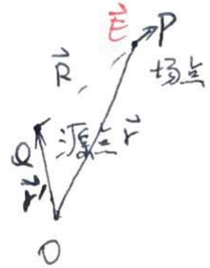
第一章 静电学

§1. 静电学一般规律

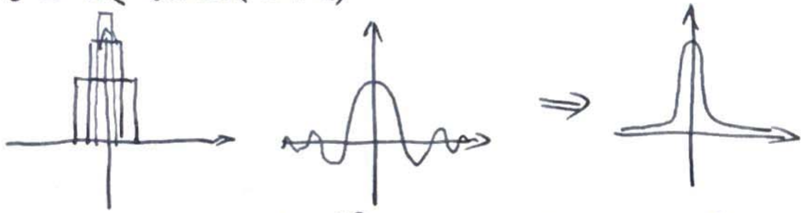
1. Coulomb 定律

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{e}_r$$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}\hat{e}_R \quad , \vec{R} = \vec{r} - \vec{r}'$$

δ函数（函数极限）



$$\delta(x) = \begin{cases} \infty, & x = 0 \\ 0 & x \neq 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

$$\nabla^2 \frac{1}{r} = -4\pi\,\delta(\vec{r})$$

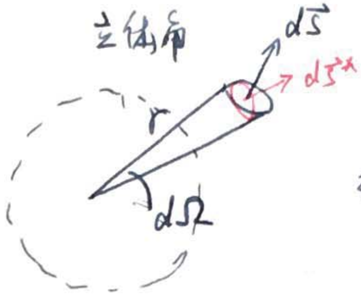
连续分布 $\vec{E} = \frac{1}{4\pi\varepsilon}\sum_i \frac{q_i}{r_i^2}\hat{e}_i$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\int_V \frac{dq}{r^2}\hat{e}_r$$

$$dq = \begin{cases} \rho\, dV \\ \sigma\, dS \\ \eta\, dl \end{cases}$$

2. Gauss 定理

立体角



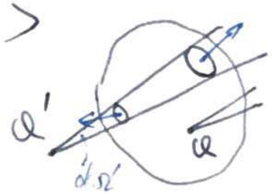
$$d\Omega = \frac{dS^*}{r^2} = \frac{d\vec{S}\cdot\hat{e}_r}{r^2}$$

球面 $\Omega = \int d\Omega = \frac{1}{r^2}\int dS^* = \frac{4\pi r^2}{r^2} = 4\pi$

$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q}{4\pi\varepsilon_0}\int \underbrace{\frac{\hat{e}_r}{r^2}\cdot d\vec{S}}_{d\Omega} = \frac{Q}{4\pi\varepsilon_0}\int d\Omega = \frac{Q}{\varepsilon_0}$$

〈Q在曲面内〉

若Q在外



一般情况 $\oiint \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint \rho(\vec{r})\, dV$

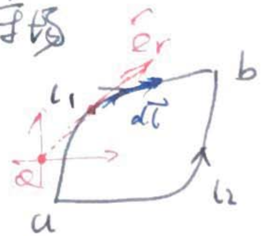
再由 Gauss 公式 $\oiint \vec{A}\cdot d\vec{S} = \iiint (\nabla\cdot\vec{A})\, dV$

$$\Rightarrow \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \quad 〈表一〉$$

3. 静电场环路定理

$\vec{F} = q\vec{E}$ 是保守力，$\vec{E}$ 是保守场

$$\int_{a\,L_1}^{b} \vec{E}\cdot d\vec{l} = \int_{a\,L_2}^{b} \vec{E}\cdot d\vec{l}$$



$$\int_{L_1} \frac{\overbrace{\hat{e}_r\cdot d\vec{l}}^{dr}}{r^2} \qquad \int_{L_2} \frac{\overbrace{\hat{e}_r\cdot d\vec{l}}^{dr}}{r^2}$$

等价于 $\oint \vec{E}\cdot d\vec{l} = 0$

再由 Stokes 公式：$\oint_L \vec{A}\cdot d\vec{l} = \iint_S \nabla\times\vec{A}\cdot d\vec{S}$

$\therefore \nabla\times\boxed{\vec{E}} = 0$ （麦二微分）

由于 $\nabla\times\boxed{\nabla\varphi} = 0$，定义标势 φ

$\vec{E} = -\nabla\varphi \Longleftrightarrow d\varphi = -\vec{E}\cdot d\vec{l}$

以点电荷 $\frac{1}{r^2}$ (场) $\longrightarrow$ ？(势)，利用 $\nabla\frac{1}{r} = -\frac{\hat{e}_r}{r^2}$

故 $\varphi = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

连续 $\varphi = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$，$dq = \begin{cases}\rho\, dV \\ \sigma\, dS \\ \eta\, dl\end{cases}$

$\boxed{\text{势方程}}$ 将 φ 定义代入麦一

$\nabla^2\varphi = -\frac{\rho}{\varepsilon_0}$ （泊松方程）

$$\begin{cases}\vec{D} = \varepsilon_0\vec{E} + \vec{P} \\ \vec{B} = \mu_0(\vec{H} + \vec{M})\end{cases}$$