TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory Report-T1

March 25th, 2021

António Cordeiro 95769

Catarina Falcão 95775

Luís Pinheiro 97230

Contents

1 Introduction

The main objective of this laboratory assignment is to analyse a circuit using a simulation and the node and mesh methods. It is important to rely on a method that simplifies a circuit as it gives us many information. So, it is equally important to understand if these methods can be applied and if the results are satisfactory. To prove the truthfulness of this analysis, the following report was developed. The circuit can be seen in Figure 1.

In the next section (2), we briefly explain the procedure to analyse the given circuit, first with the node method and then with the mesh method. In order to solve the necessary calculations and obtain the values for the tension and current in every node and branch, respectively, we used Octave maths tool. Then, we resorted to Ngspice to simulate our circuit and obtain the simulated values for the same physical quantities previously calculated. These values will be shown in Section 3. We will also compare both results and give certain notes related to our analysis. The report finishes with its conclusion, in section 4, where we resume the most important points of the lab assignment.

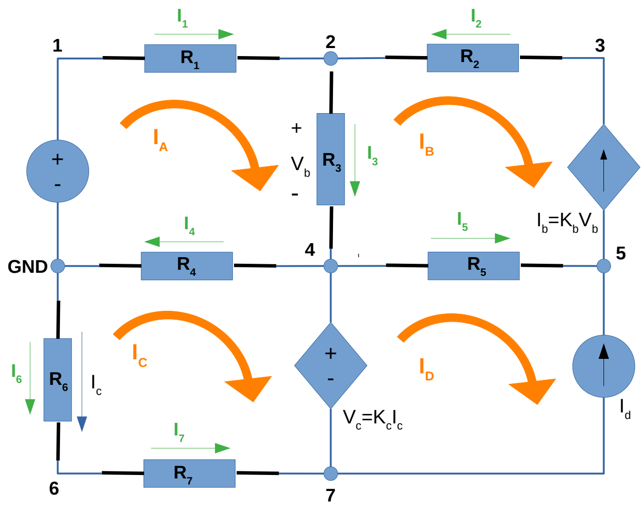

Figure 1: Circuit under analysis.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using both mesh and node methods.

2.1 Mesh Method

Starting with the mesh method, we considered four meshes and labeled them with a capital letter. The current that flows in each mesh is represented by the letter I followed by the letter that was previously given (see fig. 1). Once we find the values of the currents that flow in the four meshes we can determine all the unknowns, such as the current in every branch and the tension in every node, using Ohm's Law:

$$V = R_6 * I. \tag{1}$$

For the analysis based on the mesh method, we decided to use Kirchhoff's Voltage Law (KVL) in the two meshes that have no current sources, which means, in meshes A and C, because it diminishes the complexity of the problem. Since we had four unknowns, we needed four equations, so we added two new equations based on a previous analysis of the circuit. Given that, we obtained the four equations, each one related to a mesh.

Mesh A:

$$R1 * I_A + R3 * (I_A - I_B) + R4 * (I_A - I_C) = Va. \tag{2}$$

Mesh B:

$$I_B = -Kb * (I_A - I_B) * R3. \tag{3}$$

Mesh C:

$$Vc + R7 * I_C + R6 * I_C + R4 * (I_C - I_A) = 0. \tag{4}$$

Mesh D:

$$I_D = -Id. \tag{5}$$

Then we transformed the equations above in a linear system to be solved in $Octave$ maths tool. Solving the systems of linear equations, we calculated the vector (I_A, I_B, I_C, I_D). Then with these values for the current and the following relations using Ohm's Law (Eq. 1), the unknowns were fully determined.

$$V1 = Va. \tag{6}$$

$$V2 = V1 - (R1 * I_A). \tag{7}$$

$$V3 = V2 - (I_B * R2). \tag{8}$$

$$V4 = V2 - (I_A - I_B) * R3. \tag{9}$$

$$V5 = V4 - (I_D - I_B) * R5. \tag{10}$$

$$V6 = I_C * R6. \tag{11}$$

$$V7 = V6 + (I_C * R7). \tag{12}$$

The current in each branch was calculated using the conductance and the same relations as before.

2.2 Node Method

With this method, we labelled every node with a number (see fig. 1). Starting with node 1, with use of $V0 = 0$ (GND) and Kirchhoff's Current Law(KCL), we determined the voltage in each node (seven unknown nodal voltages). From one node to the next one we applied KCL and the given relations in the circuit shown in the introduction to calculate. We only considered the nodes that do not connect to voltage sources because, as stated before, it decreases the complexity of the problem. Once we knew all the nodal voltages, we could determine everything in the circuit.

