ECE408 Spring 2020

Applied Parallel Programming

Lecture 16
Parallel Computation Patterns – Parallel Scan (Prefix Sum)

Objective

- to learn parallel scan (prefix sum) algorithms based on reductions and reverse reductions
- to learn the concept of double buffering
- to understand tradeoffs between work efficiency and latency
- to learn how to develop hierarchical algorithms (across multiple kernels)

Scan Includes all Partial Results

Reductions are a simplified form of scans.

In scan / parallel prefix,

- we need all of the partial sums
- (or whatever the operator might be).

(Inclusive) Scan (Prefix-Sum) Definition

Definition: *The scan operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the prefix-sum array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, the scan operation
on the array [3 1 7 0 4 1 6 3],
returns [3 4 11 11 15 16 22 25].

Example: Sharing a Big Sandwich

You order a 100-inch sandwich to feed 10 people, and you know how much each person wants in inches:

[3 5 2 7 28 4 3 0 8 1] .

How do you cut the bread quickly?

How much of the sandwich is left over?

Method 1: sequentially!

Cut 3 inches, then cut 5 inches, then …

Method 2: **calculate cutting offsets with prefix-sum**

[3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 inches left)

Typical Applications of Scan

A simple and useful parallel building block.

Convert sequential recurrences

```
for(j=1;j<n;j++)
    out[j] = out[j-1] + f(j);
```

into parallel:

```
forall(j) { temp[j] = f(j) };
scan(out, temp);
```


Typical Applications of Scan

- Useful for many parallel algorithms:
  - radix sort
  - quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms
  - Etc.

Other Applications

- Assigning camp slots
- Assigning farmer market space
- Allocating memory to parallel threads
- Allocating memory buffer to communication channels
- …

An Inclusive Sequential Scan

Given a sequence $[x_0, x_1, x_2, ...]$

Calculate output $[y_0, y_1, y_2, ...]$

Such that

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$
$$...$$

Using a recursive definition

$$y_i = y_{i-1} + x_i$$

An Sequential C Implementation

```
y[0] = x[0];
for (i = 1; i < Max_i; i++) y[i] = y [i-1] + x[i];
```

Computationally efficient:

N additions needed for N elements - O(N)!

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$

"Parallel programming is easy as long as you do not care about performance."

Parallel Inclusive Scan using Reduction Trees

Calculate each output element as the reduction of all previous elements

- Some reduction partial sums will be shared among the calculation of output elements
- Based on hardware added design by Peter Kogge and Harold Stone at IBM in the 1970s – Kogge-Stone Trees
- Goal: low latency

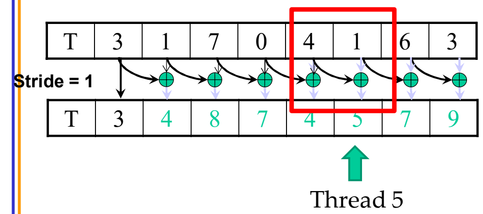
A Kogge-Stone Parallel Scan Algorithm

T	3	1	7	0	4	1	6	3

1. Load input from global memory into shared memory array T

Each thread loads one value from the input (global memory) array into shared memory array T.

A Kogge-Stone Parallel Scan Algorithm

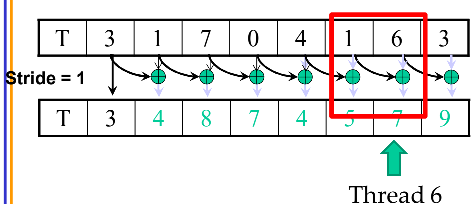


1. (previous slide)
2. Assuming n is a power of 2. Iterate log(n) times, stride from 1 to n/2. Threads *stride* to *n-1 active:* add pairs of elements that are *stride* elements apart.

