Name Key

**Exam 1 ISyE 4301**

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_______________________________________(*Signature*)

*The following multiple-choice questions are worth 5 points each. Please circle the single "best" answer. Clearly mark your answer.*

1. Flights from A to B and B to C each have a capacity of 100 seats in coach class (all seats are equivalent). The prices of tickets from A to B have 4 fare classes (F1 to F4) with prices of \$500, \$300, \$270, and \$180, respectively. Bid prices are found to be y1=200 and y2=110 (1 corresponding to the leg from A to B and 2 from B to C).


Which of the following is true for tickets sold from A to B only?

a. Close fare class F1
b. Close fare classes F1 and F2 close F4
c. Close fare classes F1, F2, and F3
d. Can't determine which fare classes to close since don't know the number of tickets in the problem specification
e. None of the above

2. A monopolist has a demand function of Q=100-2P. The marginal cost is \$5, and is independent of Q. At the optimal price:

a. Demand is unit elastic
b. Demand is elastic
c. Demand is inelastic
d. We don't have enough information to determine

$\pi = (50 - .5Q)Q - 5Q$

$\pi' = 45 - Q = 0 \rightarrow Q = 45$

$p = 50 - .5(45) = 27.5$

at $p^*$

$\epsilon = 2\left(\frac{27.5}{45}\right) = 1.22 \rightarrow$ elastic

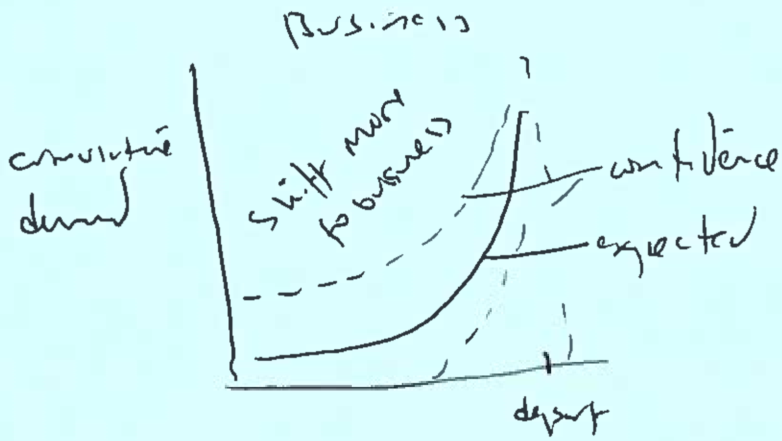
3. A store uses a $(Q,R)$ policy for an item with an order quantity of 80 (based on EOQ). The annual demand faced by the store for the item is normally distributed with a mean of 1000 and standard deviation of 300. The replenishment leadtime is 2 weeks. They desire a service level of 95%. What is their average pipeline stock for this item?
   a. 38.5
   b. 40
   c. 96.8
   d. Can't be computed from the data given
   e. None of the above

pipeline $= LD = \frac{2}{52}(1000) = 38.5$

4. (*10 points*) Answer the following:

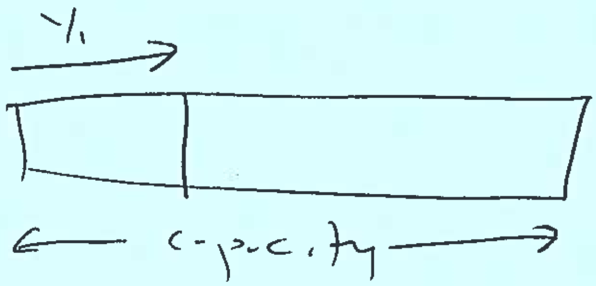
   a. Explain how booking curves work. Please be specific.

Booking curves tell you when to reallocate more seats to a class by seeing how actual cumulative demand deviates from expected. Example for business type customers shown left.




   b. What is the value of "nesting" in revenue management?

Give flexibility to deal with uncertainty as opposed to a strict partition




For example, if $y_1$ is protection level for highest paying class (as shown left), then $y_1$ seats are held, but if demand arises, more would be allowed to be sold.

