Session 6: 5/9/25

0) Quick clarification on Rules 4 & 5 for Root Locus

Some topics we've covered so far...

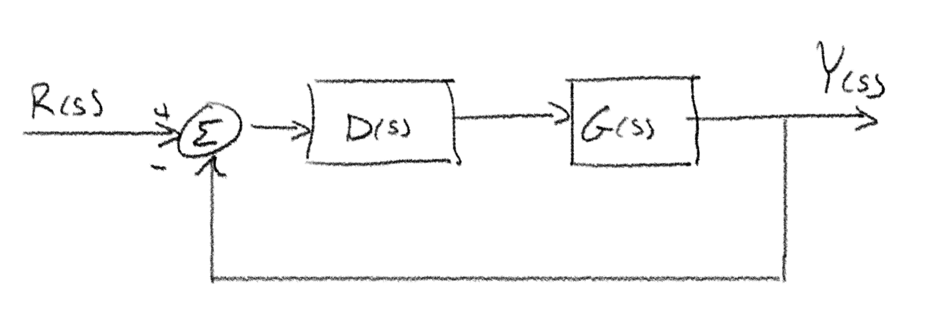
1) Deriving T.F. from Newton's 2nd law
   - Closing the loop w/ P vs PD control → pole locations, stability, etc.
2) Effect of pole/zero locations on transient response
3) Routh-Hurwitz criterion & control design (to stabilize a system)
4) Time-domain performance specifications (rise time vs settling time, overshoot, etc.)
5) Root locus & control design based on pole locations (relation to time-domain specs.) (continue ex. from last week)

Highly encourage to study the worked out examples in F&P and in the posted Hw solutions

Given:

$$G(s) = \frac{1}{ms^2+bs}$$

$m = 2000$
$b = 500$



Problem 2:

Design a controller that meets these specs:

- $t_s \leq 10$ sec
- $M_p \leq 20\%$

(Recall formulas: $t_s = \frac{4.6}{\zeta\omega_n}$, $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \rightarrow \zeta = \sqrt{\frac{\ln(M_p)^2}{\ln(M_p)^2+\pi^2}}$)

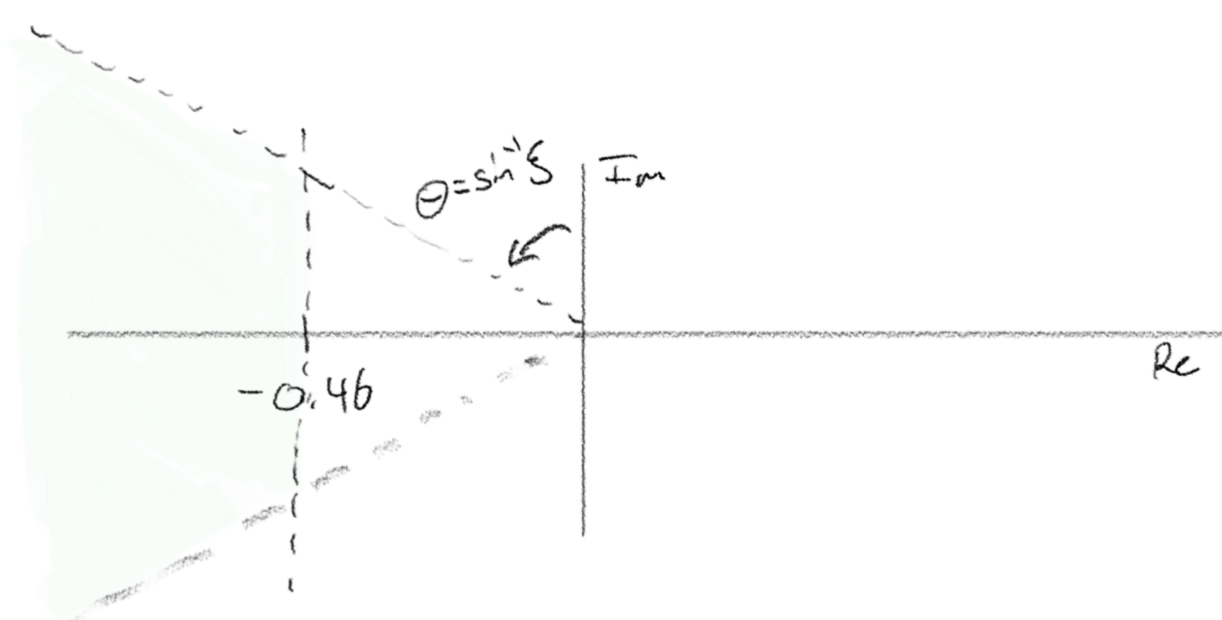
Sol'n: Assume closed-loop system behaves like a second-order system w/ no zeros, so settling time and overshoot even make sense to talk about. Then closed-loop t.f. will have the general form

