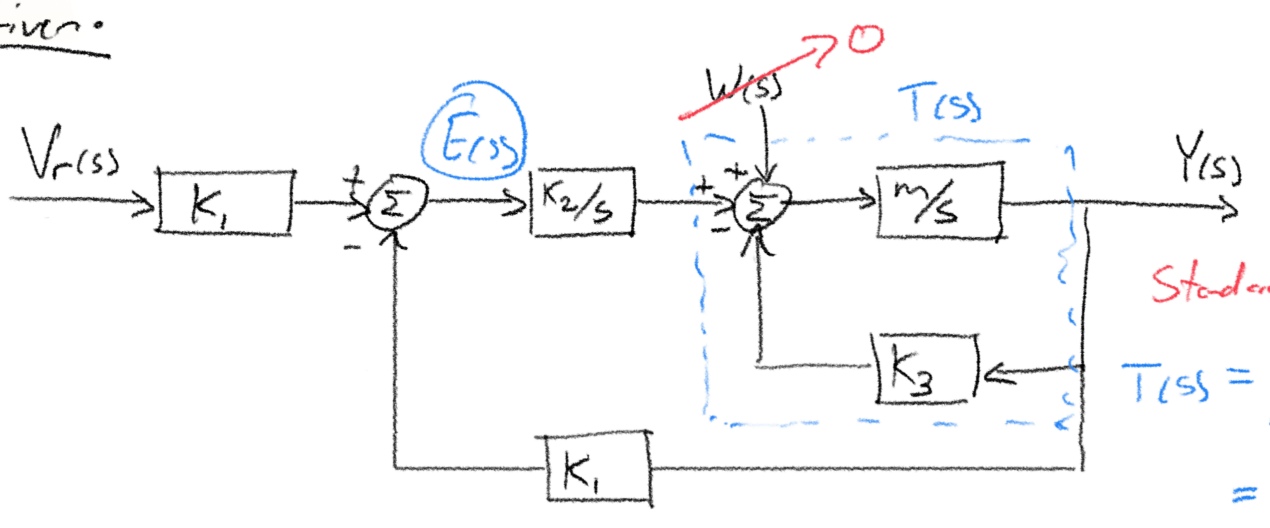
Session 5 5/2/25

Agenda:
0) Comment on Routh-Hurwitz
1) System type wrt input or disturbance

→ Block diagram → T.F. → Final Value Thm
(tedious!)

→ Trick w/ unity feedback

Given:



Standard feedback loop

$$T(s) = \frac{m/s}{1 + K_3 m/s} = \frac{m}{s + k_3 m}$$

Problem 1: Determine system type w.r.t. input $V_r(s)$.

Soln: Step 1: find t.f. $\frac{E(s)}{V_r(s)}$

Step 2: Use FVT

$$E(s) = V_r K_1 - Y K_1$$

$$Y(s) = E(s) T(s) K_2/s$$

$$\ldots \rightarrow \frac{E(s)}{V_r(s)} = \frac{K_1}{1 + K_1 T K_2/s} = \ldots = \boxed{\frac{K_1 s^2 + K_1 K_3 m s}{s^2 + K_3 m s + K_1 k_2 m}}$$

Step 2: FVT: $\underbrace{\lim_{t \to \infty} e(t)}_{e_{ss}} = \lim_{s \to 0} s E(s)$, $V_r(s) = \frac{A}{s^k}$ $\left(\begin{array}{l} k=1 \to \text{step} \\ k=2 \to \text{ramp} \\ k=3 \to \text{parabola} \end{array} \right)$

$$e_{ss} = \lim_{s \to 0} s \frac{K_1 s^2 + K_1 K_3 m s}{s^2 + k_3 m s + K_1 K_2 m} \frac{A}{s^k}$$

