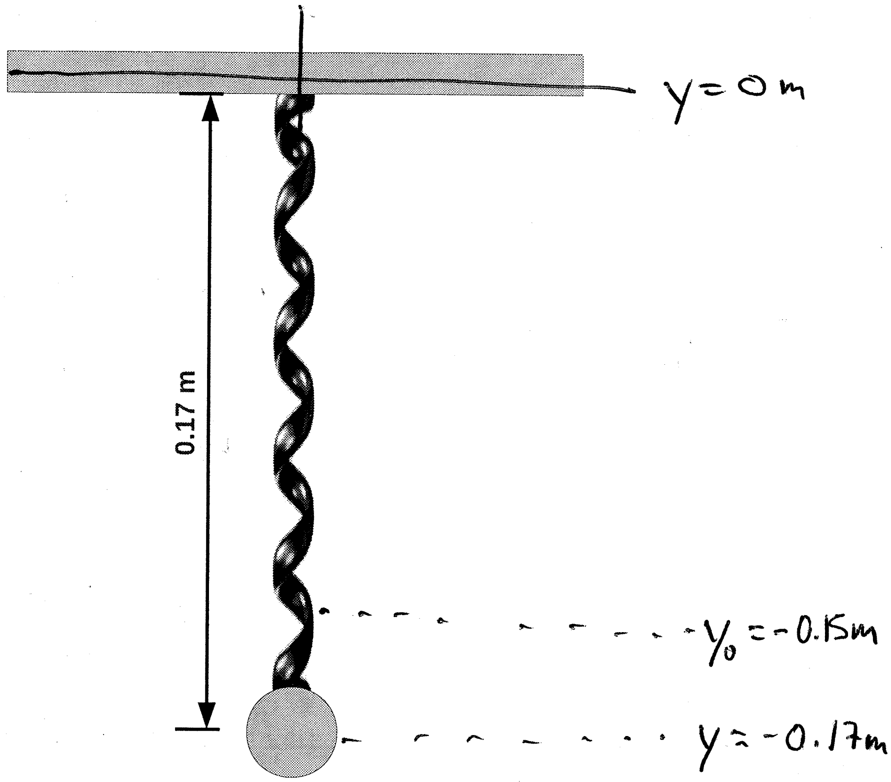
PHYS 2211 Test 1 Key (Spring '11)

Problem 1 (25 Points)

A mass of 0.03 kg is attached to a *vertically-hanging* spring with a spring stiffness of 12 N/m and a relaxed length of 0.15 m. You pull the mass downwards, so that the spring's length is 0.17m. Then you release it so that when it leaves your hand it has zero velocity.



The First Time Step

(a 5pts) What is the net force on the mass the instant after you release it? Remember to express your answer as a vector.

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring}$$
$$= \langle 0, -mg, 0\rangle + \langle 0, -k(y-y_0), 0\rangle$$
$$= \langle 0, -mg - k(y-y_0), 0\rangle$$
$$= \langle 0, -(0.03kg)(9.8ms^{-2}) - (12N/m)(-0.17m + 0.15m), 0\rangle$$
$$\boxed{\vec{F}_{net} = \langle 0, -0.054, 0\rangle N}$$

(b 5pts) What is the new velocity of the mass 0.02 seconds after you release it from rest? Remember to express your answer as a vector. You may assume the net force is constant over this relatively short time period.

$$\vec{P}_f = \underbrace{\vec{P}_i}_{0} + \vec{F}_{net}\Delta t$$
$$\Rightarrow V_f = \frac{\vec{F}_{net}\Delta t}{m} = \frac{\langle 0, -0.054, 0\rangle N \cdot 0.02s}{0.03kg}$$
$$\boxed{\vec{V}_f = \langle 0, -0.036, 0\rangle\ m/s}$$

(c 5pts) What is the new position of the mass 0.02 seconds after you release it from rest? Remember to express your answer as a vector. You may assume the net force is constant over this relatively short time period.

$$\vec{V}_{AVG} = \frac{\vec{V}_i + \vec{V}_f}{2}$$

$$= \frac{\langle 0,0,0\rangle m/s + \langle 0,-0.036,0\rangle m/s}{2}$$

$$= \langle 0,-0.018,0\rangle m/s$$

$$\vec{r}_f = \vec{r}_i + \vec{V}_{AVG}\Delta t$$

$$= \langle 0,-0.17,0\rangle m + \langle 0,-0.018 m/s,0\rangle (0.02 s)$$

$$\boxed{\vec{r}_f = \langle 0,-0.17036,0\rangle m}$$

Alternatively, let $\vec{V}_{AVG} \approx \vec{V}_f$ since Δt is ~ 0.

