Quiz 6

Name: _____

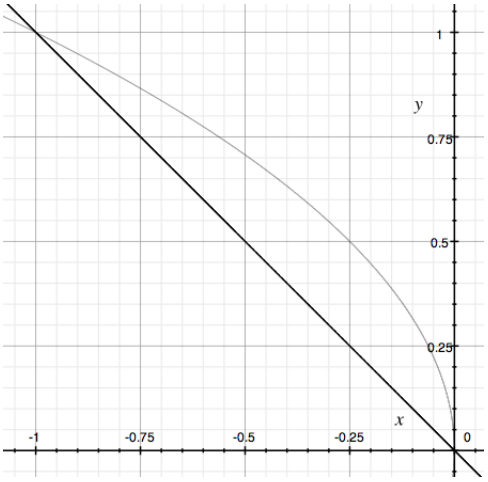
Math 2401 - Calculus III

Section (B1, B2, B3, or B4): _____

Note: *To receive full credit, your work must justify your answer*

1. Sketch the region bounded by $x = -y^2$ and $y = -x$ and calculate the region's area by evaluating the corresponding double integral.

$$\int_0^1 \int_{-y}^{-y^2} dxdy = \int_0^1 x\Big|_{-y}^{-y^2} = \int_0^1 (-y^2 + y)dy = \frac{1}{6}$$



2. Evaluate the following double integral in polar form:

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln\left(x^2 + y^2 + 1\right) \, \mathrm{d}x\mathrm{d}y$$

Step 1: Convert to polar (don't forget the jacobian!!)

$$\int_0^{2\pi} \int_0^1 r * \ln\left(r^2 + 1\right) \, \mathrm{d}r\mathrm{d}\theta$$

Step 2: Evaluate

$$\begin{aligned} \int_0^{2\pi} \int_0^1 r \ln\left(r^2 + 1\right) \, \mathrm{d}r\mathrm{d}\theta &= 2\pi \int_0^1 r \ln(r^2 + 1)\mathrm{d}r = \pi \int_0^1 \ln(u + 1)\mathrm{d}u \\ &= \pi[(u + 1)\ln(u + 1) - (u + 1)]\big|_{u=0}^{1} \\ &= \pi(2\ln 2 - 1). \end{aligned}$$

3. Find the volume of the region in the first octant enclosed by the cylinder $x^2 + z^2 = 4$ and the plane $y = 3$.

$$\int_0^3 \int_0^2 \sqrt{4 - x^2} \mathrm{d}x \mathrm{d}y = 3\pi$$