

This quiz contains 2 questions. Write neatly and show all your work.

1. Find the center and radius of the sphere $4x^2 + 4y^2 + 4z^2 - 8x + 4z + 1 = 0$. [3]

$$\begin{aligned}\Rightarrow x^2 + y^2 + z^2 - 2x + z + \frac{1}{4} &= 0 \\ \Rightarrow (x^2 - 2x) + y^2 + (z^2 + z) &= -\frac{1}{4} \\ \Rightarrow (x^2 - 2x + 1) + y^2 + (z^2 + z + \frac{1}{4}) &= -\frac{1}{4} + 1 + \frac{1}{4} \\ \Rightarrow (x-1)^2 + (y-0)^2 + (z+\frac{1}{2})^2 &= 1^2\end{aligned}$$

$$\therefore \text{Center} = (1, 0, -\frac{1}{2})$$

$$\text{radius} = 1.$$

2. Let $P = (1, 0, -1)$, $Q = (2, -2, 1)$ and $\vec{PR} = -\mathbf{i} + \mathbf{j}$.

(a) Find the unit vector in the direction of \vec{PQ} . [1.5]

$$\begin{aligned}\vec{PQ} &= \vec{OQ} - \vec{OP} = \langle 2, -2, 1 \rangle - \langle 1, 0, -1 \rangle \\ &= \langle 1, -2, 2 \rangle\end{aligned}$$

$$|\vec{PQ}| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{9} = 3$$

$$\therefore \text{Unit vector in the direction of } \vec{PQ} = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{\langle 1, -2, 2 \rangle}{3} = \langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \rangle$$

(b) Find the scalar component of \vec{PR} in the direction of \vec{PQ} . [1.5]

Scalar component of \vec{PR} in the direction of \vec{PQ}

$$= \vec{PR} \cdot \frac{\vec{PQ}}{|\vec{PQ}|}$$

$$= \langle -1, 1, 0 \rangle \cdot \langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \rangle$$

$$= -\frac{1}{3} - \frac{2}{3} + 0 = -\frac{3}{3} = -1.$$

(c) Find a vector perpendicular to the plane of $\triangle PQR$.

[2.5]

$\vec{PQ} \times \vec{PR}$ is a vector perpendicular to the plane of $\triangle PQR$.

$$\begin{aligned}\vec{PQ} \times \vec{PR} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 2 \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} -2 & 2 \\ 1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 1 & -2 \\ -1 & 1 \end{vmatrix} \vec{k} \\ &= (0-2) \vec{i} - (0+2) \vec{j} + (1-2) \vec{k} \\ &= -2\vec{i} - 2\vec{j} - \vec{k}.\end{aligned}$$

(d) Find the area of $\triangle PQR$. $= \frac{1}{2} |\vec{PQ} \times \vec{PR}|$

[1.5]

$$\begin{aligned}&= \frac{1}{2} \sqrt{(-2)^2 + (-2)^2 + (-1)^2} \\ &= \frac{1}{2} \sqrt{9} \\ &= \frac{3}{2}.\end{aligned}$$