Sample Size Determination in Cluster Randomised Trials with Bayes Factor

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Outline

1. Bayes factor
2. Bayesian power
3. Algorithm
4. Previous research
5. Results
6. CRT with multiple outcomes
7. Simulation

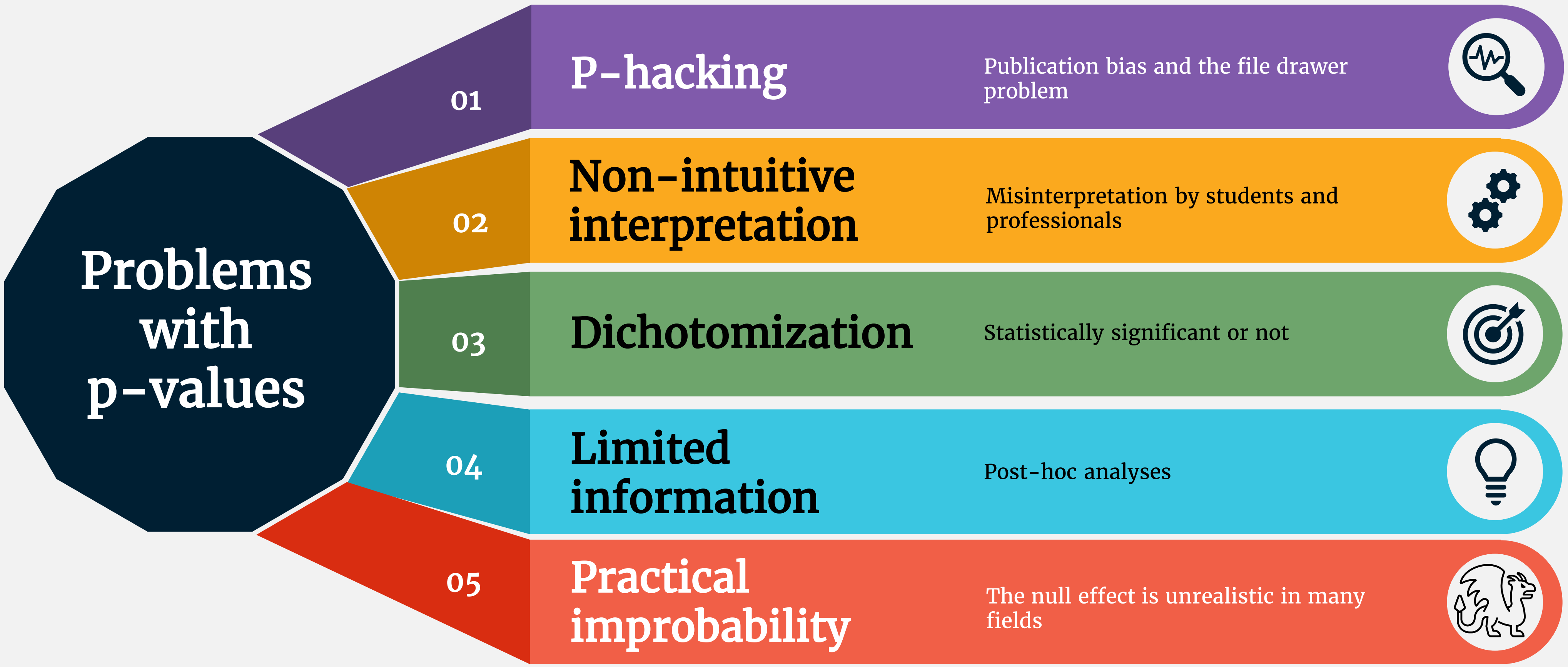
How to determine the sample sizes?

Using the design effect

Using formulae

Sample size calculator

Null Hypotheses Significance Testing


Problems with p-values
01
P-hacking
Publication bias and the file drawer problem
02
Non-intuitive interpretation
Misinterpretation by students and professionals
03
Dichotomization
Statistically significant or not
04
Limited information
Post-hoc analyses
05
Practical improbability
The null effect is unrealistic in many fields