5. (*15 points*) A hospital uses two types of surgical kits (M1 and M2). They use a $(Q,R)$ system with a service level of 96%. Replenishment leadtime is 1 week. Each time they place an order, there is a transaction cost of $500 (independent of order size). Each surgical kit costs $30 and they use a discount rate of 18%. Annual demand for each is normally distributed: M1 has a mean of 10,000 and a standard deviation of 2000; M2 has a mean of 8,000 and a standard deviation of 3000. The correlation between the two kit types equals 0.35. They are considering using only one type of surgical kit, and have found that mean demand would be approximately the same. Determine how much the safety stock, cycle stock, and pipeline stock will change if they go to a single kit type.

If independent

$$SS_T = \underset{1.75}{\underbrace{Z_{.96}}} \sqrt{\frac{1}{52}}\,(2000 + 3000) = 1214 \text{ units}$$

$$Q_1 = \sqrt{\frac{2(500)(10000)}{.18(30)}} = 1361 \qquad Q_2 = \sqrt{\frac{2(500)(8000)}{.18(30)}} = 1217$$

so cycle $= \dfrac{1361 + 1217}{2} = 1289$

pipeline $= \frac{1}{52}(10000 + 8000) = 346$

If pool

$$\sigma_T = \left[(2000^2 + 3000^2) + 2(.35)(2000)(3000)\right]^{1/2} = 4147$$

$$SS_T = 1.75\sqrt{\frac{1}{52}}\,(4147) = 1007 \quad \text{(reduction of 207)}$$

$$Q = \sqrt{\frac{2(500)(10000 + 18000)}{.18(30)}} = 1825$$

$$\text{cycle} = \frac{1825}{2} = 913 \quad \text{(reduction of 376)}$$

pipeline $= \frac{1}{52}(10000 + 8000) = 346$ (no difference)

6. (*15 points*) An agency provides a dental tourism service where patients are bused from Phoenix, Arizona to Los Algodones, Mexico each morning to receive dental procedures. They serve insured (P), government insured (G), and uninsured (U) patients, and charge a different price to each. The prices charged to P, G, and U are \$600, \$400, and \$320, respectively. The daily demands from each patient type are normally distributed with means and standard deviations of P=(mean=30, std dev=10), G=(30, 20), and U=(40, 20). Assuming the bus holds 60 patients, determine the booking limits using the EMSR heuristic. (Assume the marginal cost is \$0).

Using EMSR

$$y_P = F_P^{-1}\left(1 - \frac{400}{600}\right)$$

$$\rightarrow y_P = 30 + z_{.33}(10)$$

$$= 30 - .44(10) = 26$$

To compare P and G to U

$$\bar{p} = \frac{600(30) + 400(30)}{60} = 500$$

$$\sigma = (10^2 + 20^2)^{1/2} = 22.4$$

so $y_G = F^{-1}\left(1 - \frac{320}{500}\right)$

$$y_G = (30 + 30) + z_{.36}(22.4) = 52$$

$$\rightarrow \quad b_u = 60 - 52 = 8$$

$$b_G = 60 - 26 = 34$$

7. (*15 points*) A monopolist has a demand function of $Q$=100-2$P$. The marginal cost is $5, and is independent of $Q$. Answer the following:

   a. Determine if the good is elastic or inelastic at the optimal price.

$$\pi = (50 - .5Q)Q - 5Q$$

$$\pi' = 45 - Q = 0 \quad \longrightarrow \quad Q^* = 45$$

$$P^* = 27.5$$

$$\epsilon = -(-2)\left(\frac{27.5}{45}\right) = 1.23 \quad \text{(elastic)}$$

   b. Given you answer in a, is there a different price where the good would change its type of elasticity? (e.g., if you found the good is elastic in a, is there a price where it would become inelastic?). If so, find a price that works, and if not, explain why not.

The good will be unit elastic at $p = 22.5$ and any price below that would lead to

$$\frac{2P}{45} < 1 \qquad (\text{if } p < 22.5)$$

8. (*15 points*) A concert venue faces two types of demand. Demand 1 is given by $Q_1=100-P_1$ and demand 2 is given by $Q_2=120-2P_2$. The marginal cost of a unit is \$10. What is their optimal profit if they can't segment the market?

