PHYS 2211 Test 1
Fall 2011

Name(print)____

Instructions

- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You must show all work, including correct vector notation.
- Correct answers without adequate explanation will be counted wrong.
- Incorrect work or explanations mixed in with correct work will be counted wrong. Cross out anything you don't want us to read!
- Make explanations correct but brief. Don't write a lot of prose.
- Include diagrams!
- Show what goes into a calculation, not just the final number, e.g.: $\frac{a \cdot b}{c \cdot d} = \frac{(8\times10^{-3})(5\times10^{6})}{(2\times10^{-5})(4\times10^{4})} = 5 \times 10^4$
- Give standard SI units with your results.

Unless specifically asked to derive a result, you may start from the formulas given on the formula sheet, including equations corresponding to the fundamental concepts. If a formula you need is not given, you must derive it.

If you cannot do some portion of a problem, invent a symbol for the quantity you can't calculate (explain that you are doing this), and use it to do the rest of the problem.

Honor Pledge

"In accordance with the Georgia Tech Honor Code, I have neither given nor received unauthorized aid on this test."

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Sign your name on the line above

Problem 1 (25 Points)

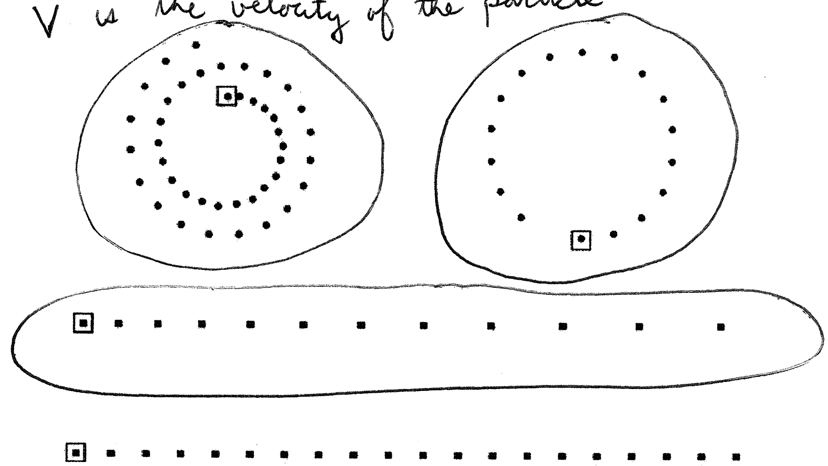
(a 5pts) Write down the definition of the momentum of a particle valid at all speeds. Please define and describe any quantities you use in your definition. Your answer must be exactly correct to receive credit, including arrows for vectors, correct subscripts, etc. There is no partial credit for this part.

$\vec{p} = \gamma m \vec{v}$ where $\gamma \equiv \frac{1}{\sqrt{1-\left(\frac{|v|}{c}\right)^2}}$

m is the mass of the particle

$\vec{v}$ is the velocity of the particle

(b 5pts) Below are several snapshots of a particle taken at equal time intervals. Circle the trajectories that indicate an interaction is taking place between the particle and its surroundings.



(c 10pts) Write down any **one** of the valid forms of the momentum principle. If you write more than one and any of them are incorrect, the whole problem will be marked as incorrect. Your answer must be exactly correct to receive credit, including arrows for vectors, correct subscripts, etc. There is no partial credit for this part.

$\boxed{\Delta \vec{p} = \vec{F}_{net} \Delta t}$ OR $\boxed{\vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t}$ or $\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{net}}$

(d 5pts) You are riding in an elevator from the top of a building to the ground floor at a constant speed. The only forces acting on the elevator are the force gravity and the force of the cable that the elevator is suspended from. Which of the statements below is correct (**circle all that apply**)?

- The magnitude of the force from the cable on the elevator is slightly greater than the force of the Earth on the elevator.
- The magnitude of the force from the cable on the elevator is slightly less than the force of the Earth on the elevator.
- The magnitude of the force from the cable on the elevator is the same as the force of the Earth on the elevator.
- The magnitude of the force from the cable is zero and the force of the Earth is mg.
- Not enough information is given to make an assessment about the forces acting on the elevator.

Problem 2 (25 Points)
A soccer ball is kicked at an angle of θ to the horizontal with an initial speed of v_0. In the following questions you can safely ignore air resistance. If you use an equation not given on the formula sheet you need to derive it from a fundamental principle (i. e. show where you equation comes from).

(a 15pts) Calculate how far down the field (horizontal distance) the soccer ball travels.

