Operator and Entanglement Dynamics in Asymmetric Quantum Systems

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May 1, 2018

Overview

Thermalization in Quantum Systems

Contradiction? Unitarity vs. Information Loss

- Quantum mechanics
- Fluid dynamics
- Thermodynamics
- Black holes

Resolution: Information Spreading

- Information not destroyed but spread
- Not accessible to local measurements

Time-Independent Hamiltonian

- Spin-$\frac{1}{2}$ sites
- Not relativistic: no limit to information speed
- *Most* information still travels at finite speed: Lieb Robinson bounds
- Exponentially small outside "lightcone"

Out-of-Time-Order Commutator

Instead use OTOC:

$$\begin{aligned} C(i,t) &= \frac{1}{2}\left\langle \left|[O_0(t), W_i]\right|^2 \right\rangle \\ O_0 &= Z \otimes I \otimes I \otimes \cdots \\ O_0(t) &= e^{-Ht} O_0 e^{-iHt} \\ W_i &= I \otimes I \otimes \cdots W \otimes \cdots I, \end{aligned}$$

with $W = X, Y, Z$ at site i.

Velocity-Dependent Lyapunov Exponent

For a set velocity, $C(i = vt, t) \sim e^{\lambda(v)t}$

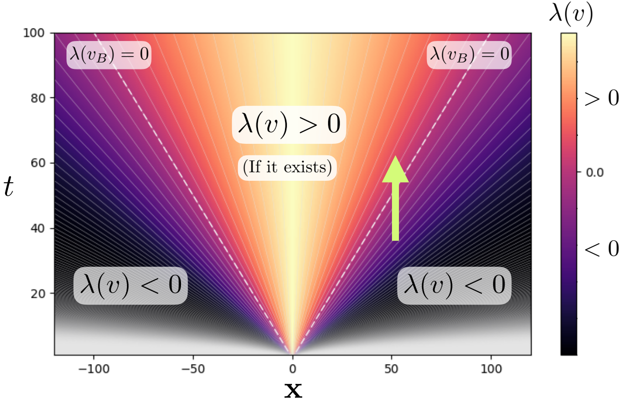


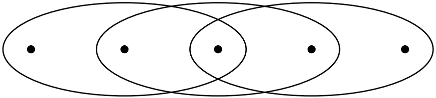
Figure: Lyapunov exponent at various velocities, from [2]

Asymmetric Hamiltonian

3-site Hamiltonian

- Want $S_{123}\left|\alpha\beta\gamma\right\rangle = \left|\gamma\alpha\beta\right\rangle$
- Use $H_3 = \frac{2\pi}{3\sqrt{3}}\,\vec{\sigma}_1\cdot\left(\vec{\sigma}_2\times\vec{\sigma}_3\right)$
- $S_{123} = \exp\left(-iH_3\right)$

Chain these together



Weight at site

Average of OTOC over $W = X, Y, Z$

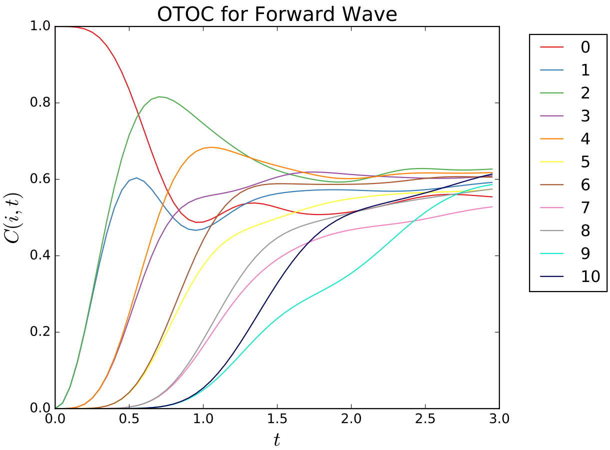


Figure: Weight of operators that begin on site i.

OTOC

Looking for asymmetry in early behavior

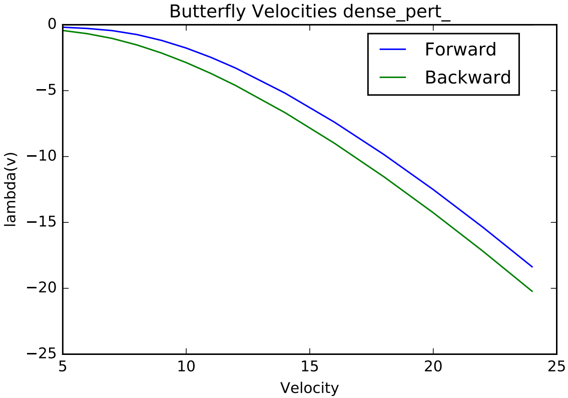


Figure: Calculating Butterfly Velocity from early time evolution.

Quantum Circuits

- Chain of spin-q sites
- Apply unitary operators at discrete times
- Information speed is set by circuit architecture

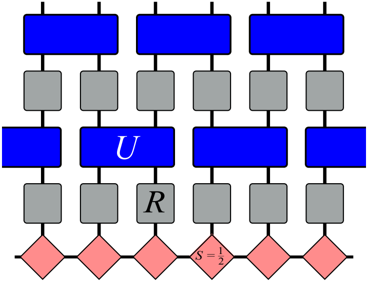


Figure: Quantum circuit, from [3]

Entanglement Entropy

- Start with $\rho_{AB} = |\Psi\rangle \langle\Psi|$, $\rho_A = \mathrm{Tr}_B\, \rho_{AB}$, $\rho_B = \mathrm{Tr}_A\, \rho_{AB}$
- Decompose ρ_{AB} into ρ_A, ρ_B
- Entanglement Entropy: $S(i) = \mathrm{Tr}_A\{\rho_A \log \rho_A\} = \mathrm{Tr}_B\{\rho_B \log \rho_B\}$

Entanglement Bounds

- For spin q, $|S(i) - S(i+1)| \leq S(1) \leq \log q$
- Take log base q, $\implies |S(i) - S(i+1)| \leq 1$
- For $q \to \infty$, arbitrary gates will saturate bound [3]

Solvable Limit

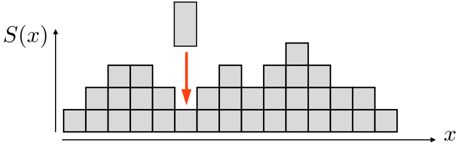


Figure: **Tetris-like model for large-q chain.** The gate at cut x adds enough entropy so that $S(x)$ is one greater than either of its neighbors. Taken from [3].

Staircase Circuits

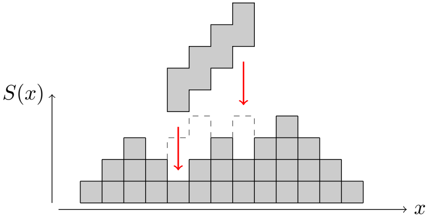


Figure: **Staircase circuit architecture**, in which the gate at site x is always followed by ones at sites x + 1,x + 2, and x + 3 making this a 4-stair. Note that not all gates are productive, only the ones that fall on sites that are local minima when they fall.

Growth Rates

After course graining, $\frac{\partial S}{\partial t}$ is a function of $m = \frac{\partial S}{\partial x}$ to first order or for linear entropy.

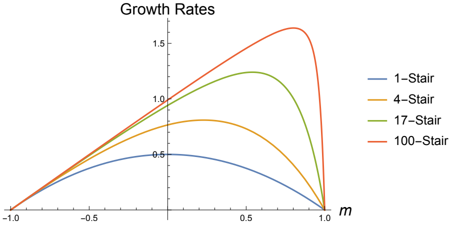


Figure: Growth rates for 1-, 4-, 17-, and 100-stairs as a function of slope m. As stair length increases, the growth rate asymptotes to the function $\frac{\partial S}{\partial t} = m + 1$.

The butterfly velocities are the extremal slopes of this curve [1].

Measured Growth Rates

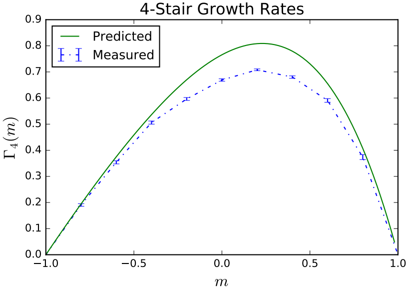


Figure: **Measured and predicted growth rates** for the 4-stair architecture. The measured rate is now significantly lower than predicted, implying that the correlations built up by the stairs act to lower their entropy production.

Correlations

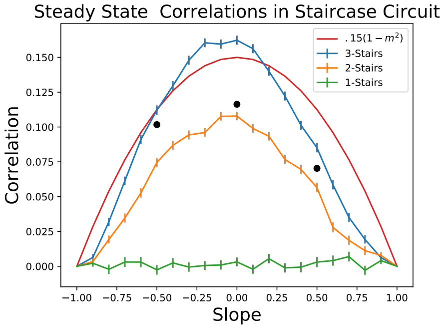


Figure: **Correlations created by 1-, 2-, and 3-stair circuits**. The correlation in 1-stair circuits is consistent with 0. Larger stairs generate significant correlation. The black dots are the correlations calculated analytically.

Summary

- Asymmetric v_B in Hamiltonian system
- Maximally asymmetric v_B in circuit models

Further Work

- More asymmetric Hamiltonian
- Understand correlations better
- Direct calculation of growth rate

References

[1] C. Jonay, D. A. Huse, and A. Nahum.
Coarse-grained dynamics of operator and state entanglement.
2018.

[2] V. Khemani, D. A. Huse, and A. Nahum.
Velocity-dependent lyapunov exponents in many-body quantum, semi-classical, and classical chaos.
2018.

[3] A. Nahum, J. Ruhman, S. Vijay, and J. Haah.
Quantum entanglement growth under random unitary dynamics.
Physical Review X, 7(3), jul 2017.