

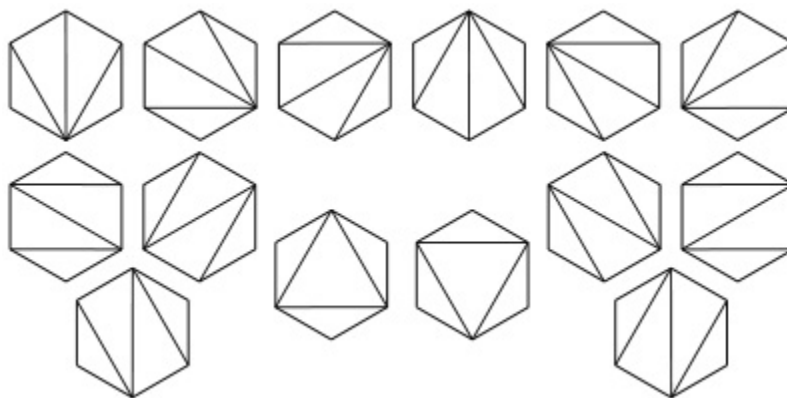
# MATH 3012, Quiz 3

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**Problem 1** (5 points).

Let  $C_n$  denote the number of different ways a convex polygon with  $n + 2$  sides can be cut into triangles by connecting vertices with straight lines. For example, the following picture shows that  $C_4 = 14$ .



We can define  $C_0 = 1$ . In general, we have

$$C_{n+1} = \sum_{i=0}^n C_i C_{n-i} \quad \text{for } n \geq 0.$$

1. Let  $c(x)$  be the generating function of  $C_n$ . Show that

$$c(x) = 1 + x \cdot c(x)^2.$$

2. Solve for  $c(x)$ .
3. Use the following Taylor expansion to expand  $c(x)$  and find  $C_n$ .

$$\sqrt{1+y} = 1 - 2 \sum_{n=1}^{\infty} \binom{2n-2}{n-1} \left(-\frac{1}{4}\right)^n \frac{y^n}{n}.$$

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