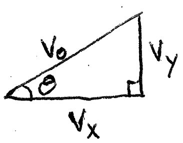
Begin with the momentum principle:

$$\Delta\vec{p} = \vec{F}_{net}\,\Delta t$$

$$\Rightarrow \vec{p}_f - \vec{p}_i = \vec{F}_{net}\,\Delta t$$

$$\Rightarrow m\vec{v}_f - m\vec{v}_i = \vec{F}_{net}\,\Delta t \quad \text{since } \vec{p} = m\vec{v}$$

$$\Rightarrow \vec{v}_f - \vec{v}_i = \frac{\vec{F}_{net}\,\Delta t}{m}$$



$$\vec{V}_i = \langle V_0\cos\theta, V_0\sin\theta, 0\rangle$$

Gravity is the only force: $\vec{F}_{net} = \langle 0, -mg, 0\rangle$

Consider only the y-component to find the time until the ball reaches its peak. This is half of the total time.

$$V_{f,y} - V_{i,y} = \frac{F_{net,y}\,\Delta t_{1/2}}{m}$$

$$0 - V_0\sin\theta = \frac{-mg\,\Delta t_{1/2}}{m}$$

$$\Rightarrow \Delta t_{1/2} = \frac{V_0\sin\theta}{g}$$

Total time of flight is $\Delta t = 2\,\Delta t_{1/2} = \dfrac{2V_0\sin\theta}{g}$

Position update formula: $\Delta\vec{r} = \vec{V}_{avg}\,\Delta t$

For x: $\Delta x = V_{x,avg}\,\Delta t$

$$\Rightarrow \Delta x = (V_0\cos\theta)\left(\frac{2V_0\sin\theta}{g}\right)$$

$$\Rightarrow \boxed{\Delta x = \frac{2V_0^2\sin\theta\cos\theta}{g}}$$

(b 10pts) Calculate how high in the air the soccer ball travels (vertical distance).

Position update formula: $\Delta\vec{r} = \vec{V}_{avg}\,\Delta t$

For y: $\Delta y = V_{y,avg}\,\Delta t_{1/2}$

$$V_{y,avg} = \frac{V_{i,y} + V_{f,y}}{2}$$

$$\Rightarrow V_{y,avg} = \frac{V_0\sin\theta + 0}{2} = \frac{V_0\sin\theta}{2}$$

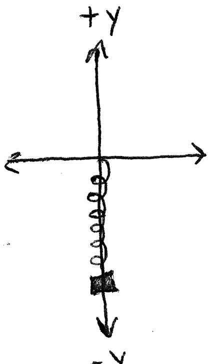
and $\Delta t_{1/2} = \dfrac{V_0\sin\theta}{g}$ from above

Thus, $\Delta y = \left(\dfrac{V_0\sin\theta}{2}\right)\left(\dfrac{V_0\sin\theta}{g}\right) = \boxed{\dfrac{V_0^2\sin^2\theta}{2g}}$

Problem 3 (25 Points)

A spring has a relaxed length of 0.2 m, and its stiffness is 15 N/m. The spring hangs vertically from the ceiling on Earth. You attach a block of mass 0.05 kg on the end of the spring and pull down on the block until the spring is 0.3 m long. You hold the block motionless on the stretched spring, then remove your hand.

(a 15pts) Choose the block as the system, and use the usual axis, with the +y axis running vertically up. At a time 0.030 seconds after releasing the block, calculate the new position of the block.



Momentum principle : $\Delta \vec{p} = \vec{F}_{net} \Delta t$

$\Rightarrow \vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$

$\Rightarrow \vec{V}_f = \vec{V}_i + \dfrac{\vec{F}_{net} \Delta t}{m}$

$\vec{V}_i = \langle 0,0,0 \rangle$ (it starts from rest)

$\vec{F}_{net} = \vec{F}_g + \vec{F}_s = \langle 0, -|mg| + |k(L-L_0)|, 0 \rangle$

($\vec{F}_g$ points down and $\vec{F}_s$ points up)

$\vec{V}_f = \langle 0, \dfrac{-|mg|\Delta t}{m} + \dfrac{|k(L-L_0)|\Delta t}{m}, 0 \rangle$

$\vec{V}_f = \langle 0, -(9.8)(0.030) + \dfrac{(15)(0.1)(0.030)}{(0.05\,kg)}, 0 \rangle$

$\vec{V}_f = \langle 0, 0.606, 0 \rangle$ m/s

Now, assume $\vec{F}_{net}$ is approximately constant over this short time period, so that

$\vec{V}_{avg} = \dfrac{\vec{V}_i + \vec{V}_f}{2} = \langle 0, 0.303, 0 \rangle$ m/s

Position update formula $\Delta \vec{r} = \vec{V}_{avg} \Delta t$

$\Rightarrow \vec{r}_f = \vec{r}_i + \vec{V}_{avg} \Delta t$

$\Rightarrow \vec{r}_f = \langle 0, -0.3, 0 \rangle + \langle 0, .303, 0 \rangle \cdot (0.030)$

$\Rightarrow \boxed{\vec{r}_f = \langle 0, -.291, 0 \rangle\, m}$

(b 5pts) Calculate the force exerted by the spring on the block at the position you found in part (a).

The block is still below the equilibrium position, so the force from the spring is still pointing up.

