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7.1

对于下面给出的每一组斯托克斯参数，在线偏振基和圆偏振基中推导出电场的幅值，和相位，并作出与Fig. (7.4) 类似的精确图形，表明其中一个椭圆的轴线长度及取向

(a) $s_0=3, \quad s_1=-1, \quad s_2=2, \quad s_3=-2$
(b) $s_0=25, \quad s_1=0, \quad s_2=24, \quad s_3=7$

(b)

联立

$$\begin{aligned} s_0 &= a_1^2+a_2^2 \\ s_1 &= \left|\epsilon_1\cdot\mathbf{E}\right|^2-\left|\boldsymbol{\epsilon}_2\cdot\mathbf{E}\right|^2=a_1^2-a_2^2 \\ s_2 &= 2a_1a_2\cos\left(\delta_2-\delta_1\right) \\ s_3 &= 2a_1a_2\sin\left(\delta_2-\delta_1\right) \end{aligned} \tag{1}$$

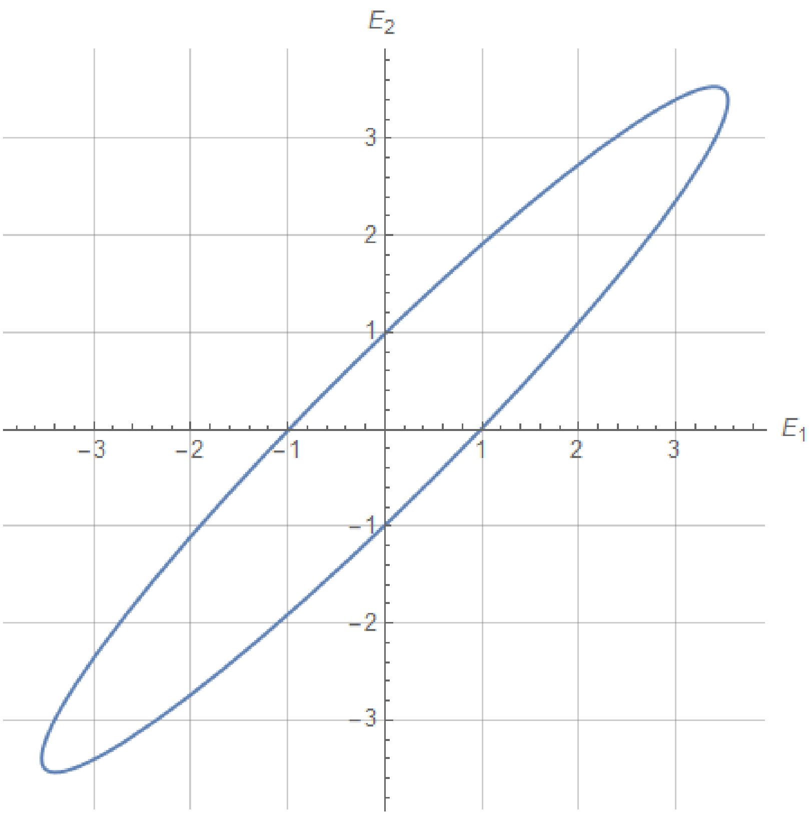
解得

$$\begin{gathered} a_1=a_2=\frac{5}{\sqrt{2}} \\ \delta_2=\left[-2\pi c_1+\delta_1+\tan^{-1}\left(\frac{7}{24}\right), c_1\in\mathbb{Z}\right] \end{gathered} \tag{2}$$

取$\delta_1=0$，有

$$\begin{gathered} \vec{E}_1=a_1e^{i\delta_1}=\frac{5e^{-i\omega t}}{\sqrt{2}}\epsilon_1 \\ \vec{E}_2=a_2e^{i\delta_2}=\frac{5e^{-i\left(\omega t-\tan^{-1}\left(\frac{7}{24}\right)\right)}}{\sqrt{2}}\epsilon_2 \end{gathered} \tag{3}$$

