Package ‘fOptions’

July 2, 2014

Version 3010.83

Revision 5524

Date 2013-06-23

Title Basics of Option Valuation

Author Diethelm Wuertz and many others, see the SOURCE file

Depends methods, timeDate, timeSeries, fBasics

Suggests RUnit, tcltk

Maintainer Yohan Chalabi <yohan.chalabi@rmetrics.org>

Description Environment for teaching ``Financial Engineering and Computational Finance''

Note Several parts are still preliminary and may be changed in the future. This typically includes function and argument names, as well as defaults for arguments and return values.

LazyData yes

License GPL (>= 2)

URL http://www.rmetrics.org

NeedsCompilation yes

Repository CRAN

Date/Publication 2013-06-24 01:54:03

R topics documented:

BasicAmericanOptions 2
BinomialTreeOptions 4
HestonNandiGarchFit 7
HestonNandiOptions 11
LowDiscrepancy 13
MonteCarloOptions 15
PlainVanillaOptions 19

Index 25

`BasicAmericanOptions` *Valuation of Basic American Options*

Description

A collection and description of functions to valuate basic American options. Approximative formulas for American calls are given for the Roll, Geske and Whaley Approximation, for the Barone-Adesi and Whaley Approximation, and for the Bjerksund and Stensland Approximation.

The functions are:

`RollGeskeWhaleyOption`	Roll, Geske and Whaley Approximation,
`BAWAmericanApproxOption`	Barone-Adesi and Whaley Approximation,
`BSAmericanApproxOption`	Bjerksund and Stensland Approximation.

Usage

```
RollGeskeWhaleyOption(S, X, time1, Time2, r, D, sigma,
    title = NULL, description = NULL)
BAWAmericanApproxOption(TypeFlag, S, X, Time, r, b, sigma,
    title = NULL, description = NULL)
BSAmericanApproxOption(TypeFlag, S, X, Time, r, b, sigma,
    title = NULL, description = NULL)
```

Arguments

`b`	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
`D`	a single dividend with time to dividend payout `t1`.
`description`	a character string which allows for a brief description.
`r`	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
`S`	the asset price, a numeric value.
`sigma`	the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.
`Time`	the time to maturity measured in years, a numeric value.
`time1, Time2`	[RollGeskeWhaley*] - the first value measures time to dividend payout in years, e.g. 0.25 denotes a quarter, and the second value measures time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
`title`	a character string which allows for a project title.
`TypeFlag`	a character string either "c" for a call option or a "p" for a put option.
`X`	the exercise price, a numeric value.

Details

Roll-Geske-Whaley Option:

The function `RollGeskeWhaleyOption` valuates American calls on a stock paying a single dividend with specified time to dividend payout according to the pricing formula derived by Roll, Geske and Whaley (1977).

`Approximations for American Options:`

The function `BSAmericanApproxOption` valuates American calls or puts on an underlying asset for a given cost-of-carry rate according to the quadratic approximation method due to Barone-Adesi and Whaley (1987). The function `BSAmericanApproxOption` valuates American calls or puts on stocks, futures, and currencies due to the approximation method of Bjerksund and Stensland (1993).

Value

`RollGeskeWhaleyOption`
`BAWAmericanApproxOption`
return the option price, a numeric value.

`BSAmericanApproxOption`
returns a list with the following two elements: `Premium` the option price, and `TriggerPrice` the trigger price.

Note

The functions implement the algorithms to valuate basic American options as described in Chapter 1.4 of Haug's Option Guide (1997).

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Barone-Adesi G., Whaley R.E. (1987); *Efficient Analytic Approximation of American Option Values*, Journal of Finance 42, 301–320.

Bjerksund P., Stensland G. (1993); *Closed Form Approximation of American Options*, Scandinavian Journal of Management 9, 87–99.

Geske R. (1979); *A Note on an Analytical Formula for Unprotected American Call Options on Stocks with known Dividends*, Journal of Financial Economics 7, 63–81.

Haug E.G. (1997); *The Complete Guide to Option Pricing Formulas*, Chapter 1, McGraw-Hill, New York.

Roll R. (1977); *An Analytic Valuation Formula for Unprotected American Call Options on Stocks with known Dividends*, Journal of Financial Economics 5, 251–258.

Examples

```
## All the examples are from Haug's Option Guide (1997)

## CHAPTER 1.4: ANALYTICAL MODELS FOR AMERICAN OPTIONS

## Roll-Geske-Whaley American Calls on Dividend Paying
   # Stocks [Haug 1.4.1]
   RollGeskeWhaleyOption(S = 80, X = 82, time1 = 1/4,
     Time2 = 1/3, r = 0.06, D = 4, sigma = 0.30)

## Barone-Adesi and Whaley Approximation for American
   # Options [Haug 1.4.2] vs. Black76 Option on Futures:
   BAWAmericanApproxOption(TypeFlag = "p", S = 100,
     X = 100, Time = 0.5, r = 0.10, b = 0, sigma = 0.25)
   Black76Option(TypeFlag = "c", FT = 100, X = 100,
     Time = 0.5, r = 0.10, sigma = 0.25)

## Bjerksund and Stensland Approximation for American Options:
   BSAmericanApproxOption(TypeFlag = "c", S = 42, X = 40,
     Time = 0.75, r = 0.04, b = 0.04-0.08, sigma = 0.35)
```

`BinomialTreeOptions` *Binomial Tree Option Model*

Description

A collection and description of functions to valuate options in the framework of the Binomial tree option approach.

The functions are:

`CRRBinomialTreeOption`	CRR Binomial Tree Option,
`JRBinomialTreeOption`	JR Binomial Tree Option,
`TIANBinomialTreeOption`	TIAN Binomial Tree Option,
`BinomialTreeOption`	Binomial Tree Option,
`BinomialTreePlot`	Binomial Tree Plot.

Usage

```
CRRBinomialTreeOption(TypeFlag = c("ce", "pe", "ca", "pa"), S, X,
    Time, r, b, sigma, n, title = NULL, description = NULL)
JRBinomialTreeOption(TypeFlag = c("ce", "pe", "ca", "pa"), S, X,
    Time, r, b, sigma, n, title = NULL, description = NULL)
TIANBinomialTreeOption(TypeFlag = c("ce", "pe", "ca", "pa"), S, X,
    Time, r, b, sigma, n, title = NULL, description = NULL)

BinomialTreeOption(TypeFlag = c("ce", "pe", "ca", "pa"), S, X,
    Time, r, b, sigma, n, title = NULL, description = NULL)
```

```
BinomialTreePlot(BinomialTreeValues, dx = -0.025, dy = 0.4,
    cex = 1, digits = 2, ...)
```

Arguments

`b`	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
`BinomialTreeValues`	the return value from the `BinomialTreeOption` function.
`cex`	a numerical value giving the amount by which the plotting text and symbols should be scaled relative to the default.
`description`	a character string which allows for a brief description.
`digits`	an integer value, how many digits should be displayed in the option tree?
`dx, dy`	numerical values, an offset fine tuning for the placement of the option values in the option tree.
`n`	number of time steps; an integer value.
`r`	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
`S`	the asset price, a numeric value.
`sigma`	the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.
`Time`	the time to maturity measured in years, a numeric value; e.g. 0.5 means 6 months.
`title`	a character string which allows for a project title.
`TypeFlag`	a character string either `"ce"`, `"ca"` for an European or American call option or a `"pe"`, `"pa"` for a put option, respectively.
`X`	the exercise price, a numeric value.
`...`	arguments to be passed.

Details

CRR Binomial Tree Model:

Binomial models were first suggested by Cox, Ross and Rubinstein (1979), CRR, and then became widely used because of its intuition and easy implementation. Binomial trees are constructed on a discrete-time lattice. With the time between two trading events shrinking to zero, the evolution of the price converges weakly to a lognormal diffusion. Within this mode the European options value converges to the value given by the Black-Scholes formula.

JR Binomial Tree Model:

There exist many extensions of the CRR model. Jarrow and Rudd (1983), JR, adjusted the CRR model to account for the local drift term. They constructed a binomial model where the first two moments of the discrete and continuous time return processes match. As a consequence a probability measure equal to one half results. Therefore the CRR and JR models are sometimes atrributed as equal jumps binomial trees and equal probabilities binomial trees.

TIAN Binomial Tree Model:

Tian (1993) suggested to match discrete and continuous local moments up to third order.

Leisen and Reimer (1996) proved that the order of convergence in pricing European options for all three methods is equal to one, and thus the three models are equivalent.

Value

The option price, a numeric value.

Note

Note, the `BinomialTree` and `BinomialTreePlot` are preliminary implementations.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Broadie M., Detemple J. (1994); *American Option Evaluation: New Bounds, Approximations, and a Comparison of Existing Methods*, Working Paper, Columbia University, New York.

Cox J., Ross S.A., Rubinstein M. (1979); *Option Pricing: A Simplified Approach*, Journal of Financial Economics 7, 229–263.

Haug E.G. (1997); *The complete Guide to Option Pricing Formulas*, McGraw-Hill, New York.

Hull J.C. (1998); *Introduction to Futures and Options Markets*, Prentice Hall, London.

Jarrow R., Rudd A. (1983); *Option Pricing*, Homewood, Illinois, 183–188.

Leisen D.P., Reimer M., (1996); *Binomial Models for Option Valuation – Examining and Improving Convergence*, Applied Mathematical Finanace 3, 319–346.

Tian Y. (1993); *A Modified Lattice Approach to Option Pricing*, Journal of Futures Markets 13, 563–577.

Examples

```
## Cox-Ross-Rubinstein Binomial Tree Option Model:
   # Example 14.1 from Hull's Book:
   CRRBinomialTreeOption(TypeFlag = "pa", S = 50, X = 50,
     Time = 5/12, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
   # Example 3.1.1 from Haug's Book:
    CRRBinomialTreeOption(TypeFlag = "pa", S = 100, X = 95,
        Time = 0.5, r = 0.08, b = 0.08, sigma = 0.3, n = 5)
   # A European Call - Compare with Black Scholes:
   CRRBinomialTreeOption(TypeFlag = "ce", S = 100, X = 100,
     Time = 1, r = 0.1, b = 0.1, sigma = 0.25, n = 50)
   GBSOption(TypeFlag = "c", S = 100, X = 100,
     Time = 1, r = 0.1, b = 0.1, sigma = 0.25)@price

## CRR - JR - TIAN Model Comparison:
   # Hull's Example as Function of "n":
```

```
  par(mfrow = c(2, 1), cex = 0.7)
  steps = 50
  CRROptionValue =  JROptionValue = TIANOptionValue =
    rep(NA, times = steps)
  for (n in 3:steps) {
    CRROptionValue[n] = CRRBinomialTreeOption(TypeFlag = "pa", S = 50,
      X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price
    JROptionValue[n] = JRBinomialTreeOption(TypeFlag = "pa", S = 50,
      X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price
    TIANOptionValue[n] = TIANBinomialTreeOption(TypeFlag = "pa", S = 50,
      X = 50, Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = n)@price
  }
  plot(CRROptionValue[3:steps], type = "l", col = "red", ylab = "Option Value")
  lines(JROptionValue[3:steps], col = "green")
  lines(TIANOptionValue[3:steps], col = "blue")
  # Add Result from BAW Approximation:
  BAWValue =  BAWAmericanApproxOption(TypeFlag = "p", S = 50, X = 50,
    Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4)@price
  abline(h = BAWValue, lty = 3)
  title(main = "Convergence")
  data.frame(CRROptionValue, JROptionValue, TIANOptionValue)

## Plot CRR Option Tree:
  # Again Hull's Example:
  CRRTree = BinomialTreeOption(TypeFlag = "pa", S = 50, X = 50,
    Time = 0.4167, r = 0.1, b = 0.1, sigma = 0.4, n = 5)
  BinomialTreePlot(CRRTree, dy = 1, cex = 0.8, ylim = c(-6, 7),
    xlab = "n", ylab = "Option Value")
  title(main = "Option Tree")
```

`HestonNandiGarchFit` *Heston-Nandi Garch(1,1) Modelling*

Description

A collection and description of functions to model the GARCH(1,1) price paths which underly Heston and Nandi's option pricing model.

The functions are:

`hngarchSim`	Simulates a Heston-Nandi Garch(1,1) process,
`hngarchFit`	MLE for a Heston Nandi Garch(1,1) model,
`hngarchStats`	True moments of the log-Return distribution,
`print.hngarch`	Print method,
`summary.hngarch`	Diagnostic summary.

Usage

```
hngarchSim(model, n, innov, n.start, start.innov, rand.gen, ...)
hngarchFit(x, model = list(lambda = -0.5, omega = var(x), alpha =
    0.1 * var(x), beta = 0.1, gamma = 0, rf = 0), symmetric = TRUE,
    trace = FALSE, title = NULL, description = NULL, ...)
hngarchStats(model)

## S3 method for class 'hngarch'
print(x, ...)
## S3 method for class 'hngarch'
summary(object, ...)
```

Arguments

`description`	a brief description of the porject of type character.
`innov`	[hngarchSim] - is a univariate time series or vector of innovations to produce the series. If not provided, `innov` will be generated using the random number generator specified by `rand.gen`. Missing values are not allowed. By default the normal random number generator will be used.
`model`	a list of GARCH model parameters with the following entries: `lambda`, `omega`, the constant coefficient of the variance equation, `alpha` the autoregressive coefficient, `beta` the variance coefficient, `gamma` the asymmetry coefficient, and `rf`, the risk free rate, numeric values.
`n`	[hngarchSim] - is the length of the series to be simulated. The default value is 1000.
`n.start`	[hngarchSim] - gives the number of start-up values to be discarded. The default value is 100.
`object`	[summary] - a fitted HN-GARCH(1,1) time series object of class `"hngarch"` as returned from the function `hngarchFit`.
`rand.gen`	[hngarchSim] - is the function which is called to generate the innovations. Usually, `rand.gen` will be a random number generator. Additional arguments required by the random number generator `rand.gen`, usually the location, scale and/or shape parameter of the underlying distribution function, have to be passed through the `dots` argument.
`start.innov`	[hngarchSim] - is a univariate time series or vector of innovations to be used as start up values. Missing values are not allowed.
`symmetric`	[hngarchFit] - a logical, if TRUE a symmetric model is estimated, otherwise the parameters are estimated for an asymmetric HN Garch(1,1) model.
`title`	a character string which allows for a project title.

`trace`	[hngarchFit] - a logical value. Should the optimizarion be traced? If `trace=FALSE`, no tracing is done of the iteration path.
`x`	[hngarchFit] - an univariate vector or time series. [print] - a fitted HN-GARCH(1,1) time series object of class `"hngarch"` as returned from the function `hngarchFit`.
`...`	additional arguments to be passed.

Details

Path Simulation:

The function `hngarchSim` simulates a Heston-Nandi Garch(1,1) process with structure parameters specified through the list `model(lambda, omega, alpha, beta, gamma, rf)`.

Parameter Estimation:

The function `hngarchFit` estimates by the maximum log-likelihood approach the parameters either for a symmetric or an asymmetric Heston-Nandi Garch(1,1) model from the log returns `x` of a financial time series. For optimization R's `optim` function is used. Additional optimization parameters may be passed throught the `...` argument.

Diagnostic Analysis:

The function `summary.hngarch` performs a diagnostic analysis and recalculates conditional variances and innovations from the time series.

Calculation of Moments:

The function `hngarchStats` calculates the first four true moments of the unconditional log return distribution for a stationary Heston-Nandi Garch(1,1) process with standard normally distributed innovations. In addition the persistence and the expectation values of sigma to the power 2, 4, 6, and 8 can be accessed.

Value

`hngarchSim`
returns numeric vector with the simulated time series points neglecting those from the first `start.innov` innovations.

`hngarchFit`
returns list with two entries: The estimated model parmeters `model`, where `model` is a list of the parameters itself, and `llh` the value of the log likelihood.

`hngarchStats`
returns a list with the following components: `persistence`, the value of the persistence, `meansigma2`, `meansigma4`, `meansigma6`, `meansigma8`, the expectation value of sigma to the power of 2, 4, 6, and 8, `mean`, `variance`, `skewness`, `kurtosis`, the mean, variance, skewness and kurtosis of the log returns.

`summary.hngarch`
returns list with the following elements: `h`, a numeric vector with the conditional variances, `z`, a numeric vector with the innovations.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Heston S.L., Nandi S. (1997); *A Closed-Form GARCH Option Pricing Model*, Federal Reserve Bank of Atlanta.

Examples

```
## hngarchSim -
   # Simulate a Heston Nandi Garch(1,1) Process:
   # Symmetric Model - Parameters:
   model = list(lambda = 4, omega = 8e-5, alpha = 6e-5,
     beta = 0.7, gamma = 0, rf = 0)
   ts = hngarchSim(model = model, n = 500, n.start = 100)
   par(mfrow = c(2, 1), cex = 0.75)
   ts.plot(ts, col = "steelblue", main = "HN Garch Symmetric Model")
   grid()

## hngarchFit -
   # HN-GARCH log likelihood Parameter Estimation:
   # To speed up, we start with the simulated model ...
   mle = hngarchFit(model = model, x = ts, symmetric = TRUE)
   mle

## summary.hngarch -
   # HN-GARCH Diagnostic Analysis:
   par(mfrow = c(3, 1), cex = 0.75)
   summary(mle)

## hngarchStats -
   # HN-GARCH Moments:
   hngarchStats(mle$model)
```

HestonNandiOptions *Option Price for the Heston-Nandi Garch Option Model*

Description

A collection and description of functions to valuate Heston-Nandi options. Included are functions to compute the option price and the delta and gamma sensitivities for call and put options.

The functions are:

`HNGOption`	Heston-Nandi GARCH(1,1) option price,
`HNGGreeks`	Heston-Nandi GARCH(1,1) option sensitivities,
`HNGCharacteristics`	option prices and sensitivities.

Usage

```
HNGOption(TypeFlag, model, S, X, Time.inDays, r.daily)
HNGGreeks(Selection, TypeFlag, model, S, X, Time.inDays, r.daily)
HNGCharacteristics(TypeFlag, model, S, X, Time.inDays, r.daily)
```

Arguments

`model`	a list of model parameters with the following entries: `lambda`, `omega`, `alpha`, `beta`, and `gamma`, numeric values.
`r.daily`	the daily rate of interest, a numeric value; e.g. 0.25/252 means about 0.001% per day.
`S`	the asset price, a numeric value.
`Selection`	sensitivity to be computed, one of `"delta"`, `"gamma"`, `"vega"`, `"theta"`, `"rho"`, or `"CoC"`, a string value.
`Time.inDays`	the time to maturity measured in days, a numerical value; e.g. 5/252 means 1 business week.
`TypeFlag`	a character string either `"c"` for a call option or a `"p"` for a put option.
`X`	the exercise price, a numeric value.

Details

Option Values:

`HNGOption`calculates the option price, `HNGGreeks` allows to compute the option sensitivity Delta or Gamma, and `HNGcharacterisitics` summarizes both in one function call.

Value

`HNGOption`
returns a list object of class `"option"` with `$price` denoting the option price, a numeric value, and

`$call` a character string which matches the function call.

`HNGOGreeks`
returns the option sensitivity for the selected Greek, either `"delta"` or `"gamma"`; a numeric value.

`HNGCharacteristics`
returns a list with the following entries:

`premium`	the option price, a numeric value.
`delta`	the delta sensitivity, a numeric value.
`gamma`	the gamma sensitivity, a numeric value.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Heston S.L., Nandi S. (1997); *A Closed-Form GARCH Option Pricing Model*, Federal Reserve Bank of Atlanta.

Examples

```
## model -
   # Define the Model Parameters for a Heston-Nandi Option:
   model = list(lambda = -0.5, omega = 2.3e-6, alpha = 2.9e-6,
     beta = 0.85, gamma = 184.25)
   S = X = 100
   Time.inDays = 252
   r.daily = 0.05/Time.inDays
   sigma.daily = sqrt((model$omega + model$alpha) /
     (1 - model$beta - model$alpha * model$gamma^2))
   data.frame(S, X, r.daily, sigma.daily)

## HNGOption -
   # Compute HNG Call-Put and compare with GBS Call-Put:
   HNG = GBS = Diff = NULL
   for (TypeFlag in c("c", "p")) {
     HNG = c(HNG, HNGOption(TypeFlag, model = model, S = S, X = X,
       Time.inDays = Time.inDays, r.daily = r.daily)$price )
     GBS = c(GBS, GBSOption(TypeFlag, S = S, X = X, Time = Time.inDays,
       r = r.daily, b = r.daily, sigma = sigma.daily)@price) }
   Options = cbind(HNG, GBS, Diff = round(100*(HNG-GBS)/GBS, digits=2))
   row.names(Options) <- c("Call", "Put")
   data.frame(Options)

## HNGGreeks -
   # Compute HNG Greeks and compare with GBS Greeks:
   Selection = c("Delta", "Gamma")
   HNG = GBS = NULL
   for (i in 1:2){
```

```
    HNG = c(HNG, HNGGreeks(Selection[i], TypeFlag = "c", model = model,
      S = 100, X = 100, Time = Time.inDays, r = r.daily) )
    GBS = c(GBS, GBSGreeks(Selection[i], TypeFlag = "c", S = 100, X = 100,
      Time = Time.inDays, r = r.daily, b = r.daily, sigma = sigma.daily) ) }
  Greeks = cbind(HNG, GBS, Diff = round(100*(HNG-GBS)/GBS, digits = 2))
  row.names(Greeks) <- Selection
  data.frame(Greeks)
```

`LowDiscrepancy` *Low Discrepancy Sequences*

Description

A collection and description of functions to compute Halton's and Sobol's low discrepancy sequences, distributed in form of a uniform or normal distribution.

The functions are:

`runif.halton`	Uniform Halton sequence,
`rnorm.halton`	Normal Halton sequence,
`runif.sobol`	Uniform scrambled Sobol sequence,
`rnorm.sobol`	Normal scrambled Sobol sequence,
`runif.pseudo`	Uniform pseudo random numbers,
`norma.pseudo`	Normal pseudo random numbers.

Usage

```
runif.halton(n, dimension, init)
rnorm.halton(n, dimension, init)

runif.sobol(n, dimension, init, scrambling, seed)
rnorm.sobol(n, dimension, init, scrambling, seed)

runif.pseudo(n, dimension, init)
rnorm.pseudo(n, dimension, init)
```

Arguments

`dimension`	an integer value, the dimension of the sequence. The maximum value for the Sobol generator is 1111.
`init`	a logical, if TRUE the sequence is initialized and restarts, otherwise not. By default TRUE.
`n`	an integer value, the number of random deviates.
`scrambling`	an integer value, if 1, 2 or 3 the sequence is scrambled otherwise not. If 1, Owen type type of scrambling is applied, if 2, Faure-Tezuka type of scrambling, is applied, and if 3, both Owen+Faure-Tezuka type of scrambling is applied. By default 0.

`seed` an integer value, the random seed for initialization of the scrambling process. By default 4711. On effective if `scrambling>0`.

Details

Halton's Low Discrepancy Sequences:

Calculates a matrix of uniform or normal deviated halton low discrepancy numbers.

Scrambled Sobol's Low Discrepancy Sequences:

Calculates a matrix of uniform and normal deviated Sobol low discrepancy numbers. Optional scrambling of the sequence can be selected.

Pseudo Random Number Sequence:

Calculates a matrix of uniform or normal distributed pseudo random numbers. This is a helpful function for comparing investigations obtained from a low discrepancy series with those from a pseudo random number.

Value

All generators return a numeric matrix of size `n` by `dimension`.

Note

The global variables `runif.halton.seed` and `runif.sobol.seed` save the status to restart the generators. Note, that only one instance of a generators can be run at the same time.

The ACM Algorithm 659 implemented to generate scrambled Sobol sequences is under the License of the ACM restricted for academic and noncommerical usage. Please consult the ACM License agreement included in the `doc` directory.

Author(s)

P. Bratley and B.L. Fox for the Fortran Sobol Algorithm 659,
S. Joe for the Fortran extension to 1111 dimensions,
Diethelm Wuertz for the Rmetrics R-port.

References

Bratley P., Fox B.L. (1988); *Algorithm 659: Implementing Sobol's Quasirandom Sequence Generator*, ACM Transactions on Mathematical Software 14, 88–100.

Joe S., Kuo F.Y. (1998); *Remark on Algorithm 659: Implementing Sobol's Quaisrandom Seqence Generator*.

Examples

```
## *.halton -
   par(mfrow = c(2, 2), cex = 0.75)
   runif.halton(n = 10, dimension = 5)
   hist(runif.halton(n = 5000, dimension = 1), main = "Uniform Halton",
     xlab = "x", col = "steelblue3", border = "white")
   rnorm.halton(n = 10, dimension = 5)
   hist(rnorm.halton(n = 5000, dimension = 1), main = "Normal Halton",
     xlab = "x", col = "steelblue3", border = "white")

## *.sobol -
   runif.sobol(n = 10, dimension = 5, scrambling = 3)
   hist(runif.sobol(5000, 1, scrambling = 2), main = "Uniform Sobol",
     xlab = "x", col = "steelblue3", border = "white")
   rnorm.sobol(n = 10, dimension = 5, scrambling = 3)
   hist(rnorm.sobol(5000, 1, scrambling = 2), main = "Normal Sobol",
     xlab = "x", col = "steelblue3", border = "white")

## *.pseudo -
   runif.pseudo(n = 10, dimension = 5)
   rnorm.pseudo(n = 10, dimension = 5)
```

`MonteCarloOptions` *Monte Carlo Valuation of Options*

Description

A collection and description of functions to valuate options by Monte Carlo methods. The functions include beside the main Monte Carlo Simulator, example functions to generate Monte Carlo price paths and to compute Monte Carlo price payoffs.

The functions are:

`sobolInnovations`	Example for scrambled Sobol innovations,
`wienerPath`	Example for a Wiener price path,
`plainVanillaPayoff`	Example for the plain vanilla option's payoff,
`arithmeticAsianPayoff`	Example for the arithmetic Asian option's payoff,
`MonteCarloOption`	Monte Carlo Simulator for options.

Usage

```
MonteCarloOption(delta.t, pathLength, mcSteps, mcLoops, init = TRUE,
    innovations.gen, path.gen, payoff.calc, antithetic = TRUE,
    standardization = FALSE, trace = TRUE, ...)
```

Arguments

`antithetic`	a logical flag, should antithetic variates be used? By default TRUE.

`delta.t` the time step interval measured as a fraction of one year, by default one day, i.e. `delta.t=1/360`.

`init` a logical flag, should the random number generator be initialized? By default TRUE.

`innovations.gen`
a user defined function to generate the innovations, this can be the normal random number generator `rnorm.pseudo` with mean zero and variance one. For the usage of low discrepancy sequences alternativey `rnorm.halton` and `rnorm.sobol` can be called. The generator must deliver a normalized matrix of innovations with dimension given by the number of Monte Carlo steps and the path length. The first three arguments of the generator are the the number of Monte Carlo steps `mcSteps`, the path length `pathLength` and the initialization flag `init`. Optional arguments can be passed through the argument `...`, e.g. the type of scrambling for low discrepancy numbers.

`mcLoops, mcSteps`
the number of Monte Carlo loops and Monte Carlo Steps. In total `mcLoops*mcSteps` samples are included in one MC simulation.

`path.gen` the user defined function to generate the price path. As the only input argument serves the matrix of innovations, the option parameters must be available as global variables.

`pathLength` the length of the price path. This may be calculated as `floor(Time/delta.t)`, where `Time` denotes the time to maturation measured in years.

`payoff.calc` a user defined function to calculate the payoff of the option. As the only input argument serves the path matrix as returned by the path generator. The option parameters must be available as global variables.

`standardization`
a logical flag, should the innovations for one loop be standardized? By default TRUE.

`trace` a logical flag, should the Monte Carlo simulation be traced? By default TRUE.

`...` additional arguments passed to the innovations generator.

Details

The Innovations:

The innovations must created by the user defined innovation generator. The Generator has to return a numeric matrix of (random) innovations of size `mcSteps` times the `pathLength`. The example section shows how to write sa function for scrambled Quasi Monte Carlo Sobol numbers. The package comes with three generators `rnorm.pseudo`, `rnorm.halton` and `rnorm.sobol` which can easily be used for simulations.

The Price Paths:

The user must provide a function which generates the price paths. In the example section the function `wienerPath` creates a Wiener Monte Carlo path from random innovations. The Wiener price path requires as input `b`, the annualized cost-of-carry rate, and `sigma`, the annualized volatility

of the underlying security, to compute the drift and variance of the path, these variables must be globally defined.

The Payoff Function:

The user must also provide a function which computes the payoff value of the option. The example sections show how to write payoff calculators for the plain vanilla option and for the arithmetic Asian Option. As the only input argument the path matrix is required. Again, the option parameters must be globally available.

The Monte Carlo Simulator:

The simulator is the heart of the Monte Carlo valuation process. This simulator performs `mcLoops` Monte Carlo loops each with `mcSteps` Monte Carlo steps. In each loop the following steps are done: first the innovation matrix is created from the specified innovation generator (usually build from the normal pseudo random number or low discrepancy generators), then anththetic innovations are added if desired (by default `anththetic=TRUE`), then the innovations can be standardized within each loop (by default `standardization=FALSE`), and finally the average payoff of all samples in the loop is computed. The simulation can be traced loop by loop setting the argument `trace=TRUE`.

Value

The user defined innovation generator
returns a numeric matrix of (random) innovations to build the Monte Carlo Paths.

