# MULTIBODY DYNAMICS HOMEWORK

(Assignment #5)

By: Charles Chano

Date: 20/1/2022

The assigned task (problem #5) was to take a given kinematic structure, with the given coordinates for each revolute joint and center of mass.

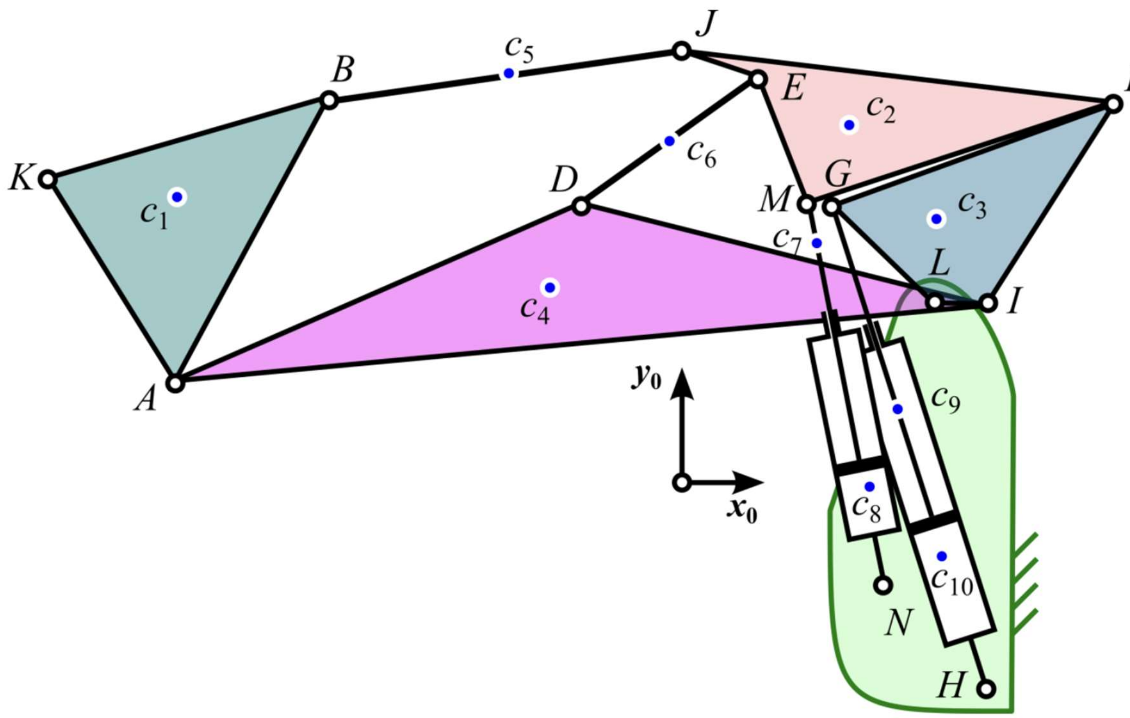



*Figure 1: Kinematic diagram for problem #5*

*Table 1: Location of joints*

| Point | A | B | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| X [m] | -1 | -0.7 | -0.2 | 0.1 | 0.9 | 0.3 | 0.7 | 0.7 | 0 | -1.2 | 0.6 | 0.2 | 0.4 |
| Y [m] | 0.2 | 0.8 | 0.5 | 0.8 | 0.7 | 0.6 | -0.4 | 0.3 | 0.9 | 0.6 | 0.3 | 0.6 | -0.2 |

The first task was to model the given schematic in MSC ADAMS, as can be seen in Figure 2. The actuators were each modeled as two rigid links with some distance on between. The translational joints were fixed to bodies of C7 and C9, then attached to the bodies of C8 and C10. So, the positive direction of motion is set to be downward.

Next some motion equations were formulated using the formula provided in the assignment:

$$x_k = l_k + a_k \sin(\omega_k t + \varphi_k)$$

The motion equations used in ADAMS were based on a grid in millimeters. The equation used for both actuators was:

$$x = -50 \sin(1.5t) \ \ [mm]$$

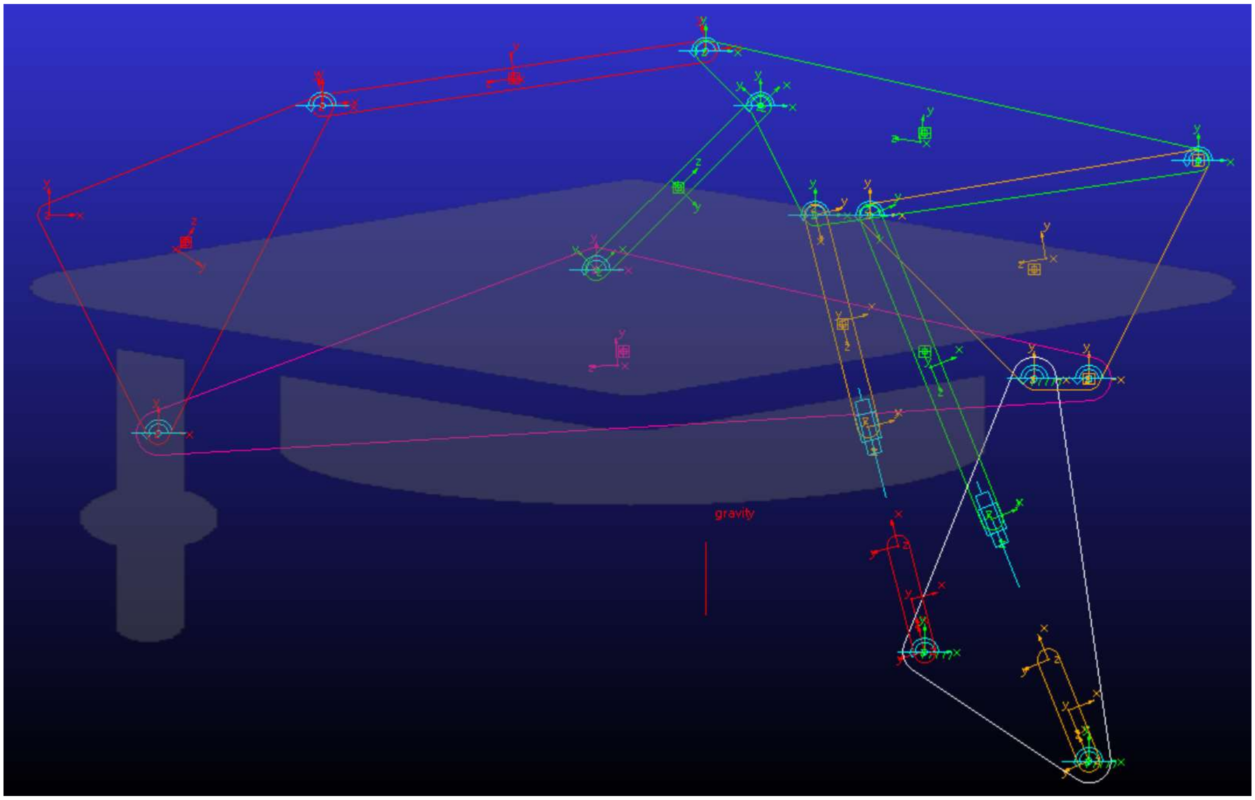



*Figure 2: Kinematic model in ADAMS*

The next task was to develop a matlab script to calculate the displacement, velocity, acceleration of all the given points.

