STA 250, Summer 2013, Test #2 - Solution

1. $Y \to$ # of days until a fire drill — Geometric $(p = .04)$

5 a) $P(Y \le 20) = 1 - (.96)^{20} \approx .558$

10 b) $E(Y) = \frac{1}{p} = \frac{1}{.04} = 25$, $V(Y) = \frac{q}{p^2} = \frac{.96}{(.04)^2} = 600$, $\sigma = \sqrt{600} \approx 24.5$

$\mu + 2\sigma = 25 + 2(24.5) = 74 \quad P(Y > 74) = (.96)^{74} \approx .049$

8 c) $P(30 \le Y \le 60) = P(Y \le 60) - P(Y \le 29) = [1 - .96^{60}] - [1 - .96^{29}] \approx .2197$

6 **2** a. $P(Y < -1) = \int_{-2}^{-1} \frac{3}{32} y^2 (2-y)\,dy \approx .789$

6 b. $E(Y) = \int y \cdot f(y)\,dy = \int_{-2}^{2} y \cdot \frac{3}{32} \cdot y^2 (2-y)\,dy = -1.2$

(this is 11:48:48 am)

8 c. $E(Y^2) = \int_{-2}^{2} y^2 \left(\frac{3}{32}\right) y^2 (2-y)\,dy = 2.4 \quad V(Y) = E(Y^2) - \mu^2 = 2.4 - (-1.2)^2$

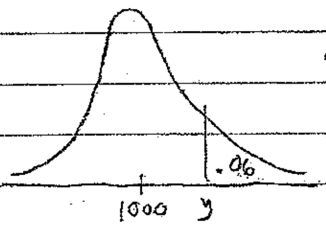
$V(Y) = E(Y^2) - \mu^2 = 2.4 - (-1.2)^2 = .96, \quad \sigma = \sqrt{V(Y)} = \sqrt{.96} = .98$ min

3 $Y \to$ Total SAT $\sim$ Normal$(\mu = 1200, \sigma = 180)$

8 a. for $y = 1200$, $z = \frac{y - \mu}{\sigma} = \frac{1200 - 1000}{180} = 1.11$

$P(Y > 1200) = .1335$

8 b.



area $= .06$, $z = 1.56 = \frac{y - \mu}{\sigma}$

So, $y = 1000 + 1.56(180) \approx 1281$

8 **4** Want $P(Y > 6\text{ mL}) = .01$

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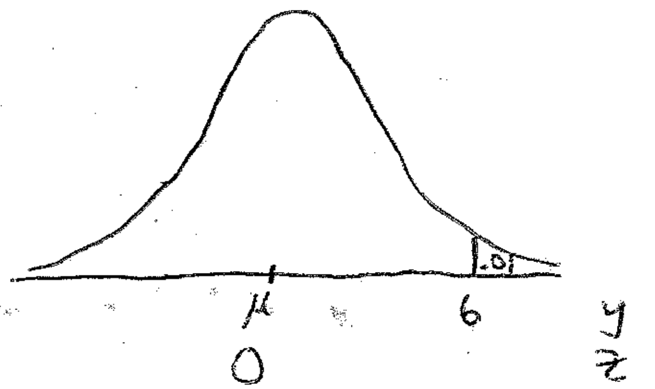
$z_{.01} = 2.33$

So, $2.33 = \frac{6 - \mu}{\sigma}$

$\to \mu = 6 - 2.33(.35)$

$= 5.1845$ mL



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5 $V(Y) = E(Y^2) - \mu^2$

$$= \int_{\theta_1}^{\theta_2} \frac{1}{\theta_2-\theta_1} y^2 dy \quad - \left(\frac{\theta_1+\theta_2}{2}\right)^2$$

$$= \left[\frac{1}{3}\frac{y^3}{(\theta_2-\theta_1)}\Big|_{\theta_1}^{\theta_2} = \frac{\theta_2^3-\theta_1^3}{3(\theta_2-\theta_1)}\right] - \left(\frac{\theta_1+\theta_2}{2}\right)^2$$

$$= \frac{\cancel{(\theta_2-\theta_1)}(\theta_2^2+\theta_1\theta_2+\theta_1^2)}{3\cancel{(\theta_2-\theta_1)}} - \left(\frac{\theta_1+\theta_2}{2}\right)^2$$

$$= \frac{4\theta_2^2+4\theta_1\theta_2+4\theta_1^2}{12} - \frac{(3\theta_1^2+6\theta_1\theta_2+3\theta_2^2)}{12}$$

$$\frac{\theta_2^2-2\theta_1\theta_2+\theta_1^2}{12} = \frac{(\theta_2-\theta_1)^2}{12}$$

10 **6** a. The distribution of ages at the time of a first heart attack is clearly skewed to the left. The average age at the time of the first heart attack is around 62 years of age. The ages range from about 30 to nearly 80 years. There are no obvious outliers in the distribution.

5 b. Location $= \frac{n+1}{2} = \frac{105+1}{2} = 53$. The median is between 60 to 64 years inclusive.

or $S \approx \frac{62-30}{3} \approx 10.7$ yrs

5 c. $S \approx \frac{\text{Range}}{4} \approx \frac{80-30}{4} = 12.5$ yrs

[redacted]

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7. $Y \to$ time to grade one page $\sim$ Exponential$(\beta = 1)$

7 a. $P(Y > 2) = 1 - P(Y \le 2) = 1 - F(2) = 1 - [1 - e^{-\frac{2}{1}}] = e^{-2} = .135$

6 b. Median is where $F(y) = .5 = 1 - e^{-\frac{y}{1}}$

$\rightarrow e^{-y} = .5$

$y = -\ln(.5) = .69$ min

12 c. $n = 4 \times 28 = 132$ pages

Sampling Dist of $\bar{y}$ $\quad \mu_{\bar{y}} = \mu = 1$; $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{128}} = .088$

Total of 150 min $\rightarrow \bar{y} = \frac{150}{128} = 1.172$ min

for $\bar{y} = 1.172$, $z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}} = \frac{1.172 - 1}{.088} = 1.944$

$P(\bar{y} < 1.172) = 1 - .0262 = .9738$

10 **8** Sampling Distribution of $\hat{p}$

$\mu_{\hat{p}} = p = .08$; $\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{(.08)(.92)}{200}} = .019$

$p \pm 3\sigma_{\hat{p}} \rightsquigarrow (.023, .137)$ within $(0, 1)$ Normal!

for $\hat{p} = .05$, $z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.05 - .08}{.019} \approx -1.56$

$P(\hat{p} < .05) = .0594$

10 **9** Sampling Distribution of $\bar{y}$

a) $\mu_{\bar{y}} = \mu$; $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{90}{\sqrt{80}} = 10.06$, Normal by C.L.T.

$\bar{y} = \mu \pm 10$, $z = \frac{10}{10.06} \approx .99$ $\quad P(\bar{y}$ ~~within~~ $10K$ of $\mu) = 1 - [.1611 + .1611]$

$= .6878$

8 b. Want $z = \frac{10}{90/\sqrt{n}} = 2.33$

So, $n = \left(\frac{2.33 \times 90}{10}\right)^2 = 439.7$

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So, need $n \ge 440$