Bayes factor

Quantifies relative evidence for a pair of competing hypotheses

$$BF_{10} = \frac{Hypothesis_1}{Hypothesis_0}$$

> 1: Hypothesis 1 is preferred

$= 1$: None of the hypotheses is preferred

< 1:Null hypothesis is preferred

Bayes factor

$$BF_{1u} = \frac{fit_1}{complexity_1}$$

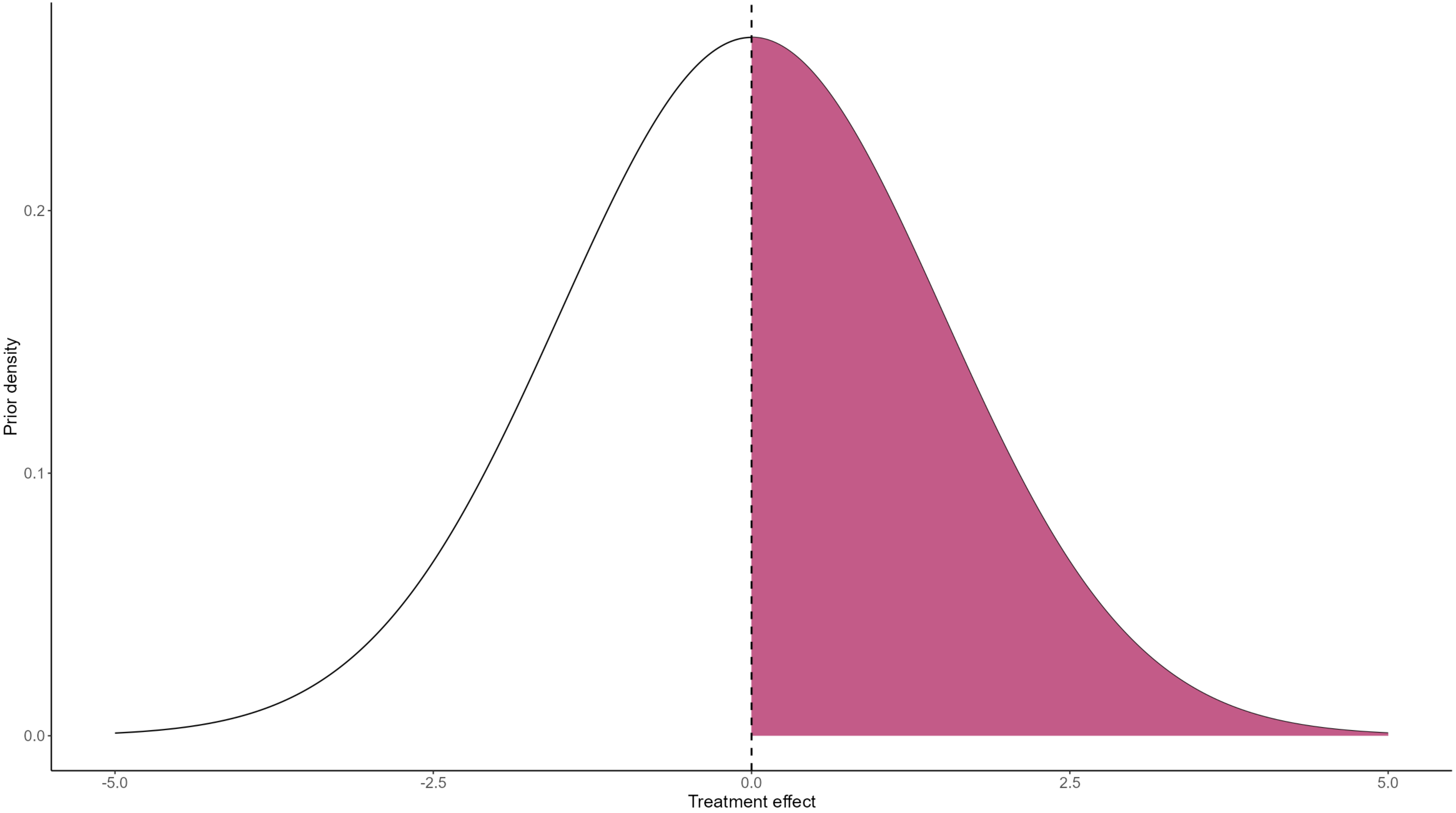
$$BF_{10} = \frac{Hypothesis_1}{Hypothesis_0}$$

$$BF_{10} = \frac{BF_{1u}}{BF_{0u}}$$

$$BF_{0u} = \frac{fit_0}{complexity_0}$$

Prior distribution

$$H_0 : \mu_C = \mu_T$$
$$H_1 : \mu_C < \mu_T$$



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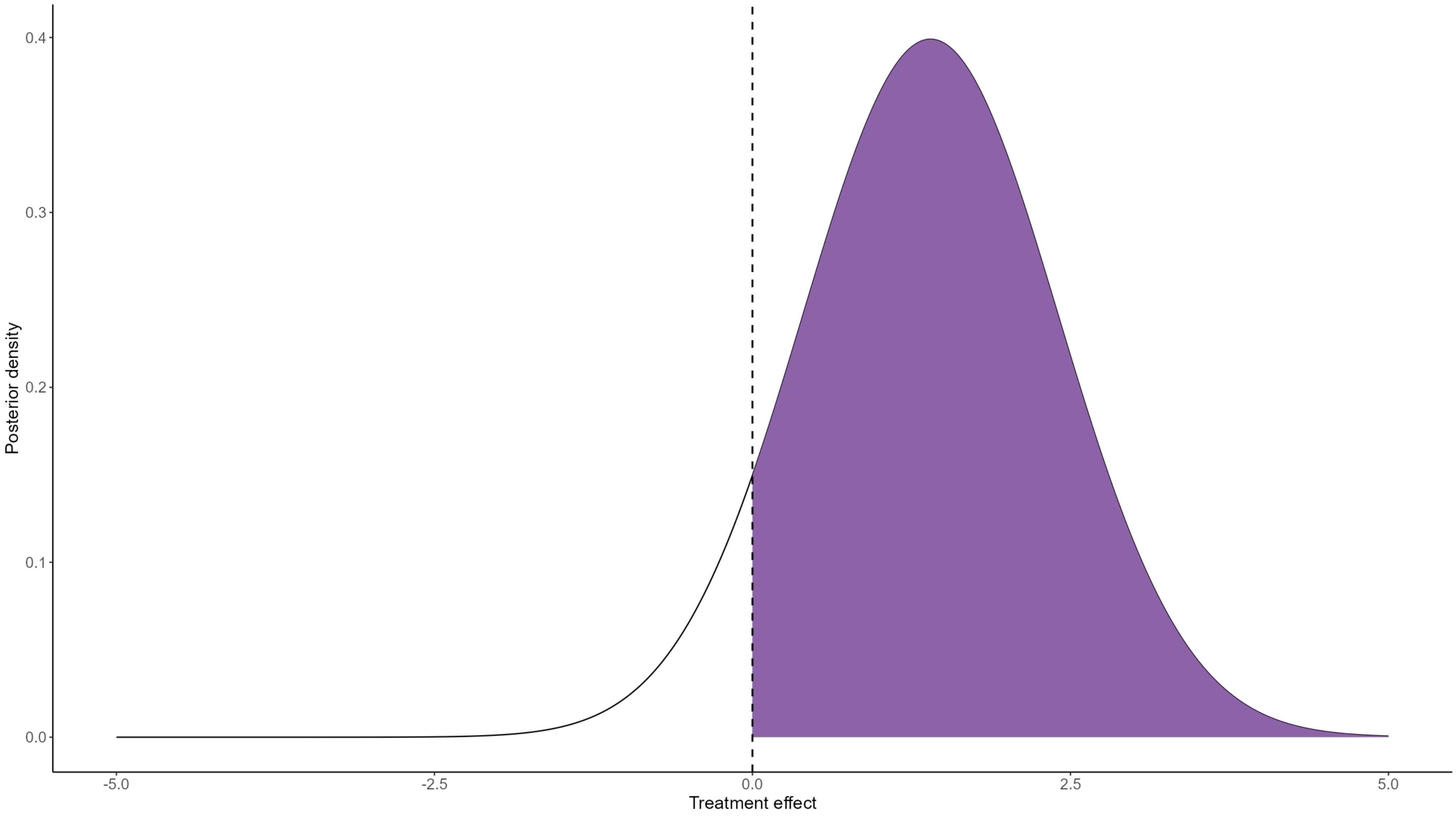
Posterior distribution



