An Efficient Algorithm for PAC Mode Estimation

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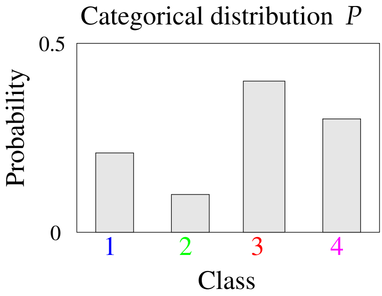
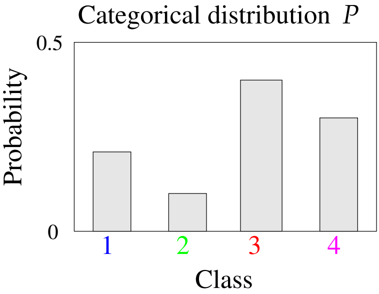
August 2023

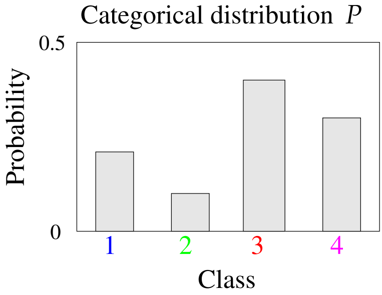

Categorical distribution P
0.5
Probability
0
1
2
3
4
Class

Mode Estimation



What is the **mode** of $\mathcal{P}$?

Mode Estimation



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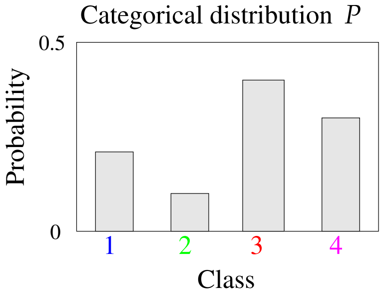

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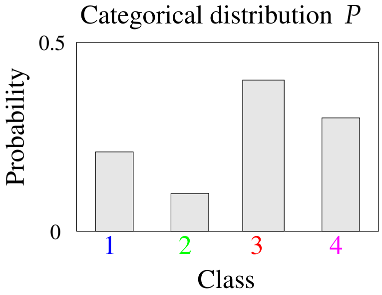

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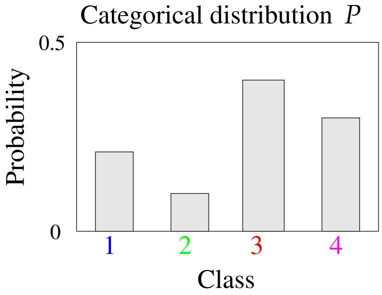

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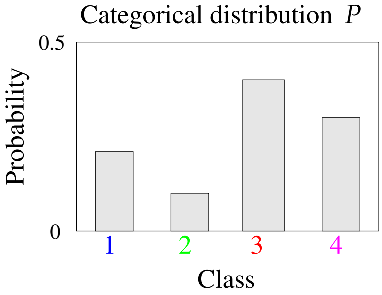

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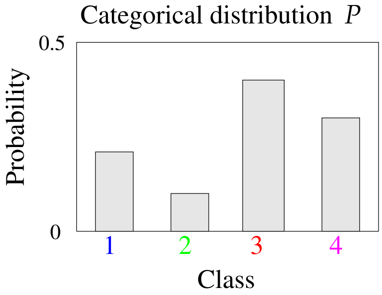

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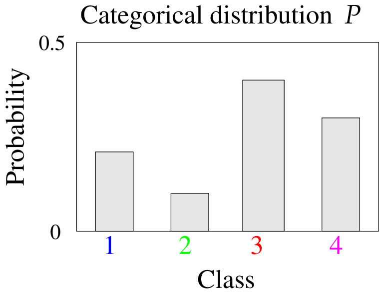

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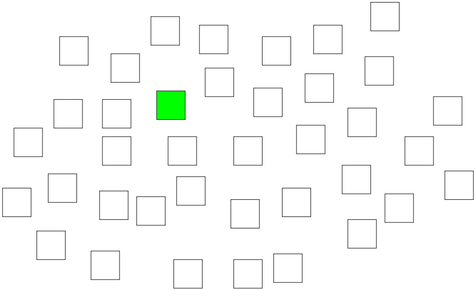
When can we confidently stop sampling and declare the mode?

