Math 1501 E, Fall 2013

Exam #2

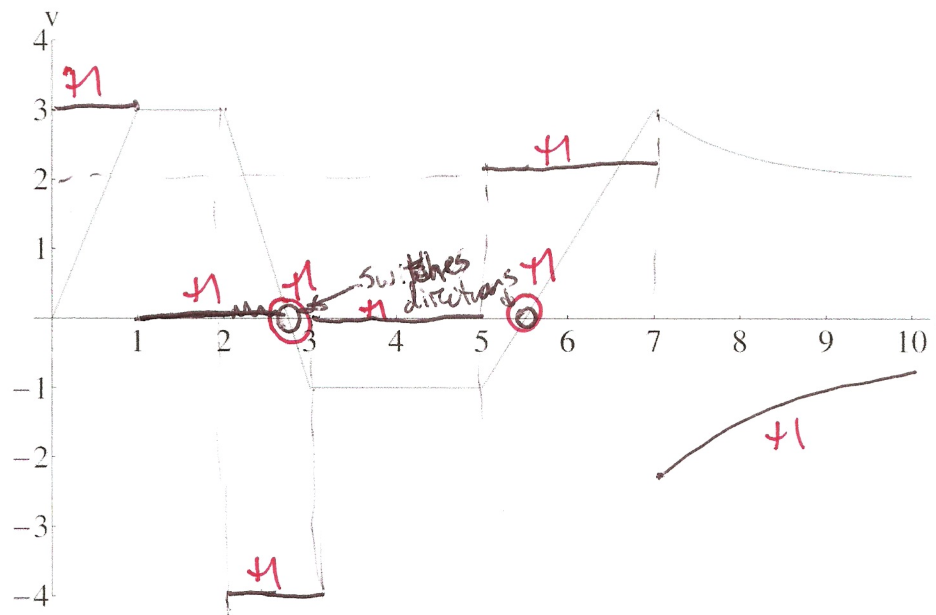
Name: Rubric

Section: ______________________

- You will have 50 minutes to complete the exam.
- No calculators, books, or notes allowed.
- Partial credit will be given. However, **no** credit will be given for a problem in which no work is shown, whether the answer is correct or not. Hence, show all applicable work.

Question:	1	2	3	4	5	Total
Points:	11	14	8	6	11	50
Score:						

1. A particle moves along the x axis with a velocity as a function of time, $v(t)$, given by the plot below:



(a) (3 points) Label the point(s) in time when the particle switches directions. Explain why you chose the point(s) you did.

At those two points, the velocity goes from positive (moving right) to negative (moving left) or vice-versa.

+1

(b) (6 points) Sketch a plot of the acceleration of the particle, $a(t)$, on the figure above. For $t < 7$, you must have the correct values for acceleration sketched, but for $t > 7$, you only need the general idea to be correct.

$a(t) = \frac{dv}{dt}$

(c) (2 points) Describe what is happening at times $t > 7$ in as much detail as you can.

+1 The particle is moving to the right, but is slowing down ($a < 0$) in a non-constant rate ($a(t)$ not constant here).

+1

2. Evaluate the following derivatives. **Be sure to show work.**

(a) (6 points) $\frac{d^2}{dx^2}\left[x^4 - 2xe^x\right]$. **Note that this is a second derivative.**

+1

$$\frac{d}{dx}\left[x^4 - 2xe^x\right] = 4x^3 - 2\frac{d}{dx}[xe^x] = 4x^3 - 2(1\cdot e^x + xe^x)$$

+1 +1

$$= 4x^3 - 2e^x(1+x)$$

+1 +1 +1

$$\frac{d}{dx}\left[4x^3 - 2e^x(1+x)\right] = 12x^2 - 2\frac{d}{dx}[e^x + xe^x] = 12x^2 - 2e^x - 2(1\cdot e^x + xe^x)$$

$$= \boxed{12x^2 - e^x[4+2x]}$$

(b) (5 points) $\frac{d}{dx}\left[\ln\left(x + \cos^2 x\right)\right]$.

+1

let $f(u) = \ln(u)$, $u = x + \cos^2 x \Rightarrow \frac{df}{du} = \frac{1}{u} = \frac{1}{x+\cos^2 x}$

$\frac{du}{dx} = 1 + \underbrace{\frac{d}{dx}[\cos^2 x]}_{-2\cos x \sin x}$ +1 let $h(z) = z^2$, $z = \cos x \Rightarrow$

$\frac{dh}{dz} = 2z = 2\cos x$, $\frac{dz}{dx} = -\sin x \Rightarrow$

+1 +1

+1 $\boxed{\frac{1 - 2\cos x \sin x}{x + \cos^2 x}}$

(c) (3 points) $\frac{d}{dx}\left[\frac{\tan x}{x^2+2}\right]$.

+1

+1

$$\frac{d}{dx}\left[\frac{\tan x}{x^2+2}\right] = \frac{\sec^2 x\,(x^2+2) - \tan x\,(2x)}{(x^2+2)^2}$$

+1

$$= \frac{\sec^2 x}{x^2+2} - \frac{2x\tan x}{(x^2+2)^2}$$

3. (8 points) Find the equations of the tangent and normal lines to the curve

$$y^2(4-x) = x^3 + y^4$$

at the point $(0, 2)$.