Can only charge a single price — charge 1 price

$$\pi = [p(100-p) - 10(100-p)] + [p(120-2p) - 10(120-2p)]$$

$$\pi' = [100 - 2p + 10] + [120 - 4p + 20] = 0$$

$$\rightarrow p^* = 41.7$$

$$\pi = \$3008$$

9. (*15 points*) Utility is given by $U(X,Y) = 10X^{0.5}Y^{0.5}$. The price of a unit of X is \$10 and a unit of Y is \$15. There is a total budget of \$100. What is the change in the demand for X if the price of a unit of X goes up to \$14 (assume the budget doesn't change)?

max $10x^{.5}y^{.5}$

s.t. $10x + 15y = 100$

$\mathcal{L} = 10x^{.5}y^{.5} + \lambda(100 - 10x - 15y)$

$\frac{\partial \mathcal{L}}{\partial x} = 5x^{-.5}y^{.5} - 10\lambda = 0$ (1)

$\frac{\partial \mathcal{L}}{\partial y} = 5x^{.5}y^{-.5} - 15\lambda = 0$ (2)

$\frac{\partial \mathcal{L}}{\partial \lambda} = 100 - 10x - 15y = 0$ (3)

Take ratio of (1) to (2) gives

$\frac{y}{x} = \frac{10}{15} \longrightarrow y = .67x$

plug into (3)

$100 - 10x - 15(.67x) = 0$

$\longrightarrow x = 5$

If $P_y = 14$ get

$\frac{y}{x} = \frac{14}{15} \longrightarrow y = .93x$

$100 - 14x - 15(.93x) = 0$

$\longrightarrow x = 3.58$

so x goes down from 5 to 3.58

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*The following multiple-choice questions are worth 5 points each. Please circle the single "best" answer. Clearly mark your answer.*

1. A store uses a $(Q,R)$ policy for an item with an order quantity of 80 (based on EOQ). The annual demand faced by the store for the item is normally distributed with a mean of 1000 and standard deviation of 300. The replenishment leadtime is 2 weeks. They desire a service level of 95%. What is their average pipeline stock for this item?
   a. 38.5
   b. 40
   c. 96.8
   d. Can't be computed from the data given
   e. None of the above

pipeline = LD

$= \frac{2}{52}(1000)$

$= 38.5$

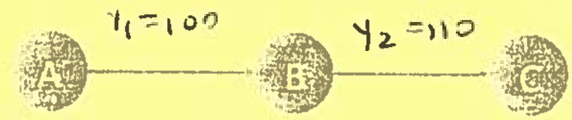
2. The BL Bike Company uses two main inputs $X$ and $Y$. Output $(Q)$ is defined by the Cobb Douglas production function: $Q = 300X^{0.7}Y^{0.4}$. The cost to BL is \$400 per unit of X and \$500 per unit of Y.
   a. Production exhibits decreasing returns to scale
   b. Production exhibits increase returns to scale
   c. The average cost per bike when Q=30 is \$930
   d. Both a. and c. are true
   e. Both b. and c. are true
   f. None of the above

$0.7 + 0.4 > 1$

$\rightarrow$ ~~[illegible]~~ increasing returns

(accepted b or e)

3. Flights from A to B and B to C each have a capacity of 130 seats in coach class (all seats are equivalent). The prices of tickets from A to B have 4 fare classes (F1 to F4) with prices of \$500, \$300, \$270, and \$180, respectively. Bid prices are found to be y1=100 and y2=110 (1 corresponding to the leg from A to B and 2 from B to C).




Which of the following is true for tickets sold from A to B only?

a. Close fare class F1
b. Close fare classes F1 and F2
c. Close fare classes F1, F2, and F3
d. Can't determine which fare classes to close since don't know the number of tickets in the problem specification
e. None of the above

