STA 250, Summer 2013, HW #3

3.2 $S \to \{HH, HT, TH, TT\}$ (your coin) $Y \to$ "winnings"

y: 2, −1, −1, 1

y	−1	1	2
$p(y)$	2/4	1/4	1/4

3.6 $N = C^5_2 = 10$, $S \to \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$

a) $Y \to$ largest

y	2	3	4	5
$p(y)$	1/10	2/10	3/10	4/10

b) $Y \to$ Sum

y	3	4	5	6	7	8	9
$p(y)$	1/10	1/10	2/10	2/10	2/10	1/10	1/10

3.9 $E \to$ error on entry $P(E) = .05$

Three random entries $Y \to$ # of errors

$S \to \{EEE, EE\bar{E}, E\bar{E}E, \bar{E}EE, \bar{E}\bar{E}E, \bar{E}E\bar{E}, E\bar{E}\bar{E}, \bar{E}\bar{E}\bar{E}\}$

prob. $(.05)(.05)(.05)$, $(.05)(.05)(.95)$, $(.05)(.95)(.05)$, $(.95)(.05)(.05)$, $(.95)(.95)(.05)$, $(.95)(.05)(.95)$, $(.05)(.95)(.95)$, $(.95)(.95)(.95)$

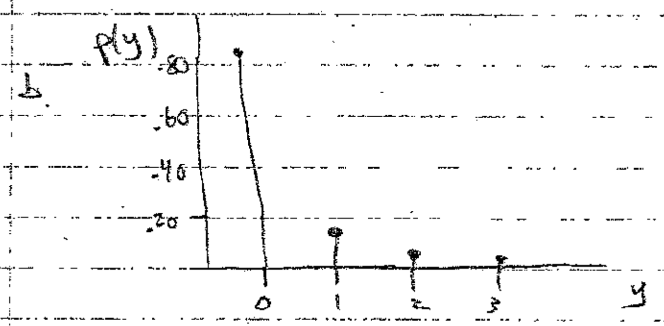
y: 3, 2, 2, 2, 1, 1, 1, 0

So,

y	$p(y)$
0	$(.95)^3 = .8574$
1	$3(.05)(.95)^2$ .1354
2	$3(.05)^2(.95) = .0071$
3	$(.05)^3 = .0001$

You should recognize this as Binomial $(n=3, p=.05)$

b.



c) $P(Y > 1) = .0071 + .0001 = .0072$

3.14 a) $E(Y) = \sum y \cdot p(y) = 3(.03) + 4(.05) + \cdots + 13(.01) = 7.9$ yrs

b) $V(Y) = \sigma^2 = E(Y^2) - \mu^2 = \sum y^2 \cdot p(y) - \mu^2$

$= 3^2(.03) + 4^2(.05) + \cdots + 13^2(.01) - 7.9^2 = 4.73 \text{ yrs}^2$

SD, $\sigma = \sqrt{\sigma^2} = \sqrt{4.73} \approx 2.2$ yrs

c) $\mu \pm 2\sigma \rightsquigarrow 7.9 \pm 2(2.2) \sim (3.5, 12.3)$

$P(Y \text{ within } \mu \pm 2\sigma) = P(4) + P(5) + \cdots + P(12)$

$= .05 + \cdots + .03 = .96$

3.15 $J \to$ prefer Jay Leno $P(J) = .52$

Three viewers, $Y \to$ # that prefer Jay Leno

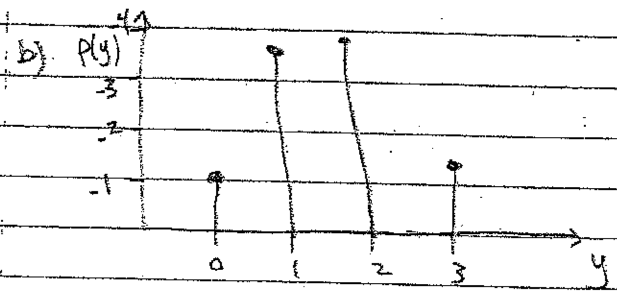
$S \to \{JJJ, JJ\bar{J}, J\bar{J}J, \bar{J}JJ, \bar{J}\bar{J}J, \bar{J}J\bar{J}, J\bar{J}\bar{J}, \bar{J}\bar{J}\bar{J}\}$

a)

y	$P(y)$	
0	$(.48)^3$	$\approx .1106$
1	$3(.52)(.48)^2$	$.3594$
2	$3(.52)^2(.48)$	$.3894$
3	$(.52)^3$	$.1406$

Again, Binomial $(n=3, p=.52)$

b)



c) $P(Y=1) = .3594$

d) $\mu = E(Y) = \sum y \cdot p(y) = 0(.1106) + 1(.3594) + \cdots + 3(.1406) = 1.56$

$V(Y) = E(Y^2) - \mu^2 = \sum y^2 \cdot p(y) - \mu^2$

$= 0^2(.1106) + 1^2(.3594) + \cdots + 3^2(.1406) - 1.56^2 = .7488$

SD, $\sigma = \sqrt{V(Y)} = \sqrt{.7488} = .865$

e) $\mu \pm 2\sigma \to 1.56 \pm 2(.865) \longrightarrow (-.17, 3.29)$

$P(Y \text{ within } \mu \pm 2\sigma) = P(0) + P(1) + \cdots + P(3) = 1$

3.24 $F \rightarrow$ flawed $P(\text{flawed}) = .10$

Select 2 bottles $S \rightarrow \{ FF, F\bar{F}, \bar{F}F, \bar{F}\bar{F} \}$

$Y \rightarrow$ # of flawed bottles

y	$p(y)$
0	$(.9)^2 = .81$
1	$2(.1)(.9) = .18$
2	$(.1)^2 = .01$

$\mu = \sum y \cdot p(y) = 0(.81) + 1(.18) + 2(.01) = .20$ flaws

$V(Y) = E(Y^2) - \mu^2 = 0^2(.81) + 1^2(.18) + 2^2(.01) - .2^2 = .18$ flaws^2

Larry, Moe, and Curly arrive at a hotel for a reunion. The stooges will each be independently and randomly assigned a room on one of the three floors. For example, the outcome (122) indicates Larry's room was on the first floor, while Moe, and Curly were both on the second floor.

A. Create the probability distribution for the random variable Y representing how many floors contain stooges.

Number in the sample space, N = 3 x 3 x 3 = 27

Probability distribution for Y

y	p(y)	Outcomes
1	3/27 = 1/9	111, 222, 333
2	18/27 = 6/9	112, 121, 211, 113, 131, 311, 221, 212, 122, 223, 232, 322, 331, 313, 133, 332, 323, 233
3	6/27 = 2/9	123, 132, 213, 231, 312, 321

B. Find the expected number of floors to be occupied by stooges.

$$\mu = E(Y) = \sum y \cdot p(y) = 1(1/9) + 2(6/9) + 3(2/9) = 19/9 \approx 2.1 \text{ floors}$$

C. Find the standard deviation for the number of floors occupied by stooges.

$$\sigma^2 = E(Y^2) - \mu^2 = 1^2(1/9) + 2^2(6/9) + 3^2(2/9) - (19/9)^2 = 26/81 \approx .321 \text{ floors}^2.$$

$$\sigma = \sqrt{V(Y)} = \sqrt{(26/81)} \approx .56 \text{ floors}$$