ISyE 4031 Regression and Forecasting
Homework 9 Solutions
Spring 2016

1. Exercise 8.5

a. $\ell_2 = \alpha y_2 + (1-\alpha)[\ell_1 + b_1] = .2(245) + .8[203.0051 + (-.2933)] = 211.1694$

$b_2 = \gamma[\ell_2 - \ell_1] + (1-\gamma)b_1 = .1[211.1694 - 203.0051] + .9(-.2933)$

$= 0.5525$ (Slight round - off)

b. $\hat{y}_3(2) = \ell_2 + b_2 = 21.1694 + .5525 = 211.7219$

$y_3 - \hat{y}_3(2) = 185 - 211.7219 = -26.7219$

c. $\ell_3 = \alpha y_3 + (1-\alpha)[\ell_2 + b_2] = .2(185) + .8[211.1694 + .5525] = 206.3775$

$b_3 = \gamma[\ell_3 - \ell_2] + (1-\gamma)b_2 = .1[206.3775 - 211.1694] + .9(.5525) = 0.0181$ (Round - off)

d. $\hat{y}_4(3) = \ell_3 + b_3 = 206.3775 + .0181 = 206.3956$

$y_4 - \hat{y}_4(3) = 169 - 206.3956 = -37.3956$ (Round - off)

2. Exercise 8.9.

a. Revised point forecast for sales in week 56 when $y_{53} = 330$

$\hat{y}_{56}(53) = \ell_{53} + 3b_{53} = 322.8089 + 3(4.7281) = 336.9932$

Revised 95% prediction interval for sales in week 56 when $y_{53} = 330$

$1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2 = 1 + (.247)^2(1+.095)^2 + (.247)^2(1+2(.095))^2 = 1.1595$

$[\hat{y}_{56}(53) \pm z_{[.025]} s\sqrt{1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2}\,]$

$= [336.9932 \pm 1.96(27.89)\sqrt{1.1595}] = [336.9932 \pm 58.86] = [278.13,\ 395.85]$

Revised point forecast for sales in week 57 when $y_{53} = 330$

$\hat{y}_{57}(53) = \ell_{53} + 4b_{53} = 322.8089 + 4(4.7281) = 341.7213$

Revised 95% prediction interval for sales in week 57 when $y_{53} = 330$

$1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2 + \alpha^2(1+3\gamma)^2$

$= 1.1595 + \alpha^2(1+3\gamma)^2 = 1.1595 + (.247)^2(1+3(.095))^2 = 1.2602$

$[\hat{y}_{57}(53) \pm z_{[.025]} s\sqrt{1 + \alpha^2(1+\gamma)^2 + \alpha^2(1+2\gamma)^2 + \alpha^2(1+3\gamma)^2}\,]$

$= [341.7213 \pm 1.96(27.89)\sqrt{1.2602}] = [341.7213 \pm 61.37] = [280.35,\ 403.09]$

b. Revised level and growth rate when $y_{54} = 320$

$\ell_{54} = \alpha y_{54} + (1-\alpha)[\ell_{53} + b_{53}] = .247(320) + .753[322.8089 + 4.7281] = 325.6754$

$b_{54} = \gamma[\ell_{54} - \ell_{53}] + (1-\gamma)b_{53} = (.095)[325.6754 - 322.8089] + .905(4.7281) = 4.5512$

3. Exercise 8.11.

a. Using the error correction form of the smoothing equations,

$$\ell_2 = \ell_1 + b_1 + \alpha[y_2 - (\ell_1 + b_1 + sn_{2-4})]$$
$$= \ell_1 + b_1 + \alpha[y_2 - \hat{y}_2(1)] = 22.3079 + 1.0286 + .2[1.1105] = 23.5586$$
$$b_2 = b_1 + \alpha\gamma[y_2 - (\ell_1 + b_1 + sn_{2-4})]$$
$$= \mathrm{b}_1 + \alpha\gamma[y_2 - \hat{y}_2(1)] = 1.0286 + (.2)(.1)[1.1105] = 1.0508$$
$$sn_2 = sn_{2-4} + (1-\alpha)\delta[y_2 - (\ell_1 + b_1 - sn_{2-4})] = sn_{-2} + (1-\alpha)\delta[y_2 - \hat{y}_2(1)]$$
$$= 6.5529 + (.8)(.1)[1.1105] = 6.6417$$

(Round - off error because spreadsheet carried more decimal places)

b. $\hat{y}_3(2) = \ell_2 + b_2 + sn_{3-4} = 23.5586 + 1.0508 + 18.5721 = 43.1815$

One-period-ahead forecast error in period 3 is $y_3 - \hat{y}_3(2) = 43 - 43.1815 = -0.1815$.

c. $\ell_3 = \ell_2 + b_2 + \alpha[y_3 - (\ell_2 + b_2 + sn_{3-4})]$
$$= \ell_2 + b_2 + \alpha[y_3 - \hat{y}_3(2)] = 23.5586 + 1.0508 + .2[-0.1815] = 24.5731$$
$$b_3 = b_3 + \alpha\gamma[y_3 - (\ell_2 + b_2 + sn_{3-4})]$$
$$= \mathrm{b}_3 + \alpha\gamma[y_3 - \hat{y}_3(2)] = 1.0508 + (.2)(.1)[-0.1815] = 1.0472$$
$$sn_3 = sn_{3-4} + (1-\alpha)\delta[y_3 - (\ell_2 + b_2 - sn_{3-4})]$$
$$= sn_{-1} + (1-\alpha)\delta[y_3 - \hat{y}_3(2)] = 18.5721 + (.8)(.1)[0.1815] = 18.5576 \quad \text{(Round - off error)}$$

d. $\hat{y}_4(3) = \ell_3 + b_3 + sn_{4-4}$
$$= \ell_3 + b_3 + sn_0 = 24.5731 + 1.0472 + (-10.9088) = 14.7115$$

One-period-ahead forecast error in period 4 is $y_4 - \hat{y}_4(3) = 16 - 14.7115 = 1.2885$.