画图有

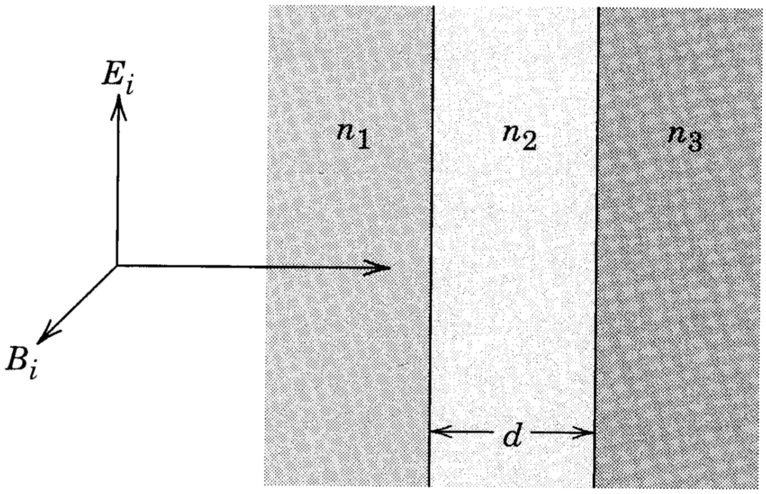


7.2

平面波入射到分界面上，如图所示。三种非渗透性介质得折射率分别为n_1, n_2, n_3，中间层厚度为d，其它每种介质都是半无线得

(a) 计算传输和反射系数（透射和反射坡印廷矢量与入射坡印廷矢量的比值），并对他们关于频率的函数在$n_1 = 1, n_2 = 2, n_3 = 3; n_1 = 3, n_2 = 2, n_3 = 1$和$n_1 = 2, n_2 = 4, n_3 = 1$情况下绘制草图，

(b) 介质n_1是光学系统（如透镜）的一部分；介质n_3为空气$(n_3 = 1)$，希望在表面涂上光学薄膜（介质n_2），使得对于频率ω_0没有反射波，需要什么厚度d和折射率n_2？



(a)

各介质内有电磁波

$$\begin{aligned}\mathbf{E}_1 &= a e^{ik_1 z - i\omega t} + b e^{-ik_1 z - i\omega t} \\ \mathbf{E}_2 &= \alpha e^{ik_2 z - i\omega t} + \beta e^{-ik_2 z - i\omega t} \\ \mathbf{E}_3 &= \eta e^{ik_3 z - i\omega t}\end{aligned} \tag{4}$$

其中

$$k_1 = n_1 c/\omega \quad k_2 = n_2 c/\omega \quad k_3 = n_3 c/\omega \tag{5}$$

入射波为振幅a的部分

界面处连续，有

> 直接一个，一阶导数一个

$$a + b = \alpha + \beta \tag{6}$$

$$a - b = \frac{n_2}{n_1}(\alpha - \beta) \tag{7}$$

$$\alpha e^{ik_2 d} + \beta e^{ik_2 d} = \eta e^{ik_3 d} \tag{8}$$

$$\alpha e^{ik_2 d} - \beta e^{ik_2 d} = \frac{n_3}{n_2}\eta e^{ik_3 d} \tag{9}$$

解得

$$\begin{aligned} b &\to \frac{a\left(\mathrm{n1}\left((\mathrm{n2}-1)e^{2id\mathrm{k2}} + \mathrm{n2}+1\right) + \mathrm{n2}\left((\mathrm{n2}-1)e^{2id\mathrm{k2}} - \mathrm{n2}-1\right)\right)}{\mathrm{n1}\left((\mathrm{n2}-1)e^{2id\mathrm{k2}} + \mathrm{n2}+1\right) + \mathrm{n2}\left((\mathrm{n2}-1)\left(-e^{2id\mathrm{k2}}\right) + \mathrm{n2}+1\right)} \\ \alpha &\to \frac{2a\mathrm{n1}(\mathrm{n2}+1)}{\mathrm{n1}\left((\mathrm{n2}-1)e^{2id\mathrm{k2}} + \mathrm{n2}+1\right) + \mathrm{n2}\left((\mathrm{n2}-1)\left(-e^{2id\mathrm{k2}}\right) + \mathrm{n2}+1\right)} \\ \beta &\to \frac{2a\mathrm{n1}(\mathrm{n2}-1)e^{2id\mathrm{k2}}}{\cdots} \end{aligned} \tag{10}$$

$$\rho \to \frac{}{\text{n1}\left((\text{n2}-1)e^{2idk2}+\text{n2}+1\right)+\text{n2}\left((\text{n2}-1)\left(-e^{2idk2}\right)+\text{n2}+1\right)}$$

$$\eta \to \frac{4a\text{n1n2}e^{-id(k-\text{k2})}}{\text{n1}\left((\text{n2}-1)e^{2idk2}+\text{n2}+1\right)+\text{n2}\left((\text{n2}-1)\left(-e^{2idk2}\right)+\text{n2}+1\right)}$$

