Name Key

Exam 2 ISyE 4301

Please read the following: This is a closed-note exam. In addition, only calculators that *do not* have the capability to send or receive data may be used (e.g., phones are not allowed). By signing the following, you are agreeing to these terms and acknowledging that all of the work on this exam is your own.

____________________________________(*Signature*)

The following true-false/multiple-choice questions are worth 5 points each. Clearly mark your answer.

1. For the two person game below, (A1,A1) is Pareto improving over any Nash equilibrium.

	A1	A2
A1	(5,4)	(-3,6)
A2	(8,2)	(4,4)

NE is A2, A2 so A1, A1 is Pareto Improving

   a. True
   b. False

2. Consider a buy-back contract for supply chain coordination for an item that satisfies newsvendor assumptions. The buy-back price that will coordinate the supply chain does not depend on the demand distribution.
   a. True
   b. False

set $\frac{P-C}{P-B} = CR$ (centralized) so False

3. Incentive compatibility constraints ensure that the agent selects the desired action
   a. True
   b. False

4. An individual is given a standard gamble where option 1 is \$130 and option 2 is \$150 with probability p and \$100 with probability 1-p. If the value of p for which the individual is indifferent between the two options is 0.63, which is the best answer?
   a. The individual is risk averse
   b. The individual is risk neutral
   c. The risk premium is \$3
   d. Both a. and c. are true
   e. There is not enough information to determine a. or b.
   f. None of the above

set $u(150)=1$
$u(100)=0$
so
$u(130)=.63$

risk neutral $\rightarrow .6$

so risk averse

5. Player A is given $100. They offer player B $X. If B accepts the offer, then B gets $X and A gets $100-$X. If B rejects the offer, they both get $0. Which is the best answer?
   a. The Nash Equilibrium is X=$0.01
   b. The Nash Equilibrium is X=$50
   c. X=$49 is Pareto improving over X=$50
   d. Both a. and c. are true
   e. Both b. and c. are true
   f. There is no Nash equilibrium

6. Currently a group of 4 stores all use (Q,R) to manage inventory with Q defined by EOQ. Each face independent annual demand that is normally distributed with the same mean and variance? If the inventory for the 4 stores is pooled, which is the best answer?
   a. Total safety stock is reduced in half
   b. Total pipeline stock is reduced in half
   c. Demand correlation will increase the amount safety stock
   d. a. and b. are true
   e. None of the above

$SS_{original} = 4\,(Z_{1-\alpha}\,\sigma_L)$

$SS_{pooled} = \sqrt{4}\,(Z_{1-\alpha}\,\sigma_L) = \tfrac{1}{2}\,SS_{original}$

Pipeline = DL so not a function of σ_L

7. (*10 points*) Evets Inc. orders product each month from a single (unique) supplier. Roughly 80% of the time the supplier delivers exactly what is needed, and 20% of the time it delivers nothing. When the supplier makes a delivery, Evets earns $100,000 for the month; otherwise it earns $0 for the month. Answer the following:

   a. (*2 points*) Determine the expected monthly earnings for Evets Inc.
   b. (*4 points*) Suppose Evets Inc. is risk averse (utility equals square root of earnings). Further, suppose that the supplier could guarantee that Evets Inc. will make its expected earnings each month. How much would Evets Inc. be willing to pay for the guarantee?
   c. (*4 points*) Suppose that we don't know the form of Evets Inc. risk aversion. However, we know that they are indifferent between $80,000 with certainty and $100,000 with probability 0.91 and $0 with probability of 0.09. What would your answer to b be in this case?

