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7.20

一均匀、各向同性、非渗透性的电介质的特征是折射率满足$n(\omega)$，为了描述吸收过程，它一般比较复杂。

(a) 证明一维平面波的一般解为

$$u(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\left[A(\omega)e^{i(\omega/c)n(\omega)x}+B(\omega)e^{-i(\omega/c)n(\omega)x}\right] \tag{1}$$

其中，$u(x,t)$是$\mathbf{E}$或$\mathbf{B}$的组成部分

(b) 如果$u(x,t)$是实的，证明$n(-\omega)=n^*(\omega)$

(c) 证明，若$u(0,t)$和$\partial u(0,t)/\partial x$是$u$及其导数在$x=0$处的边界值，则系数$A(\omega)$和$B(\omega)$为

$$\begin{Bmatrix}A(\omega)\\B(\omega)\end{Bmatrix}=\frac{1}{2}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dte^{i\omega t}\left[u(0,t)\mp\frac{ic}{\omega n(\omega)}\frac{\partial u}{\partial x}(0,t)\right] \tag{2}$$

(a)

亥姆霍次方程有解

$$u(\omega,x,t)=A(\omega,x)e^{i(\omega/c)n(\omega)x}+B(\omega,x)e^{-i(\omega/c)n(\omega)x} \tag{3}$$

傅里叶变换有

$$u(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\left[A(\omega)e^{i(\omega/c)n(\omega)x}+B(\omega)e^{-i(\omega/c)n(\omega)x}\right] \tag{4}$$

(b)

u为实，则，$u^*=u$，

根据(a)有

$$\begin{aligned}
u(x,t)|_{\omega}&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dwe^{-i\omega t}\left[A(w)e^{i\frac{w}{t}n(w)x}+B(w)e^{-i\frac{w}{c}n(w)x}\right]\\
u^*(x,t)|_{\omega}&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dwe^{iwt}\left[A(w)^*e^{-i\frac{w}{c}n^*(w)x}+B^*(w)e^{i\frac{w}{c}n^*(w)x}\right]\\
u(x,t)|_{-\omega}&=\frac{1}{\sqrt{2\pi}}\int_{+\infty}^{-\infty}-d\omega e^{i\omega t}\left[A(-\omega)e^{-i\frac{w}{2}n(-\omega)x}+B(w)e^{i\frac{w}{c}n(-\omega)x}\right]\\
&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\omega e^{i\omega t}\left[A(-\omega)e^{-i\frac{w}{c}n(-\omega)x}+B(-\omega)e^{i\frac{w}{c}n(-\omega)x}\right]
\end{aligned} \tag{5}$$

合并有

$$\begin{gathered}
\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\omega e^{i\omega t}\left[A^*(\omega)e^{-i\frac{w}{t}n(\omega)x}+B^*(\omega)e^{i\frac{w}{c}n^*(\omega)x}\right]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}d\omega e^{i\omega t}\left[A(-\omega)e^{-i\frac{u}{t}n(-\omega)x}+B(-\omega)e^{i\frac{w}{c}n(-\omega)x}\right]\\
A^*(w)e^{-i\frac{w}{c}n^*(w)x}+B^*(w)e^{i\frac{w}{c}n(w)x}=A(-w)e^{-i\frac{w}{t}n(-\omega)x}+B(-w)e^{i\frac{w}{c}n(-w)x}\\
\Downarrow\\
A^*(w)=A(-w),B^*(w)=B(-w),n^*(w)=n(-w)
\end{gathered} \tag{6}$$

(c)

$$u(0,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}[A(\omega)+B(\omega)] \tag{7}$$

$$\Rightarrow[A(\omega)+B(\omega)]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}dte^{i\omega t}u(0,t) \tag{8}$$

$$\frac{\partial u(0,t)}{\partial x}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}\left[i\frac{\omega}{c}n(\omega)A(\omega)e^{i(\omega/c)n(\omega)x}-i\frac{\omega}{c}n(\omega)B(\omega)e^{-i(\omega/c)n(\omega)x}\right]\Bigg|_{x=0} \tag{9}$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}d\omega e^{-i\omega t}i\frac{\omega}{c}n(\omega)[A(\omega)-B(\omega)] \tag{10}$$

$$\sqrt{2\pi} \quad {}_{-\infty} \qquad\qquad c$$

$$\Rightarrow [A(\omega) - B(\omega)] = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega e^{i\omega t} \frac{ic}{\omega n(\omega)} \frac{\partial u(0,t)}{\partial x} \tag{11}$$

$$\Rightarrow \begin{Bmatrix} A(\omega) \\ B(\omega) \end{Bmatrix} = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt e^{i\omega t} \left[u(0,t) \mp \frac{ic}{\omega n(\omega)} \frac{\partial u}{\partial x}(0,t) \right] \tag{12}$$