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Exam I

10:05-10:55 AM (50-MINUTE EXAM)

To receive full credit on each problem, it is advised to write down all equations and work required to reach the final answer. Label all variables and equations. Include a brief word description to explain steps when necessary (e.g. $A_1=A_2=A$), stating all assumptions (e.g. incompressible).

Please make sure to answer the question and place box around your final solution.
Numerical answers without units or explanations (work required for solution) will not receive credit.
Turn in your 1-page handwritten note sheet with your exam, along with any scrap paper.

The use of wireless devices (e.g. cell phones, IR transmitters/receivers) is not permitted during exam.

NAME: Dawson

The work presented here is solely my own.
I did not receive any assistance nor did I assist other students during the exam.
I pledge that I have abided by the above rules and the Georgia Tech Honor Code.

Signed: ____________________

Problem 1 _____/ 30
Problem 2 _____/ 30
Problem 3 _____/ 30

Total _____/ 90

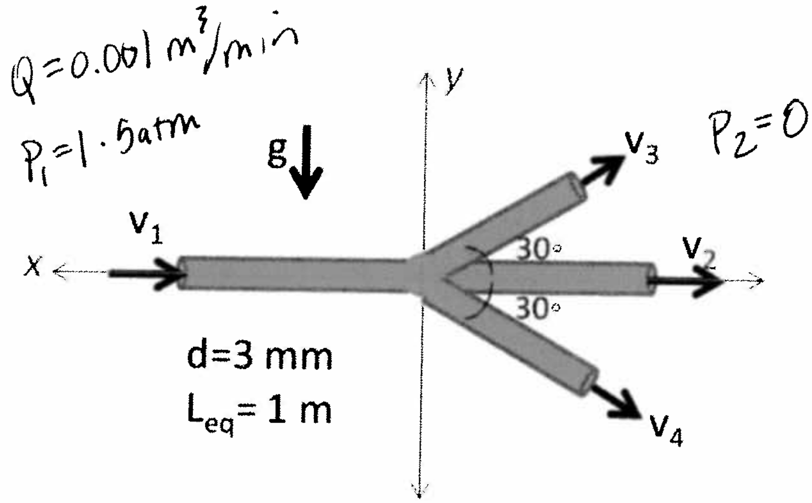
Make the following assumptions when necessary:

$g = 10\ m\ s^{-2} = 30\ ft\ s^{-2}$
$R = 8.31\ J\ K^{-1} mol^{-1} = 10.7\ ft^3\ psi\ R^{-1}\ lbm^{-1}$
$\rho_{water} = 1000\ kg/m^3$, $\mu_{water} = 1$ cP, 1 Poise (P) = 1 gram/cm s
$\rho_{air} = 1\ kg/m^3$, $\mu_{air} = 0.01$ cP
$P_{atm} = 1$ atm = 760 mm Hg = 1.01×10^5 Pa = 14.7 psi
$\gamma = \rho/\rho_{\omega}$

Problem 1 (30 points):

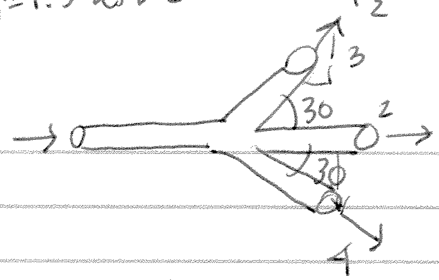
Newtonian fluid (specific gravity $\gamma = 0.85$) flows through the piping system shown on the right, which is formed from hard plastic tubing with diameter of 3 mm and total length of 1 m (equivalent length for straight tube). At the tube inlet the absolute pressure is 2.5 atm and the volumetric flow rate Q=0.001 m^3/min. Assume fluid exits tubing network (regions 2-4) at atmospheric pressure (1 atm).