for A-B, no ticket < \$100

so keep all classes open

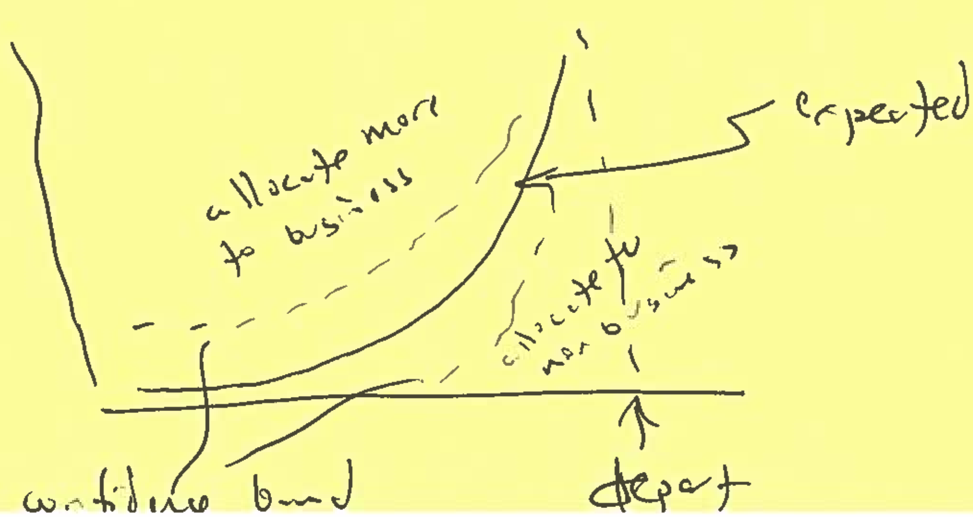
4. (*10 points*) Answer the following:

a. What is the value of "nesting" in revenue management?

Rather than a strict partition, we use nesting in revenue management through protection levels (or booking limits). The value arises from the fact that demand is not known with certainty, and so this gives us the flexibility to sell more higher priced tickets if that demand materializes.

b. Explain how booking curves work. Please be specific.

Booking curves show how cumulative tickets sold changes as we approach departure date. They are based on historical sales, and helps us to reallocate if our demand differs. An example of a business booking curve is below



5. (*15 points*) The BL Bike Company uses two main inputs *X* and *Y*. Output (*Q*) is defined by the Cobb Douglas production function: $Q = 300X^{2\alpha}Y^{-\alpha}$. The cost to BL is $400 per unit of X and $500 per unit of Y. Answer the following:

   a. What is the value of α for which the production function exhibits increasing returns to scale? Please be clear if there is more than one value, or if a value doesn't exist.

want $2\alpha - \alpha > 1 \rightarrow \alpha > 1$

   b. Suppose α=0.6 and that BL wants to make 40 bikes. Determine the average cost per bike.

$\min TC = 400X + 500Y$ $\qquad AC = \frac{TC}{Q}$

s.t. $300X^{2\alpha}Y^{-\alpha} = 40$

$\mathcal{L} = 400X + 500Y + \lambda(40 - 300X^{2\alpha}Y^{-\alpha})$

$\frac{\partial \mathcal{L}}{\partial X} = 400 - 360X^{.2}Y^{-.6} \equiv 0$ (1)

$\frac{\partial \mathcal{L}}{\partial Y} = 500 + 180X^{1.2}Y^{-1.6} \equiv 0$ (2)

$\frac{\partial \mathcal{L}}{\partial \lambda} = 40 - 300X^{1.2}Y^{-.6} \equiv 0$ (3)

Take ratio of (1) over (2)

$\frac{400}{500} = -\frac{2X}{Y} \rightarrow X = -2.5Y$

so $40 + 300Y^{-.6}(2.5Y)^{1.2} \rightarrow Y = -.044$

$X = .11$

Note: There was a typo on Q. It should have been $Q = 300X^{2\alpha}Y^{\alpha}$. The $-\alpha$ doesn't make sense in this case. Therefore, if you set it all up correctly, you received credit.

6. (*15 points*) A concert venue faces two types of demand. Demand 1 is given by $Q_1 = 140 - 2P_1$ and demand 2 is given by $Q_2 = 110 - P_2$. The marginal cost of a unit is \$15. What is their optimal profit if they can't segment the market?

$$\pi = \left[P(140 - 2P) - 15(140 - 2P)\right] + \left[P(110 - P) - 15(110 - P)\right]$$

→ charge 1 price

$$\pi' = 140 - 4P + 30 + 110 - 2P + 15 = 0$$

$$295 - 6P = 0$$

$$\rightarrow P^* = \$49.17$$

$$\pi = 2502.08$$

7. (*15 points*) Utility is given by $U(X,Y) = 10X^{0.5}Y^{0.5}$. The price of a unit of X is \$12 and a unit of Y is \$13. There is a total budget of \$100. What is the change in the demand for Y if the price of a unit of X goes up to \$14 (assume the budget doesn't change)?

