Problem Session 4 4/25/25

Agenda:

1) Homework hints
2) Wrap up of throttle valve example (position servo control)
   -> "Throttle trace" specification
   -> System type
   -> PD vs PID, pros & cons (a word of caution)

☆ 3) Block diagram algebra (more complex example)

Homework hints:

- Formula for M_p: $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$

  $\hookrightarrow \xi = \sqrt{\dfrac{\ln(M_p)^2}{\ln(M_p)^2 + \pi^2}}$ ← don't cancel $\sqrt{(\ )^2}$ leads to ± issue

Routh-Hurwitz Primer (see end of HW 4 document)

EXAMPLE 3.32 *Routh's Test*

↑ F&P 8th ed. warning! typo!

The polynomial

$$a(s) = s^6 + s^5 + s^4 + 2s^3 + 5s^2 + 2s + 3$$

satisfies the necessary condition for stability since all the $\{a_i\}$ are positive and nonzero. Determine whether any of the roots of the polynomial are in the RHP.

Solution. The Routh array for this polynomial is

$$\begin{array}{lllll}
s^6: & 1 & 1 & 5 & 3 \\
s^5: & 1 & 2 & 2 & 0 \\
s^4: & -1 = \dfrac{1\cdot 1 - 1\cdot 2}{4} & 3 = \dfrac{1\cdot 5 - 1\cdot 2}{1} & 3 = \dfrac{1\cdot 3 - 1\cdot 0}{1} & \\
s^3: & 5 = \dfrac{-1\cdot 2 - 1\cdot 3}{-1} & 5 = \dfrac{-1\cdot 2 - 1\cdot 3}{-1} & 0 & \\
s^2: & 4 = \dfrac{5\cdot 3 + 1\cdot 5}{5} & 3 = \dfrac{5\cdot 3 + 1\cdot 0}{5} & & \\
s: & \dfrac{5}{4} = \dfrac{4\cdot 5 - 5\cdot 3}{4} & 0 & & \\
s^0: & 3 = \dfrac{\frac{5}{4}\cdot 3 - 4.0}{\frac{5}{4}} & & &
\end{array}$$

$$\begin{array}{c|ccccc}
s^n & a_0 & a_2 & a_4 & a_6 & \dots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \dots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 & \dots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \\
s^1 & m_1 & 0 & 0 & 0 & \\
s^0 & n_1 & 0 & 0 & 0 &
\end{array}$$

where $a_0, a_1, \dots, a_n$ are the coefficients of the characteristic polynomial eq. (1) and

$$b_1 = -\frac{1}{a_1}\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix},\quad b_2 = -\frac{1}{a_1}\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix},\quad b_3 = -\frac{1}{a_1}\begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix},\quad \vdots$$

$$\begin{array}{c|ccccc}
s^n & a_0 & a_2 & a_4 & a_6 & \dots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \dots \\
s^{n-2} & b_1 & b_2 & b_3 & b_4 & \dots \\
s^{n-3} & c_1 & c_2 & c_3 & c_4 & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \\
s^1 & m_1 & 0 & 0 & 0 & \\
s^0 & n_1 & 0 & 0 & 0 &
\end{array}$$

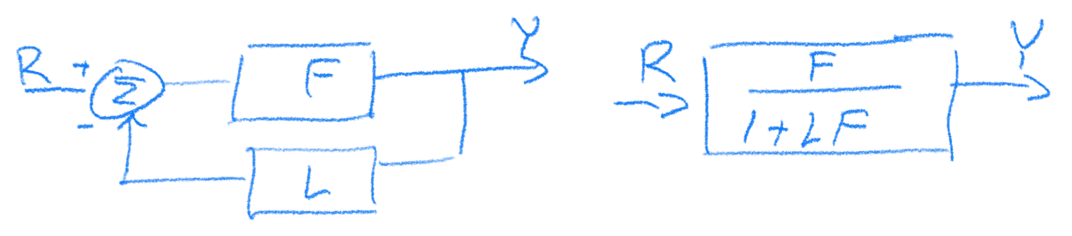
$$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix},\quad c_2 = -\frac{1}{b_1}\begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix},\quad c_3 = -\frac{1}{b_1}\begin{vmatrix} a_1 & a_7 \\ b_1 & b_4 \end{vmatrix},\quad \vdots$$

are the entries in the row corresponding to s^{n-3}, and so on.

