Session 8 5/13/25

Agenda:

0) Quick hint for Hw 6 Problem 1
1) Bode Plots from TF
2) TF from Bode Plot

Homework 6 Problem 1:

- Need to use "phase condition" to find z
- Then use "magnitude condition" to find K

} (ctrl + F) F&P Chapter 5.2

Bode Plot Primer (nothing new, will post)

At some point in the engineering process, we transition from having mathematical models of our system to having a real physical system (plant). *What matters at the end of the day is that our control design to perform well on the real plant.* We may decide to spend a significant amount of effort tuning our mathematical models in order to get them to match the real physical system so that we can use our model-based control designs on the real plant; note that tuning the model in this way requires running experiments on our system to have data to compare against. This is a *good idea*: we know a lot about physics, mathematical modeling, model-based control design, and system analysis. Often though, this model fitting process can be challenging and tedious.

It turns out that it is possible to analyze and even do control design directly with certain types of experiment data. Specifically, we can do all of this using one of the most common and straightforward types of tests: **the frequency response test**, i.e. vibration tests. The frequency response, whether it is obtained analytically from a model or by empirical means, is conveyed through a plot of the magnitude and phase of the output of a system divided by the input to the system for periodic inputs. Physically, this corresponds to exciting the plant with sinusoidal signals (vibrations). Together, the magnitude and phase plots form the **Bode Plot**. Here, we will describe how to sketch a Bode Plot given an analytical transfer function; the same process can be used in reverse to estimate a transfer function given a Bode Plot. For a thorough discussion of the topic, see Franklin & Powell Chapter 6.

Bode Plot Primer[1]

The Bode Plot is based on the putting the transfer function in **Bode form**:

$$H(s)\Big|_{s=j\omega} = K_o(j\omega)^n \frac{(j\omega\tau_1+1)(j\omega\tau_2+1)\dots}{(j\omega\tau_a+1)(j\omega\tau_b+1)\dots} \quad (1)$$

In the Bode form, there will be three classes of terms. Each of these classes of terms has rules associated for plotting its Bode Plot contributions (magnitude and phase). Since the system is linear, we can plot each component separately, and then add them together to create the composite plot. The classes and their Bode Plot rules are summarized below.

Type of term	Magnitude	Phase
Class 1: $K_o(j\omega)^n$ $\approx \frac{1}{100}$	line through $20\log_{10} K_o$ @ $\omega = 1$ w/ slope $n \times 20$ dB/dec.	constant $n \times 90°$.
Class 2: $(j\omega\tau_1+1)^{\pm 1}$ $\omega_{cf} = 1/\tau$	0 dB for $\omega \ll \omega_{cf}$, and line w/ slope ± 20 dB/dec for $\omega \gg \omega_{cf}$.	★ 0° for $\omega \ll \omega_{cf}$ and $\pm 90°$ for $\omega \gg \omega_{cf}$, through $\pm 45°$ for $\omega = \omega_{cf}$. (1 dec on either side)
Class 3: $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^{\pm 1}$	0 dB for $\omega \ll \omega_n$, and line w/ slope ± 40 dB/dec for $\omega \gg \omega_n$; peak height estimated by ζ.	0° for $\omega \ll \omega_n$ and $\pm 180°$ for $\omega \gg \omega_n$, through $\pm 90°$ for $\omega = \omega_n$.

★ for rhp zeros, only the phase changes:
- start at 180° instead of 0°
- decrease phase rather than increase

Using the rules in the table, we can plot each component separately, and then obtain the composite plot by combining all of the plots together.

Summary of Bode Plot Rules (copied from Franklin & Powell)

Instead of plotting each component separately, it is also possible to directly construct a sketch of the composite plot; this should be apparent after practicing the Bode plot a few times. Below is a summary of the procedure.