Verifying Smart Contracts in Permissionless Blockchains

- Ask a node for the result of a hard computation.
- Node returns "Result = ■".



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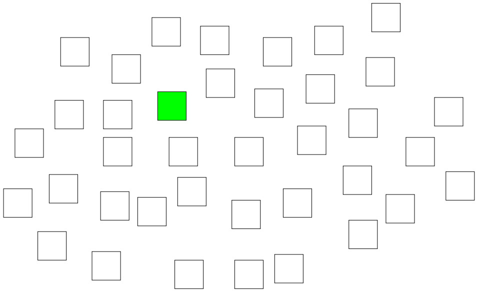

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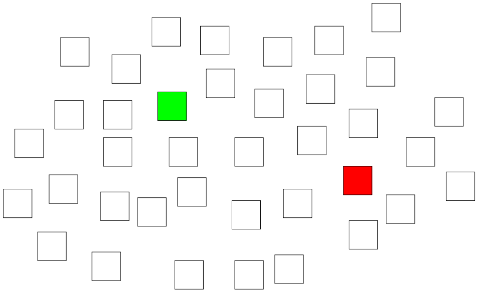

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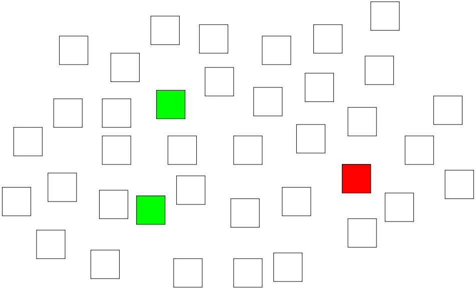

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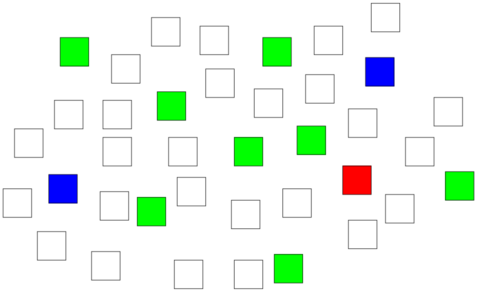

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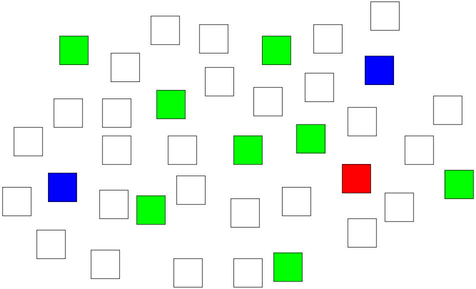
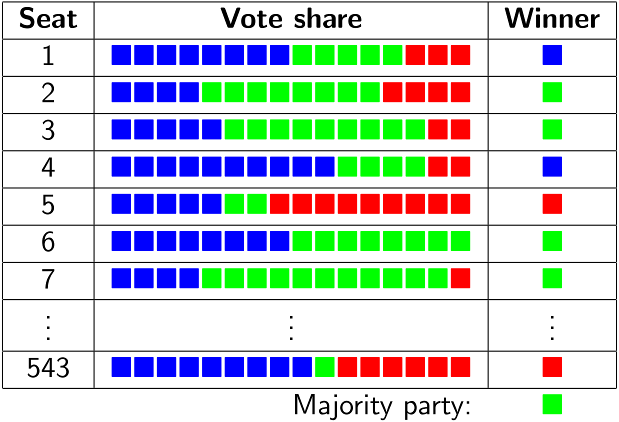


- Assume that most frequently returned result is correct.
- Determine most frequently returned result with sufficient confidence, while minimising number of queries.

