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6.3

无界电介质中，准静态失势的齐次扩散方程(5.160)对于初值问题有解

$$\mathbf{A}(\mathbf{x}, t)=\int d^{3} x^{\prime} G\left(\mathbf{x}-\mathbf{x}^{\prime}, t\right) \mathbf{A}\left(\mathbf{x}^{\prime}, 0\right) \tag{42}$$

其中， $\mathbf{A}\left(\mathbf{x}\prime, 0\right)$表示场的初值，$G$为适当的核函数

（a）利用三维空间傅里叶变换求解 $\mathbf{A}(\mathbf{x}, t)$的初值问题。格林函数有傅里叶表示

$$G\left(\mathbf{x}-\mathbf{x}^{\prime}, t\right)=\frac{1}{(2 \pi)^{3}} \int d^{3} k e^{-k^{2} t / \mu \sigma} e^{i \mathbf{k} \cdot\left(\mathbf{x}-\mathbf{x}^{\prime}\right)} \tag{43}$$

（b）通过在时间和空间上进行傅里叶分解，并在ω复平面上对频率积分来得到（a）中结论。证明$G\left(\mathbf{x}-\mathbf{x}\prime, t\right)$是满足下列非齐次方程的扩散格林函数

$$\frac{\partial G}{\partial t}-\frac{1}{\mu \sigma} \nabla^{2} G=\delta^{(3)}\left(\mathbf{x}-\mathbf{x}^{\prime}\right) \delta(t) \tag{44}$$

（c）证明，若σ在全空间是均匀的，则格林函数为

$$G\left(\mathbf{x}, t ; \mathbf{x}^{\prime}, 0\right)=\Theta(t)\left(\frac{\mu \sigma}{4 \pi t}\right)^{3 / 2} \exp \left(\frac{-\mu \sigma\left|\mathbf{x}-\mathbf{x}^{\prime}\right|^{2}}{4 t}\right) \tag{45}$$

> 5.160
>
> $$\nabla^{2} \mathbf{A}=\mu \sigma \frac{\partial \mathbf{A}}{\partial t} \tag{46}$$
>
> 傅里叶微分特性
>
> $$\mathcal{F}\left[f^{\prime}(x)\right]=i \omega \mathcal{F}[f(x)] \tag{47}$$
>
> 卷积特性
>
> $$\mathcal{F}[f * g]=\mathcal{F}[f] \cdot \mathcal{F}[g] \tag{48}$$
>
> 常用变换

时域信号	角频率表示的傅里叶变换	弧频率表示的傅里叶变换
$g(t) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t}\, \mathrm{d}\omega$	$G(\omega) \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t) e^{-i\omega t}\, \mathrm{d}t$	$G(f) \equiv \int_{-\infty}^{\infty} g(t) e^{-i2\pi f t}\, \mathrm{d}t$
$\operatorname{rect}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \operatorname{sinc}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\lvert a\rvert} \cdot \operatorname{sinc}\left(\frac{f}{a}\right)$
$\operatorname{sinc}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \operatorname{rect}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\lvert a\rvert} \cdot \operatorname{rect}\left(\frac{f}{a}\right)$
$\operatorname{sinc}^2(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \operatorname{tri}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\lvert a\rvert} \cdot \operatorname{tri}\left(\frac{f}{a}\right)$
$\operatorname{tri}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \operatorname{sinc}^2\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{\lvert a\rvert} \cdot \operatorname{sinc}^2\left(\frac{f}{a}\right)$
$e^{-\alpha t^2}$	$\frac{1}{\sqrt{2\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}}$	$\sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi f)^2}{\alpha}}$
$e^{iat^2} = e^{-\alpha t^2}\big\rvert_{\alpha=-ia}$	$\frac{1}{\sqrt{2a}} \cdot e^{-i\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)}$	$\sqrt{\frac{\pi}{a}} \cdot e^{-i\left(\frac{\pi^2 f^2}{a}-\frac{\pi}{4}\right)}$