a) $E[\,] = .8\,(100000) + .2\,(0) = 80,000$

b) $u = \sqrt{80,000} = 282.8$ (sure thing)

compare to $.8u(100000) + .2u(0)$

$= .8\sqrt{100000} + .2\sqrt{0} = 252.9$

difference $= 30 \longrightarrow 30^2 = \$900$

c) $u(80000) = .91\,u(100000) + .09\,u(0)$

$= .91$

compare to risk neutral of 0.8

difference $= .11$ utils

$\longrightarrow \$11,000$

8. (*15 points*) Haxiom Inc. produces 3 products in California that are shipped to a DC in Malaysia: P1, P2, and P3. Annual demand for each is normally distributed with mean and standard deviation $(\mu_i; \sigma_i)$ of (10,000; 2,000), (15,000; 4000), and (9,000;4000) respectively. Leadtime from California to Malaysia is 13 weeks. Demand for P1 is independent of P2 or P3, but demand between P2 and P3 is correlated with $\rho_{23} = -0.6$. Each product has a value of \$400. Answer the following:

   a. (*6 points*) Currently, items are ordered independently from Malalysia. The order cost is \$1000 and percentage used for holding cost is 20%. They use a (*Q*,*R*) model for each item, where *Q* is determine by EOQ. The service level is 97%. Determine the total average cycle, pipeline, and cycle stock.

   b. (*9 points*) Haxiom has come up with a way to postpone at the DC. It adds three days to the leadtime at the DC. Determine the average cycle, pipeline, and cycle stock in this case and comment.

a) $\text{cycle} = \frac{1}{2}\sum_{i=1}^{3}\sqrt{\frac{2(1000)D_i}{.2(400)}} = .5\left[\sqrt{\frac{2(1000)(10000)}{80}} + \sqrt{\frac{2(1000)(15000)}{80}} + \sqrt{\frac{2(1000)(9000)}{80}}\right]$

$= 1586.8/2$

$\text{pipeline} = \sum_i D_i L = \frac{13}{52}(10000 + 15000 + 9000) = 8500$

$\text{safety} = Z_{.97}\sqrt{13/52}\sum \sigma_{L_i} = 1.88\sqrt{\frac{13}{52}}(2000+4000+4000) = 9400$

b) $\text{cycle} = \sqrt{\frac{2(1000)(10000+15000+9000)}{.2(400)}} = 921.9$

$\text{pipeline} = 8500$ (assume 3 days is at DC)
okay if you added it in here

$\sigma_p = \left[2000^2 + 4000^2 + 4000^2 - 2(.6)(4000)(4000)\right]^{1/2} = 4098.8$

$\text{safety} = 1.88\sqrt{\frac{13}{52}}(4098.8) + 1.88\sqrt{\frac{3}{352}}(2000+4000+4000)$

$= 5557$

$P = \$250$ to market

9. (*15 points*) For the following problem, assume that newsvendor assumptions hold. A supplier purchases product as \$100 per unit and sells to a store for \$160 per unit. Distribution costs from the supplier to the store is \$5 per unit. The store sells to a market with uniform demand between 2000 and 3000 units. It costs them \$2 in material handling costs for each item they sell. Any items at the end of the selling period are salvaged for \$50.
   a. (*5 points*) Determine the buy-back price that will coordinate the supply chain.
   b. (*10 points*) Determine the profit for the supplier using the buy-back price found in a. (Note: solve without any consideration for the shared profit from the manufacturer that would be needed to coordinate)

a)

If centralized, $CR = \dfrac{250 - (100 + 5 + 2)}{250 - 50} = .715$

$Q_c = 2000 + .715(1000) = 2715$

so set $\dfrac{250 - (160 + 2)}{250 - B} = .715 \rightarrow B = \$126.9$

b) $E[\text{profit}] = (160 - (100 + 5))(2715)$

$$- \int_{2000}^{2715} (127 - 50)(2715 - D)\left(\frac{1}{1000}\right) dD$$

$$= 149325 - .077\left[2715D - .5D^2\right]\Big|_{2000}^{2715}$$

$= \$129,643$

(so obviously the distributor would need to share profit to make this work)

