final_test

2024-05-22

```
library(tidyverse)
```

```
## Warning in system("timedatectl", intern = TRUE): running command 'timedatectl'
## had status 1
```

```
## -- Attaching core tidyverse packages ------------------------ tidyverse 2.0.0 --
## v dplyr     1.1.4     v readr     2.1.5
## v forcats   1.0.0     v stringr   1.5.1
## v ggplot2   3.5.1     v tibble    3.2.1
## v lubridate 1.9.3     v tidyr     1.3.1
## v purrr     1.0.2
## -- Conflicts ------------------------------------------ tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
```

```
library(igraph)
```

```
##
## Attaching package: 'igraph'
##
## The following objects are masked from 'package:lubridate':
##
##     %--%, union
##
## The following objects are masked from 'package:dplyr':
##
##     as_data_frame, groups, union
##
## The following objects are masked from 'package:purrr':
##
##     compose, simplify
##
## The following object is masked from 'package:tidyr':
##
##     crossing
##
## The following object is masked from 'package:tibble':
##
##     as_data_frame
##
## The following objects are masked from 'package:stats':
##
##     decompose, spectrum
##
## The following object is masked from 'package:base':
```

```
##
##     union
```

```
#install.packages('igraph')
library(igraphdata)
library(Matrix)
```

```
##
## Attaching package: 'Matrix'
##
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
```

```
library(lattice)
library(vcd)
```

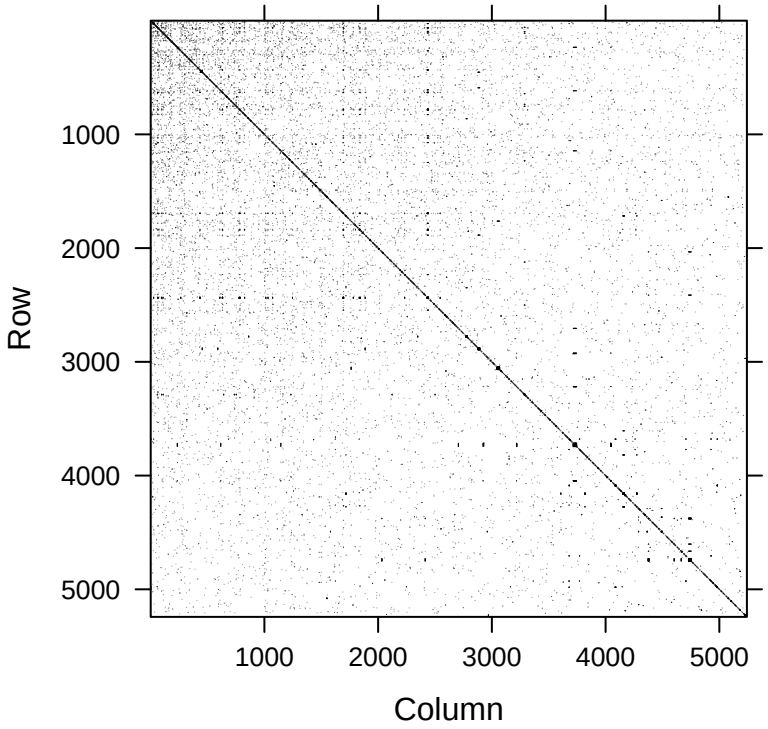
```
## Warning: package 'vcd' was built under R version 4.3.3
```

```
## Loading required package: grid
```

```
file_path <- "./data/ca-GrQc.txt"
# Load the dataset
edges <- read.table(gzfile(file_path), comment.char="#", sep="\t", col.names=c("from", "to"))
max(edges)
```

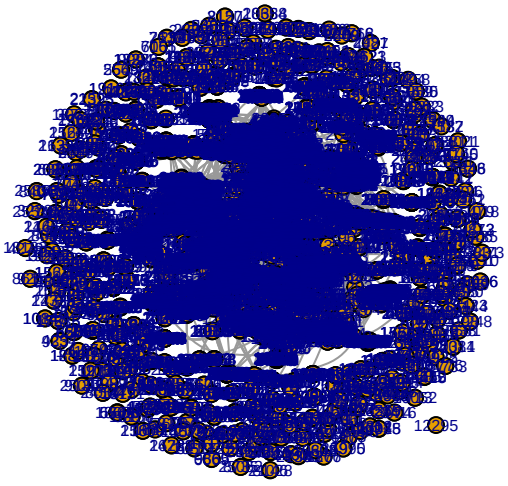
```
## [1] 26196
```

```
# Create a graph object from the edges
grqc <- graph_from_data_frame(edges, directed=FALSE)
grqc <- simplify(grqc, remove.multiple=TRUE, remove.loops=TRUE)

grqc.adjacency <- as_adj(grqc)

image(Matrix(grqc.adjacency))
```



Dimensions: 5242 x 5242

```
plot(grqc,
     vertex.size = 7,
     vertex.color = V(grqc)$color,
     vertex.label.color = V(grqc)$label.color,
     vertex.label.family = "Helvetica",
     vertex.frame.color = "black",
     vertex.label.cex = 0.5,
     edge.color = "gray60",
     edge.arrow.size = 0.2,
     edge.width = E(grqc)$weight * 0.5,
     edge.curved = 0.5,
     mark.col = "gray90",
     mark.border = "pink")
```



```r
min(degree(grqc)) # make cutofs - graph all nodes with mre than 60 edges and then graph nodes with 30-5
```

```
## [1] 0
```

```r
graph.density(grqc)
```

```
## Warning: `graph.density()` was deprecated in igraph 2.0.0.
## i Please use `edge_density()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

