QUIZ 4 FOR MATH 2401, SEP 18

1. Find and sketch the domain for the function, then find the function's range.
$f(x,y) = \ln(9 - x^2 - y^2)$

sol:
$domain = \{(x,y) \mid x^2 + y^2 < 9\}$,
$range = (-\infty, \ln 9]$.

2. Find the domain for the function, then find the function's range.
$f(x,y) = \sqrt{(x^2-4)(y^2-9)}$

sol:

It requires $x^2 - 4 \geq 0, y^2 - 9 \geq 0;$ or $x^2 - 4 \leq 0, y^2 - 9 \leq 0$.
for $x^2 - 4 \geq 0, y^2 - 9 \geq 0$, we get $\{(x,y) \mid x \geq 2 \ or \ x \leq -2, \ y \geq 3 \ or \ y \leq -3\}$;
for $x^2 - 4 < 0, y^2 - 9 < 0$, we get $\{(x,y) \mid -2 < x < 2, -3 < y < 3\}$.
Then the domain =
$\{(x,y) \mid x \geq 2 \ or \ x \leq -2, \ y \geq 3 \ or \ y \leq -3\} \bigcup \{(x,y) \mid -2 < x < 2, -3 < y < 3\}$.

The range = $[0, \infty)$

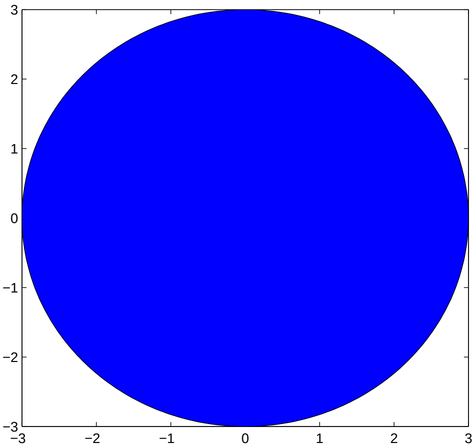

FIGURE 1. Sketch of domain for problem 1. A disc without boundary

3. Find the limit of f or show that the limit does not exist.

(1) $\lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2}$

(2) $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

Sol:

(1) $\lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{x+y-4}{\sqrt{x+y}-2} = \lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{(\sqrt{x+y})^2-2^2}{\sqrt{x+y}-2} = \lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \frac{(\sqrt{x+y}-2)(\sqrt{x+y}+2)}{\sqrt{x+y}-2}$
$= \lim\limits_{\substack{(x,y)\to(2,2)\\ x+y\neq 4}} \sqrt{x+y}+2) = \sqrt{2+2}+2 = 4$

(2) let $y = kx^2$.
$\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \lim\limits_{(x,y)\to(0,0)} \frac{x^2kx^2}{x^4+(kx^2)^2} = \lim\limits_{(x,y)\to(0,0)} \frac{kx^4}{(1+k^2)x^4} = \frac{k}{1+k^2}$. Since k varies, the limit does not exist.