对于中场(感应区)

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\sum_{l,m}\frac{4\pi}{2l+1}Y_{lm}(\theta,\phi)\frac{e^{ikr}}{r^{l+1}}\left[1+\sum_{n=1}^{l}a_n(ikr)^n\right]\int \vec{J}(\vec{r}')r'^l Y^*_{lm}(\theta',\phi')\,dV'$$

其中系数，参考(4.84)式， $a_1=-1$, $a_2=i$, $a_3\cdots$

① $l=0$ 时

$$\vec{A}^{(0)}(\vec{r}) = \frac{\mu_0}{4\pi}\cdot\frac{4\pi}{1}Y_{00}(\theta,\phi)\frac{e^{ikr}}{r}\int \vec{J}(\vec{r}')Y^*_{lm}(\theta',\phi')\,dV'$$

再利用 $\sum_m Y^*_{lm}Y_{lm} = \frac{2l+1}{4\pi}P_l(\cos\gamma)$

$$= \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(\vec{r}')\,dV' = \cdots = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\vec{p}$$

② $l=1$ 时

$$\vec{A}^{(1)}(\vec{r}) = \frac{\mu_0}{4\pi}\cdot\frac{4\pi}{3}\sum_{m=-1}^{1}Y_{1m}(\theta,\phi)\frac{e^{ikr}}{r^2}(1-ikr)\int \vec{J}(\vec{r}')r' Y^*_{1m}(\theta',\phi')\,dV'$$

$$= \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r^2}(1-ikr)\int \vec{J}(\vec{r}')\underset{\hat{e}_r\cdot\hat{e}_{r'}}{\underset{\parallel}{\cos\gamma}}\, r'\,dV'$$

$$= \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(\frac{1}{r}-ik\right)\hat{e}_r\cdot\int \vec{J}(\vec{r}')\vec{r}'\,dV'$$

其中 $\nabla\frac{e^{ikr}}{r} = \frac{ike^{ikr}\nabla r}{r} + e^{ikr}\nabla\frac{1}{r}$

$$= ik\frac{e^{ikr}}{r}\hat{e}_r - \frac{e^{ikr}}{r^2}\hat{e}_r$$

$$= ik\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}\right)\hat{e}_r$$

$$\therefore \vec{B}^{(0)} = \frac{ik\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}\right)\hat{e}_r\times\dot{\vec{p}}$$

$$或 = \frac{ck^2\mu_0}{4\pi}\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}\right)\hat{e}_r\times\vec{p}$$

进一步 由 $\vec{E} = \frac{ic}{k}\nabla\times\vec{B}$

$$\vec{E}^{(0)} = \frac{1}{4\pi\varepsilon_0}\left\{k^2(\hat{e}_r\times\vec{p})\times\hat{e}_r\frac{e^{ikr}}{r} + [3\hat{e}_r(\hat{e}_r\cdot\vec{p})-\vec{p}]\left(\frac{1}{r^3}-\frac{ik}{r^2}\right)e^{ikr}\right\}$$

§2. 多极辐射场

1. 电偶极辐射场

$$\vec{B} = \nabla\times\vec{A}^{(0)} = \frac{\mu_0}{4\pi}\nabla\times\left(\frac{e^{ikr}}{r}\dot{\vec{p}}\right) = \frac{\mu_0}{4\pi}\nabla\frac{e^{ikr}}{r}\times\dot{\vec{p}}$$

特别，对于辐射区(远)

$$\vec{B}^{(0)} = \frac{ck^2\mu_0}{4\pi}\frac{e^{ikr}}{r}\hat{e}_r\times\vec{p}, \quad \vec{E}^{(0)} = \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}(\hat{e}_r\times\vec{p})\times\hat{e}_r$$

对于近场区 $e^{ikr}\to 1$

即P(4.13)式

$$\vec{B}^{(0)} = \frac{i\omega\mu_0}{4\pi}\frac{1}{r^2}\hat{e}_r\times\vec{p}, \quad \vec{E}^{(0)} = \frac{1}{4\pi\varepsilon_0}[3\hat{e}_r(\hat{e}_r\cdot\vec{p})-\vec{p}]\frac{1}{r^3}$$

能流与功率角分布：$\vec{S} = \vec{E}\times\vec{H}^*$，$\langle \vec{S}\rangle = \frac{1}{2}(\vec{E}\times\vec{H}^*)$

作业 9.3

$$\frac{dP}{d\Omega} = r^2\langle \vec{S}\rangle\cdot\hat{e}_r = \frac{1}{2} r^2\hat{e}_r\cdot(\vec{E}\times\vec{H}^*) = \frac{c}{2\mu_0} r^2\,\hat{e}_r\cdot(\vec{B}\times\hat{e}_r)\times\vec{B}^*$$

对于偶极辐射

$$= \frac{c}{2\mu_0} r^2 |\vec{B}\times\hat{e}_r|^2$$

$\downarrow$

$$\frac{dP}{d\Omega} = \frac{c^3 k^4\mu_0}{32\pi^2}|(\hat{e}_r\times\vec{p})\times\hat{e}_r|^2$$

$$= \frac{c^3k^4\mu_0}{32\pi^2}|p|^2\sin^2\theta$$

例：线型中馈天线 $I(z)e^{-i\omega t} = I_0\left(1-\frac{2|z|}{d}\right)e^{-i\omega t}$

由电荷守恒 $\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{J}=0$，$\partial\rho$

$$\frac{\partial\rho}{\partial t} = -\frac{d}{dz}I(z)e^{-i\omega t} = \pm\frac{2I_0}{d}e^{-i\omega t}$$

$$\therefore \rho = \pm\frac{2iI_0}{d\omega}e^{-i\omega t}$$

$$\vec{p} = \int\rho\vec{r}'dV' = \int_{-\frac{d}{2}}^{\frac{d}{2}}\rho\cdot z\,dz$$

$$= \frac{2iI_0}{d\omega}e^{-i\omega t}\left(\int_0^{\frac{d}{2}}z\,dz - \int_{-\frac{d}{2}}^{0}z\,dz\right)$$

$$= \frac{iI_0d}{2\omega}$$

$$\therefore \frac{dP}{d\Omega} = \frac{ck^2\mu_0 I^2d^2}{128\pi^2}\sin^2\theta$$