

MATH 1711 TEST 2, FALL 2009, PAGE I

Print Your Name: Key-1

T.A. or Section Number: _____

WORK ALL OF PROBLEMS 1-4.

1. (14 points) Six people have a deck of 52 well-shuffled cards. Each person chooses a card at random from his/her deck. Find the probability that there is at least one match (match=exact same card). You do not need to simplify your final answer.

$$\begin{aligned} P(\geq 1 \text{ match}) &= 1 - P(\text{no matches}) \\ &= 1 - \frac{\# \text{ of ways to get 6 different cards}}{\text{total \# possible outcomes}} \\ &= 1 - \frac{P(52, 6)}{52^6} \end{aligned}$$

2. (14 points) Three cards are drawn from a deck of 52 cards. Find the probability that two cards are spades and one is a diamond, in any order, if the cards are drawn:

(a) without replacement

$$P(2 \text{ spades and } 1 \text{ diamond}) = \frac{C(13, 2) \cdot C(13, 1)}{C(52, 3)}$$

OR $\frac{3!}{2!1!} \cdot \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{13}{50}$ gives the same answer
 (ways to order S, S, D)

(b) with replacement

$$P(2 \text{ spades and } 1 \text{ diamond}) = \frac{\frac{3!}{2!1!}}{3} \cdot \left(\frac{13}{52}\right)^3$$

ways to
order S, S, D

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T.A. or Section Number: %

WORK ALL OF PROBLEMS 1-4.

1. (14 points) Six people have a deck of 52 well-shuffled cards. Each person chooses a card at random from his/ her deck. Find the probability that there is at least one match (match=exact same card). You do not need to simplify your final answer.

2. (14 points) Three cards are drawn from a deck of 52 cards. Find the probability that two cards are spades and one is a diamond, in any order, if the cards are drawn:

3. (14 points) A bag contains five red and seven white marbles. A sample of three marbles is drawn, without replacement, from the bag. If it is known that at least one marble in the sample is white, find the probability that all three marbles are white. You do not need to simplify your final answer.

$$Pr(\text{all 3 w} | \geq 1 \text{ w}) = \frac{Pr(\text{all 3 w and } \geq 1 \text{ w})}{Pr(\geq 1 \text{ w})}$$

$$= \frac{Pr(\text{all 3 w})}{1 - Pr(\text{none are w})}$$

$$= \frac{C(7,3)/C(12,3)}{1 - C(5,3)/C(12,3)}$$

OR $\frac{C(7,3)}{C(12,3) - C(5,3)}$ is an equivalent answer

4. (14 points) A coin is biased so that heads is four times as likely as tails. Find a probability distribution for this experiment.

$$Pr(H) = 4 Pr(T)$$

$$\text{and } Pr(H) + Pr(T) = 1$$

$$\text{so: } 4 Pr(T) + Pr(T) = 1$$

$$\Rightarrow Pr(T) = 1/5$$

$$\text{and } Pr(H) = 4/5$$

outcome	prob.
H	4/5
T	1/5

3. (14 points) A bag contains five red and seven white marbles. A sample of three marbles is drawn, without replacement, from the bag.

If it is known that at least one marble in the sample is white, find the probability that all three marbles are white. You do not need to

4. (14 points) A coin is biased so that heads is four times as likely as tails. Find a probability distribution for this experiment.

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WORK ONLY FOUR (4) OF THE NEXT FIVE PROBLEMS (NUMBERS 5-9). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST FOUR WILL BE GRADED.

5. (12 points) Fifty-seven percent of potato chips sold in a certain store are barbeque flavored. What are the odds that a randomly selected bag of chips will be barbeque flavored?

$$P(\text{BBQ}) = 0.57 = \frac{57}{100}$$

$$= \frac{57}{57+43}$$

So the odds are 57 to 43

6. (12 points) An experiment consists of tossing a die two times. Let $E = \{\text{the first toss is an even number}\}$, $F = \{\text{the first toss is an odd number}\}$, and $G = \{\text{the second toss is a 4}\}$. Which of the events are mutually exclusive? Which of the events are independent? Explain your answers.

Mutually exclusive: can't both happen
 E and F , since 1st toss cannot be both even and odd

Independent: no effect on each other
 E and F , and ~~and~~ F and G ,
 since outcome on 1st toss does not affect the 2nd toss

WORK ONLY FOUR (4) OF THE NEXT FIVE PROBLEMS (NUMBERS 5-9). WRITE OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH

PROBLEM TO OMIT, THEN ONLY THE FIRST FOUR WILL BE GRADED.

