Corrections to Hawking radiation from asteroid-mass primordial black holes: Formalism of the exchange interaction

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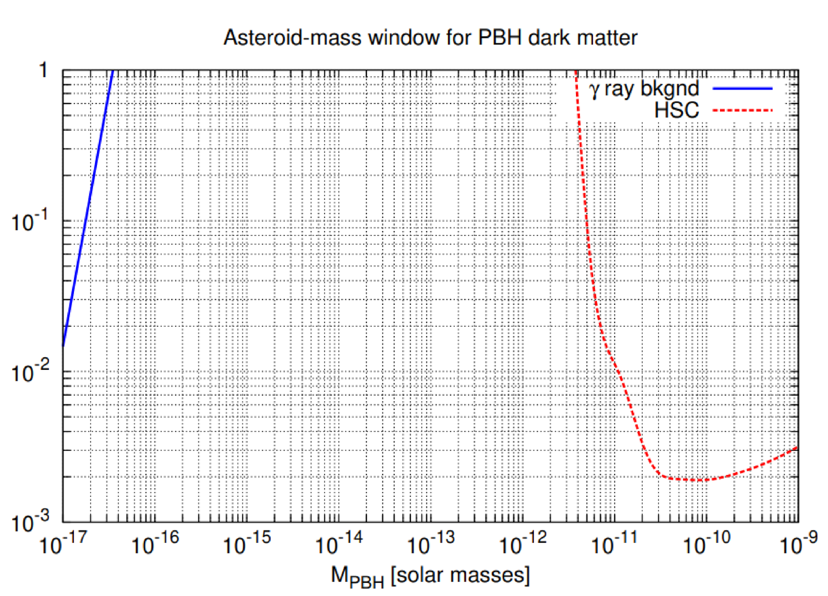
Background

This project forms part of an overall group effort to study asteroid mass primordial black holes (PBHs) as a dark matter candidate. Our group aims to produce projected spectra by which we could identify these objects if they were to be directly observed. While the 0^{th} order Hawking radiation spectra are known, the calculation of secondary interactions between particles emitted by PBHs has not yet been done in full Quantum Electrodynamics on a curved spacetime. The goal of the group is to do this calculation to first order in the fine structure constant (α) for an uncharged, non-rotating asteroid mass PBH.

The asteroid mass range includes PBHs with masses approximately between 10^{-17} and 10^{-9} solar masses. As shown below, this contains a window between the parameter space for which PBHs would have evaporated, and the parameter space for which they are detectable by microlensing observations.



Parameter space showing the observational constraints limiting what fraction of dark matter could be made up by asteroid mass PBHs, where the space to the left of the blue line is constrained by gamma (γ) ray background observations, and the space to the right and above the red line are constrained by microlensing observations. Figure credit: Paulo Montero-Camacho et al JCAP08(2019)031

Our group specifically aims to determine the photon, electron and positron spectra produced by PBHs in the 10^{-17}–10^{-16} solar mass range, since these are not too massive to produce positrons and electrons, and massive enough to make up a significant fraction of dark matter.

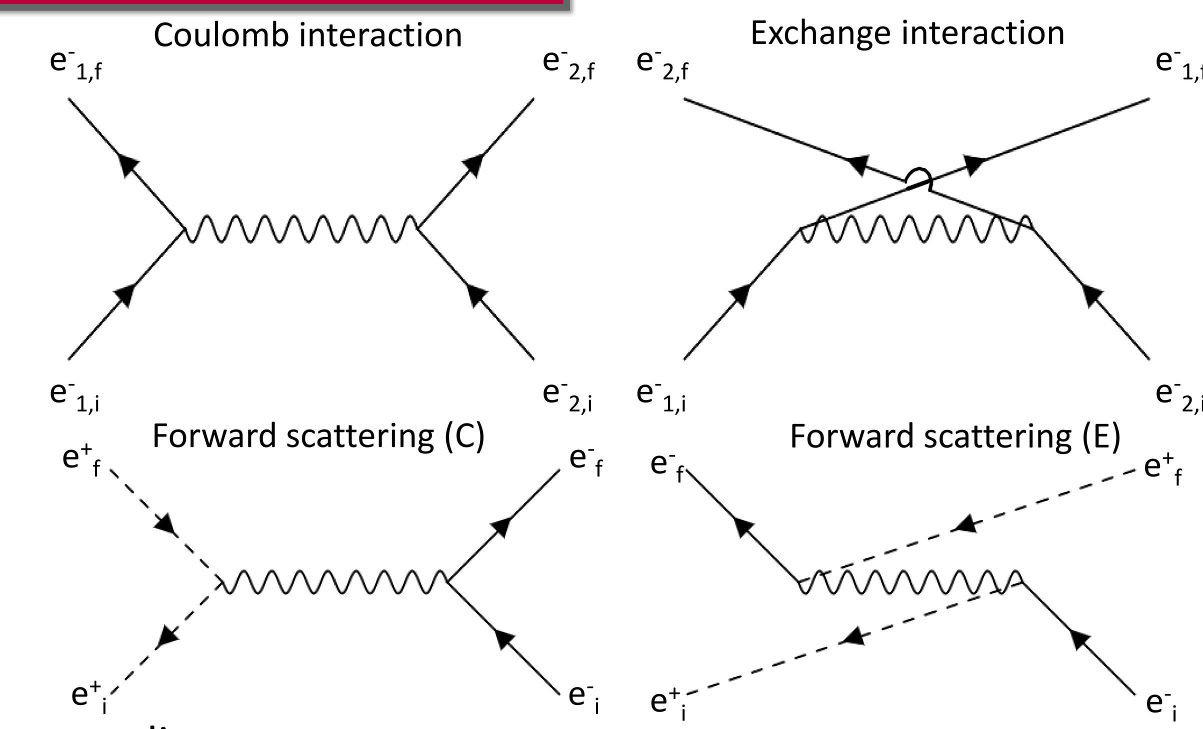
Our group has made significant progress in determining corrections for the dissipative interactions for the photon spectrum [1],[2], as well as determining the corrections due to stochastic charge emission for the electron/positron spectra [3].

