This quiz is worth a total of 100 points, and the value of each question is listed with each question.
You must show your work; answers without substantiation do not count.

Answers must appear in the box provided! No cheat!

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1. (a) (20pts) Define the continuity of a function $f(x)$ at an interior point $x = c$.
(b) (15pts) Explain why the function

$$f(x) = \frac{x^2 + x - 6}{x^2 - 4}, \ x \in [0,4]\backslash\{2\}$$

is not continuous at $x = 2$.
(c) (15pts) It is possible to extend the function's domain to include the point $x = 2$ in such a way that the function is *continuous at $x = 2$*. Find the value of $f(2)$ so that $f(x)$ is continuous on $[0, 4]$.

Answer: (a) $f(c)$ exists, $\lim\limits_{x \to c} f(x)$ exists, and $\lim\limits_{x \to c} f(x) = f(c)$.
(b) $f(2)$ does not exist.
(c) $\lim\limits_{x \to 2} \frac{x^2+x-6}{x^2-4} = \lim\limits_{x \to 2} \frac{(x+3)(x-2)}{(x+2)(x-2)} = \lim\limits_{x \to 2} \frac{(x+3)}{(x+2)} = \frac{5}{4}$.
Therefore, if we define $f(2) = \frac{5}{4}$, $f(x)$ is continuous on $[0, 4]$.
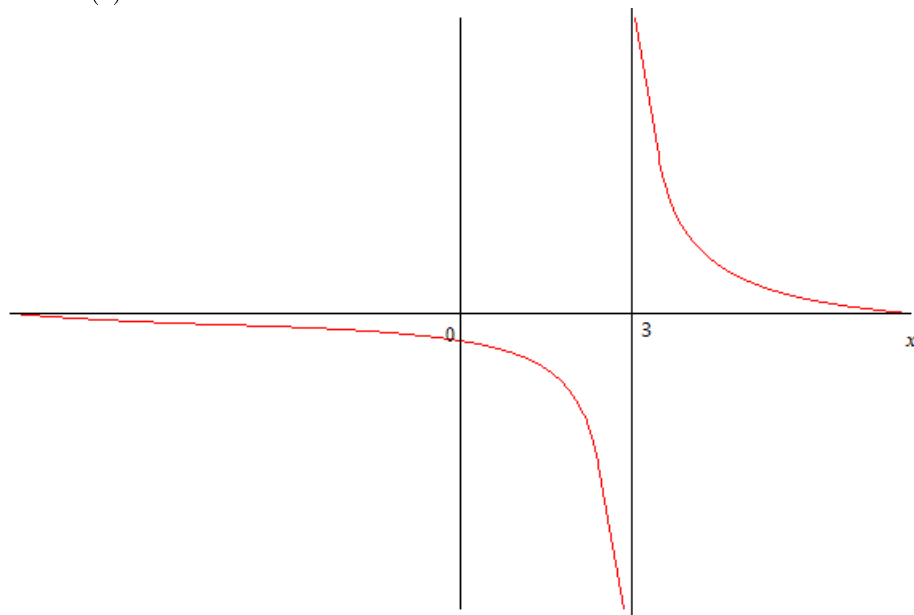
2. (25 pts) Find the limit

$$\lim_{x \to \infty} \frac{-3x^3 + 4}{\sqrt{x^6 + 9}}.$$

Answer: $\lim\limits_{x \to \infty} \frac{-3x^3+4}{\sqrt{x^6+9}} = \lim\limits_{x \to \infty} \frac{-3+4/x^3}{\sqrt{1+9/x^6}} = \lim\limits_{x \to \infty} \frac{-3+0}{\sqrt{1+0}} = -3$

3. (a) (10 pts) Draw the graph of $f(x) = \frac{2}{x-3}$, $x \in (-\infty, \infty)$.
(b) (15 pts) Find $\lim\limits_{x \to 3} \frac{2}{x-3}$.

Answer: (a)

3 should be accurately pointed!
(b) $\lim\limits_{x \to 3^+} \frac{2}{x-3} = \infty$, and $\lim\limits_{x \to 3^-} \frac{2}{x-3} = -\infty$.
Therefore, the limit does not exist since $\lim\limits_{x \to 3^+} \frac{2}{x-3} \neq \lim\limits_{x \to 3^-} \frac{2}{x-3}$.