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$$BF_{1u} = \frac{fit_1}{complexity_1}$$

$$BF_{1u} = \frac{0.92}{0.5} = 1.84$$

$$BF_{10} = \frac{1.84}{0.55} = 3.34$$

$$BF_{0u} = \frac{fit_0}{complexity_0}$$

$$BF_{0u} = \frac{0.15}{0.27} = 0.55$$

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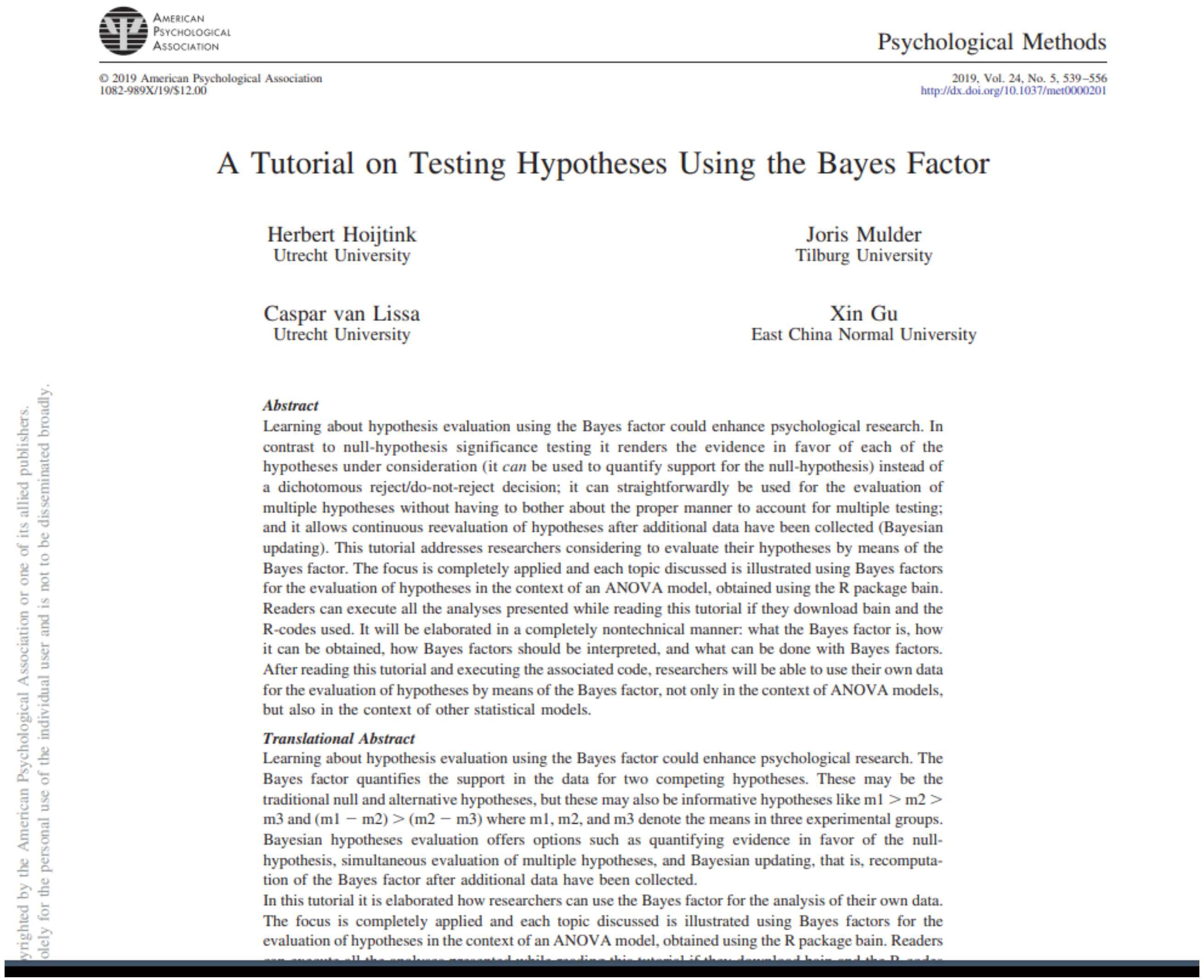

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A Tutorial on Testing Hypotheses Using the Bayes Factor

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Abstract

Learning about hypothesis evaluation using the Bayes factor could enhance psychological research. In contrast to null-hypothesis significance testing it renders the evidence in favor of each of the hypotheses under consideration (it *can* be used to quantify support for the null-hypothesis) instead of a dichotomous reject/do-not-reject decision; it can straightforwardly be used for the evaluation of multiple hypotheses without having to bother about the proper manner to account for multiple testing; and it allows continuous reevaluation of hypotheses after additional data have been collected (Bayesian updating). This tutorial addresses researchers considering to evaluate their hypotheses by means of the Bayes factor. The focus is completely applied and each topic discussed is illustrated using Bayes factors for the evaluation of hypotheses in the context of an ANOVA model, obtained using the R package bain. Readers can execute all the analyses presented while reading this tutorial if they download bain and the R-codes used. It will be elaborated in a completely nontechnical manner: what the Bayes factor is, how it can be obtained, how Bayes factors should be interpreted, and what can be done with Bayes factors. After reading this tutorial and executing the associated code, researchers will be able to use their own data for the evaluation of hypotheses by means of the Bayes factor, not only in the context of ANOVA models, but also in the context of other statistical models.

