ISyE 4232 Exam # 2
Spring 2013

Name
Please be neat and show all your work so that I can give you partial credit.
GOOD LUCK.

Question 1
Question 2
Question 3
Total

(30) **1.** A decision maker observes a discrete time system which moves between states $\{s_1, s_2, s_3, s_4\}$ according to the following transition probability matrix:

$$P = \begin{bmatrix} 0.3 & 0.4 & 0.2 & 0.1 \\ 0.2 & 0.3 & 0.5 & 0.0 \\ 0.1 & 0.0 & 0.8 & 0.1 \\ 0.4 & 0.0 & 0.0 & 0.6 \end{bmatrix}$$

At each point in time the decision maker may leave the system and receive a reward of $R = 20$ units or alternatively remain in the system and receive a reward of $r(s_i)$ units if the system occupies state s_i. If the decision maker decides to remain in the system, its state at the next decision epoch is determined by P. Assume that $r(s_i) = i$. The decision maker's objective is to maximize his discounted infinite horizon reward when the discount factor $\alpha = 0.8$.

a. (15) Formulate this as a Markov decision process problem. That is provide the state space, set of actions in each state, transition probabilities, rewards for each state action combination.

b. (20) Provide the primal and dual LP's to solve this problem.

(35) **2.** Let $S = \{s_1, s_2, s_3\}$, $A_{s_1} = \{a_{11}, a_{12}\}$, $A_{s_2} = \{a_{21}\}$, and $A_{s_3} = \{a_{31}\}$, $r(s_1, a_{11}) = r(s_1, a_{12}) = 0$, $r(s_2, a_{21}) = 3$, and $r(s_3, a_{31}) = 4$, and $p(s_1|s_1, a_{11}) = p(s_2|s_1, a_{11}) = \frac{1}{2}$, $p(s_1|s_1, a_{12}) = \frac{2}{3}$, $p(s_3|s_1, a_{12}) = \frac{1}{3}$, $p(s_1|s_2, a_{21}) = 1$, and $p(s_1|s_3, a_{31}) = 1$. Use policy iteration to compute the long run average reward optimal policy.

(30) **3.** The condition of an equipment deteriorates over time. Let the state $s \in S = \{0, 1, \cdots\}$ represent the condition of the equipment at each decision epoch. The higher the value of s, the worse the condition of the equipment is. At each decision epoch the decision maker can either operate the equipment as is for an additional period or he can replace the equipment with a new one. We assume that each period the equipment detoriorates i units with probability $p(i)$ independent of the state at the beginning of the period. if the decision maker chooses to replace the equipment a fixed cost of R is immediately incurred. In addition, each time the equipment is in state s an operating cost $C(s)$ is incurred. Formulate this problem as a discounted (with discount factor α) infinite horizon Markov Decision Process problem. That is provide the state space, set of actions in each state, transition probabilities, rewards for each state action combination.