

Locks and Barriers

Andrews Chapter 03

Critical Section Problem

- Mutual Exclusion.
- Absence of Deadlock (Livelock).
- Absence of Unnecessary Delay.
- Eventual Entry.

Three safety properties, one liveness property.

```

bool in1 = false, in2 = false;
## MUTEX:  $\neg(\text{in1} \wedge \text{in2})$  -- global invariant
process CS1 {
    while (true) {
        ⟨await (!in2) in1 = true;⟩ /* entry */
        critical section;
        in1 = false;                /* exit */
        noncritical section;
    }
}
process CS2 {
    while (true) {
        ⟨await (!in1) in2 = true;⟩ /* entry */
        critical section;
        in2 = false;                /* exit */
        noncritical section;
    }
}

```

Figure 3.1 Critical section problem: Coarse-grained solution.

```

bool lock = false;
process CS1 {
    while (true) {
        ⟨await (!lock) lock = true;⟩  /* entry */
        critical section;
        lock = false;                /* exit */
        noncritical section;
    }
}
process CS2 {
    while (true) {
        ⟨await (!lock) lock = true;⟩  /* entry */
        critical section;
        lock = false;                /* exit */
        noncritical section;
    }
}

```

Figure 3.2 Critical sections using locks.

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- Can be used by arbitrarily many processes.
- `lock` as a boolean is equivalent to $in1 \vee in2$

Test and Set

```
bool TS(bool lock) {  
    <  
    bool initial = lock;  
    lock = true;  
    return initial;  
    >  
}
```

- Implemented in hardware.

```

bool lock = false;                                /* shared lock */
process CS[i = 1 to n] {
    while (true) {
        while (TS(lock)) skip;                    /* entry protocol */
        critical section;
        lock = false;                              /* exit protocol */
        noncritical section;
    }
}

```

Figure 3.3 Critical sections using Test and Set.

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- Such a solution is called a **spin-lock**.
- In spin-lock solution, exit protocol simply resets shared variables.
- Requires strong fairness.
- Weak fairness usually enough.

```

bool lock = false;                                /* shared lock */
process CS[i = 1 to n] {
    while (true) {
        while (lock) skip;                        /* entry protocol */
        while (TS(lock)) {
            while (lock) skip;
        }
        critical section;
        lock = false;                             /* exit protocol */
        noncritical section;
    }
}

```

Figure 3.4 Critical sections using Test and Test and Set.

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- Avoids repeated unnecessary setting of lock.

Implementing Await Statements

- Any critical section solution can be used to implement unconditional atomic actions:

```
⟨ S; ⟩
```

```
CSenter;  
S;  
CSEXit;
```

- Provided all other code that could interfere with variables in **S** are also protected similarly.
- This was what we did using semaphores as **mutexes**.

Implementing Await Statements

- How should we add `await`?

```
⟨ await(B) s; ⟩
```

```
CSenter;  
while (B) { ??? }  
S;  
CSexit;
```

- If we don't do anything, deadlock is guaranteed since all processes are blocked.

Implementing Await Statements

- `< await(B) s; >`

```
CSenter;  
while (B) {  
    CExit;  
    CSenter;  
}  
S;  
CExit;
```

- Correct but inefficient.
- Good chance the scheduler will not be very fair.

Implementing Await Statements

- `< await(B) s; >`

```
CSenter;  
while (B) {  
    CSexit;  
    Delay;  
    CSenter;  
}  
S;  
CSexit;
```

- Gives more chance for other processes to change B
- Used in Ethernet **truncated binary exponential backoff protocol**.
 - After one fail, randomly wait 0 or 1 time units.
 - After two fails, randomly wait 0 or 1 or 2 or 3 time units.
 - After three fails, randomly wait 0 or 1 or ... or 7 time units.
 - ...
 - After n or greater fails, randomly wait 0 ... $2^n - 1$ time units
- Shown to be useful in critical section entry protocols, too.

Critical Sections: Fair Solutions

- Spin-lock solutions we've seen require a strongly fair scheduler.
- This is impractical, real schedulers are usually weakly fair.
- Three user-defined critical section protocols, only requiring weak fairness:
 - Tie breaker algorithm
 - Ticket algorithm
 - Bakery algorithm

```

bool in1 = false, in2 = false;
## MUTEX:  $\neg(\text{in1} \wedge \text{in2})$  -- global invariant
process CS1 {
    while (true) {
        ⟨await (!in2) in1 = true;⟩ /* entry */
        critical section;
        in1 = false;                /* exit */
        noncritical section;
    }
}
process CS2 {
    while (true) {
        ⟨await (!in1) in2 = true;⟩ /* entry */
        critical section;
        in2 = false;                /* exit */
        noncritical section;
    }
}

```

Figure 3.1 Critical section problem: Coarse-grained solution.

