Circuit Theory and Electronics Fundamentals

Integrated Master in Aerospace Engineering, Técnico, University of Lisbon

Laboratory Report-T2

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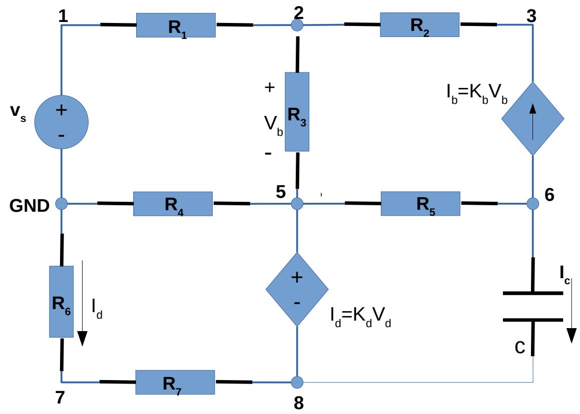
1 Introduction

The main objective of this laboratory assignment is to analyse a RC circuit in order to determine the natural and the forced responses and also a frequency analysis. The circuit under analysis is shown in Figure 1.

In the next section (2), we briefly explain the procedure to analyse the given circuit in six different subsections, each one to answer the different topics asked by the teacher. In order to solve the necessary calculations and obtain the values we used Octave maths tool. In this section, we also plotted graphics and tables to a better analysis of the results.

Then, we resorted to Ngspice to simulate our circuit and obtain the simulated values for the same physical quantities previously calculated. These values will be shown in Section 3, divided by six subsections, five of which to explain in a more proper way what we simulated, and the last one to compare both theoretical and simulated results and give certain notes related to our analysis.

The report finishes with its conclusion, in section 4, where we resume the most important topics of the lab assignment.



$$v_s(t) = V_s u(-t) + \sin(2\pi f t) u(t)$$

$$u(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}$$

Figure 1: Circuit under analysis.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically. The following subsections correspond to the answers to the respective given exercises.

2.1 Task 1

In this exercise, voltage soure V_S is constant and it is assumed that the capacitor is constant too, which means that current I_c is null. Starting by calculating the voltages in evry node, we can then determine the currents in every branch using Ohm's Law (1).

$$V = R * I. \tag{1}$$

To calculate de voltages in every node, we labelled them with a number (see fig. 1). From one node to the next one we applied KCL and the given relations in the circuit shown in the introduction to calculate the seven unknown nodal voltages. We only considered nodes that do not connect to voltage sources because it decreases the complexity of the problem. To solve the voltages in every node and current in every branch, all the unknown currents used in the node analysis were considered to be diverging from the node. Once all the nodal voltages were calculated, everything in the circuit could be determined.

In this section we needed seven equations to determine the voltages, so we considered equations related to some nodes and added more equations derived from the analysis of the circuit. In this case, we recurred to another equation using a value $Iaux$, which is the current in the voltage source Vd, and an additional equation to determine de seven nodal voltages and the value of $Iaux$. Voltage in node 4 was also considered to be zero because it is connected to the ground(GND). We then solved this system of linear equations using matrixes and $Octave$. The result was the vector (V1,V2,V3,V5,V6,V7,V8,Ix). Finally, using Ohm's Law shown in subsection 2.1 and a previous analysis, we determined the current in every branch and the values for Vb, Ib, Vc, Vd and Id (see fig. 1).

$$V1 = Vs. \tag{2}$$

$$(V2 - V1) * G1 + (V2 - V3) * G2 + (V2 - V5) * G3 = 0. \tag{3}$$

$$(V3 - V2) * G2 + (V5 - V2) * Kb = 0. \tag{4}$$

$$(V2 - V5) * Kb + (V6 - V5) * G5 = 0. \tag{5}$$

$$(V7 - V8) * G7 + V7 * G6 = 0. \tag{6}$$

$$(V5 - V8) + G6 * Kd * V7 = 0. \tag{7}$$

$$(V5 - V2) * G3 + V5 * G4 + (V5 - V6) * G5 - Ix = 0. \tag{8}$$

$$Iaux = (V7 - V8) * G7. \tag{9}$$

These equations were computed into a matrix and solved in $Octave$ as refered before, and the values of current in every branch were calculated using Ohm's Law. The obtained results are given in Table 1.

