Box #___ (Name on Back)
Math 65 Section ___
Homework 1
10/30/18

A1: (Poole, p. 441) §6.1 #4. Determine whether the given set, together with the specified operations of addition and scalar multiplication is a vector space. If it is not, list all the axioms that fail to hold:

$$V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \text{ in } \mathbb{R}^2 \text{ with } x \geq y \right\}$$

with the usual vector addition and scalar multiplication.

A2: (Poole, p. 441) §6.1 #10. Determine whether the given set, together with the specified operations of addition and scalar multiplication is a vector space. If it is not, list all the axioms that fail to hold:

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ in } M_{22} \text{ with } ad = 0 \right\}$$

with the usual matrix addition and scalar multiplication.

A3: (Poole, p. 441) §6.1 #26. Use Theorem 6.2 to determine whether W is a subspace of V.

$$V = \mathbb{R}^3, \qquad W = \left\{ \begin{bmatrix} a \\ b \\ a+b+1 \end{bmatrix} \right\} \text{ where } a, b \text{ are in } \mathbb{R}$$

A4: (Poole, p. 441) §6.1 #34. Use Theorem 6.2 to determine whether W is a subspace of V:

$$V = \mathcal{P}_2, \qquad W = \left\{bx + cx^2\right\} \text{ where } b, c \text{ are in } \mathbb{R}$$

A5: (Poole, p. 441) §6.1 #38. Use Theorem 6.2 to determine whether W is a subspace of V:

$$V = \mathcal{F}, \qquad W = \{f \text{ in } \mathcal{F} : f(-x) = f(x)\}$$

A6: (Poole, p. 441) §6.1 #46. Let V be a vector space with subspaces U and W. Prove that $U \cap W$ is a subspace of V.

A7: Test the set of matrices for linear independence in M_{22}. For those that are linearly dependent, express one matrix as a linear combination of the others:

$$\left\{ \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

A8: (Poole, p. 456) §6.2 #12. Test the set of functions for linear independence in $\mathcal{F}$. For those that are linearly dependent, express one of the functions as a linear combination of the others.

$$\{e^x, e^{-x}\}$$

A9: (Poole, p. 456) §6.2 #13. Test the set of functions for linear independence in $\mathcal{F}$. For those that are linearly dependent, express one of the functions as a linear combination of the others.

$$\{1, \ln(2x), \ln x^2\}$$

You should assume that $x > 0$.

A10: (Poole, p. 456) §6.2 #15. If $f(x)$ and $g(x)$ are in $\mathcal{C}^{(1)}$, the vector space of all functions with continuous derivatives, then the determinant

$$W(x) = \begin{vmatrix} f(x) & g(x) \\ f'(x) & g'(x) \end{vmatrix}$$

is called the *Wronskian* of f and g. Show that f and g are linearly independent if their Wronskian is not identically zero (that is, there is some x such that $W(x) \neq 0$).

A11: (Poole, p. 456) §6.2 #27. Find the coordinate vector of

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

with respect to the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

of M_{22}.

A12: Find the coordinate vector of $p(x) = 2 - x + 3x^2$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of $\mathcal{P}_2$.

A13: (Poole, p. 456) §6.2 #34. Find the dimension of the vector space V and give a basis for V:

$$V = \{p(x) \text{ in } \mathcal{P}_2 : p(0) = 0\}$$

A14: (Poole, p. 456) §6.2 #36. Find the dimension of the vector space V and give a basis for V:

$$V = \{p(x) \text{ in } \mathcal{P}_2 : xp'(x) = p(x)\}$$