Translational Abstract

Learning about hypothesis evaluation using the Bayes factor could enhance psychological research. The Bayes factor quantifies the support in the data for two competing hypotheses. These may be the traditional null and alternative hypotheses, but these may also be informative hypotheses like m1 > m2 > m3 and (m1 − m2) > (m2 − m3) where m1, m2, and m3 denote the means in three experimental groups. Bayesian hypotheses evaluation offers options such as quantifying evidence in favor of the null-hypothesis, simultaneous evaluation of multiple hypotheses, and Bayesian updating, that is, recomputation of the Bayes factor after additional data have been collected.

In this tutorial it is elaborated how researchers can use the Bayes factor for the analysis of their own data. The focus is completely applied and each topic discussed is illustrated using Bayes factors for the evaluation of hypotheses in the context of an ANOVA model, obtained using the R package bain. Readers

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Moerbeek, M. (2019). Bayesian evaluation of informative hypotheses in cluster-randomized trials. *Behavior Research Methods*, *51*(1), 126–137. https://doi.org/10.3758/s13428-018-1149-x

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Bayesian evaluation of informative hypotheses in cluster-randomized trials

Mirjam Moerbeek[1]

Published online: 22 October 2018

Abstract
Researchers often have informative hypotheses in mind when comparing means across treatment groups, such as $H_1 : \mu_A < \mu_B < \mu_C$ and $H_2 : \mu_B < \mu_A < \mu_C$, and want to compare these hypotheses to each other directly. This can be done by means of Bayesian inference. This article discusses the disadvantages of the frequentist approach to null hypothesis testing and the advantages of the Bayesian approach. It demonstrates how to use the Bayesian approach to hypothesis testing in the setting of cluster-randomized trials. The data from a school-based smoking prevention intervention with four treatment groups are used to illustrate the Bayesian approach. The main advantage of the Bayesian approach is that it provides a degree of evidence from the collected data in favor of an informative hypothesis. Furthermore, a simulation study was conducted to investigate how Bayes factors behave with cluster-randomized trials. The results from the simulation study showed that the Bayes factor increases with increasing number of clusters, cluster size, and effect size, and decreases with increasing intraclass correlation coefficient. The effect of the number of clusters is stronger than the effect of cluster size. With a small number of clusters, the effect of increasing

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How to determine the sample size when using Bayes factors?

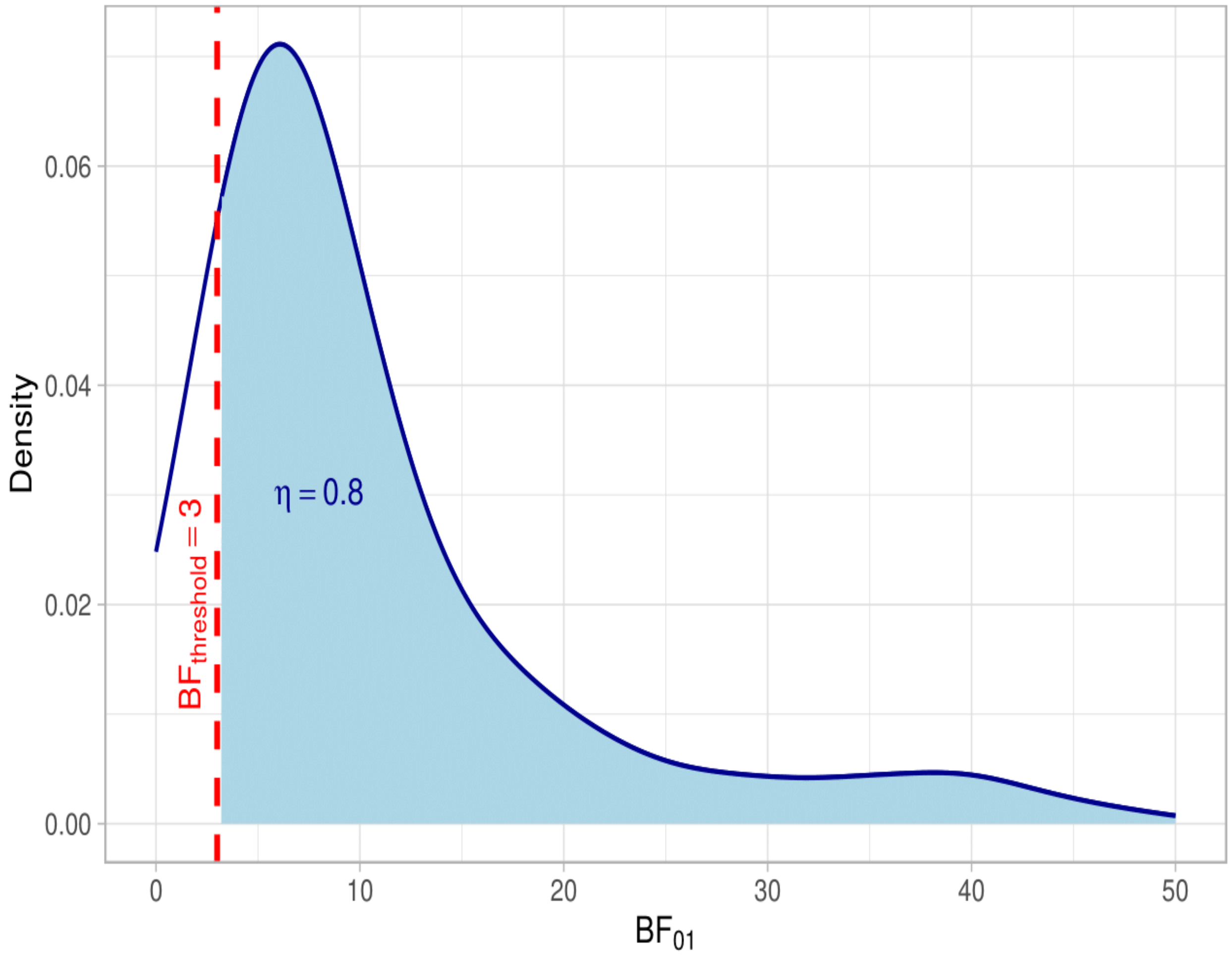
Bayesian Power

Sample size will be determined such that it ensures the probability η of the Bayes factor exceeds a user-specified threshold value ($BF_{threshold}$), when a hypothesis is true