Election Forecasting



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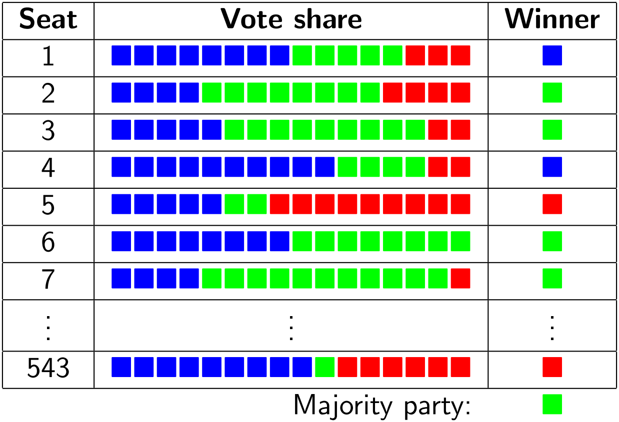

- Confidently identify winner of each seat by sampling.
- Aggregate wins to predict overall winner.

Overview of Talk

1. PAC Formulation
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- **Categorical distribution** $\mathcal{P} = (p, K)$:
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- **Prefer** δ-correct algorithm with low sample complexity.

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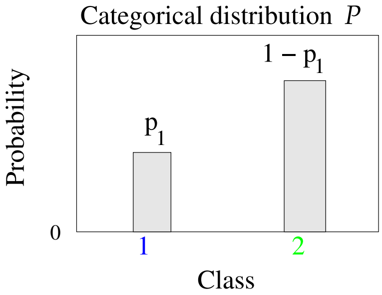
Solving the $K = 2$ Special Case



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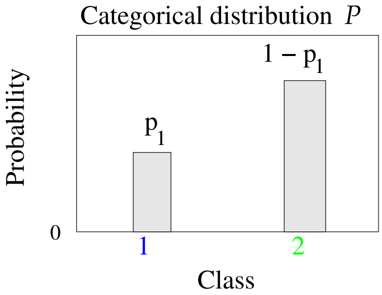

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- Maintain lower confidence bound LCB and upper confidence bound UCB on p_1, stop if $LCB > 0.5$ or $UCB < 0.5$.

Using Confidence Bounds

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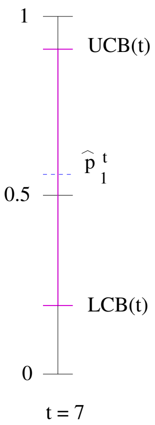

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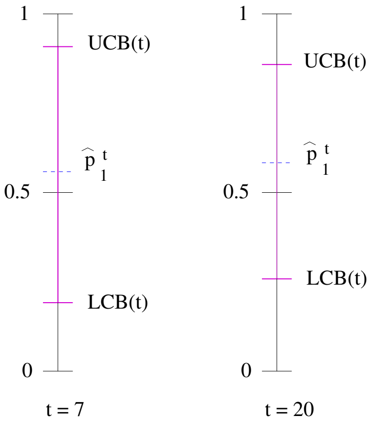

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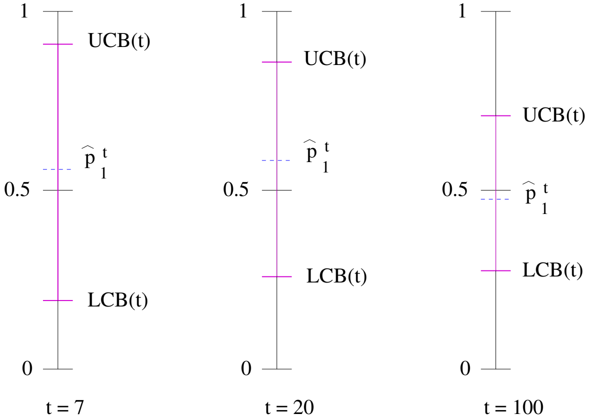

