Interaction of 2 Massive Mesoswimmers at Low Reynold's Number

Christian Neureuter[1]

[1] *Friedrich-Alexander-University Erlangen-Nürnberg (FAU)*

February 26, 2022

Abstract

Micro- and Meso-scale swimmers at low Reynold's number cannot, according to the *Scallop Theorem*, use a reciprocal stroke to produce locomotion. However, when these swimmers have asymmetric mass distributions, it breaks the symmetry of the system and allows the swimmers to propel themselves with simple stroke motions. This project investigates the interaction of 2 massive mesoswimmers. The intertial effects of the swimmers and fluid produce a strong interaction, but one that is highly dependent on distance. Through numerical simulation, it was found that the interactions are strongly dependent on configuration and distance, covering multiple regimes, from inverse linear relationship to inverse square and cubic dependant interaction strength.

1 Introduction

The scallop theorem says that at low Reynolds number ($\ll 1$), without the intertia terms in the Navier-Stokes equation, any reciprocal motion — that is, any motion that is the same forward in time as it is backward — cannot produce locomotion [5]. The name comes from the example of a scallop, which, with its single hinge, generates its locomotion by a swimming motion that is time reversal invariant and thus cannot go anywhere in a low Reynolds number fluid [5]. At the microscale, the inertial forces of fluids are more dominant, which leads to new, non-Stokesian fluid dynamics. However, for a "dumbbell swimmer" (2 masses on a spring), even with a time reciprocal stroke, an asymmetry in the masses of the beads breaks the symmetry of the system and allows them to self-propel [1]. This comes about due to the difference in coasting times (inertial interaction of the beads and the fluid) [6].

In our case, we use a force free model which simply means any applied force on one of the beads, has to be applied equally and in the opposite direction to the other bead. Here, the force of the spring, while pulling one bead, simultaneously pushes the other. This limits us to one degree of freedom which, given the time-reversibility and linearity of the Stokes equation, means that we cannot have a time-reversible stroke of the beads. However, with unequal bead masses, we introduce an asymmetry of the inertial interactions between the beads and fluid which allows a reversible stroke to generate locomotion [1].

Recent research has proven the validity of these "massive mesoswimmers"; that their inertial effects dominate those of the fluid allowing for self-propulsion [1, 3, 4]. Our goal is to look at things a step further, and explore the interaction of 2 of these swimmers in a low Reynolds number fluid. We aim to explore the interaction — one mediated by hydrodynamic interactions — of the two swimmers and their effects on locomotion.

2 Methods

For our dumbbell model, we use a matrix differential equation which encodes all of the parameters, and from which, we can obtain the equations of motion for the beads (or that of their average) [1].

$$\frac{\partial \mathbf{x}}{\partial t} = \hat{M}(x)\left(\mathbf{F}(t) + \mathbf{G}(x) - \hat{m}\frac{\partial^2 \mathbf{x}}{\partial t^2}\right) \tag{1}$$

Here the hat denotes a matrix and the bold indicates a vector. The $\mathbf{x}$ vector is the location of each bead (a concatenated vector of (x, y)), $\mathbf{F}(t)$ is our sinusoidal driving function, $\mathbf{G}(x)$ is the representation of Hooke's law of our spring, $\hat{m}$ is a mass matrix for the two beads, and finally – and perhaps most importantly – $\hat{M}(x)$ is the mobility matrix. This encodes the hydrodynamic interactions at low Reynolds number. We use an approximated version known as the

Oseen tensor. The Oseen tensor is the Green's function of the Stokes equations. The Oseen tensor can be considered as the first-order term in a far-field expansion which does not include the effects of rotations of the beads [6].

Although a single massive dumbbell style swimmer may be solved analytically, under the right assumptions; since we aim to extend it to 2 swimmers and explore their interactions, we will be using numerical methods for solving equation (1). To this end, we use Mathematica to solve the differential equation.

