STA 250, Summer 2013, Hw #2

2.72

	M	F	
A: Pass	24	36	
$\bar{A}$: fail	16	24	
			100

a) $P(A) = \frac{24+36}{100} = .6$

$P(M) = \frac{24+16}{100} = .4$

$P(A|M) = \frac{P(A \cap M)}{P(M)} = \frac{24/100}{40/100} = \frac{24}{40} = .6$

The conditional probability is still equal to the unconditional probability; so, yes A and M are independent

b) $P(\bar{A}) = 1 - P(A) = 1 - .6 = .4$

$P(F) = 1 - P(M) = 1 - .4 = .6$

$P(\bar{A}|F) = \frac{P(\bar{A} \cap F)}{P(F)} = \frac{24/100}{60/100} = \frac{24}{60} = .4$

Yes, $\bar{A}$ and F are independent

2.73 $S \rightarrow \{rr, rR, Rr, RR\}$ each with probability $\frac{1}{4}$

a) $P(\text{at least one } R) = \frac{3}{4}$

b) $P(\text{at least one } r) = \frac{3}{4}$

c) $P(\text{one } r \mid \text{at least one } R) = \frac{P(\text{one } r \text{ \cancel{\wedge} at least one } R)}{P(\text{at least one } R)} = \frac{2/4}{3/4} = \frac{2}{3}$

2.74

Child	A: Too High	B: About Right	C: Too little	
D → Yes	.20	.09	.01	.30
E → No	.41	.21	.08	.70
	.61	.30	.09	1.00

a) $P(A \cap D) = .20$ $P(A) \times P(D) = (.61)(.30) = .183$ not independent

b) $P(B \cap D) = .09$ $P(B) \times P(D) = (.3)(.3) = .09$ yes, "

c) $P(C \cap D) = .01$ $P(C) \times P(D) = (.09)(.3) = .027$ not, "

2.75 a) $A \rightarrow$ spade

$$P(A_3 A_4 A_5 \mid A_1 A_2) = \frac{P(A_3 \cap A_4 \cap A_5 \cap A_1 \cap A_2)}{P(A_1 \cap A_2)}$$

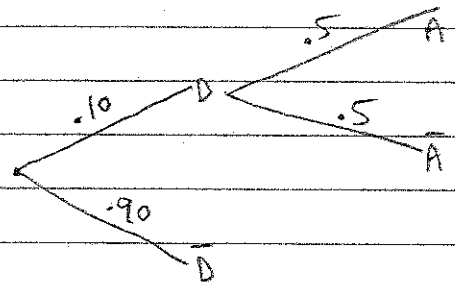
$$= \frac{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}}{\frac{13}{52} \times \frac{12}{51}} = .0084$$

b) $$P(A_4 A_5 \mid A_1 \cap A_2 \cap A_3) = \frac{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}}{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}} = .0383$$

c) $$P(A_5 \mid A_1 \cap A_2 \cap A_3 \cap A_4) = \frac{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}}{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49}} = .1875$$

2.76 $D \rightarrow$ dissatisfied $P(D) = .10$

$P(A \mid D) = .5$, $P(A) = .4$



a) $$P(D \mid A) = \frac{P(D \cap A)}{P(A)} = \frac{P(D) \cdot P(A \mid D)}{P(A)} = \frac{(.1)(.5)}{.40} = .125$$

b) $$P(\bar{D} \mid A) = 1 - P(D \mid A) = 1 - .125 = .875$$

2.90 $J \rightarrow$ injury, $P(J) = .02$ and jumps are independent

a) $P(J_1 \cup J_2) = P(J_1) + P(J_2) - P(J_1 \cap J_2) = .02 + .02 - (.02)(.02) = .0396$

or

Would you jump again if injured on first jump?

