MATH 2403, Midterm 3

11/09/2012

Name: _____ GTID:_____

Circle your section below

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Problem No.	Points
1	
2	
3	
4	
5	

TOTAL:_____

Please do show all your work including intermediate steps. Partial credit is available. You may use a basic non-graphing calculator and a handwritten one-sided 11x8.5 note sheet.

Problem 1 (20 points).

Find a fundamental matrix for the given system.

$$\mathbf{x}' = \begin{pmatrix} 4 & 1 & 3 \\ 6 & 4 & 6 \\ -5 & -2 & -4 \end{pmatrix} \mathbf{x}.$$

You may use the fact that the eigenvalues of matrix $\begin{pmatrix} 4 & 1 & 3 \\ 6 & 4 & 6 \\ -5 & -2 & -4 \end{pmatrix}$ are $\lambda = 2$ which has multiplicity 1 and $\lambda = 1$ which has multiplicity 2.

Problem 2 (15+5 points).

a) Find e^{At} for the system

$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x}$$

b) Solve the given initial problem by using the fundamental matrix e^{At}

$$\mathbf{x}' = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$$

Problem 3 (10+10 points).

For each of the given differential equation, find the solution by using the change of variable $z = \ln x$ to transform the equation into one with constant coefficients.

a) $x^2 y'' + 7xy' + 5y = 0.$

b) $x^2 y'' + 7xy' + 5y = x^2.$

Problem 4 (20 points).

Use the method of reduction of order to find a second solution y_2 of the given differential equation such that $\{y_1, y_2\}$ is a fundamental set of solutions on the given interval.

$$(x-1)y'' - xy' + y = 0, \quad x > 1; \quad y_1(x) = e^x.$$

Hint: $e^{\int \frac{x-2}{x-1} dx} = e^x/(x-1)$.

Problem 5 (20 points).

Find the general solution of the given differential equation.

$$y'' + 4y' + 4y = t^{-2}e^{-2t}, \quad t > 0.$$