1 Introduction

The modern solving strategies for solving constrained non linear optimization problems begin with identification of the Karush-Kuhn-Tucker conditions. Several NLP solvers take advantage of the fact that for a differentiable convex system where constraint qualification holds, the solution of NLP is the solution to the KKT system and vise versa. For example PATHNLP (/link) derives the KKT system of an NLP model and solves it as a MCP system. However, it is sometimes advantageous to explicitly model the KKT conditions as part of the NLP problem formulation. At time, the complexity of the model leads to error in deriving the KKT conditions, leading to incorrect or infeasible solutions. Finding the source of the error can be a particularly tedious task. In this article, we propose a systematic method for identification of errors in the explicit KKT conditions for solving an NLP. Consider a constrained optimization problem such that

$$\begin{array}{lll} \min & f(x) & \\ \text{s.t.} & g_i(x) \leqslant 0 & i = 1, 2...m \\ & h_k(x) = 0 & k = 1, 2...p \\ & d_l(x) \geqslant= 0 & l = 1, 2...q \\ & x \in R & \end{array} \tag{1}$$

The Karush Kuhn Tucker (KKT) conditions provide the necessary conditions for a local minimum at $\hat{x}$:

$$\bigtriangledown f(\hat{x}) - \sum_{i=1}^{m} u_i \bigtriangledown g_i(\hat{x}) - \sum_{k=1}^{p} v_i \bigtriangledown h_k(\hat{x}) - \sum_{l=1}^{q} w_i \bigtriangledown d_l(\hat{x}) = 0$$

$$\begin{aligned} h_k(\hat{x}) &= 0k = 1, 2...p \\ g_i(\hat{x}) &\leq 0i = 1, 2...m \\ d_l(\hat{x}) &\geq= 0l = 1, 2...q \\ &and, \\ < u_i, g_i(x) >= 0 \\ < v_i, h_k(x) >= 0 \\ < w_l, d_l(x) >= 0 \end{aligned} \tag{2}$$

where $< u_i, g_i(x) >= 0$ represent the complimentarily condition and variables u, v, and w represent the marginals of the respective constraint. It is often written as

$g_i(x) \perp L \leq u \leq U$

where symbol $\perp$(referred to as perpendicular to) indicates pair-wise complementarity between the function g() and variable u and its bounds. Details about the conditions of complementarity are described in the GAMS documentation.

The process of solving NLP with explicit KKT conditions has two major step

1. Checking for errors in KKT by solving NLP as an MCP
2. Using dummy equations for KKT system to identify equations with errors

Using dummy equations to identify the error(s) with KKT system is discussed in subsequent section.

2 Maximum Revenue- NLP formulation

Consider a simple case of a steel factory trying to maximize the revenue under budget constraints, with man-hours(h) and raw materials steel (s) as the decision variables. The revenue is a function of decision variables given by

$$R(h, s) = 200h^{(2/3)}s^{(1/3)}$$

where, budget = \$ 20,000 cost of manpower = \$ 20 /hr cost of raw material = \$ 170 / tonn

In it's standard form, the model can be written as :

$$\begin{aligned} \min \quad & R(h, s) = -200h^{2/3}s^{1/3} \\ \text{s.t.} \quad & 20h + 170s \leq 20000 \\ & L < h, s < U \end{aligned} \tag{3}$$

The GAMS program below gives the optimum value of revenue R at $h = 666.67, s = 38.21$.

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$ontext
The model below showcases using GAMS to identify / modify the KKT
     conditions for a given
NLP formulation. Our starting point is a basic minimization NLP
    model.
$offtext

scalar
    labor 'cost of labor dollar per hour' /20/
    steel 'cost of steel per ton' /170/
    budget 'total budget for production' /20000/
    ;
variables
    h 'man hours in production' /lo 1, up 50000/
    s 'tons of raw material' /lo 1, up 50000/
    R 'Revenue'
    ;
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Equations
    con1 'constraint on budget'
    obj 'objective function'
    ;
obj.. R =e= - 200 * h**(2/3) * s**(1/3) ;
con1.. 20*h + 170*s =l= budget;

h.l=10;
s.l=10;

model khan /con1,obj/;
solve khan using NLP minimizing R;
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The above model can we written in form of an MCP by explicitly adding the KKT conditions to the model. For the given system, the KKT conditions are given as

$$\begin{aligned}
dLdh: && -200*(2/3)h^{(-1/3)}s^{(1/3)} - con1_m*(200) = 0 \\
dLds: && -200*(1/3)h^{(2/3)}*(1/3)*s^{(-2/3)} - con1_m*(170) = 0 \\
con1: && 20*h + 170*s = 20000 \\
&& con1_m*con1 = 0
\end{aligned} \tag{4}$$

where con1_m is the marginal, or Lagrange multiplier of the constraint *con1* , and dLdh,dLds are the gradients of the Lagrange function on the model.

The original model with KKT can be reformulated as an MCP in accordance to the complementarity conditions described in Section 1.

$$\begin{aligned}
&\frac{dL}{dh} \perp h = 0 \\
&\frac{dL}{ds} \perp s = 0 \\
&con1_m \perp con1 = 0
\end{aligned} \tag{5}$$