```
## [1] 0.001054405
```

```r
transitivity(grqc)
```

```
## [1] 0.6298425
```

```r
average.path.length(grqc, directed = FALSE)
```

```
## Warning: `average.path.length()` was deprecated in igraph 2.0.0.
## i Please use `mean_distance()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

```
## [1] 6.048515
```

```r
diameter(grqc)
```

```
## [1] 17
```

```r
farthest_vertices(grqc)
```

```
## $vertices
## + 2/5242 vertices, named, from 7bb4bfd:
## [1] 20255 22190
##
## $distance
## [1] 17
```

```r
degree(grqc)
```

```
##  3466 10310  5052  5346 15159 19640 10243 18648 16470 17822 14265 19738  8612
##     8    13    29    20     4    25    14     4     1     1    37     5    18
## 10822 16258 21194 14123  2710 18757 16148 10794  7050  4846   824  2133  6610
##     2    13     1    13    33     8    18     2     6    22    13    12    68
##  6700 11082 14419 17330 18487 22779 23382 12928 13740 13096 22393  3872 23096
##    31    12    14    17    27    11    30    11    11    22     5     8     1
##  8862 22598  8254 17309 24833 10115 23916  7893 18051  4515 16778  8978  9017
##     7    18    13     1    17     2     1     9    10     8     8     4    28
## 15170 15455 16589  7510  8851   624  2654 10130 16032  3937 21012 22691 19215
##     4    17     5     2    11    17    38    15    16     4    81    77     2
##  2980 14376  9522  9572 17394 18924  3441 10268  7188 18866  4870  5175  8282
##    24     1     5    34     1     8     6     7    12    44     3     6     3
## 22046  8157  2120  7713 19052  8302 16484 17778 21699  8701 17379  6858  8148
##    11     1     1     1     1     1     1     5     3    10    14     3     3
## 15366  9360 11102 25758  9755 14344 24163 12691  3420  7442  7768  4624 15770
##     3    10     5    51    31     3     2     2     3    18    10    12     5
```

```
## 23099 14337 15799 11241 25271 25396 16994 21776  3345  8868 11077 17162 21389
##     2     9     9    49    16    23    20     5    15     1    14     2     5
##  6895 12968 15144 19974  8708 23776 13597 20303  3032 22415  4700  9124  4046
##     1    10    13     4     1     1    17    10    12    12     2    24     1
## 21806  3409 18001 22177 25480  8916 12334 13713  2949  7042 10113 11661 13621
##     6    16     2    21     7    11    13     2     9     4    12     8     1
## 25388   284  6427  1044  5809   811 15123 16083  2127 11712  3412 10762 13847
##     1     1     6     8     2     9     3    10     1    12     3    30     7
## 17207  7689 18457 17228 20207 17559 19206 13813  7504 12212  1545 10791 10919
##    10    32     1    12    10    13     3    13    11    15    14    15     2
## 10588  5729  1310 23186 25931  9710  7007 17536 22476  5233 17932  1000 23107
##    12     3    11     6    10    20    25    10     7     2    11    25     9
## 23939  5464 13653 26038 23576   580  1877  2042 15880 16101 20014  2846 18375
##    17     6     1    15     4     2    10    17     9    10     6    16     1
##  8335 13529  6494  6857 24781 14308 20787 23689 25143 22184  5597 23836  6355
##     9     1     7    10    10     7     6     3     2     7    12    15     2
##  7194 20960 23066 14500  3765  1508  8279  2004 21191 21663 14020 16543 16563
##     5     2     3     2     7     3     5     1     1     2     5     8     4
##   238  8887  6072   491  1818 17082 17039  3547 14707 14818  6804 19624   214
##     4     1     1    11     3    10     3    13     8     5     7     4     7
##  6512 10590 18904  2190 11801 23441   449 10358 10615 12074  3099  4254  9337
##    49     7     5     4    11    11    30     3     8     6     5     1     1
##  1497  6724  2558 15538 21910  8968 12260  4241 24924  4697 22082 13276 15409
##     2     8     2    14    11    10     2    18    22     5     5    26     2
##   639  1153 17501 16834   570 19657 26190  1727 19992  9417 24009 15961 14316
##     7     1    13     2    48    10     5     7    15     2     9     1     3
## 14690 13008  4550 13142 24330 15666 15205 24640 15415  8365 17075 23665    78
##     8    19    21    28    20     3     4     3     1    12    19    12     4
##   140 24122 18365  1248 16331 17843  4952  5060 20660 11447  7383 21432 13929
##     6     1    14     3     3     5    28     1     4     4    13    18    45
##  5136 13556  6934 20683 25419 21771 22644 10467 13175 21816 11640 14371 17443
##     2    20    14     7     5     7    12     3     2     2     2     1     6
## 16310 19517 22989 23708  5934 15066 16469  4383 13614 17245 10620  1880 15353
##    17     8     5    16    17    21    19     1    16     6     3     1     4
## 21048  8932  6709 18003 12135  3209  6863 24371  1833 12107  8177 14967 16741
##     6     1     1     2     6     2     2    22     2     2     7     5     7
## 19233 19297 20597  7542  7350  7712 19149 11275 14638 11566 11808 18560 13282
##     3    16     2     1    19    18     1    19     5     2     2     2     5
## 15249 15624  4766 12665  3310  7449 22778 23351 23943 17754 25674 16123 10841
##     1     1     2     1    16     3    10     9     4     2     1     5     7
##  1403  2368 20168  8503 18719 18867 23409 10942 23647 22119  3953  5672  3430
##     7     9    10    14    22     1     2    11    12     3     4     2     5
## 24431 16020 23986 16675 13859 16091  2081  1941 15846  2054 10355 13469 23137
##     8     7     7     1     8     3     1     7    10     5     1    12     2
##   896 21158  9629 25783 17038 19865 10134  7899 18109  3561  8231 24149 25934
##     5     3    12     7    17    26     1     5     5     9     1     1     2
## 14485 22254 21754 13635 12498  2870 22555 16368 11821 21469 23503 16429 21608
##    21     1    12     4     2    11    13    11     3     3     8     6     9
##  8476 10458  5674 13388 12739 18676  1711  6726 16931 21491 16414 25408  8378
##     1     7     1     2     7     9     8     1     8    20    21     4     1
##   135  6408  9889   773  9341 21847 14628 12842   215   185   351  1074  1858
##    11    12    20    55     4    48     2    15    17    17    17    17    17
##  3939  4512  6892  8280 14542 17751 18095 18143 18649 18943 19573 22601 22793
##    18    17    17    17    17    17    17    17    17    17    17    37    17
```

```
##  5835  6219 18160 24568  5555  7459  3804   293 25158 22436  9504 25215 13174
##     6     3     3     8     8    12    10     2     3     2     2     5     4
##   392 23344 10956 22899 23552 14093  1425  6832 18940 10235 24620 20230 22483
##     1     9     6     5    14     5     9     1    15    13     3     1     8
##  2926 22275 16159 14698  6075 20345 14499 10801 11902 23870  2607  9450  1217
##     2     6     4     1     3     2     2    12     4     5    11    10    24
##  2165  6033 23233  1172  8408 11493  5059 21530 11032 22074   379  3909  3593
##    13     6     8     8     8     3     1     2     2    10     2    17     6
## 16607  2752 18171  5413 25872 20148 19219  6094 24860 25948 11617  8774 25902
##     4     4     2     1     1    12     2     4     4     4     1     7     9
## 21823 10435  1765 20307  1194 21858  7829 19101  4252 24451 24270 25435  4868
##     3     3     8     2     1     1     4     5     1     2     7     6     6
## 25827 10912 11638 22836 13932   302 11490 13970 22028 11733 17285 19903 25469
##     2     3     2     3     1     2     5     1     5    13     6     1     7
##  2501 24559 10676 24961  7264  1736 14424 19462 12155  2569  5546  3228 15477
##    10    23     1     1    15     6     5     5     3     1     1     6     9
## 16640 23855 21635  1600 23175 25662  6023 19404 14346 10055 16230 20702 26005
##     7     8     2     2     2     1    11     6     1    17     1     3     1
##    70  4727 15559  4798 14627 17991  3875 17968  9094 19061  2080  4069  1968
##     2     4     5     6     5     2     1     6     1     1     2     1     1
##  9082 13741 25006 22530  3651  6891  4983 24183 23866 17392 14051  9829  8579
##     6     1    22     2    35     7     2     7     1     3     6    15     5
## 21206  5355 16770 21968  6556 21543 25863 25988  1595  6179  4382 14924 18408
##     1     2     3    13     3     7     1     2     1    46     1    23    10
##  2536 16041 24753 22620  2449  7586 20478 23038 12491 24503 24942  2119  7637
##     4     4    13     2    34     3    26    33    15     2     2     1     1
##  6468 11919  5287 17273  1997  2741 13060 14807 24955 16921 21646 10555 11591
##     6     4    10     5     8    65     9    60    51    12     9     6     8
##  5541  4583  8928  2420  5621  2789 12112 17440 18140 26098 10246 11891 19184
##     6     3     1     5    22     5     2     2    14     6     1     4     2
## 20879 20215 21333  7774 18008 19340   408 13834  7895  5503  5843 25877  7584
##    15     2     2     4     4    11     2     1     2     4     3     2     3
## 18097  6527   272  2912 11811 11840 15300 19012 10456 13320 10211  6030  5266
##     1     1     6    18     6     6    19     5     9     1    10     4     6
## 24113  8719 11696 18445 18511 22366 20229  4775 15784 17589 13384 22242  9483
##     1     1    19     1     9    19     2     2    11     1    16    10     2
## 12599 25250 10879 11925 13659 15082  6627 21433 17021  9066  4032  5194  1254
##     7     7     3     3     8     3     3     2     2     7     2     5     3
##  5385   197  3890 16056 15306   244 14952 16900 22246  3721 14781 17721  8925
##    13     2     9    12     1     7    12     3     3     2     2    10     3
## 20255  3743 13480 10351  6667 14562 25996  5953 23134  5230  7069 15435 20248
##     1     8     6     3     3     1     5    14    21     2     4    14     2
##  7863  7209  8547  6351  8552 17266  5712   339 11868 10711 17359 11442  5425
##     2     3     4     3     5     9    14    30     3    23     9     5     5
## 16224 13651 20886  3944 21927 25069 15718  3730  1588  5695  5901  4743  3006
##     2     2     2     3     4     2     1     3    19    17    27    25    10
##  2326 19048 26088  2770 14033 11806  5787 17403 20149  6266  8301 24240 10081
##     7    12     8     4     6     2     8     4     2     3     2     8     5
## 13291  7885 19551  4416 17414  7801 18521 18271 13702 17874    45  4511 23063
##     3     2     3    19     9     6     3     9     6     6    49    45     2
##  9616 11325 17746 25729 21281 23293 22917 20940 22879  2656 14560 24251 22832
##     8     6     6     4    79    53    11     6     5     2     2     2     2
##  4351 26171  6943 22748  3630  7715 16703 22253  5738 12042 11604   615  1490
##     9     1     2     2     8     2     2     2     6     1     5    18     1
```

```
## 20813 19781 19358  8505  4638  2115  4756  8666  9241 20792  4472  3927  6303
##     1     8     2     2     1    17     3     9    13     3     4     2     2
## 19711 20616 22574 11175 15552  9482 15003 23481 12291 24199 12032 15495 24970
##     1     2     9     9    18    21    62     5     1     6     5     4     6
##  9264 12614 19146 22265 22699 20001  2339 24594  5477 13497  1963 25125 23558
##     5     3     3    11     6     1     1     1     2     1     2     4     4
## 10524  5655 19022  3916 24614 12373  4199  6443 19932 25864 19936  8596 11863
##     2     8     6     8     7     2     4     4     4     4     6     2     2
## 12110 17944 20884  9471  2348  9994 18383 17276   178  7475 25316   523 20827
##     3     3     2    24     1     2     2     2     8     6     6     2     8
##  4211 11876 24121 22283 15824 21195 15305 15171 17864 23417 18517  3174 12720
##     9     5     4     1     1     4     6     4     4     4     9     1    13
## 12733  2455 14353 14886  2915 11426  5156  7483 13528   294  8626  3007 25537
##    10     2     2     4     3     3     5     1     2     1     1     2     3
## 11459  6706  2118  8428 10637 10983 22309  8134 12687  7615 15657  4115 12938
##     1     1     1     3     2    12     3     6    11     7     2     5     8
## 25836 17990 20003 21718 10719 18764 22433 11112 14864 23770  6183 23841  4822
##     8     3     4     6     4     3     3     4    10    10     5    10     2
## 18600 11700 24161 12141 21549 13931 17180 13675  4186 12616 16266  4485  1822
##     4     1     2     5     2     3     1     1     1     2     2     2     1
##  6503  6838 11093 17076 19018 15614 11899 20122 14282 11035 21864  3839  4284
##     5     7     2     1     2     7     1     4     2     1     3    15     2
## 10463 21379  5302  9643  8352 14004  4713  6824 24696  1840 12492  7154  8312
##     2    22    11     4     2     2     5     5    20     5     5    13     7
## 25112  7277  9517 20169  6524  5363 17122  2259 10317  9800  6448  1346 17161
##     1     9     2     2     6     4     1     1     2     9     1     1     2
##  2136  9591 17453 24372  1050 17120  9595  6744 10931 11607 19351 20765 19078
##     2    11     7     1     1     1     1     1    10     4     4    18     3
## 21089 15321  9125 15943  8678 12874  1154 12017 12930 12161 19162 23637 11293
##    11     5     2     2     2     6     2     2     4     1     1    10    13
## 11400 12587  3113 15960  7105 25286 25034 20373 12927 22790 20635 26067 15194
##    18    15    17     2     5    13    19    25    16     8    47     2     2
## 23909  3821 12545  6629  9974  8047  4013 22088 22224 12056 14599 15973 18549
##     1     1    22     1     4     2    10     1     5     1    30     2     1
##   743 23092 22457 15528 19423 14771 24127 23355 25180 24110  3677 25116 21697
##     9     3    30     4    63     1     1     3     2     7    12     3     3
## 17560 12689 12337 22826 12419  3507 19877 11626  4575 18250 23465   598 12758
##     3     3     2     5     3     3     5     2    19     2     1     1     3
## 25582  8045 15829 17979 17235  4650  6806 10600 13493 23394 16333 12287 23477
##     9    15     4     5     2     2     2     6     3     6     1     3     2
##  5225  9312  2490  3193  5134  3372 23452 23628  2404 22421  9283 12639 25916
##     6     3     5     7     7    49    22     6     8     6     7     2    10
##  3849   888  1520 13385  4017  7888   593 12627 21777 22108 10597 17149 23901
##     3     6     6     1     2     2    16    15     2     5     6     6     2
##  1498  3033  9893 20526   884 24699  3164 20892  3136 11108  1149 16234 18373
##     3     3     1     2     2     4     6     4     4     1     9     2     4
## 10251 23786 21450 25534  2287 15905 25486 17797  1488 13498 16261 25086 19445
##     5     2     5     2    14     1     3     1    31     1     1    10     1
## 16281 24728  2762 22378  2350 25405 25698  4819 22203 21658  1669 12703  3988
##     1     1     1     1     2     2    13     2     3     1     1     2     1
##  8200 12302 11964 16694 18340  6963 16696 12187 25225 11785 20595  3845 18453
##     2     2    13     9     1    12     2    10     3    21     9     1    36
##  1914 12070  1599 11052 26168 12654 15667   245 16976 10871  2556 15235  1350
##     4    10     1     6     3     8     3     2    11    14    11     8     1
```

```
##  5993  5246  8742    80 12101 20613 11746   676 20850  1787 13056 18875 20806
##    13     3     3     2    12    10     7    26    15     3    10     8     2
##  5444 15258  2072  7071 15951 10039 19457 14165 24731  3999  2340  2443  8116
##     2     6    16     3     5    11     1     8     1     1     3    13    20
## 16906 14430 21708 17724 25957 22771 10406 19723 15388 15387 16674  3955 10532
##     4    15     3     2     4     1     7     5     1     1    13     3    11
## 23470  4468 18783 25378  6390 11522 12235 22129 11495 17950  9634 25321  8530
##     6     2     2    11     1     4     6     3     3     1     3     2     2
## 21144 26170  9485  6154 15608 18540 14149 11616  7725  9306 13501  3096  7450
##     1     2     2    15     6    14     8     2     6    11     2    12     4
## 13955 18286 23967  5361  4896 13292  8876 10990 22190  4128 20511 20100 11279
##    12     7    15     1    10     1     3     1     1     1    32    12     2
## 17750 15600  7307 11047 14606 25850  2047  3632  6575 15383 24385 18894  5862
##     4     3    26     4     2     2     4     4    11     4    11    47     6
## 17113 19642 22188 22811 13190 14007  5542 11861 17396 22245 20432  9649 17626
##    12     2     8     2    11     4     1    17     1     1     5     4    16
##  4633 18415   482 26194  7625  1386  2720  7246 17240 22523 24002  4102 10376
##     1     5    13     2     6     3     3     2     2    10     1     9     5
##  5606  9207 20344 19204 23204 21017  5601  8710  2591  5083  8711  5084  9265
##     2     5     3    11    12     1     2     2     6     2     2     2     1
## 11372 11379 21860  4180 15959 21707  2316 11939   350   951 25676 15365  3034
##    17     8     8     6     9     3     3     3     2     2     2     5     2
## 19686 10895 20086 18037   925  8615 19131  8471 13483  5812 20618 17291 17426
##     3     8     2     3     1     1     1    12     1     8     3     2     2
## 24932 21324  8715  4467 11602  1685  8263 25528 23490 18983 11194  5849  8592
##     5     5     4     1     4     9     2     3     2     4     6     3     6
##  6172 14933  7204 10657 17172 16389  4068  5130 12722 24578 15998 20557  3171
##     6     4     6     2     8    26    13     4     7     4     6     1     1
## 20335 11125 23529 18579  7087  9984 25569 15419  4080 14746 19324 17331 22806
##     3     3     9    13    11     3    10     1     4    25     4     1     5
## 22951 22953  1407    26  9760  1620  7039 12166 11223    95 15524  4371 26004
##     8     2     6     5     1     7     3     3     4     2     1     6     7
## 11629 18399 19207 18973 19580 21994  9943 12130 25050  9135   543  2193  1958
##     3     1     3     3     4    17    11    11     8     1    24     6    19
## 18379  6918  1556 23154 22382  4759 12664 14345  5212  6628 19015 19444 25139
##     1     4     3     1     1     2     2     1     4     4     4     4     4
## 22023 11610  9126  9441 10388 14496 20495  5807 18782 21075 12659 14619  2784
##     1     3     3     1     2     2     2     9     5    16     9     1     3
## 16109 21808 19159  2044  4490 17001 18013  3618 21466  8854  1045 20915 18297
##     3     3     6     4     4     6     6    13     6     5     1     1     6
## 10253 20182 23918  8369 16888 23896  1347 12641 23297  1373  7245  8031  7844
##     2     7     4     4     4    15     1     1     5     5     8     4    13
## 16726 16270 12406 25354  2997  6888 11664 11026  7632 20414  5240 18920  3681
##     3     7    10     4     3     3     2     6     2     3     6     6     3
## 13328  7882  5739 16347  3048  5838  1008  2185 11824 12066 20808 21815 17208
##     8     8     4     8     5     2     8     8     8     8     8     8     2
## 21929 21149   312 15395  7958 25130  9964 23485 21615  9266 11066 10162  1073
##     2     3     2    12     1     3     4     6     1     1     3     2     3
## 20668  9098  3996  3068  7579    62 10966 13153 19493 23149  2917 16039 19675
##     1     2     5     1     2     7     6     4     5     4     2     7    16
##  4458  6878 22244 18088 18217 11812  6911 13801 16647  9769 14932 22489  8815
##    10     3     1     8     4     2     3    45     3     3     6     3     2
## 13496  3716 16955  2388 23614  8680 16817 20952 12311  6264   376  2611 16620
##     2    13     3     3    26     5     9     3     5    32     2     4     3
```

