Box #____ (Name on Back)
Math 65 Section ___
Homework 2
11/2/18

Problems for Section 6.3: Change of Basis

B1: For problems B1-B3 you will answer (a)-(e). Consider

$$\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \ \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, \ \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$$

where $\mathcal{B}$ and $\mathcal{C}$ are bases for $\mathbb{R}^2$.

(a) Find the coordinate vectors $[\vec{x}]_{\mathcal{B}}$ and $[\vec{x}]_{\mathcal{C}}$ of $\vec{x}$ with respect to the bases $\mathcal{B}$ and $\mathcal{C}$, respectively.

(b) Find the change-of-basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

(c) Use your answer to part (b) to compute $[\vec{x}]_{\mathcal{C}}$, and compare your answer with the one found in part (a).

(d) Find the change-of-basis matrix $P_{\mathcal{B}\leftarrow\mathcal{C}}$ from $\mathcal{C}$ to $\mathcal{B}$.

(e) Use your answers to parts (c) and (d) to compute $[\vec{x}]_{\mathcal{B}}$, and compare your answer with the one found in part (a).

Problem B1 Continued:

B2: Answer (a)-(e) from exercise B1 using $p(x)$ instead of $\vec{x}$. Consider

$$p(x) = 2 - x + 3x^2,\ \mathcal{B} = \{1 + x, 1 + x^2, x + x^2\},\ \mathcal{C} = \{1, x, 1 + x + x^2\}$$

where $\mathcal{B}$ and $\mathcal{C}$ are bases for $\mathcal{P}_2$.

B3: Answer (a)-(e) from exercise B1 using $f(x)$ instead of $\vec{x}$. Consider

$$f(x) = \cos x,\ \mathcal{B} = \{\sin x, \sin x + \cos x\},\ \mathcal{C} = \{\sin x + \cos x, \sin x - \cos x\}$$

where $\mathcal{B}$ and $\mathcal{C}$ are bases for the vector space $\mathcal{V} = \operatorname{span}(\sin x, \cos x)$.

B4: (Poole, p. 472) Section 6.3 #18.

Express $p(x) = 1 + 2x - 5x^2$ as a Taylor polynomial about $a = -2$.

Note: It may help to consider this as a problem in $\mathcal{P}_2$. Remember that the Taylor polynomial about $a = -2$ will take the form

$$p(x) = a_0 + a_1(x+2) + a_2(x+2)^2.$$

Finally, think about why $\mathcal{B} = \{1, x+2, (x+2)^2\}$ is a basis for $\mathcal{P}_2$.

Problems for Section 6.4: Linear Transformations

B5: (Poole, p. 480) Section 6.4 #5.

Determine whether T is a linear transformation.

$$T\colon M_{nn} \to \mathbb{R} \text{ defined by } T(A) = \operatorname{tr}(A)$$

Note: Here $\operatorname{tr}(A)$ is the trace of the matrix A. If you have forgotten the definition, try googling it.

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B6: (Poole, p. 480) Section 6.4 #6.

Determine whether T is a linear transformation.

$$T\colon M_{nn} \to \mathbb{R} \text{ defined by } T(A) = a_{11}a_{22}\cdots a_{nn}$$

B7: (Poole, p. 480) Section 6.4 #12.

Determine whether T is a linear transformation.

$$T\colon \mathcal{F} \to \mathbb{R} \text{ defined by } T(f) = f(c), \text{ where } c \text{ is a fixed scalar}$$

Note: Here $\mathcal{F}$ is the set of functions $f(x)$ defined for all $x \in \mathbb{R}$.