The first step was to determine the location and orientation of all the local frames for each body, as can be seen in Figure 3. All frames except $\pi_8$, $\pi_7$, $\pi_9$, and $\pi_{10}$ were oriented in the orientation as the global frame. Frames $\pi_8$, $\pi_7$, $\pi_9$, and $\pi_{10}$ were oriented with the X-axis along the line of translation, as shown in Figure 4. With the local frames set up, the vectors from the global frame to the local frames can be determined along with the initial angle from the global frame. This information would become the initial "q" vector, or "q_init".

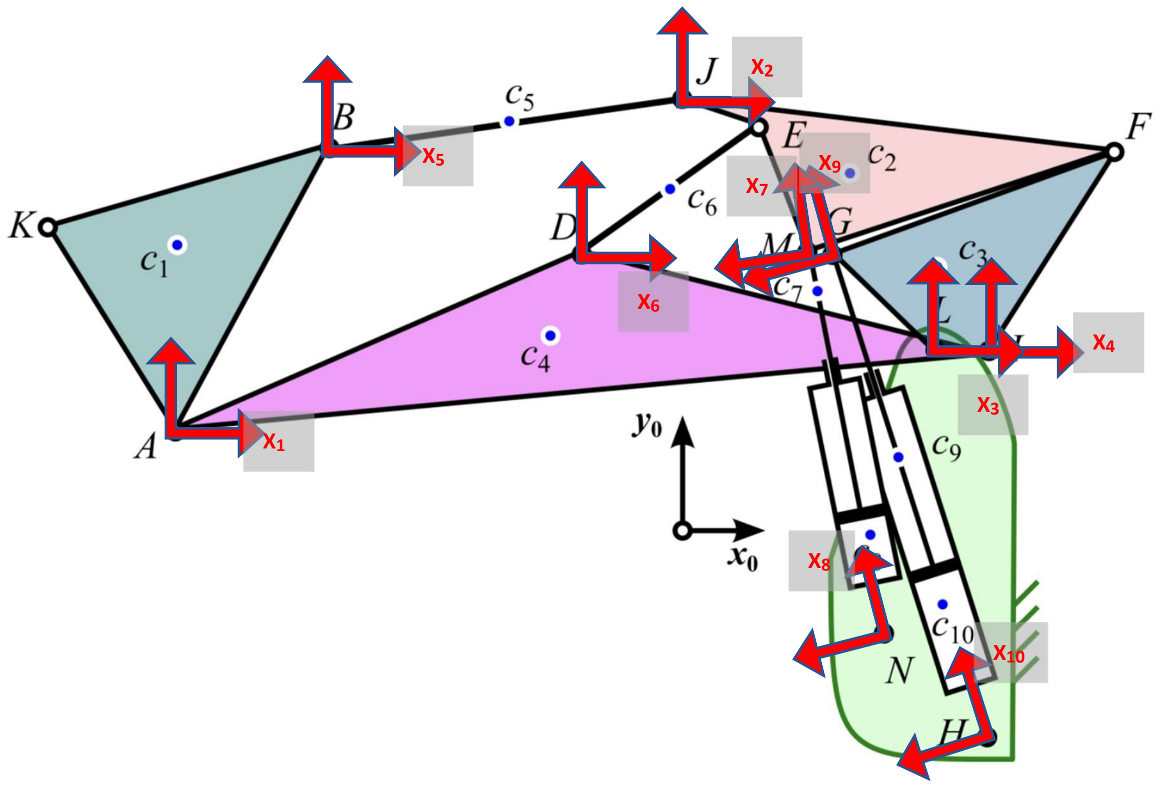



*Figure 3: Local Frame placement and orientation for each body*

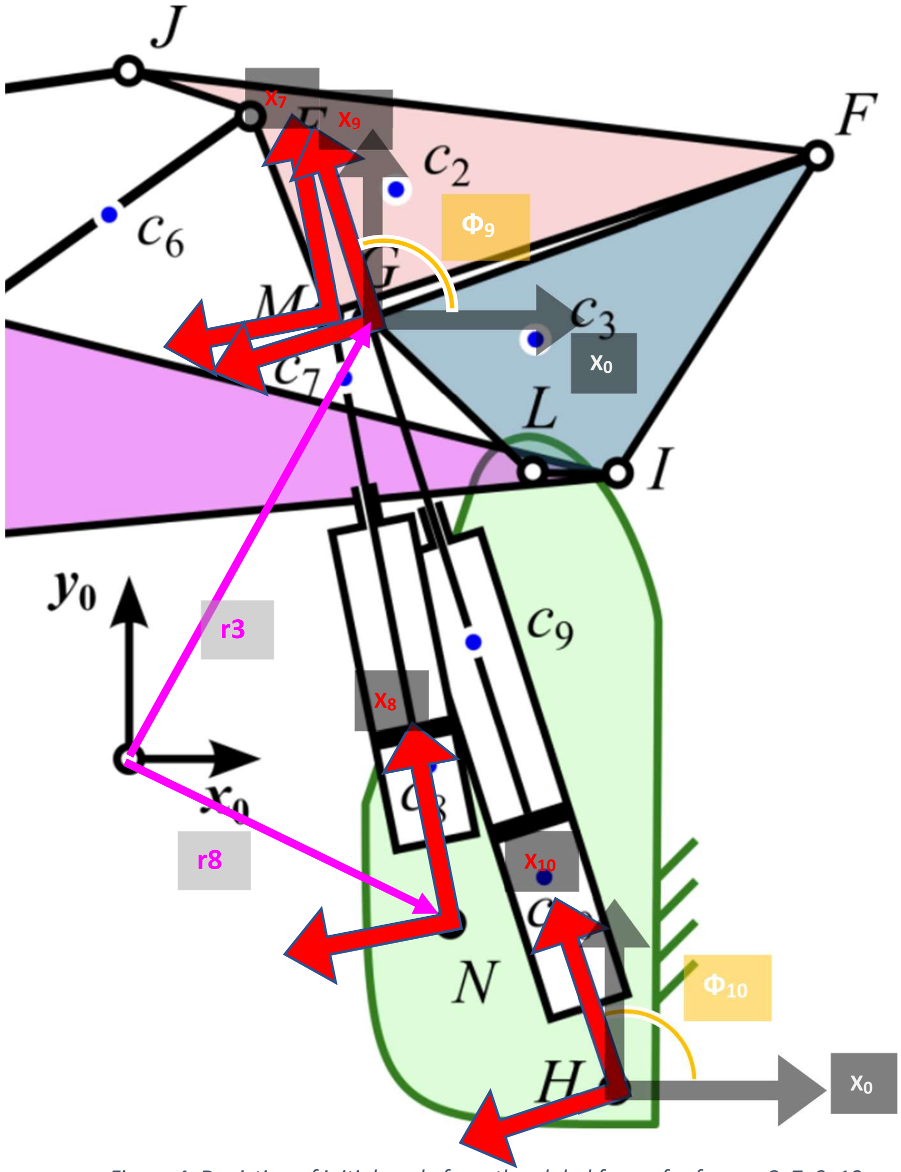



*Figure 4: Depiction of initial angle from the global frame for frames 8, 7, 9, 10*