```
## 13999 12675 14834 23248 10872 24250  2666  9630 10252 16820  4877  5570   834
##     4     9    14     1     2     2     2     2     2     2     3     3     3
## 15882 10381 22721 14159 24852   299 18746 22051  5547  5659 12313 13520  4525
##     1     9     3     2     3     5     2     5     3     3     3    16     8
##  6337 18105 10490  2511 14550 19964 16013 10923  2342  1796  3386 19980  5957
##    10     2     9     5     2     5     4     6     3     2    11     8    14
##  7716 11233 12118 14090 18397 24814 25541  8704 17591  3199 19507 21695  5564
##     8     8    15     8     8    21     8     2     2     1     2     4    10
##  8871 17090 25557  9659 25598 11816 22830  3772 23758 23720 18208 15920  1608
##     2     2     5     5     2     1     2     1     2     6    24     2     2
## 25304 14854 14512 24219 24835 17933 15891  6823  6465 25152  2124  1234 25053
##     4     3    14     2     8     1     5    21     5     2     1     7     9
##  5825  9413  5166 18543 20375 22371 13422  1613 15217 13992 16154 16505 22765
##     4     2     3     8     4     1     7     4     4     4     8     5     2
## 13220 22336  2883  4015 18066 21621 24877 12787 23622 22900 24114  2991 25382
##    11    11     2     2     3     6     8     3     1     1     1     2     3
## 11213 18557 19909 23474 24765 24097 17395 10763  2593 10433 15253  8823 22466
##     3     2     2     2     5    10     1     1     1     1     1    24     4
## 17546 25710 20243 23681 26127 17911 23480 23652  2922 16107  8208  5830 15580
##     4     3     4     6     8    14     1     1     3     1     3     2     7
## 15325 26173  1278  6218  9269 20641 15081  9712 25209 19489 10539 23751 24152
##     4     2     8    13     2     2     4     2     6    16     4     4     1
## 14003  5529   414  5184  8643 21779 10338 19378 15416  1343  3977 13310  5262
##     6     4     1     3     3     3     6     5     2     2     7     2     1
##   161  3105 21821 22719  6732 19013  1323  7985 24816  9871 17179 14286 17848
##     4     1     1     4     7     2     2    15     2     2     2     3     1
## 19996 20078 21401   380 23264 19464 23968 11132 12980 18595  9773  4164  4125
##     3     3     4     3     1     2     2     7     8     2     2    54     5
##  2239  8517  1098  7956 12365 17655 25346  1405  5485 19578 11182  5107 17800
##     4     7     3    56    77    66    56     2     3     1     4    18     3
## 14102 16521 20517 21184  8374 17465   495  1854  9458  9890 14809  2087  4364
##     7     4     3     3     3     3     4     8     5     1     1     6    33
##  4377  6271  7023 24722  6158  9922 18592 14369 17738 24499 19890 21554 19598
##     4    30    10     4     4     1     5    10     8     1     6     1     2
## 11216 13815 11892 12786 12318 13673 19924 21321  7695  4442 19871 25526 26051
##     1     1     1     6     3     7     1     6     1    10     2     2     2
## 13619 18585 22075  1075 17156 13641  3239 11577  8074 11537 17986 20196 10096
##     6     6     2     1     3     1     2     2     3     4     1     2    18
## 11734  4466   267  6666 22758  3501 19501  6583 14089  4293 24489  9511 20902
##     3     6     3     5    11    23     4    25     5     3     7     6     4
## 17099  2076 25572  3076 21629 12389 13485 23856   281 18552 12085 17141 11462
##     1     2     2     4     7     2     1     1     8     2     9     1     1
##  1653 22299 14869  4712  5814 17690  1656  5828 21531 25562 24057 20191   950
##    56     4     1     1     1     1     4     2     2     5    23     3     7
## 19895 20942  3102 12414 18562 24439 13164 14558 11788  1092 23226  9387 14615
##     4     3     3     4     3     7    13     2     3     2     1     1    10
##  4033 17194 20782 23730   574 12286 20959  5740  6315  2504 19585  4793 22964
##     3     5     3     3     2    10     1     1     2     2     2     7     3
## 15516 19004 20552 20553  4040 11929  2451  5510  6748 22018 25660 12472 21316
##     5     2     4     2     1     1     1     7     2     8     5    11     1
##  9785  2560  4146 10926 12553 16594 18668 21508 14540  5241  5631 16067 24444
##    68     9    13     9     9    19     9    67    46     1     2     2     3
## 14325  9184 21798 17187 24207 18621 17588 12551  9962 22428  6354 15188  9913
##     2    21     8     5     7     7     3     7     3     1     1     1     2
```

```
##  1838 11791 10004 17306 20052 25234 18314  9037  5395 23298  2506 10682  6364
##     2     3     5     4     7     3     9    10     3     2     2     2     6
##  7853  9946 18182  7126  9959  6008  5081 12256  5488  8877  6815 23145  9832
##     6     2     3     2     2     2     1     2     5     1     3     3     2
## 20716 12781  4138 14662  7999  1674  3323 21943  1186  9314 10655 12742  1109
##    28    57     2     2     1     6     5    13    17     1     3     4     3
## 19164 12247 12798 19997 19060 13705  5116 19080 14067  1107  6973  7125  5359
##     4     5     1     1     1     1     5     3     9     1     1    10     2
##  2950 12380 15166  3990 25402  4724  1124 14324 25607 25661 15821 22100  4960
##     1     3     3     3     7     4     7     2     8     3     4     9     2
## 20554  5125 12637 15005  7524  8721 15372 16022 18685 22083 11631 16575  3265
##     1     3     3     8     7     8    10     8     7     7     2     6     9
## 15625 19247  7010 20781 13636 22528  4451 23051 22376 24295 21027 19527 18586
##     1     1     3     2     2     2     2     8     4     4     2     4     1
## 25540 10543 14970  9325 17764 12320 12321 19222 23714  2474 18344 21650 15172
##     8     6     2     7     7     2     2     2     2     3     1     7     1
## 17121  1857  3967 12053  8298  4019 24957  5626 18946 12860 16853  9862 18030
##     1     2     2     4     5     2     2     2     1     3     3     7     9
##  8557  8308  3516  7053  1378  7877  2984 10564 16162 12040 13346  9128  3922
##     3     1     5     1     1     4     2     4     6     5    13     2    10
##  3312 15191 23227  2476 18984  8731 21614 18189 17599   187  1821 21386 12884
##     3     1     1     1     1     2     2     1     2     2     2     2     9
##   731  1116  6305  7726 14870 14872 20316  1436 18331  8768 23991 25443 14403
##     1     9     5     5     5    10     5     3    12    18     3     1    11
## 16953  7926 21340 18370  3684  6376  7824 12065 10967 16336 10133 18218 20341
##     3     9     2     4     2     2     4     3     2     2     1     1     1
##  2526 11427  3067 18721  7650  6808  7802 23555 25959   844   890 11197 26058
##     6     1     1     1    37     4     4     4     1     2     2     2     5
## 16110 13035 26130  1916 20953 24463 15706 20776 12868 19244 25449 17681 20030
##     3     3     3     2     2     2     2     1     1     1     2     7     7
## 13711 16899 14383 15582 19707  5260 19791  5519 26167 21998  2516  2055 13352
##     4     4     2     7     7     1     5     6     2     4     3     2     8
##  7025 19252  3195    75 19807 24597 17280 20116 12606  2783  9408 13013 23920
##     1     1     1     1     2     1     4     7     7     2     2     1     3
## 16006 22693  2879  9758 16210 21142  7861  2355 14543 21221 17297 25219 11842
##    17     3     1     3    13     2     2     1     2     2     3     1     1
## 25601 21404 20184 16324   232  6533  8155 24204   820  2515 12049 17382  3056
##     1     2     4     7     1     3     5     3     2     1     3     3     3
## 25319   194  8628 15974 23883 14414 13622 21125 22144 18215 22603 13832 22890
##     1     1     1     2     4     2     2    12    11    11     2     1     3
##  9074 12425  6184  4748  5851  8146 21028 19568 14170  3946 20427 18684 23418
##     4     4     1    19     3     1     1     4     2     1     4     3     1
## 21866 25958 10897  5548 25896 10986 11924 14534 16475 25565  1694 24583  6919
##     1     1     1     1     7     1     1    12     2     4    10     1     1
##  4319  5255  6641   727  9870 24475 10496 21409  6159 19554 24878  8053  8193
##     3     2     1     6     3     3     7     6     2     2     2     3     1
##  5577 23881 12387 19520 23084  3827   921  8069 22888  2664 22876 12016 16485
##     1     1     2     3     5     3     2     2     1     1    13     7     2
## 12617 17388 20434 23506 23741  9944 18736  9143 14647   345  1078 11318 20924
##     1     2     2     5     5     1    11     7     4    13     5    11     8
## 15316 10026    85 12045   800 15485 19697 20260  4275 13625 24471  4081  3196
##     3     2     1     1     2     2     6     1     5     3     3     3     1
## 18122 21853 16754 19612 16357 12411 25777  6056  9095  6280  7463 17861   253
##    13     5     6     6    10     4     8    10     1     5     6     2     3
```

```
##  4634 21051 22993  6813 20854  3725 15911  5415 20650 25719  1658 23945  1258
##     3     4     1     5     2     4     4     1     2     1     6     3     4
## 12797 24280 21341  9092 11979  6455 23683 15616 14976  2212 11231 17683 17575
##     2     3     3     3     3     1     3     3     6    47     6     4     4
##  7878  4380 16756 24402  3100 18098 20129  1519 23429  4139 20002 16729 24293
##     4     2     2     5     1     1     6     3     4     4     4     1    20
## 16332 22609 12046  8801 12695  4704 25442  8178 19380  1695 13470 19140  8726
##     3     3     1     4     3     3     8     2     1     1     2     7     2
## 15196   500 13971  8224   396 15214  9618 11285 12193 14174 24713 13703 20046
##     2     8     2     6     4     3     2     2     2     2     6     2     8
## 19107   953 13831 20328 10684 12971  2142 24601  4051 26092 21313  7533  8437
##     6     3    16    10     2     1     5     4     3     4     3     8     2
## 25718 17127  4143 20667 16837 22848   778  1048 22235 20101 19800  5826    22
##     3     3     5     1     1     2     3     3     3     1     3     1     6
## 15793 15917  3074 23416 11865  9892 11011 25785  1101  9030 17854 13664 16940
##     5     4     1     1     1     6     7     6     2     4     2     5     3
## 12696 21220 16278 22462 24705 18222 22426 19911 25487 25868  6229  7647  7911
##     1    11     3     4     7     4     6     1     1     1     2     1    10
## 20599 10847 10148 11328 11405 11284  8387 16216  2936 13404 21152 11009 20157
##     3     1     1     4     2     4     2     1     1    16     2     4     2
##  3206  6579 23412  1430 13205   252  2394  7013 17345 10824  4588 12165  9647
##     2     1     1     2     5     1     5    18     4     2     6     4     5
## 16941 19313 24340 18597 17612 12290 17307  5545  4135 16128 15125 21438 19301
##     1     2     3     1     9     2     1     1     5     4     3     1     3
##  8215  2049  9915  9138  6037 16274 21322  3073  3607 13196  8441 20053  3321
##     3     2     2     1     1     5     1     1     2     4     1     1     3
##  1908 16621 13995 11034  1058 14661  5143 25853  8404    25 22891 17124   164
##     4     2     5     3     9    13     2     1     1     1     1     5     2
## 13202 11621  7959    28  7916 16835 18677  7522 24959  4656 24841 23303 18196
##     1     7     2     1     3     2     2     4     1     3     8     3     3
## 13966   231  5055  4875 11844 12306   915 18624  4834  4835  1668 18158 21317
##     8    30     1     3     3     3     2     2    36     2     2     3     2
##  2993  1675 20914 22975  1550 21190  4493  1321 10559 18245 12763 20391    46
##     5     1     2     5     3     2     5     4     1     1     2     7    42
## 19961  2952  6830  8879 11472 12496 12851 15659 17692 20108 20562 22887 26138
##    45    45    45    45    45    48    45    45    45    45    45    47     2
##  9735 16765 10445 18724  2085 22611 22290  8718 21287  8614  7594 24126  6389
##     4     4     2     3     4     2     2     2     8     1     1     6     2
##  6914 18279  4141 17755  7468 10002 17304 18587 12041 14384  1311  3983 16876
##     2     2     4     2     1     1     4     4     1     1     3     3     7
##  9147 17289  8299 23693 16945 18443  2182  3553 19557  7563 24248 17595 22834
##     3     2     1     9     1     4     5     2     2     7     2     3     6
## 20064 13191  1029  4483  6175 13683 21203 26065 21778 17951   833   869 14709
##     2     1     2     7     3     3     4     9     3     1     1     5     6
## 17231 11557 12701 23304 17439 17131  8122  3173 25229  4826 26022  4021  6161
##     2    15     4     1    29     3     3     1     1     2     2     2     1
##  7055  6073 17294 14616 17817 19712 21024  3062 19572  7857 11968   853 24398
##     1     2     2     2     3     1     4     2     2     2     1     4     5
## 14841  6774 15850  2485 25589  7317   930 25904 24587  4422 25917 18210  3063
##     1    27     5     2     3     7     5     1     1     8     3     3     3
## 20489   409  6746 22618 25043 25849 25829 13022  8077 10041  4513 17268 20954
##     2     3     3     4     5     3     3     1     1     6    43     4     2
##  6300  2117  5608 25107  6675  5353 23153 19089 21809 15609 21177 21830 20547
##     1     1     4    10    10     4     2     5     1     1     3     3     3
```

```
## 14181  4673 15596 12803 16495 21018  5479  5392  7541 17665  8376 13067 25102
##    10    10     3     2     6     1     1     8     8     7     3     6     7
##   880  7506 10427 17729  7855 18681 24762 25668 16124 20807 24334 16415 14710
##     3     3     8     2     1     3     4     1     2     2     2     6     6
##  7046  7045 15682 17157 23093 20567 24101 15357 18913 16482 20424 24023 22321
##     6    10     6     6     6    11     5     9     3    11     4     9     7
##   941 13712 16727 23151 23383 12296 23423  1823 20421  4641 17937  7482 15681
##     9     7     7    10     7     3     5     1     3     2     2     1     2
## 16818 16553  8869  5078 10346 19383  1985  8195 12071  8452 16872  8179 10153
##     1     4     1     8     5     3     2     4     2     1     3     1     4
##  1059 21660  8728 12709 26157 24470 20644 22233 14843 20569 18193 17585 15218
##     3     1     2     2     2     1     6     7     3     3     5     1     1
## 16967 23163  7957 18895 12679 10476  6626 25628 10996 11630 12268  6190 11403
##     8     3     1     1     2     4     3     3     4     5     7     4     4
## 10656 25610  6031  1353  4765  4825   851   122  9593  4966 25094 15623 11898
##     4     4     2     4     7     6     6     8     4     3     3     3     1
## 18619 18751 15181  2331  8511 16471  4117 25910  4446 25080 19316  2200 18972
##     9     8     1     1     8     2     2    12     4    14     4     4     1
##   887  2568 14541 12458 18209 17277  6642 18899 17461 22920 11583   385 12849
##     2     5     2     5     3     3     3     5     2     2     1     6     3
##  2001  7935 11002   373 21579  6012  3188 23064 23775 11012 25089  4283 23880
##     3     4     2     1     1     5     1     1     2     2     3     3     7
## 25251 20574  2295 15108 17502 17740 25387 19922  8641  8595 14691 15938 12324
##     2    37     1    22     2     2     3     5     5     3     3     3     5
## 21944 23806  9419 15947  2487  3052 25516 23653   309 21159 11148   188  4376
##     4     3     1     1     1     5     8     1     5     1    10     2     4
## 10652  2596 25644 13990 10356 19062  4258   106 11183  1896 10186  4372 10880
##     5     3     3     2     1     1     8     4     3     8     1     2     2
## 16598  6913  4195  2797 15971  9763 10552  3185 18195 15197 18658 23302  9937
##     4     1     5     8     5     4     1     1     2     2     2     3     7
## 15913 19682 21657  9432  8219 21613  7807 11828 14128 16946 20155  2352 22502
##     1     4     1     5     2     3     1     1     4     3     2     1     3
##  6904 19945   200   492 19678  6708  8151  9020  9021  9208 16882 17022 19586
##     4     5    14    14    14    14    14    14    14    14    14    14    14
## 24861  9722  6375 18580 22770  3119 22197 24034 16819 10163  1281 25287  3653
##    14    14     5     2     1     2     4     3     2     6     6     6     6
##  6482 20772 18910 22979 16858 21293 10432 16891  2624 24202 17934 17015 15572
##    13     3     2     4     3     2     2     1     1     1     1     5     1
##  7356  5216 17419 25172 17564  6185 19605  2661 24845 18233 15931 23471 20008
##     1     3     8     8     1     1     2     7     8     3    10     2     2
##  6442 12453  3452 20587 17953 23458  5352 17988 23950  8593 17923  2155 23554
##     7    15     2     6     2     4     4     1     2     3     4    10     2
##  8811 22021  5767  6010  2332 21596 18544 17794 16783 17563 18522 22105 20059
##     1     1     4     2     2     2     2     5     2     2     1     2     7
##  5605  6711 22725  2307 12252 22404 10000 19783 18669 19866 25545 11528 19450
##    10     3     4     2     1     2     1     1     2     2     3     2     1
## 15400 11107 23620  7536  7534   113  3383 23989  1344 18126  1776 15816 13624
##     1     8     4     1     1     4     1     1     1     1     4     4     5
## 15907 14009 18667  7357 10477  3877  4354  5412  6544  7719  9397 11445 12788
##     1     3     3    20    20    20    20    20    20    20    23    20    20
## 18185 19246 19463 20780 21861 22284 22741 23514  7091  6638  1738 19440 24490
##    20    20    20    20    20    20    20    20     3     6     3     3     1
## 23403  4276 24159  6535 23387  5402  1376  4267  5995  6631  1289  4365 10918
##    11     1     3     1     2     1     1     1     3     1     3     1     3
```