In this section we needed seven equations to determine the voltages, so we considered equations related to some nodes and added more equations derived from the analysis of the circuit. Just like we did for the mesh method, we then solved this system of linear equations using matrixes and $Octave$. The result was the vector (V1,V2,V3,V4,V5,V6,V7). Finally, using

Ohm's Law shown in subsection 2.1 and a previous analysis, we determined the current in every branch and the values for Vb, Ib, Vc and Ic (see fig. 1).

$$V1 = Va. \tag{13}$$

$$-G1 * V1 + (G1 + G2 + G3) * V2 - G2 * V3 - G3 * V4 = 0. \tag{14}$$

$$(-G2 - Kb) * V2 + G2 * V3 + Kb * V4 = 0. \tag{15}$$

$$Kb * V2 + (-G5 - Kb) * V4 + G5 * V5 = Id. \tag{16}$$

$$(G6 + G7) * V6 - G7 * V7 = 0. \tag{17}$$

$$V4 + G6 * Kc * V6 - V7 = 0. \tag{18}$$

$$-G3 * V2 + (G3 + G4 + G5) * V4 - G5 * V5 - G7 * V6 + G7 * V7 = -Id. \tag{19}$$

2.3 Results

After the calculations using both methods, we obtained the following values (voltage is measured in V and Current is measured in A) :

Name	**Mesh Method Value**
I_A	2.700573e-04
I_B	2.825057e-04
I_C	-9.361162e-04
I_D	-1.048513e-03
$V1$	5.169294e+00
$V2$	4.893019e+00
$V3$	4.311489e+00
$V4$	4.932163e+00
$V5$	9.018152e+00
$V6$	-1.899119e+00
$V7$	-2.873335e+00
$I1$	2.700573e-04
$I2$	-2.825057e-04
$I3$	-1.244847e-05
$I4$	1.206173e-03
$I5$	-1.331018e-03
$I6$	9.361162e-04
$I7$	9.361162e-04
Vb	-3.914400e-02
Vc	7.805498e+00
Ib	-2.825057e-04
Ic	9.361162e-04

Table 1: Node voltages and Current in branches [A or V] using the Mesh Method. Negative values mean opposite direction to indicated in Figure 1.

Name	Node Method Value
$V1$	5.169294e+00
$V2$	4.893019e+00
$V3$	4.311489e+00
$V4$	4.932163e+00
$V5$	9.018152e+00
$V6$	-1.899119e+00
$V7$	-2.873335e+00
$I1$	2.700573e-04
$I2$	-2.825057e-04
$I3$	-1.244847e-05
$I4$	1.206173e-03
$I5$	-1.331018e-03
$I6$	9.361162e-04
$I7$	9.361162e-04
Vb	-3.914400e-02
Vc	7.805498e+00
Ib	-2.825057e-04
Ic	1.416328e-03

Table 2: Node voltages and Current in branches [A or V] using the Node Method. Negative values mean opposite direction to indicated in Figure 1.

Analysing the results seen in Tables 1 and 2 and comparing the values, we conclude that both methods give us remarkably similar values, so the calculations match each other. This result was expected, since both methods are valid and can be applied to the analysis of the given circuit.

3 Simulation Analysis

The following subsections present the obtained results when simulating the circuit and a comparison between the theoretical and simulated values.

3.1 Simulated Results

To determine whether we can or cannot rely on these methods, we simulated the given circuit using Ngspice. Table 3 shows the obtained values in the simulation for current and voltage.

3.2 Overall Results

Comparing the theoretical and simulated results, one notices the values are very similar and in line with what was expected. Small diferences were noticed due to the approximations made in the calculations using $Octave$ tool.

Closely observing the differences between the simulated and theoretical results, the maximum relative error obtained in this assignment was aroud 0,0009percent , which is a remarkably small value.

Name	Value [A or V]
v(1)	5.169294e+00
v(2)	4.893019e+00
v(3)	4.311489e+00
v(4)	4.932163e+00
v(5)	9.018152e+00
v(6)	0.000000e+00
v(7)	-1.89912e+00
@r1[i]	2.700573e-04
@r2[i]	-2.82506e-04
@r3[i]	-1.24485e-05
@r4[i]	1.206173e-03
@r5[i]	-1.33102e-03
@r6[i]	9.361162e-04
@r7[i]	9.361162e-04

Table 3: Node voltages and Current in branches. Names preceeded by "@" refer to current and values are measured in A.

4 Conclusion

In this laboratory assignment, the objective of analysing a circuit was achieved. Theoretical calculations were made using both mesh and node methods. Then, a simulation of the given circuit was produced and the results were perfectly similar to the previously calculated values. One reason for these satisfactory results is the fact that this circuit containes only linear components and so it was expected that the results would not differ. In resume, the reliability of these methods was proven, since we obtained relative errors with a maximum order of magnitude equal to (-4). Since the results were very precise, one deduces the methods can be used in order to help analysing circuits.