Solutions to Homework 6

1) We need $q_{0.5}$ such that

$$P(X \le q_{0.5}) = \frac{1}{2}$$

$$1 - e^{-\lambda q_{0.5}} = \frac{1}{2} \quad \Rightarrow \quad \frac{1}{2} = e^{-\lambda q_{0.5}}$$

$$\Rightarrow \ln \frac{1}{2} = -\lambda q_{0.5}$$

$$\Rightarrow q_{0.5} = \frac{-\ln \frac{1}{2}}{\lambda}$$

2) a)

$$F(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^{1/2} & \text{if } 0 < x < 1 \\ 1 & \text{if } x \ge 1 \end{cases}$$

b) Again we need $P(X \le q_{0.5}) = \frac{1}{2}$

$$\Rightarrow (q_{0.5})^{1/2} = \frac{1}{2} \quad \Rightarrow \quad q_{0.5} = \frac{1}{4}$$

3) Note that there is no $q_{0.5}$ such that

$$P(X \le q_{0.5}) = 0.5$$

Then we need the smallest $q_{0.5}$ such that $P(X \le q_{0.5}) \ge 0.5$ and $q_{0.5} = 5$