Session 7:

1) Bode example
2) Midterm common issues
   (& advice for rest of quarter/final)

Bode Plots: (Chapter 6.1 in F&P)

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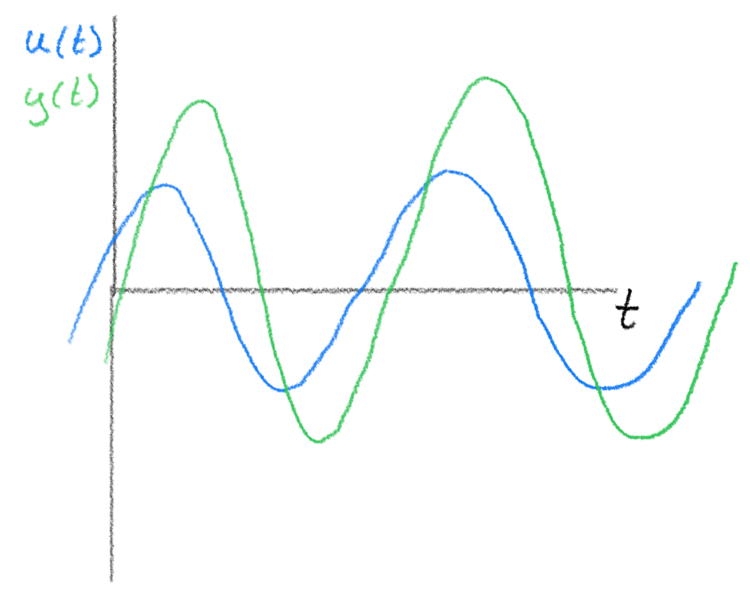
Response of LTI systems to sinusoidal input is also sinusoidal @ same frequency.
Only difference between input $u(t)$ and output $y(t)$ is <u>Magnitude</u> & <u>Phase</u>.

$$\underset{(u(t))}{U(s)} \longrightarrow \boxed{G(s)} \longrightarrow \underset{(y(t))}{Y(s)}$$



Bode form: $s = j\omega$

$$G(j\omega) = K_0 (j\omega)^n \frac{(j\omega\tau_1 + 1)\times(\ldots)}{(j\omega\tau_a + 1)\times\left[(j\omega/\omega_n)^2 + 2\zeta j\omega/\omega_n + 1\right]\times\ldots}$$

3 types of terms:

1) $K_0 (j\omega)^n$ (poles or zeros @ origin)
2) $(j\omega\tau + 1)^{\pm 1}$ (first-order terms)
3) $\left[(j\omega/\omega_n)^2 + 2\zeta j\omega/\omega_n + 1\right]$ (second-order terms)

Bode Plot Rules:

1) $K_0 (j\omega)^n$:

Magnitude: line of slope $n \times 20$dB per decade passing through $\log K_0$ @ $\omega = 1$

Phase: $n \times 90°$

2) $(j\omega\tau + 1)^{\pm 1}$ $\quad \omega_{cf} = \frac{1}{\tau}$ $\quad (\omega_{cf}\tau = 1)$

Magnitude:
- 0 dB — $\omega \ll \omega_{cf}$
- line w/ slope ± 1 (20 dB/decade) — $\omega \gg \omega_{cf}$

Phase:
- $0°$ — $\omega\tau < 0.1$
- $\pm 45°$ — $\omega = \omega_{cf}$
- $\pm 90°$ — $\omega\tau > 10$

3) $\left[(j\omega/\omega_n)^2 + 2\zeta j\omega/\omega_n + 1\right]^{\pm 1}$

Magnitude:
- 0 dB — $\omega \ll \omega_n$
- line w/ slope ± 2 (40 dB/decade) — $\omega \gg \omega_n$

Phase:
- $0°$ — $\omega \ll \omega_n$
- $\pm 90°$ — $\omega = \omega_n$
- $\pm 180°$ — $\omega \gg \omega_n$

F&P Chapter 6

Summary of Bode Plot Rules

1. Manipulate the transfer function into the Bode form given by Eq. (6.16).
2. Determine the value of n for the $K_o(j\omega)^n$ term (class 1). Plot the low-frequency magnitude asymptote through the point K_o at $\omega = 1$ with a slope of n (or $n \times 20$ db per decade).
3. Complete the composite magnitude asymptotes: Extend the low-frequency asymptote until the first frequency break point. Then step the slope by ± 1 or ± 2, depending on whether the break point is from a first- or second-order term in the numerator or denominator. Continue through all break points in ascending order.
4. The approximate magnitude curve is increased from the asymptote value by a factor of 1.4 (+3 db) at first-order numerator break points, and decreased by a factor of 0.707 (−3 db) at first-order denominator break points. At second-order break points, the resonant peak (or valley) occurs according to Fig. 6.3(a), using the relation $|G(j\omega)| = 1/2\zeta$ at denominator (or $|G(j\omega)| = 2\zeta$ at numerator) break points.
5. Plot the low-frequency asymptote of the phase curve, $\phi = n \times 90°$.
6. As a guide, the approximate phase curve changes by $\pm 90°$ or $\pm 180°$ at each break point in ascending order. For first-order terms in the numerator, the change of phase is $+90°$; for those in the denominator the change is $-90°$. For second-order terms, the change is $\pm 180°$.
7. Locate the asymptotes for each individual phase curve so their phase change corresponds to the steps in the phase toward or away from the approximate curve indicated by Step 6. Each individual phase curve occurs as indicated by Fig. 6.8 or Fig. 6.3(b).
8. Graphically add each phase curve. Use grids if an accuracy of about $\pm 5°$ is desired. If less accuracy is acceptable, the composite curve can be done by eye. Keep in mind that the curve will start at the lowest-frequency asymptote and end on the highest-frequency asymptote and will approach the intermediate asymptotes to an extent that is determined by how close the break points are to each other.