This section of the project aims to study the exchange interaction terms for the electron/positron spectra.

Exchange Interaction

This part of the overall project focuses on the set of interactions where fermion number is conserved, characterized by two fermions interacting via a photon. There will be both Coulomb terms and exchange terms for the positron and the electron spectra. The Feynman diagrams for the electron case are shown on the right.

Certain terms in this calculation will also correspond to forward scattering (right), which is captured by the same mathematics in the Dirac sea picture. These terms will be common to both the positron and electron spectra.

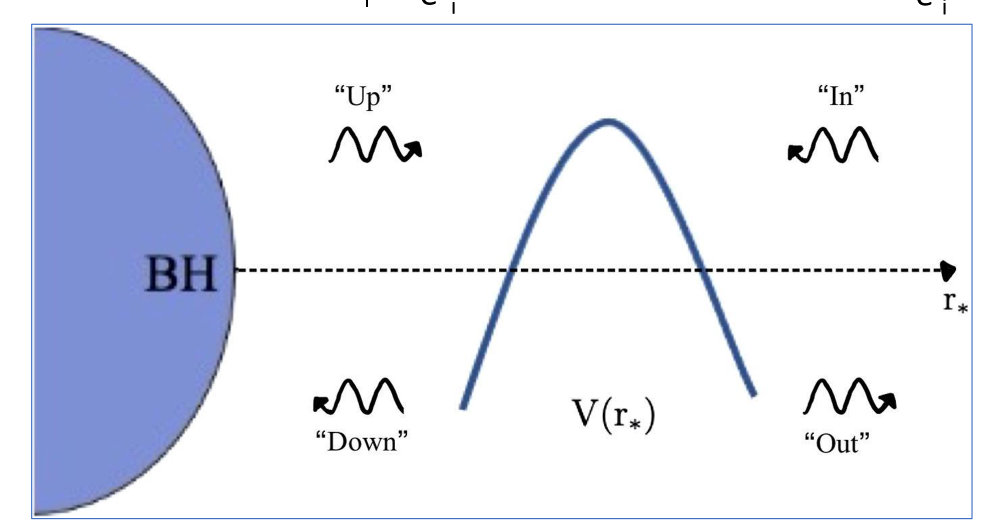


If this interaction occurs between particles in the plasma surrounding the PBH, it can change the transmission probability through the angular momentum barrier, and can therefore have an effect on the PBH electron spectrum that we would measure.

The electron spectrum correction is defined as the change in the number of electrons (N_{e^-}) per unit energy (h) per unit time (t). For this project, it is calculated as follows:

$$\frac{dN^{(1)}_{e^-}}{d\tilde{h}\,dt} = \sum_{\tilde{k},\tilde{m}} \frac{1}{2\pi} i \left\langle \left[H_\Phi, b^\dagger_{out,\tilde{k}\tilde{m}\tilde{h}}\, b_{out,\tilde{k}\tilde{m}\tilde{h}} \right] \right\rangle \qquad (1)$$

where H_Φ is the unperturbed interaction Hamiltonian between the electron field and the scalar potential field, and b† and b are creation and annihilation operators of the electron, respectively. The subscripts are explained with the help of the Fig 3 on the right.