Next, the constraint equations were developed for every joint in the kinematic structure and two driving constraints.

For the revolute joint connections, equation 5.17 of Lecture 5 was used. An example of how the parameters were defined can be seen in Figure 6.

*Equation 5.17 of Lecture 5:*

$$\mathbf{\Phi}^{K\bullet} \equiv \mathbf{r}_i + \mathbf{R}_i\mathbf{s}_A^{(i)} - \left(\mathbf{r}_j + \mathbf{R}_j\mathbf{s}_B^{(j)}\right) = \mathbf{0}$$

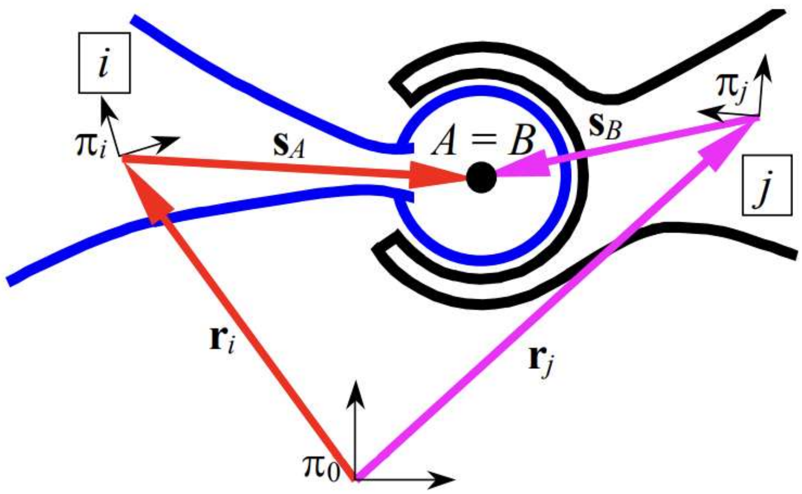



*Figure 5: Revolute joint constraint definition*

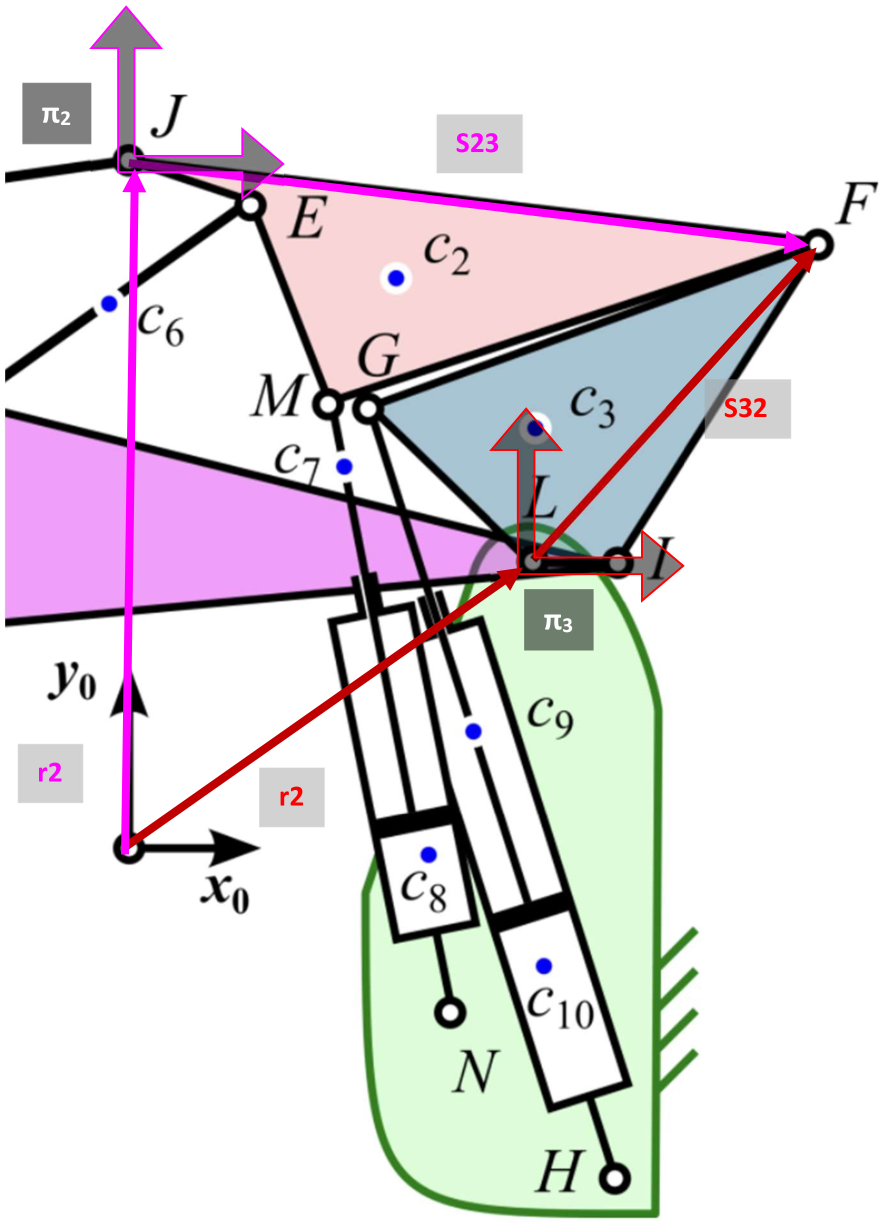



*Figure 6: Depiction of definition of revolute joint constraint for this task*

For translational joints equation 5.18 and 5.21 of Lecture 5 was used. In my implementation, since the frames were along the line of translation with points “A” and “B” on the local frame origin, simplifications were made such as:

$S_A = 0\ ; S_B = 0$

*Equation 5.18 of Lecture 5:*

$$\Phi^{K\angle} \equiv \varphi_i - \varphi_j - \varphi^0 = 0$$

*Equation 5.21 of Lecture 5:*

$$\Phi^{K\uparrow} \equiv \left(\mathbf{r}_j + \mathbf{R}_j\mathbf{s}_B^{(j)} - \mathbf{r}_i - \mathbf{R}_i\mathbf{s}_A^{(i)}\right)^T \mathbf{R}_j\mathbf{v}^{(j)} \equiv \left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T \left(\mathbf{r}_j + \mathbf{R}_j\mathbf{s}_B^{(j)} - \mathbf{r}_i - \mathbf{R}_i\mathbf{s}_A^{(i)}\right)$$

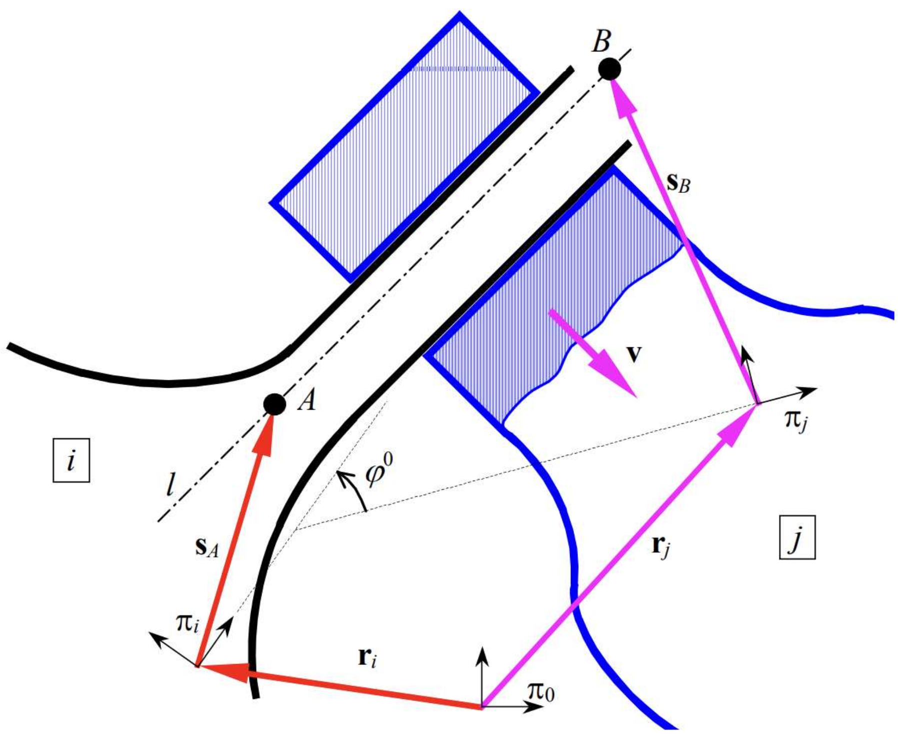



*Figure 7: Depicted definition for the translational joint constraint*

For the driving constraint equations, 5.23 of Lecture 5 was used. The equation of motion used was slightly different from that used in ADAMS. ADAMS automatically determines the starting distance, so it does not need to be considered, whereas in matlab it does. Additionally, in ADAMS all calculations were done in millimeters, whereas in Matlab, all calculations were done in meters.

*Equation of motion used in Matlab:*

$$f_{AB}(t) = L_{AB} + 0.05\sin(1.5t) \quad [m]$$

*where,*

$$L_{AB} = \sqrt{(r_A(x) - r_B(x))^2 + (r_A(y) - r_B(y))^2}$$

*Equation 5.21 of Lecture 5:*

$$\mathbf{\Phi}^{D\bullet} \equiv \mathbf{d} - \mathbf{f}_{AB}(t) \equiv \mathbf{r}_j + \mathbf{R}_j \mathbf{s}_B^{(j)} - \left(\mathbf{r}_i + \mathbf{R}_i \mathbf{s}_A^{(i)}\right) - \mathbf{f}_{AB}(t) = \mathbf{0}$$

To ensure correctness of the constraints, the initial q vector was solved for, with the expectation of getting all zeros for the solution of every constraint.

Next, the jacobian needed to be formulated. Taking the partial derivative of each constraint equation with respect to every “q” vector, a 30x30 matrix was attained. For each constraint equation, most of the partial derivatives would result in zero except for the bodies in the constraint equation.

The results of taking the partial derivative of the revolute kinematic joints are as shown in equations 5.30-5.33 of Lecture 5:

$$\mathbf{\Phi}^{K\bullet}_{\mathbf{r}_i} = \mathbf{I}_{2\times 2}$$
$$\mathbf{\Phi}^{K\bullet}_{\varphi_i} = \mathbf{\Omega}\mathbf{R}_i\mathbf{s}^{(i)}_{\mathrm{A}}$$
$$\mathbf{\Phi}^{K\bullet}_{\mathbf{r}_j} = -\mathbf{I}_{2\times 2}$$
$$\mathbf{\Phi}^{K\bullet}_{\varphi_j} = -\mathbf{\Omega}\mathbf{R}_j\mathbf{s}^{(j)}_{\mathrm{B}}$$

The results of taking the partial derivative of the translational kinematic joints are as shown in equations 5.42 - 5.45 and 5.48 – 5.51 of Lecture 5:

$$\Phi^{K\angle}_{\mathbf{r}_i} = \mathbf{0}_{1\times 2}$$
$$\Phi^{K\angle}_{\varphi_i} = 1$$
$$\Phi^{K\angle}_{\mathbf{r}_j} = \mathbf{0}_{1\times 2}$$
$$\Phi^{K\angle}_{\varphi_j} = -1$$

The results of taking the partial derivative of the translational kinematic joints are as shown in equations 5.48 – 5.51 of Lecture 5:

$$\Phi^{K\uparrow}_{\mathbf{r}_i} = -\left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T$$
$$\Phi^{K\uparrow}_{\varphi_i} = -\left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T\mathbf{\Omega}\mathbf{R}_i\mathbf{s}^{(i)}_A$$
$$\Phi^{K\uparrow}_{\mathbf{r}_j} = \left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T$$
$$\Phi^{K\uparrow}_{\varphi_j} = \left(\mathbf{r}_j - \mathbf{r}_i - \mathbf{R}_i\mathbf{s}^{(i)}_A\right)^T\mathbf{\Omega}\mathbf{R}_j\mathbf{v}^{(j)} = -\left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T\mathbf{\Omega}\left(\mathbf{r}_j - \mathbf{r}_i - \mathbf{R}_i\mathbf{s}^{(i)}_A\right)$$

Lastly, for taking the partial derivatives of the driving constraints, equations 5.48 – 5.51 of Lecture 5 are again referenced with a change from the vector “v” to the vector “u” (as seen in Figure 8). Vector “u” is a unit vector defined parallel to the motion of translation in the local frame coordinate.

