ISyE 2027C Probability with Applications

HINTS for Homework 6 for Quiz March 10 **March 5, 2015**

Simple Calculation Problems

1. $X = -1$ w.p. 0.5; $X = 0$ w.p. 0.2; $X = 3$ w.p. 0.3. Calculate $\sigma(X)$. Use Chebyshev's inequality to find an upper bound on $P(X \geq 3)$. $\sqrt{3.04}; 0.45$.

2. Continuous random variable Y has uniform distribution on the interval $[0, 3]$. Calculate $\sigma(Y)$. Use Chebyshev's inequality to find an upper bound on the probability that $|Y - 1.5| > 1.25$ $\sqrt{3}/2; 0.48$.

3. Continuous random variable Y has uniform distribution on the interval $[-11, 11]$. Use your answer to the previous question and properties of expectation and variance to find $\sigma(Y)$. $11/\sqrt{3}$.

4. $X_i : i = 1, 2, 3$ are independent Bernoulli variables equal to 1 with probabilities $1/3, 1/2, 2/3$ respectively, and equal to 0 otherwise. Calculate $\sigma(Y)$ if
$$Y = \min_{1 \leq i \leq 3} X_i$$
. $2\sqrt{2}/9$.

5. Discrete random variables $X_i : i = 1, 2, \ldots, 10$ are Bernoulli variables with parameter $p = P(X_i = 1) = 0.25$. Discrete random variables $Y_i : i = 1, 2, \ldots, 10$ are Bernoulli variables with parameter $p = P(X_i = 1) = 0.75$. All 20 variables are jointly independent. Let $Z = \sum_{i=1}^{10} X_i + Y_i$. Calculate $\sigma(Z)$. $\sqrt{15}/2$

6. Continuous random variable Y has density $1/6$ on the interval $[2, 4]$ and density $1/3$ on the interval $[6, 8]$. Calculate $\sigma(Y)$.
$\sqrt{35}/3$

7. Continuous random variable Y has density αy in the range $0 \leq y \leq 2$. Find α. Find $\sigma(Y)$.

Qualitative Problems

1. Let X and Y be independent random variables. Then $\sigma(X) + \sigma(Y) - \sigma(X + Y)$ is:
   (a) < 0
   (b) ≤ 0 and can be < 0
   (c) $= 0$
   (d) ≥ 0 and can be > 0
   (e) > 0
   (f) sometimes 0, sometimes < 0 and sometimes > 0

   Hint: try this on a couple of very simple random variables, or remember that by independence $\sigma^2(X)+\sigma^2(Y) = \sigma^2(X + Y)$ *and think about what the square root function does.*

2. Let X and Y be dependent random variables. Then $\sigma(X) + \sigma(Y) - \sigma(X + Y)$ is:
   (a) < 0
   (b) ≤ 0 and can be < 0
   (c) $= 0$
   (d) ≥ 0 and can be > 0
   (e) > 0
   (f) sometimes 0, sometimes < 0 and sometimes > 0

   Hint: The two extreme cases ought to be when $X = Y$ *(positive correlation) and when* $X = -Y$ *(negative correlation). Figure out both extreme cases.*

3. In Problem 5 above, suppose all 20 variables changed to be Bernoulli with parameter $p = \frac{1}{2}.25 + \frac{1}{2}.75 = .5$. Would $\sigma^2(Z)$ (the variance of Z, not the standard deviation of Z) change to a smaller, equal, or larger value?

   Hint: Consider the extreme case where $p = 0$ for the X_i variables and (you fill in the rest).

Problems

1. A Georgia Tech degree is worth \$100K today. Each day the value of the Tech degree increases by 1% with probability .5 and decreases by $\frac{100}{101}$% with probability .5. Let X be the number of days until the degree is again worth exactly \$100K. Prove that you can't calculate $\sigma^2(X)$. *Hint: Try to calculate $E[X]$, or to find lower bounds on $E[X]$.*

2. Random variables X and Y are independent with $E[X] = 5, E[X^2] = 49, E[Y] = 30, E[Y^2] = 1000$. Use Chebyshev's inequality to find a number β (the smallest value you can get) such that $P(|X + Y - 35| \geq \beta) \leq 0.04$. *Hint: use the independence of X and Y, and observe that $E[X + Y] = 35$.*