ISyE 3833 - HW 1 Due: Jan 27 / 28

1. (From Winston, Section 2.1, Problem 1) For

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

find:
(a) $-A$,
(b) $3A$,
(c) $A + 2B$
(d) A^T,
(e) B^T,
(f) AB, and
(g) BA.

2. (From Winston, Section 2.1, Problem 2) Only three brands of beer (beer 1, beer 2, and beer 3) are available for sale in Metropolis. From time to time, people try one or another of these brands. Suppose that at the beginning of each month, people change the beer they are drinking according to the following rules.

- 30% of the people who prefer beer 1 switch to beer 2.
- 20% of the people who prefer beer 1 switch to beer 3.
- 30% of the people who prefer beer 2 switch to beer 3.
- 30% of the people who prefer beer 3 switch to beer 2.
- 10% of the people who prefer beer 3 switch to beer 1.

For $i = 1, 2, 3$, let x_i be the number who prefer beer i at the beginning of this month and y_i be the number who prefer beer i at the beginning of next month. Use matrix multiplication to relate the following:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

3. (a) Expand the following expression,

$$\forall j = 1, 2, 3, \qquad b_j = 2^{-j} \sum_{\substack{i=1, \\ i \neq j}}^{3} 2^{(i+j)}(a_{ij} + 1),$$

where 2^{-j} denotes 2 raised the power of $-j$, i.e., $1/2^j$, and $2^{(i+j)}$ denotes 2 raised to the power of $i + j$.

(b) Write the following system of inequalities in compact form, using summation and "for all" notation:

$$\begin{aligned} 2x_1 + 4x_2 + 6x_3 + \ 8x_4 + 10x_5 &\leq b_1, \\ 3x_1 + 5x_2 + 7x_3 + \ 9x_4 + 11x_5 &\leq b_2, \\ 4x_1 + 6x_2 + 8x_3 + 10x_4 + 12x_5 &\leq b_3. \end{aligned}$$

4. (Winston, Section 3.1, Problem 1) Farmer Jones must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labor per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labor per week. All wheat can be sold at \$4 a bushel, and all corn can be sold at \$3 a bushel. Seven acres of land and 40 hours per week of labor are available. Government regulations require that at least 30 bushels of corn be produced during the current year. Let x_1 = number of acres of corn planted, and x_2 = number of acres of wheat planted. Using these decision variables, formulate an LP whose solution will tell Farmer Jones how to maximize the total revenue from wheat and corn.

5. (Winston, Section 3.1, Problem 2) In each case, briefly justify your answer.

   (a) Is $x_1 = 2$, $x_2 = 3$, in the feasible region?

   (b) Is $x_1 = 4$, $x_2 = 3$, in the feasible region?

   (c) Is $x_1 = 2$, $x_2 = -1$, in the feasible region?

   (d) Is $x_1 = 3$, $x_2 = 2$, in the feasible region?

6. (Winston, Section 3.2 Problem 6) Farmer Jane owns 45 acres of land. She is going to plant each with wheat or corn. Each acre planted with wheat yields \$200 profit; each with corn yields \$300 profit. The labor and fertilizer used for each acre are given in Table 1. One hundred workers and 120 tons of fertilizer are available. Use linear programming to determine how Jane can maximize profits from her land. Formulate the problem, then use the graphical solution method to find the optimal solution.

Table 1: Farmer Jane's labor and fertilizer use, by crop

	Wheat	Corn
Labour	3 workers	2 workers
Fertilizer	2 tons	4 tons

7. Use the graphical solution method to solve the LP below. Sketch the feasible set, and identify the optimal solution. What is the optimal value? Which constraints are binding?

$$\begin{aligned} \max \quad & 5x_1 + 4x_2 \\ \text{s.t.} \quad & 6x_1 + 4x_2 \geq 24 \\ & x_1 + 2x_2 \leq 6 \\ & -x_1 + x_2 \leq 1 \\ & x_2 \leq 2 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned}$$

8. A realtor is developing a rental housing and retail area. The housing area consists of 3 types of housing: efficiency apartments, duplexes, and single-family homes. Maximum demand by potential renters is estimated to be 500 efficiency apartments, 300 duplexes and 250 single-family homes (and there is no need to build more houses than the demand), but the number of duplexes must equal at least 50% of the total number of efficiency apartments and single-family homes. For each efficiency, duplex and single-family apartment, there must be retail space of at least 10 ft^2, 15 ft^2 and 18 ft^2 respectively. However, land availability limits retail

space to no more than 10,000 ft^2. The monthly rental income is estimated at \$600, \$750, and \$1200 for efficiency, duplex and single family units respectively. The retail space rents for \$100/ft^2. Formulate an LP that would find the amount of retail space area and number of family residences to maximize rent income.