The user defined path generator
returns a numeric matrix of the Monte Carlo paths for the calculation of the option's payoffs. To be more precise, as an example the function returns for a Wiener process the matrix `(b-sigma*sigma/2)*delta.t + sigma*sq`
where the first term corresponds to the drift and the second to the volatility.

The user defined payoff calculator,
returns the vector of the option's payoffs calculated from the generated paths. As an example this becomes for an arithmetic Asian call option with a Wiener Monte Carlo path `payoff = exp(-r*Time)*max(SM-X, 0)` where `SM = mean(S*exp(cumsum(path)))` and `path` denotes the MC price paths.

MonteCarloOption:
returns a vector with the option prices for each Monte Carlo loop.

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Birge J.R. (1994); *Quasi-Monte Carlo Approaches to Option Pricing*, Department of Industrial and Operations Engineering, Technical Report 94–19, University of Michigan.

Boyle P. (1977); *Options: A Monte Carlo approach*, Journal of Finance, 32, 323–338.

Glasserman P. (2004); *Monte Carlo Methods in Financial Engineering*, Springer-Verlag New York, Inc., 596 pp.

Jaeckel P. (2002); *Monte Carlo Methods in Finance*, John Wiley and Sons Ltd, 222 pp.

Examples

```
## How to perform a Monte Carlo Simulation?

## First Step:
   # Write a function to generate the option's innovations.
   # Use scrambled normal Sobol numbers:
   sobolInnovations = function(mcSteps, pathLength, init, ...) {
     # Create Normal Sobol Innovations:
     innovations = rnorm.sobol(mcSteps, pathLength, init, ...)
     # Return Value:
     innovations }

## Second Step:
   # Write a function to generate the option's price paths.
   # Use a Wiener path:
   wienerPath = function(eps) {
     # Note, the option parameters must be globally defined!
     # Generate the Paths:
     path = (b-sigma*sigma/2)*delta.t + sigma*sqrt(delta.t)*eps
     # Return Value:
     path }

## Third Step:
   # Write a function for the option's payoff

   # Example 1: use the payoff for a plain Vanilla Call or Put:
   plainVanillaPayoff = function(path) {
     # Note, the option parameters must be globally defined!
     # Compute the Call/Put Payoff Value:
     ST = S*exp(sum(path))
     if (TypeFlag == "c") payoff = exp(-r*Time)*max(ST-X, 0)
     if (TypeFlag == "p") payoff = exp(-r*Time)*max(0, X-ST)
     # Return Value:
     payoff }

   # Example 2: use the payoff for an arithmetic Asian Call or Put:
   arithmeticAsianPayoff = function(path) {
     # Note, the option parameters must be globally defined!
     # Compute the Call/Put Payoff Value:
     SM = mean(S*exp(cumsum(path)))
     if (TypeFlag == "c") payoff = exp(-r*Time)*max(SM-X, 0)
     if (TypeFlag == "p") payoff = exp(-r*Time)*max(0, X-SM)
     # Return Value:
     payoff }

## Final Step:
```

```
# Set Global Parameters for the plain Vanilla / arithmetic Asian Options:
TypeFlag <<- "c"; S <<- 100; X <<- 100
Time <<- 1/12; sigma <<- 0.4; r <<- 0.10; b <<- 0.1

# Do the Asian Simulation with scrambled random numbers:
mc = MonteCarloOption(delta.t = 1/360, pathLength = 30, mcSteps = 5000,
  mcLoops = 50, init = TRUE, innovations.gen = sobolInnovations,
  path.gen = wienerPath, payoff.calc = arithmeticAsianPayoff,
  antithetic = TRUE, standardization = FALSE, trace = TRUE,
  scrambling = 2, seed = 4711)

# Plot the MC Iteration Path:
par(mfrow = c(1, 1))
mcPrice = cumsum(mc)/(1:length(mc))
plot(mcPrice, type = "l", main = "Arithmetic Asian Option",
  xlab = "Monte Carlo Loops", ylab = "Option Price")

# Compare with Turnbull-Wakeman Approximation:
# TW = TurnbullWakemanAsianApproxOption(TypeFlag = "c", S = 100, SA = 100,
#    X = 100, Time = 1/12, time = 1/12, tau = 0 , r = 0.1, b = 0.1,
#    sigma = 0.4)
# print(TW)
# abline(h = TW, col = 2)
```

`PlainVanillaOptions` *Valuation of Plain Vanilla Options*

Description

A collection and description of functions to valuate plain vanilla options. Included are functions for the Generalized Black-Scholes option pricing model, for options on futures, some utility functions, and print and summary methods for options.

The functions are:

`GBS*`	the generalized Black-Scholes option,
`BlackScholesOption`	a synonyme for the GBSOption,
`Black76Option`	options on Futures,
`MiltersenSchwartzOption`	options on commodity futures,
`NDF, CND, CBND`	distribution functions,
`print`	print method for Options,
`summary`	summary method for Options.

Usage

```
GBSOption(TypeFlag, S, X, Time, r, b, sigma,
    title = NULL, description = NULL)
GBSGreeks(Selection, TypeFlag, S, X, Time, r, b, sigma)
```

```
GBSCharacteristics(TypeFlag, S, X, Time, r, b, sigma)
GBSVolatility(price, TypeFlag, S, X, Time, r, b, tol, maxiter)
BlackScholesOption(...)

Black76Option(TypeFlag, FT, X, Time, r, sigma,
    title = NULL, description = NULL)

MiltersenSchwartzOption(TypeFlag, Pt, FT, X, time, Time,
    sigmaS, sigmaE, sigmaF, rhoSE, rhoSF, rhoEF, KappaE, KappaF,
    title = NULL, description = NULL)

NDF(x)
CND(x)
CBND(x1, x2, rho)

## S4 method for signature 'fOPTION'
show(object)
## S3 method for class 'fOPTION'
summary(object, ...)

## S3 method for class 'option'
print(x, ...)
## S3 method for class 'option'
summary(object, ...)
```

Arguments

`b`	the annualized cost-of-carry rate, a numeric value; e.g. 0.1 means 10% pa.
`description`	a character string which allows for a brief description.
`FT`	[Black76*][MiltersenSchwartz*] - the futures price, a numeric value.
`KappaE, KappaF`	[MiltersenSchwartz*] - the speed of mean reversion of the forward interest rate (E), the speed of mean reversion of the convenience yield (F), a numeric value.
`maxiter, tol`	[GBSVolatility*] - the maximum number of iterations and the tolerance to compute the root of the GBS volatility equation, see `uniroot`.
`object`	an object of class `"option"`.
`price`	[GBSVolatility*] - the price of the GBS option, a numerical value.
`Pt`	[MiltersenSchwartz*] - the zero coupon bond that expires on the option maturity; a numeric value.
`r`	the annualized rate of interest, a numeric value; e.g. 0.25 means 25% pa.
`rhoSE, rhoSF, rhoEF`	[MiltersenSchwartz*] - the correlations between the spot commodity price and the future convenience

yield (SE), between the spot commodity price and the forward interest rate (SF), between the forward interest rate and the future convenience yield (EF), a numeric value.

`S` the asset price, a numeric value.

`Selection` [GBSGreeks] -
sensitivity to be computed, one of `"delta"`, `"gamma"`, `"vega"`, `"theta"`, `"rho"`, or `"CoC"`, a string value.

`sigma` the annualized volatility of the underlying security, a numeric value; e.g. 0.3 means 30% volatility pa.

`sigmaS, sigmaE, sigmaF`
[MiltersenSchwartz*] -
numeric values, the annualized volatility of the spot commodity price (S), of the future convenience yield (E), and of the forward interest rate (F), e.g. 0.25 means 25% pa.

`time, Time` the time to maturity measured in years, a numeric value.

`title` a character string which allows for a project title.

`TypeFlag` a character string either `"c"` for a call option or a `"p"` for a put option.

`x, x1, x2, rho` [NDF][CND][CBND] -
the function argument `x` for the normal distribution function `NDF` and the cumulated normal distribution `CND`. The arguments for the bivariate function are named `x1` and `x2`; `rho` is the correlation coefficient.
[print] -
the object `x` to be printed.

`X` a numeric value, the exercise price.

`...` arguments to be passed.

Details

Generalized Black Scholes Options:

`GBSOption` calculates the option price, `GBSGreeks` calculates option sensitivities delta, theta, vega, rho, lambda and gamma, and `GBScharacterisitics` does both. `GBSVolatility` computes the implied volatility.
Note, that setting `b = r` we get Black and Scholes' stock option model, `b = r-q` we get Merton's stock option model with continuous dividend yield `q`, `b = 0` we get Black's futures option model, and `b = r-rf` we get Garman and Kohlhagen's currency option model with foreign interest rate `rf`.

Options on Futures:

The `Black76Option` pricing formula is applicable for valuing European call and European put options on commodity futures. The exact nature of the underlying commodity varies and may be anything from a precious metal such as gold or silver to agricultural products.
The `Miltersen Schwartz Option` model is a three factor model with stochastic futures prices, term structures and convenience yields, and interest rates. The model is based on lognormal distributed commodity prices and normal distributed continuously compounded forward interest rates

and future convenience yields.

Miltersen Schwartz Options:

The `MiltersenSchwartzOption` function allows for pricing options on commodity futures. The model is a three factor model with stochastic futures prices, term structures of convenience yields, and interest rates. The model is based on lognormal distributed commodity prices and normal distributed continuously compounded forward interest rates and futures convenience yields.

Distribution Functions:

The functions `NDF`, `CND`, and `CBND` compute vlues for the Normal density functions, for the normal probability function, and for the bivariate normal probability functions. The functions are implemented as described in the book of E.G. Haug.

Print and Summary Method:

Thes are two methods to print and sumarize an object of class `"fOPTION"` or of `"option"`. The second is used for the older class representation.

Value

`GBSOption`
`BlackScholesOption`
returns an object of class `"fOption"`.

`GBSGreeks`
returns the option sensitivity for the selected Greek, a numeric value.

`GBSCharacteristics`
returns a list with the following entries: `premium`, the option price, `delta`, the delta sensitivity, `gamma`, the gamma sensitivity, `theta`, the theta sensitivity, `vega`, the vega sensitivity, `rho`, the rho sensitivity, `lambda`, the lambda sensitivity.

`GBSVolatility`
returns the GBS option implied volatility for a given price.

`Black76Option`,
`MiltersenSchwartzOption`
return an object of class `"fOption"`.

The option valuation programs return an object of class `"fOPTION"` with the following slots:

`@call`	the function call.
`@parameters`	a list with the input parameters.
`@price`	a numeric value with the value of the option.
`@title`	a character string with the name of the test.

`@description` a character string with a brief description of the test.

Note

The functions implement algorithms to valuate plain vanilla options and to compute option Greeks as described in Chapter 1 of Haug's Option Guide (1997).

Author(s)

Diethelm Wuertz for the Rmetrics R-port.

References

Black F., Scholes M. (1973); *The Pricing of Options and Corporate Liabilities*, Journal of Political Economy 81, 637–654.

Haug E.G. (1997); *The Complete Guide to Option Pricing Formulas*, Chapter 1, McGraw-Hill, New York.

Hull J.C. (1998); *Introduction to Futures and Options Markets*, Prentice Hall, London.

Miltersen K., Schwartz E.S. (1998); *Pricing of Options on Commodity Futures with Stochastic Term Structuures of Convenience Yields and Interest Rates*, Journal of Financial and Quantitative Analysis 33, 33–59.

Examples

```
## All the examples are from Haug's Option Guide (1997)

## CHAPTER 1.1: ANALYTICAL FORMULAS FOR EUROPEAN OPTIONS:

## Black Scholes Option [Haug 1.1.1]
   GBSOption(TypeFlag = "c", S = 60, X = 65, Time = 1/4, r = 0.08,
     b = 0.08, sigma = 0.30)

## European Option on a Stock with Cash Dividends [Haug 1.1.2]
   S0  =  100; r  =  0.10; D1  =  D2  =  2; t1  =  1/4; t2  =  1/2
   S  =  S0 - 2*exp(-r*t1) - 2*exp(-r*t2)
   GBSOption(TypeFlag = "c", S = S, X = 90, Time = 3/4, r = r, b = r,
     sigma = 0.25)

## Options on Stock Indexes [Haug 1.2.3]
   GBSOption(TypeFlag = "p", S = 100, X = 95, Time = 1/2, r = 0.10,
     b = 0.10-0.05, sigma = 0.20)

## Option on Futures [Haug 1.1.4]
   FuturesPrice  =  19
   GBSOption(TypeFlag = "c", S = FuturesPrice, X = 19, Time = 3/4,
     r = 0.10, b = 0, sigma = 0.28)

## Currency Option [Haug 1.1.5]
   r  =  0.06; rf  =  0.08
   GBSOption(TypeFlag = "c", S = 1.5600, X = 1.6000,
     Time = 1/2, r = 0.06, b = 0.06-0.08, sigma = 0.12)
```

```
## Delta of GBS Option [Haug 1.3.1]
   GBSGreeks(Selection = "delta", TypeFlag = "c", S = 105, X = 100,
     Time = 1/2, r = 0.10, b = 0, sigma = 0.36)

## Gamma of GBS Option [Haug 1.3.3]
   GBSGreeks(Selection = "gamma", TypeFlag = "c", S = 55, X = 60,
     Time = 0.75, r = 0.10, b = 0.10, sigma = 0.30)

## Vega of GBS Option [Haug 1.3.4]
   GBSGreeks(Selection = "vega", TypeFlag = "c", S = 55, X = 60,
     Time = 0.75, r = 0.10, b = 0.10, sigma = 0.30)

## Theta of GBS Option [Haug 1.3.5]
    GBSGreeks(Selection = "theta", TypeFlag = "p", S = 430, X = 405,
     Time = 0.0833, r = 0.07, b = 0.07-0.05, sigma = 0.20)

## Rho of GBS Option [Haug 1.3.5]
   GBSGreeks(Selection = "rho", TypeFlag = "c", S = 72, X = 75,
     Time = 1, r = 0.09, b = 0.09, sigma = 0.19)

## CHAPTER 1.3 OPTIONS SENSITIVITIES:

## The Generalized Black Scholes Option Formula
   GBSCharacteristics(TypeFlag = "p", S = 1.5600, X = 1.6000,
     Time = 1, r = 0.09, b = 0.09, sigma = 0.19)

## CHAPTER 1.5: RECENT DEVELOPMENTS IN COMMODITY OPTIONS

## Miltersen Schwartz Option vs. Black76 Option on Futures:
   MiltersenSchwartzOption(TypeFlag = "c", Pt = exp(-0.05/4), FT = 95,
     X = 80, time = 1/4, Time = 1/2, sigmaS = 0.2660, sigmaE = 0.2490,
     sigmaF = 0.0096, rhoSE = 0.805, rhoSF = 0.0805, rhoEF = 0.1243,
     KappaE = 1.045, KappaF = 0.200)
   Black76Option(TypeFlag = "c", FT = 95, X = 80, Time = 1/2, r = 0.05,
     sigma = 0.266)
```

Index

∗Topic **math**
BasicAmericanOptions, 2
BinomialTreeOptions, 4
HestonNandiOptions, 11
PlainVanillaOptions, 19
∗Topic **models**
HestonNandiGarchFit, 7
∗Topic **programming**
LowDiscrepancy, 13
MonteCarloOptions, 15

arithmeticAsianMCPayoff (MonteCarloOptions), 15

BasicAmericanOptions, 2
BAWAmericanApproxOption (BasicAmericanOptions), 2
BinomialTreeOption (BinomialTreeOptions), 4
BinomialTreeOptions, 4
BinomialTreePlot (BinomialTreeOptions), 4
Black76Option (PlainVanillaOptions), 19
BlackScholesOption (PlainVanillaOptions), 19
BSAmericanApproxOption (BasicAmericanOptions), 2

CBND (PlainVanillaOptions), 19
CND (PlainVanillaOptions), 19
CRRBinomialTreeOption (BinomialTreeOptions), 4

fOPTION (PlainVanillaOptions), 19
fOPTION-class (PlainVanillaOptions), 19

GBSCharacteristics (PlainVanillaOptions), 19
GBSGreeks (PlainVanillaOptions), 19
GBSOption (PlainVanillaOptions), 19
GBSVolatility (PlainVanillaOptions), 19

HestonNandiGarchFit, 7
HestonNandiOptions, 11
hngarchFit (HestonNandiGarchFit), 7
hngarchSim (HestonNandiGarchFit), 7
hngarchStats (HestonNandiGarchFit), 7
HNGCharacteristics (HestonNandiOptions), 11
HNGGreeks (HestonNandiOptions), 11
HNGOption (HestonNandiOptions), 11

JRBinomialTreeOption (BinomialTreeOptions), 4

LowDiscrepancy, 13

MiltersenSchwartzOption (PlainVanillaOptions), 19
MonteCarloOption (MonteCarloOptions), 15
MonteCarloOptions, 15

NDF (PlainVanillaOptions), 19

plainVanillaMCPayoff (MonteCarloOptions), 15
PlainVanillaOptions, 19
print.hngarch (HestonNandiGarchFit), 7
print.option (PlainVanillaOptions), 19

rnorm.halton (LowDiscrepancy), 13
rnorm.pseudo (LowDiscrepancy), 13
rnorm.sobol (LowDiscrepancy), 13
RollGeskeWhaleyOption (BasicAmericanOptions), 2
runif.halton (LowDiscrepancy), 13
runif.pseudo (LowDiscrepancy), 13
runif.sobol (LowDiscrepancy), 13

show,fOPTION-method (PlainVanillaOptions), 19
summary.fOPTION (PlainVanillaOptions), 19

summary.hngarch *(*HestonNandiGarchFit*)*, 7
summary.option *(*PlainVanillaOptions*)*, 19

TIANBinomialTreeOption
*(*BinomialTreeOptions*)*, 4

wienerMCPath *(*MonteCarloOptions*)*, 15