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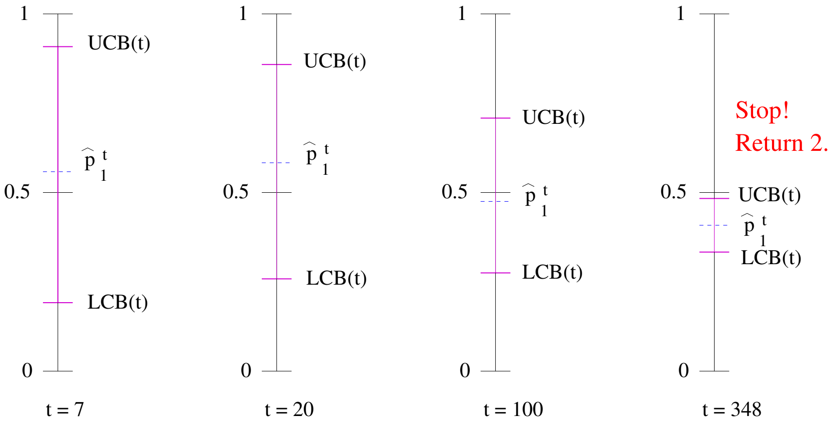

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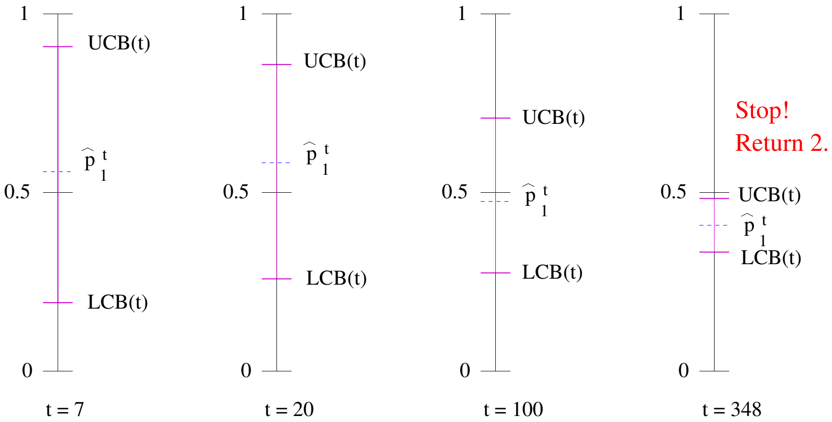

- Note that the stopping time is random.

Some Baselines

- Using Hoeffding's Inequality.

$$LCB(t), UCB(t) = \hat{p}_1^t \pm \beta(t, \delta),$$
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- "Self-normalised" inequality to avoid naïve union bound over t. KL-SN, Garivier (2013).

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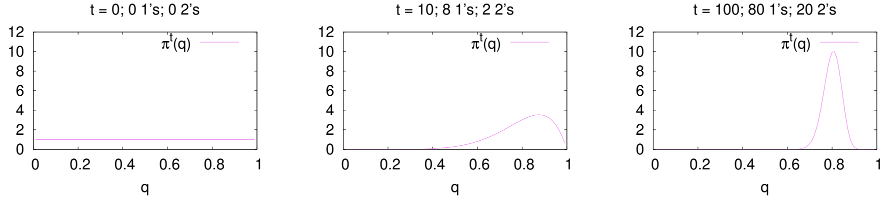

