**Homework 6**
**SOLUTIONS**

1. Solve the Farmer Jones problem (without government) USING GAMS, include the GAMS output file.

$$
\begin{aligned}
\max \quad z = & (3)(10)x_1 + (4)(25)x_2 + 0x_3 + 0x_4 + 0x_5 && (1)\\
& \text{subject to} \\
& x_1 + x_2 + x_3 = 7 \qquad \text{Acres} && (2)\\
& 4x_1 + 10x_2 + x_4 = 40 \qquad \text{Labor} && (3)\\
& x_i \geq 0 \qquad \forall i = 1,2,3,4,5 && (4)
\end{aligned}
$$

**GAMS output added as other resource. From that we have the optimal basis as $x_3, x_2$.**

$$
B = \begin{bmatrix} 1 & 1 \\ 0 & 10 \end{bmatrix}
$$

$$
B^{-1} = \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix}
$$

$$
c_{BV} = \begin{bmatrix} 0 & 100 \end{bmatrix}
$$

2. What does the price of corn need to be for it to enter the basis?

**Solution:** Require $\bar{c}_1 < 0$

$$
\begin{aligned}
\bar{c}_1 &= c_{BV} B^{-1} a_1 - c_1 \\
&= \begin{bmatrix} 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} - c_1 \\
&= 40 - c_1 \\
c_1 &\geq 40
\end{aligned}
$$

Total price per acre should be at least 40, or price per bushel of at least 4.

3. How much does your income change if you get 10 extra labor hours?

   **Solution:** Income is $c_{BV}B^{-1}b$

$$
\begin{aligned}
z &= c_{BV}B^{-1}b \\
&= \begin{bmatrix} 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 7 \\ 50 \end{bmatrix} \\
&= 500
\end{aligned}
$$

4. How much does your income change if you lose 2 acres?

   **Solution:**

$$
\begin{aligned}
z &= c_{BV}B^{-1}b \\
&= \begin{bmatrix} 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 5 \\ 40 \end{bmatrix} \\
&= 400
\end{aligned}
$$

   Note there is no change because the acres constraint was not binding, and when we lower the rhs by 2, our original basis remains feasible.

5. Suppose Farmer Jones wants to think about growing soy beans. Each acre of soy beans yields 20 bushels, and each bushel sells for \$3. What is the most amount of labor it can take to farm an acre of soy beans for Farmer Jones to grow them?

   **Solution:** Want the reduced cost of the new variable to be less than or equal to 0.

$$
\begin{aligned}
\bar{c}_s &= c_{BV}B^{-1}a_1 - c_s \\
&= \begin{bmatrix} 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ x \end{bmatrix} - 60 \\
&= 10x - 60 \\
x &\leq 6
\end{aligned}
$$

6. Suppose it only takes 3 hours to farm an acre of corn, how does this affect your optimal solution?

**Solution:** Look at how this affects the reduced cost of corn.

$$\begin{aligned}
\bar{c}_1 &= c_{BV}B^{-1}a_1 - c_1 \\
&= \begin{bmatrix} 0 & 100 \end{bmatrix} \begin{bmatrix} 1 & -1/10 \\ 0 & 1/10 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 30 \\
&= 30 - 30 \\
\bar{c}_1 &= 0
\end{aligned}$$

Because the reduced cost is 0, our old basis is still optimal, but there is another basis that is optimal that involves corn. There are multiple optimal solutions, that will yield z=400.

7. Consider the following LP:

$$\max \quad z = 5x_1 + 3x_2 + 4x_3 \tag{5}$$

subject to

$$x_1 + \frac{3}{2}x_2 + x_3 \leq 4 \tag{6}$$

$$2x_1 + x_2 + \frac{3}{2}x_3 \leq 5 \tag{7}$$

$$x_i \geq 0 \qquad \forall i = 1, 2, 3 \tag{8}$$

For each of the following choices of Basis, determine if it is optimal or not. State why. $\begin{bmatrix} x_1 \\ x_5 \end{bmatrix}, \begin{bmatrix} x_3 \\ x_2 \end{bmatrix}, \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$

**Solution:** To determine if a choice of basis is optimal, we check row 0 and the right hand side to make sure all values are $\geq 0$.

**Basis** $\begin{bmatrix} x_1 \\ x_5 \end{bmatrix}$

With this basis, the matrix B= $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

$$\begin{aligned}
\bar{c} &= c_{BV}B^{-1}A - c \\
&= \begin{bmatrix} 0 & 4.5 & 1 & 5 & 0 \end{bmatrix}
\end{aligned}$$

These values are all positive. Now need to check the right hand sides.

$$\begin{aligned}
\text{rhs} &= B^{-1}b \\
&= \begin{bmatrix} 4 \\ -3 \end{bmatrix}
\end{aligned}$$

A negative value in the right hand side means a basic variable is less than or equal to 0. Therefore this choice of basis is infeasible.

**Basis** $\begin{bmatrix} x_3 \\ x_2 \end{bmatrix}$

With this basis, the matrix B= $\begin{bmatrix} 1 & 3/2 \\ 3/2 & 1 \end{bmatrix}$.

$$\begin{aligned} \bar{c} &= c_{BV}B^{-1}A - c \\ &= \begin{bmatrix} 0.2 & 0 & 0 & 0.4 & 2.4 \end{bmatrix} \end{aligned}$$

These values are all positive. Now need to check the right hand sides.

$$\begin{aligned} \text{rhs} &= B^{-1}b \\ &= \begin{bmatrix} 2.8 \\ 0.8 \end{bmatrix} \end{aligned}$$

These are also positive. We have our optimal basis.

**Basis** $\begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$

With this basis, the matrix B= $\begin{bmatrix} 1 & 3/2 \\ 1 & 0 \end{bmatrix}$.

$$\begin{aligned} \bar{c} &= c_{BV}B^{-1}A - c \\ &= \begin{bmatrix} 1/3 & -1/3 & 0 & 0 & 8/3 \end{bmatrix} \end{aligned}$$

A negative value in Row 0 means this basis is not optimal.