Name	Voltage/Current Value
$V1$	5.169294e+00
$V2$	4.893019e+00
$V3$	4.311489e+00
$V5$	4.932163e+00
$V6$	5.799405e+00
$V7$	-1.899119e+00
$V8$	-2.873335e+00
$I1$	2.700573e-04
$I2$	2.825057e-04
$I3$	1.244847e-05
$I4$	1.206173e-03
$I5$	2.825057e-04
$I6$	9.361162e-04
$I7$	9.361162e-04
Vb	-3.914400e-02
Vc	0.000000e+00
Vd	6.756034e-06
Ib	-2.825057e-04
Id	9.361162e-04

Table 1: Node voltages and Current in branches [A or V].

2.2 Task 2

In this subsection, we consider until t=0, an infinite time has passed and so the capacitor is fully charged. This capacitor now functions as a voltage source Vx given by the expression(10) as sugested. We now have a null value for Vs, which means it can be ignored and nodes 1 and 4 can be considered as the same, so $V1 = V4 = 0$(GND).

$$Vx = V6 - V8. \tag{10}$$

The voltages $V6$ and $V8$ are the ones calculated in the previous subsection.

The main objective in this exercise is to calculate the boundary solutions, since they must be continuous, in order to calculate the equivalent resistance e the time constant for future determinations, such as the natural solution. In order to do that, we need the value Vx given in the previous equation and the current that flows in between nodes 6 and 8. Using Ohm's Law, stated before, we have,

$$Req = Vx/Ix. \tag{11}$$

Once we know this value, we can now determine the time constant for the RC circuit, with the following expression,

$$timeconstant = Req * C. \tag{12}$$

To obtain the value of Ix, a node analysis was necessary, similar to Exercise 1, calculating the voltages and currents in every node and branch, respectively. We needed, again, equations to then compute into a matrix and solve the linear equations system to obtain the vector (V2,V3,V5,V6,V7,V8). In this case, since the capacitor functioned as a voltage soure, we could no longer use the node analysis in node 6, but we could use it in node 4. Nodes 2,3 and 7 had no alterations, so we could use them again in this method. additional equations were added in order to solve the problem. The analysis resulted in the following expressions:

$$V1 = 0. \tag{13}$$

$$(V2 - V1) * G1 + (V2 - V3) * G2 + (V2 - V5) * G3 = 0. \tag{14}$$

$$(V3 - V2) * G2 + (V5 - V2) * Kb = 0. \tag{15}$$

$$(V7 - V8) * G7 + V7 * G6 = 0. \tag{16}$$

$$V6 - V8 = Vx. \tag{17}$$

$$(V5 - V8) + G6 * Kd * V7 = 0. \tag{18}$$

$$V2 * G1 + V5 * G4 + V7 * G6 = 0. \tag{19}$$

Finally, to determine Ix, one can analyse node 6 with KCl:

$$Ix + (V6 - V5) * G5 + Kb * (V2 - V5) = 0. \tag{20}$$

The results are given in the next Table:

Name	**Voltage/Current Value**
Vx	8.672740e+00
Ix	2.825161e-03
Req	3.069821e+03
TC	3.218747e-03
$V1$	0.000000e+00
$V2$	0.000000e+00
$V3$	0.000000e+00
$V5$	0.000000e+00
$V6$	8.672740e+00
$V7$	0.000000e+00
$V8$	-0.000000e+00
$I1$	0.000000e+00
$I2$	0.000000e+00
$I3$	0.000000e+00
$I4$	0.000000e+00
$I5$	2.825161e-03
$I6$	-0.000000e+00
$I7$	0.000000e+00
Vb	-0.000000e+00
Vd	0.000000e+00
Ib	-0.000000e+00
Id	0.000000e+00

Table 2: Node voltages and Current in branches [A or V], Req and $TimeConstant(TC)$.

2.3 Task 3

Using the results obtained in Exercise 2 for Req and $TimeConstant$, we can determine the natural solution:

$$V_{6n}(t) = Ae^{-\frac{t}{RC}}. \tag{21}$$

As sugested, we used the capacitor voltage Vx for $t < 0$ as the initial condition. Since there is continuity, and $V6 = Vx + V8$, we have:

$$V_{6n}(0) = A = V6 = Vx + V8. \tag{22}$$

$$V_{6n}(t) = (Vx + V8)e^{-\frac{t}{RC}}. \tag{23}$$

We now have the natural solution in fuction of time for V6. With these results we were able to plot this function, as we can see in Figure 2.