$$\vec{r}_f = \vec{r}_i + \vec{V}_f \Delta t$$

$$= \langle 0,-0.17,0\rangle m + \langle 0,-0.036 m/s,0\rangle (0.02 s)$$

$$= \langle 0,-0.17072,0\rangle m$$

The Second Time Step

(d 10pts) What is the new velocity of the mass, a second time step later (i.e. at 0.04 seconds) after you release it from rest? Remember to express your answer as a vector. You may assume the net force is constant over the relatively short time period 0.02 to 0.04 seconds.

$$\vec{F}_{net} = \vec{F}_{grav} + \vec{F}_{spring} \quad @\ t = 0.02 s$$

$$= \langle 0,-mg,0\rangle + \langle 0,-k(y_f - y_0),0\rangle$$

$$= \langle 0,-mg - k(y_f - y_0),0\rangle$$

$$= \langle 0,-(0.03 kg)(9.8 m/s^2) - (12 N/m)(-0.17036 m + 0.15 m),0\rangle$$

$$= \langle 0,-0.04968,0\rangle N$$

$$\vec{P}_f = \vec{P}_i + \vec{F}_{net}\Delta t$$

$$\Rightarrow \vec{V}_f = \vec{V}_i + \frac{\vec{F}_{net}\Delta t}{m}$$

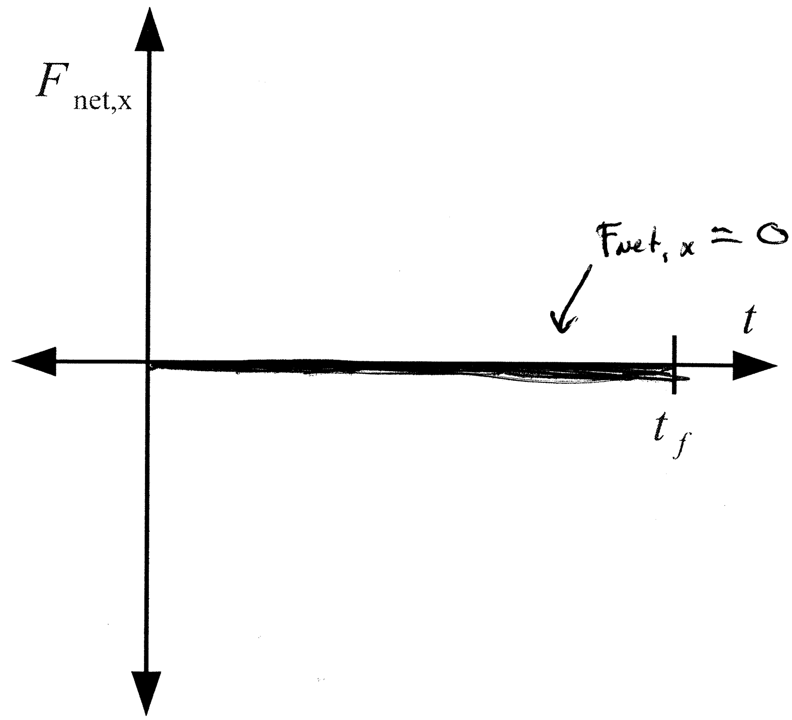
$$= \langle 0,-0.036,0\rangle m/s + \frac{\langle 0,-0.04968,0\rangle N \cdot 0.02 s}{0.03 kg}$$

