## 9.3

半径为R、电导率为无穷大的金属球壳的两半被极小的绝缘间隙隔开。在两半球之间施加一交变电势,使电势为 $\pm V\cos\omega t$ 。在长波极限下,求辐射场、辐射功率的角分布以及球体的总辐射功率

长波极限下,由(3.36),有壳层外电势

$$\Phi(r,\theta) = V \left[ \frac{3}{2} \left( \frac{R}{r} \right)^2 P_1(\cos \theta) - \frac{7}{8} \left( \frac{R}{r} \right)^4 P_3(\cos \theta) + \cdots \right]$$
 (1)

对应于, $\Phi(r, heta) = rac{1}{4\pi arepsilon_0} rac{p\cos heta}{r^2}$ ,有球体偶极矩

$$\vec{p} = 6\pi\varepsilon_0 V R^2 \hat{z} \tag{2}$$

由 (9.19) 有

$$ec{H} = rac{ck^2}{4\pi} (ec{n} imes ec{P}) rac{e^{ikr}}{r} = -rac{3}{2} \left(rac{\omega R}{c}
ight)^2 rac{V}{z_D} \sin heta rac{e^{irac{\omega}{c}r}}{r} \hat{\phi}$$
 (3)

$$ec{E}=z_0ec{H} imesec{n}=-rac{3}{2}Vigg(rac{\omega R}{c}igg)^2\sin hetarac{e^{irac{\omega}{c}r}}{r}\hat{ heta}$$

有单位立体角的辐射功率

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2} k^4 |(\vec{n} \times \vec{p}) \times \vec{n}|^2 = \frac{c^2 Z_0}{32\pi^2} k^4 \left( |\vec{p}|^2 - |\vec{p} \cdot \vec{n}|^2 \right) = \frac{9}{8} \left( \frac{wR}{c} \right)^4 \frac{v^2}{z_0} \sin^2 \theta \quad (5)$$

总共率为

$$P = \int \frac{dP}{d\Omega} d\Omega = 3\pi \left(\frac{\omega R}{c}\right)^4 \frac{v^2}{z_0} \tag{6}$$