Box #____ (Name on Back)
Math 65 Section ___
Homework 7
11/30/18

Homework #7: First-order Systems of DE's & Complex Eigenvalues/Eigenvectors

G1: Rewrite the second-order differential equation for $y(t)$.

$$y'' + 9y = 0, \qquad y(0) = 2,\ y'(0) = 1$$

as a first-order linear system. Solve the initial value problem using eigenvalues and eigenvectors. Check that your solution for $y(t)$ satisfies the original differential equation and the given initial conditions.

Box #____

G2: Write the following two differential equations,

$$x' = x - 2y, \qquad y' = 2x + y$$

in the form $\vec{x}' = A\vec{x}$. Find the general real valued solution of the system and describe the behavior of each component of the solution as time increases.

G3: Consider the system

$$\vec{x}\,' = \begin{bmatrix} -5 & -1 \\ 2 & -3 \end{bmatrix} \vec{x}.$$

(a) Find the real-valued general solution.

(b) Solve the initial value problem with $\vec{x}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

(c) What is $\lim_{t \to \infty} \vec{x}(t)$ for your answer to (b)?

G4: Find the general solution for the linear system

$$\vec{x}' = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 4 \\ 1 & -1 & 2 \end{bmatrix} \vec{x}.$$

Express your answer in the form $\vec{x}(t) = \Psi(t)\vec{c}$ where $\Psi(t)$ is a fundamental matrix.
Note: Your answer for the general solution and the fundamental matrix should be real-valued.