$P(\text{Injured}) = P(J_1) + P(\bar{J}_1 \cap J_2) = .02 + (.98)(.02) = .0396$

b) $P(\text{at one Injury}) = 1 - P(\text{no injury}) = 1 - (.98)^{50} = .635$

The friend is wrong

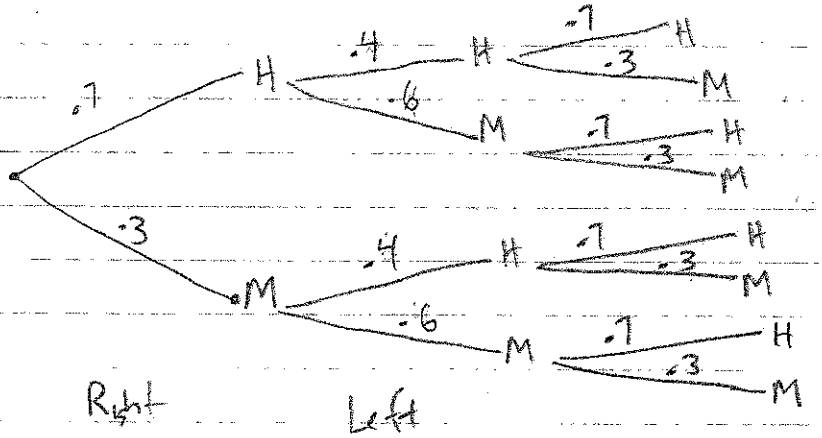
2.92 $L \rightarrow$ lying, $T \rightarrow$ truth

a) $P(L_1 \cap L_2 \cap L_3) = (.05)(.05)(.05) = .000125$

b) $P(\text{at least one } L) = 1 - P(\text{no } L\text{'s}) = 1 - P(T_1 \cap T_2 \cap T_3)$

$= 1 - (.95)(.95)(.95) = .143$

2.93



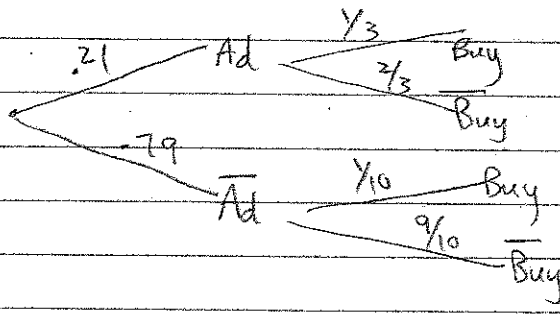
$P(\text{Wins}) = P(HHH \text{ OR } HHM \text{ OR } MHH) = (.7)(.4)(.7) + (.7)(.4)(.3) + (.3)(.4)(.4)$

$= .364$

2.101 $P(M_1) = .1$, $P(M_2 | M_1) = .5$ $P(M_1 \cap M_2) = (.1)(.5) = .05$

2.111 $P(M) = .02$, $P(T) = .20$ $P(T \cap M) = .01$

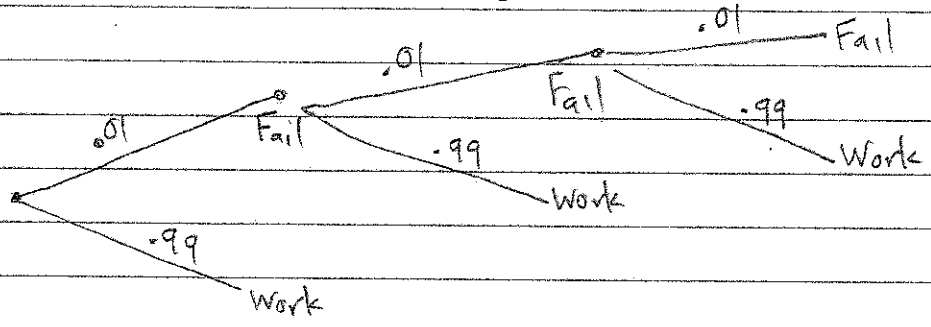
$P(\text{Seen Ad}) = P(M \cup T) = P(M) + P(T) - P(T \cap M) = .02 + .20 - .01 = .21$