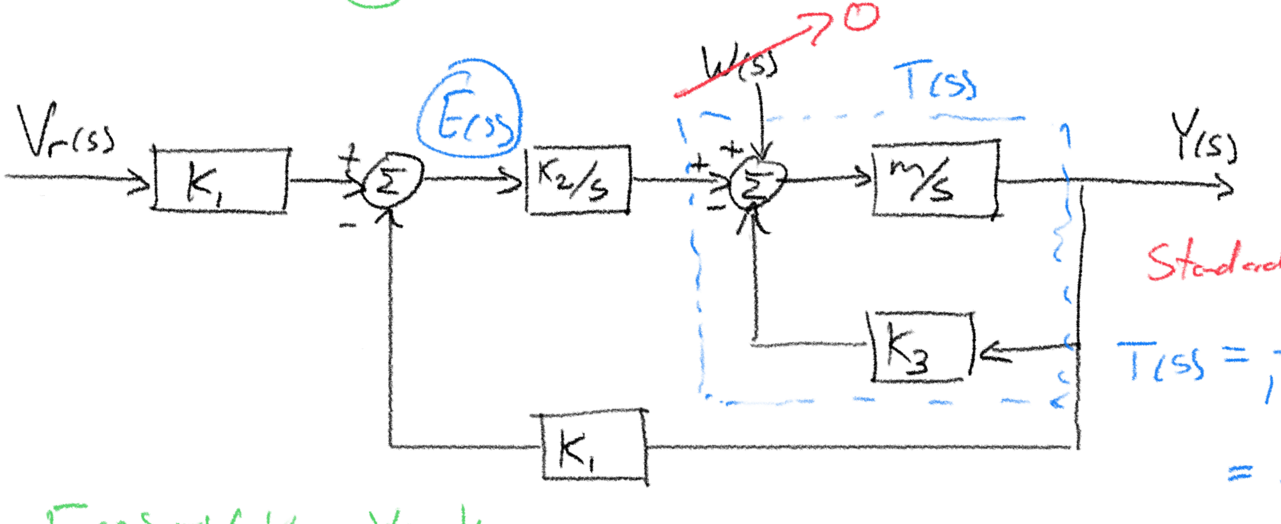
$k=1$: $\left(\frac{A}{s}\right)$ $e_{ss} = \lim_{s \to 0} \frac{A(\cancel{K_1 s^2} + \cancel{k_1 k_3 m s})}{\cancel{s^2} + \cancel{k_3 m s} + K_1 K_2 m} = 0$

$k=2$: $\left(\frac{A}{s^2}\right)$ $e_{ss} = \lim_{s \to 0} \frac{A(k_1 s^2 + k_1 k_3 m s)}{s^3 + k_3 m s^2 + k_1 k_2 m s} = \frac{A k_3}{k_2}$

$k=3$: $\rightarrow e_{ss} = \infty$

→ System is Type 1

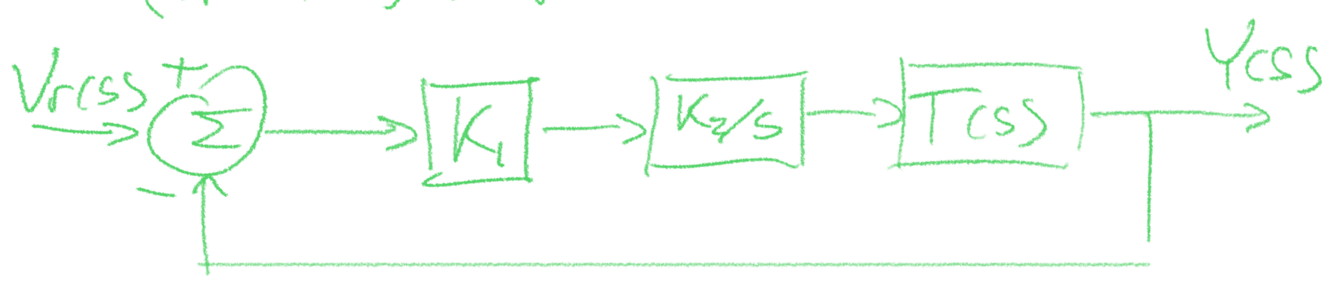
Use unity feedback if possible!



Standard feedback loop

$$T(s) = \frac{m/s}{1 + K_3 m/s} = \frac{m}{s + k_3 m}$$

$$E(s) = V_r K_1 - Y(s) K_1$$
$$(V_r - Y(s)) K_1$$



$$L(s) = K_1 K_2 T(s)/s = \frac{k_1 k_2 m}{s(s + k_3 m)}$$

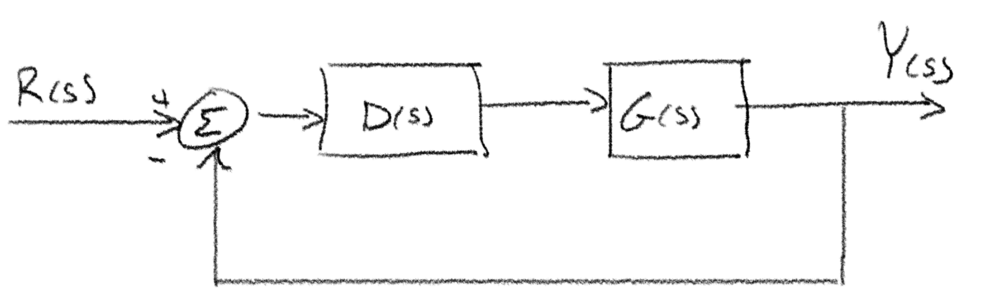
TABLE 4.1 Errors as a Function of System Type

