Asymmetric butterfly velocities in local time-independent Hamiltonians

The butterfly velocity v_B is the velocity at which initially local operators spread. In many 1-D systems this velocity is independent of the direction of spreading. This need not be the case. In fact, with arbitrarily nonlocal Hamiltonians, or arbitrarily deep circuit models, the ratio of the two butterfly velocities may be made arbitrarily large. We provide a class of circuits whose limiting behavior shows this arbitrarily large ratio. We also describe a local Hamiltonian with an asymmetric v_B, presenting various methods to measure the asymmetry.

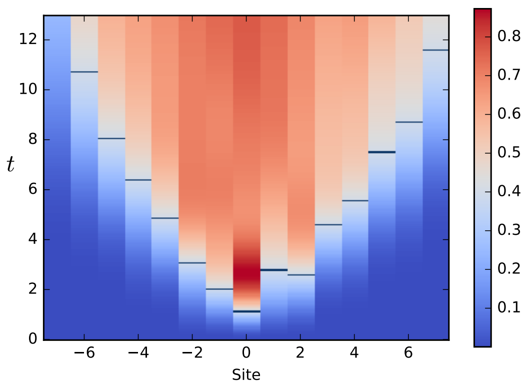

FIG. 1. Illustration of the initially local operator. The bars indicate the time at which the OTOC passes 0.4, to emphasize the asymmetry.

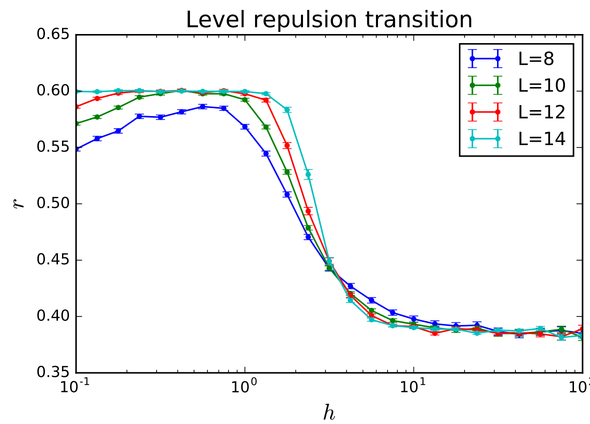

FIG. 2. Phase transition for the model, with level repulsion parameter plotted against field strength. Note that in the thermalizing phase the ratio is 0.6 instead of 0.53 because the statistics are GUE instead of GOE. I think I remember Vedika saying this but I can't find where.

INTRODUCTION

Thermalization is important because…

Asymmetric transport is seen in "staircase" and "glider" circuits, but we wanted to show it is also possible in time-independent Hamiltonians.

In a 1-D circuit, how asymmetric can the spreading be?

On the edge of a 2-D system, spreading can be chiral even with a finite circuit depth [**?**].

To be completely chiral with only 1 dimension, the circuit will have to be of infinite depth.

Given a constraint on the depth, how asymmetric can the spreading be?

In this paper we will…

LOCAL HAMILTONIANS

Motivate triple product: Has to be asymmetric-can't have 2-site interactions. Impose SU(2) as a constraint? Then our only option is the triple product.

Alone, this model is not general. Large degeneracy at $E = 0$. Explain the degeneracy is due to the $E \to -E$ (anti-)symmetry. Show parts of this degeneracy can be broken with various fields. A random Z field breaks all the degeneracy. This phase change can be seen in Fig. 2.

We will measure the asymmetry using two metrics. The first is the weight of all operators with right (left) endpoint on site i, which we will call the right (left) weight. The other is the OTOC. should we define these in this section? Make sure to point out use of initial operators as being on site 0 or $L - 1$.

In the thermalizing, generic phase, the right weights peak as the information front passes. Because of the three-site nature of each term in the Hamiltonian, the right weight and OTOC exhibit an "odd-even" effect. It is possible to account for these by averaging judiciously, or by only looking at even (or odd) sites. At $L = 13$, there are enough even sites that the asymmetry can be seen. For a picture of the rights weights with their successive peaks, see Fig. 3.