$$\boxed{\vec{V}_f = \langle 0,-0.06912,0\rangle m/s} \quad @\ t = 0.04 s$$

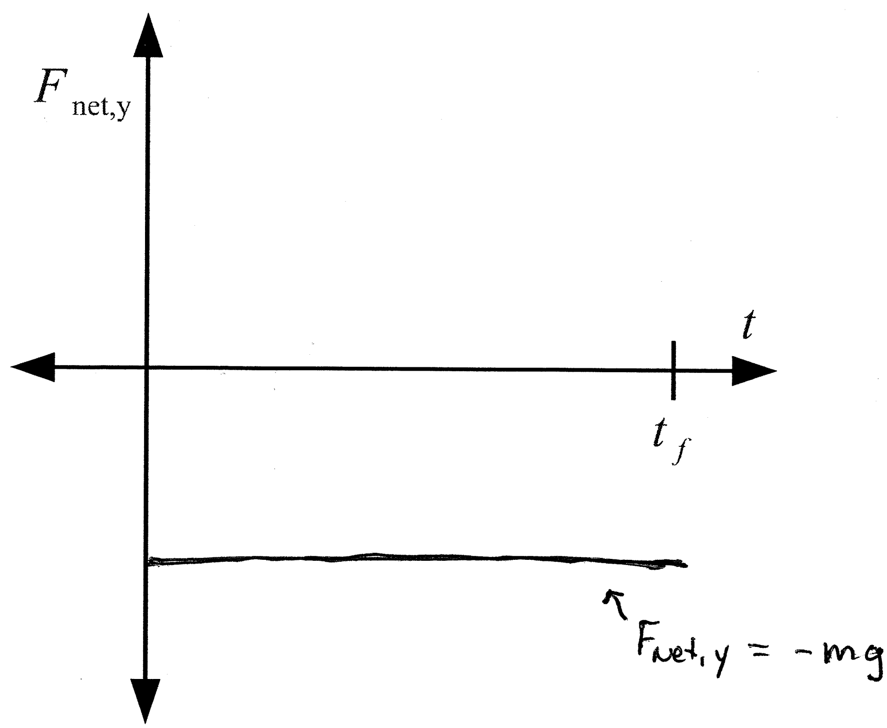
Problem 2 (30 Points)

Consider a ball of mass m that is kicked such that it has and an initial velocity of $< v_{x,i}, v_{y,i}, 0 >$ m/s, where $v_{x,i} > 0$ and $v_{y,i} > 0$ are both positive. The initial position of the ball is $< 0, 0, 0 >$ m and the only force acting on the ball is gravity (the weight). In the questions below, you will be asked to plot various components of the force, velocity and position versus time. When doing this, consider a time interval from the instant just after the ball is kicked ($t = 0$) until the instant before the ball reaches the ground ($t = t_f$).

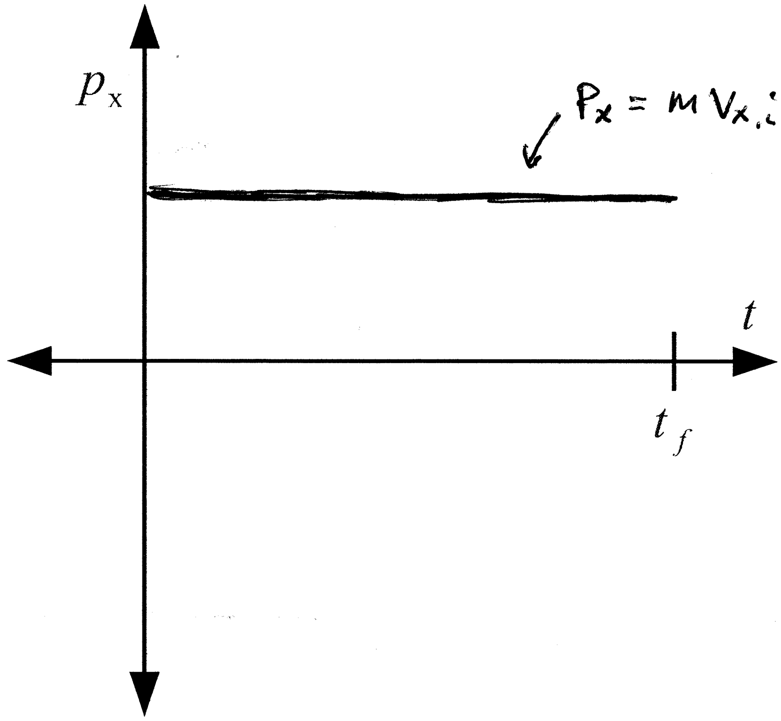
(a 5pts) Plot the x-component of the force $F_{net,x}$ versus time.



