

**STAT 500 HW6**

Using the trees data, fit a model with Volume as the response and Girth and Height as predictors.

1. Use the Box-Cox method to determine the best transformation on the response. Compare the fits with and without the transformation.

**Code:**

```
> g1 <- lm(Volume~ Girth + Height)
```

F-statistic: 255 on 2 and 28 DF, p-value: < 2.2e-16

```
> boxcox(g1, plotit=T)
```

```
boxcox(g2, plotit=T, lambda = seq(0.0, 2, by =0.05))
```

```
> boxcox(g1, lambda = seq(0.0, 0.6, by =0.05))
```

```
> summary(g2)
```

```
> g2 <- lm(I(Volume^0.3)~Girth + Height)
```

Coefficients:

```
> summary(g1)
```

Estimate Std. Error t value Pr(>|t|)

Coefficients:

(Intercept) 0.194613 0.148552 1.310 0.201

Estimate Std. Error t value Pr(>|t|)

Girth 0.121559 0.004545 26.748 < 2e-16

(Intercept) -57.9877 8.6382 -6.713 2.75e-07

Height 0.011799 0.002238 5.272 1.32e-05

Girth 4.7082 0.2643 17.816 < 2e-16

Residual standard error: 0.06676 on 28 degrees of freedom

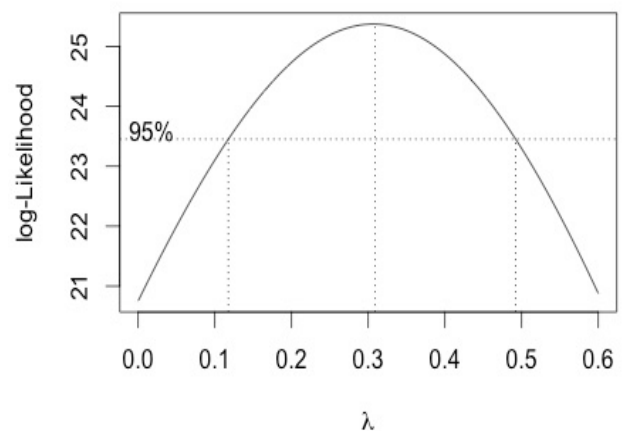
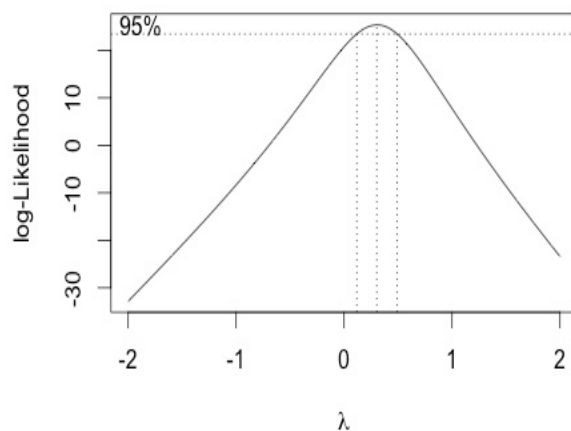
Height 0.3393 0.1302 2.607 0.0145

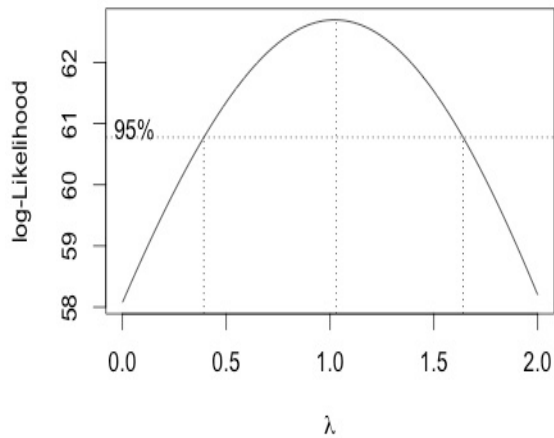
Multiple R-squared: 0.9775, Adjusted R-squared: 0.9759

Residual standard error: 3.882 on 28 degrees of freedom

F-statistic: 609.1 on 2 and 28 DF, p-value: < 2.2e-16

Multiple R-squared: 0.948, Adjusted R-squared: 0.9442





After using Box\_Cox method to determine the transformation on the response, it seems like that a cubic root transformation will make a better fit here. Also the new  $\hat{\lambda}$  is close to 1 after making cubic transformation on response. Compare with the fit without transformation, in the fit after transformation the predictor Height becomes more significant and R-squared increases slightly. So the fit after transformations is more acceptable.

2. Try adding higher order polynomial terms in the predictors to the original linear model.

Comment on the changes in fit.