Determine the <u>force B acting on the fluid in the piping network.</u>



Calculation error
2 pts

$P_1 = 1.5$ atm $\qquad P_2 = 0$



$\gamma = 0.85 = \frac{\rho}{\rho_w}$

$Q_1 = 0.001\ m^3/min$

$Q = VA$

$A = 7.06 \times 10^{-6}\ m^2$

$A_1 = A_2 = A_3 = A_4$

8

5

Continuity

$\int \rho\,(v \cdot n)\,dA = 0$

applying cos 30 sin 30 5 pt error

$-V_1 A_1 + V_2 A_1 + V_3 A_1 + V_4 A_1 = 0$

$V_1 = V_2 + V_3 + V_4 = 3 V_2$

Symmetry $\quad V_2 = V_3 = V_4$

$\boxed{V_2 = V_1/3} \qquad \boxed{Q_2 = Q_1/3}$

$V_2 = 47.2\ m/min = 0.79\ m/s \qquad V_1 = 2.36\ m/s = 141.6\ m/min$

Momentum (x, y)

10

$\Sigma F_x = P_1 A_1 + B_x = \iint v_x\,\rho\,(v \cdot n)\,dA$

$+ V_2^2 A_1 \rho$

$P_1 A_1 + B_x = -V_1^2 A_1 \rho + 2 V_2^2 \cos 30\, A_1 \rho$

$$\boxed{B_x = \overset{2}{\cancel{3}} \left(\frac{V_1}{3}\right)^2 \cos 30\, A_1 \rho - V_1^2 A_1 \rho - P_1 A_1 + \frac{V_1^2}{9} \cos 30\, A_1 \rho}$$

$\Sigma F_y = B_y - \rho g V = \cancel{V_2^2 \sin 30\, A_1 \rho}$

$\cancel{- V_2^2 \sin 30\, A_1 \rho} = 0$

10

$$\boxed{B_y = {}^{+}\rho g V} = 10\,\frac{m}{s^2}\left(1000\,\frac{kg}{m^3}\right)\left(\frac{\pi\, 0.003^2}{4}\right) m^2 \left(1\,m\right)$$

$\times\gamma \quad g \quad \rho \quad \frac{\pi D^2}{4} L$

$$B_y = 0.071^{(0.85)}\ N = 0.060\ N$$

$$B_x = A_1 \rho \left(\frac{2 V_1^2}{\cancel{9}} \cos 30 - V_1^2 + \frac{V_1^2}{9}\right) - P_1 A_1$$

$$V_1 = 0.001\,\frac{m^3}{min}\left(\frac{min}{60\,s}\right)\left(\frac{4}{\pi\, 0.003^2\ m^2}\right)$$

$$V_1 = 2.36\,\frac{m}{s}$$

$$P_1 = 1.5\,atm\left(\frac{1.013\times10^5}{atm}\frac{N}{m^2}\right) = 151{,}950\,\frac{N}{m^2}$$

$$A_1 = 7.07\times10^{-4}\ m^2$$

$$\cos 30 = \sqrt{3}/2$$

$$B_x = 0.0071\left(\cancel{-3.96}\ {-3.87}\right) - 151{,}950\left(7.07\times10^{-6}\right)$$

$$B_x = -1.1\ N$$

same

5

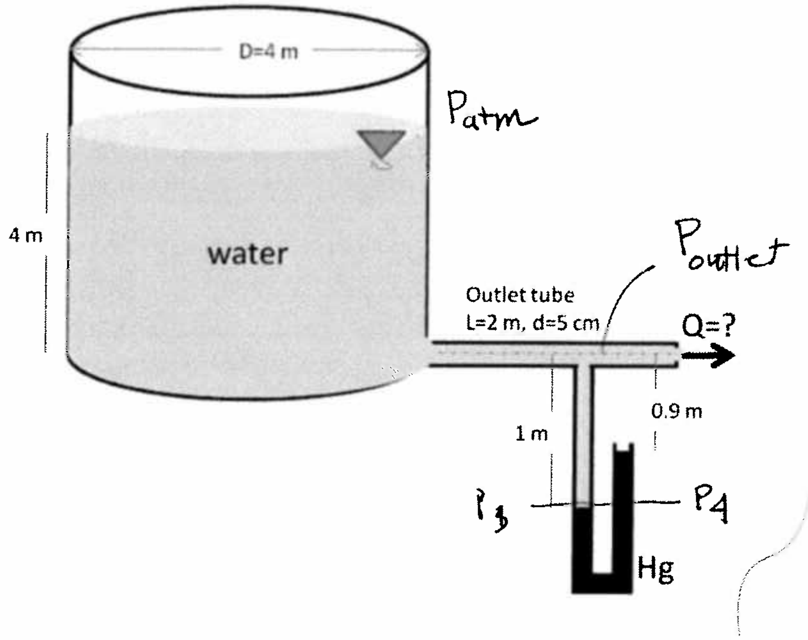
$$\boxed{\vec{B} = -1.1\,\vec{e}_x + \overset{0.060}{0.071}\,\vec{e}_y}$$

Problem 2 (30 points):

Determine the **volumetric flow rate of water exiting the tube (Q=?).**

The water flows from a large open tank as shown. Upside down triangle indicates that the water level is being maintained.

