



# Machine Learning - Module 01

## Univariate Linear Regression

*Summary: Today you will implement a method to improve your model's performance: **gradient descent**. Then you will discover the notion of normalization.*

# Notions and ressources

## Notions of the module

Gradient descent, linear regression, normalization.

## Useful Ressources

You are strongly advise to use the following resource: [Machine Learning MOOC - Stanford](#)  
Here are the sections of the MOOC that are relevant for today's exercises:

### Week 1

#### Linear Regression with One Variable

- Gradient Descent (Video + Reading)
- Gradient Descent Intuition (Video + Reading)
- Gradient Descent For Linear Regression (Video + Reading)
- Review (Reading + Quiz)

### Week 2

#### Multivariate Linear Regression

- Gradient Descent in Practice 1 - Feature Scaling (Video + Reading)

# Common Instructions

- The version of Python recommended to use is 3.7, you can check the version of Python with the following command: `python -V`
- The norm: during this piscine, it is recommended to follow the [PEP 8 standards](#), though it is not mandatory. You can install [pycodestyle](#) which is a tool to check your Python code.
- The function `eval` is never allowed.
- The exercises are ordered from the easiest to the hardest.
- Your exercises are going to be evaluated by someone else, so make sure that your variable names and function names are appropriate and civil.
- Your manual is the internet.
- You can also ask questions in the `#bootcamps` channel in the [42AI](#) or [42born2code](#).
- If you find any issue or mistakes in the subject please create an issue on [42AI repository on Github](#).
- We encourage you to create test programs for your project even though this work **won't have to be submitted and won't be graded**. It will give you a chance to easily test your work and your peers' work. You will find those tests especially useful during your defence. Indeed, during defence, you are free to use your tests and/or the tests of the peer you are evaluating.
- Submit your work to your assigned git repository. Only the work in the git repository will be graded. If Deepthought is assigned to grade your work, it will be run after your peer-evaluations. If an error happens in any section of your work during Deepthought's grading, the evaluation will stop.

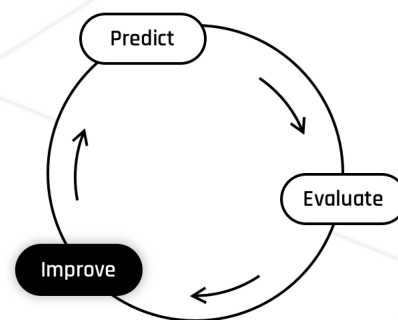
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# Chapter I

## Exercise 00

### Improve



In the previous module, you discovered the first two steps of the learning process: starting with a model that makes naive predictions and evaluating it. Now we are going to tackle the third part: improving it!

Lets take a new dataset:

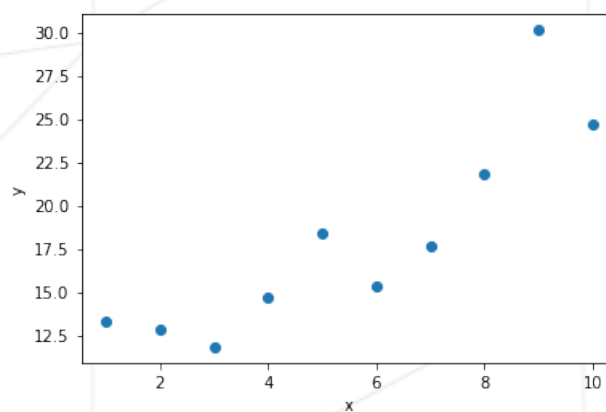


Figure I.1: Scatter plot of a given dataset

## Predict

Given our measure of performance, improvement entails **reducing the loss (or cost)** measured by the loss function. If we plot the loss of a model's predictions as a function of its  $\theta_1$  parameter (with a fixed value for  $\theta_0$ ), we obtain a curve like this one:

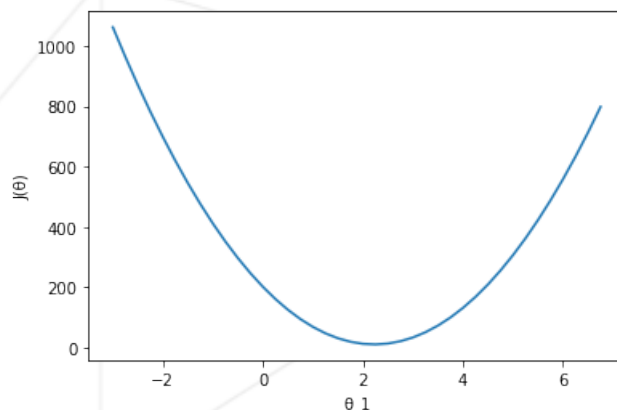


Figure I.2: Loss function given  $\theta_1$

On the graphs below, you can see that extreme  $\theta_1$  values (which modifies the slope of the hypothesis curve - in orange) correspond to a very high loss. On the other hand, as we get closer to the bottom of the curve, the loss is reduced.

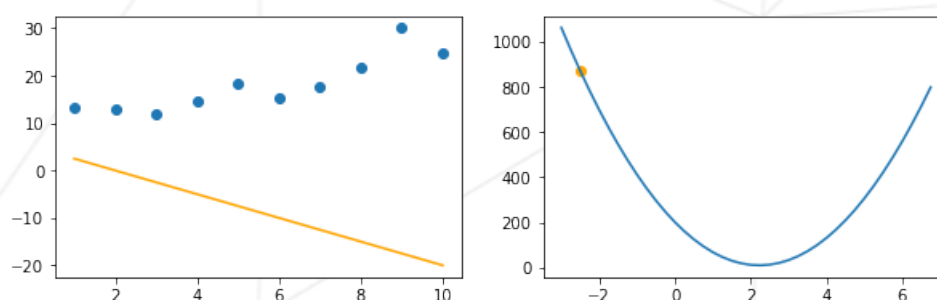


Figure I.3: A quite bad model

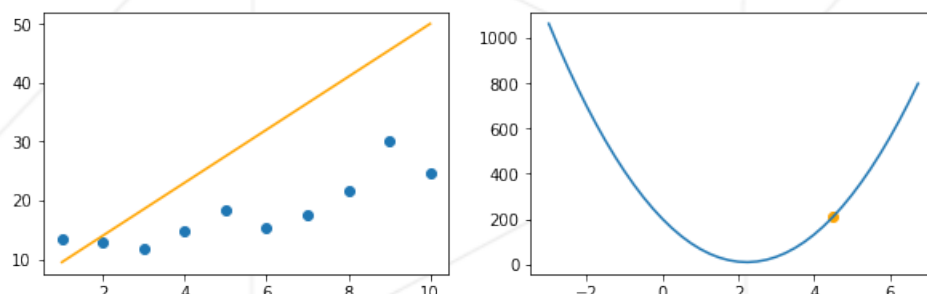


Figure I.4: A better (but still bad) model

The loss function's minimum corresponds to the bottom of the curve. We want  $\theta_1$  to get to this sweet spot. It means that wherever  $\theta_1$  starts at, as the training goes on, it needs to get closer to the value that matches  $J(\theta)$ 's minimum.

