

Quiz 3 Solution

Problem 1 : Apply the Euclidean algorithm in order to compute $\gcd(70, 18)$

	70	18
70	1	0
18	0	1
16	1	-3
2	-1	4
0		

$$\rightarrow \gcd(70, 18) = 2 = (-1) \cdot 70 + 4 \cdot 18$$

Problem 2: Let n be a natural number. Given n consecutive integers $a, a+1, a+2, \dots, a+n-1$, show that one of them is divisible by n

Proof:

Let $n \in \mathbb{N}$ and $a \in \mathbb{Z}$. We consider 2 cases:

Case 1: a is divisible by n then it is the case that of the n consecutive integers $a, a+1, a+2, \dots, a+n-1$, one of them is divisible by n

Case 2: a is not divisible by n then a must be of the form $nk - i$ for $k \in \mathbb{Z}$, $i \in \{1, 2, \dots, n-1\}$

Then $a + i = (nk - i) + i = nk$, which is divisible by n . But since $i \in \{1, 2, \dots, n-1\}$, $a + i$ must be one of the n consecutive integers $a, a+1, a+2, \dots, a+n-1$ thus we have shown that the claim holds for this case. Since the claim holds in both cases, the claim holds. ■