A drawing that demonstrates the different bases: "up" and "in" are orthogonal, as are "out" and "down". Credit: [1]

As shown above, "out" electron modes are those which are past the angular momentum barrier moving out towards spatial infinity. k and m relate to angular momentum modes.

Project Overview

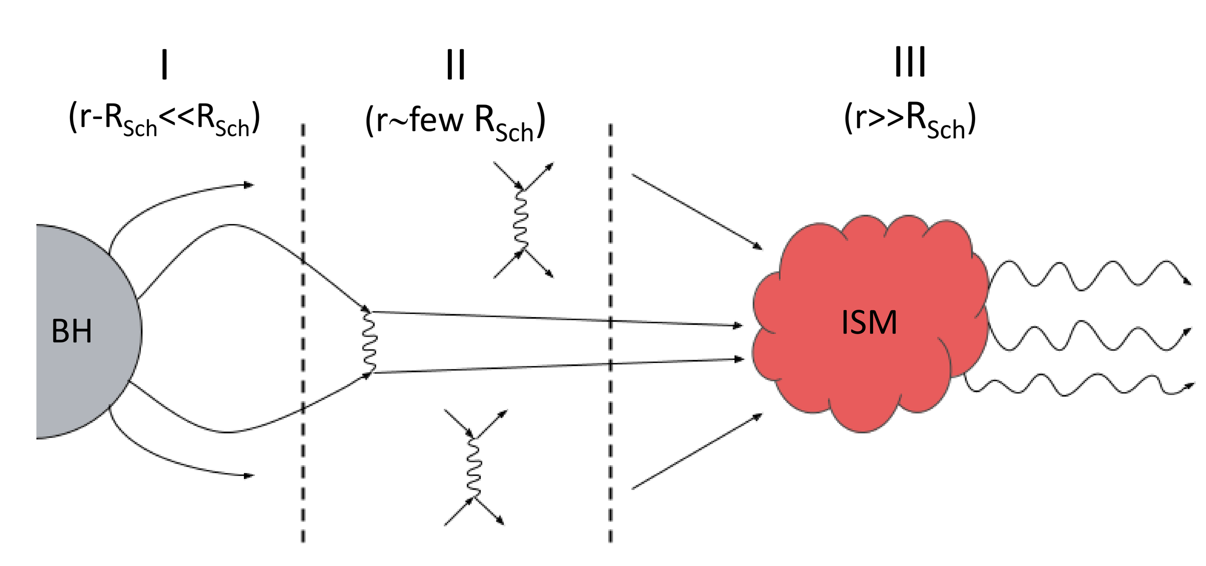
The interaction Hamiltonian introduced in equation (1) is defined in equations (2)–(4) on the right. In this project, these equations are solved formally as far as is possible, after which they will be evaluated numerically.

$$\Psi = \int_0^\infty \frac{dh}{2\pi}\frac{1}{\sqrt{2h}} \sum_{x,k,m} \frac{1}{r(1-\frac{2M}{r})^{\frac{1}{4}}\sqrt{sin\theta}} ([F_{x,k,h}(r_*)\Theta^{(F)}_{k,m}(\theta,\phi) + G_{x,k,h}(r_*)\Theta^{(G)}_{k,m}(\theta,\phi)]\hat{b}_{x,k,m,h}$$
$$+[G^*_{x,-k,h}(r_*)\Theta^{(F)}_{k,m}(\theta,\phi)) + F^*_{x,-k,h}(r_*)\Theta^{(G)}_{k,m}(\theta,\phi)]\hat{d}^\dagger_{x,k,m,h}]) \qquad (2)$$

$$D_{L,M}(r_*) = r^2\sqrt{1-\frac{2M}{r}} \int sin(\theta)d\theta d\phi Y_{L,M}(\theta,\phi) : \Psi^\dagger\Psi : \qquad (3)$$

$$H_\Phi = \frac{1}{2}e^2 \sum_{L,M} \int G_{L,s}(r_*,r'_*) D^\dagger_{L,M}(r_*) D_{L,M}(r'_*) dr_* dr'_* \qquad (4)$$

The diagram illustrates the region in which these calculations are performed. In region I (right by the event horizon) the PBH emits Hawking radiation in the form of electrons and positrons which are thermalized with the PBH horizon. As they are emitted, they form part of the plasma surrounding the PBH shown in region II. In this region, the emitted particles interact with one another, leading to changes in transmission probability through the angular momentum barrier around the PBH. Region II, therefore, is the region of interest.



