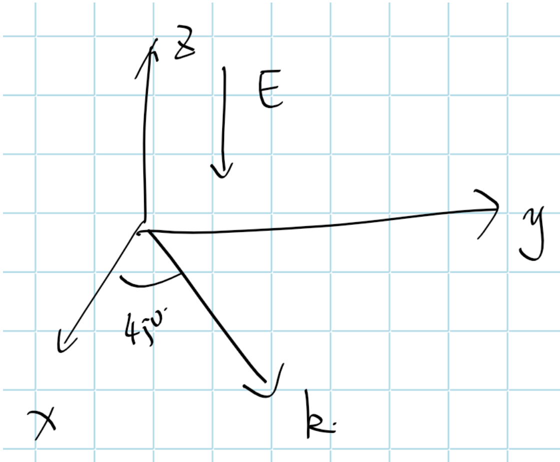
横向应用



电场施加方向与光束传播方向垂直，如上图所示

$$k_x = \sin\theta \qquad k_y = \cos\theta \tag{11}$$

以$\bar{4}3m$晶类为例，有介电张量和二阶极化率张量为

$$\begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & xyz & xyz & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & xyz & xyz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & xyz & xyz \end{bmatrix} \tag{12}$$

根据电光效应下相对介电常数张量变换，$(\varepsilon_{\mu\alpha})_{eff} = \left[\varepsilon_{r\mu\alpha} + 2\vec{\vec{\chi}}^{(2)}_{\mu\alpha\beta}(\omega, 0) \cdot E_{0\beta}\right]$，

有施加电场后，有相对介电常数变化，

$$\begin{aligned} (\varepsilon_{\mu\alpha})_{eff} &= \left[\varepsilon_{r_{\mu\alpha}} + 2\vec{\vec{\chi}}^{(2)}_{\mu\alpha\beta}(\omega, 0) \cdot E_{0\beta}\right] \\ &= \begin{pmatrix} \varepsilon_{xx} + 2\chi^{(2)}_{xxz}E_{0z} & 0 + 2\chi^{(2)}_{xyz}E_{0z} & 0 + 2\chi^{(2)}_{xzz}E_{0z} \\ 0 + 2\chi^{(2)}_{yxz}E_{0z} & \varepsilon_{xx} + 2\chi^{(2)}_{yyz}E_{0z} & 0 + 2\chi^{(2)}_{yzz}E_{0z} \\ 0 + 2\chi^{(2)}_{zxz}E_{0z} & 0 + 2\chi^{(2)}_{zyz}E_{0z} & \varepsilon_{xx} + 2\chi^{(2)}_{zzz}E_{0z} \end{pmatrix} \\ &= \begin{pmatrix} \varepsilon_{xx} & 2\chi^{(2)}_{xyz}E_{0z} & 0 \\ 2\chi^{(2)}_{yxz}E_{0z} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \end{aligned} \tag{13}$$

代入晶体光学基本方程$D = \frac{n^2}{\mu_0 c^2}[\boldsymbol{E} - \boldsymbol{k}(\boldsymbol{k} \cdot \boldsymbol{E})] = \varepsilon_{eff} \cdot \boldsymbol{E}$，有

$$\frac{n^2}{\mu_0 c^2}\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} - \frac{n^2}{\mu_0 c^2}\hat{k} \cdot (\hat{k} \cdot \vec{E}) = \varepsilon_0 \begin{pmatrix} \varepsilon_{xx} & 2\chi^{(2)}_{zxy}E_{0z} & 0 \\ 2\chi^{(2)}_{zyx}E_{0z} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} \tag{14}$$

$$\begin{pmatrix} \varepsilon_{xx} & 2\chi^{(2)}_{xyz}E_{0z} & 0 \\ 2\chi^{(2)}_{xyz}E_{0z} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} + n^2 \cdot \begin{pmatrix} k_x^2 & k_x k_y & 0 \\ k_y k_x & k_y^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} \varepsilon_{xx} + k_x^2 n^2 & 2\chi^{(2)}_{xyz}E_{0z} + k_x k_y n^2 & 0 \\ 2\chi^{(2)}_{xyz}E_{0z} + k_y k_x n^2 & \varepsilon_{xx} + k_y^2 n^2 & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} = \begin{pmatrix} n^2 & 0 & 0 \\ 0 & n^2 & 0 \\ 0 & 0 & n^2 \end{pmatrix} \begin{pmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{pmatrix} \tag{16}$$

求久期方程，有

$$\begin{vmatrix} \varepsilon_{xx} + k_x^2 n^2 - n^2 & 2\chi^{(2)}_{xyz}E_{0z} + k_x k_y n^2 & 0 \\ 2\chi^{(2)}_{xyz}E_{0z} + k_y k_x n^2 & \varepsilon_{xx} + k_y^2 n^2 - n^2 & 0 \\ 0 & 0 & \varepsilon_{xx} - n^2 \end{vmatrix} = 0 \tag{17}$$

可以看出有一个解为

$$n_1^2 = \varepsilon_{xx} \tag{18}$$

代入$k_x = \sin\theta, \quad k_y = cos\theta$，取久期方程前两阶有

$$\left(\varepsilon_{xx} + n^2\sin^2\theta - n^2\right)\left(\varepsilon_{xx} + n^2\cos^2\theta - n^2\right) = \left(2\chi^{(2)}_{xyz}E_{0z} + n^2\sin\theta\cos\theta\right)^2$$

$$n_2^2 = \frac{\sec^2(\theta)\sqrt{\left(\epsilon\left(\mathrm{Cod}(\theta)^2 + \sin^2(\theta) - 2\right) - 2a\chi\sin(2\theta)\right)^2 - 2\cos^2(\theta)\left(\epsilon^2 - 4a^2\chi^2\right)\left(-2\mathrm{Cod}(\right.}}{2\left(\mathrm{Cod}(\theta)^2 - \cos^2(\theta)\right)}$$

代回基本方程，有本征方向