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

- $p_{1,2} = -\sigma \pm j\omega_d$
- $\sigma = \zeta\omega_n$

$t_s < 10s$: $t_s = \frac{4.6}{\zeta\omega_n} < 10 \rightarrow \zeta\omega_n > 0.46$

$M_p < 20\%$: $\rightarrow \zeta > 0.46$



$D = K \rightarrow H(s) = \frac{DG}{1+DG} \rightarrow 1 + DG = 0$

$$\hookrightarrow 1 + K\underbrace{\left(\frac{1}{ms^2+bs}\right)}_{L(s)} = 0$$

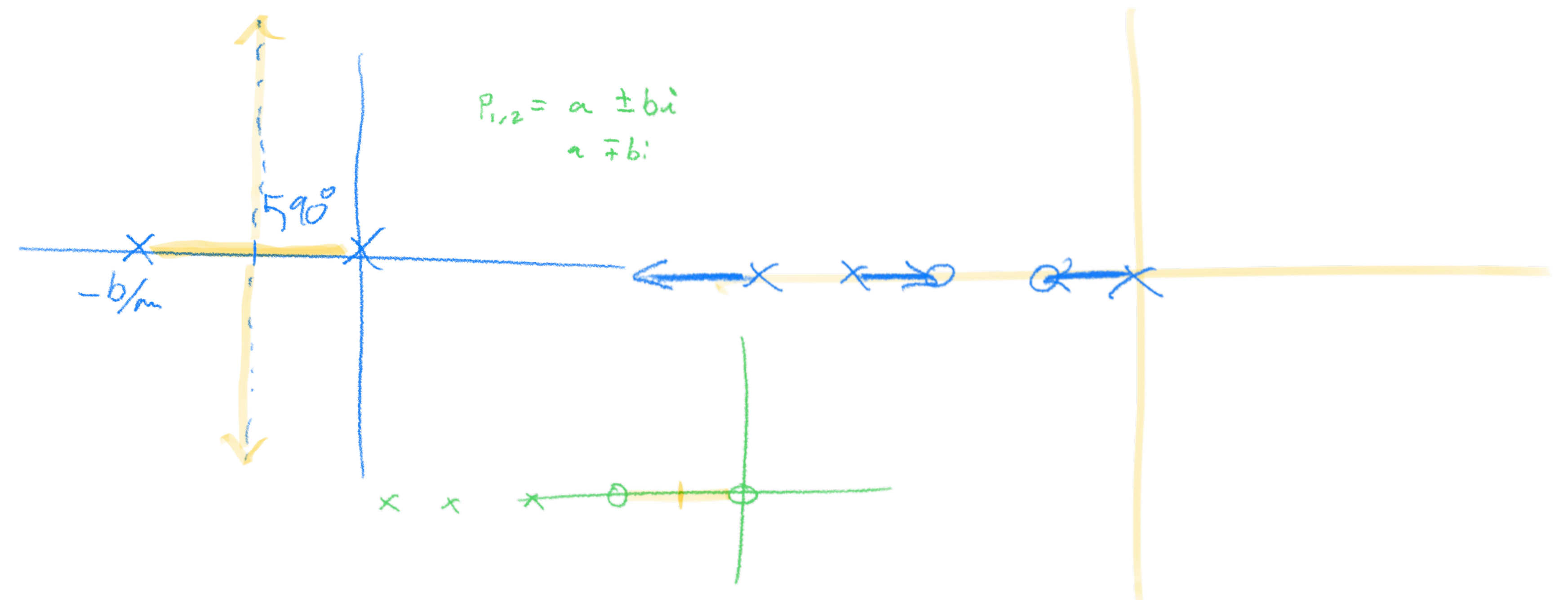
Step 1: $p_{1,2} = -\frac{b}{m}, 0$ $(n=2)$

