BME 3400 Midterm Exam # 1 September 17, 2009 Name: SOLUTION

This is a closed-book exam. Calculators are allowed, but integrals should be solved before numbers are plugged in.

Show all your work! Free-body diagrams must be present and correct for full credit. Plug in numbers only at the end of a problem.

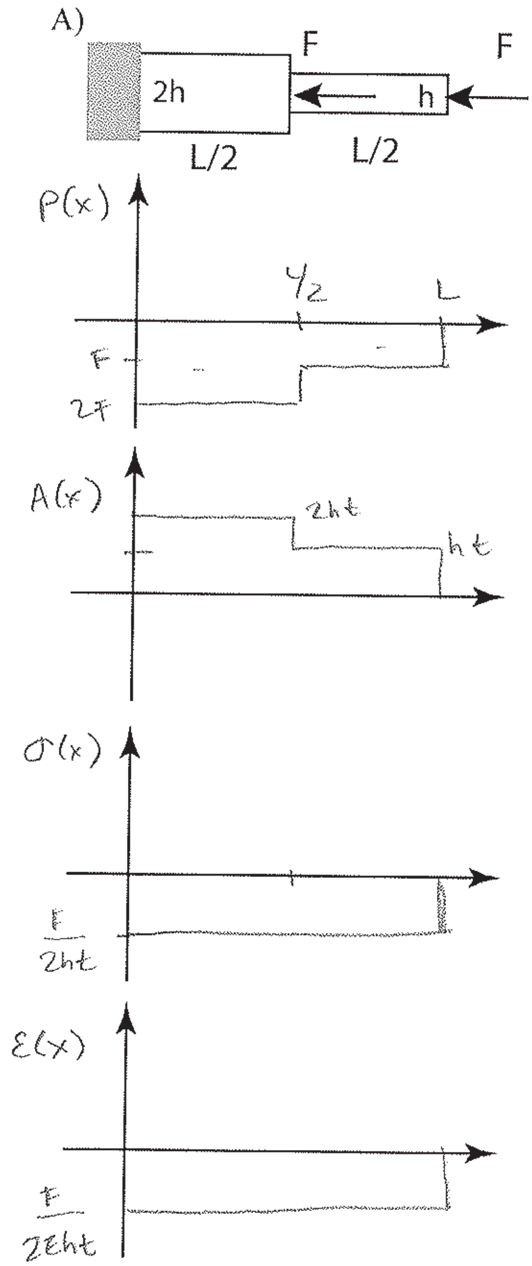
HONOR CODE
The conditions of this examination are subject to the Georgia Institute of Technology Academic Honor Code.

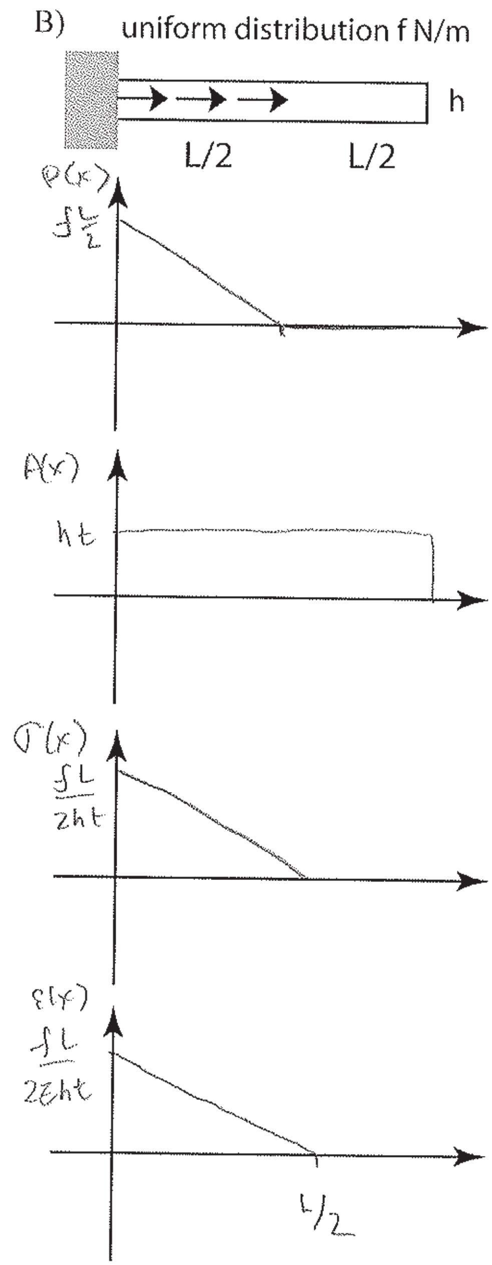
I pledge that the work in this exam represents my own, original work. I have not communicated with anyone about the contents of this exam, nor participated in or observed any conduct prohibited by the Honor Code.

