

MATH 1711, Midterm 2

10/8/2014

Name: key GTID: _____

Circle your section below

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| Problem No. | Points |
|-------------|--------|
| 1 | 15 |
| 2 | 15 |
| 3 | 15 |
| 4 | 20 |
| 5 | 15 |
| 6 | 15 |
| 7 | 5 |

TOTAL: _____

Please do show all your work including intermediate steps. Partial credit is available.

Problem 1 (15 points).

Without consultation with each other, each of ten technology companies announces a one-day convention to be held during October. Find the probability that at least two companies specify the same day for their convention. You do *not* need to simplify your final answer.

Let E = at least two companies specify the same day

$$\Pr(E) = 1 - \Pr(E') = 1 - \frac{P(31, 10)}{31^{10}}$$

Problem 2 (15 points).

When a pair of dice is rolled, what is the probability that the sum of the dice is 8, given that the sum is not 7? You do *not* need to simplify your final answer.

Let E = the sum of the dice is 8.

F = the sum of the dice is not 7.

$$\Pr(E|F) = \frac{\Pr(E \cap F)}{\Pr(F)} = \frac{\Pr(E)}{\Pr(F)} = \frac{\frac{5}{36}}{1 - \frac{6}{36}} = \frac{1}{6}$$

Turn over for more problems

Problem 5 (15 points).

Apple Inc. knows that .05% of all iPhone 6 manufactured are defective ("bendgate"). A testing machine is 99% effective, that is, 99% of good iPhone 6 will be declared fine and 99% of flawed iPhone 6 will be declared defective. If a randomly selected iPhone 6 is tested and found to be defective, what is the probability that it actually is defective? You do *not* need to simplify your final answer.

let E = iPhone is actually defective.

F = iPhone is tested to be defective

$$\Pr(E|F) = \frac{0.05\% \cdot 99\%}{0.05\% \cdot 99\% + 99.95\% \cdot 1\%}$$

Problem 6 (15 points).

It is known that ten percent of the population is left-handed. What is the probability that in a group of 10 people, at most five are left-handed? You do *not* need to simplify your final answer.

let E = at most 5 are left-handed

$$\Pr(E) = \sum_{k=0}^5 \binom{10}{k} \cdot 0.1^k \cdot 0.9^{10-k}$$

Turn over for more problems

Problem 3 (15 points).

A sequence of two cards is drawn at random (*without* replacement) from a standard deck of 52 cards. Are the events "the first card is a Queen" and "the second card is black" independent? Justify your answer.

Let E = the first card is a Queen.

F = the 2nd card is black.

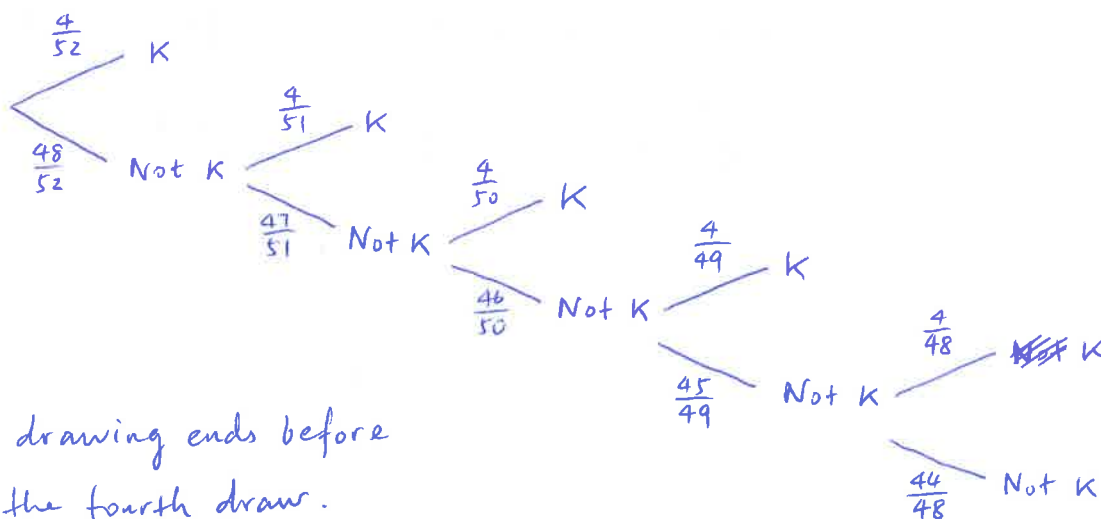
$$\text{Then } \Pr(E) = \frac{1}{13}, \quad \Pr(F) = \frac{1}{2}$$

$$\Pr(E \cap F) = \Pr(E) \cdot \Pr(F|E) = \frac{1}{13} \cdot \frac{1}{2}$$

$$\text{So } \Pr(E \cap F) = \Pr(E) \cdot \Pr(F) \quad \text{Independent!}$$

Problem 4 (20 points).

A card is drawn from a standard deck of 52 cards. We continue to draw until we have drawn a king or until we have drawn five cards, whichever comes first. Draw a tree diagram that illustrates the experiment. Put the appropriate probabilities on the tree. Find the probability that the drawing ends before the fourth draw. You do *not* need to simplify your final answer.

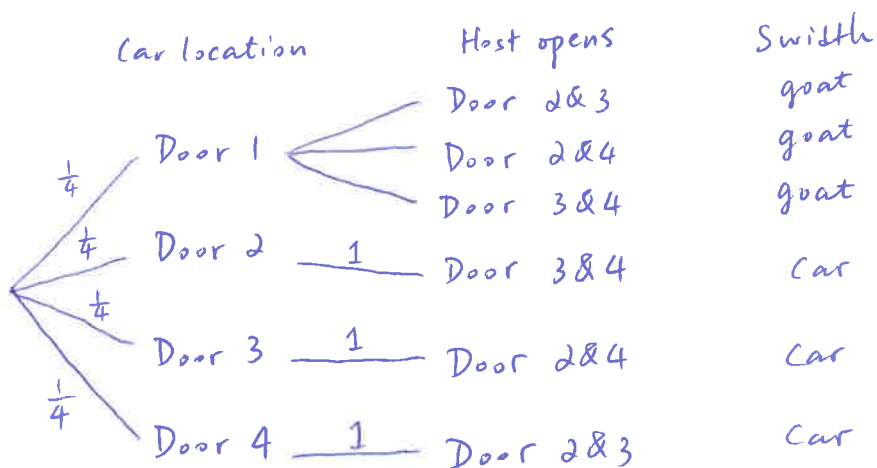


$$\Pr(E) = \frac{4}{52} + \frac{48}{52} \cdot \frac{4}{51} + \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{4}{50}$$

Turn over for more problems

Problem 7 (⁵~~15~~ points).

Suppose you are a contestant on "Let's Make a Deal", and you're given the choice of *four* doors: Behind one door is a car; behind the others, goats. Your host Monty asks you to select a door to open. So you randomly pick a door, say No. 1. Instead of opening your selected door, Monty, who knows what's behind the doors, opens two other doors that both contain goats. He then says to you, "Do you want to pick the other unopened door?" What is the probability you will win the car if you change your guess? Justify your answer.



Switch to win w/ probability $\frac{3}{4}$.

The End.

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