(Servo throttle problem on GitHub)

☆ Block Diagram Algebra

Feedback manipulation

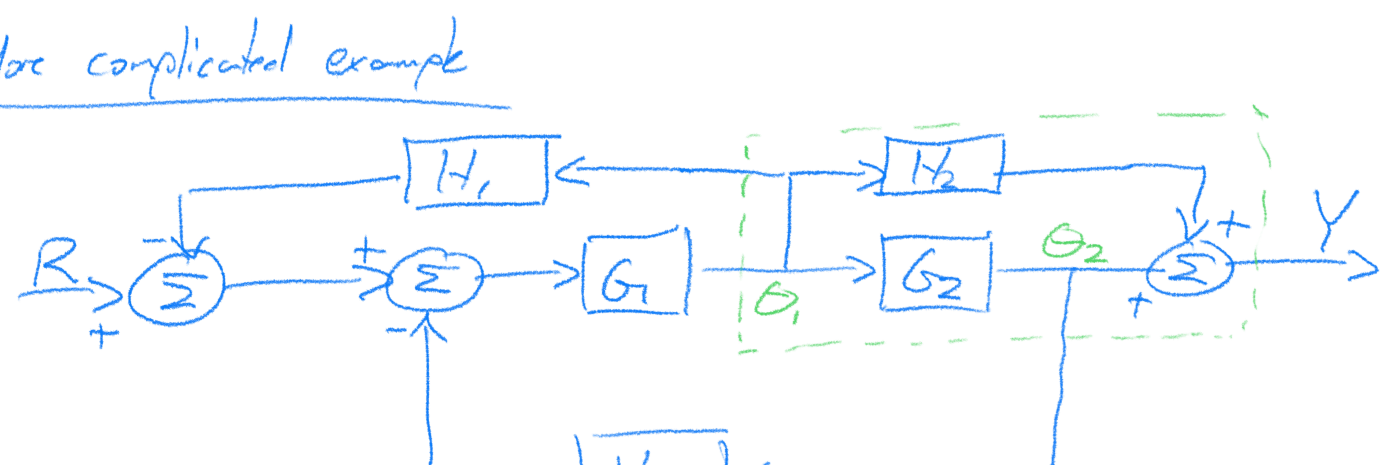


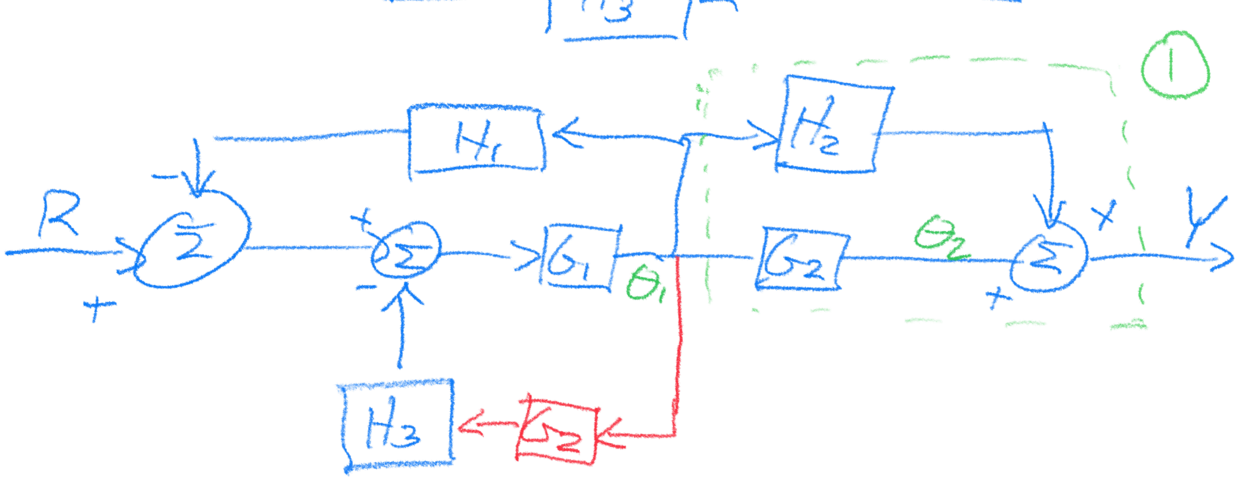
Feedforward Manipulation

General Approach:

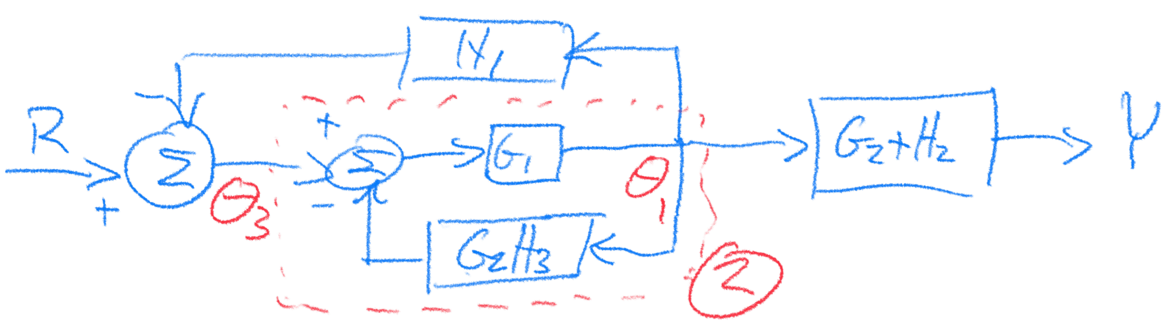
1) Isolate loops
2) Collapse loops

More complicated example

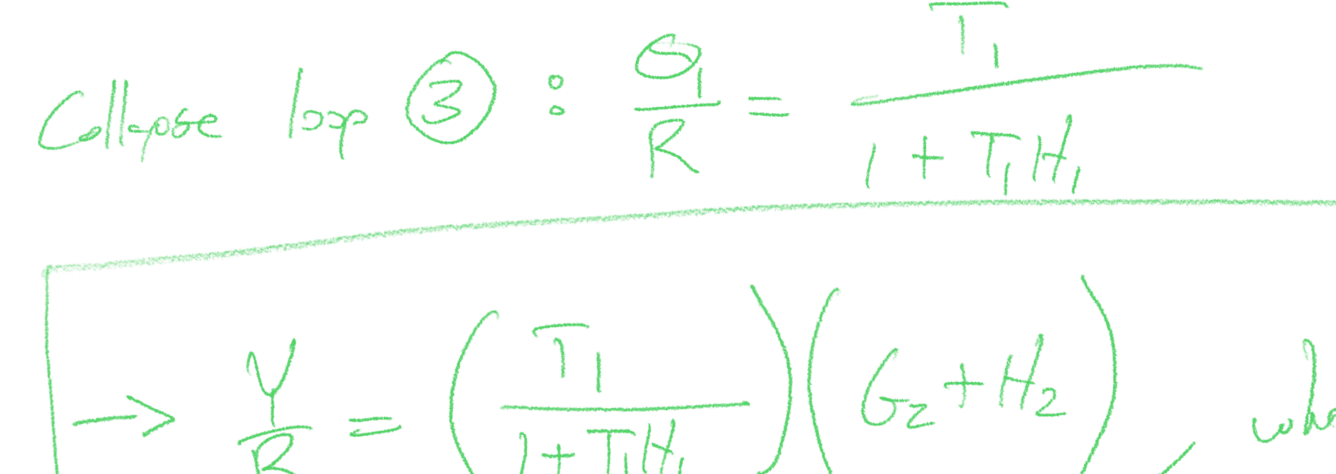




Collapse loop ①: $\dfrac{Y}{\theta_1} = G_2 + H_2$



$$\frac{\theta_3}{\theta_1} = \frac{G_1}{1 + G_1G_2H_3} =: T_1$$



Collapse loop ③: $\dfrac{\theta_1}{R} = \dfrac{T_1}{1 + T_1H_1}$

$$\boxed{\rightarrow \frac{Y}{R} = \left(\frac{T_1}{1+T_1H_1}\right)\left(G_2 + H_2\right), \text{ where } T_1 = \frac{G_1}{1 + G_1G_2H_3}}$$

$$= \frac{G_1G_2 + G_1H_2}{1 + G_1H_1 + G_1G}$$