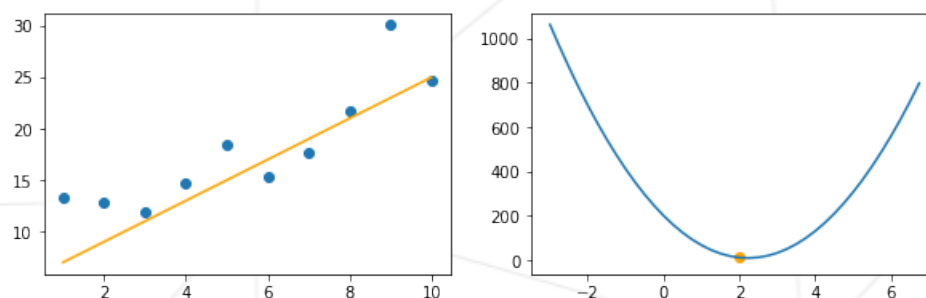



Figure I.5: A good model

## But how to get closer to the minimum?

Excellent question dear reader. We're glad you asked! First, the algorithm needs to figure out in which direction  $\theta_1$  should be moved (i.e. increased or decreased). It does so by calculating the **slope** of the  $J(\theta)$  curve at the current position of  $\theta_1$ . If the slope is positive,  $\theta_1$  must be decreased. If the slope is negative, it must be increased. If you have studied calculus, you probably sense that all of this involves calculating the derivative of the loss function.

The story gets a little more complicated, however, because we have two parameters to adjust:  $\theta_0$  and  $\theta_1$ . Not just  $\theta_1$  (as we showed in our example to simplify). This means the  $J(\theta)$  function doesn't have only one derivative, but two **partial derivatives**. One that computes the slope of  $J$  with respect to  $\theta_0$ , and a second one for the slope of  $J$  with respect to  $\theta_1$ . Finally, we package those partial derivatives in a vector of dimension  $m$ , which is called **gradient** (noted  $\nabla$ ).

Don't worry if you don't master multivariate calculus yet, we have calculated the partial derivatives for you, all you will need to do is write them in Python.

	Exercise : 00
Linear Gradient - Iterative Version	
Turn-in directory : <i>ex00/</i>	
Files to turn in : <b>gradient.py</b>	
Forbidden functions : <b>None</b>	

## Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. You must write a function that computes the **gradient** of the loss function. It must compute a partial derivative with respect to each theta parameter separately, and return the vector gradient. The partial derivatives can be calculated with the following formulas:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$\nabla(J)_1 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})x^{(i)}$$

Where:

- $\nabla(J)$  is the gradient vector of size 2, (this strange symbol :  $\nabla$  is called nabla),
- $x$  is a vector of dimension  $m$ ,
- $y$  is a vector of dimension  $m$ ,
- $x^{(i)}$  is the  $i^{\text{th}}$  component of vector  $x$ ,
- $y^{(i)}$  is the  $i^{\text{th}}$  component of vector  $y$ ,
- $\nabla(J)_j$  is the  $j^{\text{th}}$  component of  $\nabla(J)$ ,
- $h_{\theta}(x^{(i)})$  corresponds to the model's prediction of  $y^{(i)}$ .

## Hypothesis Notation

$h_{\theta}(x^{(i)})$  is the same as what we previously noted  $\hat{y}^{(i)}$ . The two notations are equivalent. They represent the model's prediction (or estimation) of the  $y^{(i)}$  value. If you follow Andrew Ng's course material on Coursera, you will see him using the former notation.

As a reminder:  $h_{\theta}(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$



## Instructions

In the `gradient.py` file create the following function as per the instructions given below:

```
def simple_gradient(x, y, theta):  
    """Computes a gradient vector from three non-empty numpy.array, without any for-loop.  
    The three arrays must have compatible shapes.  
    Args:  
        x: has to be a numpy.array, a vector of shape m * 1.  
        y: has to be a numpy.array, a vector of shape m * 1.  
        theta: has to be a numpy.array, a 2 * 1 vector.  
    Return:  
        The gradient as a numpy.array, a vector of shape 2 * 1.  
        None if x, y, or theta are empty numpy.array.  
        None if x, y and theta do not have compatible shapes.  
        None if x, y or theta is not of the expected type.  
    Raises:  
        This function should not raise any Exception.  
    """  
    ... Your code ...
```

