ISyE 3232 Final
Fall 2012

Name

Please be neat and show all your work so that I can give you partial credit. GOOD LUCK AND HAPPY HOLIDAYS.

Question 1
Question 2
Question 3
Question 4
Question 5

Total

(25) **1.** Customers arrive to a single server system with respect to a Poisson process of rate λ and require an exponential amount of service with rate μ. Customers waiting in line are impatient and if they are not in service they will leave after an exponential amount of time with rate δ independent of their position in the queue. Let $X(t)$ be the number of customers in the system at time t. Clearly, $\{X(t) : t \geq 0\}$ is a continuous time Markov chain.

(a) (10) Draw the rate diagram for$\{X(t) : t \geq 0\}$. When does the long run probabilities for the number of customers in the system exist?

(b) (15) Find the long distribution of the number of customers in the system when $\mu = \delta$.

(25) **2.** Consider a system with two servers and a capacity of at most three customers. The incoming customers belong to two classes: class 1 and class 2. Class 2 customers are allowed to enter the system if and only if there is a free server immediately available, otherwise they are turned away. Class 1 customers always enter the system (i.e., if there are less than 3 customers in the system) and wait for service if necessary. Assume that class 1 customers arrive with respect to a Poisson process of rate 10/hr and class 2 customers arrive with respect to a Poisson process of rate 15/hr. The arrival processes of class 1 and class 2 customers are independent of each other. The service times are exponential with rate 20/hr. Let $X(t)$ be the number of customers in the system at time t.

(a) (10) Is $\{X(t) : t \geq 0\}$ a continuous time Markov chain? If it is, what is the rate diagram (or the generator matrix)?

(b) (15) What is the expected number of customers in the system in the long run?

(25) **3.** Traffic on Lavista Road follows a Poisson process with rate 2/3 per minute. 10% of the vehicles are trucks, the other 90% are cars.

(a) (5) What is the probability that at least one truck passes in an hour?

(b) (10) Given that 10 trucks have passed by in an hour, what is the expected number of vehicles that have passed by in the same one hour?

(c) (10) Given that 50 vehicles have passed by in a hour, what is the probability that there were exactly 5 trucks and 45 cars?

(25) **4.** An individual has three umbrellas, some at her office and some at home. If she is leaving home in the morning (or leaving work in the evening) and it is raining, she will take an umbrella, if one is there. Otherwise, she gets wet. Assume that independent of the past, it rains at any point in time with probability 0.2. What is the long-run probability that she gets wet?

(25) **5.** Harry's boutique is a trendy clothing store on Rodeo Drive in Beverly Hills, California. Suppose that the arrival rate starts at 0 at 10:00, increases to 4 at 12:00, to 6 by 2:00, drops to 2 by 4:00 and decreases to 0 by the time the store closes at 6:00 and that the arrival rates are linear in between these time points.

(a) (10) What is the probability that no one arrives before noon?

(b) (10) What is the expected number of customers who come from 10:00 am to 6:00pm?

(c) (5) Suppose that Harry closes the store at 5:30pm instead of 6:00. What is the expected number of customers lost?