第四章 电磁波的传播

§1. 电磁波方程. 平面波

1. 电磁波的波动方程

(1) 真空中波方程

$$\begin{cases} \nabla\cdot\vec{E}=0 \\ \nabla\times\vec{E}=-\dfrac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{B}=0 \\ \nabla\times\vec{B}=\mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t} \end{cases} \longrightarrow \nabla\times(\nabla\times\vec{E})+\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=0$$

$$\downarrow$$

$$\nabla^2\vec{E}-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=0$$

同理 $\nabla^2\vec{B}-\mu_0\varepsilon_0\dfrac{\partial^2\vec{B}}{\partial t^2}=0$

熟悉一维波动方程(弦振动) $\dfrac{\partial^2\xi}{\partial x^2}-\dfrac{1}{u^2}\dfrac{\partial^2\xi}{\partial t^2}=0$ 对比

波速为 $c=\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

(2) 时谐波

$$\vec{E}(\vec{r},t)=\vec{E}(\vec{r})e^{-i\omega t}$$

$$\vec{B}(\vec{r},t)=\vec{B}(\vec{r})e^{-i\omega t} \quad \text{代入 Maxwell}$$

$$\begin{cases} \nabla\cdot\vec{E}=0, & \nabla\times\vec{E}=i\omega\mu\vec{H} \\ \nabla\cdot\vec{H}=0, & \nabla\times\vec{H}=-i\omega\varepsilon\vec{E} \end{cases}$$ ，相应方程(电)

$$\nabla^2\vec{E}+\frac{\omega^2}{c^2}\vec{E}=0 \qquad \text{Helmholtz} \quad \text{“}k=\frac{\omega}{c}\text{”}$$

对于平面波 $\vec{E}=\vec{E}_0e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\vec{H}=\vec{H}_0e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$$\begin{cases} \vec{k}\cdot\vec{E}=0, & \vec{k}\times\vec{E}=\omega\mu\vec{H} \\ \vec{k}\cdot\vec{H}=0, & \vec{k}\times\vec{H}=-\omega\varepsilon\vec{E} \end{cases}$$

平面电磁波五条性质:

① 横波性 $\vec{k}\perp\vec{E},\vec{H}$ ② 垂直性 $\vec{E}\perp\vec{H}$ ③ 右手系 ④ 同相位 $\varphi_{10}=\varphi_{20}$ ⑤ 幅值比 $\sqrt{\varepsilon_0}E_0=\sqrt{\mu_0}H_0$

规则: $\dfrac{\partial}{\partial t}\to -i\omega$, $\nabla\to i\vec{k}$

(3) 平面波的能.

$$u=\frac{1}{2}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})=\frac{1}{2}\varepsilon_0E^2+\frac{1}{2}\mu_0H^2=\varepsilon_0E^2$$

$$\vec{S}=\vec{E}\times\vec{H}=\sqrt{\frac{\varepsilon_0}{\mu_0}}E^2\hat{n}=\frac{\varepsilon_0E^2}{\sqrt{\mu_0\varepsilon_0}}\hat{n}=cu\hat{n}=u\vec{c}$$

平均值: $\langle u\rangle=\dfrac{1}{2}\varepsilon_0E_0^2$, $\langle\vec{S}\rangle=\dfrac{1}{2}\sqrt{\dfrac{\varepsilon_0}{\mu_0}}E_0^2\hat{n}$

(4) 复波矢

$$\vec{k}=\vec{k}_R+i\vec{k}_I \quad , \hat{n}=\hat{n}_R+i\hat{n}_I$$

由 $\hat{n}\cdot\hat{n}=1$ 得 $\begin{cases} n_R^2-n_I^2=1 \\ \hat{n}_R\cdot\hat{n}_I=0 \end{cases}$

参数化为 $\hat{n}_R=\cosh\theta\,\hat{e}_1$, $\hat{n}_I=\sinh\theta\,\hat{e}_2$, 则

$$\hat{n}=\cosh\theta\,\hat{e}_1+i\sinh\theta\,\hat{e}_2$$

由 $\hat{n}\cdot\vec{E}_0=0$, 可取

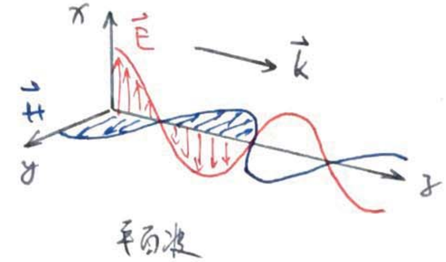
$$\vec{E}_0=(i\hat{e}_1\sinh\theta-\hat{e}_2\cosh\theta)A+\hat{e}_3A'$$

特别, 若取 $\theta=0$. , $\hat{n}=\hat{e}_1=\hat{e}_3$ 则

$$\vec{E}_0=-\hat{e}_2A+\hat{e}_3A'=\hat{e}_2A+\hat{e}_1A'$$

复波矢下, $\vec{E}=\underbrace{\vec{E}_0e^{-\vec{k}_I\cdot\vec{r}}}_{\text{衰减的振幅}}\underbrace{e^{i(\vec{k}_R\cdot\vec{r}-\omega t)}}_{\text{平面波}}$

2. 偏振波



$\vec{E}=(\hat{\epsilon}_1E_1+\hat{\epsilon}_2E_2)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

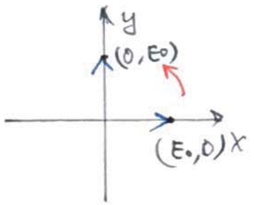
偏振态

(1) 线偏振 与 圆/椭圆偏振

设 $\vec{E}=E_0(\hat{\epsilon}_1+i\hat{\epsilon}_2)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

即 $\begin{cases} E_x = E_0\cos(\vec{k}\cdot\vec{r}-\omega t) \\ E_y = E_0\cos(\vec{k}\cdot\vec{r}-\omega t+\frac{\pi}{2}) = -E_0\sin(\vec{k}\cdot\vec{r}-\omega t) \end{cases}$

显然 $E_x^2+E_y^2=E_0^2$ 圆偏振 (取 $z=0$ 平面)



$t=0$ 时，$E_x=E_0$，$E_y=0$

$t=\frac{T}{4}$ 时，$E_x=0$，$E_y=E_0$

故为左旋。

结论. $\hat{\epsilon}_1\pm i\hat{\epsilon}_2$ 分别代表左、右旋 (基矢)

圆偏振基矢 $\hat{\epsilon}_\pm=\frac{1}{\sqrt{2}}(\hat{\epsilon}_1\pm i\hat{\epsilon}_2)$

对于椭圆偏振

$\vec{E}=(E_1\hat{\epsilon}_1+iE_2\hat{\epsilon}_2)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\frac{E_x^2}{E_1^2}+\frac{E_y^2}{E_2^2}=1$

分解圆偏振波

$\vec{E}=(E_+\hat{\epsilon}_++E_-\hat{\epsilon}_-)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$=\left[\frac{1}{\sqrt{2}}(E_++E_-)\hat{\epsilon}_1+\frac{1}{\sqrt{2}}i(E_+-E_-)\hat{\epsilon}_2\right]e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\equiv(E_1\hat{\epsilon}_1+iE_2\hat{\epsilon}_2)e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

即 $E_{1,2}=\frac{1}{\sqrt{2}}(E_+\pm E_-)$