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STAT 500 HW4

1.Check the constant variance assumption for the errors. Modify the model if necessary (see below).

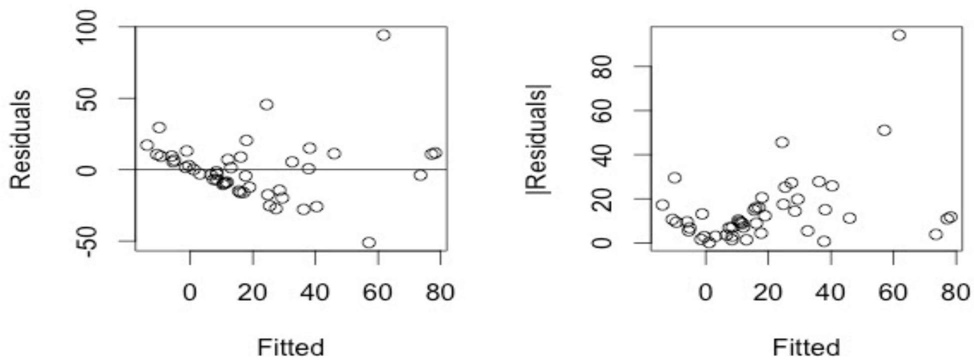


Figure 1

Code:

```
##plot residuals vs fitted values

result1 <- lm(gamble ~ sex+ status+ income+ verbal)

par(mfrow = c(1,2))

plot(result1$fitted, result1$residual, xlab= "Fitted",
ylab= "Residuals")

abline(h = 0)

##plot absolute values of residuals vs fitted values

plot(result1$fitted, abs(result1$residual), xlab=
"Fitted", ylab= "|Residuals|")

summary(lm(abs(result1$residual) ~ result1$fitted ))
```

Coefficients (regression of $|\hat{\varepsilon}|$ and $\hat{y}$)

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	9.3303	2.8789	3.241	0.00224 **
result1$fitted	0.2645	0.0968	2.732	0.00895 **

In left plot of figure1, we make a regression of $\hat{\varepsilon}$ and $\hat{y}$, it shows that the variance of $\hat{\varepsilon}$ and the scatter are not symmetrically distributed in the vertical direction, but displays a slightly downward trend. So it violates nonlinearity.

In the right plot of figure 2, we make a regression of $|\hat{\varepsilon}|$ and $\hat{y}$, we can see that the scatters are still not vertically symmetric. And since p-value of coefficients of this regression are both less than 0.05, the linear relationship between $|\widehat{\varepsilon}|$ and $\hat{y}$ is rather significant. So it violates constant variance.

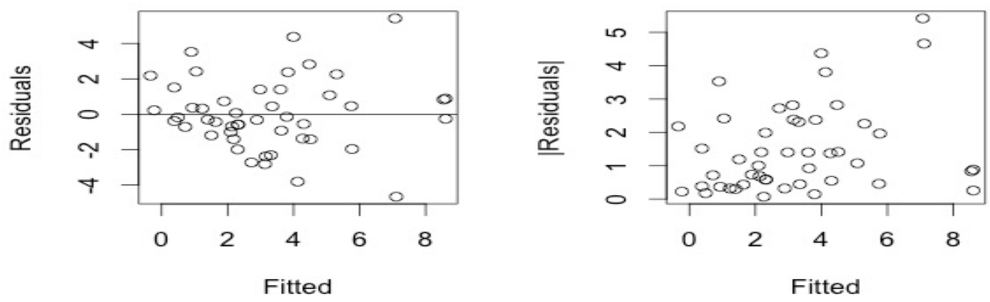


Figure 2

Codes:

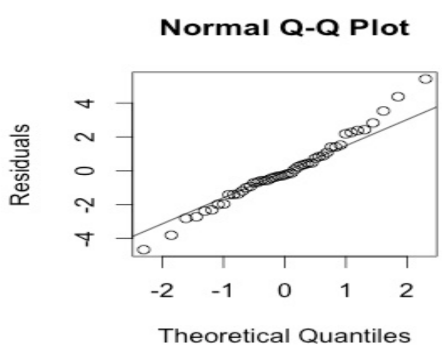
```
##square root transformation of y

result2 <- lm(sqrt(gamble) ~ sex+ status+ income+ verbal)

plot(result2$fitted, result2$residual, xlab= "Fitted", ylab= "Residuals")

abline(h = 0)

plot(result2$fitted, abs(result2$residual), xlab= "Fitted", ylab= "|Residuals|")

summary(lm(abs(result2$residual) ~ result2$fitted ))
```

Coefficients (regression of $|\hat{\varepsilon}|$ and $\mathrm{sqrt}(\hat{y})$)

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1.01136	0.32365	3.125	0.00311 **
result2$fitted	0.14957	0.08242	1.815	0.07623 .