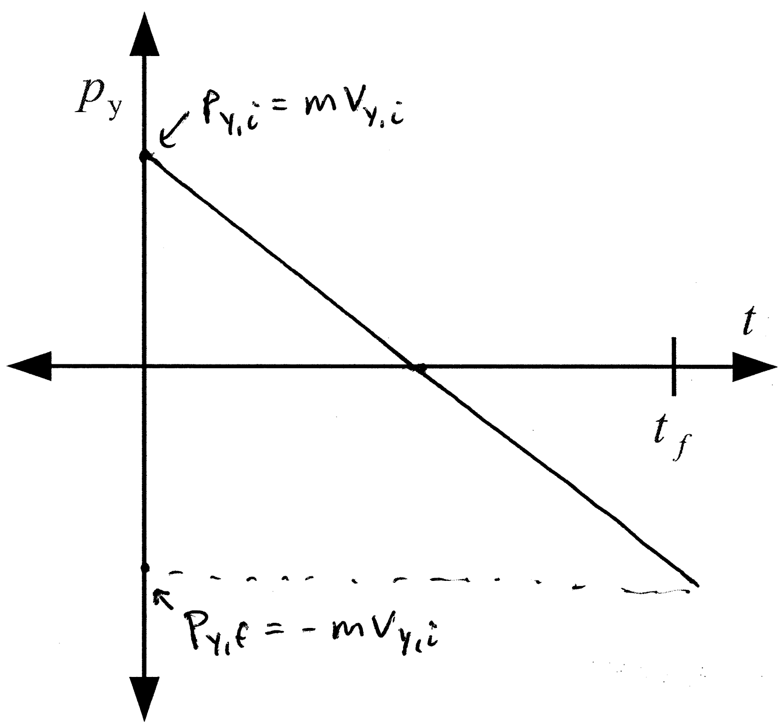
(b 5pts) Plot the y-component of the force $F_{net,y}$ versus time.



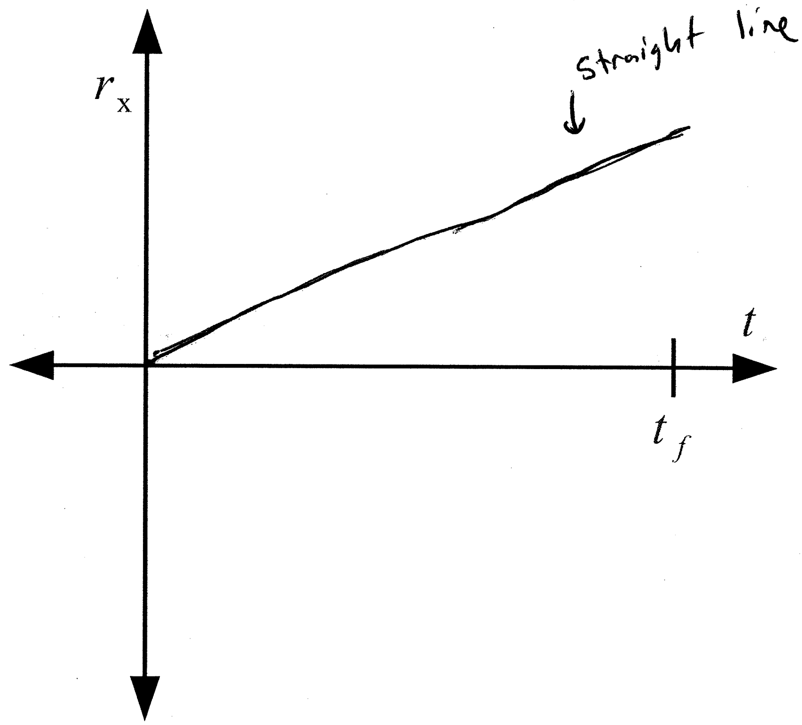
(c 5pts) Plot the x-component of the momentum p_x versus time.



