MATH 2403 H1-H3. Differential Equations. Quiz 3. Jan 28, 2014.
Instructor: Dr. Luz V. Vela-Arévalo.

Student's Name: Sol Section

Show all work to receive credit

1. A rocket sled having an initial speed of 150 mi/h is slowed by a channed of water. During the braking process, the acceleration a is $a(v) = -\mu v^2$, where v is the velocity and μ is a constant. Use the chain rule $dv/dt = v(dv/dx)$ to solve for v in terms of x. If it requires a distance of half a mile to slow the sled to 15mi/h, determine μ.

$v_0 = 150$ mi/h

$\frac{dv}{dt} = -\mu v^2$, with $\frac{dv}{dt} = v\frac{dv}{dx} \Rightarrow v\frac{dv}{dx} = -\mu v^2$

$\Rightarrow \int \frac{dv}{v} = \int -\mu \, dx$

Integrating: $\ln|v| = -\mu x + C$ or $v = Ce^{-\mu x}$

As $v(0) = v_0 = 150$, $\boxed{v = 150 e^{-\mu x}}$

Now, $15 = 150 e^{-\mu(0.5)} \Rightarrow \boxed{\mu = -\ln\left(\frac{15}{150}\right)\left(\frac{1}{.5}\right)}$

2. For the initial value problem

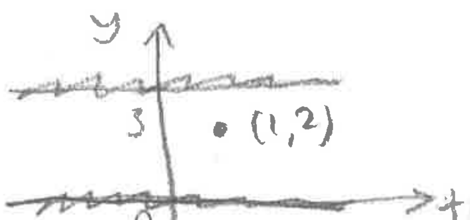
$$\frac{dy}{dt} = \frac{1+t^2}{3y - y^2}, \quad y(1) = 2,$$

provide a rectangle R where the hypotheses of the Theorem of existence and uniqueness are satisfied.

Let $f(t,y) = \frac{1+t^2}{3y-y^2}$. Then $\frac{\partial f}{\partial y} = \frac{-(1+t^2)(3-2y)}{(3y-y^2)^2}$

Both f and $\frac{\partial f}{\partial y}$ are continuous everywhere except for points for which $3y - y^2 = 0$.

This is $y(3-y) = 0$ or $y = 0, y = 3$.



For $(1,2)$, the rectangle R is $-\infty < t < \infty$
$0 < y < 3$.