## Examples

```
import numpy as np  
x = np.array([[12.4956442],[ 21.5007972],[ 31.5527382],[ 48.9145838],[ 57.5088733]])  
y = np.array([[37.4013816],[ 36.1473236],[ 45.7655287],[ 46.6793434],[ 59.5585554]])  
  
# Example 0:  
theta1 = np.array([[2],[ 0.7]])  
simple_gradient(x, y, theta1)  
# Output:  
array([[ -19.0342574],[  -586.66875564]])  
  
# Example 1:  
theta2 = np.array([[1],[ -0.4]])  
simple_gradient(x, y, theta2)  
# Output:  
array([[ -57.86823748],[ -2230.12297889]])
```

# Chapter II

## Exercise 01

### Linear Algebra Tricks II

If you tried to run your code on a very large dataset, you would find that it takes a long time to execute! That's because it doesn't use the power of Python libraries that are optimized for matrix operations.

Remember the linear algebra trick of in the previous module? Let's use it again! If you concatenate a column of 1's to the left of the  $x$  vector, you get what we called matrix  $X'$ .

$$X' = \begin{bmatrix} 1 & x^{(1)} \\ \vdots & \vdots \\ 1 & x^{(m)} \end{bmatrix}$$

This transformation is very convenient because we can rewrite each 1 as  $x_0^{(i)}$ , and each  $x^{(i)}$  as  $x_1^{(i)}$ . So now the  $X'$  matrix looks like this:

$$X' = \begin{bmatrix} x_0^{(1)} & x_1^{(1)} \\ \vdots & \vdots \\ x_0^{(m)} & x_1^{(m)} \end{bmatrix}$$

Notice that each  $x^{(i)}$  example becomes a vector made of  $(x_0^{(i)}, x_1^{(i)})$ . The 0 and 1 indices on the  $x$  features correspond to the indices of the  $\theta$  parameters with which they will be multiplied.

Why does this matter? Well, if we take the equation from the previous exercise:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

We can multiply it by 1 without changing its value:

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot 1$$

And rewrite 1 as  $x_0^{(i)}$ :

$$\nabla(J)_0 = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

This means that the equation for  $\nabla(J)_0$  is now similar to the equation we had for  $\nabla(J)_1$ , so they can both be captured by ONE **generic equation**:

$$\nabla(J)_j = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad \text{for } j = 0, 1$$

And as you probably suspected, a generic equation opens the door to vectorization...

## Vectorizing the Gradient Calculation

Now it's time to learn how to calculate the entire gradient in one short, pretty, linear algebra equation!

- First, we'll use the  $X'$  matrix and our vectorized hypothesis equation:  $h_{\theta}(x) = X'\theta$

$$\nabla(J)_j = \frac{1}{m} (X'\theta - y) X'_j \quad \text{for } j = 0, 1$$


- Second, we need to tweak the equation a bit so that it directly returns a  $\nabla(J)$  vector containing both  $\nabla(J)_0$  and  $\nabla(J)_1$ .

$$\nabla(J) = \frac{1}{m} X'^T (X'\theta - y)$$

If the equation does not seem obvious, play a bit with your vectors, on paper and in your code, until you get it.

### Notation Remark

$X'^T$ : You might wonder what the  $T$  is for. It means the  $X'$  matrix must be **transposed**. Transposing a matrix flips it on its diagonal so that its rows become its columns and vice versa. Here we need to do it so that matrix dimensions are appropriate multiplication and to multiply the right elements together.

	Exercise : 01
Linear Gradient - Vectorized Version	
Turn-in directory : <i>ex01/</i>	
Files to turn in : <b>vec_gradient.py</b>	
Forbidden functions : <b>None</b>	

## Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. You must implement the following formula as a function:

$$\nabla(J) = \frac{1}{m} X'^T (X'\theta - y)$$

Where:

- $\nabla(J)$  is a vector of dimension 2,
- $X'$  is a **matrix** of dimensions  $(m \times 2)$ ,
- $X'^T$  is the transpose of  $X'$ . Its dimensions are  $(2 \times m)$ ,
- $y$  is a vector of dimension  $m$ ,
- $\theta$  is a vector of dimension 2.

Be careful:

- the  $x$  you will get as an input is an vector (of dimension  $m$ ),
- $\theta$  is a vector (of dimension 2).