$P(\text{Buy}) = (.21)\left(\frac{1}{3}\right) + (.79)\left(\frac{1}{10}\right)$

$= .07 + .079 = .149$

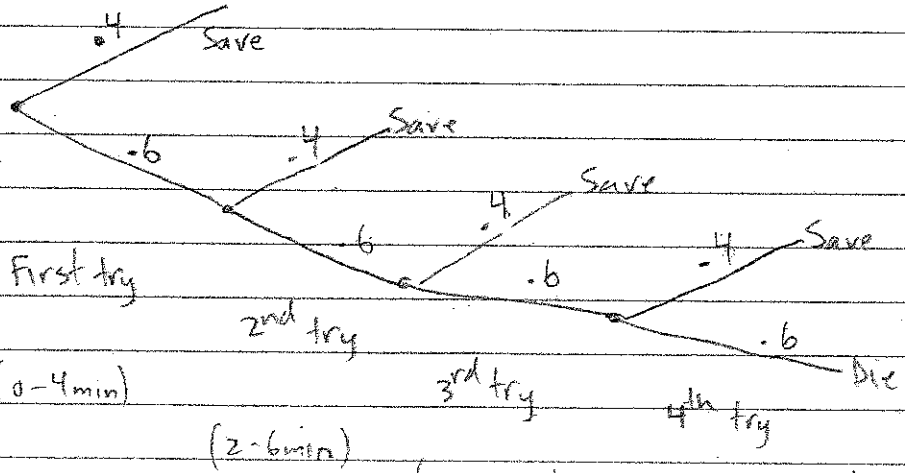
2.116



$P(\text{not completely fail}) = 1 - P(F_I \cap F_{II} \cap F_{III})$

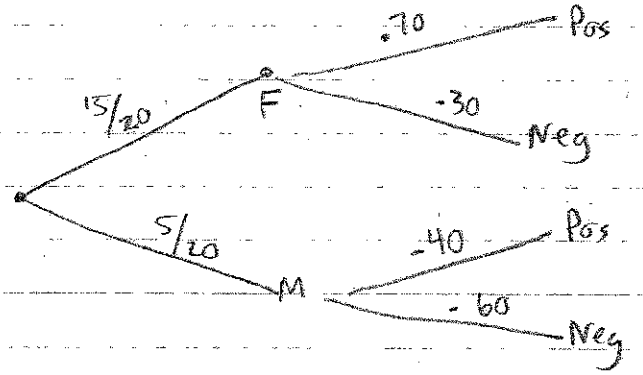
$= 1 - (.01)(.01)(.01) = 1 - (.01)^3 = .999999$

2.118



$P(\text{Saved}) = .4 + (.6)(.4) + (.6)^2(.4) + (.6)^3(.4) = .8704$

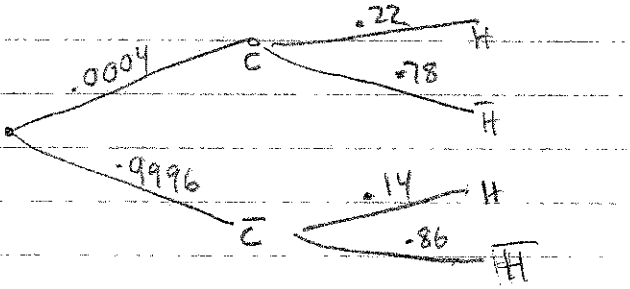
2.129



$$P(M \mid Neg) = \frac{P(M \cap Neg)}{P(Neg} = \frac{\left(\frac{5}{20}\right)(.6)}{\left(\frac{5}{20}\right)(.6) + \left(\frac{15}{20}\right)(.3)} = \frac{.15}{.15 + .225} = .40$$

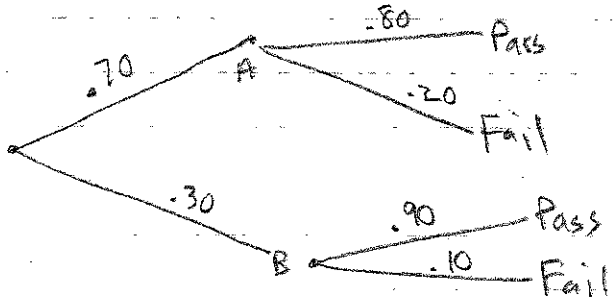
2.130 C → Cancer, H → Shipyard

$P(H \mid C) = .22 \quad P(H \mid \bar{C}) = .14 \quad P(C) = .0004$



$$P(C \mid H) = \frac{P(C \cap H)}{P(H)} = \frac{(.0004)(.22)}{(.0004)(.22) + (.9996)(.14)} = .00063$$

2.134



$$P(A \mid Fail) = \frac{P(A \cap Fail)}{P(Fail)} = \frac{(.70)(.20)}{(.70)(.20) + (.30)(.10)} = \frac{.14}{.14 + .03} = .82$$