```
## 23219 23809  2630 19978 11841  7098 16722  1229 17126  9993 18778 12409  5109
##     2     4     5     3     1     3     7     7     2     4    34     3     2
## 16174 18235  7444 14552 15585 19544 25111 22000 25136  1327 16565  2771   348
##     4     1    28     5     5     1     5     1     2     1     4     4     2
##  5660 15847 18884 16648 22696 15007   205 16568 22929 16011  7446  6095 25236
##     2     2     2     1     2     1     6     4     1     1     4     3     3
## 22439   315 21297 16659  8732 22377 13029  5629 17385 12386 13333 11969  1967
##     3     1     2     3     3     3     3     4    18     5     5     2     2
## 18996 23110  7481  6156 18381 22423  3291 10765 15712 19596  5365 15814  5144
##     2     8     3     4     4    16     3     6     1     1     1    12     2
##  8087  3292 10499 17956 18065 25597 21630 24943  1292 10341 14840  6648  9732
##     3     1     1     5     8     3     2     1     3     3     3     3     4
## 11030 23946 25611  3194 15209 11572  3948  2184  6538 16040   628 20536 25684
##     5     5     5     1     2     3     2     3     6     1     1     2     3
## 14652 21592  9968 10936 19509  4273  7355  5259 21705  6265 17293 15668 25614
##     3     3     3     3     1     4     8     5     3     1     3     3     3
## 26006 20543 21339 20647  4702 19784 25220  2465 17538 25978 10178 24133  7645
##     3     3     2     3     2     2     2     4     6     6     2     2     6
## 26044 16225  2293  1006 21187   750 19641 16262 25379  3989 17471  8523  1105
##     1     2     3     2     2     2     2     1     2     1     1     2     1
##  8589 17981 14985  2144  2774  3055  3766  3652  3655  3814  6388  7096  9249
##     7     2     1    23    23    23    23    23    23    23    23    23    23
## 12051 12423 12885 14808 15173 15189 15999 18942 19705 20250 21030 21825 25108
##    23    23    23    23    23    23    23    23    23    23    23    23    23
## 11502 12158 11227 17006 10679  8671 23757 16802   830  2877  6076  7311 12615
##     7     3     3     4     3     1     2     2     2     3     6     6     6
## 15163 17731 17939  5769  2143  9053 12504 12735  1093  3059 11461 22087 22839
##     6     7     1     2     1     1     1     5     3     3     6     3     3
## 15418 12478  6941 19350 15311  1499 20261 16486  1589  7603  5661 19525   864
##     1    10     5     3     5     2     2     2     1     1     1     3     1
## 18275 20331 22738  8128 23759 15208  2742 19143 20636 16065  2622 13989 21558
##     1     1     3     5     1     1     1     3     2     1     1    10     3
## 19683 22744  1028 25179 24582  4555  5505 19355 16177 21100  7095 15206  1981
##     3     3     7     1     2     2     3     1     3     6     2     1     7
##  5468 18034 19879  1150 25648  1322 16068  3818 23167 21755 18625   920 10677
##     5     5     5     3     6     2     2     3     3     1     1     2     3
## 17865 24885  8730   914  5162  3811  9090 10478  5119 10517 13411 19560  6039
##     1     7     3     7     7    11    10     1     1     1     8     3     1
## 23224 13704  6446  8036 25201 25189 16742  6288 11563  3243  8864 16131 16280
##     3     1     1     1    13     4     1     6     1     3     4     4     4
## 17137 15247 10640    13 11196 19102  3037 13647 25057  9335 17463  6350  4495
##     4     2     2     3     1     1     2     6     2     1     1     8     1
## 19093  6421  3526 15786 25114  5137  6884 10354 16766 11017  7597  5271 10904
##     5     6     2     2     1     1     1     1     3     1     2     4     2
## 12707 24174 25609  1855  2009 22894 23509  1013 19011 21712  8403  2527 18603
##     2     3     1     2     6     2     3     1     2     2     5     2     1
## 11613 25599 16718 19167 13068 15244 21547 24718 21588  3316  4708 23805 10158
##     3     2     2     1     1    59     3    40     1     2     3    12     1
## 20803 25231 24939 19170  1551  8058  5408 19609  8336 24464 21407 12712 16511
##     2     1     3     1     1     2     2     2     1     1     6     3     4
##  8737 19059 20004 24870 16213  8386  1339  8198 16963  8305  5367 14983 19144
##     2     6     3     4     3     4     5     3     1     3     4     4     4
## 10468 19084 16498 23267 22599 19223 21634  8739 25115 13357   748 16155  2059
##     1     5     4     1     2     1     1     2     2     3     1     1     1
```

```
##  2982 13364 22966  4036 13032 18633  9486 21523 21515 24148 19051  3200  4264
##     3     8     4     3     2     7     1     2     2     3     1     2     2
## 17468 25510  3279 21684  3058 11815 19954  2250  7717 11015 13714 14767 18971
##     2     1     1     3     2     2     6    17    17    22    17    17    17
## 19216 20534 25205 24332  9597  9188 16545 10726 18784   894  5441 14351  3718
##    17    17    17     3     1     3     2     7     1     5     8     9     2
##  7511 11593  3910 19179  9267 22395 22726 10513 25595  1104  5639  6009  6896
##     2     1     1     7     3     3     3     2     1     2     7     4     4
##  9736 12503 23511 22493  7477 19257 22961    93  2459 17389 19906 26102 19806
##     4     9     4     7     2     2     2     5     1     3     1     1     2
## 16890 25852 25571  1962 21800  6291 15687 19454 13749 13034 12121  6079 19941
##     1     1     1     2     4     2     3     3     4     2     2     1     1
##  3633  9255 19126 25844 17401 19433  6835 10024 13177  1829  6735 13325 14502
##     2     4     2     1     1     8     1     5     3     6     6     4     4
## 20926 13318  8187  2826  8857 23216   899 16338 21390  8261    82  3844 21130
##     3     3     5     5     4     3     3     1     1    18     1     1     5
## 25491   937 25204 17603  6864 15688 22692 24252  9907 20106 11903 24210   510
##     2     5     1     4     2     1     1     1    10     2     3     4     2
## 10805 22285  4955  4954  5409  8623  7592  3511  6807  5411 10390 11186 18154
##     3     1     1     1     1     1     3     2     2     3     3     3     1
## 21560 24160 21696 25854  7386  2761 24453 20315 22379  4023  1561  2169  5209
##     2     2     2     2     2     4     5     4     2     3     1     5    12
##  5839  7026 12192  1375   141 17286  4302  8475 26181  1611 21638 10438 20558
##     2     2     2     2     4     4     4     4     4     3     3     2     2
##  5794  2750 20883   787 16717  4988  2222 21309  2215 15695 24341  8349  1549
##     3     1     1     2     7     3     5     4     3     5     1     6     4
## 22766 23168  4272  2557  7596  5773 17543 22727 20422 22503  2247  9606  4298
##     3     3     1     1     1     4     4     2     2     2     1     6     4
##  1427  2385 20783  8067 26196  7072 18234 18102 19434 13371  2981  4427 21594
##     2     3     3     5     7     3     3     4     1     4     1     1     1
## 12369 22239  2010  8064  9133  6669  9865 21584 20033  8449 13490 19528 24412
##     2     2     3     3     1     1     1     2     4     3     3     3     9
## 12940 11037 14547  4289  6380  7546 18366 10501 22722 24032 16747  9216   717
##     1     1     1     2     1     4     1     2     2     2     5     3     1
## 20174  4554 10412 21723  7588  7590  3599 14130  9983 17999 14302 12676 11670
##     1     2     2     5     1     1     1     1     1     3     2     1     7
## 25962 21562 11109  8972  8625 13369 17583  1696  1026  6317 15664 25235 13521
##     2     2     2     4     7     9     6     2     2     4     4     1     1
## 11721 15221 21344 12151  4148  9863 14037 18745 17248  3411  5717 17818  5667
##     3     3     5     3     2     2     1     3     3     3     9     3     3
## 23461 17178 23215 19900 14894  1670  3362  3347 22937 23363 13355  2306  8446
##     3     4     2     4     2     1     2     1    18    10     3     4     4
##  6971 13199 10183  2938 18581  1817 16513 15617 21593 22325 18970  2409  3522
##     4     5     1     1     1     5     1     1     1     1     4     4     4
## 21031 24001 11719 12749 17984 19703 12806  8970 15187  1511 18278  2201 14868
##     2     6     1     1     6     3     6     2     2     3     1     2     2
## 16226  5211 11028 21171 22730 14982  2712  3858 23161  7309  1187  6290  4814
##     2     3     3     3     3     2     2     4     4     1     1     1    10
## 21468  8457  8391 12743 17189 16608  8184 13677 18288 23206 20533 23513 10792
##     3     2     4     2     4     3     4     2     2     2     3     4     2
##  4110 17696  7811 17308 12157 15641  5466 19387   283  9713 15423 18144  7011
##     2     1     2     2     1     3     1     4     2     2     1     1     1
## 21101 10247  8630 14600  1023  4959 24592 23907 18612  8063  1174  9721 22798
##     1     1     1     1    29     1     3     5     4     5     2     2     6
```

```
## 20793  2043  9149 20319  7595  8118  6506 11820 17019 13010  5533 12611   782
##     1     2     2     2     2     2     4     2     2     4     2     5     3
## 15227   715 14664 17135  5268   115 20231  6081 10850  7452   254  7572 16087
##     4     4     1     2     2     2     3     2     1     1     1     1     1
## 20031 22720 20150 25226 22312 15150  9273 15942 23621 11623 17116  5164 19741
##     1     2     2     2     2     1     2     1     1     1     5     4     5
## 19801 20119 19168  4484  1667 10352 10910 10500 25159  5960  8705 11662 14094
##     1     5     3     2     9     1     3     4     4     4     4     4     4
## 19014 22031 13616 21497  8787 24575 14842 25181  5938  6431 17346  9639 19870
##     4     1     1     3     1     2     1     1     3     2     2    31    21
## 20532 22527 18237 10062 16102 22463  1989  7515 17782 18231  1728  5656 22610
##    20    25     1     7     4     4     4     4     4     4     4     4     4
## 18488 14176 20257 17884 18628 17785  5181  1090 24734  1356 16676  3980 16962
##     4     2     3     2     1     3     4     6     3     4     2     2     2
##  5366 10878 26020 25870 17174   365 10991 24029 12422  3293 15010    88  1995
##     2     5     2     1     1     5     1     5     3     2    42    42    42
##  2338  2530  2535  4755  5210 23712  6340  7197  7635  8727  9656  9869 10350
##    42    42    42    42    33    34    42    42    42    33    42    41    43
## 12507 12710 14766 15245 15457 15933 16654 17807 18225 19446 19645 19939 20576
##    42    33    42    33    34    41    42    42    42    33    42    42    41
## 22504 25211 11971 15523  9987  8161 19109 23246  4685  5202 23686 16349 11121
##    33     1     1     2     1     4     5     6     1     3     1     1     1
##  9895   751  3297 24732  7636 12554  4278 18183 21727 25272 11152 15198  3231
##     1     1     3     1     1     1     1     1     2     1     3     1    15
## 17997 22369  1125 10522 10561 20550 25678 25631  1660 23068  3377  6226  6737
##    14    11     1     1     1     1     4     4     4     4     5     2     2
##  8526 23476 24060 16010 17290  3683  1265  3825 20893  1966  8048 15717 25940
##     3     3     3     6     1     1    10     2     2     3     3     3     2
## 15912  6908 25444 16751  4263  6859 18282  3851 25056  9988 11967 14620 12865
##     2     2     2     1     3     3     3     1     2     2     4     2     1
##  4631  4576   822  9518 16358  5227  1879 10807  5065 18396 11951  6160 19475
##     3     1     2     2     2     3     1     2    23    24     3     3     3
## 18238 26180  4777 15396  1606 19442 24169  2243 26191 23464 17249  1910  9316
##     3     3     4     3     5     4     2     7     1     3     1     2     2
##   360  2304  7602  8307  5057  4625  6064  9439 23648  9514 15568 18161 16554
##     3     3     3     3     2     1     6     6     1     1     6     1     1
## 17688 22734 14770  2356  1493  2452  9027 14153 25091  6867  3982  4269 26159
##     5     3     4     1    11     5     5     5     1     2     1     3     3
## 11944  1915 20253 19248  4640   749  5446  5632 20433 22199 26013 18879  5483
##     2     1     2     3     3     1     2     2     2     1     3     4     2
##  3752 11879 20305 21814 22269 17339 26193 10549  3685 15935 18276 12308  1594
##     3     1     5     2     4     1     5     4     2     5     4     4     5
## 15899 11468 17804 10818 11016 25596  6513 23214 25138  2255 16801  5668 10969
##     2     2     3     2     2     2     2     4     2     5     1     5     2
## 12645 16803  8807  8553 20089 19569  3750 19451 16932  9579 22927 20947 12863
##     3     3     4     1     3     1     3     2     2     5     1     3     1
## 18390  6635  4103 12312 18159 14711 12106  1836  8400 13387 22048 21199 17158
##     1     2     3     3     1     1     2     1     1     2     2     2     3
## 15186  5578  1280 23424  8153 15182 16728 22319  9491  8054 24504 22605 21623
##     1     7     3     4     4     4     4     4     4     1     1     1     1
##  4290  7479 13390 19710 23225  8976 25176 13389   364 25383 16935  4878 10382
##     1     5     1     3     3     2     1     1     4     1     3     2     1
## 25482 11611 20779  2223  4266 20216 16643 20035  6170  8881  8211  4760 15583
##     1     2     2     2     2     2     1     1     2     1     2     2     2
```

```
## 24272  7281  5497  3418 10906   372 14131  6534   114 12113 16886 18152  8735
##     2     1     3     5     2     1     1     1     1     1     3     1     1
## 25062  7384 11923  5731  1992 17343 16708 21154 15589 15250 17825 22147 11470
##     1     2     1     1     1     2     4     2     2     2     3     6     2
##  4847 10842  8786 14078 18790 13600 25980  8340  3754  8185 16496 21167  9039
##     3     3     6     5     2     2     2     5     3     2     2     2     1
## 11911 21391   374 17821  4771  5251 18256 18337 18613 25585   855 16348 18788
##     1     2     1     2     6     6     9     9     4     4     1     1     4
## 12612 19208  3998 10497  4301 20560  3595 23673 15192 23721  9391 21653  5575
##     4     4     3     1     2     2     4     2     2     3     2     3     3
## 16624  3873 21156   358    65  9093  6810  2620 18227 21303   734 19809 12160
##     3     2     3     2     7     1     3     8     4     1     1     3     3
##  4355  9313 14648 19114 21178 19221 17470 18772 15399 21587 19090  4552 15422
##    31    31    31    31    31     1     1     1     3     3     3     3     1
##  2959  8071 25866 22487 25891 10391 21949 18194  3852 11622   260 25358  6434
##    10     1     2     2     2     2     1     1     2     2     5     2     6
## 21162  5697 14358  7854  8558 13717   839  5848 12642  2803  4183    97 19583
##     5     1     1     2     1     1     2     4     1     1     1     2     2
##  6907  9025 15683 21824 13649 25289 20664 19923 25359  5067  3773  8534  8676
##    16    16    10    10    16    16    10    10    10    16     1     1     1
##  8504  4425  9248 22381 16800 19356 13682 16749 14317 17827 18174 17173 15252
##     1     5     5     1     1     1     3     3     2     1     4     1     1
## 12149 16433   369 16570 21772 15478 26176 23138 14991   934  1662  2613 15413
##     3     5     5     4     3     1     5     1     2     7     2     2     5
##  9224 16830  4144 18596 23771 11141 19447  5445 16622 18454 13031 13620 21550
##     3     3     2     2     2     5     2     2     1     1     3     1     4
## 24220 25554 25553  9338 16312 16314 17878 10022 17875 23095  7206 15184 22990
##     4     4     4     1     1     1     2     4     3     3     1    13     2
## 23687  8922 13837 17250  1563  2457  3534  4639  6387 14379 15562 15708 15714
##     1     1     5     5    23    23    23    19    23    23    23    23    23
## 15805 17580 18952 19677 24617 24819 17086 20515 23915 23488 23114 15678 17134
##    23    23    23    23    19    23     4     1     9     9    14     9     9
## 12266 22815 25575 21436  3878 16584 18032  3451 17850  5172  5435 23559 11416
##     9    10     9     9     3     4     1     1     1    10     3     3     7
##   547 18870 14319 23256 26100 20934  4353 19256 10907 10514  2592  8156 11991
##     5     5     3     5     2     5     6     1     2     2     2     1     1
##  3186  2532 14182 17537  4958 10884 15358 18036 18338  7094  7772  2073 11402
##     1     1     1     1     3     3     2     2     2     1     1     3     4
## 14969  1862  2503  9631 10898 19927  9209  2806 17237 19023 14175 12307 22980
##     7     1     3     2     2     4     2     2     1     1     2     2     2
## 17808  4255 17233 16644  9217 21995 16265 18622 23530  1025 19562 24477 13322
##     4     5     4     1     2     2     2     2     3     1     7     7     4
##  4249 24456 18091  5387 18156  4642 12054 16042  6738 15329 19145 23760  4052
##     2     3     3     1     1     1     2     2     1     3     1     1     1
## 12887  2386 11561 11465 17252 18142 18791 11464 20662 22837  1983 13012 20943
##     1     2     1     2     3     3     2    14     7     7     2     2     2
##  4632 20774   123   142 18626 17089 22110  5231 15145 17584 23241 24616  3819
##    12    15     1     2     2     6     1     1    21    21    21    21     1
## 15802 17882  5087  6423 22311 20084 22149   400  6576  7109  1621 18771  3843
##     1     1     2     1     1     1     1     1     2     2     2     7    19
##  4488  9018 18299 12751  6307 21029  3917 18398  1267 13486  4259 18688 15160
##     9     3     5     1     1     1     3     3     4     2     2     2     3
## 19325 22831  4630 10672  5840 17191  5636  4286  4288  3654 15322   831 11280
##     3     3     3     3     2     1     1     4     4     3     1     1     1
```