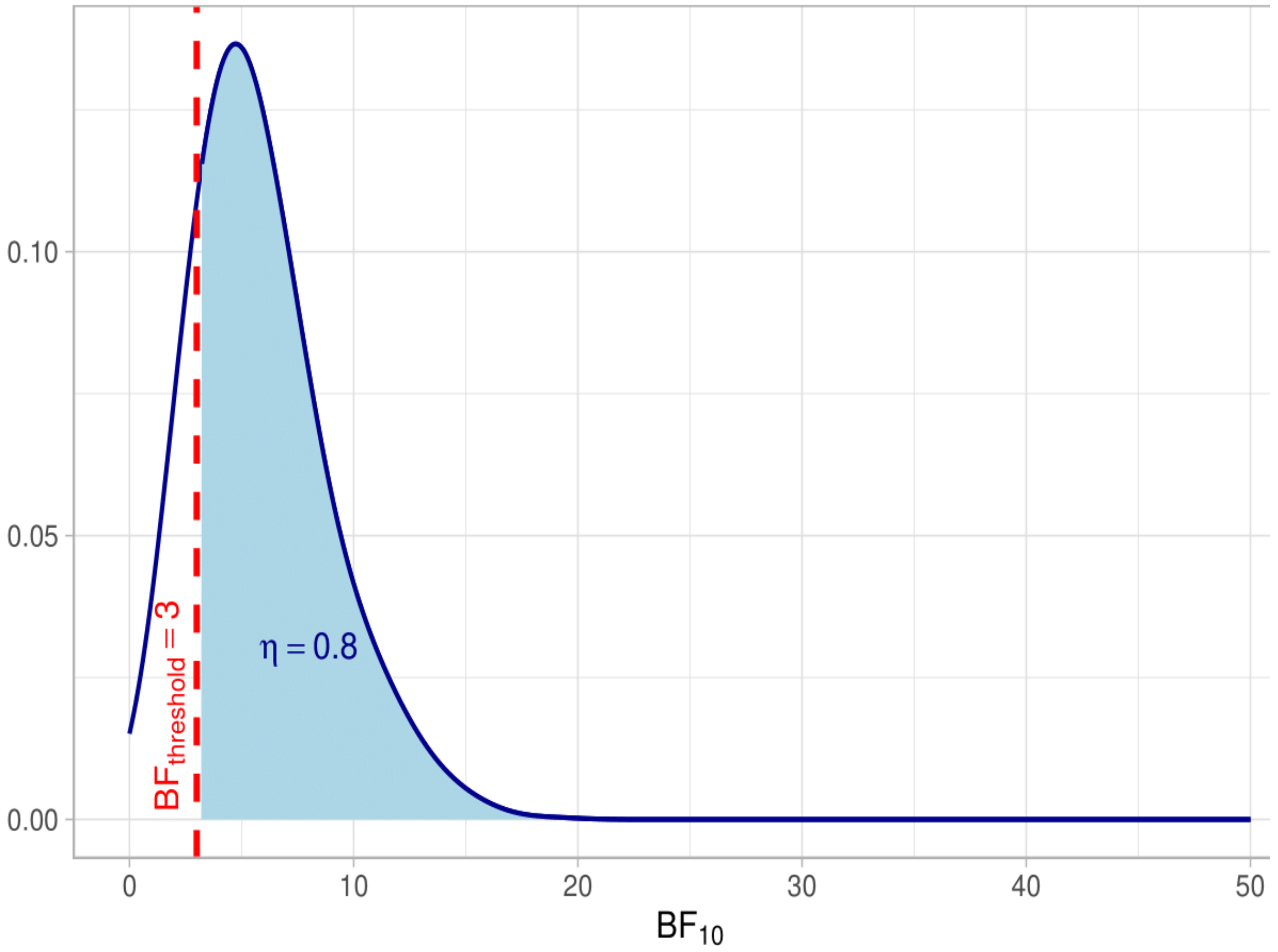
$$P(BF_{01} > BF_{thresh} | H_0) \geq \eta$$

