Instructions: *Print* your name, student ID number and recitation session in the spaces below.

Name: ____

Student ID:_____________________________________

Recitation session:____________________________________

Practice Final Exam, Calculus III (Math 2551)

Show your work clearly and completely!

No calculators are allowed.

You can bring a formula sheet of a one-side letter size paper.

Question	Points
1)	
2)	
3)	
4)	
5)	
6)	

Problem 1 (24 points)**:** a) Find $\frac{du}{dt}$, if

$$u(x,y) = 3xy^2 - x^2;\; x = t^2 + 2t,\; y = 3t.$$

b) Write an equation for the tangent plane of the surface

$$z^3 + xyz - 2 = 0$$

at the point $P(1,1,1)$.

c) Calculate the second-order partial derivatives of

$$g(x,y) = xy\sin(xy).$$

Problem 2 (16 points): Closed rectangular boxes 16 cubic feet in volume are to be constructed from three types of metal. The cost of the metal for the bottom of the box is \$0.50 per square foot, for the sides of the box \$0.25 per square foot, for the top \$0.10 per square foot. Find the dimensions that minimize cost of material.

Problem 3 (15 points)**:** Evaluate

$$\int\int_{\Omega} \cos\left(\frac{y-x}{y+x}\right)\ dxdy,$$

where Ω is the region in the first quadrant bounded by the lines $x+y=1$ and $x+y=2$.

(Hint: Use proper change of variables.)

Problem 4 (15 points)**:** Evaluate

$$\int_C y\ dx + yz\ dy + z\,(x-1)\ dz$$

where C is the intersection of the sphere $x^2 + y^2 + z^2 = 4$ with the cylinder $(x-1)^2 + y^2 = 1$ traversed from $(2,0,0)$ to $(0,0,2)$.

Problem 5 (15 points)**:** Find the area enclosed by the curve

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1.$$

(Hint: Use Green's theorem.)

Problem 6 (15 points)**:** Calculate the total flux of

$$\vec{v}\,(x, y, z) = 2x\ \mathbf{i} + xz\ \mathbf{j} + z^2\ \mathbf{k}$$

out of the solid bounded by the paraboloid $z = 9 - x^2 - y^2$ and the $xy-$plane.