记入射波振幅$a=1$

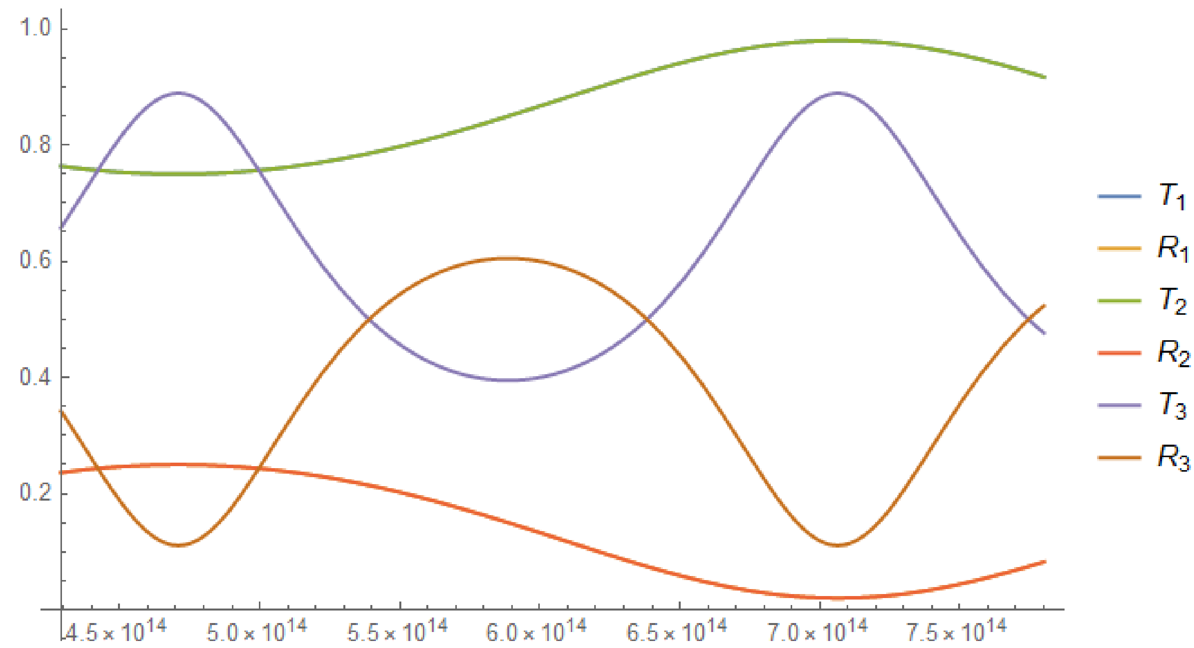
有传输系数

$$\begin{aligned} T &= \frac{\left|\eta e^{ik_3z-i\omega t}\right|^2}{\left|ae^{ik_1z-i\omega t}\right|^2}\cdot\frac{n_3}{n_1} \\ &= \frac{\eta\cdot\eta^*}{a\cdot a^*}\cdot\frac{n_3}{n_1} \\ &= \frac{8\text{n1n2}^2\text{n3}}{\left(\text{n1}^2-\text{n2}^2\right)\left(\text{n2}^2-\text{n3}^2\right)\cos\left(2d\text{k2}\right)+\text{n1}^2\left(\text{n2}^2+\text{n3}^2\right)+4\text{n1n2}^2\text{n3}+\text{n2}^2\left(\text{n2}^2+\text{n3}^2\right)} \\ &= \frac{8\text{n1n2}^2\text{n3}}{\left(\text{n1}^2-\text{n2}^2\right)\left(\text{n2}^2-\text{n3}^2\right)\cos\left(2d\frac{\text{n2}}{c}\omega\right)+\text{n1}^2\left(\text{n2}^2+\text{n3}^2\right)+4\text{n1n2}^2\text{n3}+\text{n2}^2\left(\text{n2}^2+\text{n3}^2\right)} \end{aligned} \tag{11}$$

1-传输系数即为反射系数

$$R=1-T \tag{12}$$

取$d=10^{-6}\text{m}$，在可见光频率下，有



下标分别对应题干三种情况，1，2情况下系数一致

(b)