The specific gravity of Hg in the manometer is γ=13.56. You may assume frictionless flow of water an incompressible fluid with ρ=1000 kg/m^3 and μ=0.001 Pa s.



Continuity

$$\int \rho (v \cdot n)\, dA = 0$$

$$-v_1 A_1 \rho + \underbrace{v_2 A_2}_{Q} \rho = 0$$

5

$$\boxed{v_1 A_1 = v_2 A_2 \qquad A_1 >> A_2 \qquad v_1 << v_2}$$

Momentum | Energy

Bernoulli's EQN

$$\cancel{\dot{Q}} - \cancel{\dot{W}_s} - \cancel{\dot{W}_\mu} = \iint (e + P/\rho)\, \rho (v \cdot n)\, dA + \cancel{\frac{\partial}{\partial t} \iiint e \rho\, dV}$$

$$e_1 + \frac{P_1}{\rho} = e_2 + \frac{P_2}{\rho}$$

1: surface P_{atm}, V_1 small
2 Outlet $P_{manometer}$

$$\cancel{\frac{V_1^2}{2}} + g y_1 + \cancel{u_1} + \frac{P_1}{\rho} = \frac{V_2^2}{2} + g y_2 + \cancel{u_2} + \frac{P_2}{\rho}$$

frictionless flow $u_2 - u_1 = 0$

10

$$\boxed{\frac{V_2^2}{2} = \frac{P_1 - P_2}{\rho} + g(y_1 - y_2) \qquad V_2 = \sqrt{2\left(\frac{P_1 - P_2}{\rho}\right) + 2g(y_1 - y_2)}}$$

Static Fluid Manometer

$P_3 = P_4$

$P_3 = P_{outlet} + \rho_w g\,(1\,m)$

$P_4 = P_{atm} + \frac{\rho_A}{\rho_w}\rho_w\, g\,(0.1\,m)$

$P_{outlet} = P_{atm} + \gamma \rho_w g (0.1) - \rho_w g (1)$

$P_{outlet} = 1.013\times10^5 \frac{N}{m^2} + (13.56)\left(1000\frac{kg}{m^3}\right)\left(10\frac{m}{s^2}\right)(0.1\,m) - \left(1000\frac{kg}{m^3}\right)\left(10\frac{m}{s^2}\right)(1\,m)$

$P_{outlet} = 1.05\times10^5 \frac{N}{m^2} = P_2$

$P_1 = 1.013\times10^5 \frac{N}{m^2}$

10

$$\boxed{V_2 = 8.5\ \frac{m}{s}}$$

$Q_{outlet} = V_2\,\pi\,(0.025)^2 m^2 = 0.017\frac{m^3}{s}$

5

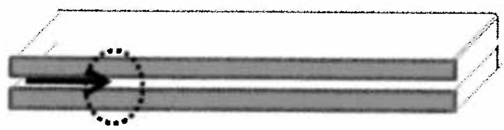
$$\boxed{Q_{outlet} = 1.0\ \frac{m^3}{min}}$$

Problem 3 (30 points):

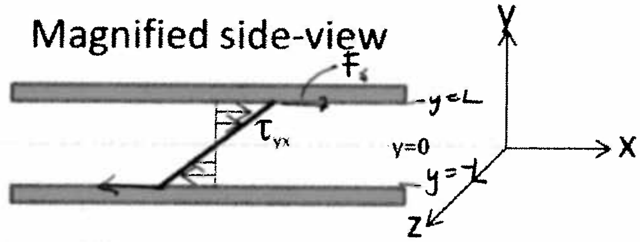
Water (an incompressible fluid with viscosity 0.001 Pa s and density 1000 kg/m^3) flows between two parallel plates (1 m in length and 50 cm in width) (*LHS – concept diagram*). The plates are separated by a distance of 1 cm. Shear force on the surface of the plates has a magnitude of 1 N. The shape of the shear profile is *illustrated in magnified side-view*.