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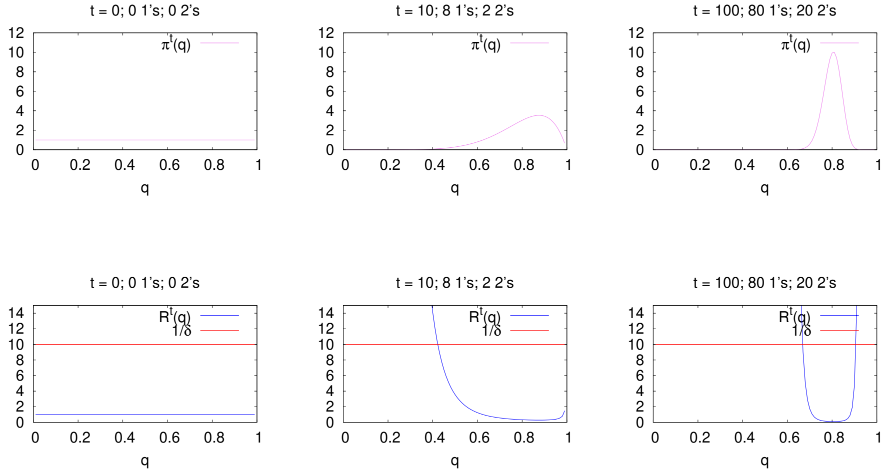
- Method more generally applicable, even for estimating multiple parameters simultaneously.

PPR Martingale Confidence Sequences: Illustration



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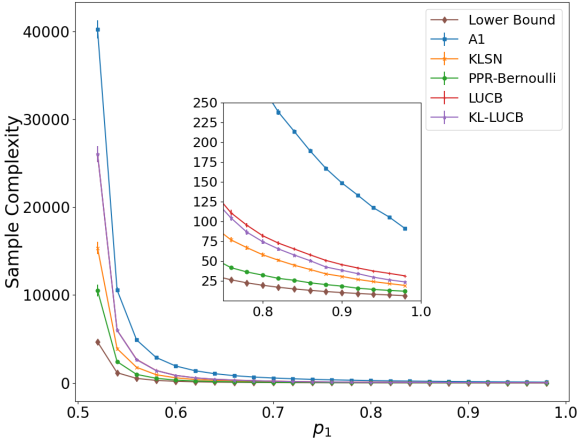
PPR-Bernoulli: Stop, declare first(t) as mode iff Beta $\left(\frac{1}{2}; s^t_{\text{first}(t)} + 1, s^t_{\text{second}(t)} + 1\right) \leq \delta$.

Comparison of Stopping Rules for $K = 2$



$$\delta = 0.01$$

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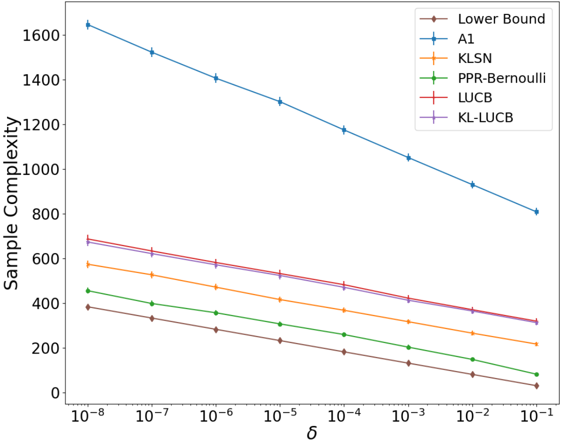

$$p_1 = 0.65$$

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Two Approaches to Generalise

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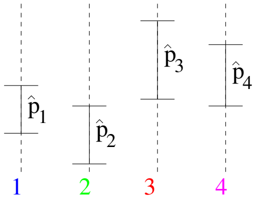
1-versus-rest approach



- For each class i, estimate underlying probability p_i.
- Stop and declare i as mode if LCB on p_i exceeds UCB on p_j for all $j \neq i$.

3, 1, 3, 4, 2, 1, 3, 2, 4, 3, 3, 1, 2, 3, . . .

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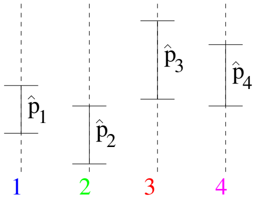

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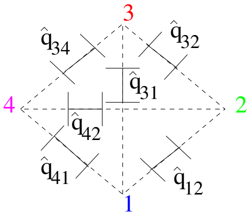

- For each pair of classes (i, j), estimate $q_{ij} = \frac{p_i}{p_i + p_j}$.
- Stop and declare i as mode if for all $j \neq i$, q_{ij} confidently exceeds 0.5.