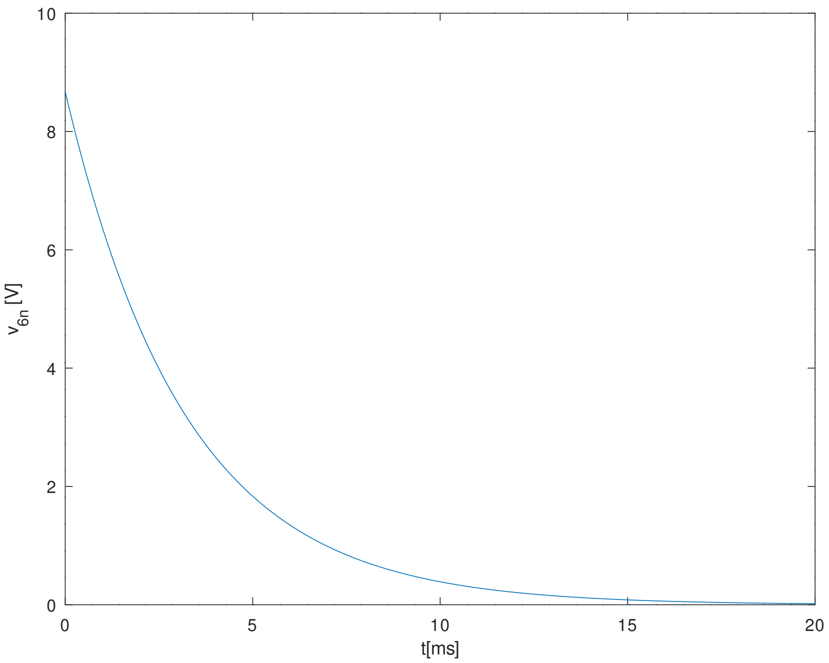


Figure 2: Natural solution of V6(t) in the interval [0,20]ms.

2.4 Task 4

In this step, the objective is to determine the forced solution for $t > 0$. We now have a frequency of $f = 1KHz$, and the phasor voltage source Vs. The capacitor no loger functions as a voltage source. For this time interval, Vs is a sinusoidal fuction of time, given by:

$$Vs(t) = sin(wt). \tag{24}$$

To simplify the problem and the calculations, we used $Vs = 1V$ as sugested. We replaced C with its impedance, to determine the current

$$Zc = 1/jwC. \tag{25}$$

$$Vc = Ic * Zc. \tag{26}$$

Similar to before, we produced a complex node analysis (Task 1) to determine the voltage in every node. Many of the equations used in this step were derived from Task 1 (2,4,4,6,7) and two other additional equations were added. This culminated in a matrix, that was then solved using $Octave$ maths tool.

$$(V1 - V2) * G1 - G7 * V5 - G6 * V7 = 0. \tag{27}$$

$$(V6 - V5) * G5 + (V6 - V8) * Yc + (V2 - V5) * Kb = 0. \tag{28}$$

The results are given in Table 3 :

Name	**Amplitude**
$V1$	1.000000e+00
$V2$	9.465546e-01
$V3$	8.340576e-01
$V5$	9.541270e-01
$V6$	5.579264e-01
$V7$	3.673846e-01
$V8$	5.558467e-01

Table 3: Complex analysis. Amplitude for each node.

2.5 Task 5

In this task, to plot the graphic of the total solution, we simply summed the solutions obtained in tasks 3 and 4 (for t¿=0) and used the values from task 1 when (t¡0):

$$V_{6t}(t) = V_{6n}(t) + V_{6f}(t). \tag{29}$$

The plotted fuction can be seen in Figure 3.