10. (15 points) Tesla and Ford each have plants that produce batteries for their electric automobiles. Each plant purchases a rare earth element from a mine in Bolivia. Because it is a scare resource, the unit cost (i.e. cost of a quantity needed for a battery) is function of the quantity demanded. In particular, the profits for each company are:

Telsa:	$\pi_T = (300 - Q)q_T - 10Q$	$Q = q_T + q_F$
Ford:	$\pi_F = (300 - Q)q_F - 11Q$	

a. (*10 points*) Consider the case where Ford chooses the quantity first, this is observed by Telsa and then they choose their quantity. What would the Nash equilibrium be?

b. (*5 points*) Explain *in words* why Ford would prefer to choose first as opposed to second (i.e., how does Ford take advantage of moving first).

a) solve for Telsa first

$$\pi_T = (300 - q_T - q_F)q_T - 10(q_T + q_F)$$

$$\pi'_T = 300 - 2q_T - q_F - 10 \equiv 0$$

$$\rightarrow q_T = \frac{290 - q_F}{2}$$

so

$$\pi_F = \left(300 - \left(\frac{290 - q_F}{2}\right) - q_F\right)q_F - 11\left(\frac{290 - q_F}{2} + q_F\right)$$

$$\pi'_F = 155 - q_F - \frac{11}{2} \equiv 0$$

$$\rightarrow q_F = 149.5$$

$$q_T = \frac{290 - 149.5}{2} = 70.25$$

b) By moving first, Ford can order a higher amount and force Tesla to order less.

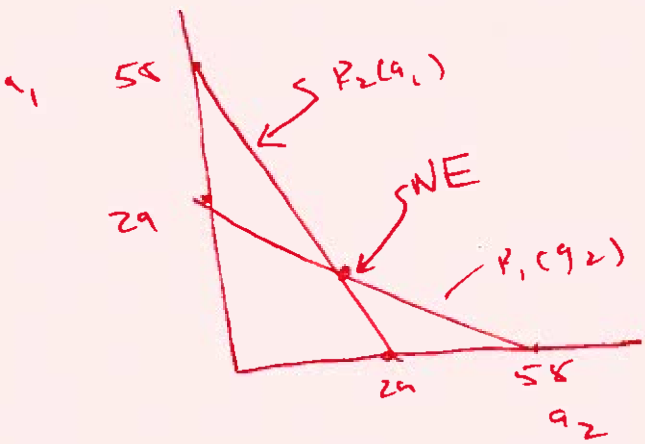
11. (15 points) Two firms compete in a Cournot game. The market demand is P=300-5Q, where q_1+q_2=Q. Unit cost for each firm is \$10. Answer the following:

   a. Sketch the best response curves (please label axes, and try to draw to scale as best possible).
   b. Solve for the Nash equilibrium.
   c. If they colluded and acted as a Monopoly, would this be Pareto improving? Show why or why not.

a) $\pi_1 = (300 - 5(q_1 + q_2))q_1 - 10q_1$

$\pi_1' = 300 - 10q_1 - 5q_2 - 10 = 0$

$R_1(q_2) = q_1 = \dfrac{290 - 5q_2}{10} \Rightarrow R_2(q_1) = \dfrac{290 - 5q_1}{10} = q_2$



b) $q_1 = \dfrac{290 - 5\left(\dfrac{290 - 5q_1}{10}\right)}{10}$

$\rightarrow q_1 = 14.5 + .25q_1$

$q_1 = 19.3$

symmetric so $q_2 = 19.3$

$\pi_1 = \pi_2 = (300 - 5(2(19.3)))19.3 - 10(2(19.3)) = 1869$

c) If Monopoly $\pi = (300 - 5Q)Q - 10Q$

$\pi' = 300 - 10Q - 10 = 0 \rightarrow Q = 29$

$\rightarrow \pi = (300 - 5(29))29 - 10(29) = 4205$ which they would split)

So Pareto improving

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_________________________________(*Signature*)

The following true-false/multiple-choice questions are worth 5 points each. Clearly mark your answer.