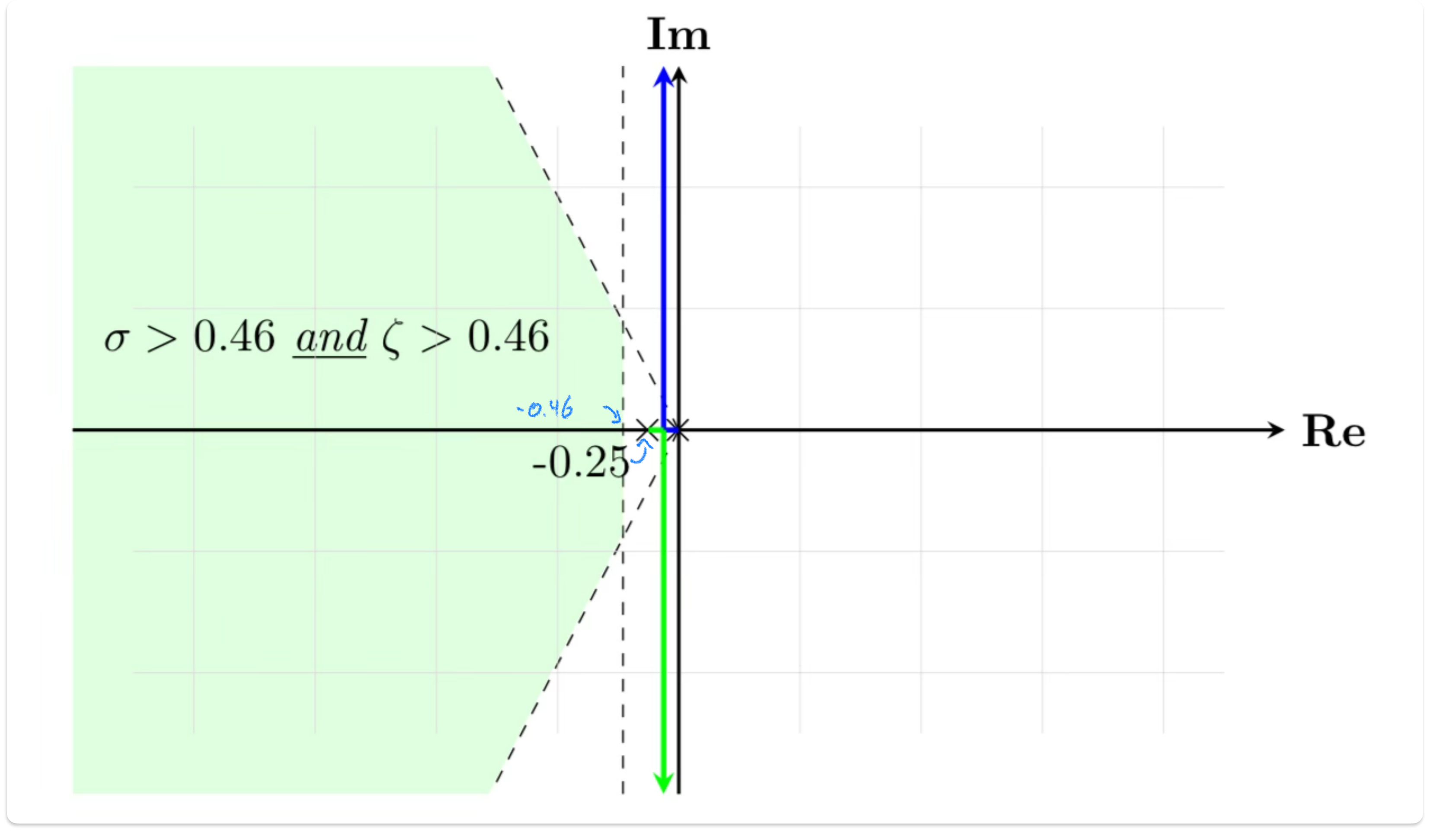
z: none $(m=0)$

Rule 2: real axis rule

Rule 3: $\alpha = \frac{-b/m}{2}$

$\phi_1 = \frac{180^\circ}{2} = 90^\circ$, $\phi_2 = \frac{180^\circ + 360^\circ}{2} = 270^\circ$





PD control R.L.:

PD controller: $D(s) = K_p + K_d s = k_d\left(s + \frac{k_p}{k_d}\right)$

★ Fix ratio k_p/k_d to be 1 for now

$\rightarrow D(s) = k\left(\frac{s+1}{1}\right)$

Intuition: Poles → zeros "poles want to go to zeros"

$1 + K\left(\frac{s+1}{1}\right)\left(\frac{1}{ms^2+bs}\right) = 0$

$$\Rightarrow 1 + K\underbrace{\frac{s+1}{ms^2+bs}}_{L(s)} = 0$$

Step 1: $p_{1,2} = -b/m, 0$ $(n=2)$

z : $z_1 = -1$ $(m=1)$

Rule 1: start @ x, end @ o or ∞

Rule 2: real axis rule

Rule 5: $\frac{180^\circ + 360(l-1)}{q}$ $(l = 1, 2, \dots, q)$, $q = 2 \rightarrow \underbrace{\frac{180}{2}}_{l=1}, \underbrace{\frac{540}{2}}_{l=2}$

Rule 3:
$\alpha = \frac{-b/m - (-1)}{1}$
$= -\frac{b}{m} + 1$

$\phi_1 = \frac{180 + \cancel{360(l-1)}}{1} = 180^\circ$