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MATH 1501 Sample Quiz Questions for Test 3, Fall 2007 WTT

1. The first five problems all pertain the the plane region R bounded by the curve y = sinx and the line πy = 2x. Set up an integral for the area of R using a partition on the x-axis of the interval [0,π/2]. Then evaluate the integral. Note: On Test 3, you will sometimes be asked to set up a definite integral to answer a question—without actually evaluating the integral. However, in working through these sample test questions, it is a good idea to evaluate the integrals, whenever you can.

2. Set up an integral for the area of R using a partition on the y-axis of the interval [0,1]. Evaluate the integral.

3. The region R is revolved about the y-axis. Set up an integral, using “washers”, for the volume of the resulting solid. Evaluate the integral.

4. Repeat the preceding problem, but this time use “cylindrical shells.” Evaluate the integral and verify that the answer agrees with the computation in the preceding problem.

5. The region R is revolved about the x-axis. Set up an integral, using “washers”, for the volume of the resulting solid. Evaluate the integral.

6. Repeat the preceding problem, but this time use “cylindrical shells.” Evaluate the integral and verify that the answer agrees with the computation in the preceding problem.

7. Set up integrals to find the centroid of R. Then evaluate these integrals. Finally, verify the correctness of Pappus’ Theorem for the two solids of revolution.

8. Hook’s Law asserts that the force F required to compress (or stretch) a spring a distance d satisfies the simple equation F = kd, where k is a constant—called the “spring constant.” If it takes 800 pounds to compress an automotive coil 2 inches, set up an integral for the work done in compressing it 5 inches from its natural length. Evaluate the integral.

9. A cylindrical tank of height 6 feet and radius 2 feet is filled with water to a depth of 5 feet. Set up an integral for the work done in emptying the tank through an outlet at the top of the tank. Then evaluate the integral. Repeat the problem for a second outlet which is one foot below the top of the tank.

10. A chain weighing 2 pounds per foot is used to haul a 4000 bag of concrete to the top of a 100 building. Set up an integral for the work done. Evaluate the integral.

11. Repeat the preceding problem if the concrete is spilling out a hole and when raised to a height h, the weight of the remaining concrete is 4000 − 20h.

12. The two ends of a water trough are shaped like the region of the plane bounded by the curve y = x2 and the line y = 4. If the depth of the water in the trough is 3 feet, set up an integral for the fluid force on one of the ends. Evaluate the integral.

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13. Use integration by parts to calculate:

a.

∫

xe3x dx

b.

∫

x2e3x dx

c.

∫

e3x sin 5x dx

d.

∫

ln 3x dx

e.

∫

xln 3x dx

f.

∫

sin(ln 3x) dx

14. Calculate:

a.

∫

b.

sin2 3x dx (Hint: ∫ sin2 3xcos 2x;dx

use the formula sin2 θ = 1 2

− 1 2

cos 2θ.

c.

∫

sin2 3xcos3 2x;dx

15. Use a trig subsitution to calculate:

a.

∫

x2(25 − x2)1/2 dx

b.

∫

x3(9 + x2)−1/2 dx

c.

∫

x2(x2 − 4)−1/2 dx

16. Use partial fractions to evaluate

∫

(2x3 − 4x + 7)/(x2 − 5x + 6).

17. Use partial fractions to evaluate

∫

(3x − 4)/(x2 − 6x + 9).

18. Use partial fractions to evaluate

∫

(5x − 2)/(x2 − 6x + 10).

19. Set up a system of linear equations to find numbers A, B, C and D so that

2x2 − 4x + 8 3x(x − 1)(x + 2)(x − 3)

=

A x

+

x B

− 1

+

x C

+ 2

+

x D

− 3

Then solve this system of equations and find the associated antiderivative.

20. Write the form of the partial fraction decomposition associated with a rational function f(x) = p(x)/q(x) where p(x) and q(x) are polynomials, deg(p(x)) < deg(q(x)) and

q(x)=3x3(x − 2)4(x − 3)(x2 + 1)3(x2 + 13x + 42)3(x2 + 2x + 7)2

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21. The next three problems involve the complex numbers z =2+3i and w = 5−6i. Find z +w, z − w, zw, z/w, |z|, |w|. arg(z), arg(w) and arg(zw).

22. Illustrate graphically the way vector addition works in the complex plane by using the vectors z and w.

23. Illustrate graphically the way multiplication of complex numbers works (in particular, how arguments behave) by using the vectors z and w.

24. Find six distinct complex numbers satisfying the equation z6 = 1.

25. True-False

a. In Z, (4,7) = (2,5).

b. In Q, (4,7) = (2,5).

c. In C, (4,7) = (2,5).

d. In Z, (4,7) + (2,5) = (1,7).

e. In Q, (4,7) + (2,5) = (68,70).

f. In Q, (4,7) + (2,5) = (6,12).

g. In Z, (4,7) × (2,5) = (8,35).

h. In Q, (4,7) × (2,5) = (8,35).

i. In Q, (4,7) × (2,5) = (8,35).

j. The natural number six is denoted 6. We consider six as an integer, but the integer six is

really (11,5).

k. We also consider six as a fraction (rational number), but now the rational number six is really

((27,3),(9,5)).