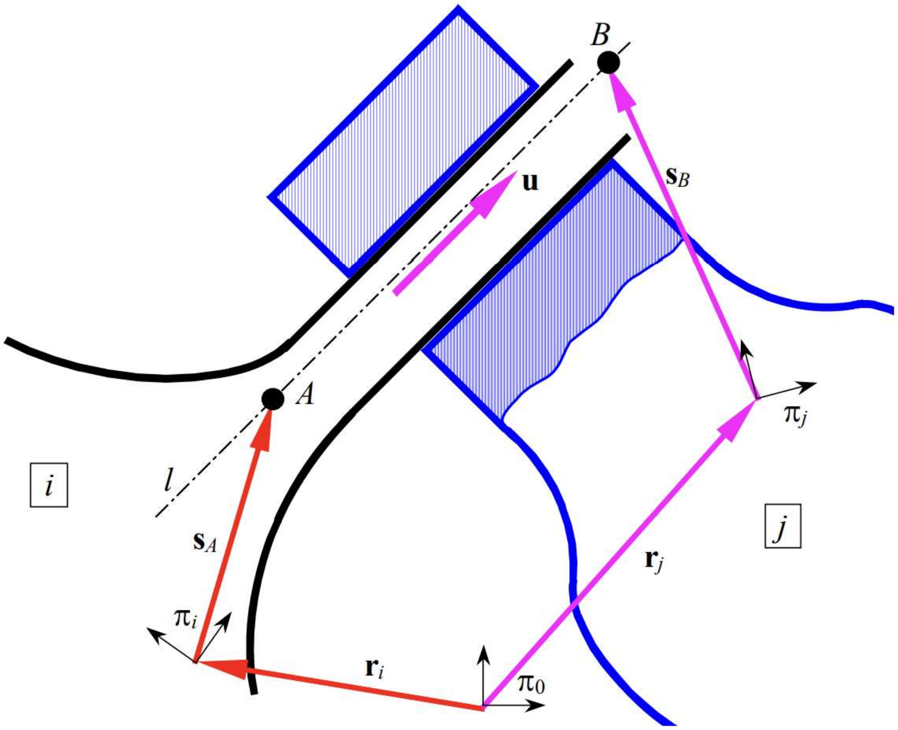



*Figure 8: Relative displacement in translational joint*

Next, the velocity constraints were determined. These constraints had a simple solution where all the results for the kinematic constraints equated to zero, whereas the driving constraints equated to the first derivative with respect to time of the motion equation:

$$\frac{df_{AB}(t)}{dt} = (1.5) \times 0.05\cos(1.5t) \quad [m/s]$$

Lastly, the acceleration constraints were determined. Taking the second derivative with respect to time, the following equations results:

*For revolute kinematic joints, Equation 5.41 of Lecture 5:*

$$\mathbf{\Gamma}^{K\bullet}_{2\times1} = \mathbf{R}_i\mathbf{s}_A^{(i)}\dot{\varphi}_i^{\,2} - \mathbf{R}_j\mathbf{s}_B^{(j)}\dot{\varphi}_j^{\,2}$$

*For translational kinematic joints, Equation 5.47 and 5.48 of Lecture 5:*

$$\Gamma^{K\angle}_{1\times1} = 0$$

$$\Gamma^{K\uparrow}_{1\times1} = \left(\mathbf{R}_j\mathbf{v}^{(j)}\right)^T\left(2\mathbf{\Omega}\left(\dot{\mathbf{r}}_j - \dot{\mathbf{r}}_i\right)\dot{\varphi}_j + \left(\mathbf{r}_j - \mathbf{r}_i\right)\dot{\varphi}_j^{\,2} - \mathbf{R}_i\mathbf{s}_A^{(i)}\left(\dot{\varphi}_j - \dot{\varphi}_i\right)^2\right)$$

*For translational driving acceleration constraints, Equation 5.48 of Lecture 5 was again used with a few changes. The* vector "v" was changed to the vector "u" (as seen in Figure 8) and the second time derivative of the motion equation was subtracted from the whole equation.

*The equation of motion for acceleration:*

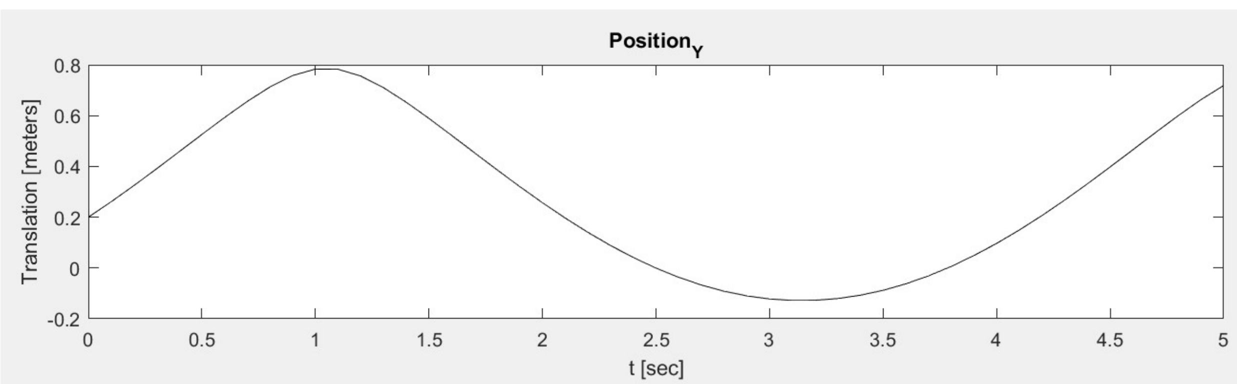
$$\frac{d^2 f_{AB}(t)}{dt^2} = (1.5)^2 \times 0.05 \sin(1.5t) \quad [m/s]$$