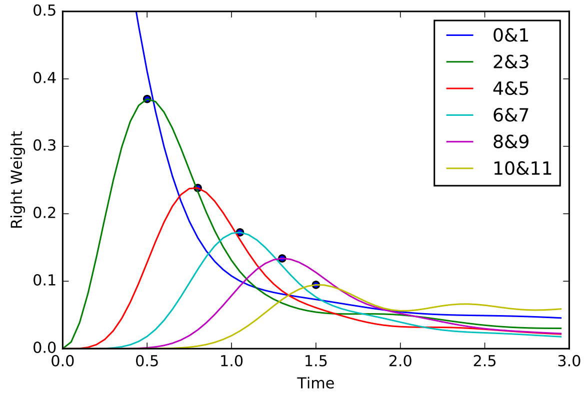

FIG. 3. Right weight at even sites for $L = 13$. The peak travels ballistically. Later peaks are smaller Is this due to broadening?

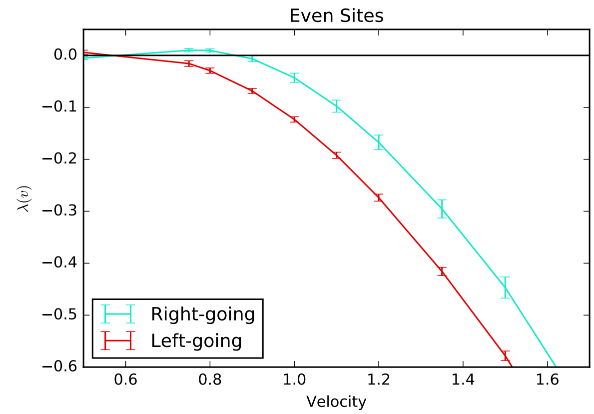

FIG. 5. Velocity-dependent Lyapunov exponents extracted from the OTOC on even sites. Since $\lambda_r(v) > \lambda_l(v)$, we know $v_{B,r} > v_{B,l}$.

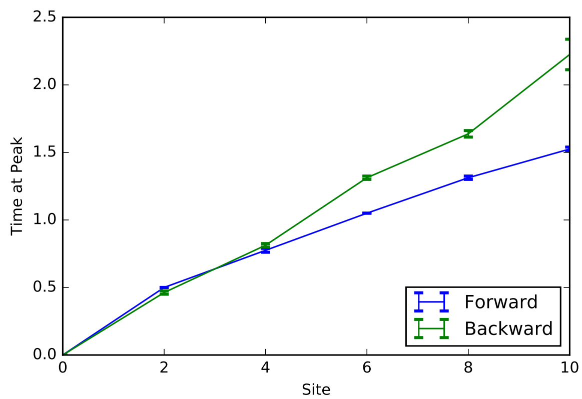

FIG. 4. Time of peak vs. site. Since this is plot of time as a function of distance, the larger slope in the left weight means that v_B is larger for propagation to the right. This has the wrong normalization but I'm working on it.

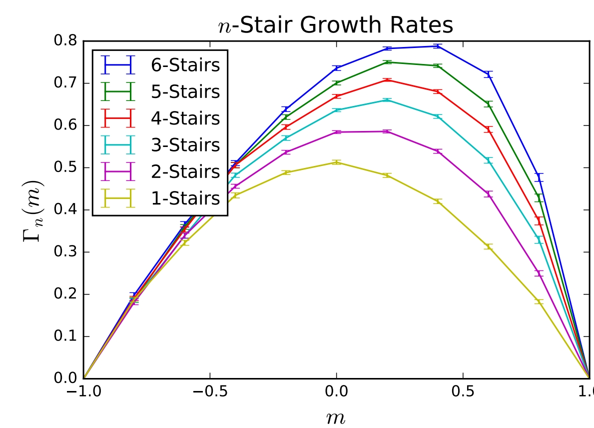

FIG. 6. Empirical growth rate as a function of slope for n-stair circuits. The right/forward and left/backward butterfly velocities are the slopes of these curves at their endpoint, indicating that as the left v_B stays constant, the right v_B increases. The appendix includes an argument that the right v_B is unbounded in the large-n limit.

Fig. 4 shows the peaks traveling ballistically. The peaks reach equivalent sites at later times for the left-moving wave, implying $v_{B,l} < v_{B,r}$. We can extract $v_{B,l}$ and $v_{B,r}$ from these curves by fitting linear functions to the peak timings.

