# MATH 1552 - SPRING 2016
# QUIZ 2 - SHOW YOUR WORK

NAME: ______________________________ TA: _______________________________

1. (10 points) Use a Riemann Sum with 4 equally spaced subintervals and the right endpoints to approximate $\int_0^1 x^3\, dx$. In doing your calculations, do not simplify your fractions. For example, write $\frac{2}{4}$ instead of $\frac{1}{2}$. Leave your answer as a fraction (you do not need to simplify the final fraction)

Since there are 4 equally spaced subintervals, $\Delta x = \frac{1}{4}$. Also, the right endpoints are

$x_1 = \frac{1}{4}$, $x_2 = \frac{2}{4}$ , $x_3 = \frac{3}{4}$, $x_4 = \frac{4}{4}$

$\Rightarrow f(x_k) = \left(\frac{k}{4}\right)^3$

**Riemann Sum =**

$$\sum_{k=1}^{4} f(x_k)\, \Delta x = \left(\frac{1}{4}\right)^3 \left(\frac{1}{4}\right) + \left(\frac{2}{4}\right)^3 \left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)^3 \left(\frac{1}{4}\right) + \left(\frac{4}{4}\right)^3 \left(\frac{1}{4}\right) = \frac{100}{256}$$

2. (20 points) Find $\int_0^1 x^3\,dx$, by finding $\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x_k$, where the partition of the interval [ 0, 1 ] has $n$ equally spaced points $x_k$.

$$x_k = \frac{k}{n} \Rightarrow f(x_k) = \left(\frac{k}{n}\right)^3 \quad \& \quad \Delta x = \frac{1}{n}$$

$$\int_0^1 x^3\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x_k = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^3\left(\frac{1}{n}\right) = \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3}{n^3}\left(\frac{1}{n}\right) =$$

$$\lim_{n\to\infty}\left(\frac{1}{n^4}\right)\sum_{k=1}^{n} k^3$$

$$= \lim_{n\to\infty}\left(\frac{1}{n^4}\right)\left(\frac{n(n+1)}{2}\right)^2 = \lim_{n\to\infty}\left(\frac{1}{n^4}\right)\left(\frac{n^2(n+1)^2}{4}\right) = \frac{1}{4}\lim_{n\to\infty}\left(\frac{n^2(n+1)^2}{n^4}\right) = \frac{1}{4}$$

**Since degree Numerator = 4 & degree Denomenator = 4,**

$$\lim_{n\to\infty}\left(\frac{n^2(n+1)^2}{n^4}\right) = 1$$

$$\Rightarrow \int_0^1 x^3\,dx = \frac{1}{4}$$