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MATH 1501 Sample Quiz Questions for Test 2, Fall 2007 WTT

Note 1: Just as was the case with sample problems for Test 1, there are more questions—by far— than you can expect on an hour exam. I am hoping this more comprehensive version will be of greater assistance to students in studying for the test. Also, I haven’t left as much space on the pages as will be the case on an actual hour test.

1. For what values of x is the function f(x) = sin2 x+sinx concave upwards? Concave downwards? Find all inflection points. Find where it achieves local maxima and local minima. 2. •

Use √

36.7

differentials to approximate:

• sin(.02).

• (27.15)−2/3.

3. Use the Newton-Raphson method to find x

1

, x

2

and x

3

when f(x) = x2 − 5x + 6 and x

0

= 4. To what value will this sequence converge? 4. ∫

a b

f(x) Let dx:

f be a function that is continuous on an interval [a, b]. Define the Reiman integral:

Answer

∫

b

a

f(x) dx = lim

µ(P)→0

Σn

i=1

f(t

i

)∆(x

i

)

Note: You should also know this answer implicitly includes

• P = [x

0

,x

1

,...,x

n

] is a partition of [a, b];

• ∆(x

i

) = x

i

− x

i−1

;

• for each i = 1,2,...,n, the number t

i

belongs to the sub-interval [x

i−1

,x

i

] (Note. The textbook uses the notation x∗ i

rather than t

i

); and

• µ(P), the mesh of P, is max{∆(x

i

):1 ≤ i ≤}.

5. State the First Fundamental Theorem: Answer Let f and G be continuous on [a, b] with G differentiable on (a, b). If G (x) = f(x) for all x from (a, b), then

∫

b

a

f(t) dt = G(b) − G(a)

Note. If you see yourself taking advanced math courses as an EE, CS or Math major, you should understand the role played by the Mean Value Theorem (applied to G) in proving the Second Fundamental Theorem. 6. State the Second Fundamental Theorem: Answer Let f be continuous on [a, b] and let c be any number in [a, b]. Also, let F be the funtion defined on [a, b] by setting

F(x) =

∫

x

c

f(t) dt

Then F is continuous on [a, b] and differentiable on (a, b). Also

F (x) = f(x) for all x ∈ (a, b).

7. Evaluate:

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∫

•

3 −2

y2(y3)+1 dy

•

∫

0

π

sinu du

•

∫

0

π/2

sinz dz

•

∫

0

2π

sinw dw

•

∫

0

π/4

sec2 t dt.

8. Define a function g by

g(x) =

∫

x

2.34π

sin(ecos 2u) tan(e− sinu du

Find f (x)

9. Calculate;

•

∫

(

√

y) dy

•

√

y − 1/ ∫

sin 3w cos 3w dw

•

∫

u2/(1 − u3)2/3 du

•

∫

sin(1/z)z−2 dz

•

∫

et(1 + et)−8/5 dt

•

∫

(lny)3/y dy

10. Define the function lnx.

Answer: lnx =

∫

x

1

1 t

dt

11.

• What is the domain of the natural logarithm function lnx?

• Why is lnx strictly increasing?

• What is the formula for the derivative of lnx? Why does this formula hold?

• Minimum knowledge: What is the formula for ln(xy) when x and y are both positive. Higher level: Explain your answer.

• Minimum knowledge: What is the formula for lnxq Where x is positive and q is a rational number. Higher level: Explain your answer.

• What is the range of lnx. Can you explain your answer? Hint. First show that ln2 > 1/2. Then show that ln 2n > n/2 for every positive integer n. This shows that lnx is not bounded from above.

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• What is the definition of the exponential function Exp(x)? Answer: Exp(x) is the composi- tional inverse of lnx.

• What is the derivative of Exp(x) Minimum knowledge. Know the answer. Higher level. Explain your answer.

• What is the domain of Exp(x)?

• What is the range of Exp(x)?

• Why is Exp(x) strictly increasing?

• What is the number e? Answer. e is the unique solution to lnx = 1.

• Explain why Exp(q) = eq for every rational number q. Hint. Take natural logarithms of the two expressions.

• What is meant by ex when x is not rational? Answer. When x is not rational, set ex = Exp(x).

• What is the formula for ex+y? Minimum knowledge. Know the answer. Higher level. Be able to epxlain your answer.

• What is the formula for exy? Minimum/higher as before.

• What is the definition of AT(x)?

Answer AT(x) =

∫

0

x

1 + 1

t2

dt

• What is the domain of AT(x)?

• What is the formula for the derivative of AT(x)?

• Why is AT(x) strictly increasing.

• Why is AT(x) an odd function?

• Minimum knowledge: AT(x) is bounded and has range (−π/2,π/2). Higher level. First, explain why AT(x) is bounded. Second, the number π can be defined by

π =2l.u.b.{AT(x) : x > 0}

• Higher level. The compositional inverse of AT(x), denoted Tan(x) has as its derivative the function 1 + Tan2(x). The function Tan(x) is just the restriction of tanx to the interval (−π/2,π/2). All our familiar trig functions and their derivatives can be defined from this starting point. In a certain sense, this means that AT(x) (also called arctan(x) can be con- sidered as the first trig function!

12. Calculate the derivatives of the following functionss:

• ln sinx

• ecos x

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• cos(ex)

• sinelnx

• (cosx)sinx

• xx.

• coshx + e−2x

• (coshx)(sinhx)

13. Calculate:

•

∫

esin 4x cos 4x dx

•

∫

2x dx

•

∫

x5x2 dx

14. A sum of 2,364 dollars is invested in Acme Trust and Savings which will pay 7% compounded continuously. How long will it take for the original investment to triple in value? Suppose an error is detected an the original investment turns out to be 2,549. How does this change the answer?

15. A radio-active substance of quantity 5.37 grams reduces to 4.97 grams after 183 days. How many days does it take to reduce to 50% of its original amount?

16. A local farmer is struggling in his efforts to grow a successful crop of corn. One day a wizard comes through town and upon learning of the farmer’s dilemma offers to help. He proposes a consulting operation to last 60 days for which he is paid one penny for day one, two pennies for day two, four pennies for day three, etc. In other words, the salary for each day is twice the salary for the preceding day. Suppose the wizard manages to make the farmer the most successful corn grower on the planet. Is it a good deal for the farmer? Suppose the farmer counter-offers with a 20 day plan, during which the same end result is achieved. Now is it a good deal?

17. Derive a derivative formula for

• arcsinx

• arccosx

• arctanx You shouldn’t have to work hard to answer this!

• ln(cosh 2x).

• esinh 2x.

18. Calculate:

•

∫

0

ln 2

ex/(1 + e2x) dx

• lim

x→0

arcsinx/x

• Show that cosh2 x − sinh2 x = 1 for all x.