Step 1. *Manipulate the transfer function into the Bode form given by eq. (1).*

Step 2. *Determine the value of n for the $K_o(j\omega)^n$ term (Class 1). Plot the low-frequency magnitude asymptote through the point K_o at $\omega = 1$ with a slope of n (or $n\times$ 20 db per decade).* (20 log K_o)

Step 3. *Complete the composite magnitude asymptotes:*
- *Extend the low-frequency asymptote until the first frequency break point.*
- *Then step the slope by ± 1 or ± 2, depending on whether the break point is from a first- or second-order term in the numerator or denominator.*
- *Continue through all break points in ascending order.*

Step 4. *Complete the composite phase asymptotes:*
- *Plot the low-frequency asymptote of the phase curve, $\phi = n \times 90°$.*
- *As a guide, the approximate phase curve changes by $\pm 90°$ or $\pm 180°$ at each break point (depending on the class type) in ascending order.*
- *Add each phase curve based on the guides.*

[1]This write-up is based on Franklin, Powell, & Naeini's "Feedback Control of Dynamic Systems".

Ex. 1: $H(s) = \frac{s+0.1}{(s+1)(s+10)}$

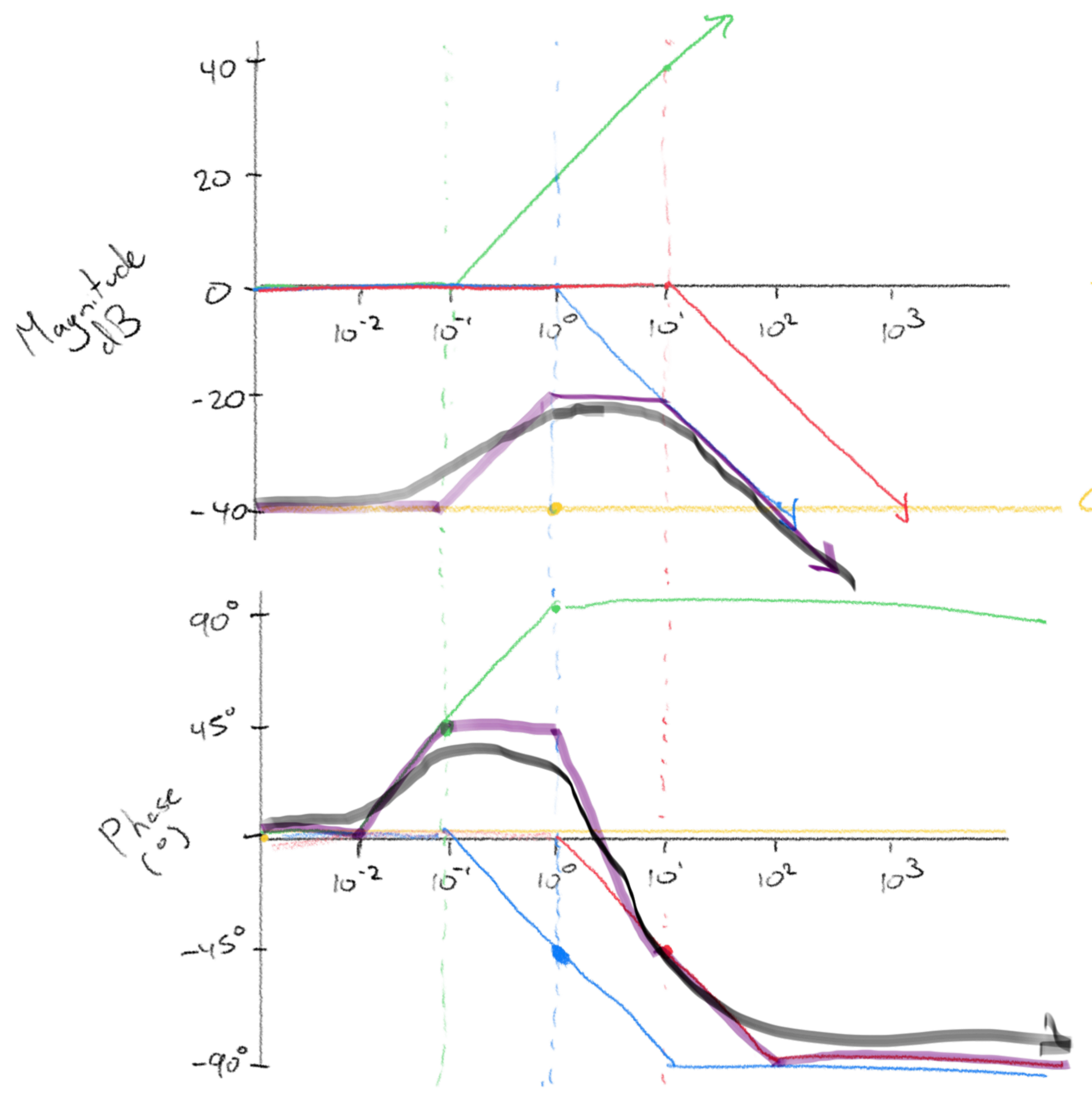
✓Step 1) convert to Bode form: $H(s) = \frac{0.1\left(\frac{s}{0.1}+1\right)}{(s+1)\cdot 10\cdot\left(\frac{s}{10}+1\right)} \overset{(s=j\omega)}{=} \frac{1}{100} \times \frac{\left(\frac{j\omega}{0.1}+1\right)}{(j\omega+1)(j\omega/10+1)}$

