Machine Learning
Exercise 1

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There will be no credit points for the first exercise – we'll do them on the fly. (*Präsenzübung*) For those of you that had lectures with me before this is redundant—you're free to skip the tutorial.

1 Reading: Pedro Domingos

Read at least until section 5 of Pedro Domingos's *A Few Useful Things to Know about Machine Learning* http://homes.cs.washington.edu/~pedrod/papers/cacm12.pdf. Be able to explain roughly what generalization and the bias-variance-tradeoff (Fig. 1) are.

2 Matrix equations

a) Let X, A be arbitrary matrices, A invertible. Solve for X:

$$XA + A^\top = \mathbf{I}$$

b) Let X, A, B be arbitrary matrices, $(C - 2A^\top)$ invertible. Solve for X:

$$X^\top C = [2A(X + B)]^\top$$

c) Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, A \in \mathbb{R}^{d\times n}$. A obviously *not* invertible, but let $A^\top A$ be invertible. Solve for x:

$$(Ax - y)^\top A = \mathbf{0}_n^\top$$

d) As above, additionally $B \in \mathbb{R}^{n\times n}$, B positive-definite. Solve for x:

$$(Ax - y)^\top A + x^\top B = \mathbf{0}_n^\top$$

3 Vector derivatives

Let $x \in \mathbb{R}^n, y \in \mathbb{R}^d, A \in \mathbb{R}^{d\times n}$.
a) What is $\frac{\partial}{\partial x} x$? (Of what type/dimension is this thing?)
b) What is $\frac{\partial}{\partial x}[x^\top x]$?
c) Let B be symmetric (and pos.def.). What is the minimum of $(Ax - y)^\top(Ax - y) + x^\top Bx$ w.r.t. x?

4 Coding

Future exercises will need you to code some Machine Learning methods. You are free to choose your programming language. If you're new to numerics we recommend Matlab/Octave or Python (SciPy & scikit-learn). I'll support C++, but recommend it really only to those familiar with C++.
To get started, try to just plot the data set http://ipvs.informatik.uni-stuttgart.de/mlr/marc/teaching/data/dataQuadReg2D.txt, e.g. in Octave:

```
D = importdata('dataQuadReg2D.txt');
plot3(D(:,1),D(:,2),D(:,3), 'ro')
```

Or in Python

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

D = np.loadtxt('dataQuadReg2D.txt')

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(D[:,0],D[:,1],D[:,2], 'ro')
plt.show()
```

Or you can store the grid data in a file and use gnuplot, e.g.:

```
splot 'dataQuadReg2D.txt' with points
```

For those using C++, download and test http://ipvs.informatik.uni-stuttgart.de/mlr/marc/source-code/15-MLcourse.tgz. In particular, have a look at examples/Core/array/main.cpp with many examples on how to use the array class. Report on problems with installation.