(49)

$\cos\left(at^2\right)$	$\frac{1}{\sqrt{2a}}\cos\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$	$\sqrt{\frac{\pi}{a}}\cos\left(\frac{\pi^2 f^2}{a}-\frac{\pi}{4}\right)$
$\sin\left(at^2\right)$	$\frac{-1}{\sqrt{2a}}\sin\left(\frac{\omega^2}{4a}-\frac{\pi}{4}\right)$	$-\sqrt{\frac{\pi}{a}}\sin\left(\frac{\pi^2 f^2}{a}-\frac{\pi}{4}\right)$
$\mathrm{e}^{-a\|t\|}$	$\sqrt{\frac{2}{\pi}}\cdot\frac{a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2 f^2}$
$\frac{1}{\sqrt{\|t\|}}$	$\frac{1}{\sqrt{\|\omega\|}}$	$\frac{1}{\sqrt{\|f\|}}$

2

函数	傅立叶变换 正，普通的频率	傅立叶变换 么正，角频率	傅立叶变换 非自正，角频率
	$\hat{f}(\xi)=$ $\int_{-\infty}^{\infty}f(x)e^{-2\pi ix\xi}dx$	$\hat{f}(\omega)=$ $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(x)e^{-i\omega x}dx$	$\hat{f}(\nu)=$ $\int_{-\infty}^{\infty}f(x)e^{-i\nu x}dx$
$a\cdot f(x)+b\cdot g(x)$	$a\cdot\hat{f}(\xi)+b\cdot\hat{g}(\xi)$	$a\cdot\hat{f}(\omega)+b\cdot\hat{g}(\omega)$	$a\cdot\hat{f}(\nu)+b\cdot\hat{g}(\nu)$
$f(x-a)$	$e^{-2\pi ia\xi}\hat{f}(\xi)$	$e^{-ia\omega}\hat{f}(\omega)$	$e^{-ia\nu}\hat{f}(\nu)$
$e^{2\pi iax}f(x)$	$\hat{f}(\xi-a)$	$\hat{f}(\omega-2\pi a)$	$\hat{f}(\nu-2\pi a)$
$f(ax)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\nu}{a}\right)$
$f(ax)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{\|a\|}\hat{f}\left(\frac{\nu}{a}\right)$
$\frac{\hat{f}(x)}{d^n f(x)}$	$f(-\xi)$	$f(-\omega)$	$2\pi f(-\nu)$
x^n	$(2\pi i\xi)^n\hat{f}(\xi)$	$(i\omega)^n\hat{f}(\omega)$	$(i\nu)^n\hat{f}(\nu)$
$(f*g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$	$\sqrt{2\pi}\hat{f}(\omega)\hat{g}(\omega)$	$\hat{f}(\nu)\hat{g}(\nu)$
$f(x)g(x)$	$(\hat{f}*\hat{g})(\xi)$	$\frac{(\hat{f}*\hat{g})(\omega)}{\sqrt{2\pi}}$	$\frac{1}{2\pi}(\hat{f}*\hat{g})(\nu)$

(50)

