

Name \_\_\_\_\_ Student Number \_\_\_\_\_

Homework #3, CSCI 480, Winter 2015

Give justifications for each line in the following proofs. Include line numbers wherever appropriate.

- |    |   |                       |
|----|---|-----------------------|
| 1. | $P \Rightarrow (P \Rightarrow Q)$                                 | Assumption            |
| 2. | $P$   | Assumption            |
| 3. | $P \Rightarrow Q$   | 1,2,Modus Ponens      |
| 4. | $Q$   | 2,3,Modus Ponens      |
| 5. | $P \Rightarrow Q$   | 2-4,Conditional proof |
| 6. | $(P \Rightarrow (P \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$ | 1-5,Conditional proof |

- |    |   |                       |
|----|---|-----------------------|
| 1. | $P \Rightarrow Q$   | Assumption            |
| 2. | $P \wedge R$  | Assumption            |
| 3. | $P$   | 2,Simplification      |
| 4. | $R$   | 2,Simplification      |
| 5. | $Q$   | 1,3,Modus Ponens      |
| 6. | $Q \wedge R$  | 4,5,Conjunction       |
| 7. | $(P \wedge R) \Rightarrow (Q \wedge R)$                                 | 2-6,Conditional proof |
| 8. | $(P \Rightarrow Q) \Rightarrow ((P \wedge R) \Rightarrow (Q \wedge R))$ | 1-7,Conditional proof |

- |     |  |                        |
|-----|--|------------------------|
| 1.  | $(P \Rightarrow R) \wedge (Q \Rightarrow S)$   | Assumption             |
| 2.  | $P \Rightarrow R$  | 1, Simplification      |
| 3.  | $Q \Rightarrow S$  | 1, Simplification      |
| 4.  | $P \wedge Q$   | Assumption             |
| 5.  | $P$  | 4, Simplification      |
| 6.  | $Q$  | 4, Simplification      |
| 7.  | $R$  | 2,5,Modus Ponens       |
| 8.  | $S$  | 3,6,Modus Ponens       |
| 9.  | $R \wedge S$   | 7,8,Conjunction        |
| 10. | $(P \wedge Q) \Rightarrow (R \wedge S)$  | 4-9,Conditional proof  |
| 11. | $((P \Rightarrow R) \wedge (Q \Rightarrow S)) \Rightarrow ((P \wedge Q) \Rightarrow (R \wedge S))$ | 1-10,Conditional proof |

1.	$P \Rightarrow (Q \Rightarrow R)$	Assumption
2.	$Q$	Assumption
3.	$P$	Assumption
4.	$Q \Rightarrow R$	1,3,Modus Ponens
5.	$R$	2,4,Modus Ponens
6.	$P \Rightarrow R$	3-5,Conditional proof
7.	$Q \Rightarrow (P \Rightarrow R)$	2-6,Conditional proof
8.	$(P \Rightarrow (Q \Rightarrow R)) \Rightarrow (Q \Rightarrow (P \Rightarrow R))$	1-7,Conditional proof

1.	$(P \Rightarrow Q) \Rightarrow Q$	Assumption
2.	$Q \Rightarrow P$	Assumption
3.	$\neg P$	Assumption
4.	$\neg Q$	2,3,Modus Tollens
5.	$\neg(P \Rightarrow Q)$	1,4,Modus Tollens
6.	$P \wedge \neg Q$	5, Equivalent by Axiom
7.	$P$	6,Simplification
8.	$P$	3,7,Indirect proof
9.	$(Q \Rightarrow P) \Rightarrow P$	2-8,Conditional proof
10.	$((P \Rightarrow Q) \Rightarrow Q) \Rightarrow ((Q \Rightarrow P) \Rightarrow P)$	1-9,Conditional proof

**Due date: Monday, February 2, at the start of class.**