- Active threads: *stride* to *n*-1 (*n - stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j-stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - make sure that input is in place
  - make sure that all input elements have been used

Iteration #1
Stride = 1

A Kogge-Stone Parallel Scan Algorithm



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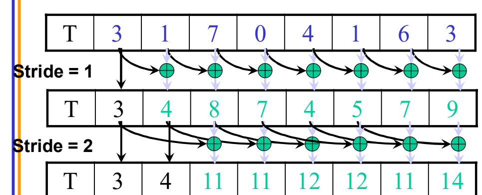
- Active threads: *stride* to *n*-1 (*n - stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j-stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - syncthreads(); // make sure that input is in place
  - float temp = T[*j*] + T[*j-stride*];
  - syncthreads(); // make sure that previous output has been consumed
  - T[*j*] = temp;

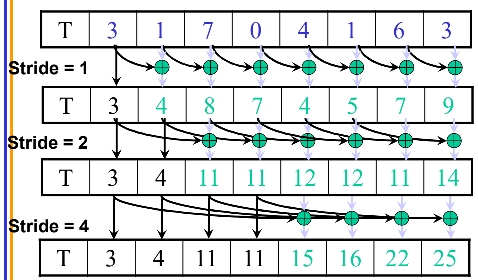
Iteration #1
Stride = 1

A Kogge-Stone Parallel Scan Algorithm



Iteration #2
Stride = 2

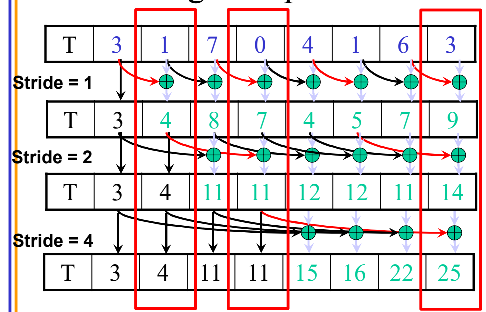
A Kogge-Stone Parallel Scan Algorithm



1. …
2. …
3. Write output from shared memory to device memory

Iteration #3
Stride = 4

Sharing Computation in Kogge-Stone

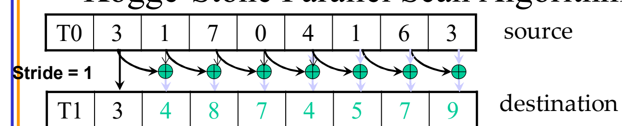


Iteration #3
Stride = 4

Double Buffering

- Use two copies of data T0 and T1
- Start by using T0 as input and T1 as output
- Switch input/output roles after each iteration
  - Iteration 0: T0 as input and T1 as output
  - Iteration 1: T1 as input and T0 and output
  - Iteration 2: T0 as input and T1 as output
- This is typically implemented with two pointers, source and destination that swap their contents from one iteration to the next
- This eliminates the need for the second __syncthreads() call

A Double-Buffered Kogge-Stone Parallel Scan Algorithm

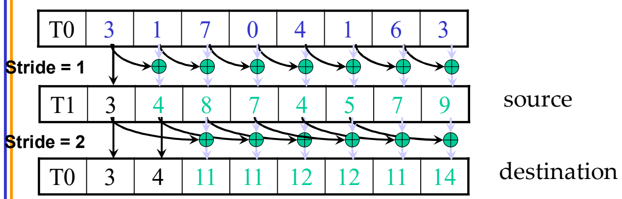


- source = &T0[0]; destination = &T1[0];
- Each iteration requires only one syncthreads()
  - syncthreads(); // make sure that input is in place
  - float destination[*j*] = source[*j*] + source[*j-stride*];
  - temp = destination; destination = source; source = temp;

•After the loop, write destination contents to global memory

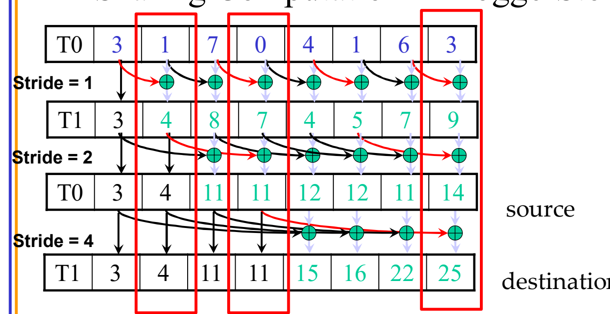
Iteration #1
Stride = 1

A Kogge-Stone Parallel Scan Algorithm



Iteration #2
Stride = 2

Sharing Computation in Kogge-Stone



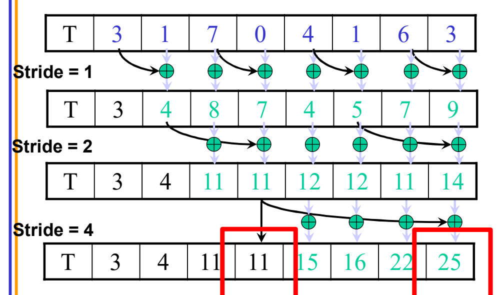
Iteration #3
Stride = 4

- Each iteration requires only one syncthreads()
  - syncthreads(); // make sure that input is in place
  - float destination[*j*] = source[*j*] + source[*j-stride*];
  - temp = destination; destination = source; source = temp;
- After the loop, write destination contents to global memory