3

时域信号	角频率表示的 傅里叶变换	弧频率表示的 傅里叶变换
$g(t)\equiv$ $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}G(\omega)e^{i\omega t}d\omega$	$G(\omega)\equiv$ $\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}g(t)e^{-i\omega t}dt$	$G(f)\equiv$ $\int_{-\infty}^{\infty}g(t)e^{-i2\pi ft}dt$
1	$\underline{\sqrt{2\pi}\cdot\delta(\omega)}$	$\delta(f)$
$\delta(t)$	$\frac{1}{\sqrt{2\pi}}$	1
e^{iat}	$\sqrt{2\pi}\cdot\delta(\omega-a)$	$\delta\left(f-\frac{a}{2\pi}\right)$
$\cos(at)$	$\sqrt{2\pi}\frac{\delta(\omega-a)+\delta(\omega+a)}{2}$	$\frac{\delta\left(f-\frac{a}{2\pi}\right)+\delta\left(f+\frac{a}{2\pi}\right)}{2}$
$\sin(at)$	$\sqrt{2\pi}\frac{\delta(\omega-a)-\delta(\omega+a)}{2i}$	$\frac{\delta\left(f-\frac{a}{2\pi}\right)-\delta\left(f+\frac{a}{2\pi}\right)}{2i}$
t^n	$i^n\sqrt{2\pi}\delta^{(n)}(\omega)$	$\left(\frac{i}{2\pi}\right)^n\delta^{(n)}(f)$
$\frac{1}{t}$	$-i\sqrt{\frac{\pi}{2}}\operatorname{sgn}(\omega)$	$-i\pi\cdot\operatorname{sgn}(f)$
$\frac{1}{t^n}$ $\operatorname{sgn}(t)$	$-i\sqrt{\frac{\pi}{2}}\cdot\frac{(-i\omega)^{n-1}}{(n-1)!}\operatorname{sgn}(\omega)$ $\sqrt{\frac{2}{\pi}}\cdot\frac{1}{i\omega}$	$-i\pi\frac{(-i2\pi f)^{n-1}}{(n-1)!}\operatorname{sgn}(f)$ $\frac{1}{i\pi f}$
$u(t)$	$\sqrt{\frac{\pi}{2}}\left(\frac{1}{i\pi\omega}+\delta(\omega)\right)$	$\frac{1}{2}\left(\frac{1}{i\pi f}+\delta(f)\right)$

(51)

$e^{-at}u(t)$	$\frac{1}{\sqrt{2\pi}(a+i\omega)}$	$\frac{1}{a+i2\pi f}$

(a)

根据傅里叶微分特性，空间域矢势傅里叶变换到频域矢势后场方程有

> 此处频率为角频率

$$\begin{aligned}
\nabla^2\cdot\vec{A}(\vec{x},t) &= \mu\sigma\frac{\partial\vec{A}}{\partial t}\\
(ik)^2\times\vec{A}(\vec{k},t) &= \mu\sigma\frac{\partial\vec{A}}{\partial t}\\
-\frac{k^2}{\mu\sigma}dt &= \mu\sigma\frac{d\vec{A}}{\vec{A}}\\
\int-\frac{k^2}{\mu\sigma}dt &= \int\frac{d\vec{A}}{\vec{A}}\\
\ln\vec{A}(\vec{k},t) &= -\frac{k^2}{\mu\sigma}t\\
\vec{A}(\vec{k},t) &= \vec{A}(\vec{k},0)e^{-k^2t/\mu\sigma}
\end{aligned} \tag{52}$$

做逆傅里叶变换有

$$\begin{aligned}
\vec{A}(\vec{x},t) &= \frac{1}{(2\pi)^3}\int\vec{A}(\vec{k},t)e^{-\vec{k}\cdot\vec{x}}d^3k\\
&= \frac{1}{(2\pi)^3}\int\vec{A}(\vec{k},0)e^{-k^2t/\mu\sigma}e^{i\vec{k}\cdot\vec{x}}d^3k\\
&= \frac{1}{(2\pi)^3}\int\int\vec{A}\left(\vec{x}',0\right)e^{-i\vec{k}\cdot\vec{x}'}d^3x'e^{-k^2t/\mu\sigma}e^{i\vec{k}\cdot\vec{x}}d^3k
\end{aligned} \tag{53}$$

类比于(42)，有

$$G\left(\vec{x}-\vec{x}',t\right) = \frac{1}{(2\pi)^3}\int e^{-k^2t/\mu\sigma}e^{i\vec{k}\cdot\left(\vec{x}-\vec{x}'\right)}d^3k \tag{54}$$

(b)