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- Not guaranteed for weakly fair scheduler.

Tie breaking attempted protocol 1

```
while (in2) skip;  
in1 = true;
```

```
while (in1) skip;  
in2 = true;
```

- Problem?

Tie breaking attempted protocol 2

```
in1 = true;  
while (in2) skip;
```

```
in2 = true;  
while (in1) skip;
```

- Problem?

Solution to deadlock problem

- Add a variable **last** to break ties when deadlocked.


```

bool in1 = false, in2 = false;
int last = 1;
process CS1 {
    while (true) {
        last = 1; in1 = true;    /* entry protocol */
        ⟨await (!in2 or last == 2);⟩
        critical section;
        in1 = false;            /* exit protocol */
        noncritical section;
    }
}
process CS2 {
    while (true) {
        last = 2; in2 = true;    /* entry protocol */
        ⟨await (!in1 or last == 1);⟩
        critical section;
        in2 = false;            /* exit protocol */
        noncritical section;
    }
}

```

Figure 3.5 Two-process tie-breaker algorithm: Coarse-grained solution.

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- Are the **await** statements at-most-once?

```

bool in1 = false, in2 = false;
int last = 1;
process CS1 {
    while (true) {
        last = 1; in1 = true;    /* entry protocol */
        while (in2 and last == 1) skip;
        critical section;
        in1 = false;            /* exit protocol */
        noncritical section;
    }
}
process CS2 {
    while (true) {
        last = 2; in2 = true;    /* entry protocol */
        while (in1 and last == 2) skip;
        critical section;
        in2 = false;            /* exit protocol */
        noncritical section;
    }
}

```

Figure 3.6 Two-process tie-breaker algorithm: Fine-grained solution.

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- At-most-once not required here.

```

int in[1:n] = ([n] 0), last[1:n] = ([n] 0);
process CS[i = 1 to n] {
    while (true) {
        for [j = 1 to n] {          /* entry protocol */
            /* remember process i is in stage j and is last */
            last[j] = i; in[i] = j;
            for [k = 1 to n st i != k] {
                /* wait if process k is in higher numbered stage
                   and process i was the last to enter stage j */
                while (in[k] >= in[i] and last[j] == i) skip;
            }
        }
        critical section;
        in[i] = 0;                    /* exit protocol */
        noncritical section;
    }
}

```

Figure 3.7 The n -process tie-breaker algorithm.

```

int number = 1, next = 1, turn[1:n] = ([n] 0);
## predicate TICKET is a global invariant (see text)
process CS[i = 1 to n] {
    while (true) {
        ⟨turn[i] = number; number = number + 1;⟩
        ⟨await (turn[i] == next);⟩
        critical section;
        ⟨next = next + 1;⟩
        noncritical section;
    }
}

```

Figure 3.8 The ticket algorithm: Coarse-grained solution.

TICKET

next > 0

\wedge

$(\forall_{1 \leq i \leq n} :$

$(\text{CS}[i] \text{ in its critical section}) \Rightarrow (\text{turn}[i] == \text{next})$

\wedge

$(\text{turn}[i] > 0) \Rightarrow (\forall_{1 \leq j \leq n, j \neq i} \text{turn}[i] \neq \text{turn}[j])$

)

Fetch and Add

```
FA(var, incr):  
  <  
  int tmp = var;  
  var = var + incr;  
  return(tmp);  
  >
```

- Implemented in hardware.

```

int number = 1, next = 1, turn[1:n] = ([n] 0);
process CS[i = 1 to n] {
    while (true) {
        turn[i] = FA(number,1);          /* entry protocol */
        while (turn[i] != next) skip;
        critical section;
        next = next + 1;                  /* exit protocol  */
        noncritical section;
    }
}

```

Figure 3.9 The ticket algorithm: Fine-grained solution.