Work Efficiency Analysis

- A Kogge-Stone scan kernel executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) add operations each
  - Total # of add operations: n * log(n) - (n-1) → O(n*log(n)) work
- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 1,000,000 elements!
  - Typically used within each block, where n ≤ 1,024
- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

A Kogge-Stone Parallel Scan Algorithm



Improving Efficiency

- A common parallel algorithm pattern:

  Balanced Trees

  – Build a balanced binary tree on the input data and sweep it to and from the root

  – Tree is not an actual data structure, but a concept to determine what each thread does at each step

- For scan:

  – Traverse down from leaves to root building partial sums at internal nodes in the tree

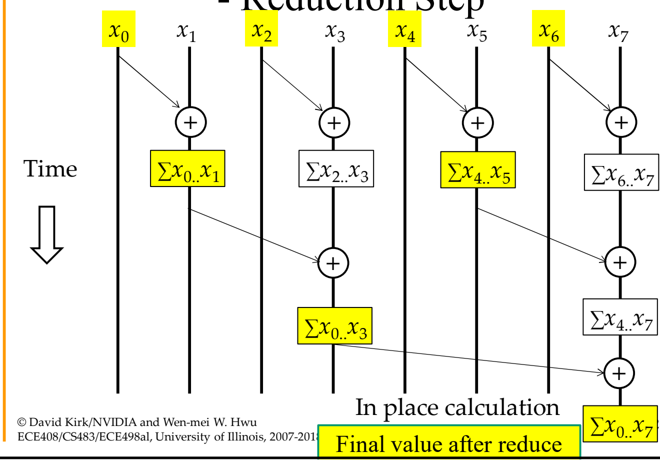
    - Root holds sum of all leaves

  – Traverse back up the tree building the scan from the partial sums

25

25

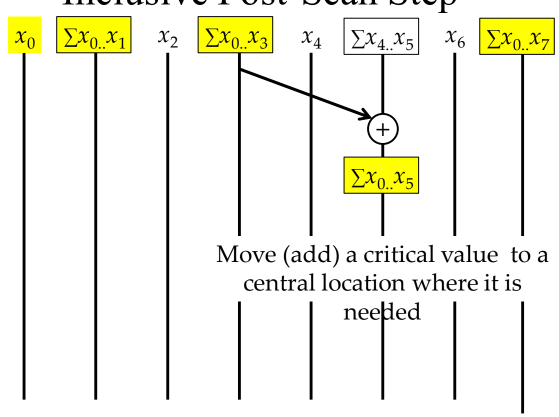
Brent-Kung Parallel Scan - Reduction Step



26

26

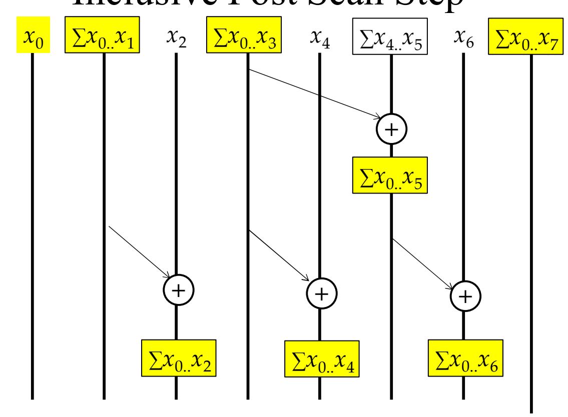
Inclusive Post-Scan Step



27

27

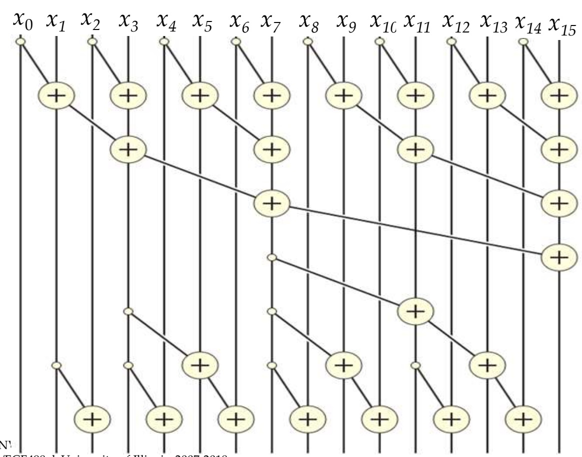
Inclusive Post Scan Step



28

28

Putting it Together (Data View)