Code:

```
> g3 <- lm(Volume~
Girth+Height+I(Girth*Height)+I(Girth^2)+I(Height^2
))
```

```
> summary(g3)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	6.60706	62.90855	0.105	0.9172
Girth	-5.12160	2.46674	-2.076	0.0483
Height	0.29491	1.77852	0.166	0.8696
I(Girth * Height)	0.06628	0.05671	1.169	0.2535
I(Girth^2)	0.16393	0.10089	1.625	0.1167
I(Height^2)	-0.00494	0.01312	-0.376	0.7097

Residual standard error: 2.655 on 25 degrees of freedom

Multiple R-squared: 0.9783, Adjusted R-squared: 0.9739

F-statistic: 225 on 5 and 25 DF, p-value: < 2.2e-16

```
> ##remove Height^2
```

```
> g4 <- lm(Volume~
Girth+Height+I(Girth*Height)+I(Girth^2))
```

```
> summary(g4)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	26.48906	33.61492	0.788	0.4378
Girth	-4.58977	1.98854	-2.308	0.0292
Height	-0.32992	0.62857	-0.525	0.6041
I(Girth * Height)	0.0570	0.05024	1.135	0.2668
I(Girth^2)	0.17071	0.09762	1.749	0.0921

Residual standard error: 2.611 on 26 degrees of freedom

Multiple R-squared: 0.9781, Adjusted R-squared: 0.9748

F-statistic: 290.8 on 4 and 26 DF, p-value: < 2.2e-16

```
> ##remove Girth^2
```

```
> g5 <- lm(Volume~ Girth+Height+I(Girth*Height))
```

```
> summary(g5)
```

Coefficients:					Residual standard error: 2.709 on 27 degrees of freedom	
	Estimate	Std. Error	t value	Pr(> t )	Multiple R-squared: 0.9756,	Adjusted R-squared: 0.9728
(Intercept)	69.39632	23.83575	2.911	0.00713	F-statistic: 359.3 on 3 and 27 DF, p-value: < 2.2e-16	
Girth	5.85585	1.92134	-3.048	0.00511		
Height	-1.29708	0.30984	-4.186	0.00027		

I(Girth \* Height) 0.13465 0.02438 5.524 7.48e-06

After adding all of the quadratic and linear terms of predictor Girth and Height, we find the Height^2 is the highest order term with the largest p-value, so we remove Height^2 and refit. Then remove Girth^2 similarly, and refit again, as a result all of the predictors left are significant with much smaller p-value than before, but the R-squared dropped slightly in the removing process.

3. Try to improve on the best model from (a) by adding higher order polynomial terms in the predictors to the model. Comment on the changes in fit.

> g6 <- lm(I(Volume^0.3)~ Girth+Height+I(Girth*Height)+I(Girth^2)+I(Height^2))					Estimate	Std. Error	t value	Pr(> t )
> summary(g6)					(Intercept)	-0.6288141	1.5838634	-0.397 0.6946
Coefficients:					Girth	0.1259222	0.0587580	2.143 0.0416
					Height	0.0332372	0.0456470	0.728 0.4730
					I(Girth * Height)	-0.0000543	0.0007483	-0.073 0.9427
					I(Height^2)	-0.0001390	0.0003349	-0.415 0.6815
					Residual standard error: 0.06887 on 26 degrees of freedom			
					Multiple R-squared:	0.9778,	Adjusted R-squared:	0.9744
					F-statistic: 286.2 on 4 and 26 DF, p-value: < 2.2e-16			
					> ##remove Height^2			
					> g8 <- lm(I(Volume^0.3)~ Girth+Height+I(Girth*Height))			
					> summary(g8)			
					Coefficients:			
					Estimate	Std. Error	t value	Pr(> t )
					(Intercept)	-0.0214129	0.5967006	-0.036 0.97164
					Girth	0.1394738	0.0480984	2.900 0.00734
					Height	0.0145742	0.0077566	1.879 0.07109 .
					I(Girth * Height)	-0.0002284	0.0006103	-0.374 0.71118

```

---Residual standard error: 0.06781 on 27 degrees of freedom
Multiple R-squared: 0.9776, Adjusted R-squared: 0.9752
F-statistic: 393.6 on 3 and 27 DF, p-value: < 2.2e-16

> ##remove Girth*Height

> g9 <- lm(I(Volume^0.3)~ Girth+Height)

> summary(g9)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.194613   0.148552   1.310    0.201
Girth  0.121559    0.004545  26.748 < 2e-16
Height 0.011799    0.002238   5.272 1.32e-05

```

After adding higher order polynomial terms to the best model, we find almost all of the predictors are not significant, then we remove the most non-significant and highest order term  $Girth^2$ . The p-values of left predictors change a little, and  $Height^2$  and  $Girth*Height$  are still extremely insignificant, so we remove  $Height^2$  at first and refit, then find  $Girth*Height$  becomes the only insignificant predictor. So we remove it and all the predictors left are significant. The R-squared is more stable in this removing process. The final model is a linear one.