$F_s = \langle 0, |ks|, 0 \rangle = \langle 0, k|L-L_0|, 0 \rangle = \langle 0, (15)|0.291-0.2|, 0 \rangle$

$= \langle 0, (0.091)(15), 0 \rangle = \boxed{\langle 0, 1.365, 0 \rangle \;\; N}$

(e 5pts) After an additional 0.030 seconds (0.060 seconds since you released the block), determine the momentum of the block.

Momentum principle : $\Delta \vec{p} = \vec{F}_{net} \Delta t$

from (a)

$\Rightarrow \vec{p}_f = \vec{p}_i + \vec{F}_{net} \Delta t$ where $\vec{p}_i = m \vec{V}_f = (0.05)\langle 0, 0.606, 0 \rangle = \langle 0, 0.0303, 0 \rangle \frac{kg \cdot m}{s}$

$\vec{F}_{net} = \langle 0, 1.365, 0 \rangle N - \langle 0, (0.05)(9.81), 0 \rangle N = \langle 0, 0.8745, 0 \rangle N$

$\Delta t = 0.030\, s$

$\vec{p}_f = \langle 0, 0.0303, 0 \rangle + \langle 0, 0.8745, 0 \rangle (0.030) = \boxed{\langle 0, 0.0565, 0 \rangle \frac{kg \cdot m}{s}}$

Problem 4 (25 Points)
Suppose you are navigating a spacecraft far from other objects. The mass of the spacecraft is m. The rocket engines are shut off, and you're coasting along with a constant velocity of $< v_i, 0, 0 >$. As you pass the location $< 0, y_i, 0 >$ you fire thruster rockets, so that your spacecraft experiences a net force of $< -F_x, F_y, 0 >$ for T_1 seconds. The ejected gases have a mass that is small compared to the mass of the spacecraft. You then turn off the thruster rockets so that no net force is acting on the spacecraft. Where are you T_2 seconds after the rockets are turned off?

Momentum principle: $\Delta\vec{p} = \vec{F}_{net}\,\Delta t$

$$\Rightarrow \vec{p}_f = \vec{p}_i + \vec{F}_{net}\,\Delta t$$

$$\Rightarrow \vec{v}_f = \vec{v}_i + \frac{\vec{F}_{net}\,\Delta t}{m}$$

Calculate the final velocity after T_1 seconds:

$$\vec{v}_f = \langle v_i, 0, 0\rangle + \frac{\langle -F_x, F_y, 0\rangle T_1}{m}$$

$$\vec{v}_f = \left\langle v_i - \frac{F_x T_1}{m}, \frac{F_y T_1}{m}, 0\right\rangle$$

Calculate the average velocity during T_1:

$$\vec{v}_{avg} = \frac{\vec{v}_i + \vec{v}_f}{2} = \frac{\langle v_i, 0, 0\rangle + \langle v_i - \frac{F_x T_1}{m}, \frac{F_y T_1}{m}, 0\rangle}{2}$$

$$\vec{v}_{avg} = \left\langle v_i - \frac{F_x T_1}{2m}, \frac{F_y T_1}{2m}, 0\right\rangle$$

Calculate the new position after T_1:

$$\Delta\vec{r} = \vec{v}_{avg}\,\Delta t$$

$$\Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\,\Delta t$$

$$\Rightarrow \vec{r}_f = \langle 0, y_i, 0\rangle + \left\langle v_i - \frac{F_x T_1}{2m}, \frac{F_y T_1}{2m}, 0\right\rangle T_1$$

$$\Rightarrow \vec{r}_f = \left\langle v_i T_1 - \frac{F_x T_1^2}{2m}, y_i + \frac{F_y T_1^2}{2m}, 0\right\rangle$$

During T_2, the rockets are turned off, so the spacecraft will move with a constant velocity, $\vec{v}_f = \left\langle v_i - \frac{F_x T_1}{m}, \frac{F_y T_1}{m}, 0\right\rangle$.

We must do one more position update, for T_2:

$\Delta\vec{r} = \vec{v}_{avg}\,\Delta t$, $\vec{v}_{avg} = \vec{v}_f$ since it is constant

$$\Rightarrow \vec{r}_f = \vec{r}_i + \vec{v}_{avg}\,\Delta t$$

$$\Rightarrow \vec{r}_f = \left\langle v_i T_1 - \frac{F_x T_1^2}{2m}, y_i + \frac{F_y T_1^2}{2m}, 0\right\rangle + \left\langle v_i - \frac{F_x T_1}{m}, \frac{F_y T_1}{m}, 0\right\rangle T_2$$

$$\Rightarrow \boxed{\vec{r}_f = \left\langle v_i T_1 + v_i T_2 - \frac{F_x T_1^2}{2m} - \frac{F_x T_1 T_2}{m},\; y_i + \frac{F_y T_1^2}{2m} + \frac{F_y T_1 T_2}{m}, 0\right\rangle}$$