无反射时，有

$$T=\frac{8\text{n1n2}^2\text{n3}}{\left(\text{n1}^2-\text{n2}^2\right)\left(\text{n2}^2-\text{n3}^2\right)\cos\left(2d\frac{\text{n2}}{c}\omega\right)+\text{n1}^2\left(\text{n2}^2+\text{n3}^2\right)+4\text{n1n2}^2\text{n3}+\text{n2}^2\left(\text{n2}^2+\text{n3}^2\right)}=1 \tag{13}$$

解得

$$n_2=\sqrt{n_1} \qquad d=\frac{c\pi\left(2n-1\right)}{2n_2\omega} \qquad n\in Z \tag{14}$$

7.3

两个相同、均匀、各向同性、非渗透性、折射率为n的无损耗介质的平面半无限平板，有一个宽度为d的气隙$(n=1)$分隔开，一个频率为ω平面电磁波从一个入射角为i的板条间隙入射。对于平行和垂直与入射平面的线偏振情况，

(a) 计算入射到第二块平板的功率与入射功率的比值，以及反射与入射功率的比值

(a)

记入射角为θ_1,

各介质内有电磁波

$$\begin{aligned}
\mathbf{E}_1 &= a e^{ik_1\mathbf{x}} + b e^{-ik_1\mathbf{x}} \\
\mathbf{E}_2 &= \alpha e^{ik\mathbf{x}} + \beta e^{-ik\mathbf{x}} \\
\mathbf{E}_3 &= \eta e^{ik_1\mathbf{x}}
\end{aligned} \tag{15}$$

考虑边界连续性有

$$\begin{array}{ll}
a+b=\alpha+\beta & \alpha e^{ik\lambda}+\beta e^{ik\lambda}=\eta e^{ik_1\lambda}; \\
a-b=\frac{\cos\theta_2}{n\cos\theta_1}(\alpha-\beta) & \alpha e^{ik\lambda}-\beta e^{ik\lambda}=\frac{n\cos\theta_1}{\cos\theta_2}\eta e^{ik_1\lambda}
\end{array} \tag{16}$$

其中,

$$\theta_2=\arccos\left(1-n^2\sin\left(\theta_1\right)^2\right) \qquad \lambda=d/\cos\theta_2 \tag{17}$$

解得

$$\begin{aligned}
b &\to \frac{a\left(-1+e^{2ik\lambda}\right)(n-\sec(\theta 1)\cos(\theta 2))(\sec(\theta 1)\cos(\theta 2)+n)}{\sec^2(\theta 1)\cos^2(\theta 2)\left(-1+e^{2ik\lambda}\right)+n^2\left(-1+e^{2ik\lambda}\right)-2n\sec(\theta 1)\cos(\theta 2)\left(1+e^{2ik\lambda}\right)} \\
\alpha &\to -\frac{2an(\sec(\theta 1)\cos(\theta 2)+n)}{\sec^2(\theta 1)\cos^2(\theta 2)\left(-1+e^{2ik\lambda}\right)+n^2\left(-1+e^{2ik\lambda}\right)-2n\sec(\theta 1)\cos(\theta 2)\left(1+e^{2ik\lambda}\right)} \\
\beta &\to \frac{2ane^{2ik\lambda}(n-\sec(\theta 1)\cos(\theta 2))}{\sec^2(\theta 1)\cos^2(\theta 2)\left(-1+e^{2ik\lambda}\right)+n^2\left(-1+e^{2ik\lambda}\right)-2n\sec(\theta 1)\cos(\theta 2)\left(1+e^{2ik\lambda}\right)} \\
\eta &\to -\frac{4an\sec(\theta 1)\cos(\theta 2)e^{i\lambda(k-\mathrm{k1})}}{\sec^2(\theta 1)\cos^2(\theta 2)\left(-1+e^{2ik\lambda}\right)+n^2\left(-1+e^{2ik\lambda}\right)-2n\sec(\theta 1)\cos(\theta 2)\left(1+e^{2ik\lambda}\right)}
\end{aligned} \tag{18}$$

取$a=1$，有

$$\begin{aligned}
T &= \frac{|\eta|^2}{|a|^2} \\
&= -\frac{8n^2\cos^2(\theta 1)\left(n^2\sin^2(\theta 1)-1\right)}{-\cos(2k\lambda)-n^4\cos(2k\lambda)+2n^2\cos(2k\lambda)+n^4\cos(4\theta 1)+4n^2\cos(2\theta 1)+2n^2+1}
\end{aligned} \tag{19}$$

其中，$k=\frac{\omega}{c}$

反射比为

$$R=1-T \tag{20}$$