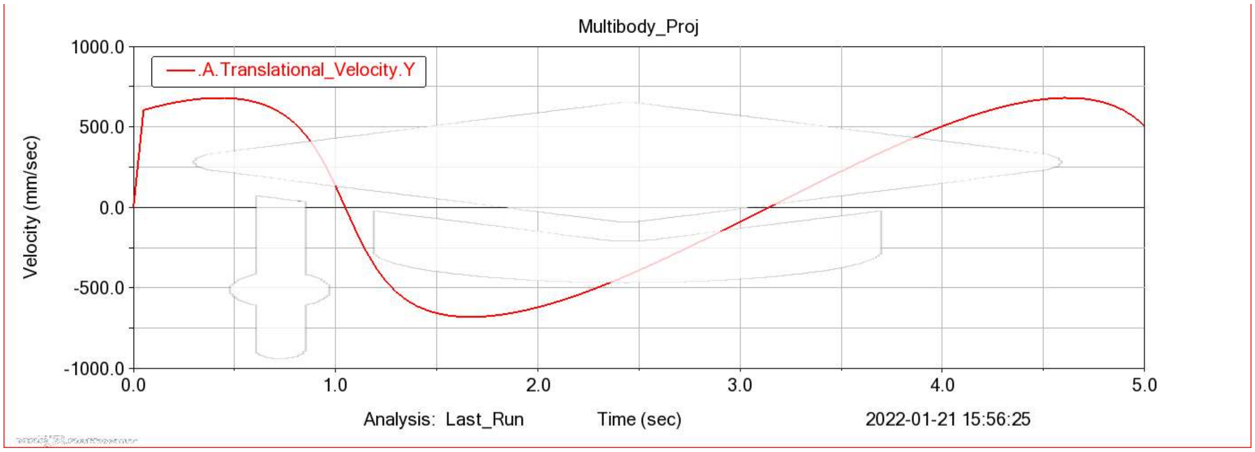
*Driving constraint equation for acceleration:*

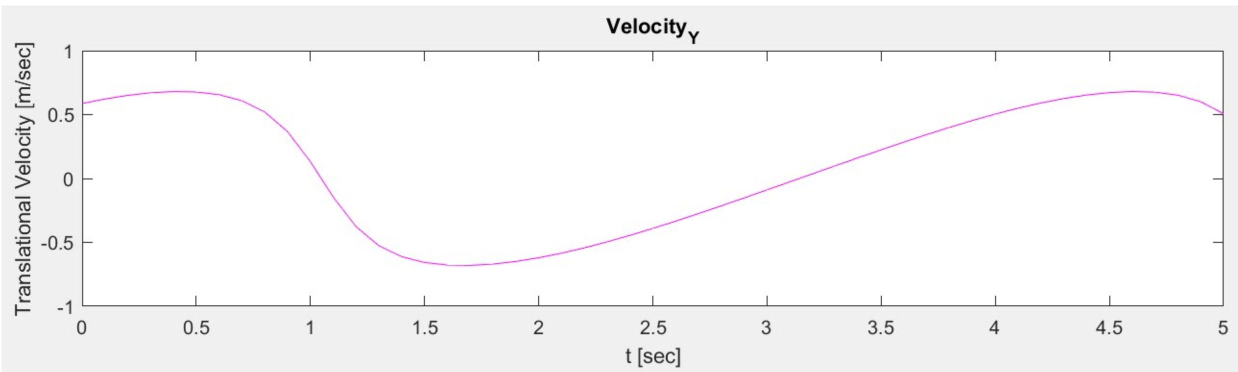
$$\Gamma_{1\times1}^{D} = \left(R_j u\right)^T \left(2\Omega\left(\frac{dr_j}{dt} - \frac{dr_i}{dt}\right)\frac{d\varphi_j}{dt} + \left(r_j - r_i\right)\left(\frac{d\varphi_j}{dt}\right)^2 - R_i S_A \left(\frac{d\varphi_j}{dt} - \frac{d\varphi_i}{dt}\right)^2\right) - \frac{d^2 f_{AB}(t)}{dt^2}$$

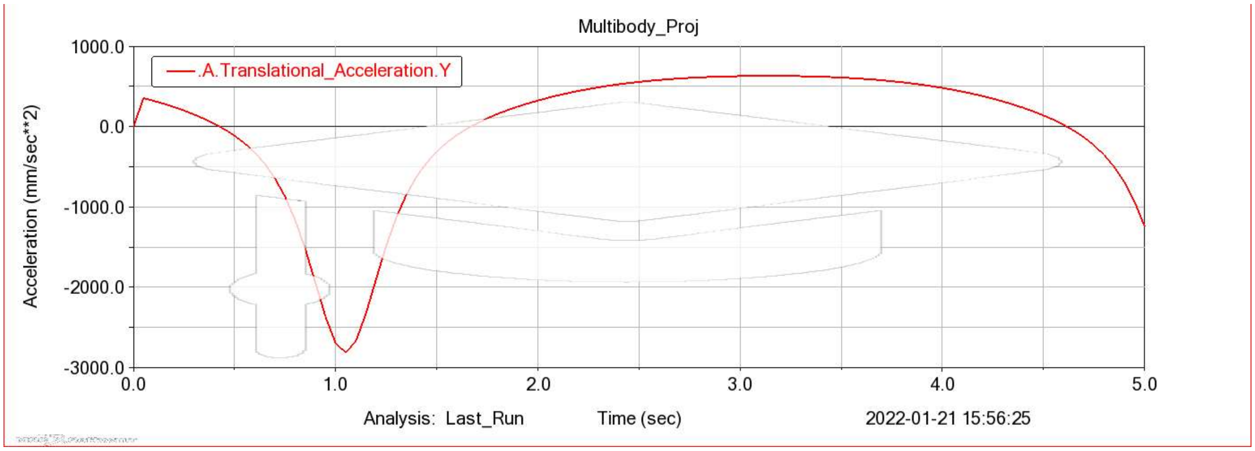
Once the Matlab script was functioning, the values were compared to the ADAMS solution.

