STA 250, Summer 2013, HW #4

4.8 a $f(y) = k \cdot y(1-y) \quad 0 \le y \le 1$

$$1 = \int_0^1 ky(1-y)\,dy = k\left[\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3 \Big|_0^1 = \tfrac{1}{6}\right] \rightarrow 1 = k\left(\tfrac{1}{6}\right) \rightarrow k = 6$$

b) $$P(.4 < y < 1) = \int_{.4}^1 6y(1-y)\,dy = 3y^2 - 2y^3 \Big|_{.4}^1 = (3-2) - (.48 - .128) = 1 - .352 = .648$$

4.12 a) $F(y) = 0 \quad y < 0$

$1 - e^{-y^2} \quad y \ge 0$

$F(-\infty) = 0$, $F(\infty) = 1$; $F(y)$ is increasing

b) Find 30^{th} percentile Solve $F(y) = .30$

$.30 = 1 - e^{-y^2} \rightarrow .70 = e^{-y^2}$

$\rightarrow \ln(.7) = -y^2; \quad y^2 = -\ln(.7) \quad y = +\sqrt{-\ln(.70)}$

$\approx .597$ hundred hours

c) $f(y) = F'(y) = 2y e^{-y^2} \quad y \ge 0$

$0 \quad y < 0$

d) $P(Y \ge 2) = 1 - P(Y \le 2) = 1 - F(2) = 1 - [1 - e^{-2^2}] = e^{-4} \approx .0183$

4.15 a) $f(y) \ge 0 \quad \int_b^\infty f(y)\,dy = \int_b^\infty \frac{b}{y^2}\,dy = -\frac{b}{y}\Big|_b^\infty 0 - (-1) = 1$

b) $F(y) = 0 \quad y < b$

$$= \int_b^y f(t)\,dt = \int_b^y \frac{b}{t^2}\,dt = -\frac{b}{t}\Big|_b^y = -\frac{b}{y} - (-1) = 1 - \frac{b}{y} \quad y \ge b$$

4.20 $f(y) = \frac{1}{2}(2-y) \quad 0 \le y \le 2$

$$\mu = E[Y] = \int_0^2 y \cdot \frac{1}{2}(2-y)\,dy = \frac{y^2}{2} - \frac{y^3}{6}\Big|_0^2 = \left(2 - \frac{8}{6}\right) = \frac{2}{3}$$

$$E(Y^2) = \int_0^2 y^2 \frac{1}{2}(2-y)\,dy = \frac{y^3}{3} - \frac{y^4}{8}\Big|_0^2 = \left(\frac{8}{3} - 2\right) = \frac{2}{3}$$

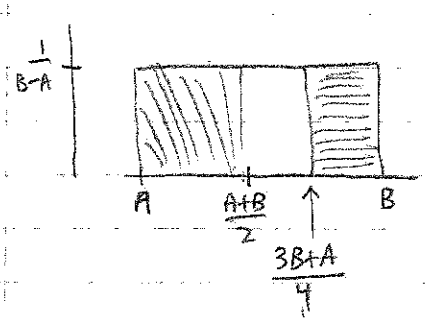
$$V(Y) = E(Y^2) - E(Y)^2 = \frac{2}{3} - \left(\frac{2}{3}\right)^2 = \frac{2}{9}$$

4.28 a) $1 = \int_0^1 c y^2 (1-y)^4\,dy \rightarrow 1 = c\left[\frac{1}{105}\right] \rightarrow c = 105$

b) $E(Y) = \int_0^1 y \cdot f(y)\,dy = \int_0^1 y \cdot 105\, y^2 (1-y)^4\,dy = \frac{3}{8} = .375$

4.31 $E(Y) = \int y \cdot f(y)\,dy = \int_2^6 y \cdot \frac{3}{32}(y-2)(6-y)\,dy = 4$ hundred calories

4.39

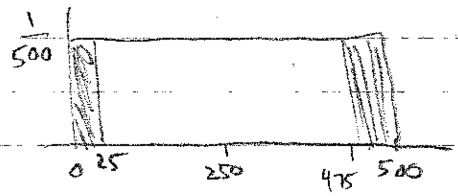


$$P(\text{closer to } A) = \left(\frac{B+A}{2} - A\right)\left(\frac{1}{B-A}\right)$$

$$= \left(\frac{B-A}{2}\right)\left(\frac{1}{B-A}\right) = \frac{1}{2}$$

$$P(\text{more than 3 times farther from } A) = \left(B - \frac{3B+A}{4}\right)\left(\frac{1}{B-A}\right) = \left(\frac{B-A}{4}\right)\left(\frac{1}{B-A}\right) = \frac{1}{4}$$

4.48 $Y \sim$ uniform over $(0, 500)$



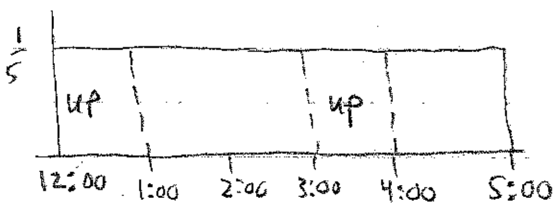
a) $P(\text{within 25 feet of end})$

$= 25/500 = .05$

b) $P(\text{within 25 feet of start}) = \frac{25}{500} = .05$

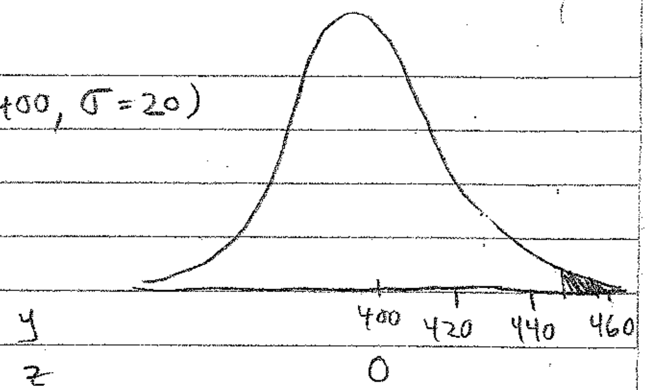
c) $P(\text{closer to start than end}) = \frac{250}{500} = .5$

