Practice test 2, MA1501 Calculus I Seong Jun Kim

This sample test is to guide your review for the Exam 2 (Tuesday, October 8).
In the exam, *you must show your work. Answers without substantiation do not count.*

Keywords: Several interpretations of the derivative (tangent line, linearization, rate of change–position, velocity, acceleration), Differentiation rules, The chain rule, Finding the derivative of various functions (trigonometric, inverse, logarithm, inverse trigonometric), Computing higher-order derivative, Techniques of differentiations (implicit differentiation, logarithmic differentiation), Related rates and Computing differentials.

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1. Definitions:
- The derivative of the function $f(x)$ with respect to the variable x is defined by

- If f is differentiable at $x = a$, the linearization of f at a is defined by

2. It is strongly recommended to memorize all of the differentiation rules:

- If c is a constant, $\frac{d}{dx}(c) =$
- If n is any real number, $\frac{d}{dx}x^n =$
- $\frac{d}{dx}a^x =$
- If u and v are differentiable at x, then $\frac{d}{dx}(uv) =$, $\frac{d}{dx}(\frac{u}{v}) =$
- $\frac{d}{dx}(\sin x) =$, $\frac{d}{dx}(\cos x) =$, $\frac{d}{dx}(\tan x) =$, $\frac{d}{dx}(\sec x) =$, $\frac{d}{dx}(\csc x) =$, $\frac{d}{dx}(\cot x) =$
- $(f \circ g)'(x) =$
- $(f^{-1})'(x) =$
- $\frac{d}{dx}\log_a x =$
- $\frac{d}{dx}(\sin^{-1} x) =$, $\frac{d}{dx}(\cos^{-1} x) =$, $\frac{d}{dx}(\tan^{-1} x) =$, $\frac{d}{dx}(\cot^{-1} x) =$

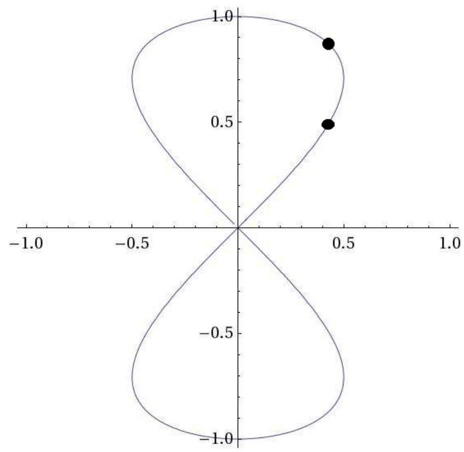
3. Calculate.
a. $\frac{df}{dx}$ for $f(x) = (x^2 + 4)^{7/5}$
b. $\frac{d}{dx}(\frac{\sin(x)}{1+\cos(x)})$
c. $\frac{d^2}{dt^2}(e^t \cos(t) + e^{\sqrt{t}})$
d. $\frac{d}{dx}\left(\log_3(\tan^{-1}(3x^4))\right)$
e. Use logarithmic differentiation to find the derivative of y with respect to t: $y = t(t+1)(t+2)^2$.
f. Find dy :

$$y = xe^{-x} + \ln\left(\frac{x+1}{\sqrt{x-1}}\right)$$

4. Sketched below is the set of all points satisfying

$$y^4 - y^2 + x^2 = 0$$

with the points $(\frac{\sqrt{3}}{4}, \frac{1}{2})$ and $(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2})$ indicated with dots.



a. Calculate $\frac{dy}{dx}$ at each of these two points.

b. Find the equation of the line tangent to the graph at $(\frac{\sqrt{3}}{4}, \frac{1}{2})$ and sketch it on the graph above.

5. Answer the following questions:

a. "Differentiability implies continuity" theorem says that if f has a derivative at $x = c$, then f is continuous at $x = c$. The converse of this theorem is false. Give an example to show that the converse is false.

b. Show that

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & x \neq 0, \\ 0, & x = 0 \end{cases}$$

is differentiable at $x = 0$ and find $f'(0)$.

6. A dynamic blast blows a heavy straight up with a launch velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t sec.

a. How high does the rock go?

b. What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? on the way down?

c. What is the acceleration of the rock at any time t during its flight?

d. When does the rock hit the ground again?

7. Let $f(x) = x^3 - 3x^2 - 1$, $x \geq 2$. Find the value of $\frac{df^{-1}}{dx}$ at the point $x = -1$ where $f(3) = -1$.

8. A conical paper cup is 12 inches deep and 8 inches across the top. It is being held "point down" while liquid is being poured in slowly from the top, at a rate of 2 cubic inches per minute. At what rate is the fluid level rising at the time when the depth of fluid in the cup is

a. 4 inches?

b. 9 inches?

See more problems regarding related rates in Exercises 3.10.