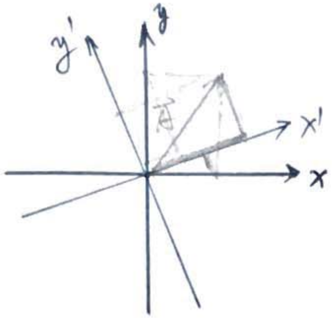
(2) 转动矩阵

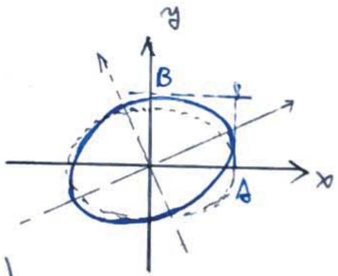
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$



即 $\begin{cases} x = x'\cos\beta - y'\sin\beta \\ y = x'\sin\beta + y'\cos\beta \end{cases}$

故 $\begin{cases} x' = x\cos\beta + y\sin\beta \\ y' = -x\sin\beta + y\cos\beta \end{cases}$

椭偏 $\begin{cases} E_x = a e^{i(\vec{k}\cdot\vec{r} - \omega t)} \\ iE_y = ib\, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \end{cases}$



$$\begin{pmatrix} E_x \\ iE_y \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} a \\ ib \end{pmatrix} =: \begin{pmatrix} A \\ iB \end{pmatrix}$$ 此时为右旋 左旋 $(b \to -b)$

$$\begin{cases} A = a\cos\beta - ib\sin\beta \\ B = b\cos\beta - ia\sin\beta \end{cases}$$

定义 $r = \dfrac{a-b}{a+b}$ 或 $\dfrac{b}{a} = \dfrac{1-r}{1+r}$

$$\frac{B}{A} = \frac{b\cos\beta - ia\sin\beta}{a\cos\beta - ib\sin\beta} = \frac{(b-a)\cos\beta + a e^{-i\beta}}{(a-b)\cos\beta + b e^{-i\beta}}$$

$$= \frac{(b-a)(e^{i\beta} + e^{-i\beta}) + 2a e^{-i\beta}}{(a-b)(e^{i\beta} + e^{-i\beta}) + 2b e^{-i\beta}}$$

$$= \frac{(b-a)e^{i\beta} + (b+a)e^{-i\beta}}{(a-b)e^{i\beta} + (a+b)e^{-i\beta}}$$

$$= \frac{-re^{i\beta} + e^{-i\beta}}{re^{i\beta} + e^{-i\beta}} = \frac{-re^{2i\beta} + 1}{re^{2i\beta} + 1}$$

定义 $\alpha = 2\beta$ ，则 $= \dfrac{1 - re^{i\alpha}}{1 + re^{i\alpha}}$

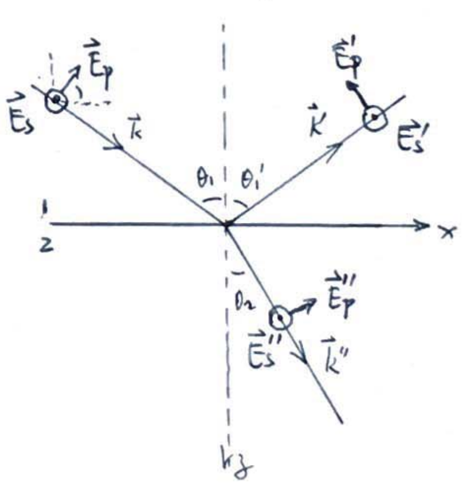
圆偏基下

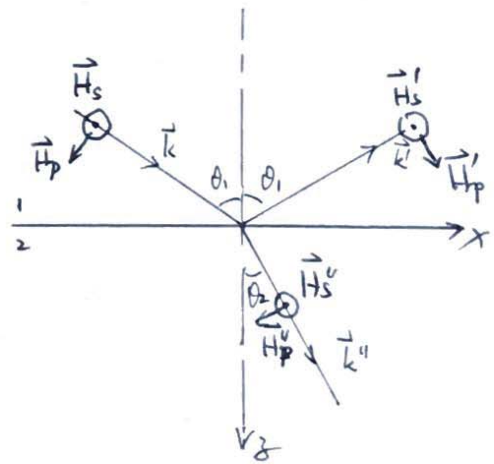
$$\frac{E_-}{E_+} = \frac{E_1 - E_2}{E_1 + E_2} = \frac{(1+re^{i\alpha}) - (1-re^{i\alpha})}{(1+re^{i\alpha}) + (1-re^{i\alpha})} = re^{i\alpha}$$

(3) Stokes 参数

§2. 电磁波在界面的反射和折射

1. 反射、折射定律





Maxwell 在边值上（介质界面）

$$\begin{cases} \hat{n}\cdot(\varepsilon_2\vec{E}_2-\varepsilon_1\vec{E}_1)=0 \\ \hat{n}\times(\vec{E}_2-\vec{E}_1)=0 \quad \checkmark \\ \hat{n}\cdot(\mu_2\vec{H}_2-\mu_1\vec{H}_1)=0 \\ \hat{n}\times(\vec{H}_2-\vec{H}_1)=0 \quad \checkmark \end{cases}$$

入: $\vec{E}=\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

反: $\vec{E}'=\vec{E}_0' e^{i(\vec{k}'\cdot\vec{r}-\omega' t)}$

透: $\vec{E}''=\vec{E}_0'' e^{i(\vec{k}''\cdot\vec{r}-\omega'' t)}$

则 $\hat{n}\times(\vec{E}+\vec{E}')\big|_s=\hat{n}\times\vec{E}''\big|_s$

$$\hat{n}\times\left[\vec{E}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}+\vec{E}_0' e^{i(\vec{k}'\cdot\vec{r}-\omega' t)}\right]\Big|_{z=0}=\hat{n}\times\vec{E}_0'' e^{i(\vec{k}''\cdot\vec{r}-\omega'' t)}\Big|_{z=0}$$

