Circuit Theory and Electronics Fundamentals

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory Report-T2

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1 Introduction

The main objective of this laboratory assignment is to analyse a RC circuit in order to determine the natural and the forced responses and also a frequency analysis. The circuit under analysis is shown in Figure **??**.

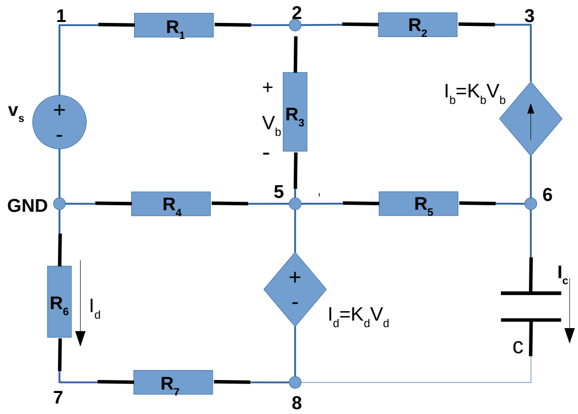
In the next section (**??**), we briefly explain the procedure to analyse the given circuit in six different subsections, each one to answer the different topics asked by the teacher. In order to solve the necessary calculations and obtain the values we used Octave maths tool. In this section, we also plotted graphics eand tables to a better analysis of the results.

Then, we resorted to Ngspice to simulate our circuit and obtain the simulated values for the same physical quantities previously calculated. These values will be shown in Section **??**, divided by six subsections, five of which to explain in a more proper way what we simulated, and the last one to compare both theoretical and simulated results and give certain notes related to our analysis.

The report finishes with its conclusion, in section **??**, where we resume the most important topics of the lab assignment.

2 Theoretical Analysis

In this section, the circuit shown in Figure **??** is analysed theoretically. The following subsections correspond to the answers to the respective given exercises.



$$v_s(t) = V_s u(-t) + \sin(2\pi f t) u(t)$$

$$u(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

Figure 1: Circuit under analysis.

2.1 Task 1

In this exercise, voltage soure V_S is constant and it is assumed that the capacitor is constant too, which means that current I_c is null. Starting by calculating the voltages in evry node, we can then determine the currents in every branch using $Ohm'sLaw$ **??**

$$V = R * I. \tag{1}$$

To calculate de voltages in every node, we labelled them with a number (see fig. **??**). From one node to the next one we applied KCL and the given relations in the circuit shown in the introduction to calculate the seven unknown nodal voltages. We only considered nodes that do not connect to voltage sources because it decreases the complexity of the problem. To solve the voltages in every node and current in every branch, all the unknown currents used in the node analysis were considered to be diverging from the node. Once all the nodal voltages were calculated, everything in the circuit could be determined.

In this section we needed seven equations to determine the voltages, so we considered equations related to some nodes and added more equations derived from the analysis of the circuit. In this case, we recurred to another equation using a value $Iaux$, which is the current in the voltage source Vd, and an additional equation to determine de seven nodal voltages and the value of $Iaux$. Voltage in node 4 was also considered to be zero because it is connected to the ground(GND). We then solved this system of linear equations using matrixes and $Octave$. The result was the vector (V1,V2,V3,V5,V6,V7,V8,Ix). Finally, using Ohm's Law shown in subsection 2.1 and a previous analysis, we determined the current in every branch and the values for Vb, Ib, Vc, Vd and Id (see fig. **??**).

$$V1 = Vs. \tag{2}$$

$$(V2 - V1) * G1 + (V2 - V3) * G2 + (V2 - V5) * G3 = 0. \tag{3}$$

$$(V3 - V2) * G2 + (V5 - V2) * Kb = 0. \tag{4}$$

$$(V2 - V5) * Kb + (V6 - V5) * G5 = 0. \tag{5}$$

$$(V7 - V8) * G7 + V7 * G6 = 0. \tag{6}$$

$$(V5 - V8) + G6 * Kd * V7 = 0. \tag{7}$$

$$(V5 - V2) * G3 + V5 * G4 + (V5 - V6) * G5 - Ix = 0. \tag{8}$$

$$Iaux = (V7 - V8) * G7. \tag{9}$$

These equations were computed into a matrix and solved in $Octave$ as refered before, and the values of current in every branch were calculated using Ohm's Law. The obtained results are given in Table **??**.

Name	Voltage/Current Value
$V1$	5.112089e+00
$V2$	4.893938e+00
$V3$	4.453446e+00
$V5$	4.923777e+00
$V6$	5.601031e+00
$V7$	-1.954351e+00
$V8$	-2.950698e+00
$I1$	2.083276e-04
$I2$	2.178511e-04
$I3$	9.523472e-06
$I4$	1.173013e-03
$I5$	2.178511e-04
$I6$	9.646853e-04
$I7$	9.646853e-04
Vb	-2.983903e-02
Vc	0.000000e+00
Vd	7.043047e-06
Ib	-2.178511e-04
Id	9.646853e-04

Table 1: Node voltages and Current in branches [A or V].

2.2 Task 2

In this subsection, we consider until t=0, an infinite time has passed and so the capacitor is fully charged. This capacitor now functions as a voltage source Vx given by the expression **??** as sugested. We now have a null value for Vs, which means it can be ignored and nodes 1 and 4 can be considered as the same, so $V1 = V4 = 0$(GND).

$$Vx = V6 - V8 \tag{10}$$

The voltages $V6$ and $V8$ are the ones calculated in the previous subsection.

The main objective in this exercise is to calculate the boundary solutions, since they must be continuous, in order to calculate the equivalent resistance e the time constant for future determinations, such as the natural solution. In order to do that, we need the value Vx given in the previous equation and the current that flows in between nodes 6 and 8. Using Ohm's Law, stated before, we have,

$$Req = Vx/Ix. \tag{11}$$

Once we know this value, we can now determine the time constant for the RC circuit, with the following expression,

$$timeconstant = Req * C. \tag{12}$$

To obtain the value of Ix, a node analysis was necessary, similar to Exercise 1, calculating the voltages and currents in every node and branch, respectively. We needed, again, equations to then compute into a matrix and solve the linear equations system to obtain the vector (V2,V3,V5,V6,V7,V8). In this case, since the capacitor functioned as a voltage soure, we could no longer use the node analysis in node 6, but we could use it in node 4. Nodes 2,3 and 7 had no alterations, so we could use them again in this method. additional equations were added in order to solve the problem. The analysis resulted in the following expressions:

$$V1 = 0 \tag{13}$$

$$(V2 - V1) * G1 + (V2 - V3) * G2 + (V2 - V5) * G3 = 0. \tag{14}$$

$$(V3 - V2) * G2 + (V5 - V2) * Kb = 0. \tag{15}$$

$$(V7 - V8) * G7 + V7 * G6 = 0. \tag{16}$$

$$V6 - V8 = Vx. \tag{17}$$

$$(V5 - V8) + G6 * Kd * V7 = 0. \tag{18}$$

$$V2 * G1 + V5 * G4 + V7 * G6 = 0. \tag{19}$$

Finally, to determine Ix, one can analyse node 6 with KCl

$$Ix + (V6 - V5) * G5 + Kb * (V2 - V5) = 0. \tag{20}$$

The results are given in the next Table: