Logic: axioms

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$
$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$\begin{aligned}
\neg\neg A &\equiv A \\
A \vee A &\equiv A \\
A \vee \neg A &\equiv true \\
A \vee true &\equiv true \\
A \vee false &\equiv A \\
A \wedge true &\equiv A \\
A \wedge false &\equiv false \\
A \wedge A &\equiv A \\
A \wedge \neg A &\equiv false
\end{aligned}$$

$$\begin{aligned}
A \Rightarrow true &\equiv true \\
A \Rightarrow false &\equiv \neg A \\
true \Rightarrow A &\equiv A \\
false \Rightarrow A &\equiv true \\
A \Rightarrow A &\equiv true \\
A \Rightarrow B &\equiv \neg A \vee B \\
A \Rightarrow B &\equiv \neg B \Rightarrow \neg A \\
\neg(A \Rightarrow B) &\equiv A \wedge \neg B \\
A \wedge (A \vee B) &\equiv A \\
A \vee (A \wedge B) &\equiv A \\
A \wedge (\neg A \vee B) &\equiv A \wedge B \\
A \vee (\neg A \wedge B) &\equiv A \vee B \\
\neg(A \wedge B) &\equiv \neg A \vee \neg B \\
\neg(A \vee B) &\equiv \neg A \wedge \neg B
\end{aligned}$$

Logic: inference rules

- Modus Ponens
  $$\frac{A \Rightarrow B, A}{B}$$
- Modus Tollens
  $$\frac{A \Rightarrow B, \neg B}{\neg A}$$
- Conjunction
  $$\frac{A, B}{A \wedge B}$$
- Simplification
  $$\frac{A \wedge B}{A}$$
- Addition
  $$\frac{A}{A \vee B}$$
- Disjunctive syllogism
  $$\frac{A \vee B, \neg A}{B}$$
- Hypothetical syllogism
  $$\frac{A \Rightarrow B, B \Rightarrow C}{A \Rightarrow C}$$
- Constructive dilemma
  $$\frac{A \vee B, A \Rightarrow C, B \Rightarrow D}{C \vee D}$$
- Destructive dilemma
  $$\frac{\neg C \vee \neg D, A \Rightarrow C, B \Rightarrow D}{\neg A \vee \neg B}$$

Axiomatic Semantics

Assignment Axiom

$$\{P_{x\leftarrow t}\}\ \texttt{x = t}\ \{P\}$$

Inference Rules

- Composition
$$\frac{\{P\}\ \texttt{S1}\ \{R\}, \{R\}\ \texttt{S2}\ \{Q\}}{\{P\}\ \texttt{S1;S2}\ \{Q\}}$$
- Consequence
$$\frac{P \Rightarrow R, \{R\}\ \texttt{S}\ \{Q\}}{\{P\}\ \texttt{S}\ \{Q\}} \qquad\qquad \frac{\{P\}\ \texttt{S}\ \{T\}, T \Rightarrow Q}{\{P\}\ \texttt{S}\ \{Q\}}$$
- If-then
$$\frac{\{P \wedge C\}\ \texttt{S}\ \{Q\}, P \wedge \neg C \Rightarrow Q}{\{P\}\ \texttt{if C then S}\ \{Q\}}$$
- If-then-else
$$\frac{\{P \wedge C\}\ \texttt{S1}\ \{Q\}, \{P \wedge \neg C\}\ \texttt{S2}\ \{Q\}}{\{P\}\ \texttt{if C then S1 else S2}\ \{Q\}}$$
- While
$$\frac{\{P \wedge C\}\ \texttt{S}\ \{P\}}{\{P\}\ \texttt{while C do S}\ \{P \wedge \neg C\}}$$

Sematics of Concurrent Execution

- Await rule
$$\frac{\{P \wedge B\}\ \texttt{S}\ \{Q\}}{\{P\}\ \langle\texttt{await (B) S;}\rangle\ \{Q\}}$$
- Co rule
$$\frac{\{P_i\}\ \texttt{Si}\ \{Q_i\}\ \text{are interference free}}{\{P_1 \wedge ... \wedge P_n\}\ \texttt{co S1; // ... // Sn; oc}\ \{Q_1 \wedge ... \wedge Q_n\}}$$
- One process **interferes** with another if it executes an assignment that invalidates an assertion in the other process.