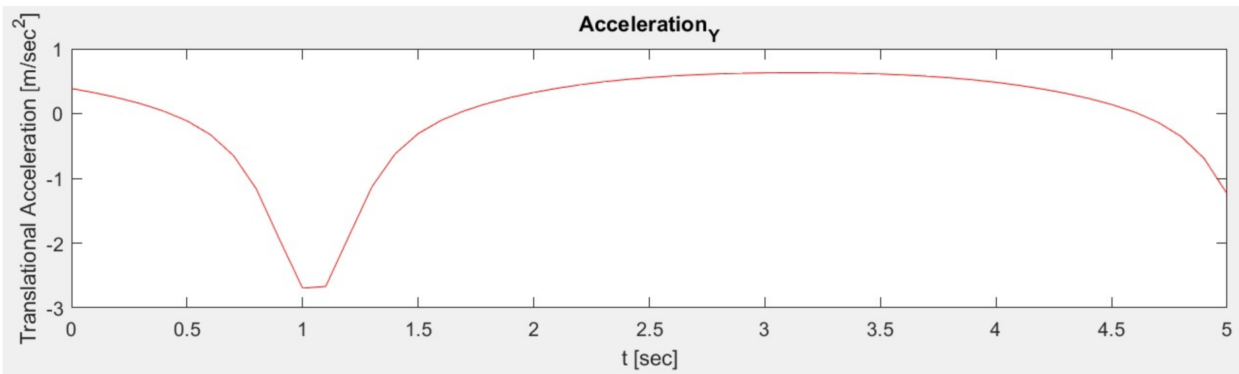
The following figures are for motion in the Y direction of point "A" (which was also the local frame $\pi_1$) for translation, velocity, and acceleration:

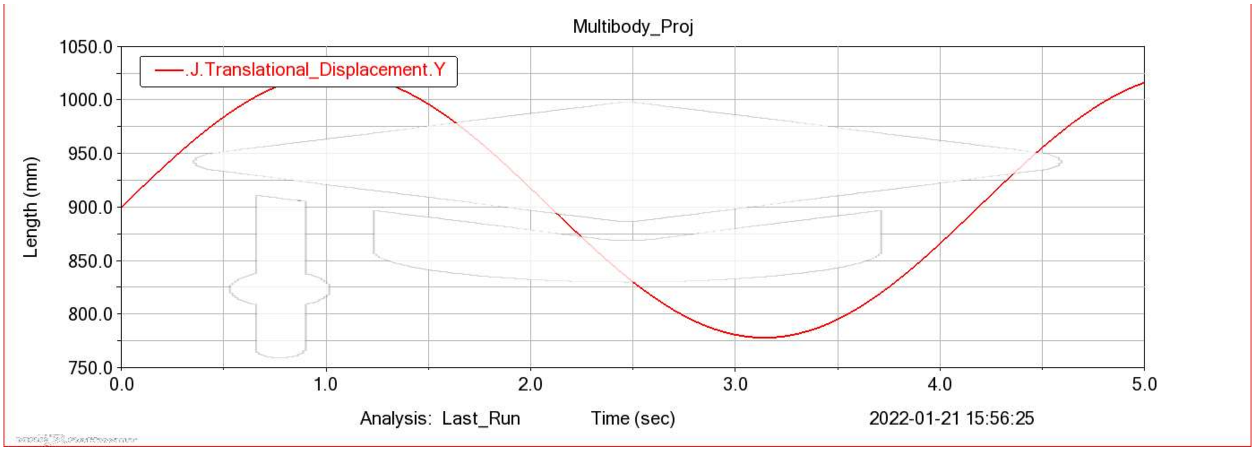

Multibody_Proj
.A.Translational_Velocity.Y
Velocity (mm/sec)
1000.0
500.0
0.0
-500.0
-1000.0
0.0
1.0
2.0
3.0
4.0
5.0
Analysis: Last_Run
Time (sec)
2022-01-21 15:56:25

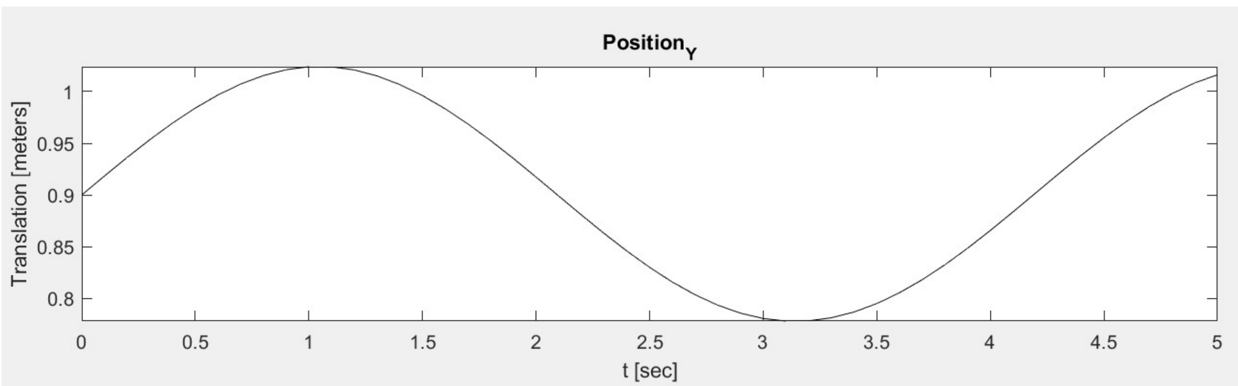

Velocity$_Y$
Translational Velocity [m/sec]
1
0.5
0
-0.5
-1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t [sec]

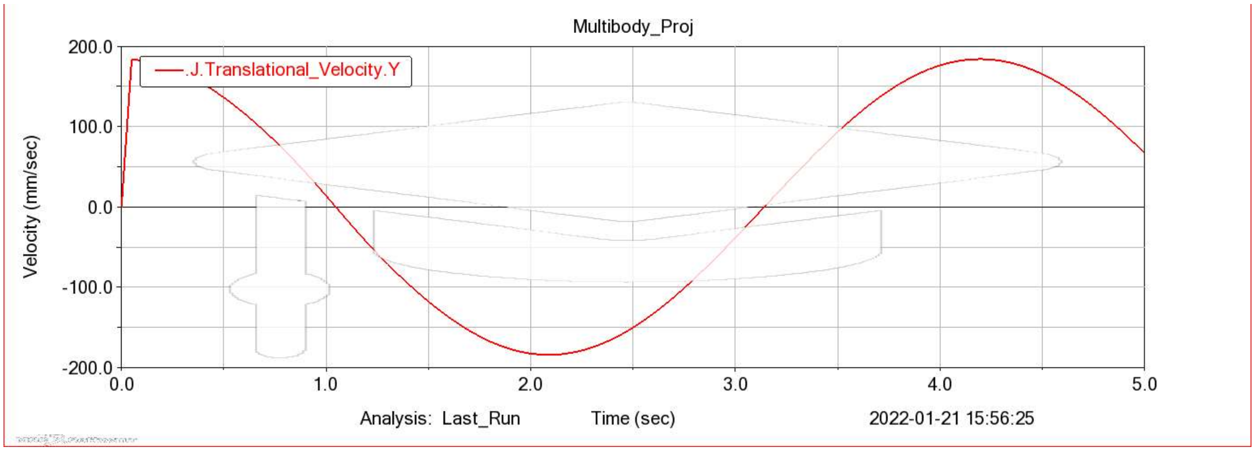

Multibody_Proj
.A.Translational_Acceleration.Y
Acceleration (mm/sec**2)
1000.0
0.0
-1000.0
-2000.0
-3000.0
0.0
1.0
2.0
3.0
4.0
5.0
Analysis: Last_Run
Time (sec)
2022-01-21 15:56:25