1. For the two person game below, (A1,A1) is Pareto improving over any Nash equilibrium.

	A1	A2
A1	(7,4)	(-3,6)
A2	(6,2)	(5,5)

NE is A2, A2

A1, A1 is not pareto improving

   a. True
   b. False

2. Participation constraints ensure that the agent selects the desired action from the principal
   a. True
   b. False

   ensures takes contract

3. Consider a buy-back contract for supply chain coordination for an item that satisfies newsvendor assumptions. The buy-back price that will coordinate the supply chain does not depend on the demand distribution.
   a. True
   b. False

   $CZ = \frac{p-c}{p-s}$ doesn't depend on D.

4. An individual is given a standard gamble where option 1 is \$130 and option 2 is \$150 with probability p and \$100 with probability 1-p. If the value of p for which the individual is indifferent between the two options is 0.63, which is the best answer?
   a. The individual is risk averse
   b. The individual is risk neutral
   c. The risk premium is \$3
   d. Both a. and c. are true
   e. There is not enough information to determine a. or b.
   f. None of the above

$u(150) = 1$

$u(100) = 0$

$u(130) = .63(1) = .63$

if risk neutral $= .6$

5. Player A is given \$100. They offer player B \$*X*. If B accepts the offer, then B gets \$*X* and A gets \$100-\$*X*. If B rejects the offer, they both get \$0. Which is the best answer?
    a. The Nash Equilibrium is X=\$99.99
    b. The Nash Equilibrium is X=\$50
    c. X=\$49 is Pareto improving over X=\$50
    d. Both a. and c. are true
    e. Both b. and c. are true
    f. None of the above

NE is \$0.01

6. Currently a group of 9 stores all use (Q,R) to manage inventory with Q defined by EOQ. Each face independent annual demand that is normally distributed with the same mean and variance? If the inventory for the 9 stores is pooled, which is the best answer?
    a. Total safety stock is reduced to one third as much
    b. Total cycle stock is reduced to one third as much
    c. Demand correlation will increase the amount safety stock
    d. Both a. and b. are true
    e. None of the above

original $\rightarrow$ $SS = Z_{1-\alpha}(9\sigma_L)$

$Q = \frac{1}{2} 9 \sqrt{\frac{2AD}{h}}$

new $\rightarrow$ $SS = Z_{1-\alpha}\sqrt{9}\,\sigma_L$

$Q = \frac{1}{2}\sqrt{\frac{2A(9D)}{h}} = \frac{1}{2}\sqrt{9}\sqrt{\frac{2AD}{h}}$

7. (15 points) Tesla and Ford each have plants that produce batteries for their electric automobiles. Each plant purchases a rare earth element from a mine in Bolivia. Because it is a scare resource, the unit cost (i.e. cost of a quantity needed for a battery) is function of the quantity demanded. In particular, the profits for each company are:

Telsa:	$\pi_T = (250 - Q)q_T - 10Q$	$Q = q_T + q_F$
Ford:	$\pi_F = (250 - Q)q_F - 9Q$	

a. (*10 points*) Consider the case where Telsa chooses the quantity first, this is observed by Ford and then they choose their quantity. What would the Nash equilibrium be?

b. (*5 points*) Explain *in words* why Tesla would prefer to choose first as opposed to second (i.e., how does Tesla take advantage of moving first).

a) solve for Ford first

$$\pi_F = (250 - q_T - q_F)q_F - 9(q_T + q_F)$$

$$\pi'_F = 250 - q_T - 2q_F - 9 \equiv 0 \rightarrow q_F = \frac{241 - q_T}{2}$$

so

$$\pi_T = \left(250 - q_T - \frac{241 - q_T}{2}\right)q_T - 10\left(q_T + \frac{241 - q_T}{2}\right)$$

$$\pi'_T = 250 - 2q_T - 120.5 + q_T - 10 + 5 \equiv 0$$

$$\rightarrow q_T = 124.5 \qquad q_F = \frac{241 - 124.5}{2} = 58.3$$

b) By choosing first Tesla can choose a larger quantity, which forces Ford to choose a smaller one.

