Homework 2
Solutions

1. Recall the diet problem discussed in class.

$$\begin{aligned}
\max \quad & 50x_1 + 200x_2 + 100x_3 + 100x_4 && (1)\\
& \text{subject to} \\
& 10x_1 + 50x_2 + 15x_3 + 40x_4 \geq 200 && (2)\\
& 0x_1 + 20x_2 + 5x_3 + 10x_4 \geq 30 && (3)\\
& 0x_1 + 0x_2 + 5x_3 + 20x_4 \geq 30 && (4)\\
& 50x_1 + 10x_2 + 30x_3 + 10x_4 \geq 70 && (5)\\
& x_i \geq 0 \qquad \forall i = 1, 2, 3, 4 && (6)
\end{aligned}$$

   State whether the following points are feasible or infeasible. If infeasible, state one of the constraints that is violated.

   (a) (5,0,1,1) **Infeasible, Violates 2,3,4**

   (b) (5,3,2,1) **Feasible**

   (c) (1,-1,4,5) **Infeasible, Violates 6**

   (d) (1.5,2.2,3,1) **Feasible**

2. Model the following problem: You run a company and can make two items, paper and graph paper. One ream of paper and one ream of graph paper each take 1 cord of wood to make. A ream of paper requires one cartridge of ink to prink, while a ream of graph paper takes 2 ink cartridges. You also know that only 4 people want to buy a ream of graph paper. Your factory has 5 cords of wood and 6 ink cartridges on hand, and you want to maximize your revenue if one ream of paper sells for $2 and one ream of graph paper sells for $3.

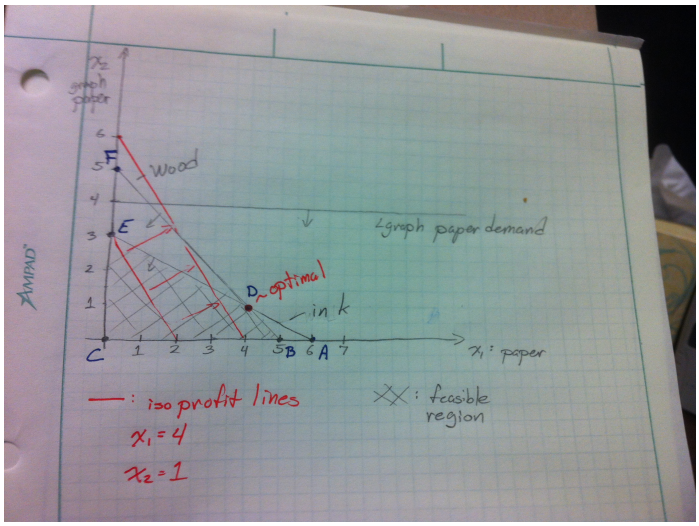
   Decision Variables

   - x_1: Number of reams of paper you make
   - x_2: Number of reams of graph paper you make

   Note: you could have labeled x_1 as graph paper and had the same equations with variables switched, as long as it is clearly labeled.

$$\begin{aligned}
\max \quad & 2x_1 + 3x_2 && && (7)\\
& \text{subject to} \\
& x_1 + x_2 \leq 5 && \text{Wood Constraint} && (8)\\
& x_1 + 2x_2 \leq 6 && \text{Ink Constraint} && (9)\\
& 0x_1 + x_2 \leq 4 && \text{Graph Paper Demand Constraint} && (10)\\
& x_i \geq 0 \qquad \forall i = 1, 2 && && (11)
\end{aligned}$$

3. Given the previous problem's model, graph the feasible region.



Note: the graph paper demand constraint does not form a boundary of the feasible region. We call this a redundant constraint.

4. Based on the graph of the feasible region and objective function, give an optimal solution.

   Draw iso-profit lines at slope = -3/2. Max line crosses at (4,1), optimal solution is 4 reams of paper and 1 ream of graph paper