ISyE 3232 Final Exam
Fall 2011

Name

Please be neat and show all your work so that I can give you partial credit. HAVE A WONDERFUL BREAK.

Question 1
Question 2
Question 3
Question 4
Question 5
Total

(20) **1.** Consider a system with two servers and a capacity of at most three customers. The incoming customers belong to two classes: class 1 and class 2. Class 2 customers are allowed to enter the system if and only if there is a free server immediately available, otherwise they are turned away. Class 1 customers always enter the system (i.e., if there are less than 3 customers in the system) and wait for service if necessary. Assume that class 1 customers arrive with respect to a Poisson process of rate 10/hr and class 2 customers arrive with respect to a Poisson process of rate 15/hr. The arrival processes of class 1 and class 2 customers are independent of each other. The service times are exponential with rate 20/hr. Let $X(t)$ be the number of customers in the system at time t.

(10) (a) Is $\{X(t) : t \geq 0\}$ a continuous time Markov chain? If it is, what is the rate diagram (or the generator matrix)?

(10) (b) What is the expected number of customers in the system in the long run?

(20) **2.** A car comes equipped with one spare tire. The lifetimes of the four tires at the beginning of a long distance journey are independent identically distributed exponential random variables with a mean of 5000 miles. The spare tire has an exponential lifetime with a mean of 1000 miles. Compute the expected number of miles that can be covered without having to go to a tire shop. Note that when one of the regular tires fails, the driver will replace it with the spare one and then when one of the remaining three regular tires or the spare tire fails, he will go to the body shop.

(20) **3.** There are two tennis courts. Pairs of players arrive at rate 3 per hour with respect to a Poisson process and play for an exponentially distributed amount of time with mean 1 hour. If there are already two pairs of players waiting new arrivals will leave. What is the long run probability that both courts are occupied?

(20) **4.** Jobs submitted for execution on a central computer at a computation center are divided into four priority classes indexed 1,2,3 and 4. The interarrival times between two consecutive jobs of priority i are independent exponential random variables with means m_i minutes, with $m_1 = 10$, $m_2 = 15$, $m_3 = 30$ and $m_4 = 60$. Assume all the priority classes behave independently of each other.

(10) (a) Let $N(t)$ be the total number of jobs of all classes that arrive during $(0, t]$. What kind of a process is $N(t)$? What is the mean and variance of $N(t)$?

(10) (b) What is the probability that no jobs arrive in a 10 minute interval?

(20) **5.** Suppose that a store manager checks his inventory at the end of each week. If his inventory level is less than or equal to 3, then by immediate procurement, the inventory level is brought up to 5. On the other hand if the inventory level is greater than 3, no replenishment is undertaken. Suppose that the weekly demand D has a geometric distribution with parameter 0.3. That is $P\{D = k\} = 0.7^k 0.3$. Assume that no backlogs are allowed. Let X_n be the inventory at the end of week n (before a procurement is made).

(10) (a) Is $\{X_n : n \geq 0\}$ a Markov chain? If it is, what is the state space and the probability transition matrix?

(10) (b) Compute $P\{X_2 \geq 4 | X_1 = 5\}$.