Homework 7
SOLUTIONS

1. State the dual of the max problem:

$$\begin{aligned}
\max \quad z = & 10x_1 + 6x_2 + 7x_3 \\
\text{subject to} & \\
& 5x_1 + 2x_2 + 3x_3 && \leq 100 \\
& 3x_1 + x_2 + x_3 && = 45 \\
& 2x_1 - 3x_3 && \geq 2 \\
& x_1 && \geq 0 \\
& x_2 && \geq 0 \\
& x_3 && \text{u.r.s}
\end{aligned}$$

Solution:

$$\begin{aligned}
\min \quad w = & 100\pi_1 + 45\pi_2 + 2\pi_3 \\
\text{subject to} & \\
& 5\pi_1 + 3\pi_2 + 2\pi_3 && \geq 10 \\
& 2\pi_1 + 1\pi_2 + 0\pi_3 && \geq 6 \\
& 3\pi_1 + 1\pi_2 - 3\pi_3 && = 7 \\
& \pi_1 && \geq 0 \\
& \pi_2 && \text{u.r.s} \\
& \pi_3 && \leq 0
\end{aligned}$$

2. Solve the following LP using Dual Simplex.

$$\begin{aligned}
\min \quad z = & 8x_1 + 6x_2 + 15x_3 \\
\text{subject to} & \\
& x_1 + 2x_2 && \geq 5 \\
& x_1 + x_2 + x_3 && \geq 6 \\
& 2x_1 + x_3 && \geq 4 \\
& x_i && \geq 0 \qquad \forall i = 1, 2, 3
\end{aligned}$$

8	6	15	0	0	0	0
-1	-2	0	1	0	0	-5
-1	$[-1]$	-1	0	1	0	-6
-2	0	-1	0	0	1	-4

2	0	9	0	6	0	-36
1	0	2	1	-2	0	7
1	1	1	0	1	0	6
$[-2]$	0	-1	0	0	1	-4

0	0	8	0	6	1	-40
0	0	3/2	1	-2	1/2	5
0	1	1/2	0	1	1/2	4
1	0	1/2	0	0	-1/2	2

3. Suppose I have a maximization problem with 3 variables and 2 constraints. Let $x = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ be a feasible solution to the problem after adding the slack variables. Let $\pi = \begin{bmatrix} 1 & 4 & 0 & 2 & 0 \end{bmatrix}$ be a feasible solution to the dual problem after adding the slack variables. What can you say about the relation between z and w? (where z is the objective value obtained at x, and w the objective value of the dual at π).

   Solution: the weak duality theorem tells us that $z \leq w$ for any x, π. Complementary slackness tells us that if $x_i e_i = 0$ and $\pi_j s_j = 0$. We are told that the maximization problem has 3 variables and 2 constraints. This means the x we are given corresponds to $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ s_1 \\ s_2 \end{bmatrix}$ and the π is $\pi = \begin{bmatrix} \pi_1 & \pi_2 & e_1 & e_2 & e_3 \end{bmatrix}$. From this, we find that $x_2 e_2 = 6 \neq 0$. We can therefore rule out $z = w$, which gives us $z < w$.

4. Suppose I have a maximization problem with 3 variables and 2 con-

straints. Let $x = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 0 \\ 0 \end{bmatrix}$ be a feasible solution to the problem after adding the slack variables. Let $\pi = \begin{bmatrix} 3 & 4 & 1 & 0 & 0 \end{bmatrix}$ be a feasible solution to the dual problem after adding the slack variables. What can you say about the relation between z and w? (where z is the objective value obtained at x, and w the objective value of the dual at π).

Solution: Using the same logic as the previous question, we find that the conditions for complementary slackness hold. Therefore $z = w$.