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STATS 500 - Homework 5

Using the sat data, fit a model with total as the response and takers, ratio, salary and expend as predictors using the following methods:

1. Ordinary least squares

```
##Ordinary least squares

library(faraway)

##read in the data

data(sat)

attach(sat)

g1 <- lm (total~ takers+ratio+salary+expend, sat)

summary(g1, cor= T)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	1045.9715	52.8698	19.784	< 2e-16
takers	-2.9045	0.2313	-12.559	2.61e-16
ratio	-3.6242	3.2154	-1.127	0.266
salary	1.6379	2.3872	0.686	0.496
expend	4.4626	10.5465	0.423	0.674

Residual standard error: 32.7 on 45 degrees of freedom Multiple R-squared: 0.8246,Adjusted R-squared: 0.809 F-statistic: 52.88 on 4 and 45 DF, p-value: < 2.2e-16

Correlation of Coefficients:

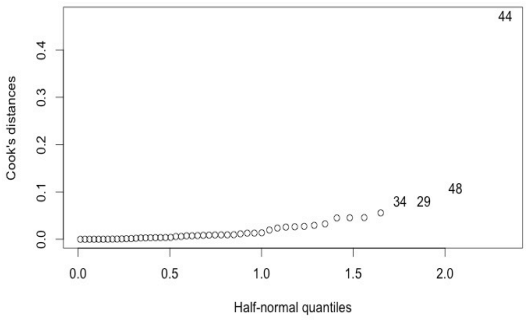
	(Intercept)	takers	ratio	salary
takers	0.04			
ratio	-0.80	0.25		
salary	0.29	-0.35	-0.72	
expend	-0.53	0.09	0.75	-0.91

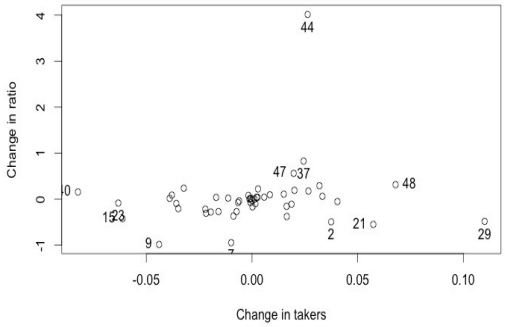
When fitting the model with ordinary least squares, we noticed that takers is very significant, while salary and expend is not really significant. And the correlation for ratio and salary is typically negative, the correlation for expend and salary is also strongly negative. The Residual standard error is so large.

```
##check for outlier-leverage points

>ti <- rstudent(g1)

> which.max(ti)

Utah

   44

> 2*(1-pt(m,df = 50-4-1))

[1] 0.003114645

> 0.05/50

[1] 0.001

## Compute Cook's distance

cook <- cooks.distance(g1)

halfnorm (cook, nlab=4, ylab = "Cook's distances")

## Compute changes in coefficients

result.inf <- lm.influence(g1)

plot(result.inf$coef[,2], result.inf$coef[,3],
xlab="Change in takers", ylab="Change in ratio")

## interactive tool to identify points by clicking
```

```
identify (result.inf$coef[, 2], result.inf$coef[, 3])

##romove #29,#44,#48 points

df1 = df[-c(29,44,48),]
```

```
result <- lm(total~ takers+ratio+salary+expend, data =
df1)

summary(result)
```





First we check the p-value of the largest (externally) studentized residual is 0.0031, which is larger than level 0.001, we conclude that the point is not an outlier, Then no outlier can be seen in the regression model.

Then we calculate the cook's distance of each point, in the left plot #29, #34, #44 and #48 points have larger cook's distance. The right plot shows the leaveout-one differences in the coefficients related to takers and ratio. We find that lots of points stick out on the plot. Then we examine the effects of removing #29, #44 and #48 points below.

Coefficients:

	Estimate	Std. Error	t value	Pr(>ltl)
(Intercept)	1091.5571	45.6268	23.924	<2e-16
takers	-3.1062	0.1902	-16.334	<2e-16
ratio	-7.4880	2.9037	-2.579	0.0135
salary	2.4870	1.9702	1.262	0.2138
expend	3.7532	8.6665	0.433	0.6672

Residual standard error: 26.07 on 42 degrees of freedom

Multiple R-squared: 0.8902, Adjusted R-squared: 0.8797

F-statistic: 85.12 on 4 and 42 DF, p-value: < 2.2e-16

After removal, the absolute value of coefficient of ratio increases more than 100%. Also predictor ratio, salary and expend all become more significant, especially for ratio. The residual standard error becomes smaller and R-squared increases slightly.

2. Least absolute deviations