$$P(BF_{10} > BF_{thresh} | H_1) \geq \eta$$

H_0= true



H_1= true



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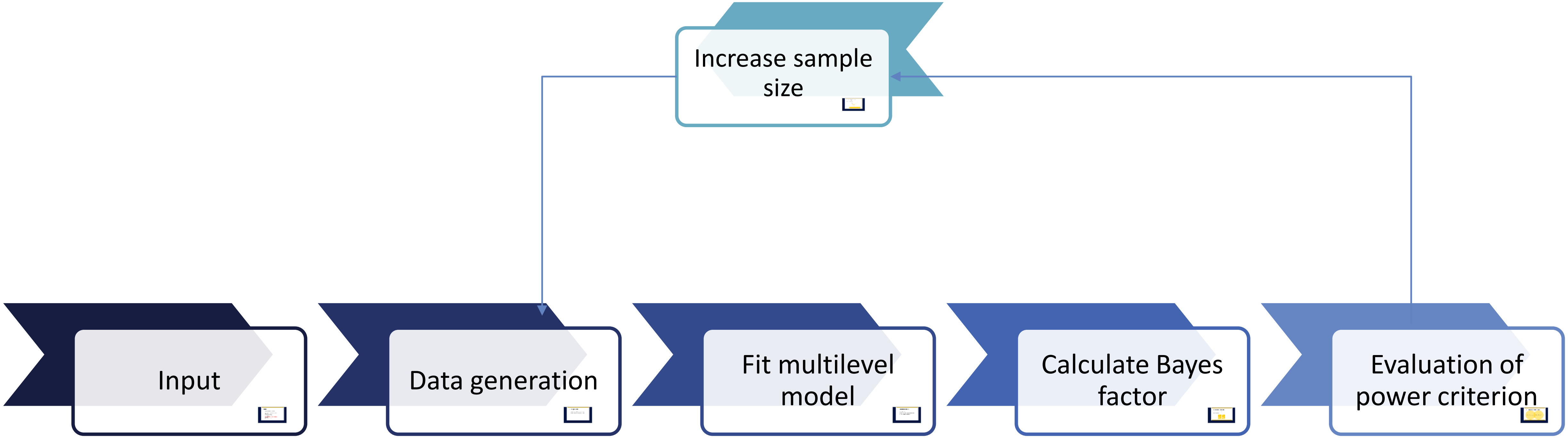

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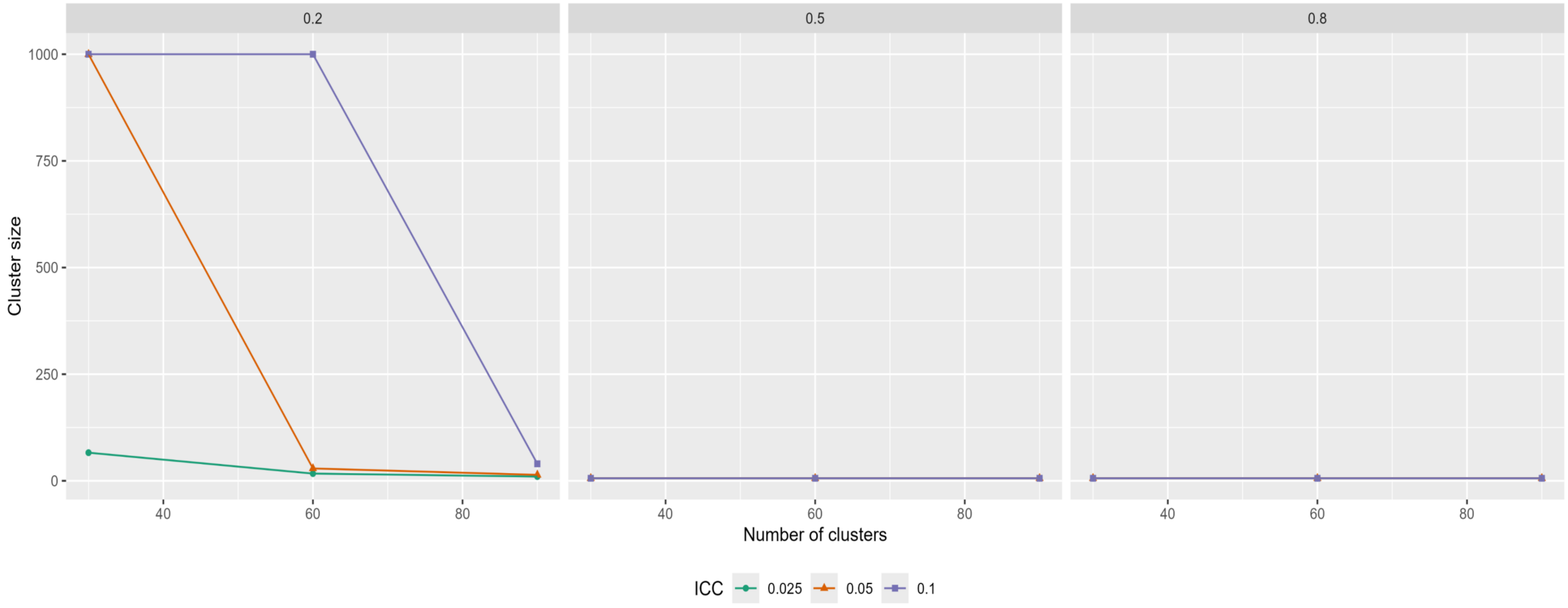
Results: Find cluster size



