4. 张量

矢量 $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\overleftrightarrow{T} = \vec{A}\vec{B}$ 一般　　$\vec{B} = B_x\hat{i} + B_y\hat{j}$

$= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})(B_x\hat{i} + B_y\hat{j})$

$= A_xB_x\hat{i}\hat{i} + A_xB_y\hat{i}\hat{j} + A_yB_x\hat{j}\hat{i} + A_yB_y\hat{j}\hat{j} + A_zB_x\hat{k}\hat{i} + A_zB_y\hat{k}\hat{j}$

记 $\overleftrightarrow{T} = \sum_{ij} T_{ij}\hat{e}_i\hat{e}_j$　类似于 $\vec{A} = \sum_i A_i\hat{e}_i$

其中(分量) $T_{ij} = \begin{pmatrix} A_xB_x & A_xB_y & 0 \\ A_yB_x & A_yB_y & 0 \\ A_zB_x & A_zB_y & 0 \end{pmatrix}$

$\overleftrightarrow{T}$ 为二阶张量，一般可记为

$$T_{ij} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}$$

类似，三阶 $\overleftrightarrow{T} = \sum_{ijk} T_{ijk} \overbrace{\hat{e}_i\hat{e}_j\hat{e}_k}^{张量基矢}$

张量的阶数和维数

标量(0阶张量)　矢量(1阶张量)

(二阶)单位张量

$$\overleftrightarrow{I} = \hat{e}_1\hat{e}_1 + \hat{e}_2\hat{e}_2 + \hat{e}_3\hat{e}_3$$

$$I_{ij} = \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

运算性质:　$\vec{A}\vec{B} \neq \vec{B}\vec{A}$

$(\vec{A}\vec{B})\cdot\vec{C} = \vec{A}(\vec{B}\cdot\vec{C})$

$(\vec{A}\vec{B}):(\vec{C}\vec{D}) = (\vec{B}\cdot\vec{C})(\vec{A}\cdot\vec{D})$

$(\vec{A}\cdot\nabla)\vec{B} = \vec{A}\cdot(\nabla\vec{B})$

$\nabla\cdot(\vec{A}\vec{B}) = (\nabla\cdot\vec{A})\vec{B} + (\vec{A}\cdot\nabla)\vec{B}$

$\nabla\times(\vec{A}\vec{B}) = (\nabla\times\vec{A})\vec{B} - (\vec{A}\times\nabla)\vec{B}$

$(\vec{A}\times\nabla)\cdot\overleftrightarrow{T} = \vec{A}\cdot(\nabla\times\overleftrightarrow{T})$

第~~零~~三章　电动力学引论

1. Maxwell 方程组

真 $\begin{cases} \nabla\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} \end{cases}$　　介 $\begin{cases} \nabla\cdot\vec{D} = \rho_f \\ \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B} = 0 \\ \nabla\times\vec{H} = \vec{J}_f + \dfrac{\partial\vec{D}}{\partial t} \end{cases}$

Lorentz 力 $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$$\text{Maxwell} \xleftrightarrow{\text{互推}} \text{电荷守恒：} \boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0}$$

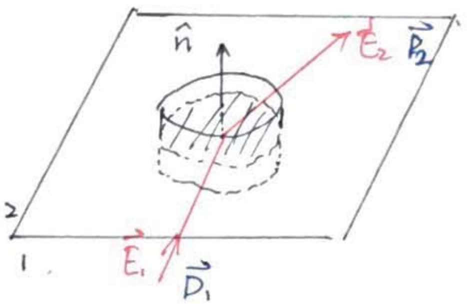
$$\xrightarrow{\text{积分}} \frac{\partial}{\partial t}\int \rho \, dV + \int \nabla \cdot \vec{J} \, dV = 0$$

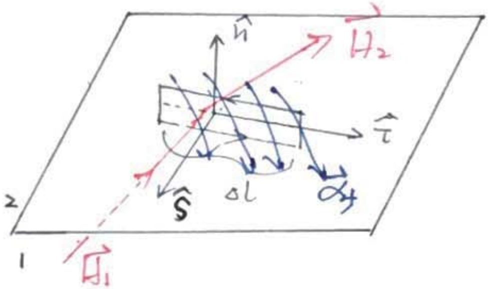
$$\frac{dQ}{dt} + \oint \underbrace{\vec{J} \cdot d\vec{S}}_{dI} = 0$$

$$\therefore \frac{dQ_{内增}}{dt} + I_{出} = 0$$

2、Maxwell 方程组的边值形式

$$\begin{cases} \oint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint \rho \, dV = \frac{Q}{\varepsilon_0} \\ \oint \vec{E} \cdot d\vec{l} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \\ \oint \vec{B} \cdot d\vec{S} = 0 \\ \oint \vec{B} \cdot d\vec{l} = \mu_0 \underset{\substack{\| \\ \mu_0 I}}{\iint \vec{J} \cdot d\vec{S}} + \mu_0 \varepsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \cdot d\vec{S} \end{cases}$$





①通：$\oiint \vec{D} \cdot d\vec{S} = Q_f$

$$(\vec{D}_2 - \vec{D}_1) \cdot \Delta S \hat{n} = \sigma_f \Delta S$$

$$\boxed{\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f}$$

或 $D_{2n} - D_{1n} = \sigma_f$

同理对于 $\vec{B}$ $\boxed{\hat{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0}$

②环：足够窄 $\iint \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \to 0$

$$\oint \vec{H} \cdot d\vec{l} = I_f + \frac{d}{dt}\iint_S \vec{D} \cdot d\vec{S}$$

注 $I = \iint \vec{J} \cdot d\vec{S}$ (体)
$= \int \vec{\alpha}_f \cdot d\vec{l}$ (面)

$$(\vec{H}_2 - \vec{H}_1) \cdot (\underbrace{-\hat{\tau}}_{\hat{s} \times \hat{n}} \cancel{\Delta l}) = \vec{\alpha}_f \cdot \cancel{\Delta l} \, \hat{s}$$

$$\hat{s} \cdot \hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}_f \cdot \hat{s}$$

$$\therefore \boxed{\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{\alpha}_f}$$ 或 $H_{2s} - H_{1s} = \alpha_f$

同理对于 $\vec{E}$ $\boxed{\hat{n} \times (\vec{E}_2 - \vec{E}_1) = 0}$