Find y': $\frac{d}{dx}[y^2(4-x)] = \frac{d}{dx}[x^3+y^4]$

+1 +1 +1 +1

$2yy'(4-x) + y^2(-1) = 3x^2 + 4y^3y'$

+1

$y'[2y(4-x) - 4y^3] = 3x^2+y^2 \Rightarrow y' = \dfrac{3x^2+y^2}{2y(4-x)-4y^3}$

+1

at $(0,2)$, $y' = m = \dfrac{2^2}{2\cdot 2\cdot 4 - 4\cdot 2^3} = \dfrac{4}{16-32} = -\dfrac{4}{16} = -\dfrac{1}{4}$, so

+1

tangent is $(y-2) = -\frac{1}{4}(x)$ normal has slope $-\frac{1}{m} = 4 \Rightarrow (y-2) = 4x$

$\boxed{y = -\frac{1}{4}x + 2}$ +1 $\boxed{y = 4x+2}$

4. (6 points) Consider the function $f(x) = e^x/x^2$ for $x \geq 2$. Find the value of $\frac{d}{dx}f^{-1}(x)$ at the point at $x = e^4/16$. **Hint:** Note that $f(4) = e^4/16$.

+1 +1 +1 +1

$\frac{d}{dx} f^{-1}(x) = \dfrac{1}{f'(f^{-1}(x))}$; $f(x) = \dfrac{e^x}{x^2} \Rightarrow f'(x) = \dfrac{e^x x^2 - e^x(2x)}{x^4}$

+1

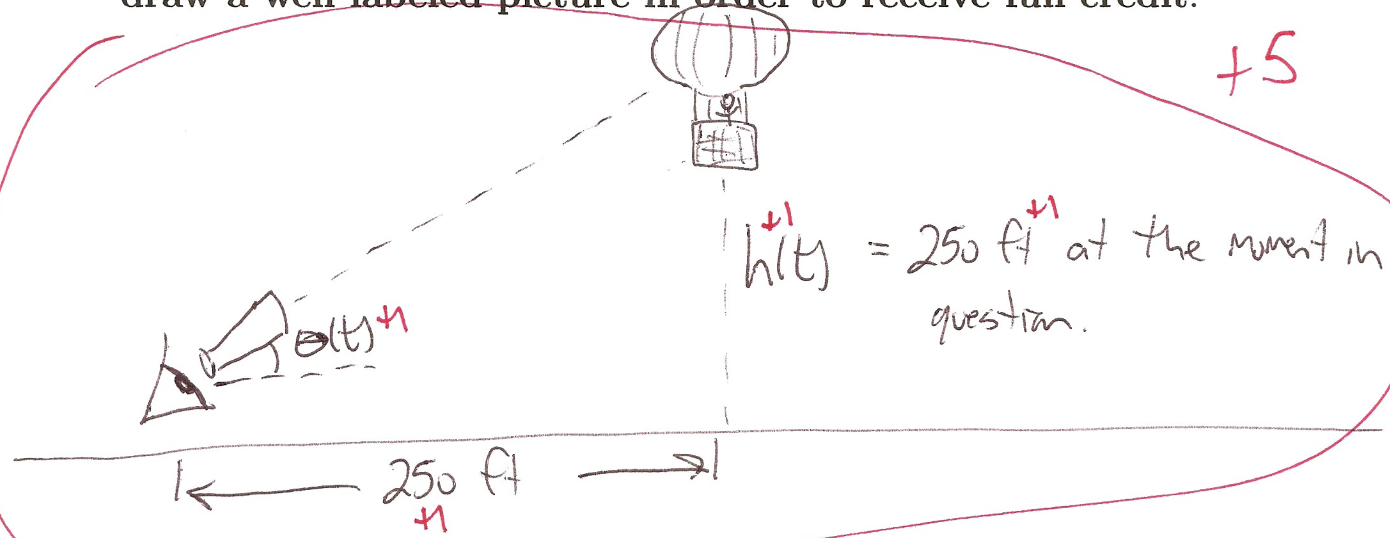
$= \dfrac{e^x}{x^3}(x-2)$; $f^{-1}\left(\frac{e^4}{16}\right) = 4$, since $f(4) = \frac{e^4}{16} \Rightarrow$

+1

$\frac{d}{dx} f^{-1}(x)\Big|_{x=\frac{e^4}{16}} = \dfrac{1}{f'(4)} = \dfrac{1}{\frac{e^4}{4^3}(4-2)} = \boxed{\dfrac{32}{e^4}}$

5. (11 points) A hot air balloon rising straight up from a level field is tracked by a range finder (a telescope with an angle indicator) located 250 ft from the balloon's liftoff point. At the moment the balloon's height above the field is 250 ft, the range finder's angle is increasing at a rate of 0.2 rad/min. How fast is the balloon rising at that moment? **Be sure to draw a well labeled picture in order to receive full credit.**

+5



+1 $\tan\theta(t) = \dfrac{h(t)}{250} \Rightarrow \dfrac{d}{dt}\left[\tan\theta(t)\right] = \dfrac{d}{dt}\left[\dfrac{h(t)}{250}\right]$

+1 $\sec^2\theta(t)\cdot\dfrac{d\theta}{dt} = \dfrac{dh}{dt}/250$ +1 $\Rightarrow \dfrac{dh}{dt} = 250\,\text{ft}\sec^2\theta(t)\,\dfrac{d\theta}{dt}$ +1

when $h(t) = 250$, $\theta = \frac{\pi}{4}$, since $\tan\frac{\pi}{4} = 1 = \frac{250}{250}$; +1

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \Rightarrow \sec^2\frac{\pi}{4} = 2$

$\dfrac{dh}{dt} = 250\,\text{ft}\cdot 2\cdot\dfrac{0.2\,\text{rad}}{\text{min}} = \boxed{100\ \dfrac{\text{ft}}{\text{min}}}$ +1