The first step is to break down equation (1) and understand the elements to be programmed. As stated, this is a matrix equation with the Oseen tensor ($\hat{M}$) being constructed from elements

$$\hat{M}_{ii} = \frac{1}{6\pi\eta a_i}\hat{1} \tag{2}$$

$$\hat{M}_{ij} = \frac{1}{8\pi\eta|x_i - x_j|}\left(1 + \frac{(x_i - x_j)\otimes(x_i - x_j)}{|x_i - x_j|^2}\right)\hat{1} \tag{3}$$

where a is the bead radius, η is the fluid viscosity, and $\hat{1}$ is the 3x3 identity matrix [1].

Similarly, F and G are constructed from

$$F_{ij} = F\cos(2\pi f t)\frac{x_i - x_j}{|x_i - x_j|} \tag{4}$$

$$G_{ij} = -k(|x_i - x_j| - L)\frac{x_i - x_j}{|x_i - x_j|} \tag{5}$$

where f is the frequency of oscillation, k the spring constant, and L the natural length of the spring [2].

Finally, the mass matrix looks like

$$\hat{m} = \begin{pmatrix} m_1\hat{1} & 0 \\ 0 & m_2\hat{1} \end{pmatrix} \tag{6}$$

Once these definitions and equations are put into Mathematica, we are able to numerically solve (1) and obtain solutions for the positions and velocities of the beads.

Much care has to be taken when expanding these matrices into multiple dimensions as their elements go from scalars to 2x2 and 3x3 matrices themselves. Similar care has to be taken for adding swimmers, since they have interaction matrix elements for each bead.

3 Results

First I simulated one swimmer and used the same parameters as [1], ($r_1 = 5, r_2 = 8, \eta = 1/6, k = 1/200, L = 28, \rho_{swim} = 8$, given in Boltzmann lattice units) so that I could verify my results against theirs. As seen in Figure 3, I was able to successfully compare my numerical results to the analytical model presented by [1, 2]. Once I knew that my implementation was correct, I proceeded to expand the system from a 1D to 2D system. Figure 1 shows that the 1D and 2D cases are identical and hence I successfully extended my code into 2 dimensions. I then continued to implement the second swimmer, first in a pseudo-1D manner for debugging purposes but then to a full 2D 2 swimmer model.

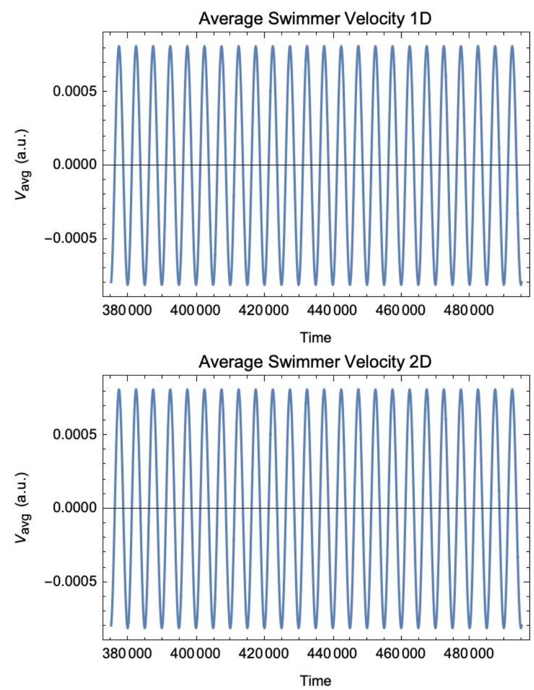


Figure 1: Comparison of 1D and 2D swimmer velocity.

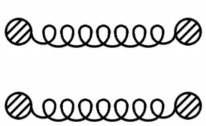
To examine the effect of the interaction of the swimmers specifically, I used the relative velocity of each swimmer compared to the one swimmer case.

$$v_{rel} = \frac{v_i - v_0}{v_0} \tag{7}$$

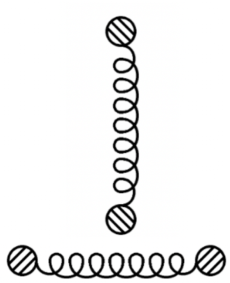
Where v_i is the given swimmer and v_0 is the single swimmer's velocity — simulated with the parameters given by [1]. A positive v_{rel} indicates an increase in the swimmers speed compared to the single swimmer case while a negative value indicates they are slower.

