ISYE 3232A Fall 2014 Quiz 1 - A

I, _____, do swear that I abide by the Georgia Tech Honor Code. I understand that any honor code violations will result in a failure (an F).

Signature: _____

- You will have 1 hour.
- This quiz is closed book and closed notes. Calculators are not allowed. No scrap paper is allowed. Make sure that there is nothing on your desk except pens and erasers.
- If you need extra space, use the back of the page and indicate that you have done so.
- **Do not remove any page from the original staple**. Otherwise, there will be 3 points off.
- **Show your work on the test sheet. However, when asked, you "have to" show your work on the answer sheet**. If you do not show your work for a problem, we will give zero point for the problem even if your answer is correct.
- **We will not select among several answers**. Make sure it is clear what part of your work you want graded. If two answers are given, zero point will be given for the problem.
- Throughout, you will receive full credit if someone with no understanding of probability, set theory, and calculus could simplify your answer to obtain the correct numerical solution. **However, you must give a numerical answer where asked**.

Some formula that you may need:

- A continuous uniform random variable X from a to b has pdf $f(x) = \frac{1}{b-a}$ for $a \leq x \leq b$ and CDF $F(x) = \frac{x-a}{b-a}$.
- A discrete uniform random variable X in $\{x_1, x_2, \ldots, x_n\}$ has pmf $p(x) = 1/n$.
- An exponential r.v. X with rate λ has pdf $f(x) = \lambda \exp^{-\lambda x}$ for $x > 0$ and CDF $F(x) = 1 - \exp^{-\lambda x}$ for $x > 0$.
- A poisson r.v. X with mean λ has pmf $p(x) = \frac{\exp^{-\lambda} \lambda^x}{x!}$.
- For a newsvendor problem, the optimal quantity that maximizes the expected profit (or minimizes the expected cost) is y^* such that $y^* = \min\{y : F(y) \geq \frac{c_p + p - c_v}{c_p + p - c_s}\}$ where $F(\cdot)$ is the CDF of demand.

Table B.1. Right tail probabilities $1 - \Phi(a) = P(Z \geq a)$ for an $N(0,1)$ distri random variable Z.

a	0	1	2	3	4	5	6	7	8	9
0.0	5000	4960	4920	4880	4840	4801	4761	4721	4681	4641
0.1	4602	4562	4522	4483	4443	4404	4364	4325	4286	4247
0.2	4207	4168	4129	4090	4052	4013	3974	3936	3897	3859
0.3	3821	3783	3745	3707	3669	3632	3594	3557	3520	3483
0.4	3446	3409	3372	3336	3300	3264	3228	3192	3156	3121
0.5	3085	3050	3015	2981	2946	2912	2877	2843	2810	2776
0.6	2743	2709	2676	2643	2611	2578	2546	2514	2483	2451
0.7	2420	2389	2358	2327	2296	2266	2236	2206	2177	2148
0.8	2119	2090	2061	2033	2005	1977	1949	1922	1894	1867
0.9	1841	1814	1788	1762	1736	1711	1685	1660	1635	1611
1.0	1587	1562	1539	1515	1492	1469	1446	1423	1401	1379
1.1	1357	1335	1314	1292	1271	1251	1230	1210	1190	1170
1.2	1151	1131	1112	1093	1075	1056	1038	1020	1003	0985
1.3	0968	0951	0934	0918	0901	0885	0869	0853	0838	0823
1.4	0808	0793	0778	0764	0749	0735	0721	0708	0694	0681
1.5	0668	0655	0643	0630	0618	0606	0594	0582	0571	0559
1.6	0548	0537	0526	0516	0505	0495	0485	0475	0465	0455
1.7	0446	0436	0427	0418	0409	0401	0392	0384	0375	0367
1.8	0359	0351	0344	0336	0329	0322	0314	0307	0301	0294
1.9	0287	0281	0274	0268	0262	0256	0250	0244	0239	0233
2.0	0228	0222	0217	0212	0207	0202	0197	0192	0188	0183
2.1	0179	0174	0170	0166	0162	0158	0154	0150	0146	0143
2.2	0139	0136	0132	0129	0125	0122	0119	0116	0113	0110
2.3	0107	0104	0102	0099	0096	0094	0091	0089	0087	0084
2.4	0082	0080	0078	0075	0073	0071	0069	0068	0066	0064
2.5	0062	0060	0059	0057	0055	0054	0052	0051	0049	0048
2.6	0047	0045	0044	0043	0041	0040	0039	0038	0037	0036
2.7	0035	0034	0033	0032	0031	0030	0029	0028	0027	0026
2.8	0026	0025	0024	0023	0023	0022	0021	0021	0020	0019
2.9	0019	0018	0018	0017	0016	0016	0015	0015	0014	0014
3.0	0013	0013	0013	0012	0012	0011	0011	0011	0010	0010
3.1	0010	0009	0009	0009	0008	0008	0008	0008	0007	0007
3.2	0007	0007	0006	0006	0006	0006	0006	0005	0005	0005
3.3	0005	0005	0005	0004	0004	0004	0004	0004	0004	0003
3.4	0003	0003	0003	0003	0003	0003	0003	0003	0003	0002

1. (24 points) Short-answer questions.