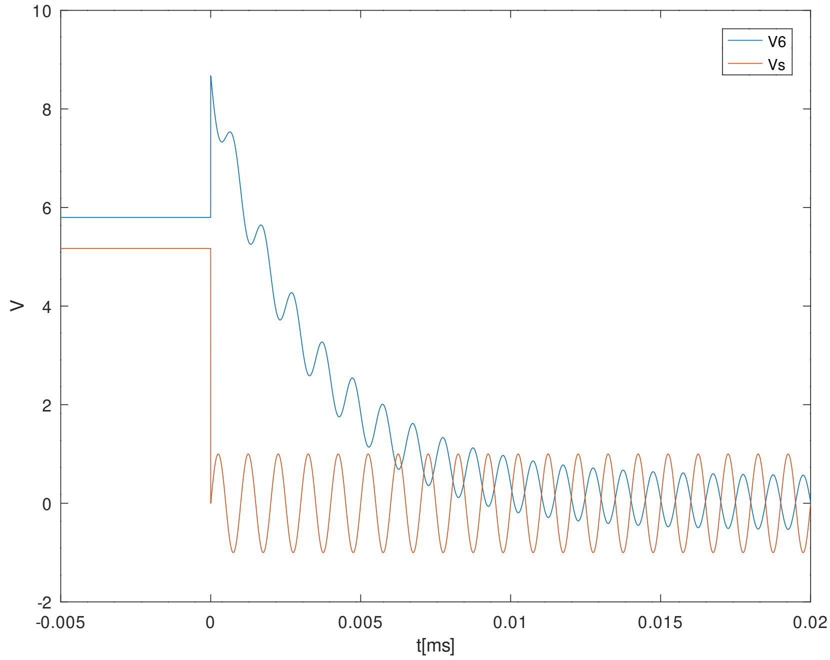


Figure 3: Total solution of V6(t) in the interval [-5,20]ms.

These results were expected, since Vs is a voltage source and has no frequency response. Contrary, it was also expected $V6t(t)$ had a frequency response, since it depends on the voltage source. The results in the figure match the predictions.

2.6 Task 6

In this subsection the objective was to observe the frequency responses of Vc and $V6$. In order to do so we did a symilar analysis to task 4 but with a frequency vector. The values for the magnitude and phase were plotted using $Octave$. We can see the results in figures 4 and 5

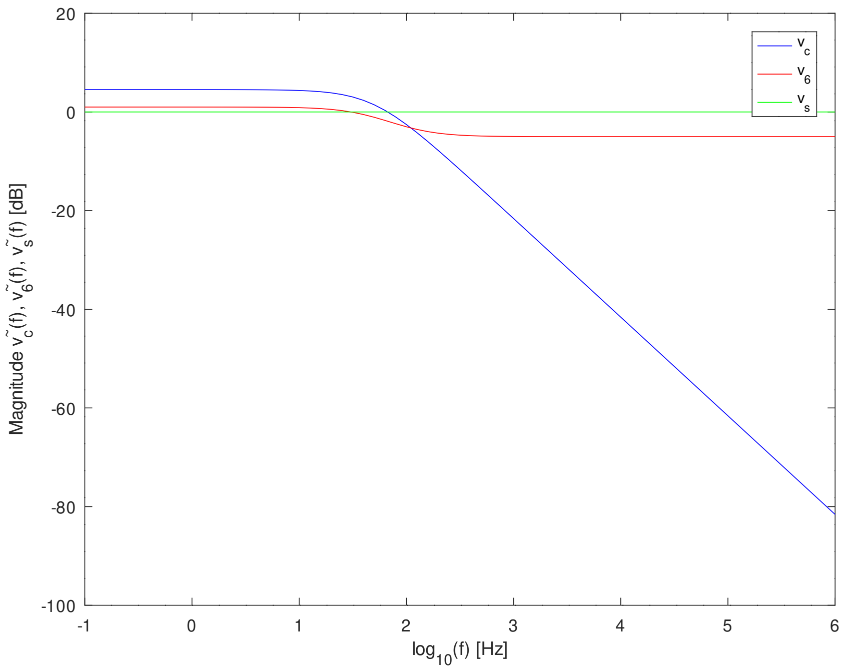


Figure 4: Magnitude response of Vs, V6 and Vc [dB].

Looking at the plotted results, one can confirm that they match the expectations. Since we have a simple RC circuit and $Vs = 1$, we know that Vc follow an inverse relation with frequency which leads us to conclude that when frequency increases, Vc must decrease. Once $V8$ is a frequency independent value, one notices that $V6$ must follow the same dependence, just as we have seen in both figures.