Once particles are transmitted through the angular momentum barrier into region III, their interactions become outside of the scope of this project. At this point particles are streaming outwards from the black hole and can interact with the interstellar medium.

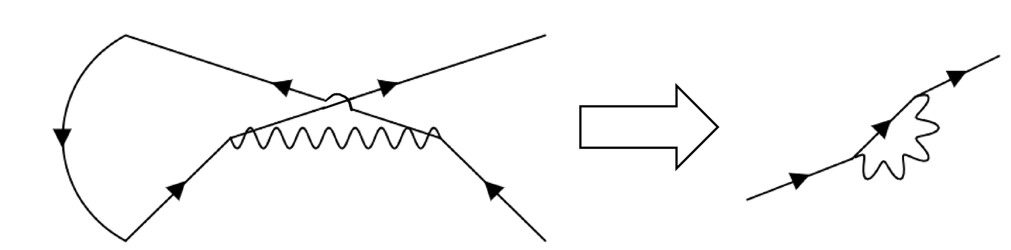
Results

The results are split into "Coulomb" terms and "Exchange terms" where forward scattering terms are classified as shown on the left.

We found that the Coulomb terms together have no contribution to the first order corrections to the spectrum:

$$\frac{dN^{(1)C}_{e^-}}{d\tilde{h}dt} = 0 \qquad (5)$$

Before the exchange term is formally evaluated, some terms are removed because they correspond to the following interaction, which forms a part of the electron self-energy interaction.



This interaction requires renormalization, which will be done later in this series of projects. Once these terms are removed, the first order corrections to the spectrum are as follows:

Short range Green's function

Transmission and reflection coefficients

3j-symbol (Related to Clebsch-Gordan coefficient)

F, G: Electron mode functions (Radial solutions to the Dirac equation)

$$\frac{dN^{(1)E}_{e^-}}{d\tilde{h}dt} = \frac{e^2}{16\pi^3\tilde{h}} \mathrm{Im}(\sum_{\tilde{k},k,L} \int \frac{dh}{h} dr_* dr'_* G_{L,s}(r_*,r'_*)$$
$$\frac{T^*_{\tilde{k},\tilde{h}} R_{\tilde{k},\tilde{h}} (2\tilde{j}+1)(2j+1)}{(e^{8\pi M\tilde{h}}+1)(e^{8\pi Mh}+1)} \begin{pmatrix} j & \tilde{j} & L \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{pmatrix}^2$$
$$((F^*_{in,\tilde{k},\tilde{h}}(r_*)G^*_{up,-k,h}(r_*) + G^*_{in,\tilde{k},\tilde{h}}(r_*)F^*_{up,-k,h}(r_*))$$
$$(G_{up,-k,h}(r'_*)F_{up,\tilde{k},\tilde{h}}(r'_*) + F_{up,-k,h}(r'_*)G_{up,\tilde{k},\tilde{h}}(r'_*))$$
$$-(F^*_{in,\tilde{k},\tilde{h}}(r_*)F_{up,k,h}(r_*) + G^*_{in,\tilde{k},\tilde{h}}(r_*)G_{up,k,h}(r_*))$$
$$(F^*_{up,k,h}(r'_*)F_{up,\tilde{k},\tilde{h}}(r'_*) + G^*_{up,k,h}(r'_*)G_{up,\tilde{k},\tilde{h}}(r'_*)))) \qquad (6)$$

Next, we will evaluate these corrections numerically for an appropriate set of PBH masses.

Related Work & References

For more on this project, see **Bowen Chen's poster** (246) now: *Corrections to the Electron Spectrum of Hawking Radiation from Asteroid Mass Primordial Black Holes: Formalism and numerical evaluation of dissipative interactions* and **Emily Koivu's talk** at 4:33PM: *First Order QED corrections to Hawking Radiation from Asteroid Mass Primordial Black Holes*

References

[1] Silva, M., Vasquez, G., Koivu, E., Das, A., & Hirata, C. M. (2023). Corrections to Hawking radiation from asteroid mass primordial black holes: Formalism of dissipative interactions in quantum electrodynamics. Physical Review D, 107(4), 045004.

[2] Koivu, E., Kushan, J., Silva, M., Vasquez, G., Das, A. and Hirata, C.M. (2025). Corrections to Hawking radiation from asteroid-mass primordial black holes: Numerical evaluation of dissipative effects. Physical Review D, 111(4), 045011.

[3] Vasquez, G., Kushan, J., Silva, M., Koivu, E., Das, A., & Hirata, C. M. (2024). Corrections to Hawking radiation from asteroid-mass primordial black holes: description of the stochastic charge effect in quantum electrodynamics. arXiv preprint arXiv:2407.09724.