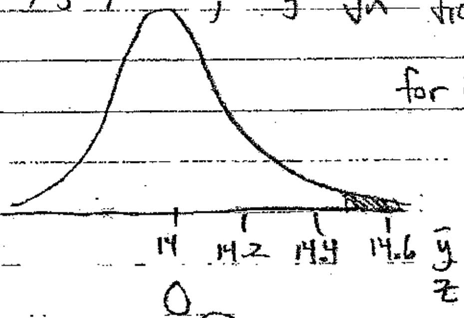
STA 250, Summer 2013, HW #5

7.42 Y → strength ~ ($\mu = 14$ kpsi, $\sigma = 2$)

a) Sample $n = 100$

Sampling Distribution of $\bar{y}$

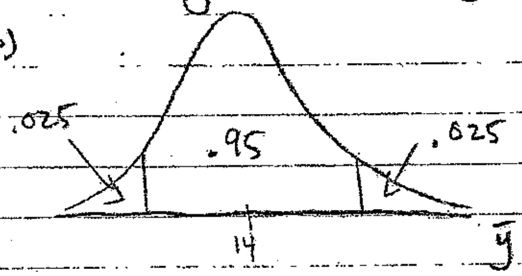
$\mu_{\bar{y}} = \mu = 14$; $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{2}{\sqrt{100}} = .2$; Normal distribution by CLT.



for $\bar{y} = 14.5$, $z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}} = \frac{14.5 - 14}{.2} = 2.5$

$P(\bar{y} > 14.5 \text{ kpsi}) = .0062$

b)



$z = \pm 1.96$

$= \frac{\bar{y} - 14}{.2}$

$\bar{y} = 14 - 1.96(.2) \approx 13.61$ kpsi

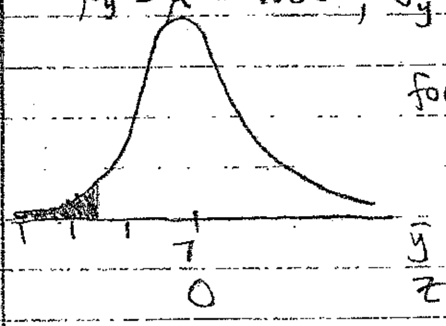
$\bar{y} = 14 + 1.96(.2) \approx 14.39$ kpsi

7.45 Y → Wage ~ ($\mu = \$7.00$, $\sigma = \$.50$)

Sample of ethnic workers finds $\bar{y} = 6.90$ ($n = 64$).

Sampling Distribution of $\bar{y}$

$\mu_{\bar{y}} = \mu = \$7.00$; $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{.5}{\sqrt{64}} = \$.0625$; normal by CLT.



for $\bar{y} = 6.90$, $z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}}$

$= \frac{6.90 - 7.00}{.0625} = -1.60$

$P(\bar{y} < 6.90) = .0548$

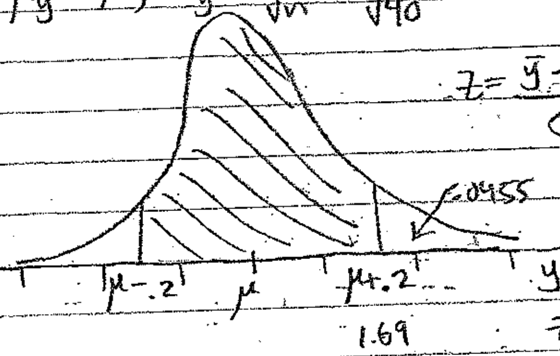
This a little unlikely but no so unusual to conclude the ethnic group is being discriminated accept.

7.46 $Y \to pH \sim (\mu, \sigma)$ $\quad \sigma \approx \frac{\text{Range}}{4} = \frac{8-5}{4} = .75$

Sample $n=40$

Sampling Distribution of $\bar{y}$

$\mu_{\bar{y}} = \mu$; $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{.75}{\sqrt{40}} = .1186$; Normal distribution by C.L.T.



$z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}} = \frac{.2}{.1186} = 1.69$

$P(\bar{y} \text{ within } .2 \text{ units of } \mu) = 1 - [.0455 + .0455] = .909$

7.73 $p \to$ true proportion of "no-shows" $= .05$

"Sell" a sample of $n=160$ tickets $\quad \hat{p} \to$ sample proportion of no-shows

Sampling distribution of $\hat{p}$

1) $\mu_{\hat{p}} = p = .05$

2) $\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{(.05)(.95)}{160}} = .017$

3) $p \pm 3\sigma_{\hat{p}} \sim (-.001, .101)$ $\quad$ Technically not normal, but...