$p = \$200$ to market

8. (*15 points*) For the following problem, assume that newsvendor assumptions hold. A supplier purchases product as \$80 per unit and sells to a store for \$150 per unit. Distribution costs from the supplier to the store is \$10 per unit. The store sells to a market with uniform demand between 2000 and 3000 units. It costs them \$5 in material handling costs for each item they sell. Any items at the end of the selling period are salvaged for \$30.
    a. (*5 points*) Determine the buy-back price that will coordinate the supply chain.
    b. (*10 points*) Determine the profit for the supplier using the buy-back price found in a. (Note: solve without any consideration for the shared profit from the manufacturer that would be needed to coordinate)

a) If centralized $CR = \dfrac{200 - (80 + 10 + 5)}{200 - 30} = .62$

so set $\dfrac{200 - (150 + 5)}{200 - B} = .62 \longrightarrow B = \$127.4$

$Q = 2000 + .62(1000) = 2620$

b)

$$E[\text{profit}] = (150 - 90)(2620) - \int_{2000}^{2620} (127.4 - 30)(2620 - D)\frac{1}{1000}\,dD$$

$$= 157200 - .0974\left(2620D - .5D^2\right)\Big|_{2000}^{2620}$$

$$= \$132,843$$

and so distributor will need to share profit to make work.

9. (15 points) Two firms compete in a Cournot game. The market demand is $P=200-4Q$, where $q_1+q_2=Q$. Unit cost for each firm is \$10. Answer the following:
   a. Solve for the Nash equilibrium.
   b. If they colluded and acted as a Monopoly, would this be Pareto improving? Show why or why not.
   c. Sketch the best response curves (please label axes, and try to draw to scale as best possible).

