Name: key

ChBE 2120, Numerical Methods, Paravastu Section, Fall 2015
Quiz 4: 20 points possible

1) (12.5 points) Setup a matrix problem that could be used to solve the Boundary Value Problem below, describing a counter-current heat exchanger. To approximate the derivatives, use the following finite difference formula: $f'(x) \cong \frac{f(x_{i+1}) - f(x_i)}{h}$. Use a step size, $h = 10$.

$\frac{dT_2}{dx} = 2(T_1 - T_2)$, $T_2(0) = 55$

$\frac{dT_1}{dx} = 1.5(T_1 - T_2)$, $T_1(30) = 25$

$i = 1$	2	3	4
$x = 0$	10	20	30
$T_1 = T_{1,1}$	$T_{1,2}$	$T_{1,3}$	25
$T_2 = 55$	$T_{2,2}$	$T_{2,3}$	$T_{2,4}$

$i=1$

$\frac{T_{2,2} - 55}{10} = 2T_{1,1} - 2(55)$ +2.5

$\Rightarrow 2T_{1,1} - 0.1T_{2,2} = -110 + 5.5 = -104.5$

$\frac{T_{1,2} - T_{1,1}}{10} = 1.5T_{1,1} - 1.5(55)$

$\Rightarrow \underbrace{(1.5 + 0.1)}_{1.6}T_{1,1} - 0.1T_{1,2} = 82.5$

$i=2$

$\frac{T_{2,3} - T_{2,2}}{10} = 2T_{1,2} - 2T_{2,2}$,

$\Rightarrow 2T_{1,2} + \underbrace{(-2+0.1)}_{-1.9}T_{2,2} + (-0.1)T_{2,3} = 0$

$\frac{T_{1,3} - T_{1,2}}{10} = 1.5T_{1,2} - 1.5T_{2,2}$

$\Rightarrow \underbrace{(1.5 + 0.1)}_{1.6}T_{1,2} + (-1.5)T_{2,2} + (-0.1)T_{1,3} = 0$

$i = 3$:

$\frac{T_{2,4} - T_{2,3}}{10} = 2T_{1,3} - 2T_{2,3}$

$\Rightarrow 2T_{1,3} + \underbrace{(-2+0.1)}_{-1.9}T_{2,3} + (-0.1)T_{2,4} = 0$

$\frac{25 - T_{1,3}}{10} = 1.5T_{1,3} - 1.5T_{2,3}$

$\underbrace{(1.5 + 0.1)}_{1.6}T_{1,3} + (-1.5)T_{2,3} = 2.5$

+5 +2.5 +2.5

$$\begin{bmatrix} 2 & 0 & -0.1 & 0 & 0 & 0 \\ 1.6 & -0.1 & 0 & 0 & 0 & 0 \\ 0 & 2 & -1.9 & 0 & -0.1 & 0 \\ 0 & 1.6 & -1.5 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 2 & -1.9 & -0.1 \\ 0 & 0 & 0 & 1.6 & -1.5 & 0 \end{bmatrix} \begin{bmatrix} T_{1,1} \\ T_{1,2} \\ T_{2,2} \\ T_{1,3} \\ T_{2,3} \\ T_{2,4} \end{bmatrix} = \begin{bmatrix} -104.5 \\ 82.5 \\ 0 \\ 0 \\ 0 \\ 2.5 \end{bmatrix}$$

2) (7.5 points) Using the functions defined in the headers below, write Matlab code to solve Problem (1) using the Shooting Method.

Script +4

```
T10 = SecantMethod(50, 50.5, @f, 0.001);
[Xseries, Tseries] = ODERungeKutta4(@InitialValueODE, [0 30], [55; T10], 0.01);
T2series = Tseries(1,:);
T1series = Tseries(2,:);
```

f.m +3.5

```
function [IVPError] = f(alpha)
    [~, Tseries] = ODERungeKutta4(@InitialValueODE, [0 30], [55; alpha], 0.01);
    IVPError = Tseries(2,end) - 25;
end
```

Function Header 1: `function [ Tprime ] = InitianValueODE( x, T )`

Function Header 2: `function [ tSolution, Ysolution ] = ODERungeKutta4( Yprime, tRange, Y0, h )`
`%Y = [T2; T1]`

Function Header 3: `function [ xRoot ] = SecantMethod( x0, x1, f, EaMax )`