Type Input	Step (position)	Ramp (velocity)	Parabola (acceleration)
Type 0	$\frac{1}{1+K_p}$	∞	∞
Type 1	0	$\frac{1}{K_v}$	∞
Type 2	0	0	$\frac{1}{K_a}$

Given:

$$G(s) = \frac{1}{ms^2 + bs}$$

$m = 2000$
$b = 500$



Problem 2:

Design a controller that meets these specs:

- $t_s < 10$ sec
- $M_p \leq 20\%$

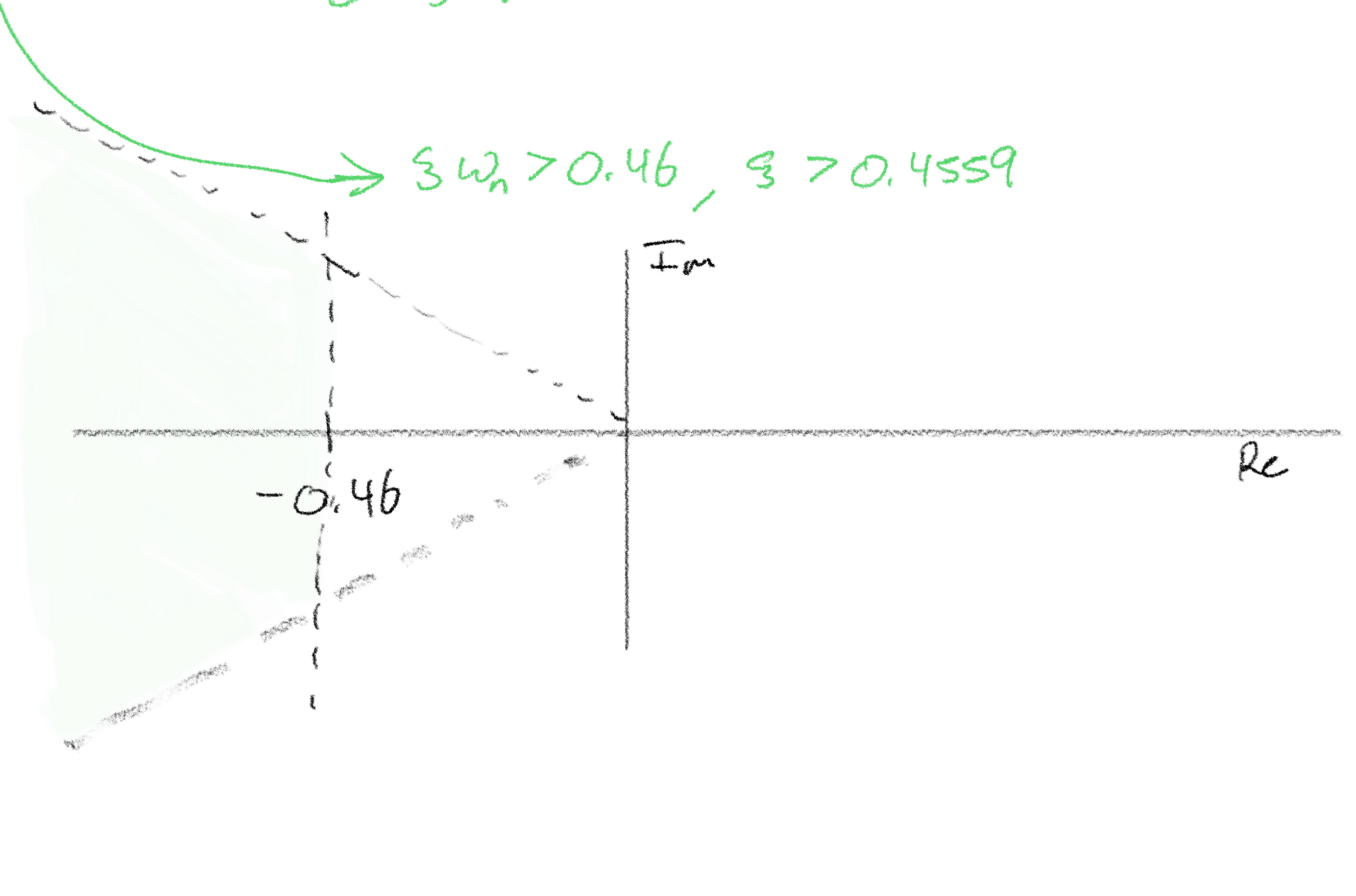
(Recall formulas: $t_s = \frac{4.6}{\zeta \omega_n}$
$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \rightarrow \zeta = \sqrt{\frac{\ln(M_p)^2}{\ln(M_p)^2 + \pi^2}}$)

