Andrews Figures, Chapter 01

Geoffrey Matthews
Western Washington University

January 28, 2013

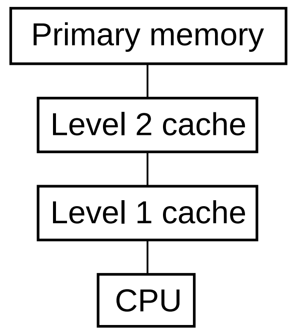


Figure 1.1 Processors, cache, and memory in a modern machine.

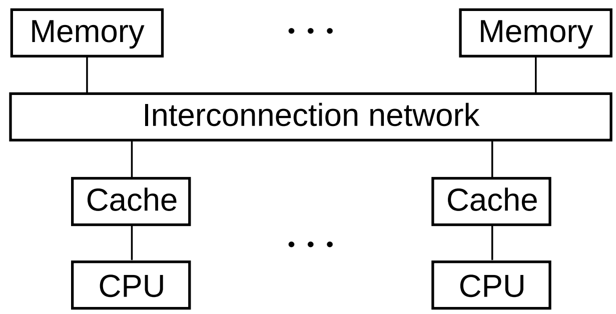


Figure 1.2 Structure of Shared-Memory Multiprocessors.

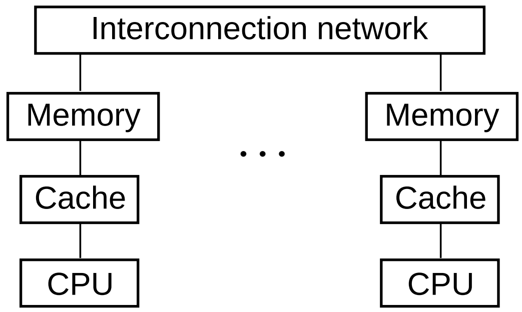


Figure 1.3 Structure of distributed-memory machines.

```
double a[n,n], b[n,n], c[n,n];

for [i = 0 to n-1] {
  for [j = 0 to n-1] {
    # compute inner product of a[i,*] and b[*,j]
    c[i,j] = 0.0;
    for [k = 0 to n-1]
      c[i,j] = c[i,j] + a[i,k]*b[k,j];
  }
}
```

Sequential Matrix Multiplication

Embarrassingly Parallel

- **Read set**: set of variables read by a process
- **Write set**: set of variables written to by a process
- Two operations are **independent** is the write set of each is disjoint from both the read and write sets of the other.

```
co [i = 0 to n-1] {  # compute rows in parallel
  for [j = 0 to n-1] {
    c[i,j] = 0.0;
    for [k = 0 to n-1]
      c[i,j] = c[i,j] + a[i,k]*b[k,j];
  }
}
```

Parallel Matrix Multiplication by Rows

```
co [j = 0 to n-1] {  # compute columns in parallel
  for [i = 0 to n-1] {
    c[i,j] = 0.0;
    for [k = 0 to n-1]
      c[i,j] = c[i,j] + a[i,k]*b[k,j];
  }
}
```

Parallel Matrix Multiplication by Columns

```
co [i = 0 to n-1, j = 0 to n-1] { # all rows and
  c[i,j] = 0.0;                   # all columns
  for [k = 0 to n-1]
    c[i,j] = c[i,j] + a[i,k]*b[k,j];
}
```

Parallel Matrix Multiplication by Rows and Columns

```
co [i = 0 to n-1] {    # rows in parallel then
  co [j = 0 to n-1] {  # columns in parallel
    c[i,j] = 0.0;
    for [k = 0 to n-1]
      c[i,j] = c[i,j] + a[i,k]*b[k,j];
  }
}
```

Parallel Matrix Multiplication Using Nested co Statements

```
process row[i = 0 to n-1] { # rows in parallel
  for [j = 0 to n-1] {
    c[i,j] = 0.0;
    for [k = 0 to n-1]
      c[i,j] = c[ij] + a[i,k]*b[k,j];
  }
}
```

Parallel Matrix Multiplication Using a Process Declaration

```
process worker[w = 1 to P] {   # strips in parallel
  int first = (w-1) * n/P;     # first row of strip
  int last = first + n/P - 1;  # last row of strip
  for [i = first to last] {
    for [j = 0 to n-1] {
      c[i,j] = 0.0;
      for [k = 0 to n-1]
        c[i,j] = c[i,j] + a[i,k]*b[k,j];
    }
  }
}
```

Parallel Matrix Multiplication by Strips (Blocks)

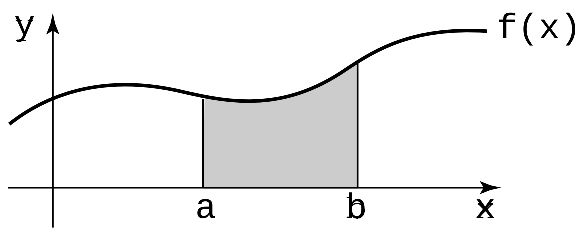


Figure 1.4 The quadrature problem.

```
double fleft = f(a), fright, area = 0.0;
double width = (b-a) / INTERVALS;
for [x = (a + width) to b by width] {
  fright = f(x);
  area = area + (fleft + fright) * width / 2;
  fleft = fright;
}
```

