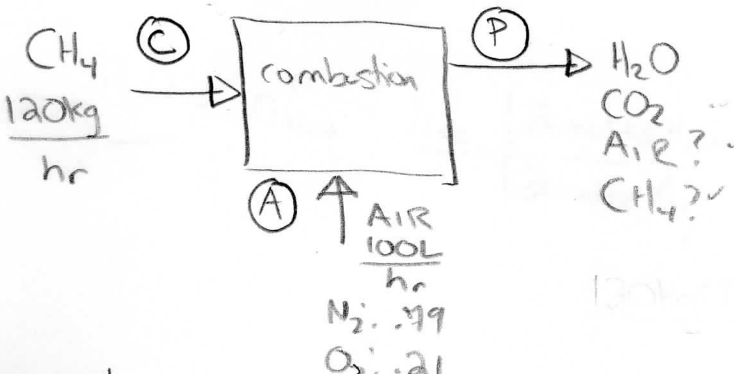
Name: ______________________________

Problem #1 **(25 points)**

If 2kg/min of CH_4 and 100L/hour of air enter and exit from a combustion engine at room temperature and react, what is the flow rate of each of the molecular species exiting the combustion engine?



RXN

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

BASIS

1 hr

Assumptions

- Air is an ideal gas
- reaction goes to completion

Legend

n_i^j = moles of species i in stream j

Find Limiting reactant

is $\frac{n^C_{CH_4}}{n^A_{O_2}}$ >, <, or $= \frac{1}{2}$?

$n^C_{CH_4} = \frac{120\,kg}{} \left| \frac{1\,kg\,mol}{16\,kg} \right. = 7.5\ kgmol = 7500$ mole

$n^A_{O_2} = \frac{PV}{RT} = \frac{(101{,}300\,Pa)\left(\frac{100L}{} \left| \frac{.001\,m^3}{1L} \right.\right)(.21)}{8.314\,\frac{Pa\,m^3}{mol\,K}\ 298K} = .859$ moles O_2

$\rightarrow O_2$ is limiting reagent $n^A_{O_2} \ll n^C_{CH_4}$

Material Balance

CH_4: $n^C_{CH_4} - n^{reacted}_{CH_4} = n^P_{CH_4}$

$n^{reacted}_{CH_4} = \frac{n^A_{O_2}}{} \left| \frac{1\,mol\,CH_4}{2\,mol\,O_2} \right. = .4295$ moles CH_4 react

7500 moles − .4295 moles = $\boxed{7000\ \frac{moles}{hour}\ CH_4\ exit}$

O_2: $n^A_{O_2} - n^{reacted}_{O_2} = n^P_{O_2}$ $\rightarrow$ $\boxed{\emptyset \text{ moles } O_2 \text{ exit}}$

$n^A_{O_2} = n^{react}_{O_2}$

N_2: $n^A_{N_2} = N^P_{N_2}$ = $\longrightarrow$ $\boxed{3\ \frac{moles}{hour}\ N_2\ exit}$

$n^A_{N_2} = \frac{PV}{RT} = \frac{(101{,}300\,Pa)\left(\frac{100L}{} \left| \frac{.001\,m^3}{1L} \right.\right)(.79)}{8.314\,\frac{Pa\,m^3}{mol\,K}\ 298K} = 3.2$ moles N_2

Problem #1 (cont'd)

CO_2: $n^P_{CO_2} = n^A_{O_2} \left| \frac{1 \text{ mol } CO_2}{2 \text{ moles } O_2} \right. = .4295$ moles CO_2 out

$= \boxed{.4 \text{ moles } \frac{CO_2}{hr}}$

H_2O: $n^P_{H_2O} = n^A_{O_2} \left| \frac{2 \text{ moles } H_2O}{2 \text{ moles } O_2} \right. = .859$ moles H_2O out

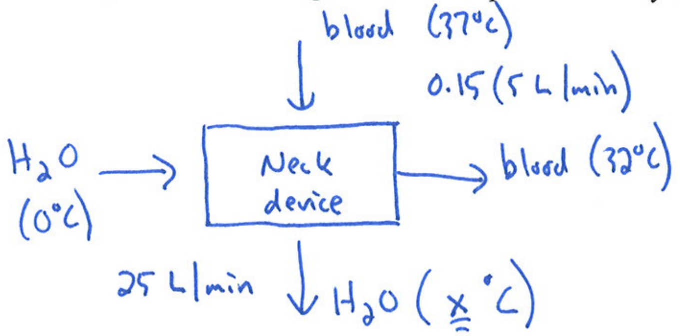
$= \boxed{.9 \frac{\text{moles } H_2O}{hr}}$

Name: Solution

Problem #2 (25 points)

In order to cryoprotect the brain prior to neurosurgery, ice-cold water is pumped at a rate of 25 L/min through a device that wraps around the neck and is 75% efficient. If the brain is to be cooled by 5°C and 15% of the total blood volume goes to the head, what is the temperature of the water as it exits the device?