max $10x^{.5}y^{.5}$

s.t. $12x + 13y = 100$

$$\mathcal{L} = 10x^{.5}y^{.5} + \lambda(100 - 12x - 13y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 5x^{-.5}y^{.5} - 12\lambda \equiv 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = 5x^{.5}y^{-.5} - 13\lambda \equiv 0 \quad (2)$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 100 - 12x - 13y \equiv 0 \quad (3)$$

take ratio of (1) to (2)

$$\frac{y}{x} = \frac{12}{13} \rightarrow y = \frac{12}{13}x$$

plug into (3)

$$100 - 12x - 13\left(\frac{12}{13}\right)x = 0$$

$$\rightarrow x = 4.17 \quad y = 3.85$$

If $P_x \rightarrow 14$

$$\rightarrow \frac{y}{x} = \frac{14}{13} \rightarrow y = \frac{14}{13}x$$

$$\rightarrow 100 - 14x - 13\left(\frac{14}{13}x\right) = 0$$

$$\rightarrow x = 3.57 \quad y = 3.84$$

$\rightarrow$ y decreases by .01

8. (*15 points*) A hospital uses two types of surgical kits (M1 and M2). They use a ($Q$,$R$) system with a service level of 94%. Each time they place an order, there is a transaction cost of \$500 (independent of order size). Each surgical kit costs \$60 and they use a discount rate of 18%. Annual demand for each is normally distributed: M1 has a mean of 8,000 and a standard deviation of 2000; M2 has a mean of 7,000 and a standard deviation of 3000. The correlation between the two kit types equals -0.25. They are considering using only one type of surgical kit, and have found that mean demand would be approximately the same. Determine how the safety stock, cycle stock, and pipeline stock will change if they go to a single kit type.

If independent

$$SS_T = Z_{.94}\sqrt{\frac{1}{52}}\,(2000 + 3000) = 1075$$

$$\uparrow$$
$$1.55$$

$$Q_1 = \sqrt{\frac{2(500)(8000)}{.18(60)}} = 861 \qquad Q_2 = \sqrt{\frac{2(500)(7000)}{.18(60)}} = 805$$

$$\text{cycle} = \frac{861 + 805}{2} = 833$$

$$\text{pipeline} = \frac{1}{52}(8000 + 7000) = 288$$

If pool

$$\sigma_T = \left(2000^2 + 3000^2 - 2(.25)(2000)(3000)\right)^{1/2} = 3162$$

$$SS_T = 1.55\sqrt{\frac{1}{52}}\,(3162) = 680$$

$$Q_T = \sqrt{\frac{2(500)(7000 + 8000)}{.18(60)}} = 1178.5$$

$$\text{cycle} = 1179/2 = 589$$

$$\text{pipeline} = \frac{1}{52}(8000 + 7000) = 288$$

9. (*15 points*) An agency provides a dental tourism service where patients are bused from Phoenix, Arizona to Los Algodones, Mexico each morning to receive dental implants. They serve insured (P), government insured (G), and uninsured (U) patients, and charge a different rate to each. The prices charged to P, G, and U are \$700, \$400, and \$220 per procedure, respectively. The daily demands from each patient type are normally distributed with means and standard deviations of P=(20, 10), G=(25, 20), and U=(50, 20). Assuming the bus holds 60 patients, determine the booking limits using the EMSR heuristic. (Assume the marginal cost is \$0).

Using EMSR

$$y_p = F^{-1}\left(1 - \frac{400}{700}\right) = F^{-1}(.428)$$

$$y_p = 20 + z_{.428}(10) = 20 - .181(10)$$

$$= 18$$

To compare P and G to U

$$\bar{p} = \frac{700(20) + 400(25)}{20 + 25} = 533.3$$

$$\sigma = (10^2 + 20^2)^{1/2} = 22.4$$

so

$$y_{P+G} = \left(1 - \frac{220}{533.3}\right) = .587$$

$$y_G = (20 + 25) + z_{.587}(22.4)$$

$$= 50$$

$$\rightarrow b_u = 60 - 50 = 10$$

$$b_G = 60 - 18 = 42$$