✓Step 2) plot class 1 terms

Step 3) plot class 2 terms

Step 4) plot class 3 terms

Step 5) combine (Mag & phase)



$\frac{1}{100} \times \frac{\left(\frac{j\omega}{0.1}+1\right)}{(j\omega+1)(j\omega/10+1)}$

class 1 term

Method 2:

Summary of Bode Plot Rules (copied from Franklin & Powell)

Instead of plotting each component separately, it is also possible to directly construct a sketch of the composite plot; this should be apparent after practicing the Bode plot a few times. Below is a summary of the procedure.

Step 1. Manipulate the transfer function into the Bode form given by eq. (1).

Step 2. Determine the value of n for the $K_o(j\omega)^n$ term (Class 1). Plot the low-frequency magnitude asymptote through the point K_o at $\omega = 1$ with a slope of n (or $n\times$ 20 db per decade). (20 log K_o)

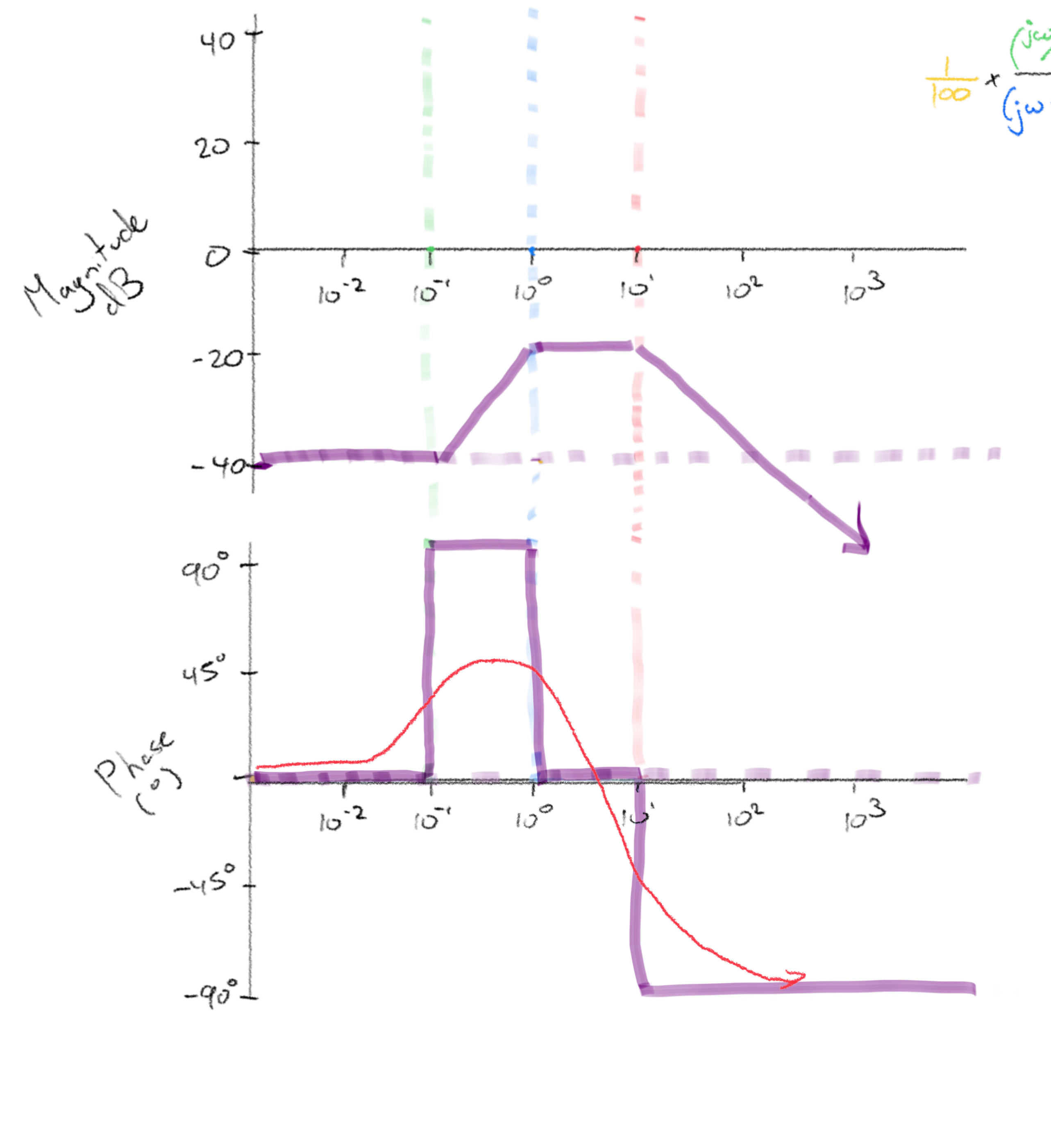
Step 3. Complete the composite magnitude asymptotes:
- Extend the low-frequency asymptote until the first frequency break point.
- Then step the slope by ± 1 or ± 2, depending on whether the break point is from a first- or second-order term in the numerator or denominator.
- Continue through all break points in ascending order.

