## Quiz 4 Solution

1. Find the equation for the plane containing the intersecting lines:

$$\ell_1(t) = (-1,2,1) + t(1,1,-1),\ t \in \mathbb{R}; \qquad \ell_2(s) = (1,1,2) + s(-4,2,-2),\ s \in \mathbb{R}.$$

**Solution:** The normal vector of the plane, is perpendicular to the two lines,

$$\vec{\mathbf{n}} = (1,1,-1) \times (-4,2,-2) = (0,6,6)$$

Any point on the lines is also on the plane, for example $(-1,2,1)$ (corresponding to $t = 0$ in the first line). The equation of the plane is

$$(0,6,6) \cdot (x,y,z) = (0,6,6) \cdot (-1,2,1) \Longrightarrow 6y + 6z = 18$$

2. Using row reduction, find the solution for the following system of equations

$$\begin{cases} x_1 \qquad\quad - 3x_3 & = 8 \\ 2x_1 + 2x_2 + 9x_3 & = 7 \\ \qquad\quad x_2 + 5x_3 & = -2 \end{cases}$$

**Solution:** Row reducing the matrix of the system we get

$$\begin{pmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 0 & 5 & -5 \\ 0 & 1 & 5 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Therefore the solution is $x_1 = 5$, $x_2 = 3$, $x_3 = -1$.

3. Determine the value(s) of $h$ such that the following matrix is the augmented matrix of a consistent system. Justify your answer. (6 points)

$$\begin{pmatrix} 1 & -4 & 7 & h \\ 0 & 3 & -5 & 2 \\ -2 & 5 & 9 & 6 \end{pmatrix}$$

**Solution:** Using row reduction

$$\begin{pmatrix} 1 & -4 & 7 & h \\ 0 & 3 & -5 & 2 \\ -2 & 5 & 9 & 6 \end{pmatrix} \xrightarrow{r_3 \to r_3 + 2r_1} \begin{pmatrix} 1 & -4 & 7 & h \\ 0 & 3 & -5 & 2 \\ 0 & -3 & 23 & 6+2h \end{pmatrix} \xrightarrow{r_3 \to r_3 + r_2} \begin{pmatrix} 1 & -4 & 7 & h \\ 0 & 3 & -5 & 2 \\ 0 & 0 & 18 & 8+2h \end{pmatrix}$$

Note that the range of the augmented matrix is the same as the range of the non-augmented matrix, regardless of the value of $h$. Therefore, the system is consistent for any value of $h$.