对于(44)做傅里叶变化有

$$\begin{aligned}
i\omega G(\vec{k}-\vec{k}',\omega)+\frac{k^2}{\mu\sigma}G(\vec{k}-\vec{k}',\omega) &= e^{-i\vec{k}\cdot\vec{x}'}\cdot\frac{1}{\sqrt{2\pi}}\sqrt{2\pi}\\
\left[(-i\omega)^2-|i\vec{k}|^2/\mu\sigma\right]G &= e^{-i\vec{k}\cdot\vec{x}'}\\
G(\vec{k}-\vec{k}',\omega) &= \frac{e^{-i\vec{k}\cdot\vec{x}'}}{k^2/\mu\sigma-i\omega}
\end{aligned} \tag{55}$$

逆变换有

$$\begin{aligned}
G(\vec{k}-\vec{k}',t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}G(\vec{k},\omega)e^{-i\omega t}d\omega\\
&= \frac{ie^{-i\vec{k}\cdot\vec{x}'}}{2\pi}\int\frac{e^{-i\omega t}}{\omega+ik^2/\mu\sigma}d\omega\\
&= \frac{ie^{-i\vec{k}\cdot\vec{x}'}}{2\pi}\cdot e^{-\frac{k^2t}{\mu\sigma}}\mathrm{Ei}\left(\frac{k^2t}{\mu\sigma}-it\omega\right)
\end{aligned} \tag{56}$$

> 没用这个Jordan's lemma - Wikipedia

对(43)空间傅里叶变换有

$$G\left(\mathbf{k}-\mathbf{k}',t\right)=\frac{1}{(2\pi)^3}\iint d^3ke^{-k^2t/\mu\sigma}e^{-i\mathbf{k}\cdot\mathbf{x}'}d^3x \tag{57}$$

对于其中含时部分$e^{-k^2t/\mu\sigma}$有逆变换

$$e^{-k^2t/\mu\sigma}=\int\delta(\omega+i\frac{k^2}{\mu\sigma})e^{i\omega t}d\omega \tag{58}$$

所以G在ω域中，仅在$\omega=-i\frac{k^2}{\mu\sigma}$处有成分存在，

所以(56)中指数积分化为$\mathrm{Ei}\,(0)=1$，有

$$G(\vec{k}-\vec{k}',t)=e^{-k^2t/\mu\sigma}e^{-i\vec{k}\cdot\vec{x}'} \tag{59}$$

逆变换有

$$G\left(\mathbf{x}-\mathbf{x}',t\right)=\frac{1}{(2\pi)^3}\int d^3ke^{-k^2t/\mu\sigma}e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\qquad t>0 \tag{60}$$

QED

(c)

$t>0$有

$$\begin{aligned}G\left(\vec{x}-\vec{x}',t\right)&=\frac{1}{(2\pi)^3}e^{-\mu\sigma\left|\vec{x}-\vec{x}'\right|^2/4t}\int e^{-t\left|\vec{k}-i\mu\sigma(\vec{x}-\vec{x}')/2t\right|^2/\mu\sigma}d^3k\\&=\frac{1}{(2\pi)^3}\left(\frac{\pi\mu\sigma}{t}\right)^{3/2}e^{-\mu\sigma\left|\vec{x}-\vec{x}'\right|^2/4t}\\&=\left(\frac{\mu\sigma}{4\pi t}\right)^{3/2}e^{-\mu\sigma\left|\vec{x}-\vec{x}'\right|^2/4t}\end{aligned} \tag{61}$$

6.5

一局域电荷分布产生静电场($\mathbf{E}=-\nabla\Phi$)。在该场内部有一小的、与时间无关的局域电流密度($\mathbf{J}(\mathbf{x})$)，该电流密度产生的磁场为($\mathbf{H}$)