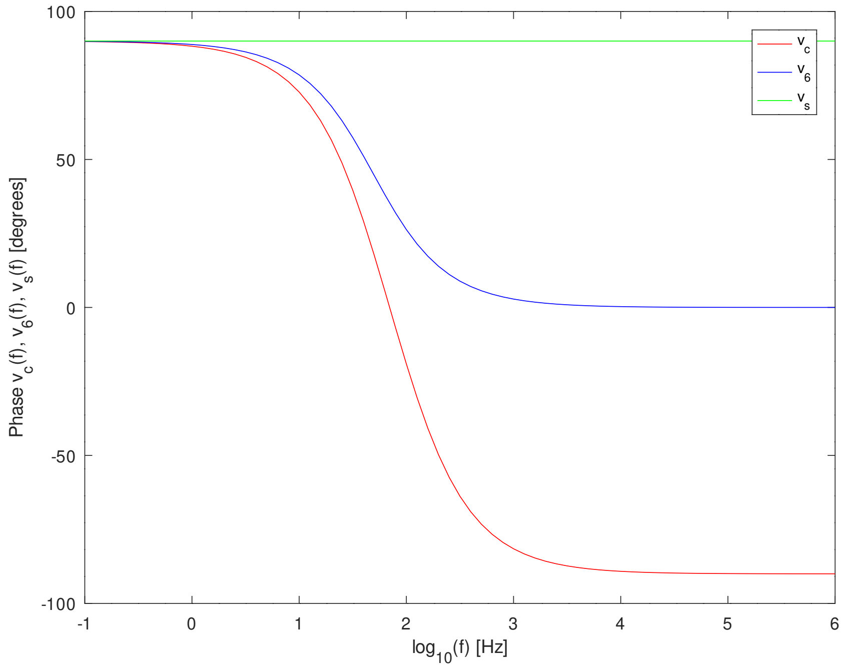


Figure 5: Angle response of Vs, V6 and Vc [degrees].

3 Simulation Analysis

The following subsections present the obtained results when simulating the circuit using $Ngspice$.

3.1 Task 1

Table 4 shows the simulated operating point results when t¡0.

Name	Value [A or V]
v(1)	5.169294e+00
v(2)	4.893019e+00
v(3)	4.311489e+00
v(5)	4.932163e+00
v(6)	5.799405e+00
v(7)	-1.89912e+00
v(8)	-2.87334e+00
@r1[i]	-2.70057e-04
@r2[i]	-2.82506e-04
@r3[i]	-1.24485e-05
@r4[i]	1.206173e-03
@r5[i]	2.825057e-04
@r6[i]	9.361162e-04
@r7[i]	-9.36116e-04

Table 4: Node voltages and Current in branches. Names preceeded by "@" refer to current and values are measured in A.

3.2 Task 2

In this subsection, an operating point analysis was made with Vs = 0 and replacing the capacitor with a voltage source, whose value derives from Vc = V6 - V8. These nodal voltages were the ones obtained in the previous task. The results can be seen in Table 5.

Name	Value [A or V]
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(5)	0.000000e+00
v(6)	8.672740e+00
v(7)	0.000000e+00
v(8)	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.825161e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00

Table 5: Node voltages and Current in branches. Names preceeded by "@" refer to current and values are measured in A.

3.3 Task 3

Task 3 shows the natural response of the circuit when t=[0,20]ms. With use of previous values for $V6$ and $V8$, a transient analysis was made and the results for the natural response are shown in Figure6.