Iterative Quadrature Program

```
double quad(double left,right,fleft,fright,lrarea) {
  double mid = (left + right) / 2;
  double fmid = f(mid);
  double larea = (fleft+fmid) * (mid-left) / 2;
  double rarea = (fmid+fright) * (right-mid) / 2;
  if (abs((larea+rarea) - lrarea) > EPSILON) {
    # recurse to integrate both halves
    larea = quad(left, mid, fleft, fmid, larea);
    rarea = quad(mid, right, fmid, fright, rarea);
  }
  return (larea + rarea);
}
```

Recursive Procedure for Quadrature Problem

Independent procedure calls

- If a procedure does not reference global variables and has only value parameters, then every call of the procedure will be independent.
- Functional programming has these features.
- For example, quicksort.

```
double quad(double left,right,fleft,fright,lrarea) {
  double mid = (left + right) / 2;
  double fmid = f(mid);
  double larea = (fleft+fmid) * (mid-left) / 2;
  double rarea = (fmid+fright) * (right-mid) / 2;
  if (abs((larea+rarea) - lrarea) > EPSILON) {
    # recurse to integrate both halves in parallel
    co larea = quad(left, mid, fleft, fmid, larea);
    // rarea = quad(mid, right, fmid, fright, rarea);
    oc
  }
  return (larea + rarea);
}
```

Recursive Parallel Adaptive Quadrature

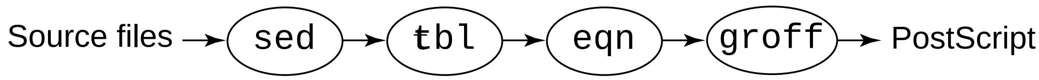


Figure 1.5 A pipeline of processes.

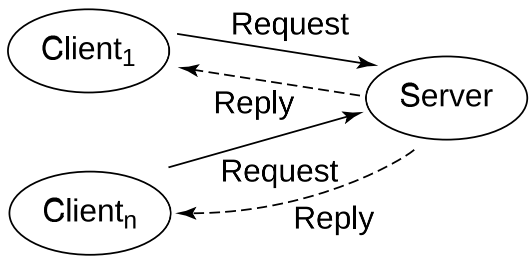
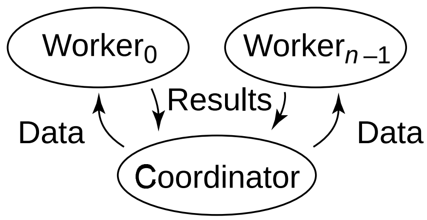
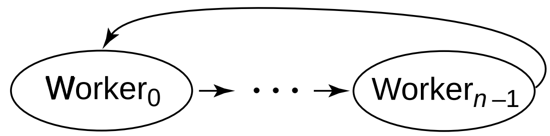


Figure 1.6 Clients and servers.



(a) Coordinator/worker interaction



(b) A circular pipeline

Figure 1.7 Matrix multiplications using message passing.

```
process worker[i = 0 to n-1] {
  double a[n];    # row i of matrix a
  double b[n,n];  # all of matrix b
  double c[n];    # row i of matrix c
  receive initial values for vector a and matrix b;
  for [j = 0 to n-1] {
    c[j] = 0.0;
    for [k = 0 to n-1]
      c[j] = c[j] + a[k] * b[k,j];
  }
  send result vector c to the coordinator process;
}

process coordinator {
  double a[n,n];  # source matrix a
  double b[n,n];  # source matrix b
  double c[n,n];  # result matrix c
  initialize a and b;
  for [i = 0 to n-1] {
    send row i of a to worker[i];
    send all of b to worker[i];
  }
  for [i = 0 to n-1]
    receive row i of c from worker[i];
  print the results, which are now in matrix c;
}
```

Matrix Multiplication Using Coordinator/Worker Interaction

```
process worker[i = 0 to n-1] {
  double a[n];       # row i of matrix a
  double b[n];       # one column of matrix b
  double c[n];       # row i of matrix c
  double sum = 0.0;  # storage for inner products
  int nextCol = i;   # next column of results
  receive row i of matrix a and column i of matrix b;
  # compute c[i,i] = a[i,*] × b[*,i]
  for [k = 0 to n-1]
    sum = sum + a[k] * b[k];
  c[nextCol] = sum;
  # circulate columns and compute rest of c[i,*]
  for [j = 1 to n-1] {
    send my column of b to the next worker;
    receive a new column of b from the previous worker;
    sum = 0.0;
    for [k = 0 to n-1]
      sum = sum + a[k] * b[k];
    if (nextCol == 0)
      nextCol = n-1;
    else
      nextCol = nextCol-1;
    c[nextCol] = sum;
  }
  send result vector c to coordinator process;
}
```

Matrix Multiplication Using a Circular Pipeline