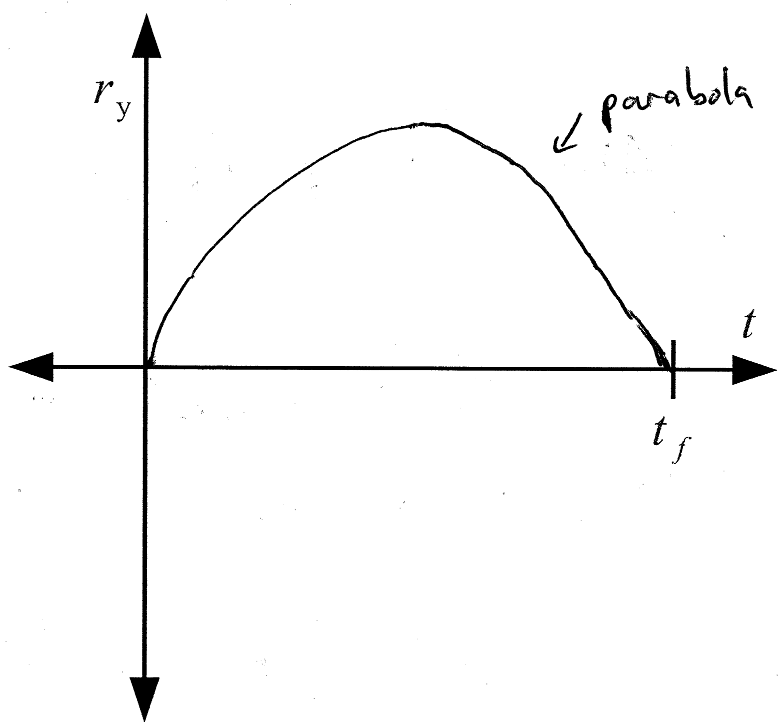
(d 5pts) Plot the y-component of the momentum p_y versus time.



(e 5pts) Plot the x-component of the position r_x versus time.

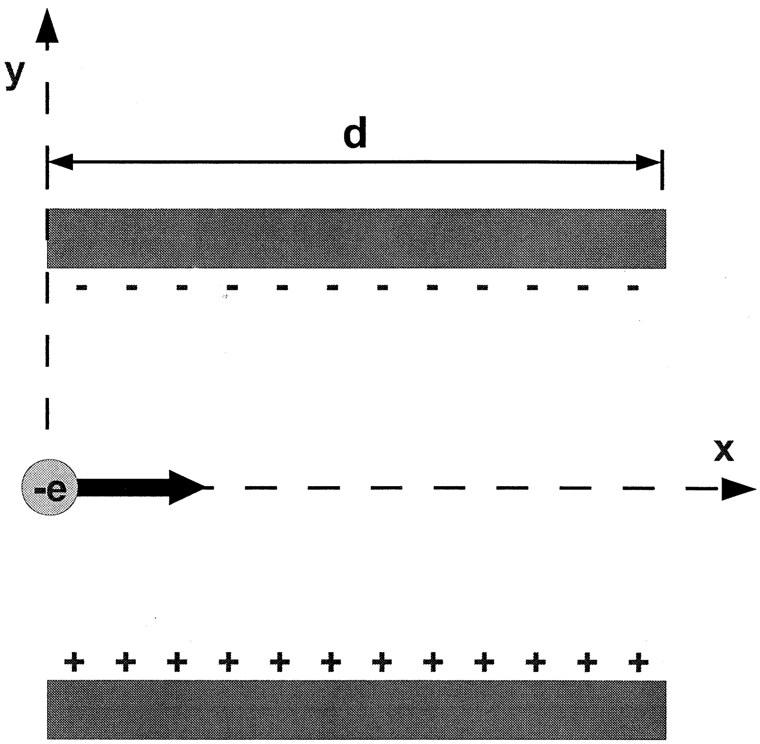


(f 5pts) Plot the y-component of the position r_y versus time.



Problem 3 (25 Points)

An electron of mass m_e enters a set of charged parallel plates with an initial velocity $\vec{v}_0 =< v_{ix}, 0, 0 >$. The electron experiences a constant force $\vec{F} =< 0, -F, 0 >$ in the negative y direction. The gravitational force on the electron is small compared to this force and can be neglected. The length of the plates is d. You may assume that the electron is moving, at all times, with a speed much less than c.



(a 5pts) How long does it take for the electron to reach the end of the parallel plates (i. e. at $x = d$)?

$\vec{F}_{net,x} = 0$ so $\Delta P_x = 0$ and $V_x = \text{constant} = V_{i,x}$

Therefore

$$X_f = X_i + V_x \Delta t$$

Let $x_i = 0$, then

$$\Delta t = \frac{d}{V_{i,x}}$$

(b 10pts) What is the electron's final velocity when it reaches the end of the parallel plates (i. e. at $x = d$)? Remember to express your answer as a vector.