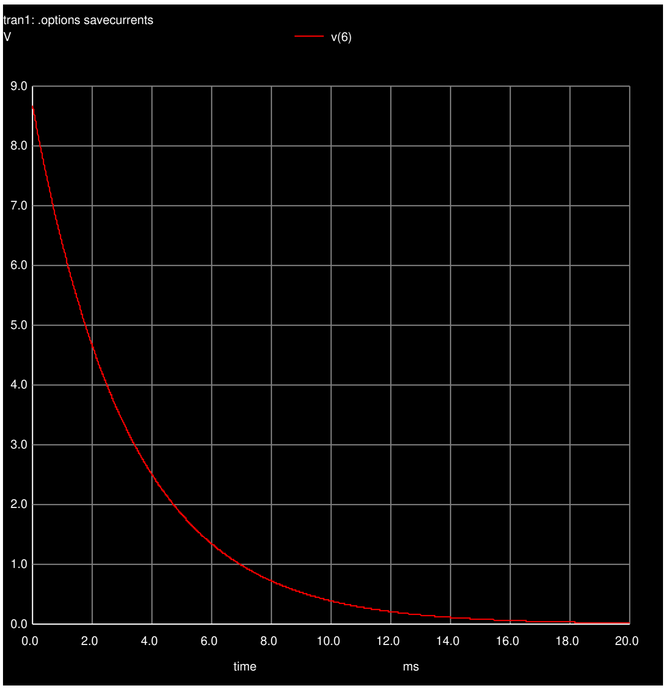


Figure 6: Natural Solution of $V6$.

3.4 Task 4

As stated in the theorethical analysis, the total response of the circuit is the sum of natural and forced solutions. Considering Vs a sinusoidal voltage source and $f = 1kHz$, we have Figure 7 of the results for V_6 and Vs.

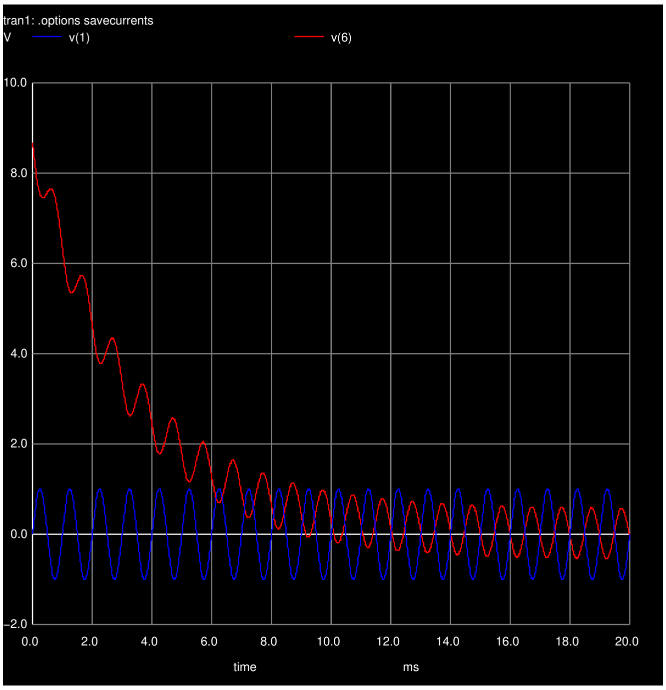


Figure 7: Total solution of $V6$ and Vs.

3.5 Task 5

In this subsection, a simulated frequency response was made. Figures 8 and 9 show the plotted results for the magnitude and phase of the frequency responses for Vc, Vs and V_6.

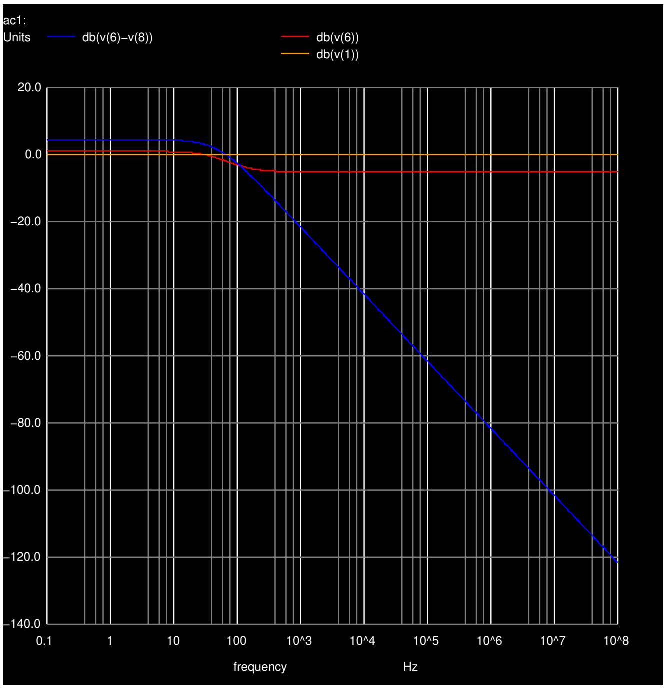


Figure 8: Magnitude response of $V6$, Vs and Vc [dB].