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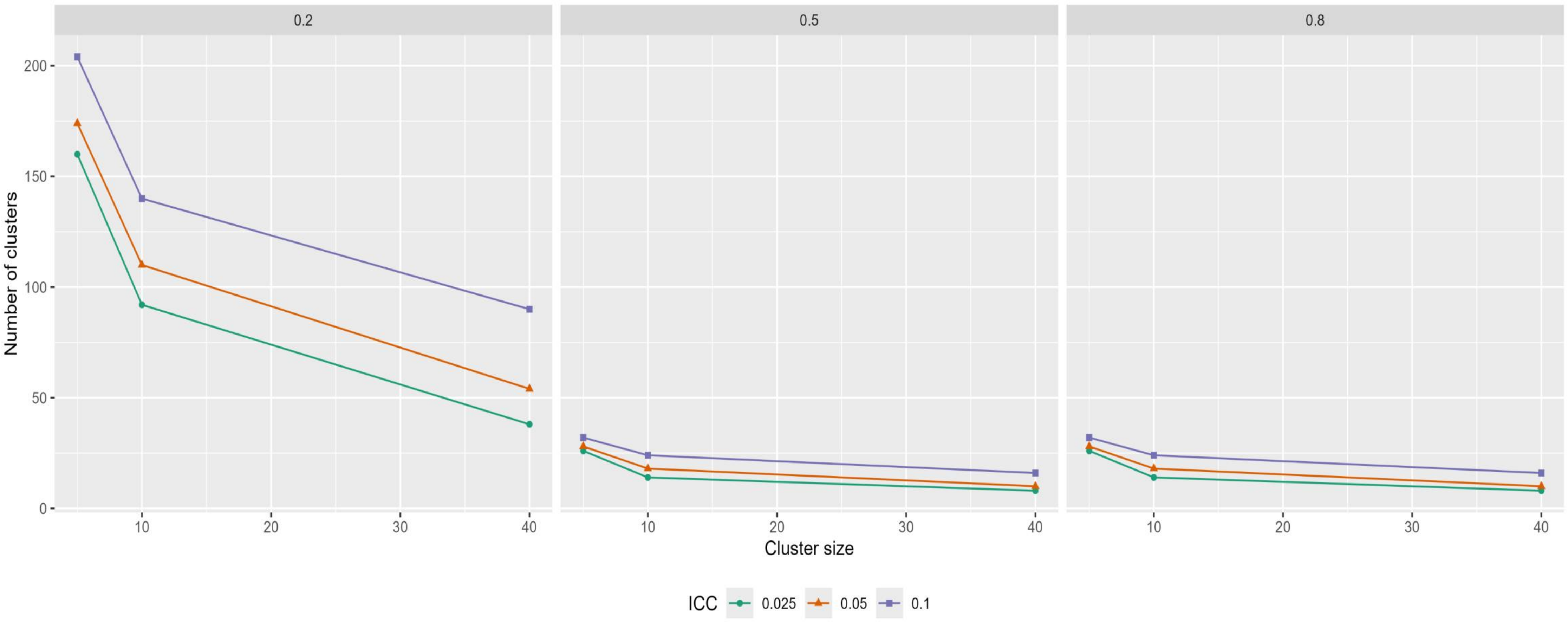
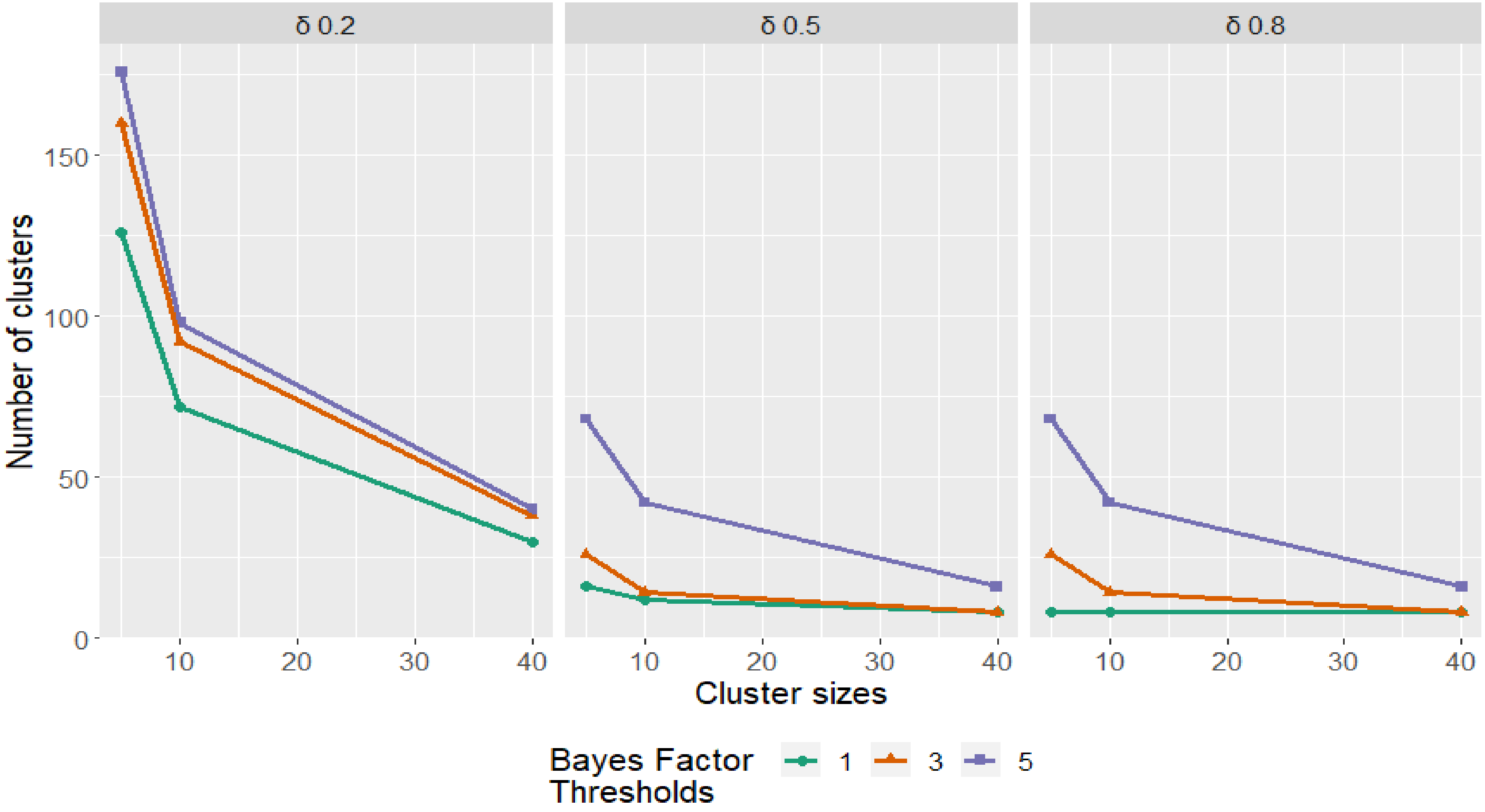
Results: Find number of clusters



