mC\Delta T$$

$$\Delta E_{\text{thermal}} = (2000\text{g})(4.2\ \text{J/gK})(80\text{K})$$

$$\boxed{\Delta E_{\text{thermal}} = 6.72 \times 10^5\ \text{J}}$$

(b 10pts) What was the transfer of energy Q due to a temperature difference into the water from the surroundings?

$$\Delta E_{\text{thermal}} = W + Q$$

$$\Rightarrow Q = \Delta E_{\text{thermal}} - W$$

$$\Rightarrow Q = 6.72 \times 10^5\ \text{J} - (100\ \text{J/s})(5\ \text{min})(60\ \text{s/min})$$

$$\Rightarrow \boxed{Q = 6.42 \times 10^5\ \text{J}}$$

(c 5pts) What was the change $\Delta E_{\text{surroundings}}$ in the rest of the Universe (the surroundings, including the stove and the beater)?

The energy that was gained by the water came from the surroundings.

Therefore, $\boxed{\Delta E_{\text{surroundings}} = -6.72 \times 10^5\ \text{J}}$