ISyE 4232 Ayhan
Springl 2014
Due: February 4, 2014

Homework 2

1. At registration at a very small college, students arrive at the English table with respect to a Poisson process of rate 10/hr and the Math table with respect to a Poisson process of rate 5/hr. A student who completes service at the English table goes to the Math table with probability 1/4 and to the cashier with probability 3/4. A student who completes service at the Math table goes to the English table with probability 2/5 and to the cashier with probability 3/5. Students who reach the cashier leave the system after they pay. Suppose that the service times for the English table, Math table, and cashier are exponentially distributed with rates 25/hr, 30/hr, and 20/hr, respectively.
   a. Does the stationary joint distribution of the number of students at the English table, Math table, and cashier exist? If it does, compute it.
   b. What is the expected number of students in the system in the long-run?
   c. What is the expected time that a student spends while registering for the classes?

2. Consider a taxi station at an airport where taxis and customers arrive with respect to Poisson processes of rates 2/min and 3/min, respectively. Suppose that a taxi will wait no matter how many other taxis are present. However, if an arriving person does not find a taxi waiting he leaves to find an alternative transportation.
   a. What is the long run probability that an arriving customer gets a taxi?
   b. What is the average number of taxis waiting?

3. Consider a production system consisting of three single-server stations in series. Customer orders arrive at the system according to a Poisson

process with rate 1 per hour. Each customer order immediate triggers a job that is released to the production system to be processed at station 1 first, and then at station 2. After being processed at station 2, a job has $p_1 = 10\%$ probability going back to station 1 for rework and $1 - p_1$ probability continuing onto station 3. After being processed at station 3, a job has $p_2 = 5\%$ probability going back to station 1, $p_3 = 10\%$ probability going back to station 2, and $1 - p_2 - p_3$ probability leaving the production system as a finished product. Assume that the processing times of jobs at each station are iid, having exponential distribution, regardless of the history of the jobs. The average processing times at stations 1, 2 and 3 are $m_1 = 0.8$, $m_2 = 0.70$ and $m_3 = 0.8$ hours, respectively.

a. Find the long-run fraction of time that there are 2 jobs at station 1, 1 job at station 2 and 4 jobs at station 3.

b. Find the long-run average (system) size at station 3.

c. Find the long-run average time in system for each job.

d. Reduce p_1 to 5%. Answer 1(c) again. What story can you tell?