Results

ICC = 0.025



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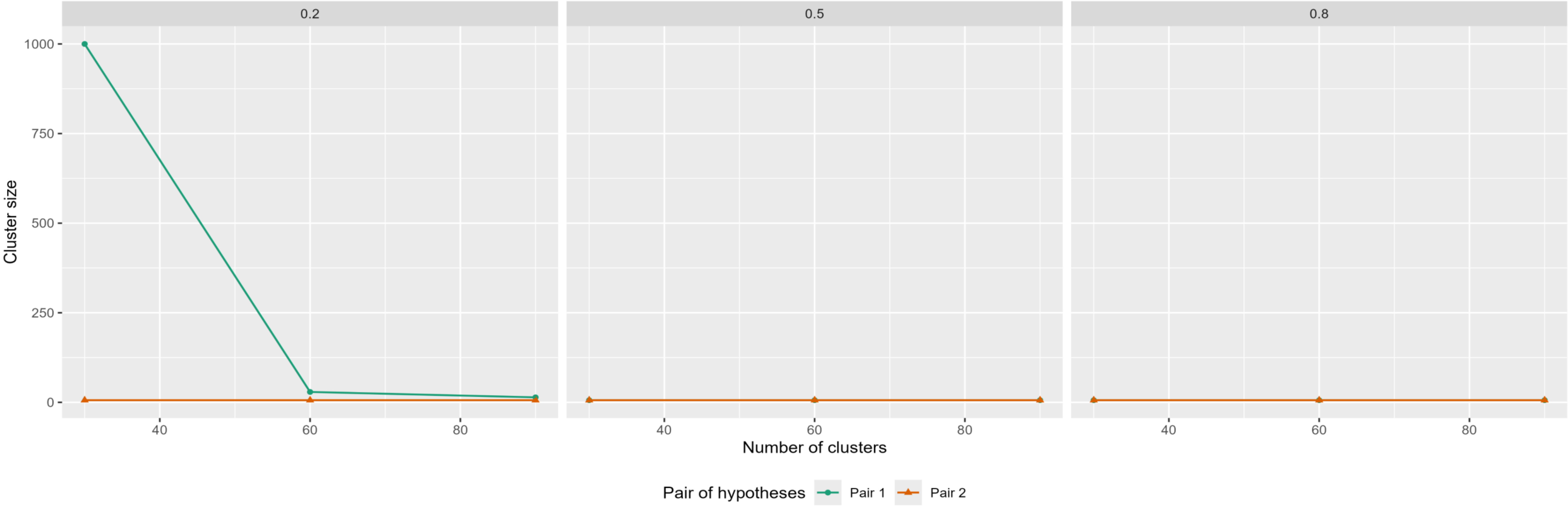
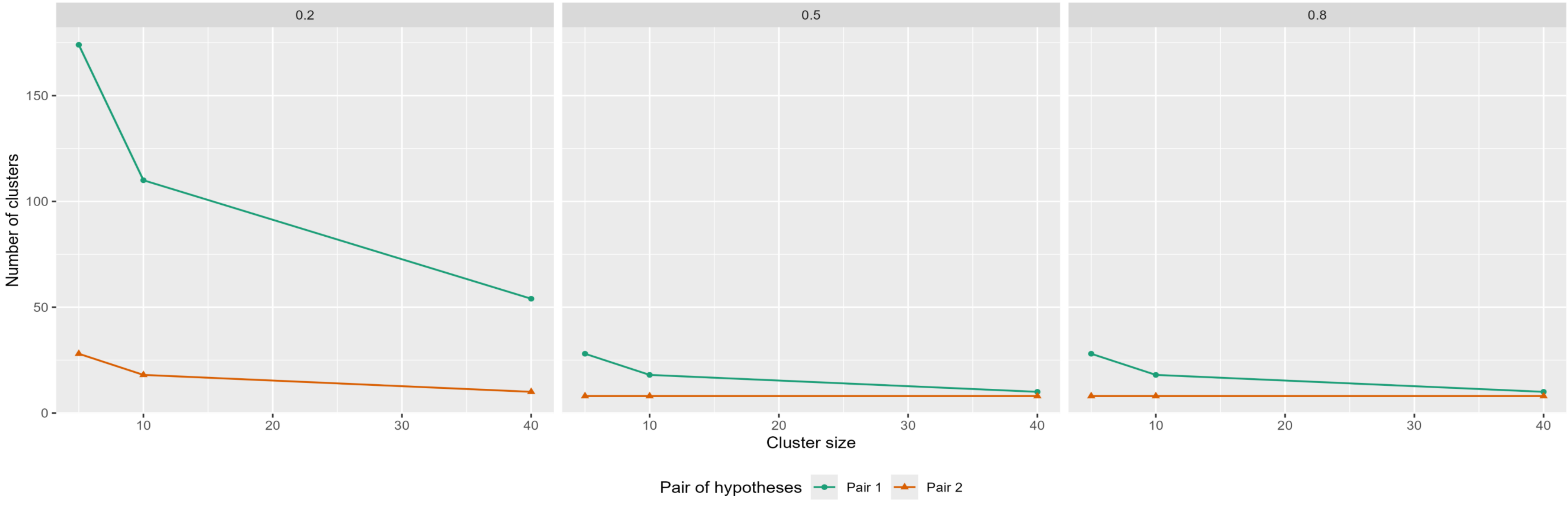
Pair 1 $H_0: \mu_C = \mu_T$
$H_1: \mu_C < \mu_T$

Pair 2 $H_1: \mu_C < \mu_T$
$H_2: \mu_C > \mu_T$



Pair 1
$$H_0: \mu_C = \mu_T$$
$$H_1: \mu_C < \mu_T$$

Pair 2
$$H_1: \mu_C < \mu_T$$
$$H_2: \mu_C > \mu_T$$



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How to determine the sample size with multiple outcomes when using Bayes factors?

Multiple outcomes

- Homogeneous effect test
- Intersection-union test
- Omnibus test

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Thank you for your attention!