a) $\pi_1 = (200 - 4q_1 - 4q_2)q_1 - 10q_1$

$\pi_1' = 200 - 8q_1 - 4q_2 - 10 \equiv 0 \to q_1 = \frac{190-4q_2}{8}$

by symmetry $q_2 = \frac{190-4q_1}{8}$

$= \frac{190 - 4\left(\frac{190-4q_2}{8}\right)}{8}$

$\to q_2 = 15.83 \quad q_1 = 15.83$

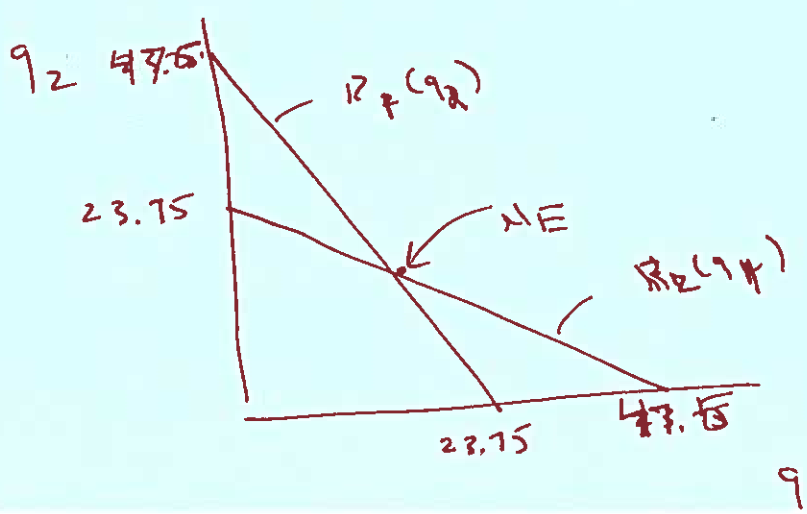
b) $\pi = (200-4Q)Q - 10Q$

$\pi' = 200 - 8Q - 10 \to Q = 23.8$

$\pi = 2256$, which they would split

→ Pareto improving

c) $R_1(q_2) = \frac{190-4q_2}{8} \quad R_2(q_1) = \frac{190-4q_1}{8}$



10. (*15 points*) Haxiom Inc. produces 3 products in California that are shipped to a DC in Malaysia: P1, P2, and P3. Annual demand for each is normally distributed with mean and standard deviation $(\mu_i; \sigma_i)$ of (10,000; 3,000), (15,000; 5000), and (9,000;4000) respectively. Leadtime from California to Malaysia is 13 weeks. Demand for P1 is independent of P2 or P3, but demand between P2 and P3 is correlated with $\rho_{23} = 0.4$. Each product has a value of \$400. Answer the following:

   a. (*6 points*) Currently, items are ordered independently from Malalysia. The order cost is \$800 and percentage used for holding cost is 20%. They use a (Q,R) model for each item, where Q is based on EOQ. The desired service level is 94%. Determine the total average cycle, pipeline, and cycle stock.

   b. (*9 points*) Haxiom has come up with a way to postpone at the DC. It adds four days to the leadtime at the DC. Determine the average cycle, pipeline, and cycle stock in this case and comment.

a) $\text{cycle} = \frac{1}{2}\sum_{i=1}^{3}\sqrt{\frac{2(800)(D_i)}{.2(400)}} = \frac{1}{2}\left[\sqrt{\frac{2(800)(10000)}{80}} + \sqrt{\frac{2(800)(15000)}{80}} + \sqrt{\frac{2(800)(9000)}{80}}\right]$

$= 709.6$

$\text{pipeline} = \sum_{i=1}^{3} D_i L = \frac{13}{52}(10000 + 15000 + 9000) = 8500$

$\text{safety} = Z_{1-\alpha}\sqrt{\frac{13}{52}}\sum_i \sigma_i = 1.56\sqrt{\frac{13}{52}}(3000 + 5000 + 4000) = 9360$

b) $\sigma_p = (3000^2 + 5000^2 + 4000^2 + 2(.4)(5000)(4000)) = 8124$

$\text{cycle} = \frac{1}{2}\sqrt{\frac{2(800)(10000 + 15000 + 9000)}{80}} = 412.3$

pipeline (doesn't change if we assume 4 days are in the DC... okay if you added in)

$\text{safety} = 1.56\sqrt{\frac{13}{52}}(8124) + 1.56\sqrt{\frac{4}{365}}(3000 + 5000 + 4000)$

$= 8296$

11. (*10 points*) Evets Inc. orders product each month from a single (unique) supplier. Roughly 75% of the time the supplier delivers exactly what is needed, and 25% of the time it delivers nothing. When the supplier makes a delivery, Evets earns $150,000 for the month; otherwise it earns $0 for the month. Answer the following:

   a. (*2 points*) Determine the expected monthly earnings for Evets Inc.
   b. (*4 points*) Suppose Evets Inc. is risk averse (utility equals the natural log of earnings). Further, suppose that the supplier could guarantee that Evets Inc. will make its expected earnings each month. How much would Evets Inc. be willing to pay for the guarantee?
   c. (*4 points*) Suppose that we don't know the form of Evets Inc. risk aversion. However, we know that they are indifferent between $100,000 with certainty and $150,000 with probability 0.91 and $0 with probability of 0.09. What would your answer to b be in this case?

a) $E[\text{earnings}] = .75(150\,000) + .25\,(0) = 112500$

b) utility of sure thing $= u(112500) = \ln(112500) = 11.63$

~~utility of~~ expected utility $= .75\,u(150\,000) + .25\,u(0)$

$= .75 \ln(150\,000) = 8.93$

so willing to pay $e^{(11.63 - 8.93)} = \$14.9$

c) $u(100{,}000) = \ln(100\,000) = 11.51$

$.91 \ln(150{,}000) = 10.85$

so pay $= e^{(11.51 - 10.85)} \approx \$1.93$