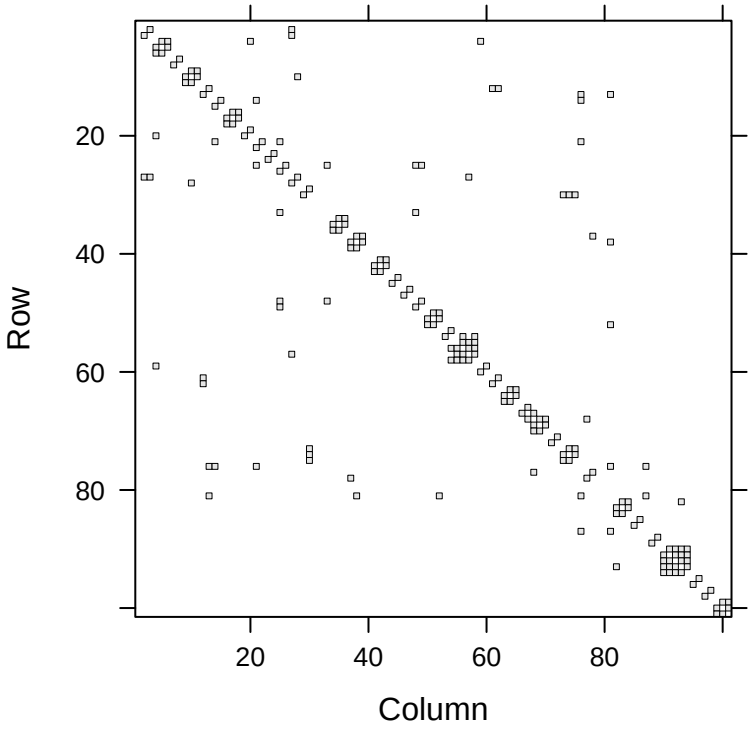
```
## 25615 15154 14132  4027 19940  1825 16523 21157 17337 16393 17918  1279  4034
##     2     1     1     1     2     2     1     1     2     4     2     2     2
##  9717  9715 26042  5427 23462  1798  4828  5404 20217 25436  8458  1839 23864
##     2     5     1     1     1     2     2     2     3     3     2     9     9
##  3502 16881 15126 17828  8973  4774  5673 19603 13615 24462 11641 22464 17125
##     9     9     9     9     1     4     4     4     3     1     1     3     3
## 14338 22729  2805  4265  5267 17192  2505   885 16070 22903 25998 15251 14154
##     1     1     2     3     3     3     3     3     1    15    15     1     1
## 17592  6703  3207 23420 10896 15200  3283  6222 13526 15148 22324 24474 20435
##     2     3     1     1     1     1    34    34    34    34    34    34     2
## 20479  5432  9194  8920 14096  7015   192 21310  1699  2303 25875 21583 16115
##     2     1     2     1     1     3     4     1     4     6     4     4     6
## 24589 12704 25977 18634 12618 23454   163 22394 11926 14621 19608   199  3449
##     4     1     2     2     2     1     2     2     1     2     2     2     2
##  9921 16106 17670 16232  4242  8222 21495 12512 17275 19466  7543 24128   450
##     3     4     4     2     2     4     5     3     1     2     2     2     1
## 14357 17581  3524 10819   193  2655 10446 18723 14972 20519 23912 11411 24242
##     4     1     2     4     4     1     1     4     3     3     3     1     1
##  6541  8216 13684 18283  5853  5492  7578 22278  1841 16611 15898  2773  6543
##     3     3     3     3     1     1     5     3     6     6     3     5     5
##  6529  4213 18094  6825 23553  3964  9414 12640  1560 15977 21288 26141  2512
##     5     3     2     2     1     3     3     3     1     1     1     4     3
## 19510  1782  9831 12473 13144 23306  5360 16264  5771 18741 25439  5434   320
##     1     2     1     1     3     4     1     7     7     7     7     4     6
## 15685  9392 21315 14079 21032 10550 22497  4428 21131 26132 10526  9151 17924
##     2     2     1     3     3     4     4     3     1     1     1     1     1
##  8969  2328  3679 23674 22032 24294  1151 15414 18743 24706  2410  6301 12801
##     1     1     3    13    13    13    13    13    13    13    13    13     2
## 19548 15663 21169 25856 19739 20568  5579  3532 19064 18605 18388 20654 13864
##     2     1     1     1     2     2     1     1     1     1     1     1     1
##  2335 16108 18052 18151  8888  5730 16957  3853 20317 20956 19495 14747 16576
##     2     2     3     1     3     1     1     1     1     1     1     3     1
##  8629 16483 10825 18447 24129 18739 15382  4433  8398  9327 18145 21799  2003
##     2     2     2     2     1     4     3     2     3     4     2     2     2
## 13036  4196 15401  4967 22109  1051  4245 25664  5478  4975 16823 19553 25447
##     1     1     1     2     2     2     2     2     3     5     3     3     1
## 15962 17393  7024 24203 18720 19607 16235 23810 21546 13167 19892 18417  7324
##     1     3     3     2     3     4     2     2     1     2     1     1     4
## 22746 14135 20000  4247  4250 17739  4643 20320 14990 25975 24506 24595  8150
##     2     1     5     3     3     3     1     1     1     1     3     2     2
##  9979  6530  6707  8372 13628 23710 11823  8037  3450   836 18042  7090 11792
##     1     3     3     6     6     4     4     1     2     3     4     5     3
## 16831  5218  8520  5417 20513 11444  2341  9867  1672  8079 16755 16651 24708
##     3     2     2     3     3     3     2     1     4     4     3     4     3
## 14157 23492 11965 16227 20645  8367 12552  1024    81  8395  8725 11054 14763
##    37     3     1     2     2     1     2     2     2     2     2     2     1
##  4432  2559  5543 17017 19019  9426  7573  3431 22225 11053 14431  6828 20766
##     1     3     3     1     1     3     1     1     2     6     2     1     1
## 11718 24593  8541 13319 10383 15553 15374 20346  5400 19111  3826   269  8206
##     1     4     8     4     4     1     1     2     1     2     2     3     1
## 19613  5490  8474 13321  2075  4182 10439 17114 15479 19931 16609 23863 12866
##     3     1     1     1     3     3     3     2     3     1     1     2     3
##  5481 20065  4379 12678  1976 23672 12295  5465 18141 20663  8448  2186  9189
##     2     2     3     4     1     1     0     3     3     3    34     1     5
```

```
##  2116  9765 12034 17935  9774 23723 20062  5182 13311 18125 15665 18246  9986
##     5     5     5     5     5     8     4     2     2     3     1     3     1
## 15642   882  1860 23419  3744 17016 16719  9075 10925 15195 17467 11410 10623
##     2     4     1     1     2     2     2     2     3     3     3     2     6
## 21665 19810  1859  2713 17274  1157 16334  6632 23299 12248    98 16352  8549
##     4     2     2     3     2     3     3     3     1     1     1    15    11
##  6411  5469 20236 10116  4181 17992 14377  5571 13802 17341  3286   375   179
##    15     3     3     1     1     1     3     3     3     1     3     1     3
##  2851 23944 23727 14667 23135 21314  9023 10519 24140  9433 17824  1293  2621
##     2     1     1     8     1     1     1     3     1    34    34    34    34
##  2623  3812  3820  3824  5131 24615  6525  6868  7525  6456 18000 12802 13482
##    34    34    34    34    34    34    34    34    34    34    34    34    34
## 14373 17600 17819 17823 18444 22901 24479 26039   938 15384 16937 20258  8538
##    34    34    34    34    34    34    34    34     2     2     2     2     1
##  2614 12981   677  4426  8409  2248   866 20173  2852 25721   128  6920 16877
##     2     1     2     2     1     1     2     2     2     2     4     3     3
##  5827 12547 17994   939  7601  8442 11579 15073 18582  4781 21713 25470 17936
##     1     1     2     1     2     2     1     1     1     3     2     2     1
## 16922 20925  7523   124  8055 18880 22645 21811  2123  8702 14639  8513 13415
##     1     1     3     2     2     2     1     2     1     1     1     2     2
## 15386 16335  7104 22375  7093  8000  4018 20383   573 23772   735 23385  7014
##     2     1     2     3     2     2     2     5     4     1     1     1     2
##  7103 13283 23858  3508  9924 12262 22507 23346 15941  4000  6356  2566 24454
##     1     4     4     1     5     5     3     1     5     1     1     3     3
## 20853 19581  5062 20379    74  2298  8717 10638  6082 15571 15715  3631  7585
##     6     1     1     1     2     2     1     1     1     6     1     1     2
##  7727 17182 22556  3442 20775 13558 24888  8669 11878  8738 13334  9103  7038
##     3     1     2     7     1     6     6     6     1     2     2     2     2
## 11418 12370  1982 19161 21831 17590 13026 26023  1897  2561 23094 13481 19724
##     1     3     3     4     3     2     4     3     2     1     1     1     3
## 25911  1969  8472 24141  4750 19147 19234 19379  2563 25903 16971  5228 20170
##     2     2     2     2     2     2     1     1     1     1     2     2     2
## 18608 22816 14339  5634  8870 13604 19962 11910  7280 20309  7267 18067 19881
##     2     1     1     2     1     1     2     2     2     2     2     1     1
##  8891 25667 22189 18995  4431  4104  1793  4261 10881 21165 18941  6776 24733
##     2     2     2     4     4     4     2     1     1     2     2     1     1
## 12644 23859 22976  5698   546 18455 18909 25784 15175 20620 15127 18524  2334
##     1     1     1     1     2     1     1     2     3     1     2     2     2
## 10180 17587 21016  4782 17472 21930 25217 22372 11215 23156 23616  4773 21931
##     2     2     1     7     7     7     1     2     1     1     1    15    11
## 19808  4705  2203 21090 23970  7718 19502  2450 11539 16655  7088  3178  4989
##    11    11     1     3     2     2     2     2     3     3     2     5     4
##  7265 10056  2754  5511  2848 22621 24726 19081    27 11114 21205 21289 13680
##     4     4     4     4     2     3     4     4     4     4     3     2     1
##  3748 17503 11401 19469 15259 10602 20805 11467 19575  4038  7058 23811 15626
##     1     1     2     2     1     1     1     1     1     2     2     2     1
##  1878 12338 15800    71 12660 26178  6944 14372  3731  5407  9099 23577 23649
##     2     2     2     1     4     4     4     4     9     9     9     9     9
##  3719  8072  8375 19545 25060  9130 25095  7638  7648  1383 17859 20066 25059
##    11    11    11    11    11     1     1     2     2     1     3     1     1
##  1010  2749  4189  5829  8049 20949 15161 12263 18905 21563  6078 11642 25230
##     1     1     2     1     8     3     2     1     2     2     1     1     1
##   733 13862   403 25228   321  8147  7991  3066 11609 14385  1832  9980  5256
##     2     3     1     5     2     1     4     6     8     3     3     2     2
```

```
##  5257 22827 15301 10601 16014  9488 16015 22938  5602  3204 20528 18758    24
##     2     2     2     2     1     1     1     1     2     6     1     3     4
## 15774  7485  9389 20333  4262  3280  6701 10407 14498  7769 17118  4469 12861
##     1     3     3     1     1     2     1     1     1     2     2     1     3
##  3876 14497 15972  5406 22435  9759 15669  6176 26021 26059 12745 23240 11120
##     3     3     1     1     1     1     3     2     2     3     1     1     3
## 20233 20538 20537 16340  3053  2329 24846 12460  3594 23713  4366   410 11713
##     1     2     2     1     1     2     2     4     4     4     1     1     1
##  3388 21544 18558  9211 13413  3850  2249  6077  6611  8672 19955 21832 19473
##     3     1     1     1     1     1     8     8     8     8     8     8     1
## 23812  6024  2074  4037 25663  6702 19443 24214  4116  8255  4106 22728  3197
##     1     2     2     2     2     1     3     1     1     1     4     1     3
## 25839 25649  9026  6316 18623 25496 19314 25543 26048 26019 19880 24820 20707
##     2     2     1     1     1     5     5     5     5     5     5     1     1
##  5486  8366 16353 14844 20332 15246  7883  5261  6282    29  1959  9723 16958
##     1     1     3     1     4     4     3     1     1     1     2     2     1
## 16649  9459  7247 11829 16889  8824  2202 24271 17251 23910  6905 19687 14374
##     3     3     3     3     1     2     4     4     1     1     2     4     4
## 11612 24944  1404  2562 11199  8310 25511  8512 25492 23623 19150  1014 18908
##     2     2     4     4     4     1     3     3     3     2     4     4     1
## 25531   195  6558 14865 11822 21548 17163 22954 16779   165 16506 12043  2309
##     1     3     4     4     1     1     3     3     2     2     3     2     3
##  6532 22791 25937  1341  5973 12122 11491 18742 18489  1834  5056  3965  5622
##     3     2     6     4     2     2     3     1     1     2     2     2     2
## 14559 16129  8443 15248   357 23300 24960  9038 22318 12213 21710 12050  5364
##     2     2     2     2     4     4     4     6     6     6     6     1     1
## 23146 11893   352 17569  1408 25437   183 16084  2823  9127 17851  5061 12688
##     8     2     3     1     3     1     1     1     4     4     1     1     3
##  1552 15420  2475 21928 16743 23266  4635  2810 16892 19110 22577 25440    14
##     1     2     3     3     3     2     2     3     2     1     2     2     1
## 14171  7278 12852 14845  5232 19521  2785  3682 19956 10117  9764  4703  3387
##     1     1     1     2     2     2     3     3     3    15    15    15    15
##   832  7447 14340 14370 15006 16231 16590 16730 19957 21848 22320  5774  4836
##    15    15    15    15    15    15    15    15    15    15    15     1     1
## 17464 10154 11113
##     2     2     3
```