```
>library(quantreg)
 > glad <- rq(total~ takers+ratio+salary+expend, data =
sat)
 > summary(glad)
```

Coefficients:

	coefficients	lower bd	upper bd

(Intercept)	1090.89886	920.17149	1151.85075
takers	-3.13961	-3.38485	-2.6647
ratio	-7.26632	-10.73796	1.62341
salary	3.18313	-0.15788	5.41909
expend	-0.79753	-8.88001	20.92522

Compared to least squares, most of the coefficients, except for coefficient of expend, are more close to those in OLS method after removing influential points. So identifying and removing bad and unusual points when applying OLS method can achieve a better fit as in LAD method. It also shows that only takers is a significant predictor, while in least squares method takers and ratio can be considered significant.

3. Huber's robust regression

```
> library(MASS)

> gr <- rlm(total~ takers+ratio+salary+expend, data = sat)

> summary(gr)
```

Coefficients:

	Value	Std. Error	t value
(Intercept)	1060.2074	49.8845	21.2533
takers	-2.9778	0.2182	-13.6470
ratio	-5.1254	3.0339	-1.6894
salary	2.0933	2.2525	0.9293
expend	3.9158	9.9510	0.3935

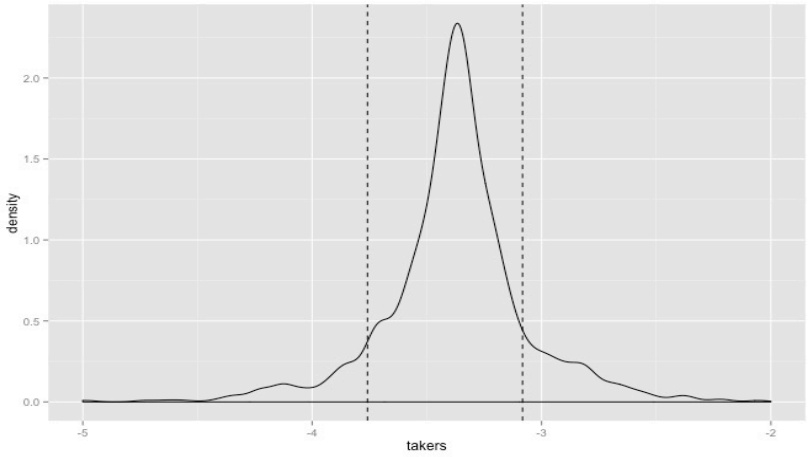
Residual standard error: 25.58 on 45 degrees of freedom

The numerical values of coefficients have changed a small amount when compared to those in OLS without unusual points, but in general they are close to each other and the general significance of the variables is almost the same.

4. Least trimmed squares

```
set.seed(123)
glts <- ltsreg(total~ expend+ratio+salary+takers, data = sat)
x <- df [, 1:4]
bcoef <- matrix(0, nrow = 1000, ncol = 5)
for (i in 1:1000){

newy <- glts$fit + glts$resid[sample(20, rep =T)]
bcoef[i,] <- ltsreg(x, newy, nsamp = "best")$coef

}
colnames(bcoef) <- names(coef(glts))
apply(bcoef, 2, function(x) quantile(x, c(0.025,0.975)))

library(ggplot2)

bcoef <- data.frame(bcoef)

p1 <- ggplot(bcoef, aes(x= takers))+geom_density()+xlim(-5,-2)

p1 + geom_vline(xintercept = c(-3.759,-3.083),linetype= "dashed")
```

	(Intercept)	expend	ratio	salary	takers
2.5%	1080.546	-4.536154	-19.413370	-0.8484121	-3.759272
97.5%	1261.343	26.047694	-8.614763	5.7348908	-3.083768



By LTS method, it is clear that both takers and ratio are significant, since 0 is outside confidence intervals of the two's coefficients. The fitting result is much close to that of OLS method without unusual points, as ratio becomes much more significant after removing some influential points. But in OLS, the predictor salary also show significance, though not that obvious. Maybe it is because some bad or unusual points have not been removed, and LTS excluded these points.

Also, from the density distribution of coefficient of takers, we see that the distribution has longish tails, which suggests that the error may not be normally distributed. Robust regression like LAD, Buber's regression and LTS can help solve the problem of lacking of fit, or we can use regression diagnostics in conjunction with least squares to identify bad and unusual points.