Signature ____________________

Problem 1A-C (40 points)

Plot the internal load, cross-sectional area, normal stress, and normal strain in each bar (of thickness t, and modulus of elasticity E). ***Be sure to label all axes and key values.***





Give an equation for the total deformation of each bar.

A)

$$\delta = \frac{PL}{EA}$$

$$= \frac{2F \cdot \frac{L}{2}}{E\,2A} + \frac{F\,\frac{L}{2}}{EA} = \boxed{\frac{FL}{EA}}$$

B)

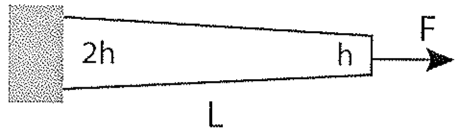
$$\delta = \int \frac{P(x)}{EA}\,dx$$

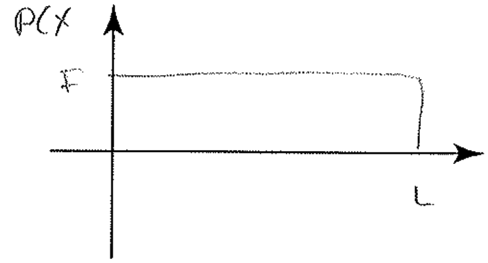
$$= \int_0^{L/2} \frac{\frac{fL}{2} - fx}{EA}\,dx$$

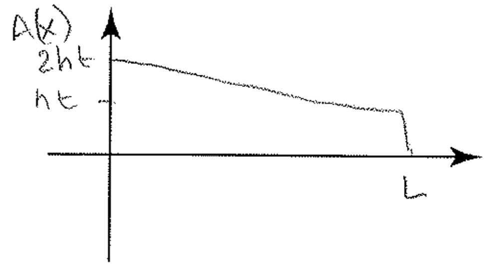
$$= \frac{fL}{2EA}x - \frac{1}{2}\frac{fx^2}{EA}\Big|_0^{L/2}$$

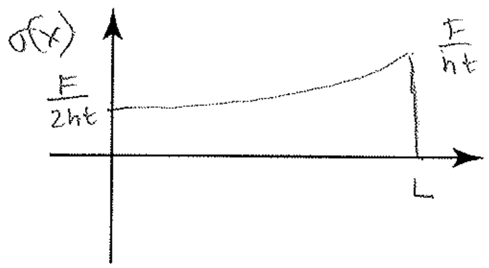
$$\delta = \frac{fL^2}{4EA} - \frac{1}{8}\frac{fL^2}{EA} = \boxed{\frac{1}{8}\frac{fL^2}{EA}}$$

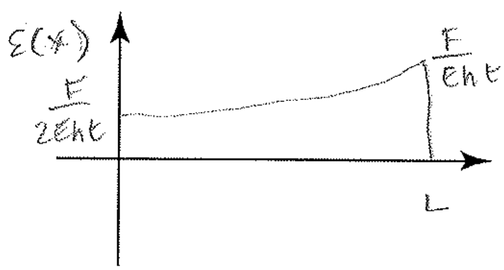
C)











Give an equation for the total deformation of each bar.

C)

$$\delta = \int \frac{P(x)}{E\,A(x)}\,dy$$

$$= \int_0^L \frac{F}{E\,A(x)}\,dx$$

$$A(x) = 2ht - \frac{ht}{L}x$$

$$\delta = \int_0^L \frac{F}{E\left(2ht - \frac{ht}{L}x\right)}\,dx$$

$$\delta = \frac{F}{Eht}\int_0^L \frac{dx}{2 - \frac{x}{L}}$$

$$= \frac{F}{Eht}(-L)\ln\left(2 - \frac{x}{L}\right)\Big|_0^L$$

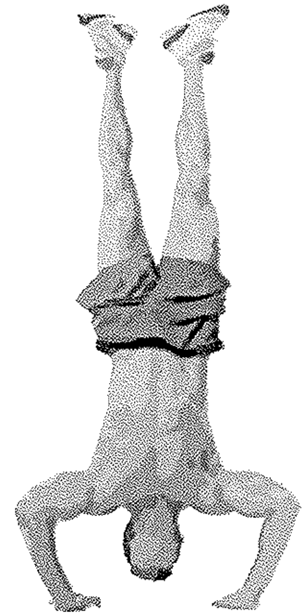
$$\delta = \frac{FL}{Eht}\ln(2)$$

Problem 2 (30 points)

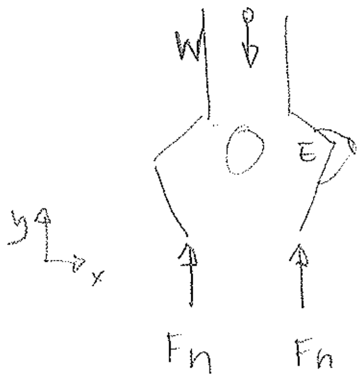
A man is doing a handstand with hands placed 0.5 m apart and forearms inclined 30° from vertical. His forearm is 30 cm in length. The moment arm of the triceps muscle is 3 cm.