Soln: $H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$$s = -\sigma \pm j\omega_d$$

$$\sigma = \zeta\omega_n$$

→ $\zeta\omega_n > 0.46$, $\zeta > 0.4559$



162 Chapter 3 Dynamic Response

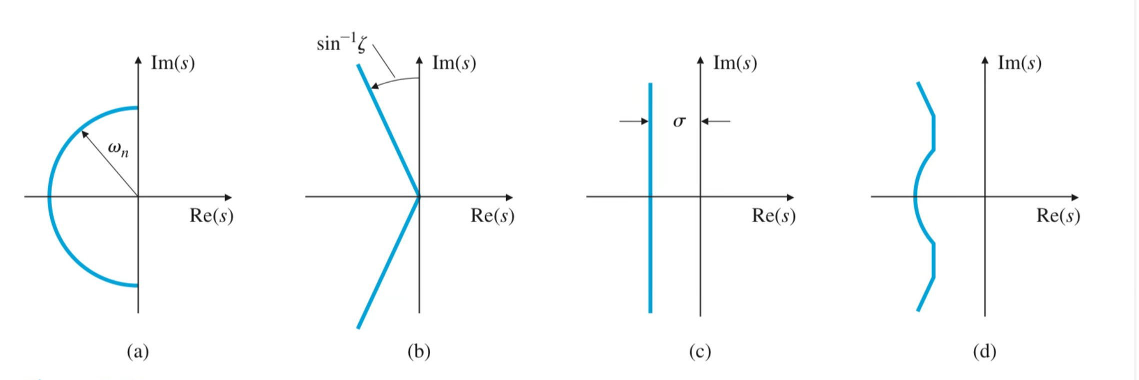
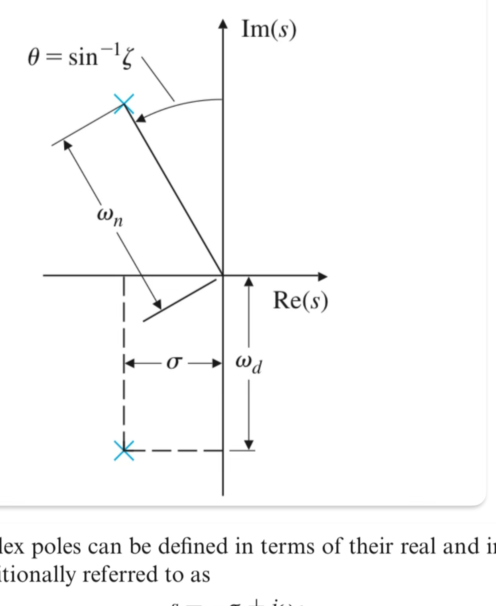


Figure 3.25
Graphs of regions in the s-plane delineated by certain transient requirements: (a) rise time; (b) overshoot; (c) settling time; (d) composite of all three requirements

Figure 3.18
s-plane plot for a pair of complex poles



Complex poles can be defined in terms of their real and imaginary parts, traditionally referred to as

$$s = -\sigma \pm j\omega_d.$$

This means a pole has a negative real part if σ is positive. Since complex poles always come in complex conjugate pairs, the denominator corresponding to a complex pair will be

$$a(s) = (s + \sigma - j\omega_d)(s + \sigma + j\omega_d) = (s + \sigma)^2 + \omega_d^2. \quad (3.62)$$

When finding the transfer function from second-order differential equations, we typically write the result in the polynomial form

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}. \quad (3.63)$$

By multiplying out the form given by Eq. (3.62) and comparing it with the coefficients of the denominator of $H(s)$ in Eq. (3.63), we find the correspondence between the parameters to be

$$\sigma = \zeta\omega_n \quad \text{and} \quad \omega_d = \omega_n\sqrt{1 - \zeta^2}, \quad (3.64)$$

Damping ratio; damped and undamped natural frequency

where the parameter ζ is the **damping ratio**[10] and ω_n is the **undamped natural frequency**. The poles of this transfer function are located at