Adjacency matrix with sub-matrix

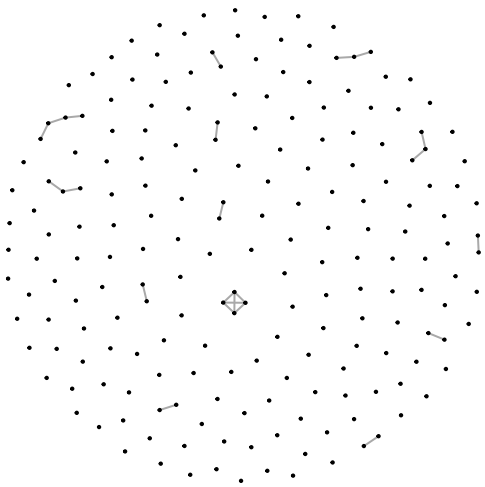
```
node_indices <- 100:200
submatrix <- grqc.adjacency[node_indices, node_indices]
image(Matrix(submatrix))
```



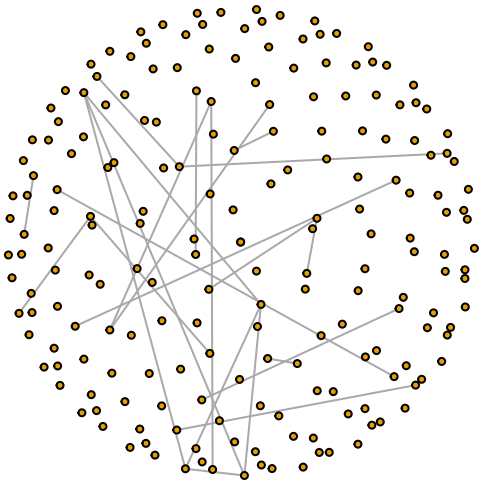
Dimensions: 101 x 101

Random Sub-Graphs

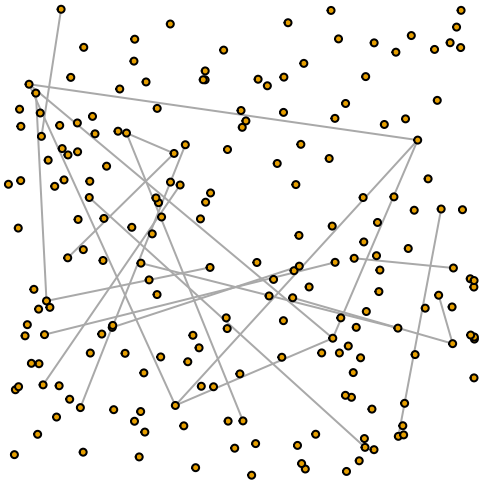
```
subnetwork1 = induced_subgraph(grqc, sample(V(grqc), 200))
subnetwork2 = induced_subgraph(grqc, sample(V(grqc), 200))
subnetwork3 = induced_subgraph(grqc, sample(V(grqc), 200))
subnetwork4 = induced_subgraph(grqc, sample(V(grqc), 200))

plot(subnetwork1,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



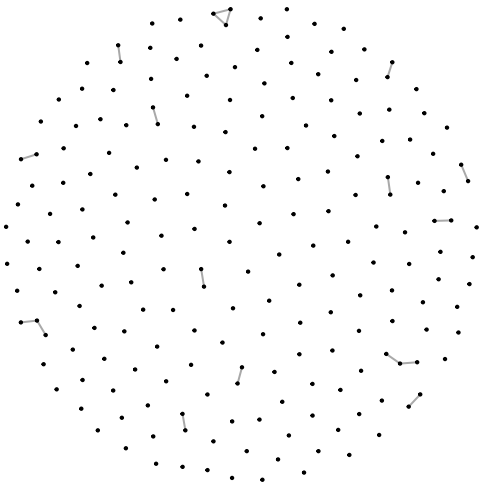
```
plot(subnetwork1,
     layout = layout.sphere(subnetwork1),
     vertex.label = NA,
     vertex.size = 3,
     vertex.color = V(grqc)$color)
```



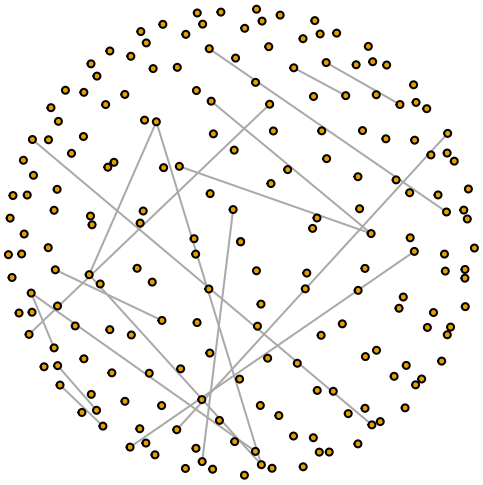
```
plot(subnetwork1,
     layout = layout.random(subnetwork1),
     vertex.label = NA,
     vertex.size = 3,
     vertex.color = V(grqc)$color)
```



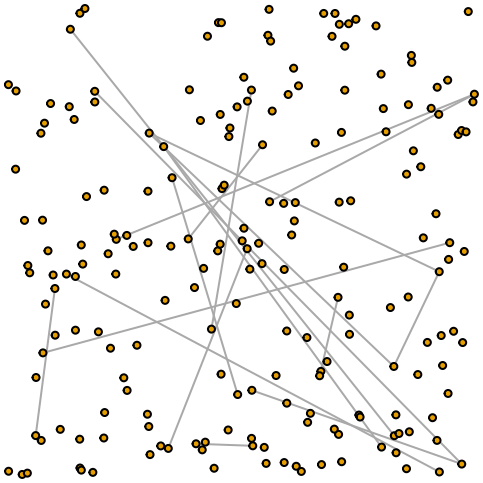
```
plot(subnetwork2,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