5. (12 points) Fifty-seven percent of potato chips sold in a certain store are barbeque flavored. What are the odds that a randomly selected bag of chips will be barbeque

6. (12 points) An experiment consists of tossing a die two times. Let $E = \{\text{the first toss is an even number}\}$, $F = \{\text{the first toss is an odd number}\}$, and $G = \{\text{the second toss is } 1 \text{ or } 4\}$. Which of the events are mutually exclusive? Which of the events are independent?

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7. (12 points) Find the coefficient of x^3y^2 in the expansion of $(x - 2y)^5$. Simplify your final answer as far as possible.

$$\begin{aligned}(x-2y)^5 &= [x + (-2y)]^5 \\ \text{" } x^3y^2 \text{ " term looks like: } & \binom{5}{2} x^3 (-2y)^2 \\ &= 10x^3 (4y^2) \\ &= 40x^3y^2 \\ \text{So the coefficient is } & \boxed{40}\end{aligned}$$

8. (12 points) A group of 25 new freshmen are randomly assigned to one of three dorms, A, B, and C. 10 students are assigned to dorm A, 8 are assigned to dorm B, and 7 are assigned to dorm C. In how many ways can these assignments be made? You do not need to simplify your final answer.

Permutation with repetition:

$$\binom{25}{10, 8, 7} = \frac{25!}{10! 8! 7!} \text{ ways}$$

OR $C(25, 10) \cdot C(15, 8) \cdot C(7, 7)$ gives the same answer

9. (12 points) Suppose E and F are two events in a sample space with $Pr(E) = 0.4$, $Pr(F) = 0.3$, and $Pr(E \cup F) = 0.7$. Find $Pr(E|F)$.

$$\begin{aligned}Pr(E) &= 1 - Pr(E') = 0.4 \\ Pr(E \cap F) &= Pr(E \cup F) + Pr(E) + Pr(F) \\ &= -0.7 + 0.4 + 0.3 = 0.2 \\ Pr(E|F) &= \frac{Pr(E \cap F)}{Pr(F)} = \frac{0.2}{0.3} = \boxed{\frac{2}{3}}\end{aligned}$$

7. (12 points) Find the coefficient of $2:33/2$ in the expansion of $(:1: - 2y)^5$. Simplify yom final answer as far as possible. '5'

8. (12 points) A group of 25 new freshmen are randomly assigned to one of three dorms, A, B, and C. 10 students are assigned to dorm A, 8 are assigned to dorm B, and 7 are assigned to dorm C. In how many ways can these assignments be made? You do not need

to simplify your final answer. H 6

9. (12 points) Suppose E and F are two events in a sample space with $\Pr(E') = 0.4$, $\Pr(F) = 0.3$, and $\Pr(E \cup F) = 0.7$. Find

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WORK ALL OF PROBLEMS 1-4.

1. (14 points) A coin is biased so that tails is five times as likely as heads. Find a probability distribution for this experiment.

$$\begin{aligned}
 P(T) &= 5P(H) \\
 \text{and } P(H) + P(T) &= 1 \\
 \text{So: } P(H) + 5P(H) &= 1 \Rightarrow P(H) = 1/6 \\
 &P(T) = 5/6
 \end{aligned}$$

outcome	prob
H	1/6
T	5/6

2. (14 points) Five people have a deck of 52 well-shuffled cards. Each person chooses a card at random from his/her deck. Find the probability that there is at least one match (match=exact same card). You do not need to simplify your final answer.

$$\begin{aligned}
 P(\geq 1 \text{ match}) &= 1 - P(\text{no matches}) \\
 &= 1 - \frac{\# \text{ of ways to have 5 diff. cards}}{\text{total \# of outcomes}} \\
 &= 1 - \frac{P(52, 5)}{52^5}
 \end{aligned}$$

Print Your Name: __1i%_ii__

T.A. or Section Number: _____

WORK ALL OF PROBLEMS 1-4.

1. (14 points) A coin is biased so that tails is five times as likely as heads. Find a probability distribution for this experiment.

2. (14 points) Five people have a deck of 52 wellshuffled cards. Each person chooses a card at random from his/ her deck. Find the probability that there is at least one match (matchzexact same card). You do not need to simplify your final answer.

....._..._....._.