Determine the equations for the shear and velocity profiles (hint: you should be able to solve for all constant terms).

Overall concept



Magnified side-view



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$$\tau_{yx} = \mu \frac{dv_x}{dy}$$

$$\frac{dv_x}{dy} = \alpha \frac{y}{L} + \beta$$

3

$$\tau_{yx} = 0 \quad y = 0$$

$$\beta = 0$$

$$\mu \frac{dv_x}{dy} = \frac{F_s}{A} \quad \text{at } y = L$$

$$\frac{dv_x}{dy} = \frac{F_s}{A\mu}$$

5

$$\frac{F_s}{A\mu} = \alpha\left(\frac{L}{L}\right) = \alpha$$

$$\alpha = \frac{1N}{(1 \times 0.50)m^2}\left(\frac{m^2}{0.001 \, N s}\right)$$

$$\alpha = 2000$$

5

$$\frac{dv_x}{dy} = \alpha \frac{y}{L}$$

$$v_x = \frac{\alpha y^2}{2L} + \gamma$$

4

$$v_x = 0 \quad y = L$$

$$0 = \frac{\alpha L^2}{2L} + \gamma$$

$$v_x = 0 \quad y = -L$$

$$0 = \frac{\alpha L^2}{2L} + \gamma$$

same BC

$$\gamma = -\frac{\alpha L}{2}$$

L = 0.5 cm

$$\gamma = -1000\,(0.005)$$

$$\gamma = -5$$

10 linear parabolic τ, v_x
12 BCs (4 F/A = τ at surface) (8 - others)
8 — ideas

$$\tau_{yx} = \mu \alpha \frac{y}{L}$$

$L = 0.5\,cm = 0.005\,m$ $\qquad \alpha = 2000$

$$\tau_{yx} = \frac{2000\,\mu}{0.005} y$$

$\mu = 0.001$ Pas

4 $\boxed{\tau_{yx} = 400{,}000\,\mu y = 400\,y}$

$$v_x = \frac{\alpha y^2}{2L} + \gamma$$

$\gamma = -5$

$$v_x = \frac{2000}{2(0.005)} y^2 - 5$$

4 $\boxed{v_x = 200{,}000\,y^2 - 5}$

<u>Check if time allows</u>

$y = 0.005\,cm$

$\tau_{yx} = 2$

$\frac{F}{A} = \frac{1N}{1 \times 0.5\,m^2} = 2$

$y = 0.005\,cm$

$v_x = 0$

$200{,}000(0.005^2) = 5$

$v_x = 0$

$V = a + by + cy^2$

same $v = 0$ at $y = L$

$v = 0$ at $y = -L$

$V = V_{max}$ at $y = 0$

$V_{max} = a$

$0 = V_{max} + bL + cL^2$

$0 = V_{max} - bL + cL^2$

$0 = 2V_{max} + 2cL^2$

$-V_{max} = cL^2$

$C = \frac{-V_{max}}{L^2}$

$b = 0$ Symmetry

$V = V_{max}\left(1 - \frac{y^2}{L^2}\right)$

$V = 5\left(1 - \frac{y^2}{(0.005)^2}\right)$

$\cancel{V = 5\left(1 - 400{,}000\, y^2\right)}$

$V = 5(1 - 40{,}000\, y^2)$ ✓

$\tau = \mu \frac{dv_x}{dy}$

$\tau = \mu\left(\frac{-2V_{max}\, y}{L^2}\right)$

$\frac{F}{A} = \tau$ at wall $= 2 \frac{N}{m^2}$

$\mu = 0.001$ Pa s $\qquad y = L = -0.005$

$V_{max} = \frac{2 \, N/m^2 \,(-0.005)}{-2\,(0.001)}$ F/A

$V_{max} = 5$ m/s

$\tau = \frac{(0.001)(2)(5)}{(.005)^2}\, y$

$\tau = 400\, y$ ✓