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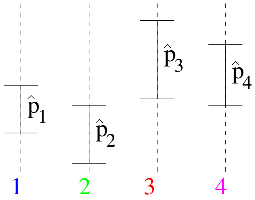

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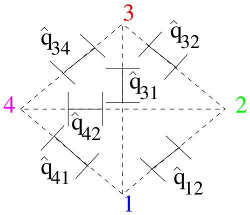

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1-versus-1 appears to waste samples, but . . .

Empirical Comparison: 1vr and 1v1

Sample complexity

Distribution	K	Type	$\mathcal{A}_1$	KL-SN	PPR
$\mathcal{P}_1$: .5, .25 $\times$ 2	3	1vr	1344$\pm$20	418$\pm$14	262$\pm$12
		1v1	1158$\pm$19	346$\pm$13	**218$\pm$11**
$\mathcal{P}_2$: .4, .2 $\times$ 3	4	1vr	1919$\pm$29	632$\pm$18	397$\pm$15
		1v1	1516$\pm$24	468$\pm$15	**298$\pm$13**
$\mathcal{P}_3$: .2, .1 $\times$ 8	9	1vr	5082$\pm$51	1900$\pm$42	1201$\pm$29
		1v1	3340$\pm$43	1138$\pm$31	**789$\pm$28**
$\mathcal{P}_4$: .1, .05 $\times$ 18	19	1vr	12015$\pm$129	4686$\pm$81	2850$\pm$55
		1v1	7352$\pm$88	2554$\pm$57	**1840$\pm$53**
$\mathcal{P}_5$: .35, .33, .12, .1 $\times$ 2	5	1vr	155277$\pm$2356	63739$\pm$2238	38001$\pm$1311
		1v1	117988$\pm$2078	47205$\pm$1291	**33660$\pm$1125**
$\mathcal{P}_6$: .35, .33, .04 $\times$ 8	10	1vr	158254$\pm$2442	66939$\pm$2241	41963$\pm$1330
		1v1	121150$\pm$2183	49576$\pm$1341	**36693$\pm$1185**

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- Computationally light compared to PPR-1vr, and other KL-divergence-based rules (which have to perform a numerical computation at each step).

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5. Summary and outlook

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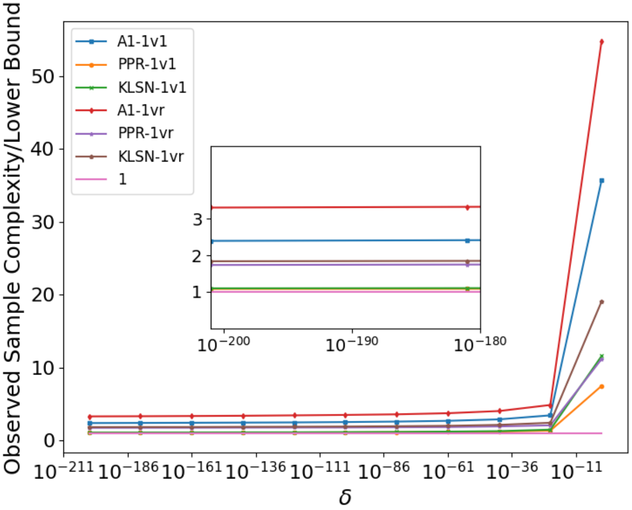
- Proof uses probability, functions, some results from Garivier and Kaufmann (2016) (who obtain similar result for PAC bandits).