(a) 证明电磁场的动量(6.117)可以变形为

$$\mathbf{P}_{\text{field}}=\frac{1}{c^2}\int\Phi\mathbf{J}d^3x \tag{62}$$

假设$\Phi\mathbf{H}$ 的乘积在远距离情况下足够快的衰减，多么快叫做“足够快”？

(b) 假设电流分布的尺度相对于电场的变化的尺度局域在一个较小的区域内，对静电势做泰勒展开，证明

$$\mathbf{P}_{\text{field}}=\frac{1}{c^2}\mathbf{E}(0)\times\mathbf{m} \tag{63}$$

其中，$\mathbf{E}(0)$为电流分布处的电场，$\mathbf{m}$为电流引起的磁矩

(c) 假设电流分布被放置在均匀电场$\mathbf{E}_0$中（充满全空间）。证明，无论局域电流J多么复杂，(a)中结果都是无穷远处的(b)中结果减去三分一的曲面积分，即

$$\mathbf{P}_{\text{field}}=\frac{2}{3c^2}\mathbf{E}_0\times\mathbf{m} \tag{64}$$

将此结果与 (6.117) 比较，并考虑5.6节末的思考

> 6.117
>
> $$\mathbf{P}_{\text{field}} = \epsilon_0 \int_V \mathbf{E} \times \mathbf{B} d^3x = \mu_0 \epsilon_0 \int_V \mathbf{E} \times \mathbf{H} d^3x \tag{65}$$

(a)

对于真空

$$\vec{P}_{\text{field}} = \frac{1}{c^2} \int_V \vec{E} \times \vec{H} d^3x = -\frac{1}{c^2} \int_V \nabla\Phi \times \vec{H} d^3x. \tag{66}$$

利用$\nabla \times (\psi\vec{a}) = \nabla\psi \times \vec{a} + \psi\nabla \times \vec{a}$有

$$\begin{aligned} c^2\vec{P}_{\text{field}} &= \int_V \Phi\nabla \times \vec{H} d^3x - \int_V \nabla \times \Phi\vec{H} d^3x \\ &= \int_V \Phi\nabla \times \vec{H} d^3x - \int_S \Phi\vec{H} d\vec{a} \end{aligned} \tag{67}$$

对于第二项表面积分，只要$\Phi\mathbf{H}$的衰减比$\mathbf{r}^2$增长的更快，则该项为0，有

$$\vec{P}_{\text{field}} = \frac{1}{c^2} \int_V \Phi\vec{J} d^3x \tag{68}$$

(b)

电势坐标原点附近泰勒展开有

$$\begin{aligned} \phi &= \phi(0) + \nabla\phi(0) \cdot \mathbf{x} \\ &= -E(0) \cdot \mathbf{x} \end{aligned} \tag{69}$$

带入(a)结论有

$$\mathbf{P}_{\text{field}} = -\frac{1}{c^2} \int (\mathbf{E}(\mathbf{0}) \cdot \mathbf{x})\mathbf{J}\Big) d^3x \tag{70}$$

利用$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$和Jackson（5.54）有

$$\begin{aligned} \mathbf{P}_{\text{field}} &= \frac{1}{c^2}\left(\int \mathbf{E}(\mathbf{0}) \times (\mathbf{x} \times \mathbf{J}) d^3x - \int (\mathbf{E}(\mathbf{0}) \cdot \mathbf{J})\mathbf{x} d^3x\right) \\ 2\mathbf{P}_{\text{field}} &= \frac{1}{c^2} \int \mathbf{E}(\mathbf{0}) \times (\mathbf{x} \times \mathbf{J}) d^3x \\ 2\mathbf{P}_{\text{field}} &= \frac{2}{c^2} \mathbf{E}(\mathbf{0}) \times \mathbf{m} \\ \mathbf{P}_{\text{field}} &= \frac{1}{c^2} \mathbf{E}(\mathbf{0}) \times \mathbf{m} \end{aligned} \tag{71}$$

(c)

电场均匀，根据Jackson(6.117)有

$$\vec{P}_{\text{field}} = \epsilon_0 \vec{E}_0 \times \int_V \vec{B} d^3x \tag{72}$$