Skipping Bakery Algorithm

Barrier Synchronization

- Inefficient solution, too many tasks starting and stopping:

```
while (true) {  
  co [i = 1 to n]  
    code for task i  
  oc  
}
```

- Much more costly to create and destroy processes than to synchronize them.
- More efficient model:

```
process Worker[i = 1 to n] {  
  while (true) {  
    code for task i  
    wait for all n tasks to complete  
  }  
}
```

```

int count = 0;
process Worker[i = 1 to n] {
    while (true) {
        code to implement task i;
        <count = count + 1;>
        <await (count == n);>
    }
}

```

Simple counter barrier in display (3.11)

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- Can implement barrier with:

```

FA(count, 1);
while (count != n) skip;

```

- Problem is resetting **count** and looping
- **count** is a global variable for each process

Flags and Coordinators

- Distribute `count` over `arrive[1:n]`

- Global invariant becomes:

`count == (arrive[1] + ... + arrive[n])`

- Waiting on this is just as bad:

`<await ((arrive[1] + ... + arrive[n]) == n); >`

- Use a coordinator task.

Task i

- `arrive[i] = 1;`
`< await (continue[i] == 1);`

Coordinator

- `for [i = 1 to n] < await (arrive[i] == 1); >`
`for [i = 1 to n] continue[i] = 1;`

Flag Synchronization Principles

- The process that waits for a synchronization flag to be set is the one that should clear that flag.
- A flag should not be set until it is known that it is clear.

```

int arrive[1:n] = ([n] 0),  continue[1:n] = ([n] 0);
process Worker[i = 1 to n] {
    while (true) {
        code to implement task i;
        arrive[i] = 1;
        ⟨await (continue[i] == 1);⟩
        continue[i] = 0;
    }
}
process Coordinator {
    while (true) {
        for [i = 1 to n] {
            ⟨await (arrive[i] == 1);⟩
            arrive[i] = 0;
        }
        for [i = 1 to n] continue[i] = 1;
    }
}

```

Figure 3.12 Barrier synchronization using a coordinator process.

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- Avoids memory contention.
- Is not symmetric.
- Coordinator spends most of its time waiting.
- Tasks have a linear time wait for coordinator.

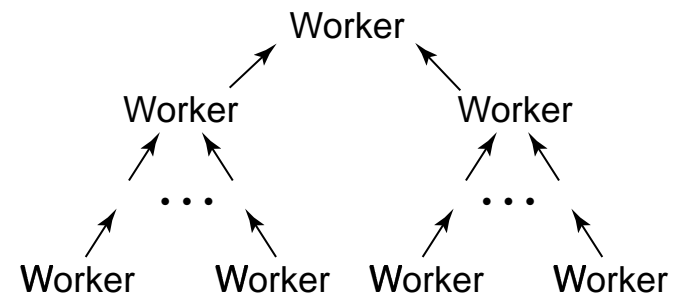


Figure 3.13 Tree-structured barrier.

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- Combining tree barrier.

```

leaf node L:  arrive[L] = 1;
               <await (continue[L] == 1);>
               continue[L] = 0;

interior node I: <await (arrive[left] == 1);>
                  arrive[left] = 0;
                  <await (arrive[right] == 1);>
                  arrive[right] = 0;
                  arrive[I] = 1;
                  <await (continue[I] == 1);>
                  continue[I] = 0;
                  continue[left] = 1; continue[right] = 1;

root node R:  <await (arrive[left] == 1);>
               arrive[left] = 0;
               <await (arrive[right] == 1);>
               arrive[right] = 0;
               continue[left] = 1; continue[right] = 1;

```

Figure 3.14 Barrier synchronization using a combining tree.

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- More symmetric, each task does some real computation.
- But still three different kinds of nodes.

```

/* barrier code for worker process W[i] */
<await (arrive[i] == 0);> /* key line -- see text */
arrive[i] = 1;
<await (arrive[j] == 1);>
arrive[j] = 0;

/* barrier code for worker process W[j] */
<await (arrive[j] == 0);> /* key line -- see text */
arrive[j] = 1;
<await (arrive[i] == 1);>
arrive[i] = 0;

```

Two-process symmetric barrier in display (3.15)

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- A symmetric two-process barrier.

Wait clearing own flag.

Set own flag.

Wait setting other flag.

Clear other flag.

- First line is necessary to prevent a process racing around.