3. (14 points) Three cards are drawn from a deck of 52 cards. Find the probability that two cards are hearts and one is a club, in any order, if the cards are drawn:

(a) without replacement

$$Pr(2 \text{ hearts and } 1 \text{ club}) = \frac{C(13,2) \cdot C(13,1)}{C(52,3)}$$

OR
$$= \frac{\left[\begin{array}{c} \text{\# ways to order H,H,C} \\ 3! \\ \hline 2!1! \end{array} \right] \cdot \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{13}{50}}{\text{gives the same answer}}$$

(b) with replacement

$$P(2 \text{ H and } 1 \text{ C}) = \frac{3!}{2!1!} \cdot \left(\frac{13}{52} \right)^3$$

\downarrow
 # of orderings independence

4. (14 points) A bag contains eight red and six white marbles. A sample of three marbles is drawn, without replacement, from the bag. If it is known that at least one marble in the sample is white, find the probability that all three marbles are white. You do not need to simplify your final answer.

$$Pr(\text{all } w \mid \geq 1 w) = \frac{Pr(\text{all } w \text{ and } \geq 1 w)}{Pr(\geq 1 w)}$$

$$= \frac{Pr(\text{all } w)}{1 - Pr(\text{none } w)}$$

$$= \frac{C(6,3)/C(14,3)}{1 - C(8,3)/C(14,3)}$$

OR this equals ...
$$\frac{C(6,3)}{C(14,3) - C(8,3)}$$

3. (14 points) Three cards are drawn from a deck of 52 cards. Find the probability that two cards are hearts and one is a club, in any

order, if the cards are drawn: (a) without replacement

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4. (14 points) A bag contains eight red and six white marbles. A sample of three marbles is drawn, without replacement, from the bag. If it is known that at least one marble in the sample is White, find the probability that all three marbles are White. You do not need to simplify your final answer.

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Print Your Name: Key-2

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WORK ONLY FOUR (4) OF THE NEXT FIVE PROBLEMS (NUMBERS 5-9). WRITE "OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT, THEN ONLY THE FIRST FOUR WILL BE GRADED.

5. (12 points) An experiment consists of tossing a die two times. Let $E = \{\text{the second toss is an odd number}\}$, $F = \{\text{the second toss is an even number}\}$, and $G = \{\text{the first toss is a 3}\}$. Which of the events are mutually exclusive? Which of the events are independent? Explain your answers.

E and F are mutually exclusive because the second toss cannot be both even and odd.
 E and G and F and G are independent since the first toss does not affect the outcome of the second toss.

6. (12 points) Suppose E and F are two events in a sample space with $Pr(E') = 0.2$, $Pr(F) = 0.6$, and $Pr(E \cup F) = 0.9$. Find $Pr(E|F)$.

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} \leftarrow \text{we need to find this}$$

$$Pr(E) = 1 - Pr(E') = 0.8$$

$$Pr(E \cap F) = Pr(E) + Pr(F) - Pr(E \cup F) \\ = 0.8 + 0.6 - 0.9 = 0.5$$

$$\text{so } Pr(E|F) = \frac{0.5}{0.6} = \boxed{\frac{5}{6}}$$

"OMIT" OVER THE PROBLEM YOU DO NOT WANT GRADED. IF YOU DO NOT INDICATE WHICH PROBLEM TO OMIT,

THEN ONLY THE FIRST FOUR WILL BE GRADED.

5. (12 points) An experiment consists of tossing a die two times. Let $E = \{\text{the second toss is an odd number}\}$, $F = \{\text{the second toss is an even number}\}$, and $G = \{\text{the first toss is a 3}\}$. Which of the events are mutually exclusive? Which of the events are independent? Explain

6. (12 points) Suppose E and F are two events in a sample space with $\Pr(E) = 0.2$,

7. (12 points) Find the coefficient of x^2y^3 in the expansion of $(2x - y)^5$. Simplify your final answer as far as possible.

" x^2y^3 " term looks like:

$$\binom{5}{3} (2x)^2 (-y)^3$$

$$= 10 \cdot 4x^2 \cdot (-y^3) = -40x^2y^3$$

So the coefficient is $\boxed{-40}$

8. (12 points) A group of 30 new freshmen are randomly assigned to one of three dorms, A, B, and C. 14 students are assigned to dorm A, 10 are assigned to dorm B, and 6 are assigned to dorm C. In how many ways can these assignments be made? You do not need to simplify your final answer.

Permutation w/ repetition:

$$\binom{30}{14, 10, 6} = \frac{30!}{14! 10! 6!} \text{ ways}$$

OR

$C(30, 14) \cdot C(16, 10) \cdot C(6, 6)$ gives the same answer

9. (12 points) Sixty-one percent of potato chips sold in a certain store are barbeque flavored. What are the odds that a randomly selected bag of chips will be barbeque flavored?

$$P(BBQ) = 0.61 = \frac{61}{100} = \frac{61}{61 + 39}$$

So the odds are $\boxed{61 \text{ to } 39}$

7. (12 points) Find the coefficient of in the expansion of $(2x$

y)5. Simplify your

final answer as far as possible.

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