Entanglement Speeds in Asymmetric Systems

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Overview

Quantum Circuits

- Chain of spin-q sites
- Apply unitary operators at discrete times
- Information speed is set by circuit architecture

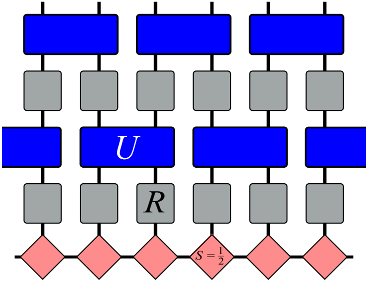

Figure: Quantum circuit, from [3]

Entanglement Entropy

- Start with $\rho_{AB} = |\Psi\rangle \langle\Psi|$, $\rho_A = \mathrm{Tr}_B\, \rho_{AB}$, $\rho_B = \mathrm{Tr}_A\, \rho_{AB}$
- Decompose ρ_{AB} into ρ_A, ρ_B
- Entanglement Entropy: $S(i) = \mathrm{Tr}_A\{\rho_A \log \rho_A\} = \mathrm{Tr}_B\{\rho_B \log \rho_B\}$

Entanglement Bounds

- For spin q, $|S(i) - S(i+1)| \leq S(1) \leq \log q$
- Take log base q, $\implies |S(i) - S(i+1)| \leq 1$
- For $q \to \infty$, arbitrary gates will saturate bound [3]

Staircase Model

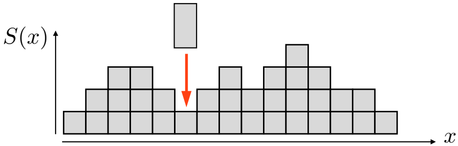

Figure: **Tetris-like model for large-q chain.** The gate at cut x adds enough entropy so that $S(x)$ is one greater than either of its neighbors. Taken from [3].

Growth Rates

After course graining, $\frac{\partial S}{\partial t}$ is a function of $m = \frac{\partial S}{\partial x}$ to first order or for linear entropy.

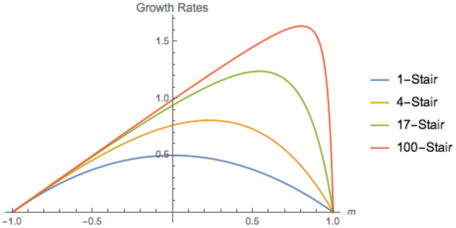

Figure: Growth rates for 1-, 4-, 17-, and 100-stairs as a function of slope m. As stair length increases, the growth rate asymptotes to the function $\frac{\partial S}{\partial t} = m + 1$.

The butterfly velocities are the extremal slopes of this curve [1].

Time-Independent Hamiltonian

- Back to spin-$\frac{1}{2}$
- Not relativistic: no limit to information speed
- *Most* information still travels at finite speed
- Use $H_3 = \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$, the three-site swap

Out-of-Time-Order Commutator

Instead use OTOC:

$$
\begin{aligned}
C(i,t) &= \frac{1}{2}\left\langle \left|[O_0(t), W_i]\right|^2 \right\rangle \\
O_0 &= Z \otimes I \otimes I \otimes \cdots \\
O_0(t) &= \mathrm{e}^{-Ht} O_0 \mathrm{e}^{-iHt} \\
W_i &= I \otimes I \otimes \cdots W \otimes \cdots I,
\end{aligned}
$$

with $W = X, Y, Z$ at site i.

Velocity-Dependent Lyapunov Exponent

For a set velocity, $C(i = vt, t) \sim \mathrm{e}^{\lambda(v)t}$

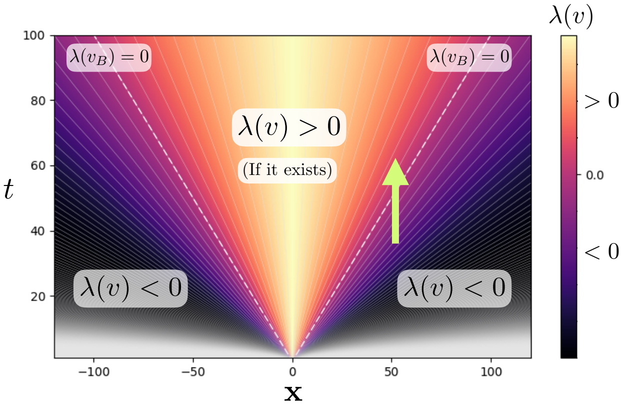

Figure: Lyapunov exponent at various velocities, from [2]

Weight at site

Average of OTOC over $W = X, Y, Z$

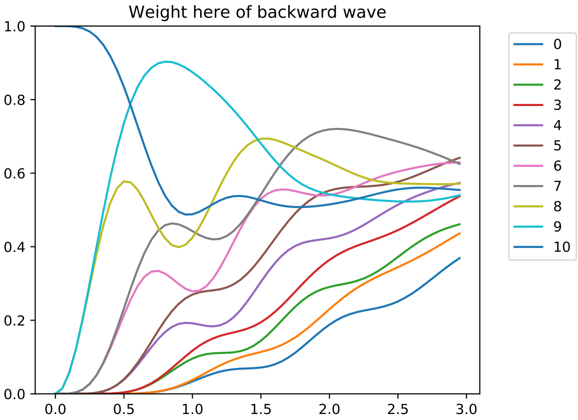

Figure: Weight of operators that begin on site i.

OTOC

Looking for asymmetry

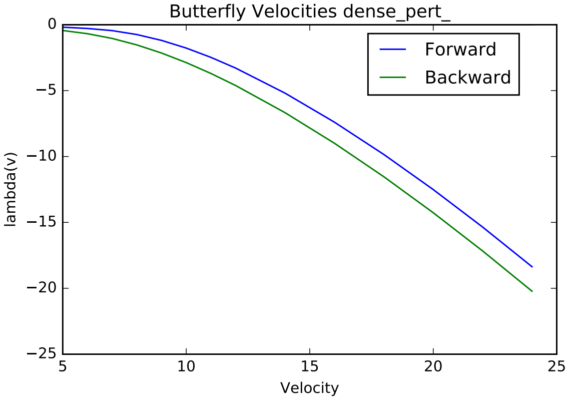

Figure: Calculating Butterfly Velocity from early time evolution.

References

[1] C. Jonay, D. A. Huse, and A. Nahum.
Coarse-grained dynamics of operator and state entanglement.
2018.

[2] V. Khemani, D. A. Huse, and A. Nahum.
Velocity-dependent lyapunov exponents in many-body quantum, semi-classical, and classical chaos.
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[3] A. Nahum, J. Ruhman, S. Vijay, and J. Haah.
Quantum entanglement growth under random unitary dynamics.
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