$P(\text{every one gets a seat})$

$= P\left(\hat{p} \geq \frac{5}{160} = .03125\right)$

$= P(z \geq -1.09) = 1 - .1379 = .8621$

7.15 $p \to$ true proportion of voters favoring the bond
$\{ p = .20 \}$

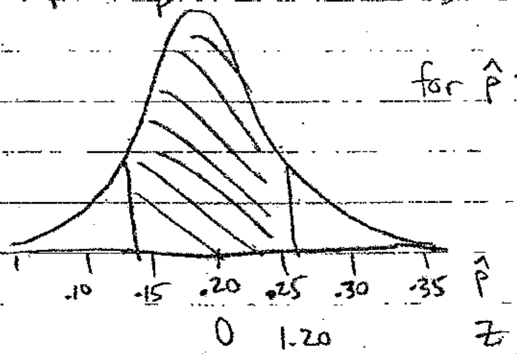
Sample $n = 64$

Sampling Distribution of $\hat{p}$

1) $\mu_{\hat{p}} = p = .20$

2) $\sigma_{\hat{p}} = \sqrt{\dfrac{p \cdot q}{n}} = \sqrt{\dfrac{(.2)(.8)}{64}} = .05$

3) $p - 3\sigma_{\hat{p}} = .20 - 3(.05) = .05 > 0$ Approx Normal
$p + 3\sigma_{\hat{p}} = .20 + 3(.05) = .35 < 1$ by CLT.



for $\hat{p} = .26$, $z = \dfrac{\hat{p} - p}{\sigma_{\hat{p}}} = \dfrac{.26 - .20}{.05} = 1.20$

$P(\hat{p} \text{ within } .06 \text{ of } p) = 1 - [.1151 + .1151] = .7698$

7.80 $p \to$ true proportion under 31 yrs $= .5$ (since 31 is median)

Sample $n = 100$

Sampling Distribution of $\hat{p}$

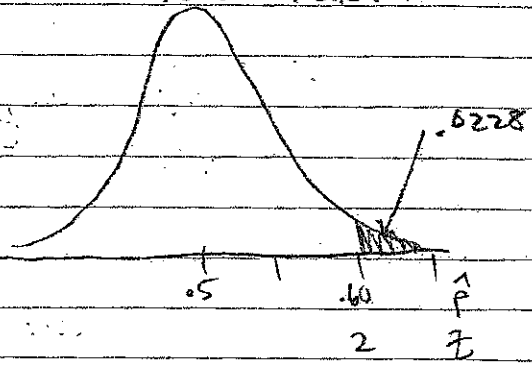
1) $\mu_{\hat{p}} = p = .5$

2) $\sigma_{\hat{p}} = \sqrt{\dfrac{p \cdot q}{n}} = \sqrt{\dfrac{(.5)(.5)}{100}} = .05$

3) $p \pm 3\sigma_{\hat{p}} \rightsquigarrow (.35, .65)$ So approximately a normal distribution

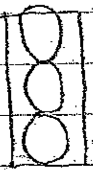
$P\left(\hat{p} \geq \frac{60}{100} = .6\right)$

$= P(z \geq 2.00) = .0228$



1.

$Y \to$ Diameter $\sim$ Normal$(\mu = 6.6, \sigma = .11)$



Sample $n=3$

Sampling Distribution of $\bar{y}$

1) $\mu_{\bar{y}} = \mu = 6.6$

2) $\sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{.11}{\sqrt{3}} = .0635$

3) Normal Dist since pop has a Normal dist

$P(\text{lid does not fit}) = P(\text{Total} > 20.35)$

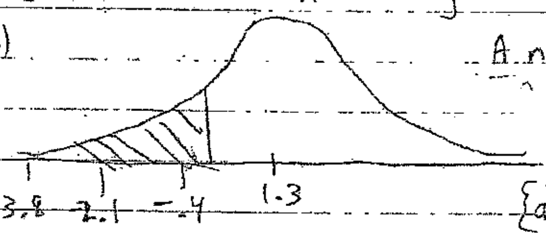
$= P\left(\bar{y} > \frac{20.35}{3} = 6.78\bar{3}\right)$

for $\bar{y} = 6.78\bar{3}$, $z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}} = \frac{6.78\bar{3} - 6.6}{.0635} = 2.89$

$P(\bar{y} > 6.78\bar{3}) = P(z > 2.89) = .0019$

2. $y \to$ # of crimes in one day $\sim (\mu = 1.3, \sigma = 1.7)$

a)



A normal distribution does not make sense here "negative" crimes!

{also this is a <u>discrete</u> variable}

b) The sampling distribution of $\bar{y}$ will still be approximately normal by C.L.T.

c) $\mu_{\bar{y}} = \mu = 1.3$ $\quad \sigma_{\bar{y}} = \frac{\sigma}{\sqrt{n}} = \frac{1.7}{\sqrt{50}} \approx .24$

d) for $\bar{y} = 1$, $z = \frac{\bar{y} - \mu_{\bar{y}}}{\sigma_{\bar{y}}} = \frac{1 - 1.3}{.24} \approx -1.25$

$P(\bar{y} < 1) = .1056$

3 $p \to$ true proportion that graduate within 4 yrs $\{p = .57\}$

for $n = 200$

Sampling Distribution of $\hat{p}$

$\mu_{\hat{p}} = p = .57$; $\sigma_{\hat{p}} = \sqrt{\frac{p \cdot q}{n}} = \sqrt{\frac{(.57)(.43)}{200}} \approx .035$

$p \pm 3\sigma_{\hat{p}} \rightsquigarrow (.465 , .675)$ within $(0, 1)$

So, approximately a normal dist

a) for $\hat{p} = .5$, $z = \frac{\hat{p} - p}{\sigma_{\hat{p}}} = \frac{.5 - .57}{.035} \approx -2.00$

$P(\hat{p} > .5) = 1 - .0228 = .9772$

b) $\hat{p}$ within .05 of $p \to \hat{p}$ within .52 to .62

for $\hat{p} = .52$, $z = -1.43$

$\hat{p} = .62$, $z = +1.43$

$P(\hat{p}$ within .05 of $p) = 1 - [.0764 + .0764] = .8472$