MATH 2403 H1-H3. Differential Equations. Quiz 4. Feb 18, 2014.
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Student's Name: Sol Section

Show all work to receive credit

1. Show that the equation
$$y\,dx + (2x - e^y)\,dy = 0$$
is not exact but it becomes exact with the integrating factor $\mu = y$. Find the solution.

$y^2\,dx + (2xy - ye^y)\,dy = 0$

$\frac{\partial M}{\partial y} = 2y$, $\frac{\partial N}{\partial x} = 2y$ ← exact.

There is ψ such that

① $\frac{\partial \psi}{\partial x} = y^2$, ② $\frac{\partial \psi}{\partial y} = 2xy - ye^y$

From ①: $\psi = xy^2 + h(y) \rightarrow \frac{\partial \psi}{\partial y} = 2xy + h'(y)$

From ②: $h'(y) = -ye^y \rightarrow h(y) = -ye^y + \int e^y$
$= -ye^y + e^y$

$\therefore \boxed{\psi = xy^2 - ye^y + e^y = C}$

2. Write the system using matrix notation:
$$x' = x + y + 4, \qquad y' = -2x + (\sin t)y.$$

Is the system autonomous? Yes No

Is the system homogeneous? Yes No

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ -2 & \sin t \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$