In order to offset the influence of non-constant variance, we make a square root transformation of response and then regress $\hat{\varepsilon}$ and the new response. It shows in figure 2 that, both $\hat{\varepsilon}$ *and* $\widehat{|\varepsilon|}$ become more vertically symmetric. And the linear relationship between $\widehat{|\varepsilon|}$ and $\hat{y}$ is no more significant, as the p-value=0.07623 > 0.05. Then we may consider the variance of $\hat{\varepsilon}$ as constant.

2. Check the normality assumption.




```
qqnorm (residuals (result2), ylab="Residuals")

qqline (residuals (result2))
```

In the plot of figure 3, we compare $\hat{\varepsilon}$ to "ideal" normal observations by Q-Q plot. qqline joining first and third quartiles is not influenced by outliers and $\hat{\varepsilon}$ follows the line approximately, except for slightly heavy tails. So $\hat{\varepsilon}$ can be considered normal.

3. Check for large leverage points.

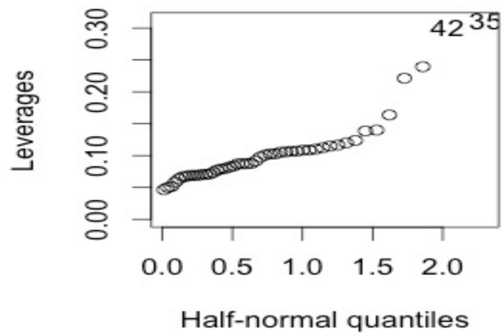


Figure 4

	sex	status	income	verbal	gamble
35	0	28	1.5	1	14.1
42	0	61	15.0	9	69.7

Half-normal plot in figure 4 shows that the #35 and #42 points diverge substantially from the rest of data, thus the two points have large leverages.

4. Check for outliers.

Code:

```
##problem 4
> ## compute (externally) studentized residuals
> ti <- rstudent(result2)
> max(abs(ti))
[1] 3.037005
> which(ti == max(abs(ti)))
24
24
> ## compute p-value
> 2*(1-pt(max(abs(ti)),df = 47-5-1))
[1] 0.00414277
> ## compare to alpha/n
> 0.05/47
[1] 0.00106383
```

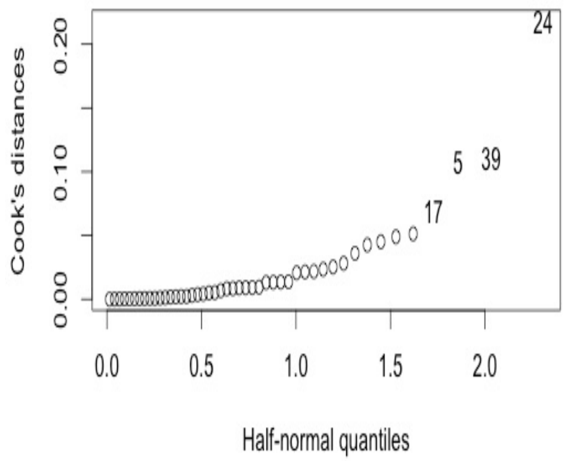
Since the p-value of the largest (externally) studentized residual is 0.00414277, which is larger than level 0.00106383, we conclude that the #24 point is not an outlier. Then no outlier can be seen in the regression model.