29

Reduction Step Kernel Code

```
// float T[2*BLOCK_SIZE] is in shared memory
// for previous slide, BLOCK_SIZE is 8
int stride = 1;
while(stride < 2*BLOCK_SIZE)
  {
     __syncthreads();
     int index = (threadIdx.x+1)*stride*2 - 1;
     if(index < 2*BLOCK_SIZE && (index-stride) >= 0)
        T[index] += T[index-stride];
     stride = stride*2;
  }
```

 30

30

Reduction Step Kernel Code

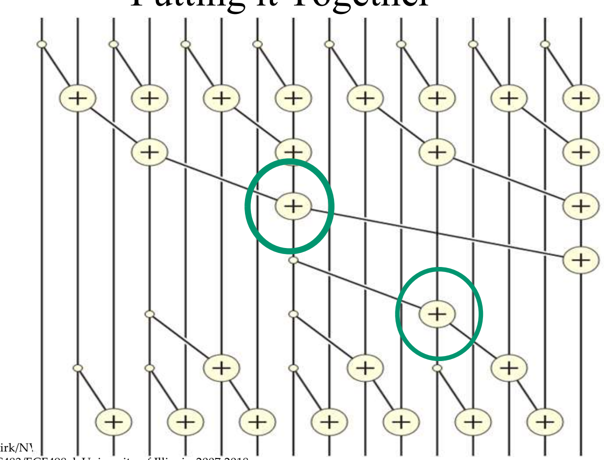
```
// float T[2*BLOCK_SIZE] is in shared memory

int stride = 1;
while(stride < 2*BLOCK_SIZE)
{
     int index = (threadIdx.x+1)*stride*2 - 1;
     if(index < 2*BLOCK_SIZE && (index-stride) >=0)
        T[index] += T[index-stride];
     stride = stride*2;        // For previous example,
                               // threadIdx.x+1    = 1, 2, 3, 4, 5, 6, 7,8
                               // stride = 1, index = 1, 3, 5, 7, 9, 11, 13, 15
     __syncthreads();
  }
```

 31

31

Putting it Together



32

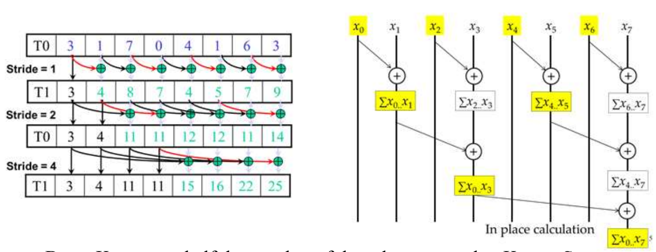
Post Scan Step

```
int stride = BLOCK_SIZE/2;
while(stride > 0)
  {
     __syncthreads();
     int index = (threadIdx.x+1)*stride*2 - 1;
     if((index+stride) < 2*BLOCK_SIZE)
     {
        T[index+stride] += T[index];
     }
     stride = stride / 2;        // for the previous example,
                                 // BLOCK_SIZE is 8
                                 // stride will go 4, 2, 1
                                 // for the first iteration, the active thread
                                 // will be thread 0, with index = 7 and
  }                              // index+stride = 11
```


Work Analysis

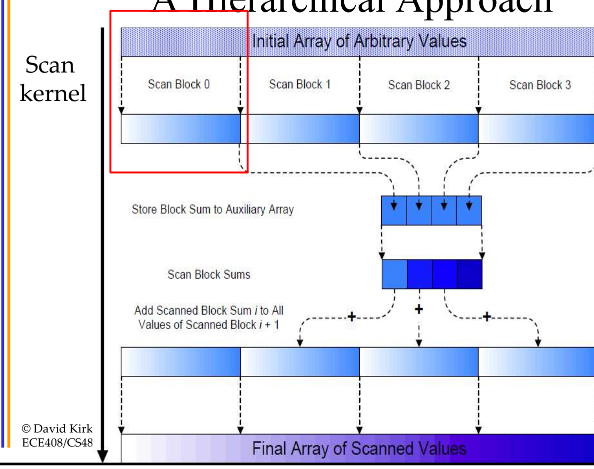
- The parallel Inclusive Scan executes 2* log(n) parallel iterations
  - log(n) in reduction and log(n) in post scan
  - The iterations do n/2, n/4,..1, (2-1), …., (n/4-1), (n/2-1) useful adds
  - In our example, n = 16, the number of useful adds is 16/2 + 16/4 + 16/8 + 16/16 + (16/8-1) + (16/4-1) + (16/2-1)
  - Total adds: (n-1) + (n-2) – (log(n) -1) = 2*(n-1) – log(n)→ O(n) work
- The total number of adds is no more than twice of that done in the efficient sequential algorithm
  - The benefit of parallelism can easily overcome the 2X work when there is sufficient hardware