According to recent research [4], microswimmers have been shown to be highly dependent on their size, shape, configuration, and other starting parameters. All of the preliminary simulations were done in a parallel side-by-side arrangement. In Figures 4-6, I investigated the dependence on distance and relative phase difference between the swimmers in various configurations (seen in Figure 2). In Figures 4-6 it is clear that there is a strong dependence on distance and phase. To get a better visual representation, the relative velocity vs distance is shown on a double logarithmic plot, revealing that there are various regimes for each configuration where the interaction is inverse linear, quadratic, cubic, or something else. Primarily though, they all seem to have a sizable region of inverse square dependence especially in the mid-range distances from 1-10 swimmer lengths (L). Interestingly, there seems to be a greater dependence on the configuration and phase when it comes to the sign of v_{rel}, however there are still regimes seen in Figures 4-6 where the sign changes with distance indicating that at certain distances the 2 swimmers help speed each other up, while at other distances they slow each other down. This could be due to the types of fluid dynamic interactions going on and the scale of the fluid's molecules in respect to the swimmers' size and distances. The phase dependence has been investigated in [7] where they described the swimmers as not only influenced by passive effects of the fluid flow field but also by what they term active interactions. According to [7], the strong phase dependence in these simulations indicate active interactions wherein the driving frequency of one swimmer causes periodic fluctuations in the flow field that affect the second swimmer; the sign and magnitude of this effect is then governed by the relative phase difference between the two swimmers.

(a) Parallel side-by-side



(b) Parallel above



(c) Perpendicular

Figure 2: The swimmer configurations examined.

Finally, through the course of investigating the relative velocities of the swimmers in these various configurations, it was noticeable that there were configurations and parameters that slow down the swimmers. In turn, I ran through various parameters in the most synergistic and antagonistic configurations; this data is shown in Figures 7 and 8. The most optimal setup found was a parallel above configuration with ($\rho_s = 10, R = 1, f = 2.198\text{E} - 3, \phi = 0.9\pi$), while the worst setup was found to be a perpendicular configuration with ($\rho_s = 10, R = 12, f = 2.0\text{E} - 2, \phi = 0.1\pi$) – where ϕ is the relative phase difference between the swimmers. The swimmer can optimally septuple ($v_{rel} = 7.8$) their speed when in a favorable setup, while nearly stopping each other ($v_{rel} = -0.99$) in the worst setup, showing that these microswimmers are extremely dependent on configuration and starting parameters. It was suspected that this was the case but our investigation sought to quantitatively analyze the system. It should be noted that after the interaction dependence on distance was examined, all successive calculations were done at 1L distance to keep that variable constant wile examining the effects of the other parameters.

4 Conclusion and Outlook

This project investigated the interaction of 2 massive meso-scale swimmers. The intertial effects of the swimmers and fluid produce a strong interaction, but one that is highly dependent on distance and phase. There are various regimes in the interaction plot but there does seem to be a considerable inverse square dependence on distance for each configuration. Interestingly, through the course of the investigation, it was found that the interaction of the swimmers need not always be beneficial. In certain configurations and with certain feasible starting parameters, the interaction between swimmers can be almost entirely antagonistic and even stop the swimmers locomotion.

Parameters were varied in order to discern their relative effects in different configurations. From these parameter sweeps, we discovered that mass—-rather than working to speed-up the swimmers, as previously expected—-merely amplifies the effect of the given configuration. To expound, in a synergistic configuration like the parallel side-by-side swimmers, increasing mass increases the relative velocities by way of amplifying the strength of their interaction. Meanwhile, in an antagonistic configuration, such as perpendicular swimmers, as mass increases relative velocity actually decreases, seemingly in opposition to our original expectation. However, with a change of perspective it is easy to deduce that rather than increasing velocity, as it does for a single swimmer, that mass in a 2 swimmer environment, more accurately,