It is also possible to extract butterfly velocities from the the velocity-dependent Lyapunov exponents. Fig. 5 shows the VDLEs for the right-going and left-going OTOCs. v_B is defined by $\lambda(v_B) = 0$, so the plot shows that $v_{B,r} > v_{B,l}$.

CIRCUIT MODELS WITH ASYMMETRIC v_B

Define $\Gamma(ds/dx)$, reference Nahum and Cheryne papers. Explain how to get v_B from this quantity.

Define the staircase circuits. Motivate the large-q approximation, describe effect of finite q (Nahum).

For the growth rate curves of n-stair circuits for $n \leq 6$ see Fig. 6.

For small n, we can simulate the circuit directly. This is particularly easy in the large q limit, where q is the dimension of the Hilbert space at each site. Again, how in-depth should this section be?.

For large n, approaching the size of the system, we can approximate the entanglement curve as being uncorrelated. In that limit, the growth rate is $\Gamma(ds/dt) = ds/dt + 1$, so that for spreading to the left $v_B = 1$ and for spreading to the right $v_B = \infty$.

CONCLUSION

Advantages of this model: Time-independent Hamiltonian. Only ingredients are chains of two-level systems.

Further work: How do the velocities depend on h? What happens at the phase transition? Maximally asymmetric three-site Hamiltonians? 2-D systems?

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Note somewhere about arXiv:1809.02614v1

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[64] Here $\tilde{v}_B(\hat{\mathbf{n}})$, with a tilde, denotes the normal propagation speed of a straight front whose normal is parallel to $\hat{\mathbf{n}}$. In Ref. [43] this was denoted $v_B(\hat{\mathbf{n}})$, but here we use $v_B(\hat{\mathbf{n}})$ to denote the speed at which an initially local operator spreads away from the origin in the direction $\hat{\mathbf{n}}$. These differ because in the absence of rotational symmetry the operator's front is not in general perpendicular to the radial vector, but they are related by a geometrical construction known from classical droplet growth [43, 83, 84].

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[72] In random circuits related random walk pictures underlie the calculation of both the OTOC and the second Renyi entropy [43, 44]. In these random systems this yields a relation between $\lambda(v)$ and the "entanglement line tension" defined in [59], specifically the line tension $\mathcal{E}_2(v)$ for the second Renyi entropy. This motivates the conjecture, for non-random systems, that $\lambda(v)|_{\mathrm{cont}} = -s_{\mathrm{eq}}(\mathcal{E}_2(v) - v)$, where s_{eq} is the thermal entropy density. The left hand side denotes the analytic continuation of $\lambda(v)$ from $v > v_B$ to values $v < v_B$. In random circuits we must distinguish different kinds of averages. The line tension extracted from a calculation of $\overline{e^{-S_2}}$ determines $\lambda(v)$ for the average OTOC $\overline{C(x,t)}$ by the above formula. It is natural to expect that the line tension determined by the more natural direct average $\overline{S_2}$ determines $\lambda(v)$ for the typical value of the OTOC, $\exp \overline{\ln C(x,t)}$. The average and typical values of the OTOC are parametrically close in the region close to the front, but they may differ significantly in the far-front regime where both are exponentially small.

[73] In some circuit models in $d > 1$ (which do not have continuous spatial rotation symmetry) some sections of the operator's front can be "glued" to the strict lightcone defined by the discrete time circuit [43]. This is a peculiar case where $v_B(\hat{\mathbf{n}}) = v_{\mathrm{LC}}(\hat{\mathbf{n}})$ for some directions $\hat{\mathbf{n}}$ in space, so that no nontrivial $\lambda(\mathbf{v})$ can be defined for these directions of $\mathbf{v}$.

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[82] Let the probability distribution for weak-link "waiting times" be $P(\tau) \sim \tau^{-a-2}$. At weak disorder ($1 < a$) the broadening of the operator's front [50] is diffusive, as in the clean system. At intermediate disorder ($0 < a < 1$) the front broadens more strongly, giving $\lambda(v) \sim -(v - v_B)^{(a+1)/a}$. For strong disorder ($-1 < a < 0$) the butterfly speed vanishes: in this regime $\lambda(v) \sim -|v|^{1-|a|}$. In the disordered system the definition of $\lambda(v)$ depends on whether we consider e.g. the mean or the typical value of the OTOC, but this should not change these exponents.

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