Step 4. Complete the composite phase asymptotes:
- Plot the low-frequency asymptote of the phase curve, $\phi = n \times 90°$.
- As a guide, the approximate phase curve changes by $\pm 90°$ or $\pm 180°$ at each break point (depending on the class type) in ascending order.
- Add each phase curve based on the guides.

Step 1: $H(s) = \frac{1}{100} \times \frac{(j\omega/0.1+1)}{(j\omega+1)(j\omega/10+1)}$

Step 2:

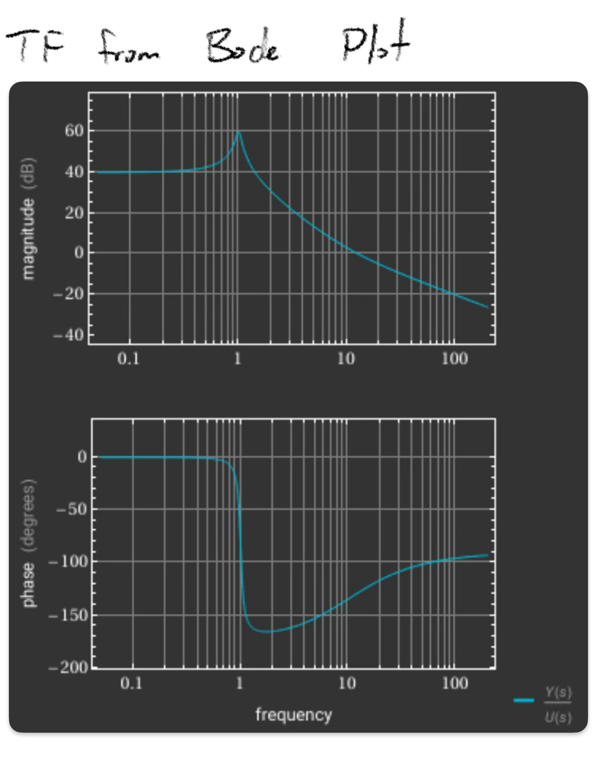
Step 3: }
Step 4: } break points:



$\frac{1}{100} \times \frac{(j\omega/0.1+1)}{(j\omega+1)(j\omega/10+1)}$

The process w/ class 3 terms is mostly the same; things generally are 2x, and there is a peak due to resonance (if $\zeta < 1$)

TF from Bode Plot



(you can practice this easily w/ Matlab or Wolfram Alpha)

observations: