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Timber Harvesting with Fluctuating Prices

RICHARD BRAZEE
ROBERT MENDELSOHN

ABSTRACT. Because of volatility in demand, timber prices tend to fluctuate from year to year. Timber owners know today's price but are uncertain about tomorrow's prices. Traditional Faustmann harvesting ignores these random annual price fluctuations and prescribes harvests on the basis of expected prices. In this paper, we adapt an asset sale model to forestry and solve for the optimal schedule of reservation prices. When current price is above the reservation price, owners should cut that age class, otherwise they should wait another year. This flexible price harvest policy significantly increases the present value of expected returns over the more rigid Faustmann model. FOR. SCI. 34(2):359–372.

ADDITIONAL KEY WORDS: Asset sale model, forest management, price uncertainty, dynamic programming, Douglas-fir, loblolly pine.

THAT STUMPAGE PRICES TEND TO FLUCTUATE from year to year is a well-known fact about the forest products market. Some decisions, such as planting and fertilizing, must often be done so far in advance that only expected prices can usefully guide management. However, with timber harvest decisions (and to a lesser extent commercial thinning), the current price of timber is known each year. When market demand and supply conditions are such that timber prices are relatively high, individual land owners should respond by cutting more. By tailoring harvests to variations in prices, the present value of all future timber revenues can be greatly enhanced over the standard Faustmann model. Although some owners may already practice a version of this price-sensitive harvesting strategy (see Gould (1984)), this strategy can be finely tuned by developing an asset sale model for forestry.

The formal asset sale model was first presented by Karlin (1962) and later developed in papers of price search (see Lippman and McCall 1976 for a review). The basic intuition of this model is that the owner should cut timber today only if today's timber value is higher than what he expects the present value of future timber value to be. The expected future timber value, in turn, depends on a set of uncertain future prices as well as the relevant age related effective yield function.[1] By committing to a policy of harvesting timber only when prices are above some minimum, the expected value of future timber value can be increased. The present value of this maximum expected future timber value becomes the current reservation (minimum) price.

The optimal reservation price depends on stumpage volume and growth and so is different for each age class and species. When the current price is

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[1] The effective yield function describes the amount of equal quality timber per acre by age of the stand. The concept thus includes both physical measures of stumpage as well as differentials in stumpage value associated with size.

below the reservation price, the owner delays harvest for that age class for an additional period. When current price exceeds the reservation price, the owner harvests the entire age class. Using this policy, owners can increase their expected returns by tending to avoid selling timber at relatively low prices. In this paper, we develop optimal reservation prices by age for several examples and demonstrate the expected advantage of this optimal harvesting strategy compared to Faustmann rotations.

In the next section of the paper, a formal asset sale model for timber is developed. This model is then calibrated in Section III for two important tree species: Douglas-fir (*Pseudotsuga menziessi* [Mirb.] Franco) and loblolly pine (*Pinus taeda* L.) under both good and marginal growing conditions. The purpose of showing these multiple examples is to demonstrate the range of effects likely under flexible price harvesting. Because the model cannot be solved analytically, we turn to numerical solution. A sensitivity analysis concerning the variability of prices, the interest rate, and the permitted range of harvest ages is also conducted. In the final section, we discuss the implications of these results for individual companies, the public sector, and the timber market.

TIMBER ASSET SALE MODEL

In this paper, we are interested in exploring how harvest strategies ought to adjust to uncertain future timber prices. Prices are assumed to be exogenous. That is, each forest owner is assumed not to have sufficient economic power to affect market prices. Prices, however, vary from year to year. Part of the reason that they vary is because of forces that can be anticipated such as population-driven shifts in demand or remaining supplies of old-growth (see Berck 1979). As shown by Hardie et. al. (1984), these predictable changes in price can be incorporated into the Faustmann model by comparing the present value of harvesting trees at different ages (given their predicted future prices). However, even with our best forecasting models, there remains substantial unpredictable short-run fluctuations in demand. These short-run demand fluctuations, in turn, result in varying short-run prices. The focus of this paper is on how to adjust harvest policies with respect to these unpredictable shifts in prices. We abstract from predictable changes in prices in order to focus our attention on these unpredictable short-run price fluctuations.[2]

A common assumption in economics is that the error structure around an empirical model's predictions is normally distributed. We adopt this convention and assume a normal distribution as well. Examining the residuals from a regression of Douglas-fir and loblolly pine over time, we find that the normal distribution provides a reasonable approximation of the observed error structure with respect to prices. Although alternative distributions may sometimes be appropriate, they would affect the quantitative but not the qualitative results.

From past prices, we assume that owners can determine the expected mean and variance of the random component of future prices. However, actual future prices are assumed to be unpredictable, drawn randomly from this known distribution. In contrast, the current price $P(t)$ is known (in year

[2] Incorporating expected price changes into this model is possible but cumbersome and would tend to conceal the price fluctuation part of the model.

t) and is assumed constant for the entire year.[3] A new random price is established each year, and we assume it is statistically independent of last year's price.[4] This last assumption may be unrealistic in that stumpage prices are sometimes serially correlated; this year's price can be correlated with last year's price. Further development of the timber asset sale model should certainly explore the implications of serially correlated prices.

We assume that the timber owner has sufficient wealth that he is risk-neutral with respect to the timber price in any one year. This, of course, may not apply to small nonindustrial owners whose primary wealth may be invested in one age class.[5] With risk-neutral owners, the management objective should be to maximize the expected present value of net returns.

In order to focus the model on price fluctuations, we assume that the yield table and inventory associated with the forest are known. That is, the owner is presumed to know the amount of standing timber he has, the age classes, and the rate of growth in each age class. In principle, however, these parameters could be uncertain as well.[6] We describe the yield table facing the forest owner in terms of a yield function $Q\ (T)$ which describes the timber volume at each age (T) of the trees. The rate of growth of the forest is the change in volume from year to year $\dot{Q}(T)$ divided by the volume that year $Q(T)$. Empirical evidence suggests that over the relevant range in which one might harvest, the rate of growth declines with age: $d(\dot{Q}/Q)/dT < 0$.

In a Faustmann model with constant prices, the slowing rate of growth of timber encourages owners to choose a finite rotation length. The optimal harvest date is the moment when the value of continued growth just equals the opportunity cost of waiting—the rent on the standing stock plus the rent on the land:

$$P\ \dot{Q}(T) = r\ P\ Q(T) + r\ W \tag{1}$$

where r is the interest or discount rate, P is a constant stumpage price, and W is the value of the bare land. Equation (1) is the first-order condition of the well-known Faustmann formula. With constant prices, the rotation age of the forest should be determined by (1). In practice, because prices are not constant, the expected value of prices $E[P]$ (where $E[\ ]$ is the expected value operator) or $\bar{P}$ is used as the estimate of price in the Faustmann equation.

With fluctuating prices, the owner must decide between a certain price today and unknown prices in the future. The underlying forces of the Faustmann model are still at work but they are affected by the variation in future prices. In order to provide some insight into how this search model works, we examine how the model applies to a single age class as it moves through

[3] Prices must be constant over the current period. If prices tend to vary significantly within a year, the period of decision could be shorter (for example, a season).

[4] See Nordstrom 1975 for a treatment of this problem.

[5] See Johansson and Lofgren 1985, chapter 12, for a discussion of price uncertainty with risk aversion.

[6] See Miller and Voltaire 1983, Brock, Rothschild and Stiglitz 1979, and Johansson and Lofgren 1985 for a discussion of uncertain growth each period.

time. With respect to this single age class, the owner's problem is when to cut the entire class.

In a simple price search model, the reservation price is based on the expected return one could earn by delaying harvest. Whenever the offered price this period is above the reservation price, the owner accepts the price, cuts all the timber (of this age), and the land is freed for its next best use. Whenever the offered price is below the reservation price, the owner delays harvest another period. The expected present value of the revenue from accepted prices determines the reservation price in each period. Because prices below the reservation price tend to be refused (truncated), the accepted prices have a higher expected value than the offered prices. Compared to a more rigid harvest rule such as the Faustmann model, the flexible harvest strategy can increase expected revenue by more fully utilizing available information.

Without any costs to waiting and an infinite time horizon, the outcome of the flexible price model becomes trivial. The owner simply waits for an infinite price to appear. In the early stages of a rotation, there are no costs to waiting since the trees are growing more than enough to compensate for the low opportunity costs. During this period, the owner invariably waits to harvest because the reservation price is very high. This condition, however, is short-lived. As the stand matures past the Faustmann age, the growth of the forest no longer compensates for the opportunity cost of the land and standing timber. Beyond this date, the owner must pay for the luxury of delaying harvest in higher and higher opportunity costs. These increasing opportunity costs depress the reservation price, making the owner more and more likely to accept the next offered price.

The optimal decision rule under varying prices is for the owner to set a reservation price so that today's revenue would equal the expected net present value of waiting. The reservation price reflects the value of delaying harvest and thus depends on the growth and opportunity costs of the timber (which are age dependent), the costs of the land, and the likelihood of obtaining a higher price in the future. Thus, an unusually high offered price can encourage owners to cut their timber before the Faustmann age, and a set of unusually low offered prices can encourage owners to delay harvest until after the Faustmann age. The actual length of rotation thus depends on the prices that happen to occur as the timber nears financial maturity.

The expected present value of a flow of net revenue with varying prices is the sum of the discounted net revenue generated by the product of accepted prices times volume in each period weighted by the likelihood that the owner sells his timber in that period. Since an owner who accepts a given price will always accept a higher price that period, we can characterize the prices accepted by the owner in each period t in terms of a minimum accepted price $P_{m,t}$ (the reservation price). The expected value of a sale in year t given a reservation price of $P_{m,t}$ is:

$$R_t = \int_{P_{m,t}}^{\infty} (p\,Q(T) + E[W])\,f(p)dp$$

where $f(p)$ is the probability of price p occurring that year. The first element of this payoff is the expected revenue in the first period from selling the standing timber: the product of accepted prices times first period volume weighted by the likelihood of being offered those prices. The second element in the expected payoff is the present value of bare land.

The value of waiting includes not just the revenue from next period but also the possible revenue from all future harvest dates which the owner may end up waiting for. The present value of all future expected revenue starting with bare land is:

$$E[W] = -C + R_1e^{-r1} + F(1)\,R_2e^{-r2} + F(1)F(2)\,R_3e^{-r3} + \ldots + F(1)F(2)F(3)\ldots F(Z-1)R_Ze^{-rZ}, \tag{2}$$

where C is planting costs, $F(t)$ is the probability that a tree is left standing at age t, and Z is the oldest age the owner is willing to allow before harvesting. Starting with bare land, the first action is to plant the forest. In each ensuing period, the owner then has the option to harvest standing trees. The probability that a tree is not harvested at age t is:

$$F(t) = \int_{-\infty}^{P_{m,t}} f(p)dp.$$

The probability that a tree will have survived to age K is the product of the probabilities that it has not been cut from age 1 through K-1. The second term in the right-hand side of (2) is the expected payoff at age one. The payoff at age one is followed by a series of similar terms representing the payoffs at other ages. Of course, the contribution of older ages is conditional on the probability that previous prices have been rejected (that the forest is still standing).

The model above is assumed to have a finite maximum harvest age. In principle, the maximum harvest age could be infinite, but such a maximum is unnecessary since forest stands have finite lives and with-well behaved yield functions, it is known that the value of the infinite series approaches a limiting value.[7] Provided Z is chosen sufficiently far into the future, the resulting solutions of the model will provide accurate land values and reservation prices (across the relevant ages). Of course, practical constraints may encourage a land owner to harvest within a more narrow range of ages (see "Numerical Solutions").

In order to solve (2), the owners must choose the reservation price for each age. Because the objective is to maximize the expected net present value, on the margin, owners should choose a reservation price at each age so that current revenue would be equal to the expected net present value of revenue from delaying harvest. For example, the revenue from the reservation price at age k should equal the present value of harvesting from age k + 1 through Z:

$$(P_{m,k}Q(k) + E[W])e^{-rk} = \int_{P_{m,k+1}}^{\infty} [(pQ(k+1)) + E[W])f(p)dp]e^{-r(k+1)} + \ldots + F(k+1)F(k+2)\ldots F(Z-1)\,(\overline{P}Q(Z) + E[\mathrm{W}])e^{-rZ}. \tag{3}$$

Note that the reservation price at age k depends on the reservation prices

[7] See Brazee 1987 for a detailed proof.

for future ages. Equation (3) is a dynamic programming problem that can be solved backwards (recursively) from age Z.

Because we assume a maximum harvest age (Z), the expected price at age Z is the expected value of offered prices, $\bar{P}$. The reservation price in age $Z - 1$ is:

$$P_{m,Z-1} = [(\bar{P}Q[Z] + E[W])e^{-r1} - E[W]]/Q[Z - 1]. \tag{4}$$

The reservation price for age $Z - 1$ can be solved with the above equation. Using (3), the reservation price schedule can be solved recursively from $Z - 1$ back to the present.

Note that one term in (3) and (4) that cannot always be determined from the end conditions is the expected value of bare land $E[W]$. If the land is to be converted to another use after the rotation is completed, $E[W]$ is simply the expected value of this use. In this case, $E[W]$ is known in advance. However, if the land is to stay in forestry, the value of future rotations depends on the unknown value of engaging in price-flexible harvesting. The recursive equations (3) and (4) must be solved simultaneously with the expected value of bare land (2). Because the analytical solution to this simultaneous system of equations is intractable, we turn to an iterative approach. We begin by assuming the land is equal to the Faustmann predicted value of bare land. Solving the recursive system using (3) and (4), we produce a new expected value of bare land. This new value is an improved estimate of bare land value. The new bare land value is then reintroduced into the model and the entire process is repeated. The solution is obtained when the initial value of bare land assumed at the beginning of the recursive process is the resulting value of bare land that appears once the recursive system is solved.[8]

Despite the intractibility of the analytical solution, comparative static analyses provide some qualitative insights concerning the optimal solution. The formal proofs for each of these results are available in Brazee (1987). In this section, we review these results and provide an intuitive explanation. First, as long as the owner chooses a reservation price that actually leads to the rejection of some offered prices, the expected net present value from a flexible price harvest strategy will exceed the present value of revenue associated with Faustmann rotations. This point is trivial in that the owner could choose the Faustmann rotation. If the owner ever finds it desirable to cut his timber earlier or later than the Faustmann rotation, he does so only to increase expected profits.

Second, the reservation price should decline with age. Early in the rotation, the forest is growing so rapidly that the owner is actually being paid to wait. Only an exceptionally high price should convince the owner to sell early. As the stand growth rate declines, the net cost of waiting increases, encouraging the owner to accept lower and lower reservation prices.

Obviously, increases in the mean of prices will increase the present value of timber owners. It is perhaps less obvious that increases in the spread of the price distribution around the mean will also increase the present value of

[8] The described iterative process converges to the solution (see Brazee 1987).

future forest revenue to the owner.[9] The greater the spread of offered prices, the higher prices can reach in good times. Since the increased number and severity of lower prices can be truncated and therefore ignored, increases in spread tend to raise the expected revenue from accepted prices. Holding the reservation price constant, a pure increase in spread of prices (preserving the mean) would result in a greater spread of prices above the reservation price. The expected value of the accepted price will therefore increase with the spread of the offered prices. This in turn increases the present value of future revenue and therefore raises the reservation price.

Although landowners become more wealthy with increased price variation, this does not prove that price variation increases social welfare. The increased spread may decrease welfare as consumers, manufacturers, and wage earners are forced to make unplanned adjustments. The forest owner practicing flexible price harvesting, however, is made better off with increased price variation.

Other important features of the model such as the frequency distribution of rotation lengths, the expected rotation age, the size of the expected value of search, and the effect of tree species and site quality on the value of search can only be determined by explicitly solving the model. In order to provide relevant quantitative answers to the above questions, we numerically solve the search model in the next section to illustrate the results for two key commercial timber species.

NUMERICAL SOLUTIONS

In this section, we solve the model described in the previous section using a numerical approach. The numerical simulation is accurate given the chosen parameters. The results, however, are limited to the range of parameters explored. We therefore attempt to include a broad range of parameters in the simulation exercise so that readers can assess the implications of the model in a variety of situations. For example, two different species, Douglas-fir and loblolly pine, are used to illustrate the difference in outcome when using a slow-maturing versus fast-maturing species. Two different qualities of land are explored to illustrate the importance of site quality and also as a proxy for the effects of management intensity.[10] In order to understand the importance of the magnitude of short-term price variation, a sensitivity analysis altering the size of price variation was also performed. A sensitivity analysis of real interest rates also was tried proving the results are robust with respect to reasonable ranges of this variable. Finally, we analyze the impact of restricting the range of allowable harvest ages upon the benefits of flexible harvesting.

In order to calculate the size of historic price fluctuations, we begin with National Forest stumpage prices between 1975–84 for second growth Douglas-fir (Haynes 1986) and 1970–79 for loblolly pine (Ulrich 1981). These nominal prices are deflated using the GNP deflator to 1986 constant

[9] This result holds for all well-behaved probability density functions (Brazee 1987).

[10] The effects of management actions such as thinning, planting, and fertilizing can be modelled at least in the short run as increases in the site index of the land.

dollars. The mean real price over this time period for Douglas-fir is \$259.20 with a standard deviation of \$86 (variance of \$7341). For loblolly pine, the mean price is \$167.40 with a standard deviation of \$40 (variance of \$1633). In the following simulation, we use these historic means and variances for the price distribution for each species. However, because various factors may alter the variance over time, we also explore the sensitivity of the results to changes in the variance.

We rely on the yield tables of McArdle (1949) and USDA (1929) for estimates of the growth and yields of Douglas-fir and loblolly pine respectively. Because it is more convenient to work with a continuous formula rather than decade tables, we convert the table into the following form:

$$V(R) = e^{a - b/r}. \tag{5}$$

Upon taking the logs of both sides of (5), the parameters a and b were calculated from the yield tables using simple linear regression analysis.[11]

The critical factor in all these yield functions is the volume and growth of timber in the range where the timber is likely to be cut. By estimating several regressions for different site quality lands, we present a probable range of expected results one would get with each species in most commercial stands. Thus, although these relatively old yield tables may not reflect the yields for each age of modern stands, the range of growth and volume near harvest age used in these simulations is indicative of the range in modern managed stands. Of course, for individual managers, they would get superior results by including the actual yield tables for their own stands. Nonetheless, the outcomes presented here will bracket the results individual managers are likely to achieve with price flexible management.

The choice of a final age in which one will commit to cutting the forest is somewhat arbitrary in this context. From simulation experiments with flexible price management, it became obvious that almost all harvesting will occur before the age that maximizes mean annual increment (the MAI rotation). Choosing the MAI rotation as the maximum harvest age, however, will slightly distort the reservation prices in the years prior to MAI. In order to be as accurate as possible, we set Z equal to twice the MAI rotation.

Starting the model at twice the MAI rotation and solving backwards, we calculate the reservation price and expected value of harvest in each time period. This series is then converted to present value estimates upon the final solution of the model. The resulting frequency distribution of harvest dates is presented in Figure 1 for high quality Douglas-fir and loblolly pine.

As illustrated in these figures, about 42% and 62% of the harvests for Douglas-fir and loblolly pine, respectively, are expected to occur within 5 years of the Faustmann rotation. The fraction increases to 71% and 90% for the two species, plus or minus 10 years of the Faustmann rotation age. Stands would reach the MAI rotation length less than 0.0008% and 1.6% of the time for Douglas-fir and loblolly pine, respectively. The simulations sug-

[11] The resulting regression coefficients for the yield table are:

	Low Site Index	High Site Index
Douglas-fir	$LNQ = 12.91 - 223.71/R$	$LNQ = 13.06 - 145.61/R$
	$(S = 110)$	$(S = 160)$
Loblolly pine	$LNQ = 12.11 - 92.71/R$	$LNQ = 12.09 \pm 52.9/R$
	$(S = 80)$	$(S = 120)$

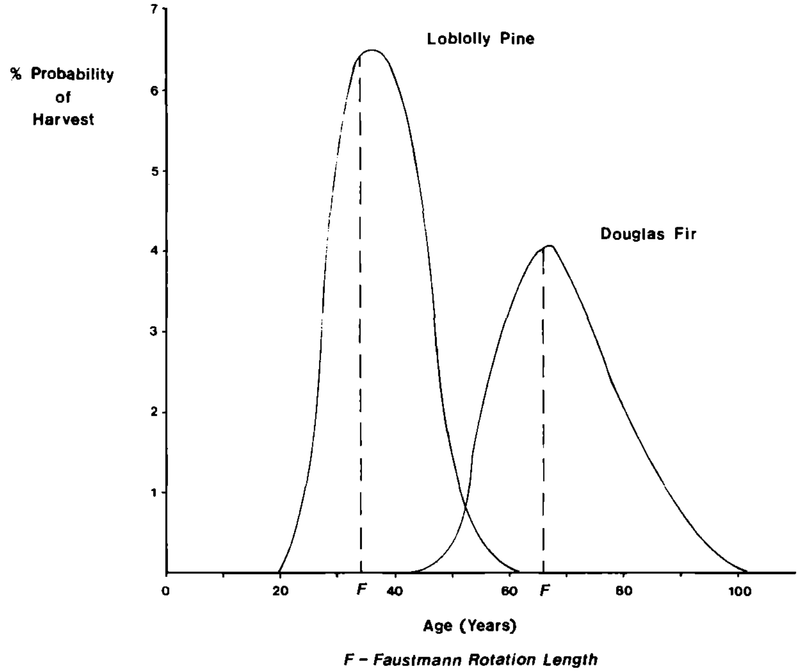

FIGURE 1.

gest that harvests will tend to occur in the vicinity of the Faustmann rotation and that harvests beyond the MAI rotation are very rare.

The expected age of harvest is slightly longer than the Faustman rotation. As shown in Table 1, the Faustmann harvest age is the rotation length that occurs with zero price variation. When the current level of price variation is simulated, the resulting expected rotation length is about 2 years greater than the Faustmann age for Douglas-fir and 1 year greater than the Faustmann age for loblolly pine. In all the cases we examined, the expected harvest age increased slightly with increasing variance. However, in all cases, this change was quite small so that the long-run timber flows under price flexible harvesting are almost identical to those which would occur under the Faustmann rule.

As indicated by the comparative statics, reservation prices decline over time. The plot between reservation prices and time for Douglas-fir and loblolly pine are shown in Figure 2. When the trees are very young, the reservation price is essentially infinite because there is very little timber to harvest. The reservation price falls at a decreasing rate from these early high figures. The reservation price remains two standard deviations above the mean price for Douglas-fir until it reaches age 55. At the Faustmann rotation, the reservation price is still 1.5 standard deviations above the mean for Douglas-fir and 1.1 standard deviations above the mean for loblolly pine. The reservation price appears to be approaching the mean price asymptotically with both schedules.

The most important measure of the desirability of flexible price harvesting is the resulting increase in the expected net present value of future timber revenue (see Table 1). When there are no fluctuations in prices (the Faustmann model), high site index Douglas-fir bare land yields a net present value of \$1975 per acre. With current levels of price variation, the price flexible model yields an expected net present value of \$3316 per acre. For low site

TABLE 1. Net Present Value of Flexible Price Management.

Douglas-fir (Mean Stumpage Price $259.20/mbf; interest rate 3%)

		Site Index 160		
Variance	NPV of bare land	Percentage gain in NPV	Expected harvest age	Reservation price at 65
0	1975	—	64.9	—
1835	2643	34%	66.2	317.65
7341	3316	68%	67.1	387.19
29364	4700	138%	68.2	531.88
		Site Index 110		
Variance	NPV of bare land	Percentage gain in NPV	Expected harvest age	Reservation price at 84
0	476	—	83.6	—
1835	687	44%	85.1	319.81
7341	894	88%	85.9	391.17
29364	1319	177%	86.9	539.31

Loblolly Pine (Mean Stumpage Price $167.40/mbf; interest rate 3%)

		Site Index 120		
Variance	NPV of bare land	Percentage gain in NPV	Expected harvest age	Reservation price at 34
0	3313	—	34.0	—
408	3999	21%	34.7	190.95
1633	4712	42%	35.4	219.91
6532	6199	87%	35.7	281.38
		Site Index 80		
Variance	NPV of bare land	Percentage gain in NPV	Expected harvest age	Reservation price at 50
0	1175	—	49.7	—
408	1466	25%	50.7	191.66
1633	1760	50%	51.3	221.97
6532	2370	102%	52.0	286.09

quality land, the net present value increases from $476 (for the Faustmann rotation) to $894 per acre given current price variation.

Price flexible management has a slightly smaller impact on the bare land value of loblolly pine. The present value of future harvests of loblolly pine increases from $3313 (Faustmann) to $4712 (given current price variation) per acre on high site quality land. On low site quality land, the comparable increase is from $1175 (Faustmann) to $1760 per acre. We believe that flexible price management has a bigger impact on Douglas-fir than loblolly pine primarily because of the greater observed variance of stumpage prices for Douglas-fir.

Although the absolute increase in net value due to flexible price management is larger for high site land than low site land, the relative increase is larger for low site quality land. That is, the percentage gain from price flexible management of low site quality land is larger than the gain from high quality lands. Thus, per dollar of investment, it is more important to manage low site quality lands flexibly than high site quality lands. This is especially evident for Douglas-fir where the value of low quality site lands can be increased by 88% over Faustmann compared to the figure for high site quality lands of 68%.

Qualitative analysis indicates that any increase in mean preserving spread will increase the reservation prices over the relevant range of ages and will also increase the net present value of future timber revenue under price

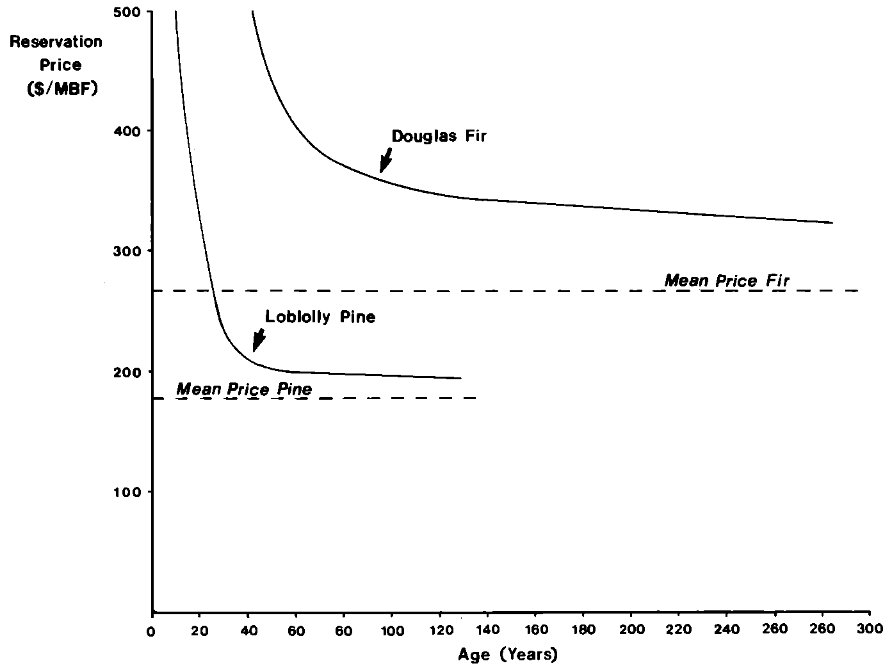

FIGURE 2.

flexible harvesting. The magnitude of this effect, however, cannot be determined from analytical methods. Using numerical methods, it is evident that as the spread of prices increases, the reservation prices become higher (see Table 1). For example, Douglas-fir on high site index land has a reservation price of $387 when it reaches the Faustmann rotation age of 65 given the current variance in prices. However, if the price variance increases fourfold, the reservation price at age 65 becomes $532. A similar though less dramatic result applies to loblolly pine.

For the specific parameters tested in this model, one can see in Table 1 that increasing the spread of prices substantially increases the present value of the land. The magnitude of the increase in expected present value is proportional to the standard deviation. Thus, a doubling of the standard deviation of prices from 43 to 86 results in a doubling of the relative gain from flexible price management from 34% to 68% for high site Douglas-fir and from 44% to 88% for low site Douglas-fir. The same pattern of proportional increase is evident with loblolly pine.

The analysis above is conducted with a real interest rate of 3% throughout to facilitate comparisons across results. We did, however, conduct sensitivity analyses and found that the results were generally robust with respect to the interest rate. The interest rate, of course, does affect the absolute size of the net present value of bare land. However, the impact of flexible price management with respect to Faustmann management remains similar regardless of the interest rate.

This general analysis permits harvests across a broad range of ages. For some types of owners, this range may need to be restricted because of cash flow constraints, the needs of operating large industrial plants, and employment concerns. We therefore examine more narrow allowable harvest ranges for constrained owners. The net present values associated with narrower ranges of harvest ages are displayed in Table 2.

Obviously, the less flexibility, the lower the net present value of the land. However, it is striking how little flexibility is needed to capture most of the gains of flexible price harvesting. For example, if the harvest ages are permitted to vary within 2 years plus or minus of the Faustmann age, one can capture over half of the gains associated with flexible price management. With a slightly larger window of 5 years plus or minus, one can capture three-fourths of the potential gains. Even modest response to prices can produce substantial increases in net present value compared to a strict Faustmann approach.

CONCLUSION

This paper applies an asset sale model to forestry in order to adapt harvest policies towards unpredictable price fluctuations near harvest time. Optimal reservation prices are calculated given these random future price fluctuations. By cutting only when current prices exceed the reservation price,

TABLE 2. Flexible Price Management With Constrained Rotation Range.

Permissible harvest range	NPV of bare land	% potential gains
Douglas-fir (mean stumpage price $257.20; stumpage price variance 7341; interest rate 3%)		
	Site Index 160	
Permissible harvest range	NPV of bare land	% potential gains
65	1975	—
63–67	2713	55
60–70	3002	77
55–75	3189	91
1–292	3316	100
	Site Index 110	
Permissible harvest range	NPV of bare land	% potential gains
84	476	—
82–86	702	54
79–89	790	75
74–94	849	89
1–446	894	100
Loblolly pine (mean stumpage price mean $167.40; stumpage price variance 1633; interest rate 3%)		
	Site Index 120	
Permissible harvest range	NPV of bare land	% potential gains
34	3313	—
32–36	4178	62
29–39	4494	84
24–44	4650	96
1–106	4712	100
	Site Index 80	
Permissible harvest range	NPV of bare land	% potential gains
50	1175	—
48–52	1519	59
45–55	1649	81
40–60	1725	94
1–185	1760	100

owners can systematically avoid selling during low price periods. The results indicate that an owner can increase the expected net present value of his land considerably by adjusting his harvest schedules to randomly fluctuating prices. Even modest harvest flexibility within a few years of the Faustmann rotation can substantially increase expected present values.

A number of other interesting results are presented. First, holding the mean of the price distribution constant, additional spread in that distribution increases the present value of price flexible harvesting by increasing the mean of accepted prices. With higher expected future accepted prices, the reservation price schedule is also higher. Second, with a normal distribution of prices, the benefits of flexible price harvesting tend to increase proportionally with the standard deviation of real prices. Third, the relative increase in the present value of future revenue from flexible price harvesting is larger for lower site quality lands. This suggests that price flexible harvesting is especially suited for marginal commercial timber land. Fourth, the reservation price for each age (the minimum acceptable price) declines over the rotation. While trees are growing rapidly, owners can afford to wait for a high price before harvesting. Fifth, at least in these simulations, reservation prices are almost always above the expected mean price of the distribution. Given current parameters, price flexible harvesting consequently leads owners to sell only when prices are above long-run average prices.

As managers take the results in this paper and prepare to apply them to their own lands, they must review the assumptions of the model. First, we assume that known changes in future prices are already being compensated for in harvesting decisions. Thus, managers must start with a model where harvest decisions have already adjusted to predicted changes in real prices. Second, the volume and growth function for each age class is assumed to be known. The manager must determine the appropriate volume and yield table for his lands. Third, unpredictable fluctuations in future prices are assumed to be normally distributed. If this assumption is not valid, the appropriate random distribution function must be inserted. Fourth, prices are assumed to be independently distributed from period to period. If this is not the case, the model must be modified to account for serial correlation amongst prices. Fifth, the model assumes that the cost of harvesting any given acre is independent of the number of acres the owner wants to harvest. Obviously, for some owners, their desired harvest in years with high prices may be large enough to affect harvesting costs. The flexible price harvesting model would have to be modified to take into account these higher harvesting costs.

The focus of this paper is upon the optimal behavior of a single landowner faced with fluctuating prices. The model, however, does have implications for market behavior as well. As prices fall below reservation prices, the model predicts that owners will delay harvesting. Although the actions of any single owner will not affect prices, the actions of many owners begin to have a cumulative impact. As many owners withdraw from harvesting, current supply shrinks sharply thereby increasing prices. Similarly, when current prices exceed reservation prices, many owners will harvest, thus increasing current supply and diminishing prices. Price flexible management thus makes the short-run supply of timber relatively more elastic compared to Faustmann harvest rules. This more flexible short-run supply helps to moderate the price shocks introduced by unanticipated demand shifts for timber.

Although the principle beneficiary of this paper will be private timber owners, the lessons from this analysis apply as well to public management of timber. Maintaining a constant flow of timber without regard to market conditions is a costly policy. For Douglas-fir and other high price variation species, flexible price management will increase the value of the flow of timber by about 75% over price inflexible harvest policies. Although this shift in policy would increase the variation in the flow of timber off the National Forest year by year, it would help stabilize stumpage prices. The Forest Service could institute more price flexibility in actual harvests by extending the length of time on harvest contracts and thus giving the contractors increased opportunity to practice flexible price harvest strategies. The contractors could then afford to bid more on public timber contracts yielding more revenue for the public treasury.

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