How many times bodyweight is the force in his triceps?

How does the force change as the forearms become more vertical? Why?



whole body FBD

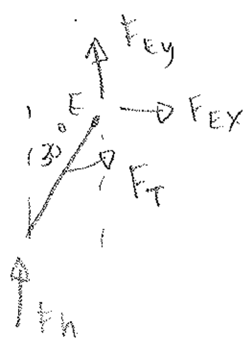


$\Sigma F_y = 0$

$2F_h - W = 0$

$\boxed{F_h = \frac{W}{2}}$

cut at elbow joint (a pin joint)



$\Sigma M/_E = 0$

$F_T \cdot d - F_h L \sin\theta = 0$

$L_{arm} = 0.3\ m$

$d = 0.03\ m$

$F_T = \frac{F_h L \sin\theta}{d}$

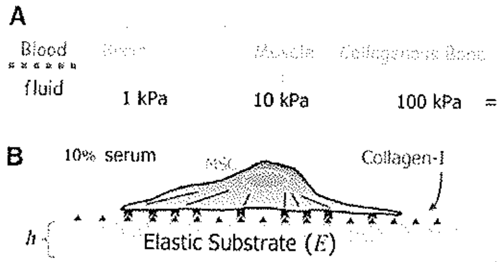
$= \frac{\frac{W}{2} \cdot (0.3\ m)(\frac{1}{2})}{0.03\ m}$

$\boxed{F_T = 2.5\ W}$

triceps force decreases as forearms go vertical because horizontal distance between hand and elbow decreases

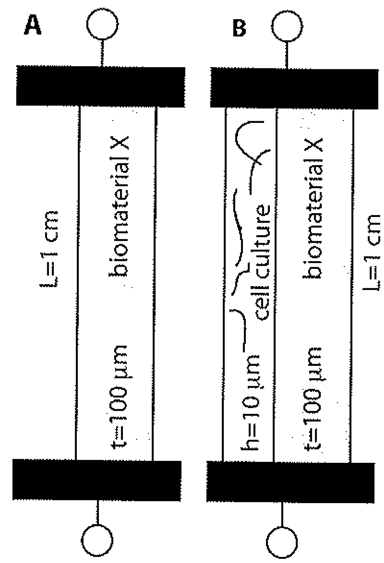
Problem 3 (30 points)

You are working in Dr. Barker's lab trying to regenerate blood vessels. You are growing mesenchymal stem cells (MSCs) in a culture dish on biomaterial 'X'. When stem cells are grown in a culture, their eventual cell type and elasticity depends upon the elasticity of the substrate they grow on.



Engler et al., *Cell* 2006

The culture dishes are 1 cm x 1 cm square
- Dish A is an empty culture dish of biomaterial X
- Dish B has a 10 micron layer of mature cells that have grown on a 100 micron layer of biomaterial X.



When you hang a mass of 1 gm from the end cuture dish, the distance between the rigid endplates increased by 2.4 mm in Dish A and 2.3 mm in Dish B.

What is the elasticity of the cell culture, and what kind of cells are likely to be growing in your dish?

empty dish

$$\delta_A = \frac{PL}{E_X A_X} = 0.0024 \text{ m}$$

$$E_Y = \frac{PL}{\delta_A A_Y} = \frac{(0.001 \text{ kg} \times 9.8 \text{ m/s}^2)(0.01 \text{ m})}{(0.0024 \text{ m})(0.01 \text{ m} \times 100 \times 10^{-6} \text{ m})}$$

$$E_X = 41 \text{ kPa}$$

cultured dish

$$\delta_B = \frac{P_X L}{E_Y A_Y} = \frac{P_C L}{E_C A_C} = 0.0023 \text{ mm}$$

P_X P_C

P

$$\Sigma F = 0$$

$$P = P_X + P_C$$

. Same deformation in both materials

. static equilibrium

can solve for P_X because I know δ_B, E_X, A_Y, L

$$P_X = \frac{\delta_B E_X A_X}{L} = \frac{(0.0023)(41 \times 10^3)(0.01 \text{ m} \times 100 \times 10^{-6} \text{ m})}{0.01} = 0.0094 \text{ N}$$

$$P_C = P - P_X = (0.001 \text{ kg} \times 9.8 \text{ m/s}^2) - 0.0094 \text{ N} \approx 4 \times 10^{-4} \text{ N}$$

$$E_c = \frac{P_c L}{S_B A_c} = \frac{(4 \times 10^{-4}\,N)(0.01\,m)}{(0.0023\,m)(0.01\,m \times 10 \times 10^{-6}\,m)}$$

$$= \boxed{E_c = 17\ kPa}$$

probably muscle cells

good since blood vessels are made of smooth muscle.