Box #____ (Name on Back)
Math 65 Section ___
Homework 9
12/7/18

Homework #9: Matrix Exponential

I1: Consider the matrix

$$M = \begin{bmatrix} 0 & q \\ -q & 0 \end{bmatrix}.$$

Use the power series definition of e^{At} to show that

$$e^{Mt} = \begin{bmatrix} \cos qt & \sin qt \\ -\sin qt & \cos qt \end{bmatrix}.$$

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I2: We know for real numbers a, b, that $e^{at}e^{bt} = e^{(a+b)t} = e^{bt}e^{at}$. This is not necessarily true for matrices. Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

(a) Compute e^{At}, e^{Bt}, and $e^{(A+B)t}$.

(b) Show $e^{At}e^{Bt} \neq e^{(A+B)t}$.

(c) Show $e^{At}e^{Bt} \neq e^{Bt}e^{At}$.

Hint: For e^{At} you will want to use the power series definition. For e^{Bt} and $e^{(A+B)t}$ either the power series method or using eigenvalues/vectors should get you to a solution quickly.

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I3: Consider the linear system

$$
\begin{aligned}
x' &= 5x + 3y && (1)\\
y' &= -6x - 4y && (2)
\end{aligned}
$$

The matrices

$$
\Psi(t) = \begin{bmatrix} e^{-t} & e^{2t} \\ -2e^{-t} & -e^{2t} \end{bmatrix} \quad \text{and} \quad \Phi(t) = \begin{bmatrix} e^{2t} - e^{-t} & e^{2t} \\ -e^{2t} + 2e^{-t} & -e^{2t} \end{bmatrix}
$$

are both fundamental matrices for this system.

(a) Show $\Psi(t)\Psi^{-1}(0) = \Phi(t)\Phi^{-1}(0)$.

(b) Compute e^{At} for the given system.

(c) Solve the initial-value problem for (1)-(2) when $x(0) = 1$ and $y(0) = 1$.

I4: Compute e^{At} where

$$A = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}.$$

Note: You do **not** want to use the power series definition.

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I5: Compute e^{At} where

$$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$$

and solve the initial-value problem $x' = Ax$, $x(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.