3.6 Overall Results

Comparing the theoretical and simulated results, one notices the values are very similar and in line with what was expected. Small diferences were noticed due to the approximations made in the calculations using $Octave$ tool and $Ngspice$.

Closely observing the differences between the simulated and theoretical results, the maximum relative error obtained in this assignment is a remarkably small value.

We can observe these results side by side in the following tables and graphics to better understand if the results are similar or not.

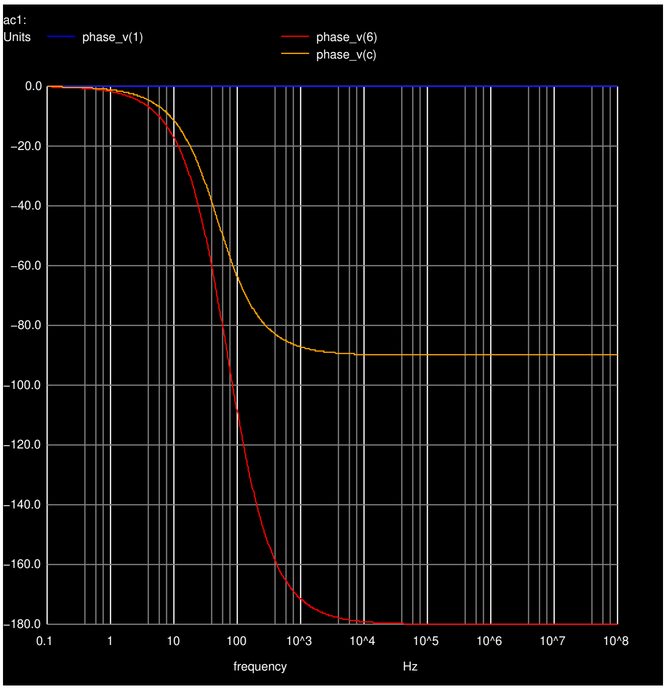


Figure 9: Angle response of $V6$, Vs and Vc [degrees].

4 Conclusion

In this laboratory assignment, the objective of analysing an RC circuit was achieved. Theoretical calculations were made following the different tasks proposed. Then, a simulation of the given circuit was produced and the results were perfectly similar to the previously calculated values. One reason for these satisfactory results is the fact that this circuit containes only one capacitor and linear components and so it was expected that the results would not differ.

Name	Value [A or V]
$V1$	5.169294e+00
$V2$	4.893019e+00
$V3$	4.311489e+00
$V5$	4.932163e+00
$V6$	5.799405e+00
$V7$	-1.899119e+00
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$I6$	9.361162e-04
$I7$	9.361162e-04
Vb	-3.914400e-02
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Vd	6.756034e-06
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Name	Value [A or V]
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@r5[i]	2.825057e-04
@r6[i]	9.361162e-04
@r7[i]	-9.36116e-04

Table 6: Operating point for $t < 0$ in Octave and NGSpice, respectivily. Names preceeded by "@" refer to current and values are measured in A.

Name	Value [A or V]
Vx	8.672740e+00
Ix	2.825161e-03
Req	3.069821e+03
TC	3.218747e-03
$V1$	0.000000e+00
$V2$	0.000000e+00
$V3$	0.000000e+00
$V5$	0.000000e+00
$V6$	8.672740e+00
$V7$	0.000000e+00
$V8$	-0.000000e+00
$I1$	0.000000e+00
$I2$	0.000000e+00
$I3$	0.000000e+00
$I4$	0.000000e+00
$I5$	2.825161e-03
$I6$	-0.000000e+00
$I7$	0.000000e+00
Vb	-0.000000e+00
Vd	0.000000e+00
Ib	-0.000000e+00
Id	0.000000e+00

Name	Value [A or V]
v(1)	0.000000e+00
v(2)	0.000000e+00
v(3)	0.000000e+00
v(5)	0.000000e+00
v(6)	8.672740e+00
v(7)	0.000000e+00
v(8)	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.825161e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00

Table 7: Operating point for $t = 0$ in Octave and NGSpice, respectivily. Names preceeded by "@" refer to current and values are measured in A.