Asymptotic Optimality (continued)



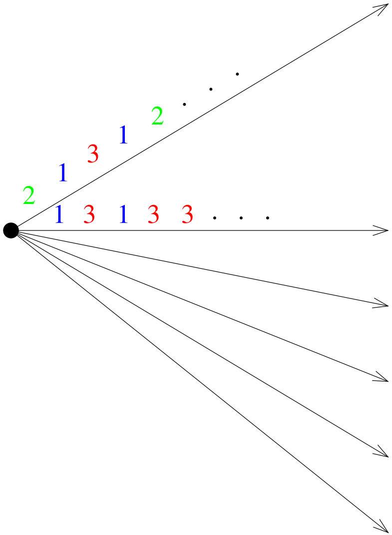
Result from $\mathcal{P}_3$

Uniform Domination

- Let $X = x^1, x^2, \ldots$ be an infinite sequence of samples from $\mathcal{P}$.
- For algorithm L, let $T(L, X)$ denote the stopping time of L on X.
- Define algorithm L_1 to uniformly dominate algorithm L_2 if for every sequence X, $\delta \in (0, 1)$,

$$T(L_2, X) < \infty \implies T(L_1, X) \leq T(L_2, X).$$

Uniform Domination



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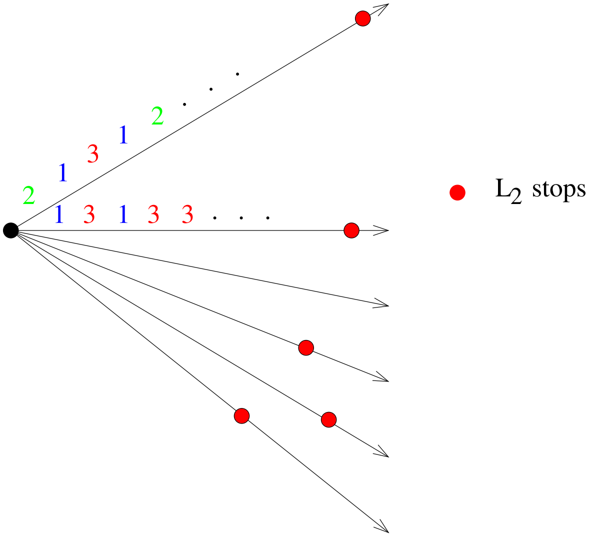

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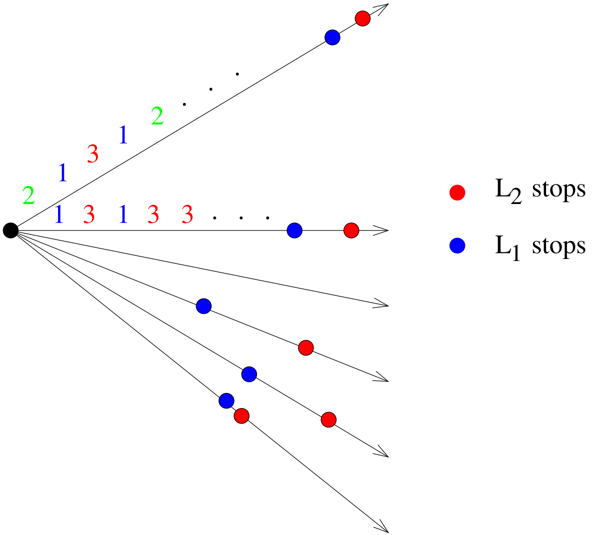

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- These proofs done by showing

$$L_2 \text{ terminates on } X \implies L_1 \text{ terminates on } X.$$

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Concluding Notes

- Have addressed PAC mode estimation problem.
- PPR an elegant approach to handle random stopping time.
- 1v1 outperforms 1vr to generalise to $K \geq 2$ classes.
- PPR-1v1 simple, asymptoticaly optimal, efficient in practice.
- Flexible to apply in practice, validated on two distinct applications.

- Code base released along with AISTATS 2022 paper.

- Conjecture on PPR-1v1 termination preceding PPR-1vr termination remains unresolved.
- Can PPR help in bandits, too?
- Do you have a practical application of mode estimation?