把Jackson（5.62）代入, 有

> $$\int_{r<R} \mathbf{B} d^3x = \frac{2\mu_0}{3}\mathbf{m} \tag{73}$$

$$\vec{P}_{field} = \frac{2}{3c^2}\vec{E}_0 \times \vec{m} \tag{74}$$

QED

6.10

与6.9中假设相同，讨论角动量守恒。证明守恒律的微分形式和积分形式分别为

$$\frac{\partial}{\partial t}(\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}}) + \boldsymbol{\nabla} \cdot \overleftrightarrow{\mathbf{M}} = 0 \tag{75}$$

和

$$\frac{d}{dt}\int_V (\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}})d^3x + \int_S \mathbf{n} \cdot \overleftrightarrow{\mathbf{M}} da = 0 \tag{76}$$

其中，场角动量密度为

$$\mathscr{L}_{\text{field}} = \mathbf{x} \times \mathbf{g} = \mu \in \mathbf{x} \times (\mathbf{E} \times \mathbf{H}) \tag{77}$$

角动量的通量用张量描述为

$$\overleftrightarrow{\mathbf{M}} = \overleftrightarrow{\mathbf{T}} \times \mathbf{x} \tag{78}$$

注：此处我们对M_{ij} 和T_{ij}使用了并失记号，双箭头传达了相当明确的含义。例如，$\mathbf{n} \cdot \overleftrightarrow{\mathbf{M}}$是一个向量，其第$j$个分量为$\sum_i n_i M_{ij}$。二阶的$\overleftrightarrow{\mathbf{M}}$可以写作三阶张量，$M_{ijk} = T_{ij}x_k - T_{ik}x_j$。但对于$i$和$j$指标是反对称的，只有三个独立的元素。包含标号$i$、$M_{ijk}$，因此有九个组成部分，并能携程上述的二阶伪张量形式

根据Jackson(6.122)，有

$$\begin{aligned} \frac{d}{dt}\int_\Omega (\mathbf{r} \times \mathbf{g}_{\text{mech}} + \mathbf{r} \times \mathbf{g}_{\text{field}})d^3x &= \oint_{\partial\Omega} \mathbf{r} \times T_{\alpha\beta}n_\beta da \\ \frac{d}{dt}\int_\Omega (\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}})d^3x &= \oint_{\partial\Omega} \mathbf{r} \times \overleftrightarrow{\mathbf{T}} \cdot \mathbf{n} da \\ \frac{d}{dt}\int_\Omega (\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}})d^3x &= -\oint_{\partial\Omega} \mathbf{n} \cdot \overleftrightarrow{\mathbf{M}} da \\ \frac{d}{dt}\int_\Omega (\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}})d^3x + \oint_{\partial\Omega} \mathbf{n} \cdot \overleftrightarrow{\mathbf{M}} da = 0 \end{aligned} \tag{79}$$

微分形式即为

$$\frac{\partial}{\partial t}(\mathscr{L}_{\text{mech}} + \mathscr{L}_{\text{field}}) + \nabla \cdot \overleftrightarrow{\mathbf{M}} = 0 \tag{80}$$

6.11

横向平面波在真空中正常入射到一个完全吸收的平板屏上。

(a) 从线性动量守恒定律出发，证明，在屏上施加的压力（辐射压）等于平面波中单位体积内的场的能量

在单位时间dt内，作用到单位面积ds上的动量$p = gc\mathrm{d}s\mathrm{d}t$

有单位面积上的压力

$$\frac{dF}{ds} = \frac{d}{ds}\frac{dp}{dt} = gc \tag{81}$$

考虑gc量纲有

$$\frac{kg\cdot m/s}{m^3}\cdot\frac{m}{s}=\frac{kg\cdot m^2/s^2\cdot s}{m^3\cdot m}\cdot\frac{m}{t}=\frac{J}{m^3} \tag{82}$$

显然，单位面积上的压力=场单位体积内的能量