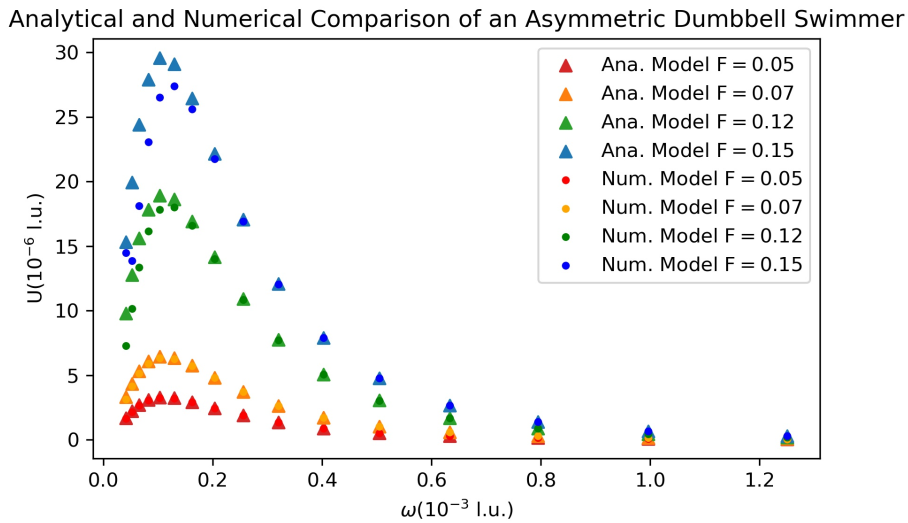


Figure 3: Swimmer speeds at various frequencies and forces, analytical model from [2] compared to numerical simulation of eq (1)

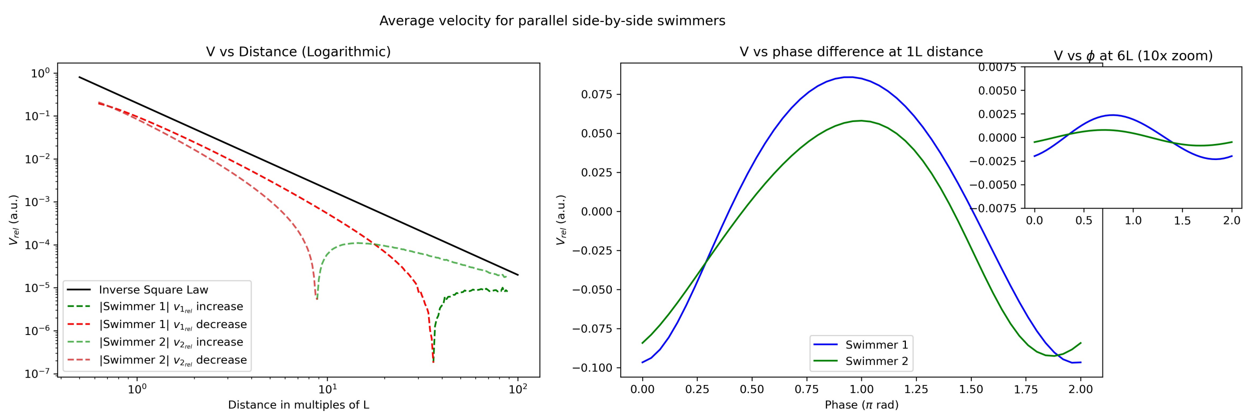


Figure 4: Horizontally parallel swimmers' interaction, phase and distance, dependence. Parameters from [1]

