MATH 1552 - SPRING 2016
QUIZ 5 - SHOW YOUR WORK

NAME: ______________________________ TA: ___________________________

1. (5 points) Does the series $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converge or diverge? Use the Direct Comparison Test.

$$n^3 \le n^3 + 1 \;\Rightarrow\; \frac{1}{n^3+1} \le \frac{1}{n^3} \;\Rightarrow\; \frac{n}{n^3+1} \le \frac{n}{n^3} = \frac{1}{n^2}$$

The series $\sum_{m=1}^{\infty} \frac{1}{n^2}$ converges ($p - series, p = 2 > 1$). **By the DCT $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converges**

2. (10 points) Does the series $\sum_{n=1}^{\infty} \frac{n\ln(n)}{n^2+1}$ converge or diverge? Use the Limit Comparison Test, with $b_n = \frac{1}{n}$

$a_n = \frac{n\ln(n)}{n^2+1}$ and Let $b_n = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2\ln(n)}{n^2+1}$. This is the indeterminant form $\frac{\infty}{\infty}$. Applying L'Hopital Rule

$$\lim_{n\to\infty} \frac{n^2\ln(n)}{n^2+1} = \lim_{n\to\infty} \frac{(2n\ln(n)+n)}{2n} = \lim_{n\to\infty} \frac{n(2\ln(n)+1)}{2n} = \lim_{n\to\infty} \ln(n) + \frac{1}{2} = \infty$$

By the LCT, since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges then $\sum_{n=1}^{\infty} \frac{n\ln(n)}{n^2+1}$ diverges

TA's: The important thing is evaluating: $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2\ln(n)}{n^2+1}$

3. (10 points) $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{n^2(n+2)}{3^n}$. a. (7 points) Does this series converge absolutely? b. (3 points) What can you say about the convergence or divergence of the series? Explain. **DO NOT USE THE AST**.

a. Use the ratio test:

$$\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{(n+1)^2(n+3)}{3^{n+1}} \quad \frac{3^n}{n^2\,(n+2)} = \lim_{n \to \infty} \frac{(n+1)^2\,(n+3)}{3\,n^2\,(n+2)} = \frac{1}{3} \Rightarrow$$

$$\textbf{\textit{ratio test}} \Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2(n+2)}{3^n} \ \textbf{converges absolutely}$$

b. $A\,C \Rightarrow C$ and so $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{n^2(n+2)}{3^n}$ **converges (Student must say something like** $\boldsymbol{AC \Rightarrow C}$)

4. (5 points) Does the series $\sum_{n=1}^{\infty} \left[\ln\left(e^2 + \dfrac{1}{n}\right)\right]^n$ converge or diverges? **Use the root test.**

Use the root test :

$$\left\{ \left[\ln\left(e^2 + \frac{1}{n}\right)\right]^n \right\}^{\frac{1}{n}} = \ln\left(e^2 + \frac{1}{n}\right) \to \ln\left(e^2\right) = 2 > 1 \Rightarrow \sum_{n=1}^{\infty} \left[\ln\left(e^2 + \frac{1}{n}\right)\right]^n \ \textbf{diverges}$$