(Note: the average cardiac output of healthy adults is 5 L/min & C_v $H_2O(l) = 4.18$ J/g•K)



Basis: 1 min

open, steady-state

Assume: $C_v^{H_2O} \approx C_v^{blood}$ $\left(= 4.18 \frac{J}{g \cdot K}\right)$

$\rho_{H_2O} \approx \rho_{blood}$ $(= 1.0 \text{ g/mL})$

$C_v = C_p$ or $\Delta H = \Delta U$

system stationary, insulated, no work, no height diff, no velocity change

$$\Delta E = 0 = \Delta\left(U + \cancel{KE} + \cancel{PE}\right) = \cancel{Q} + \cancel{W} - \Delta\left(H + \cancel{PE} + \cancel{KE}\right)$$

$$\Rightarrow 0 = \Delta U, \quad 0 = \Delta H = \Delta H_{blood} + \underbrace{\Delta H_{H_2O}}_{\times\, 0.75 \text{ (efficiency)}}$$

switch order

$$\Rightarrow 0 = (0.75 \text{ kg})\left(4.18 \frac{J}{g \cdot K}\right)(-5 K) + 0.75\left[(25 \text{ kg})\left(4.18 \frac{J}{g \cdot K}\right)(x - 273 K)\right]$$

$$0 = m \int_{T_i}^{T_i - 5} C_p^{blood}\, dT + \left[m \int_{T_i}^{T_f} C_p^{H_2O}\, dT\right] 0.75$$

$$15.675 \text{ kJ} = 78.375 \frac{kJ}{K}(x - 273 K)$$

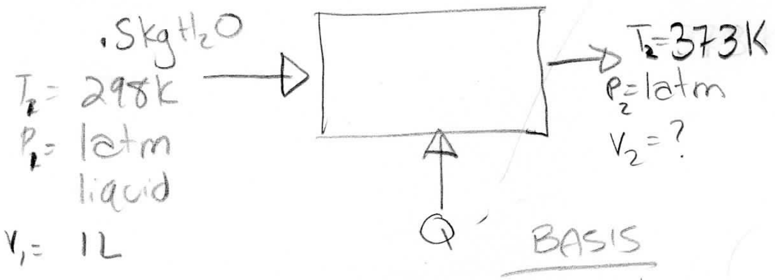
$$\boxed{0.2°C} = x - 273 K$$

Problem #3 (25 points)

A piston chamber originally at 1 L and containing 0.5 kg of water at room temperature is heated to the point that all of the water evaporates and the chamber expands to maintain constant atmospheric pressure. In this case, how much energy enters the chamber?

$\Delta\hat{H}_{vaporization}$ (100°C) =40.65kJ/(g mol) $\qquad$ C_p H_2O (l) = 75.4 J/(g mol)

$\Delta\hat{H}_{fusion}$ (0°C) = 60.1kJ/(g mol) $\qquad$ C_p H_2O (v) =33.9 J/(g mol)



BASIS

.5kg H_2O

Assume

- energy change due to heating bone dry air in vessel is negligible
- no heat lost to surroundings
- Kinetic energy, potential energy changes are negligibe

Energy Balance

$Q = \Delta U$ (closed, unsteady)

$\Delta U = \Delta H - \Delta(PV)$

$$Q = \Delta U = n_{H_2O}\int_{T_1}^{T_2} C_p\,{H_2O}_l\,dT + n_{H_2O}\Delta\hat{H}_{VAP} - (P_2V_2 - P_1V_1)$$

Find n_{H_2O} (moles H_2O)

$n = \frac{500 g H_2O}{} \left| \frac{1 mol}{18 g} \right. = 27.8$ moles

Find ΔV

$$V_2 = \frac{nRT}{P} = \frac{27.8 \text{moles}\left(8.314 \frac{Pa\, m^3}{gmol\, K}\right)(373K)}{101,300\ Pa} = .851\ m^3$$

$V_1 = 100L \left| \frac{.001 m^3}{1L} \right. = .1 m^3$

$$\Delta U = 27.8 \text{moles}\left(75.4 \frac{J}{gmol\,K}\right)(373-298K) + 27.8\text{moles}\left(40650 \frac{J}{gmol}\right) - 101300Pa\,(.851 m^3 - .1 m^3)$$

$= 157209\ J + 1,130,000\ J - 76076\ J$

$= 1,211,000\ J = 1211\ kJ \approx \Rightarrow \boxed{Q = 1000\ kJ}$

Name: Solution

Problem #4 **(25 points)**

If the heat of combustion for 1 mole of glucose ($C_6H_{12}O_6$) at standard conditions is -2825.8 kJ/g mol, and the heats of formation for water and carbon dioxide are -285.8 kJ/g mol and -393.5 kJ/g mol (respectively), what is the heat of formation for glucose?

$$C_6H_{12}O_6 + 6O_2 \rightarrow 6H_2O + 6CO_2$$

Basis: 1 g·mol $C_6H_{12}O_6$

$$\Delta H_{rxn} = \sum \nu_i \cdot \hat{H}_f\,(\text{prod}) - \sum \nu_i \cdot \hat{H}_f\,(\text{react})$$

$$-2825.8\text{ kJ} = \left[\underbrace{6(-285.8\text{ kJ})}_{-1714.8} + \underbrace{6(-393.5)}_{-2361}\right] - \left[\hat{H}_f^{glucose} + 6(\cancel{0})\right]$$

$$\Rightarrow \hat{H}_f^{glucose} = \boxed{1250\text{ kJ/g·mol}}$$