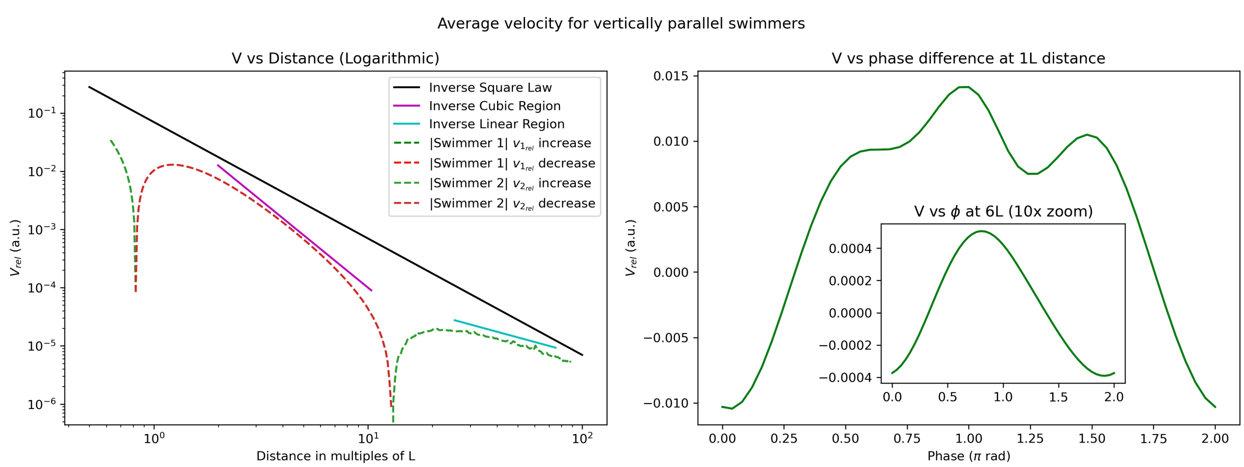


Figure 5: Vertically parallel swimmers' interaction, phase and distance, dependence. Parameters from [1]

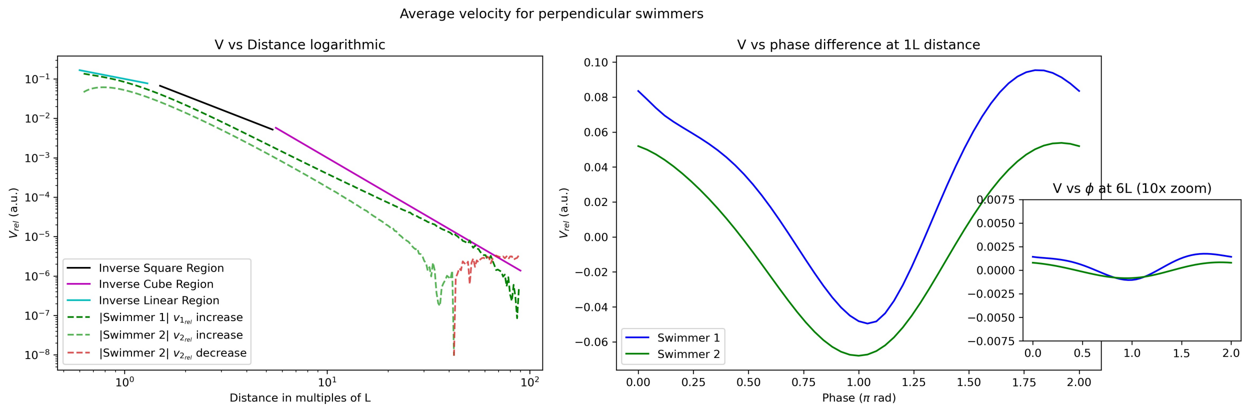


Figure 6: Perpendicular swimmers' interaction, phase and distance, dependence. Parameters from [1]

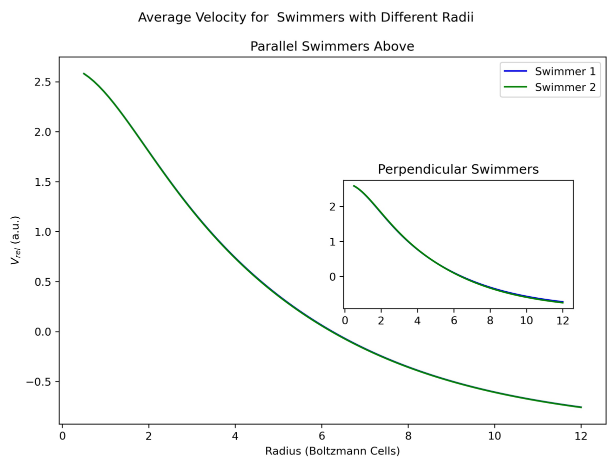


Figure 7: Effect of radius on relative velocity for 2 swimmers, done at 1L distance.

magnifies the interaction between the swimmers.

