Box #____ (Name on Back)
Math 65 Section ___
Homework 3
11/6/18

Problems for Section 6.4: Linear Transformations

C1: (Poole, p. 480) Section 6.4 #18.

Let $T\colon M_{22} \to \mathbb{R}$ be a linear transformation for which

$$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = 1, \quad T\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = 2,$$
$$T\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = 3, \quad T\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 4$$

Find $T\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$ and $T\begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

C2: (Poole, p. 480) Section 6.4 #22.

Let $\{\vec{v}_1, \ldots, \vec{v}_n\}$ be a basis for a vector space V and let $T \colon V \to V$ be a linear transformation. Prove that if $T(\vec{v}_1) = \vec{v}_1, T(\vec{v}_2) = \vec{v}_2, \ldots, T(\vec{v}_n) = \vec{v}_n$, then T is the identity transformation on V.

C3: Consider the linear transform T defined on polynomials, $p(x)$, of degree n where $n = 1, 2, 3\cdots$,

$$T\colon \mathcal{P}_n \to \mathcal{P}_{n-1} \text{ defined by } T(p) = \frac{dp}{dx}.$$

Is T invertible? If so, find its inverse. If not, explain why not.

Problems for Section 6.5: Kernel, Range & Rank Theorem

C4: Let $T\colon M_{22} \to \mathbb{R}$ be defined by $T(A) = \operatorname{tr}(A)$.

(a) Which, if any, of the following matrices are in $\ker(T)$?

(i) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & -2 \\ 2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

(b) Which, if any, of the following scalars are in $\operatorname{range}(T)$?

(i) 0 (ii) 5 (iii) $-\sqrt{2}$

(c) Describe $\ker(T)$ and $\operatorname{range}(T)$.

C5: Find bases for the kernel and range of the linear transformation T in Exercise C5. State the nullity and rank of T and verify the Rank Theorem.

C6: (Poole, p. 495) Section 6.5 #14.

Find either the nullity or the rank of T and then use the Rank Theorem to find the other.

$$T\colon M_{33} \to M_{33} \text{ defined by } T(A) = A - A^T$$