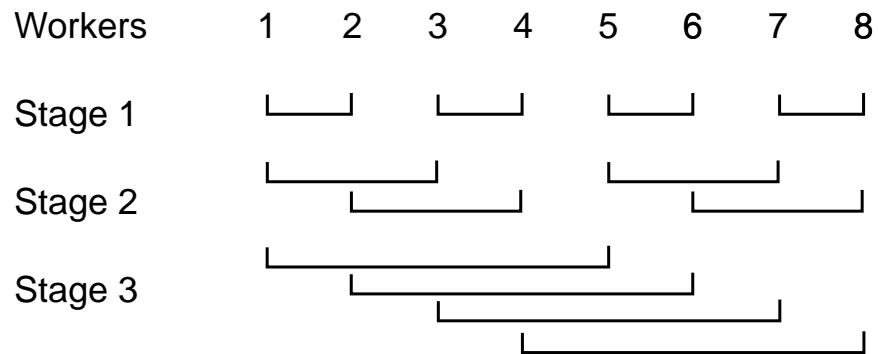


Figure 3.15 Butterfly barrier for 8 processes.

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- Combining 2-process synchronization.
- At stage s synchronize with process 2^{s-1} away.
- n must be power of 2.

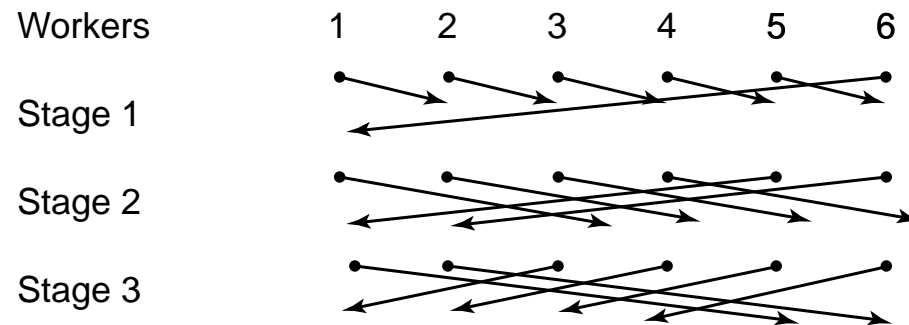


Figure 3.16 Dissemination barrier for 6 processes.

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- At stage s synchronize with process 2^{s-1} away.
- **Dissemination barrier:**
 - Set arrival flag of worker to right.
 - Wait on own flag.
 - Clear own flag.

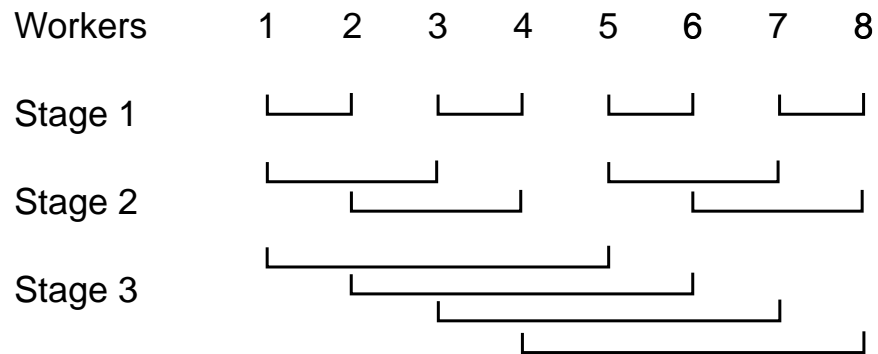


Figure 3.15 Butterfly barrier for 8 processes.

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- Need to avoid global flags:

Suppose 1 finishes, but 2 is slow.

Now 3 and 4 finish.

Now 3 tries to synchronize with 1, and thinks it is ready.

- Could use different flags for each level.
- Or use integer flags.

Data Parallel Algorithms

- Many processes execute the same code and work on different parts of shared data.
- Usually associated with parallel hardware, *e.g.* graphics cards.
- Barrier synchronization usually in hardware.
- Can be useful on asynchronous processors when granularity of the processes is large enough to compensate for synchronization overhead.

Partial sums of an array

Sequential solution

```
sum[0] = a[0];  
for [i = 1 to n-1]  
    sum[i] = sum[i-1] + a[i]
```

initial values of a	1	2	3	4	5	6
partial sums	1	3	6	10	15	21

```

int a[n], sum[n], old[n];
process Sum[i = 0 to n-1] {
    int d = 1;
    sum[i] = a[i];    /* initialize elements of sum */
    barrier(i);
    ## SUM:  sum[i] = (a[i-d+1] + ... + a[i])
    while (d < n) {
        old[i] = sum[i];    /* save old value */
        barrier(i);
        if ((i-d) >= 0)
            sum[i] = old[i-d] + sum[i];
        barrier(i);
        d = d+d;    /* double the distance */
    }
}

```

Figure 3.17 Computing all partial sums of an array.

