Homework 4
SOLUTIONS

1. Problem 1 on page 92 in the text book. (Hint: Let x_ij be the amount of material type j you put into final product i).

   Let I = {1,2} represent {Slugger, Easy Out}. J={1,2,3} represent {sugar,nuts,chocolate}

$$
\begin{aligned}
\max \quad z = & (0.20)\sum_{j\in J} x_{1j} + (0.25)\sum_{j\in J} x_{2j} \\
& \text{subject to} \\
& x_{11} + x_{21} \leq 100 && \text{Total Sugar} \\
& x_{12} + x_{22} \leq 20 && \text{Total Nuts} \\
& x_{13} + x_{23} \leq 30 && \text{Total Chocolate} \\
& 0.2x_{21} - 0.8x_{22} + 0.2x_{23} \leq 0 && \text{Easy Out Nuts} \\
& 0.1x_{11} - 0.9x_{12} + 0.1x_{13} \leq 0 && \text{Slugger Nuts} \\
& 0.1x_{11} + 0.1x_{12} - 0.9x_{13} \leq 0 && \text{Slugger Chocolate} \\
& x_{ij} \geq 0 \qquad \forall i = 1,2 j = 1,2,3
\end{aligned}
$$

2. Solve the Farmer Jones problem USING SIMPLEX METHOD (without the government constraint).

$$
\begin{aligned}
\max \quad z = & (3)(10)x_1 + (4)(25)x_2 + 0x_3 + 0x_4 + 0x_5 && (1) \\
& \text{subject to} \\
& x_1 + x_2 + x_3 = 7 && \text{Acres} \quad (2) \\
& 4x_1 + 10x_2 + x_4 = 40 && \text{Labor} \quad (3) \\
& x_i \geq 0 \qquad \forall i = 1,2,3,4,5 && (4)
\end{aligned}
$$

We look at the following starting Tableau. Pivot will be bracketed:

-30	-100	0	0	0
1	1	1	0	7
4	[10]	0	1	40

10	0	0	10	400
6/10	0	1	-1/10	3
4/10	1	0	1/10	4

Optimal: $x_1 = 0, x_2 = 4, x_3 = 3, x_4 = 0, z = 400$

3. Solve the Farmer Jones problem USING SIMPLEX METHOD (with the government constraint).

$$
\begin{aligned}
\max \quad z =& (3)(10)x_1 + (4)(25)x_2 + 0x_3 + 0x_4 + 0x_5 && (5)\\
& \text{subject to} \\
& x_1 + x_2 + x_3 = 7 && \text{Acres } (6)\\
& 4x_1 + 10x_2 + x_4 = 40 && \text{Labor } (7)\\
& 10x_1 - x_5 = 30 && \text{Government } (8)\\
& x_i \geq 0 \qquad \forall i = 1,2,3,4,5 && (9)
\end{aligned}
$$

New Starting Tableau:

-30	-100	0	0	0	0
1	1	1	0	0	7
4	10	0	1	0	40
10	0	0	0	-1	30

Note that this is not a feasible solution with just slack variables (x_5 would be -30). Instead choose x_1 to be in the basis instead of x_5. Doing the row operations gives the following table:

0	-100	0	0	-3	90
0	1	1	0	1/10	4
0	[10]	0	1	4/10	28
1	0	0	0	-1/10	3

0	0	0	10	1	370
0	0	1	-1/10	6/100	1.2
0	1	0	1/10	4/100	2.8
1	0	0	0	-1/10	3

This solutions is optimal. Note there are several ways you can choose your initial basis, but the final solution will be the same.

4. Solve the following minimization problem USING SIMPLEX METHOD.

$$
\begin{aligned}
\min \quad z =& 10x_1 + 5x_2 && && (10)\\
& \text{subject to} && && \\
& x_1 + 2x_2 \geq 8 && \text{Acres} && (11)\\
& 3x_1 + 2x_2 \geq 12 && \text{Labor} && (12)\\
& x_2 \geq 1 && \text{Government} && (13)\\
& x_i \geq 0 \qquad \forall i = 1,2 && && (14)
\end{aligned}
$$

$$
\begin{array}{ccccc|c}
10 & 5 & 0 & 0 & 0 & 0 \\
\hline
1 & 2 & -1 & 0 & 0 & 8 \\
3 & 2 & 0 & -1 & 0 & 12 \\
0 & 1 & 0 & 0 & -1 & 1
\end{array}
$$

Here again, you cannot just let all the slack variables be basic. I chose to let x_2 to be basic because it contributes less to the cost. Then I noticed that if it equals 6, x_4 becomes 0 and the other slack variables are positive.

$$
\begin{array}{ccccc|c}
5/2 & 0 & 0 & 5/2 & 0 & -30 \\
\hline
2 & 0 & 1 & -1 & 0 & 4 \\
3/2 & 1 & 0 & -1/2 & 0 & 6 \\
3/2 & 0 & 0 & -1/2 & 1 & 5
\end{array}
$$

This is optimal. The reason I chose x_2 and not x_1 is because of the third constraint. No value of x_1 would make all the slack variables non-negative.