4.50



$P(\text{up}) = \frac{2}{5} = .4$

4.64 $Y \rightarrow$ weekly repair cost Normal($\mu = 400$, $\sigma = 20$)



a) for $y = 450$,

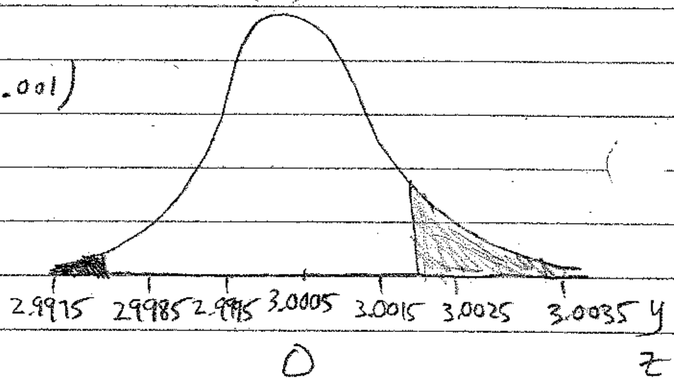
$$z = \frac{y-\mu}{\sigma} = \frac{450-400}{20} = 2.5$$

$P(Y > 450) = .0062$

4.65 Want y so that $P(Y > y) = .10$ {area = .1003}

So, $z = 1.28$, Solving $y = \mu + z \cdot \sigma = 400 + 1.28(20) = \$425.60$

4.66 Diameters Normal($\mu = 3.0005$, $\sigma = .001$)



a) $y = 2.998$, $z = -2.5$

$y = 3.002$, $z = 1.5$

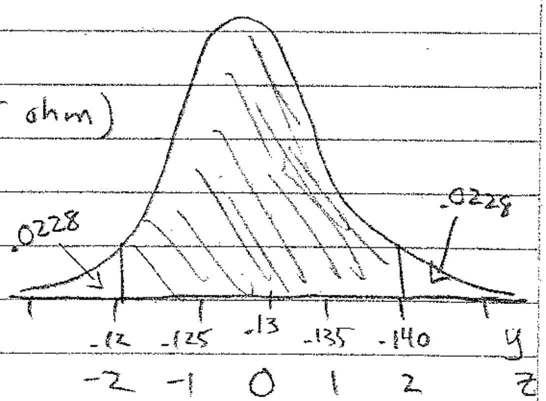
$P(\text{Scrap}) = P(Y < 2.998) + P(Y > 3.002)$

$= .0062 + .0668 = .0730$

4.67 To minimize "area" for scrap, we need equal tail areas

So, the mean must be set to 3.000 in

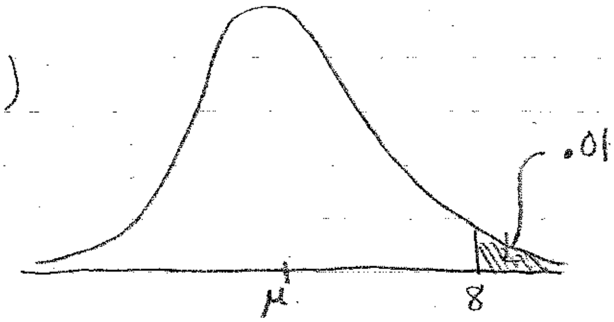
4.71 $Y \rightarrow$ resistance Normal($\mu = .13$ ohm, $\sigma = .005$ ohm)



P(meets specification)

$1 - [.0228 + .0228] = .9544$

4.75 $Y \to$ Liquid Normal(μ, $\sigma = .3$ oz)



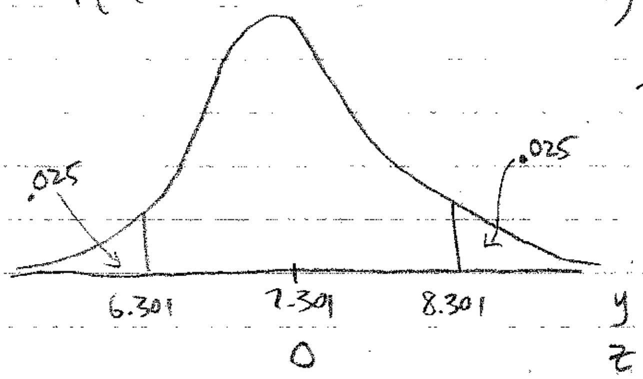
So, for $y = 8$, $z = 2.33$

$$2.33 = \frac{8-\mu}{.3}$$

$$\to \mu = 8 - 2.33(.3) = 7.301 \text{ oz}$$

4.76 $\mu = 7.301$ oz

Want P(Y between 6.301 and 8.301) $\approx .95$



for area = .025, $z = 1.96$

So, $1.96 = \frac{8.301 - 7.301}{\sigma} \to \sigma = \frac{1}{1.96} \approx .51$ oz