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initial values of a	1	2	3	4	5	6
sum after distance 1	1	3	5	7	9	11
sum after distance 2	1	3	6	10	14	18
sum after distance 4	1	3	6	10	15	21

- A $\log(n)$ concurrent solution using **doubling**.

```

int link[n], end[n];
process Find[i = 0 to n-1] {
    int new, d = 1;
    end[i] = link[i];    /* initialize elements of end */
    barrier(i);
    ## FIND: end[i] == index of end of the list
    ##          at most  $2^{d-1}$  links away from node i
    while (d < n) {
        new = null;      /* see if end[i] should be updated */
        if (end[i] != null and end[end[i]] != null)
            new = end[end[i]];
        barrier(i);
        if (new != null)    /* update end[i] */
            end[i] = new;
        barrier(i);
        d = d + d;          /* double the distance */
    }
}

```

Figure 3.18 Finding the end of a serially linked list.

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- Find the end of linked list in $\log(n)$ time.

```

real grid[n+1,n+1], newgrid[n+1,n+1];
bool converged = false;
process Grid[i = 1 to n, j = 1 to n] {
    while (not converged) {
        newgrid[i,j] = (grid[i-1,j] + grid[i+1,j] +
                        grid[i,j-1] + grid[i,j+1]) / 4;
        check for convergence as described in the text;
        barrier(i);
        grid[i,j] = newgrid[i,j];
        barrier(i);
    }
}

```

Figure 3.19 Grid computation for solving Laplace's equation.

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- Convergence can be checked with partial sums algorithm.
- Unroll into two stages to avoid copying back.
- Use red-black successive relaxation (Chapter 11).
- Partition grid into blocks (on asynchronous machines).


```

int a[n], sum[n];
process Sum[i = 0 to n-1] {
    sum[i] = a[i];    /* initialize elements of sum */
    while (d < n) {
        if ((i-d) >= 0)    /* update sum */
            sum[i] = sum[i-d] + sum[i];
        d = d + d;        /* double the distance */
    }
}

```

Computing partial sums on a SIMD machine.

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- **Single Instruction Multiple Data**
- Every processor executes exactly the same instructions in lock step.
- Barriers not needed since all finish before looping.
- Every process fetches old **sum** before writing new one.
- Parallel assignments thus appear to be atomic.
- **if** statements always take the maximum time.

```
while (true) {  
    get a task from the bag;  
    if (no more tasks)  
        break;      # exit the while loop  
    execute the task, possibly generating new ones;  
}
```

Outline of worker processes using the bag-of-tasks paradigm.

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- **Bag of Tasks**

- Can be used with recursive parallelism (calls are tasks).
- Scalable: use any number of processors.
- Automatic load balancing.

```

int nextRow = 0; # the bag of tasks
double a[n,n], b[n,n], c[n,n];

process Worker[w = 1 to P] {
    int row;
    double sum; # for inner products
    while (true) {
        # get a task
        < row = nextRow; nextRow++; >
        if (row >= n)
            break;
        compute inner products for c[row,*];
    }
}

```

Figure 3.20 Matrix multiplication using a bag of tasks.

```

type task = (double left, right, fleft, fright, lrarea);
queue bag(task);      # the bag of tasks
int size;              # number of tasks in bag
int idle = 0;          # number of idle workers
double total = 0.0;    # the total area

compute approximate area from a to b;
insert task (a, b, f(a), f(b), area) in the bag;
count = 1;

process Worker[w = 1 to PR] {
    double left, right, fleft, fright, lrarea;
    double mid, fmid, larea, rarea;
    while (true) {
        # check for termination
        < idle++;
        if (idle == n && size == 0) break; >
        # get a task from the bag
        < await (size > 0)
        remove a task from the bag;
        size--; idle--; >
        mid = (left+right) / 2;
        fmid = f(mid);
        larea = (fleft+fmid) * (mid-left) / 2;
        rarea = (fmid+fright) * (right-mid) / 2;
        if (abs((larea+rarea) - lrarea) > EPSILON) {
            < put (left, mid, fleft, fmid, larea) in the bag;
            put (mid, right, fmid, fright, rarea) in the bag;
            size = size + 2; >
        } else
            < total = total + lrarea; >
    }
    if (w == 1)    # worker 1 prints the result
        printf("the total is %f\n", total);
}

```

Typo: remove if (w == 1)

Figure 3.21 Adaptive quadrature using a bag of tasks.