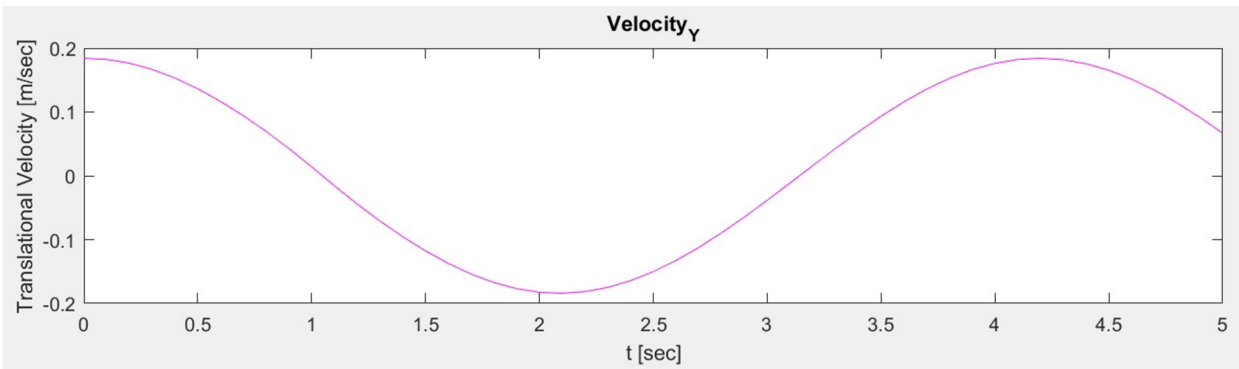

Acceleration$_Y$
Translational Acceleration [m/sec$^2$]
1
0
-1
-2
-3
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t [sec]

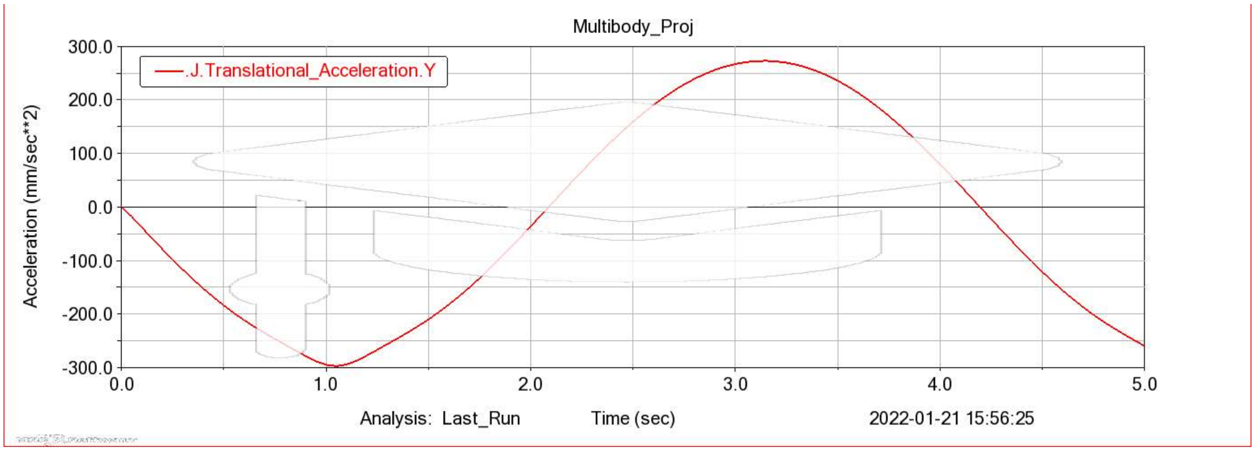
The following figures are for motion in the Y direction of point “J” (which was also the local frame $\pi_2$) for translation, velocity and acceleration:

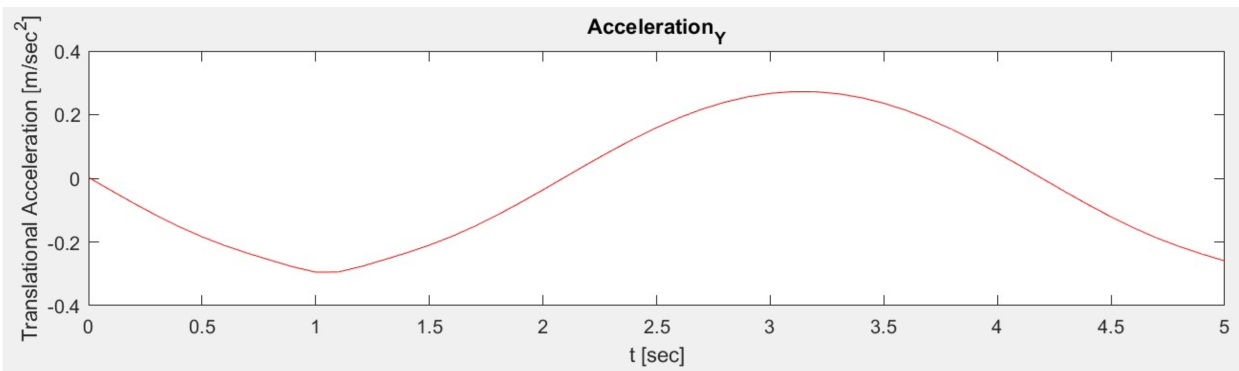



Multibody_Proj
.J.Translational_Velocity.Y
Velocity (mm/sec)
200.0
100.0
0.0
-100.0
-200.0
0.0
1.0
2.0
3.0
4.0
5.0
Analysis: Last_Run
Time (sec)
2022-01-21 15:56:25


Velocity_Y
Translational Velocity [m/sec]
0.2
0.1
0
-0.1
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t [sec]


Multibody_Proj
.J.Translational_Acceleration.Y
Acceleration (mm/sec**2)
300.0
200.0
100.0
0.0
-100.0
-200.0
-300.0
0.0
1.0
2.0
3.0
4.0
5.0
Analysis: Last_Run
Time (sec)
2022-01-21 15:56:25


Acceleration$_Y$
Translational Acceleration [m/sec$^2$]
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t [sec]