Kogge-Stone vs. Brent-Kung



- Brent-Kung uses half the number of threads compared to Kogge-Stone
  - Each thread should load two elements into the shared memory
- Brent-Kung takes twice the number of steps compared to Kogge-Stone
  - Kogge-Stone is more popular for parallel scan with blocks in GPUs

Overall Flow of Complete Scan
A Hierarchical Approach



Using Global Memory Contents in CUDA

- Data in registers and shared memory of one thread block are not visible to other blocks
- To make data visible, the data has to be written into global memory
- However, any data written to the global memory are not visible until a memory fence. This is typically done by terminating the kernel execution
- Launch another kernel to continue the execution. The global memory writes done by the terminated kernels are visible to all tead blocks.

 37

37

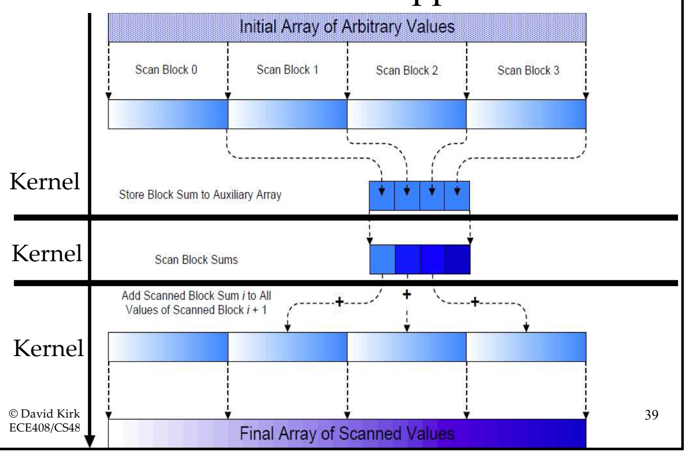
Scan of Arbitrary Length Input

- Build on the scan kernel that handles up to 2*blockDim.x elements from Brent-Kung
  - For Kogge-Stone, have each section of blockDim.x elements assigned to a block
- Have each block write the sum of its section into a Sum array using its blockIdx.x as index
- Run parallel scan on the Sum array
  - May need to break down Sum into multiple sections if it is too big for a block
- Add the scanned Sum array values to the elements of corresponding sections

 38

38

Overall Flow of Complete Scan
A Hierarchical Approach



 39

39

(Exclusive) Scan Definition

Definition: *The exclusive* scan *operation takes a binary associative operator* $\oplus$, *and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}]$$

and returns the array

$$[0, x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-2})].$$

Example: If $\oplus$ is addition, then the exclusive scan operation on [3 1 7 0 4 1 6 3],
would return [0 3 4 11 11 15 16 22].

 40

40

Why Exclusive Scan

- To find the beginning address of allocated buffers
- Inclusive and Exclusive scans can be easily derived from each other; it is a matter of convenience

	[3 1 7 0 4 1 6 3]
Exclusive	[0 3 4 11 11 15 16 22]
Inclusive	[3 4 11 11 15 16 22 25]

41

A simple exclusive scan kernel

- Adapt an inclusive, Kogge-Stone scan kernel
  - Block 0:
    - Thread 0 loads 0 into (shared) XY[0]
    - Other threads load (global) X[threadIdx.x-1] into XY[threadIdx.x]
  - All other blocks:
    - All thread load X[blockIdx.x*blockDim.x+threadIdx.x-1] into XY[threadIdex.x]
- Similar adaption for Brent-Kung kernel but pay attention that each thread loads two elements
  - Only one zero should be loaded
  - All elements should be shifted by only one position
- Intellectual contribution vs. practical contribution

42

ANY MORE QUESTIONS?

43