Final Exam
Version 1

1. Consider the following Linear Programming Problem

$$\begin{aligned} \max \quad & z = 3x_1 - 2x_2 \\ \text{subject to} \quad & \\ & x_1 + 2x_2 \geq 3 \\ & 2x_1 + x_2 \leq 4 \\ & x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0 \end{aligned}$$

   (a) **[3 points]** Solve the problem for the optimal solution.

   (b) **[2 points]** Write the Dual of the problem.

   (c) **[2 points]** Solve for the dual optimal solution (using Complementary Slackness).

2. Consider the following problem:

$$\begin{aligned} \max \quad & z = 10x_1 + 16x_2 + 20x_3 \\ \text{subject to} \quad & \\ & 4x_1 + 8x_2 + 8x_3 \leq 50 \\ & 4x_1 + 5x_2 + 9x_3 \leq 45 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

   Where the optimal solution is: $x_1 = 9\frac{1}{6}, x_2 = 1\frac{2}{3}, x_3 = 0, z = 118\frac{1}{3}$.

   (a) **[3 points]** What is the change to the optimal solution (variables and objective) if the right hand side of the first constraint is increased by 1 (to 51)?

   (b) **[2 points]** Does the optimal basis change if the objective coefficient of variable x_2 is changed from 16 to 15? (Must show work).

3. **[3 points]** Solve for the max flow on the given graph, where the numbers above the edges is the capacity of that edge.

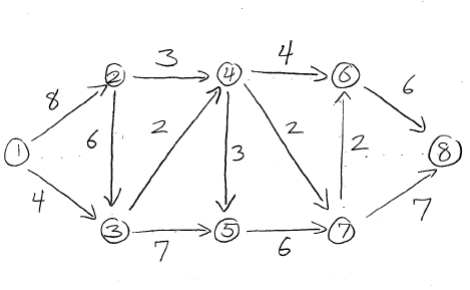

Figure 1: Max Flow Diagram

4. You are going camping and can only carry 20 pounds of equipment in addition to the essential items. The items you have available are listed in the table below, along with their weight and value to you.

Item	Stove	Grill	Table	Radio	Chair	Fan	Coffee Pot
Value	40	54	15	12	20	10	10
Weight	5	6	5	6	8	4	3

   (a) **[2 points]** Assuming you can take fractional amounts of an item, what is the most value you could pack?

   (b) **[2 points]** How does this answer change if you add the constraint that you cannot take BOTH the stove and the grill?

   (c) **[1 points]** What is the constraint in the integer formulation that would give you this condition?

5. **[2 points]** Consider the Branch and Bound chart pictured below. For each of the labeled leaves, identify a pair of possible branches. If it is not required to branch off a certain leaf, explain why. Both x_1, x_2 are required to be integers in the original formulation.

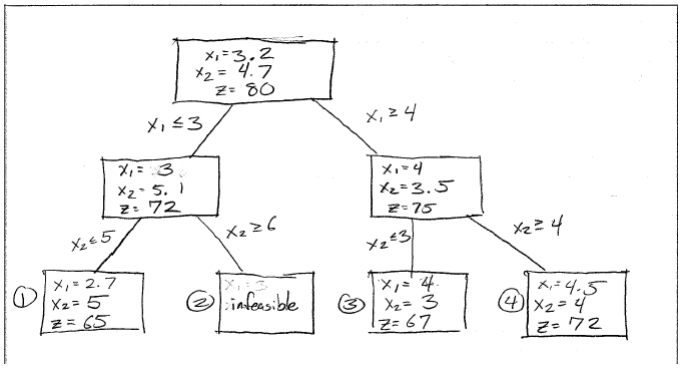

Figure 2: Branch and Bound

6. A company is deciding on several locations to place a warehouse. These warehouses must provide goods to several factories. The cost to open a warehouse is given in the table below as well as the total cost to supply a factory from said warehouse.

	Open Cost	Fact 1	Fact 2	Fact 3	Fact 4	Fact 5	Fact 6
1	10	2	3	2	1	2	3
2	5	4	2	3	2	3	4
3	8	2	3	1	2	2	1
4	7	4	3	3	1	1	2

(a) **[6 points]** Create an integer programming model that will minimize the cost of providing material to every factory from a set of warehouses. Be sure to identify your decision variables and what they mean as well as labeling all your constraints.

(b) **[2 points]** What constraint(s) (and change(s) to objective function) need to be added if there is an added condition that if you open warehouse 1, you cannot open warehouse 4

(c) **[3 points]** What constraint(s) (and change(s) to objective function) need to be added if you include the condition that a warehouse can only supply at most 2 factories, unless you pay half of the opening cost to expand the warehouse, which would allow you to service a total of 5 factories from that warehouse. .

7. Consider a similar problem to the last but now the warehouses have some capacity and the factories have a demand. The cost given to supply a factory from a warehouse is now treated as the cost to ship one unit from the warehouse to a factory, and new fixed costs are given for opening a warehouse.

	Fact 1	Fact 2	Fact 3	Fact 4	Fact 5	Fact 6
Demand	100	200	100	50	75	150

Warehouse	1	2	3	4
Capacity	300	200	300	250
Fixed Cost	400	200	300	350

   (a) **[5 points]** Create an integer programming model to minimize your costs while satisfying all factory demands. Be sure to identify all decision variables, as well as parameters you may use to shorten notation.

   (b) **[2 points]** Suppose you can pay \$20 to increase the warehouse capacity by 20 units, and you can continue to do this until the warehouse doubles in size, for all warehouses. What constraint(s) (and change(s) to objective function) need to be added?

8. **Extra Credit:** If 80% of the class completes the CIOS survey, everyone gets +1 to their score on this exam. Also, please complete the TAOS survey if you have interacted with James, I am sure he would appreciate the feed back.