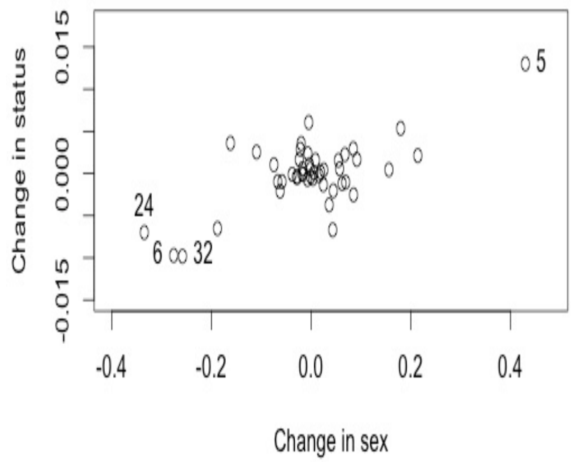
5. Check for influential points.

```
> ## Compute Cook's distance
> cook <- cooks.distance(result2)
> halfnorm (cook, nlab=4, ylab = "Cook's distances")
> ## Compute changes in coefficients
> result.inf <- lm.influence(result2)
```

```
> plot(result.inf$coef[,2], result.inf$coef[,3],
xlab="Change in sex", ylab="Change in status",
xlim=c(-0.4, 0.48), ylim=c(-0.015, 0.018))
```

```
> identify (result.inf$coef[, 2], result.inf$coef[, 3])
```






```
> ## interactive tool to identify points by clicking
```

In the first plot of figure 5, #24, #5 and #39 points have larger cook's distance from other points. The second plot of figure 5 shows the leaveout-one differences in the coefficients related to sex and status. We find that #24, #5, #32, # 6 points stick out on the plot. Then we examine the effects of removing #24 and #5 points below.

```
> summary(result2)

> result.24 <- lm(sqrt(gamble) ~ sex+ status+ income+ verbal, data = teengamb,subset =
(row.names(teengamb) !="24" ))

> summary(result.24)

Call:

lm(formula = sqrt(gamble) ~ sex + status + income +
verbal)

Coefficients:

             Estimate Std. Error t value Pr(>|t|)

(Intercept) 2.97707      1.57947     1.885    0.06638 .

sex        -2.04450      0.75416    -2.711    0.00968 **

status      0.03688      0.02582     1.428    0.16057

income      0.47938      0.09418     5.090 7.94e-06 ***

verbal     -0.42360      0.19950    -2.123    0.03967 *
Residual standard error: 2.084 on 42 degrees of
freedom    Multiple R-squared:    0.5646, Adjusted
R-squared:    0.5231 F-statistic: 13.61 on 4 and 42 DF,
p-value: 3.362e-07


Call:

lm(formula = sqrt(gamble) ~ sex + status + income +
verbal, data = teengamb, subset =
(row.names(teengamb) != "24"))

Coefficients:

             Estimate Std. Error t value Pr(>|t|)

(Intercept)    2.11915      1.47175     1.440     0.1575
```

```
sex         -1.70997       0.69840    -2.448      0.0187 *
status       0.04387       0.02372     1.849      0.0716 .
income    0.44312       0.08695      5.096 8.22e-06 ***
verbal      -0.35706       0.18375    -1.943      0.0589 .
Residual standard error: 1.906 on 41 degrees of freedom         Multiple R-squared:    0.5503, Adjusted R-squared:    0.5065    F-statistic: 12.55 on 4 and 41 DF,    p-value: 9.403e-07
> ##check for #5 point
> result.5 <- lm(sqrt(gamble) ~ sex+ status+ income+ verbal, data = teengamb,subset = (row.names(teengamb) !="5" ))
> summary(result.5)
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.56531       1.56567      2.277    0.02806 *
sex       -2.47471       0.76745    -3.225    0.00248 **
status     0.02388       0.02601      0.918    0.36397
income    0.47517       0.09150      5.193 6.01e-06 ***
verbal    -0.40768       0.19395    -2.102    0.04174 *
Residual standard error: 2.024 on 41 degrees of freedom Multiple R-squared:    0.5976,   Adjusted R-squared:    0.5584    F-statistic: 15.22 on 4 and 41 DF,    p-value: 1.041e-07
```

Comparing the data fit without #24 to the full data fit, we notice that the coefficient for sex increases about 15% and the verbal term is no longer significant.

In the data fit without #5, the coefficient for sex decreases about 20%, but the multiple R-squared increased slightly.