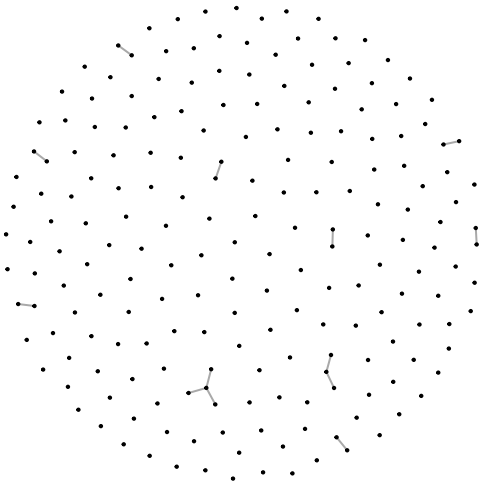
```
plot(subnetwork2,
     layout = layout.sphere(subnetwork2),
     vertex.size = 3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



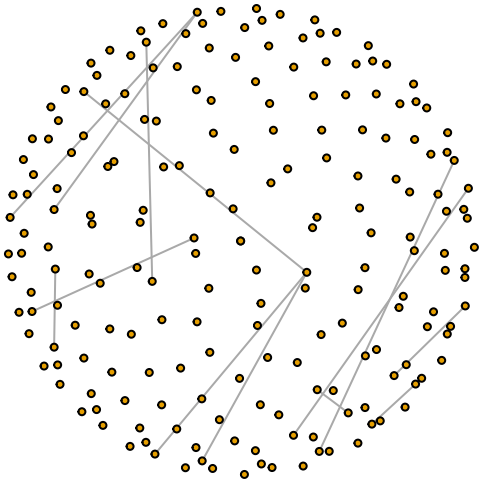
```
plot(subnetwork2,
     layout = layout.random(subnetwork2),
     vertex.size =3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



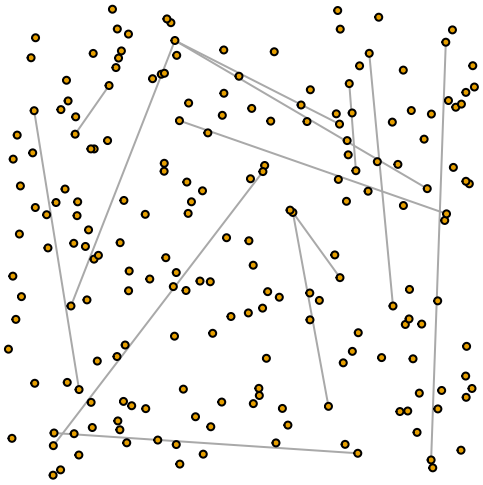
```
plot(subnetwork3,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



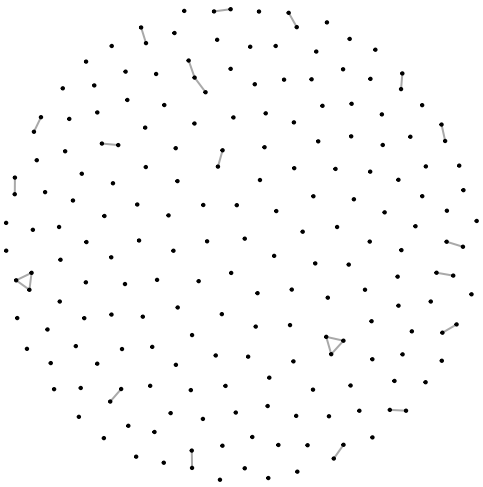
```
plot(subnetwork3,
     layout = layout.sphere(subnetwork3),
     vertex.size = 3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



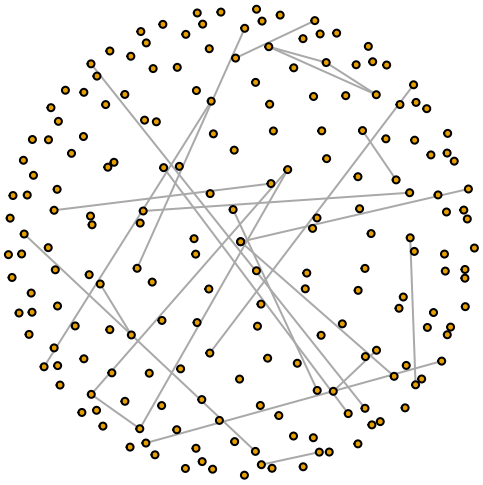
```
plot(subnetwork3,
     layout = layout.random(subnetwork3),
     vertex.size =3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



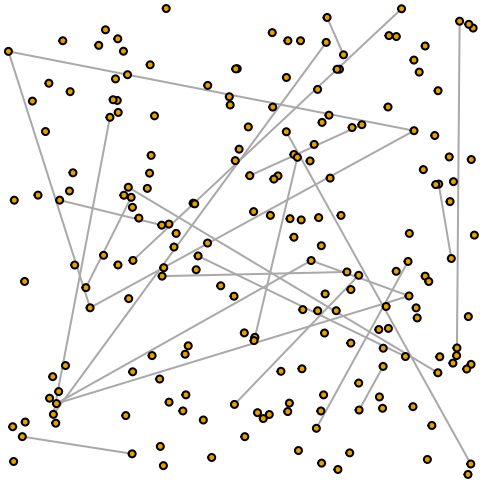
```
plot(subnetwork4,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



```
plot(subnetwork4,
     layout = layout.sphere(subnetwork4),
     vertex.size = 3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```

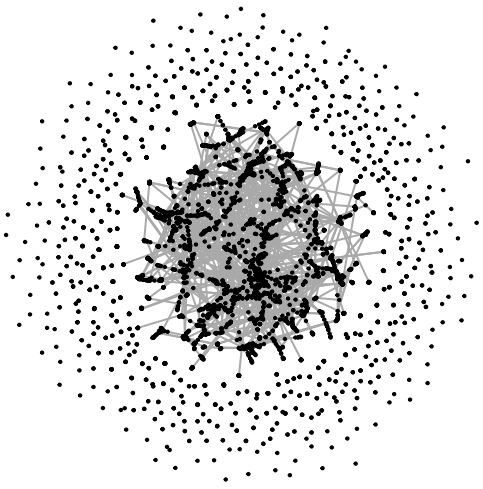


```
plot(subnetwork4,
     layout = layout.random(subnetwork4),
     vertex.size =3,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



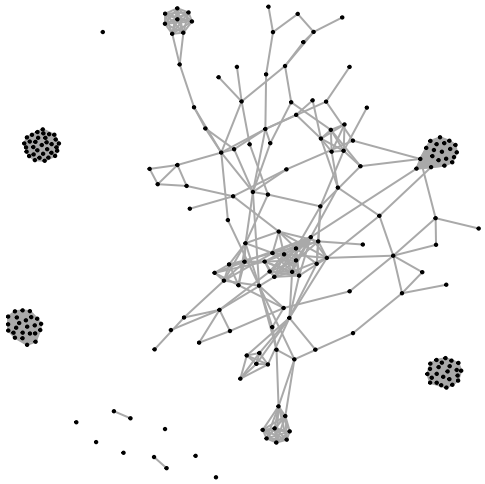
Subgroups by Degree

```
node_degrees <- degree(grqc)
low_degree_nodes <- which(node_degrees >= 1 & node_degrees <= 18)
q2_degree_nodes <- which(node_degrees >= 19 & node_degrees <= 36)
q3_degree_nodes <- which(node_degrees >= 37 & node_degrees <= 54)
high_degree_nodes <- which(node_degrees > 54)

low_degree_subgraph <- induced_subgraph(grqc, vids = low_degree_nodes)
q2_degree_subgraph <- induced_subgraph(grqc, vids = q2_degree_nodes)
q3_degree_subgraph <- induced_subgraph(grqc, vids = q3_degree_nodes)
high_degree_subgraph <- induced_subgraph(grqc, vids = high_degree_nodes)

plot(low_degree_subgraph,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```

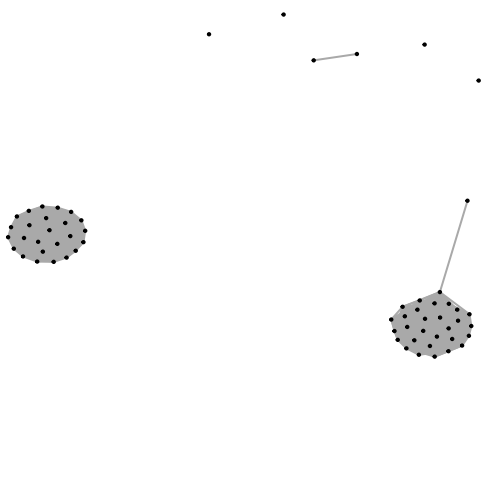


```
plot(q2_degree_subgraph,
     vertex.size = 1,
     vertex.label = NA,
```

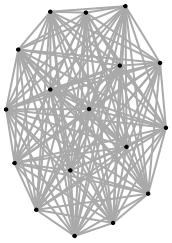
```
     vertex.color = V(grqc)$color)
```



```
plot(q3_degree_subgraph,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```

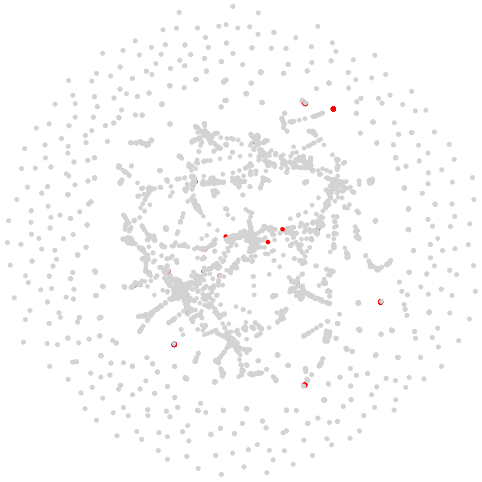


```
plot(high_degree_subgraph,
     vertex.size = 1,
     vertex.label = NA,
     vertex.color = V(grqc)$color)
```



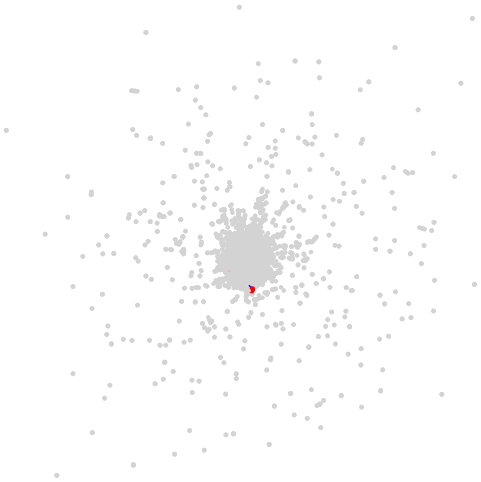
.

```
color = c('lightgrey', 'red', 'blue', 'hotpink')

V(grqc)$color[low_degree_nodes] <- color[1]
V(grqc)$color[q2_degree_nodes] <- color[2]
V(grqc)$color[q3_degree_nodes] <- color[3]
V(grqc)$color[high_degree_nodes] <- color[4]


plot(grqc, #nodes all close together
     vertex.size = 2,
     vertex.label = NA,
     vertex.frame.color = NA,
     vertex.color = V(grqc)$color,
     edge.color = NA)
```



```
layout <- layout_with_fr(grqc)

plot(grqc, #nodes more spread out to make it (very slightly) easier to see
     layout = layout,
     vertex.size = 2,
     vertex.label = NA,
     vertex.frame.color = NA,
     vertex.color = V(grqc)$color,
```

```
     edge.color = NA)
```