一定有 $k_x x+k_y y-\omega t=k_x' x+k_y' y-\omega' t=k_x'' x+k_y'' y-\omega'' t$

$$\hat{n}\times(\vec{E}_0+\vec{E}_0')=\hat{n}\times\vec{E}_0''$$

对于任意 x, y, t 都成立，则 $\begin{cases} k_x=k_x'=k_x'' \\ k_y=k_y'=k_y'' \\ \omega=\omega'=\omega'' \end{cases}$

由于 1.2 两侧 $k=\frac{\omega}{v_1}=k'$，$v_1=\frac{1}{\sqrt{\mu_1\varepsilon_1}}$，而 $k''=\frac{\omega''}{v_2}=\omega''\sqrt{\mu_2\varepsilon_2}$

取入射面为 $y=0$ 平面，则

$$k\sin\theta_1=k'\sin\theta_1'=k''\sin\theta_2$$

有 $\sin\theta_1=\sin\theta_1'$ 及 $\omega\sqrt{\mu_1\varepsilon_1}\sin\theta_1=\omega\sqrt{\mu_2\varepsilon_2}\sin\theta_2$

反之 $\theta_1=\theta_1'$ 折 $\frac{\sin\theta_1}{\sin\theta_2}=\frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}=\frac{n_2}{n_1}$ 即 $n_1\sin\theta_1=n_2\sin\theta_2$

其中 $n_1=\sqrt{\mu_1\varepsilon_1}$ 相对折 $n_{12}=\frac{n_1}{n_2}=\sqrt{\frac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$，对于大多数透明介质: $\mu=\mu_0$，因 $n_1=\sqrt{\varepsilon_1}$

2. Fresnel 公式

由 Maxwell 两切向连续条件

$$\begin{cases} E_{0s}+E_{0s}'=E_{0s}'' & ① \\ E_{0p}\cos\theta_1-E_{0p}'\cos\theta_1=E_{0p}''\cos\theta_2 & ② \\ H_{0s}+H_{0s}'=H_{0s}'' \\ H_{0p}\cos\theta_1-H_{0p}'\cos\theta_1=H_{0p}''\cos\theta_2 \end{cases}$$

由幅值也 $\sqrt{\varepsilon}E=\sqrt{\mu}H$，后两式改写

$$\begin{cases} \sqrt{\frac{\varepsilon_1}{\mu_1}}E_{0p}+\sqrt{\frac{\varepsilon_1}{\mu_1}}E_{0p}'=\sqrt{\frac{\varepsilon_2}{\mu_2}}E_{0p}'' & ③ \\ \sqrt{\frac{\varepsilon_1}{\mu_1}}E_{0s}\cos\theta_1-\sqrt{\frac{\varepsilon_1}{\mu_1}}E_{0s}'\cos\theta_1=\sqrt{\frac{\varepsilon_2}{\mu_2}}E_{0s}''\cos\theta_2 & ④ \end{cases}$$

结合 ①④

$$\frac{E_{0s}'}{E_{0s}}=\frac{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1-\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_2}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1+\sqrt{\frac{\varepsilon_2}{\mu_2}}\cos\theta_2}$$

$$=\frac{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1-\sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1}}{\sqrt{\frac{\varepsilon_1}{\mu_1}}\cos\theta_1+\sqrt{\frac{\varepsilon_2}{\mu_2}}\sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_1}}$$

利用 $n_2=\sqrt{\mu_2\varepsilon_2}$，$n_1=\sqrt{\mu_1\varepsilon_1}$

有

$$=\frac{n_1\cos\theta_1-\frac{\mu_1}{\mu_2}\sqrt{n_2^2-n_1^2\sin^2\theta_1}}{n_1\cos\theta_1+\frac{\mu_1}{\mu_2}\sqrt{n_2^2-n_1^2\sin^2\theta_1}}$$

类似，对于透射波

$$\frac{E''_{0s}}{E_{0s}} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + \frac{\mu_1}{\mu_2}\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}$$

相应的，平行分量

$$\frac{E'_{0p}}{E_{0p}} = \frac{\frac{\mu_1}{\mu_2} n_2^2\cos\theta_1 - n_1\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}{\frac{\mu_1}{\mu_2} n_2^2\cos\theta_1 + n_1\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}$$

$$\frac{E''_{0p}}{E_{0p}} = \frac{2n_1 n_2\cos\theta_1}{\frac{\mu_1}{\mu_2} n_2^2\cos\theta_1 + n_1\sqrt{n_2^2 - n_1^2\sin^2\theta_1}}$$

另外，取 $\mu_1 = \mu_2 = \mu_0$（透明）保留 θ_2，则

$$\frac{E'_{0s}}{E_{0s}} = \frac{\sqrt{\varepsilon_1}\cos\theta_1 - \sqrt{\varepsilon_2}\cos\theta_2}{\sqrt{\varepsilon_1}\cos\theta_1 + \sqrt{\varepsilon_2}\cos\theta_2}$$

此时 $n = \sqrt{\mu_0\varepsilon} \propto \sqrt{\varepsilon}$，而 $n_1\sin\theta_1 = n_2\sin\theta_2$

$$= \frac{\sin\theta_2\cos\theta_1 - \sin\theta_1\cos\theta_2}{\cdots + \cdots}$$

$$= \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_2+\theta_1)} = -\frac{\sin(\theta_1-\theta_2)}{\sin(\theta_1+\theta_2)}$$

类似于其他三个式 见《郭硕鸿》

作业： 7.1 (b) 只求线偏振下的参数

7.2 7.3 (a)