   (a) (2 points) What is a good distribution to represent the number of absent students among 50 students? Choose one among Bernoulli, Geometric, Binomial, Poisson, Uniform, Exponential, Normal and Gamma.

   (b) (2 points) What is a good distribution to represent the number of items to inspect until a defective item is found? Choose one among Bernoulli, Geometric, Binomial, Poisson, Uniform, Exponential, Normal and Gamma.

   (c) (6 points) For random variables X, Y and Z, $\mathsf{E}[X] = 3$, $\mathsf{E}[Y] = 5$, $\mathsf{E}[Z] = 1$, $\mathsf{Var}[X] = 4$, $\mathsf{Var}[Y] = 9$, and $\mathsf{Var}[Z] = 1$. Also, $\mathsf{Cov}(X,Y) = 3$, $\mathsf{Cov}(X,Z) = -1$ and $\mathsf{Cov}(Y,Z) = 2$.

      i. (1.5 point) Calculate the correlation between X and Y.

      ii. (1.5 point) Calculate $\mathsf{E}[X - Y + Z]$.

      iii. (3 points) Calculate $\mathsf{Var}(X - Y + Z)$.

   (d) (6 points, 2 points each) Service times at a small coffee shop are exponentially distributed with mean 5 minutes.

      i. What is the probability that service takes exactly 10 minutes?

      ii. What is the probability that a customer service time takes more than 10 minutes? Leave your answer in terms of exp or ln.

      iii. Find the 0.6 quantile of the service time. Leave your answer in terms of exp or ln. Show your work.

   (e) (4 points, 2 points each) Suppose that processing times are iid normally distributed with mean 15 gallons and variance 9 gallons^2 (i.e., standard deviation 3 gallons).

      i. Let X_i represent the processing time of item i. Standardize $\sum_{i=1}^{10} X_i$.

      ii. Let $\bar{X}$ represent a sample average of processing times of item 1 to item 10. Standardize $\bar{X}$.

   (f) (4 points) Demand X for a book is approximately normally distributed with mean 1000 books and variance 100 books^2. How many books should be ordered to cover demands 90% of time? Show your work.

2. (11 points) Suppose that during a football game, lemonade sells for \$15 per gallon but only costs \$5 per gallon to make. If they run out of lemonade during the game, it will be impossible to get more. On the other hand, leftover lemonade will be sold to a local food organization at \$1 per gallons. Let X represent the demand and X has a uniform distribution from 30 gallons to 50 gallons.

   (a) (3 points) If 40 gallons are prepared, what is the expected amount of lemonade that are sold from a game? No numerical answer but you should not leave your answer with integrals.

   (b) (3 points) If 40 gallons are prepared, what is the expected amount of lemonade that are leftover from a game? No numerical answer but you should not leave your answer with integrals.

   (c) (5 points) What is the optimal amount of lemonade to be prepared for a game? Show your work on the answer sheet. A numerical answer is expected.

3. (15 points) A camera store specializes in a particular popular and fancy camera. Assume that these cameras become obsolete at the end of the month. They guarantee that if they are out of stock, they will special-order the camera and promise delivery the next day. In fact, what the store does is to purchase the camera from an out of state retailer and have it delivered through an express service. Thus, when the store is out of stock, they actually lose the sales price of the camera and the shipping charge, but they maintain their good reputation. The retail price of the camera is $600, and the special delivery charge adds another $30 to the cost. At the end of each month, there is an inventory holding cost of $70 for each camera in stock (for doing inventory etc). Wholesale cost for the store to purchase the cameras is $200 each. (Assume that the order can only be made at the beginning of the month.)

   Assume that the demand has a discrete uniform distribution from 10 to 19 cameras a month and 16 cameras are ordered at the beginning of a month.

   (a) (3 points) What is the expected overstock cost?

   (b) (3 points) What is the expected understock or shortage cost?

   (c) (1 point) What is the expected total cost?

   (d) (5 points) What is optimal number of cameras to order to minimize the expected total cost? Show your work.

   (e) (3 points) Let the demand have a Poisson distribution with mean 30. When 16 cameras are ordered, what is the expected number of cameras sold? No numerical answer. You can leave your answer with $\sum_a^b$ for integers a and b.