In y: $\vec{F}_{net,y} = -F = \frac{\Delta P_y}{\Delta t} = \frac{P_{f,y} - \cancel{P_{i,y}}^{0}}{\Delta t} \rightarrow V_{f,y} = -\frac{F\Delta t}{m}$

In x: $F_{net,x} = 0 \rightarrow V_x = \text{constant} = V_{i,x}$

In z: $F_{net,z} = 0 \rightarrow V_z = \text{constant} = 0$

Therefore $\vec{V}_f = \langle V_{i,x}, -\frac{F\Delta t}{m}, 0 \rangle$

$$\boxed{\vec{V}_f = \langle V_{i,x}, \frac{-Fd}{mV_{i,x}}, 0 \rangle}$$

(c 10pts) What is the electron's final position relative to its starting position (i. e. at $r_0 = \langle 0, 0, 0 \rangle$) when it reaches the end of the parallel plates (i. e. at $x = d$)? Remember to express your answer as a vector.

$$\vec{V}_{AVG} = \frac{\vec{V}_i + \vec{V}_f}{2}$$

$$= \frac{\langle V_{i,x}, 0, 0 \rangle + \langle V_{i,x}, -\frac{Fd}{mV_{i,x}}, 0 \rangle}{2}$$

$$\vec{V}_{AVG} = \langle V_{i,x}, \frac{-Fd}{2mV_{i,x}}, 0 \rangle$$

$$\vec{r}_f = \vec{r}_i + \vec{V}_{AVG}\Delta t$$

$$= \langle 0, 0, 0 \rangle m + \langle V_{i,x}, \frac{-Fd}{2mV_{i,x}}, 0 \rangle d/V_{i,x}$$

$$\boxed{\vec{r}_f = \langle d, -\frac{Fd^2}{2mV_{i,x}^2}, 0 \rangle}$$

Problem 4 (20 Points)

Recall that, in last week's lab, you studied the motion of a fan cart, and you wrote a computer model (VPython script) to predict a fancart's motion. The script given below, which is nearly identical to your computer model from lab, is missing a few lines of code. In the space provided in the body of the script, add the statements necessary to complete the code.

```
#************BEGIN COMPUTER MODEL OF FANCART*******************
from __future__ import division
from visual import *
track = box(pos = vector(0, -.05, 0), size = (2.0, 0.05, .10), color = color.white)
cart = box(pos=vector(0.081,0,0), size=(.1,.04,.06), color=color.green)
mcart = .2395
vcart = vector(.375, .368, 0)
pcart = mcart*vcart
print 'cart momentum =', pcart
deltat = 0.01
t = 0
Fair = vector(-0.062, 0, 0)
while t < 5.01:
    rate(100)
```

(a 12pts) Add statements **here** to update the momentum and the position of the fancart.

pcart = pcart + Fair * deltat # $\vec{P}_f = \vec{P}_i + \vec{F}_{net} \Delta t$

cart.pos = cart.pos + (pcart/mcart)*deltat # $\vec{r}_f = \vec{r}_0 + \vec{V}_f \Delta t$

$\vec{V}_{AVG} \simeq \vec{V}_f$

since Δt is small

```
    t = t + deltat
print 'after the loop'
print 'final position is', cart.pos
#************ END COMPUTER MODEL OF FANCART*******************
```

This problem continues on the next page.

Refer to the code above to answer the following four questions:

(b 2pts) What is the initial position of the fancart? (Answer should be a vector with units.)

$\vec{r}_i = \langle 0.81, 0, 0 \rangle$ m

(c 2pts) What is the initial momentum of the fancart? (Answer should be a vector with units.)

$\vec{p}_i = m\vec{v}_i$

$= $ mcart * vcart

$= 0.2395\ kg\ \langle .375, .368, 0 \rangle m/s = \langle 0.090, 0.088, 0 \rangle\ kg\,m\,s^{-1}$

(d 2pts) What is the net force on the fancart? (Answer should be a vector with units.)

$\vec{F}_{Net} = \vec{F}_{Air} = \langle -0.062, 0, 0 \rangle$ N

(e 2pts) The animation from the computer model, as written above, shows motion of a fancart that is not typically observed for fancarts in lab experiments. (Hint: at the end of last week's fancart lab, you modified your computer model so that your animation showed the same untypical behavior.) Identify the source of this untypical behavior in the code and state briefly how you would change the model so that the animation of fancart motion would look like typical fancart lab observations.

The initial velocity has a non-zero y component. To replicate typical experiment set vcart = vector(0.375, 0, 0)