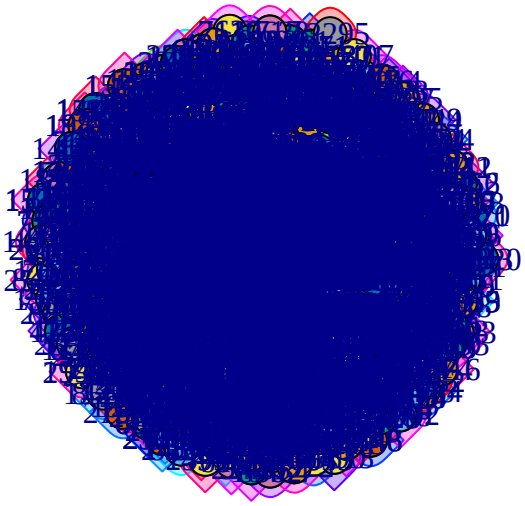


Fast-Greedy Communities

```
grqc_fc_communities = fastgreedy.community(grqc)
```

```
## Warning: `fastgreedy.community()` was deprecated in igraph 2.0.0.
## i Please use `cluster_fast_greedy()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

```
plot(grqc_fc_communities, grqc)
```



```
modularity(grqc_fc_communities)
```

```
## [1] 0.8197796
```

```
length(grqc_fc_communities)
```

```
## [1] 416
```

```r
sizes(grqc_fc_communities)
```

```
## Community sizes
##   1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20
## 423 986 264 337 194  82 136  90 102  86 125 128 129 181  34  57  43  55  41  37
##  21  22  23  24  25  26  27  28  29  30  31  32  33  34  35  36  37  38  39  40
##  39  42  76  37  36  47  46  16  12  28  15  10  10   9  14   9   7   7  10   8
##  41  42  43  44  45  46  47  48  49  50  51  52  53  54  55  56  57  58  59  60
##   7  12   7   6   6   6   8   6  14   9   5   8   7  11  11   5   5   5   6   6
##  61  62  63  64  65  66  67  68  69  70  71  72  73  74  75  76  77  78  79  80
##   5   5   7   5   8   9   5   7   7  10   6   7   7   6   5   5   5   7   8   5
##  81  82  83  84  85  86  87  88  89  90  91  92  93  94  95  96  97  98  99 100
##   5   5   5  12   4   6   4   5   9   7   4   6   4   4   8   5   8   6   4   6
## 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
##   8   4   4   6   4   4   5   4   7   8   5   6   6  11   4   4   4   4   4   4
## 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140
##   4   4   4   3   3   5   5   4   3   4   3   3   3   3   3   3   4   3   3   4
## 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160
##   3   5   4   4   3   3   3   3   3   3   3   3   3   4   3   3   4   3   3   4
## 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180
##   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3
## 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200
##   3   3   4   3   3   3   4   3   3   3   3   3   3   3   3   3   4   3   4   3
## 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220
##   3   3   4   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3
## 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240
##   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   3   2   2
## 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2
## 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416
##   2   2   2   2   2   2   2   2   2   2   2   2   2   2   2   1
```

```r
membership <- membership(grqc_fc_communities)

community_sizes <- sizes(grqc_fc_communities)

largest_communities <- order(community_sizes, decreasing = TRUE)[1:5]
```

MultiLevel Communities

```
#running multilevel (louvain) on entire graph with lower resolution parameter
grqc_ml_communities <- cluster_louvain(grqc, resolution = 0.25)
#0.9 modularity, higher than fast greedy
modularity(grqc_ml_communities)
```

```
## [1] 0.9050198
```

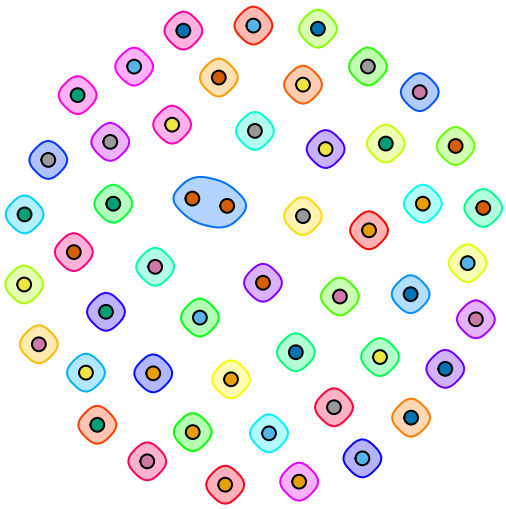
```
#370 communities
length(grqc_ml_communities)
```

```
## [1] 369
```

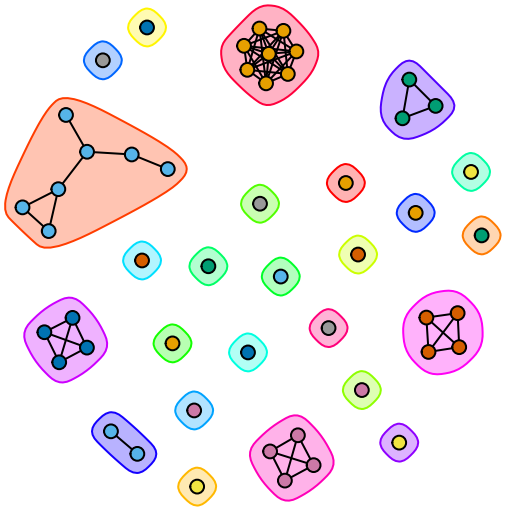
```
#low degree
ld_node_1 <- sample(low_degree_nodes, 50)
ld_graph_1 <- induced_subgraph(grqc, vids = ld_node_1)
grqc_ml_ld <- multilevel.community(ld_graph_1, resolution = 0.25)
```

```
## Warning: `multilevel.community()` was deprecated in igraph 2.0.0.
## i Please use `cluster_louvain()` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```

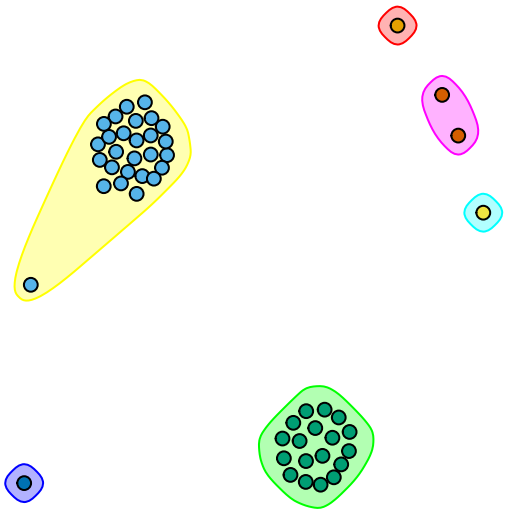
```
#q2
q2_node_1 <- sample(q2_degree_nodes, 50)
q2_graph_1 <- induced_subgraph(grqc, vids = q2_node_1)
grqc_ml_q2 <- multilevel.community(q2_graph_1, resolution = 0.25)

#q3
q3_node_1 <- sample(q3_degree_nodes, 50)
q3_graph_1 <- induced_subgraph(grqc, vids = q3_node_1)
grqc_ml_q3 <- multilevel.community(q3_graph_1, resolution = 0.25)

#high degree
high_node_1 <- high_degree_nodes
high_graph_1 <- induced_subgraph(grqc, vids = high_node_1)
grqc_ml_high <- multilevel.community(high_graph_1, resolution = 0.25)

plot(grqc_ml_ld, ld_graph_1,
     vertex.size = 6,
     vertex.label = NA,
     edge.color=NA)
```



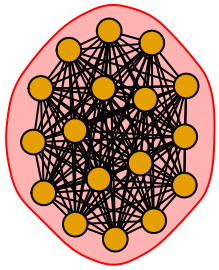
```
plot(grqc_ml_q2, q2_graph_1,
     vertex.size = 6,
     vertex.label = NA)
```



```
plot(grqc_ml_q3, q3_graph_1,
     vertex.size = 6,
     vertex.label = NA,
     edge.color=NA)
```



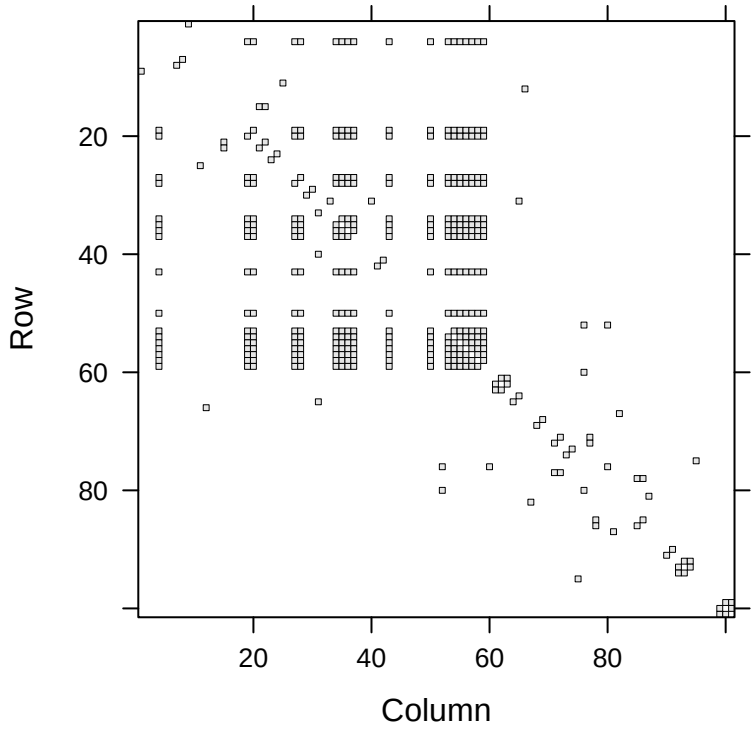
```
plot(grqc_ml_high, high_graph_1,
     vertex.size = 11,
     vertex.label = NA)
```



```
#fc_clusters <- grqc_fc_communities.as_clustering()
fc_membership <- membership(grqc_fc_communities)
ml_membership <- membership(grqc_ml_communities)
#eg_membership <- grqc_eg_communities.membership

ordered_fc <- order(fc_membership)
fc_reordered <- permute(grqc, ordered_fc)

ordered_ml <- order(ml_membership)
ml_reordered <- permute(grqc, ordered_ml)

#ordered_eg <- sorted(range(len(gc_membership)), key = lambda x: #eg_membership[x])
#gc_reordered <- grqc.permute_vertices(ordered_eg)
```

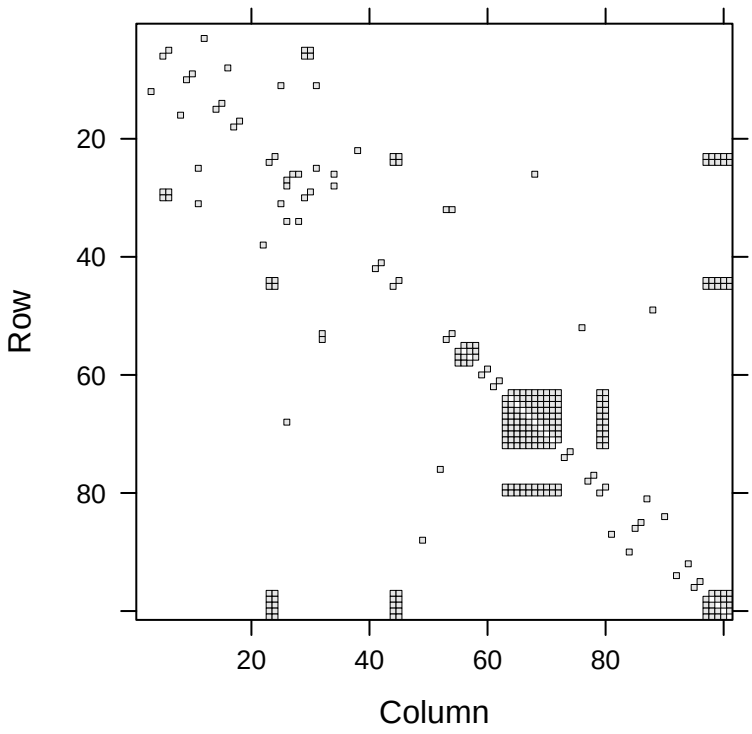
```
node_indices <- 100:200
fc_reordered.adjacency <- as_adj(fc_reordered, sparse = TRUE)

submatrix <- fc_reordered.adjacency[node_indices, node_indices]
```

```
image(Matrix(submatrix))
```



Dimensions: 101 x 101

```
# Right now, I see that there are less connections down the middle, which makes sense because all the n

ml_reordered.adjacency <- as_adj(ml_reordered, sparse = TRUE)
submatrix <- ml_reordered.adjacency[node_indices, node_indices]
image(Matrix(submatrix))
```



Dimensions: 101 x 101

```
#There are no dense areas of connection, more sparse
```

```
# layout with black background and color basd off membership of the components
g1<-grqc
clg <- components(grqc)
comps1<-clg$membership
colbar<-rainbow(max(comps1))
par(bg="black",new=FALSE)
V(g1)$color <- colbar[comps1]
plot(g1,
             edge.color="lightsteelblue1",
         edge.width=0.1, vertex.shape="circle",
             vertex.frame.color=V(g1)$color,
             vertex.color=V(g1)$color,  vertex.size=1.5,
         vertex.label=NA)
```