The overarching theme is that micro- and mesoscale swimmers, which live in a low Reynold's number environment, strongly interact with one another. That interaction, though, is highly dependent on configuration and starting parameters. The system is also very sensitive to changes in parameters. We still don't know the full extent of these interactions, nor their exact dynamics. Future considerations and research should look into a more detailed investigation of the parameter space of swimmers and the extension of these interactions to many swimmers. Does the effect still have regimes of inverse square dependence? How do multiple swimmers interact if some are in synergistic configurations while others, antagonistic? Finally further investigations into theoretical fluid dynamics that mediate these interactions should be researched.

Supplementary information

The code used to produce the presented results is supplied at `https://github.com/cneureuter/Massive-Mesoswimmers-ACP2-Project`.

References

[1] M. Hubert, O. Trosman, Y. Collard, A. Sukhov, J. Harting, N. Vandewalle, and A.-S. Smith. Scallop theorem and swimming at the mesoscale. *Phys. Rev. Lett.*, 126:224501, Jun 2021. doi: 10.1103/PhysRevLett.126.224501. URL `https://link.aps.org/doi/10.1103/PhysRevLett.126.224501`.

[2] M. Hubert, O. Trosman, Y. Collard, A. Sukhov, J. Harting, N. Vandewalle, and A.-S. Smith. Scallop theorem and swimming at the mesoscale si. *Phys. Rev. Lett.*, 126, Jun 2021.

[3] A. Najafi and R. Golestanian. Simple swimmer at low reynolds number: Three linked spheres. *Phys. Rev. E*, 69:062901, Jun 2004. doi: 10.1103/PhysRevE.69.062901. URL `https://link.aps.org/doi/10.1103/PhysRevE.69.062901`.

[4] J. Pande and A.-S. Smith. Forces and shapes as determinants of micro-swimming: effect on synchronisation and the utilisation of drag. *Soft Matter*, 11(12):2364–2371, 2015. doi: 10.1039/C4SM02611J. URL `http://dx.doi.org/10.1039/C4SM02611J`. Publisher: The Royal Society of Chemistry.

[5] E. M. Purcell. Life at low Reynolds number. *American Journal of Physics*, 45(1):3–11, Jan. 1977. ISSN 0002-9505, 1943-2909. doi: 10.1119/1.

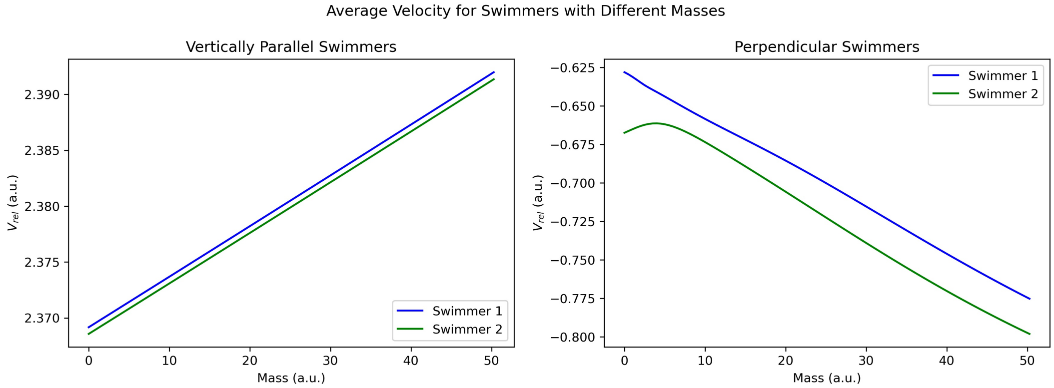


Figure 8: Effect of mass on relative velocity of 2 swimmers in different configurations, done at 1L distance.

10903. URL `http://aapt.scitation.org/doi/10.1119/1.10903`.

[6] M. Reichert. *Hydrodynamic Interactions in Colloidal and Biological Systems*. PhD thesis, Universitat Konstanz, July 2006.

[7] S. Ziegler, T. Scheel, M. Hubert, J. Harting, and A.-S. Smith. Theoretical framework for two-microswimmer hydrodynamic interactions. *New J. Phys.*, 23(7):073041, July 2021. ISSN 1367-2630. doi: 10.1088/1367-2630/ac1141. URL `https://iopscience.iop.org/article/10.1088/1367-2630/ac1141`.