## Instructions

In the `vec_gradient.py` file create the following function as per the instructions given below:

```
def gradient(x, y, theta):
    """Computes a gradient vector from three non-empty numpy.array, without any for loop.
    The three arrays must have compatible shapes.
    Args:
        x: has to be a numpy.array, a matrix of shape m * 1.
        y: has to be a numpy.array, a vector of shape m * 1.
        theta: has to be a numpy.array, a 2 * 1 vector.
    Return:
        The gradient as a numpy.array, a vector of shape 2 * 1.
        None if x, y, or theta is an empty numpy.array.
        None if x, y and theta do not have compatible shapes.
        None if x, y or theta is not of the expected type.
    Raises:
        This function should not raise any Exception.
    """
    ... Your code ...
```

## Examples

```
import numpy as np
x = np.array([[12.4956442],[ 21.5007972],[ 31.5527382],[ 48.9145838],[ 57.5088733]])
y = np.array([[37.4013816],[ 36.1473236],[ 45.7655287],[ 46.6793434],[ 59.5585554]])

# Example 0:
theta1 = np.array([[2],[ 0.7]])
gradient(x, y, theta1)
# Output:
array([[ -19.0342574],[  ],[ -586.66875564]])

# Example 1:
theta2 = np.array([[1],[ -0.4]])
gradient(x, y, theta2)
# Output:
array([[ -57.86823748],[ -2230.12297889]])
```

# Chapter III

## Exercise 02

### Interlude - Gradient Descent

So far we've calculated the *gradient*, which indicates whether and by how much we should increase or decrease  $\theta_0$  and  $\theta_1$  in order to reduce the loss. What we have to do next is update the theta parameters accordingly, step by step, until we reach the minimum. This iterative process, called **Gradient Descent**, will progressively improve the performance of your regression model on the training data.


The gradient descent **algorithm** can be summarized like this: for a certain number of cycles, at each step, both  $\theta$  parameters are slightly moved in the opposite directions than what the gradient indicates.

The algorithm can be expressed in pseudocode as the following:

```
repeat until convergence: {  
    compute  $\nabla(J)$   
     $\theta_0 := \theta_0 - \alpha \nabla(J)_0$   
     $\theta_1 := \theta_1 - \alpha \nabla(J)_1$   
}
```

Few remarks on this algorithm:

- If you directly subtracted the gradient from  $\theta$ , your steps would be too big and you would quickly overshoot past the minimum. That's why we use  $\alpha$  (alpha), called the *learning rate*. It's a small float number (usually between 0 and 1) that decreases the magnitude of each update.
- The pseudocode says "repeat until convergence", but in your implementation, you will not actually check for convergence at each iteration. You will instead set a number of cycles that is sufficient for your gradient descent to converge.
- When training a linear regression model on a new dataset, you will have to choose appropriate alpha and the number of cycles through trial and error.

	Exercise : 02
Gradient Descent	
Turn-in directory : <i>ex02/</i>	
Files to turn in : <b>fit.py</b>	
Forbidden functions : <b>any function that calculates derivatives for you</b>	

## Objective

Understand and manipulate the notion of gradient and gradient descent in machine learning. Be able to explain what it means to **fit** a Machine Learning model to a dataset. Implement a function that performs **Linear Gradient Descent** (LGD).

## Instructions

In this exercise, you will implement linear gradient descent to fit your model to the dataset.

The pseudocode for the algorithm is the following:

```

repeat until convergence: {
    compute  $\nabla(J)$ 
     $\theta_0 := \theta_0 - \alpha \nabla(J)_0$ 
     $\theta_1 := \theta_1 - \alpha \nabla(J)_1$ 
}

```

Where:

- $\alpha$  (alpha) is the *learning rate*. It's a small float number (usually between 0 and 1),
- For now, "repeat until convergence" will mean to simply repeat for `max_iter` (a number that you will choose wisely).

You are expected to write a function named `fit_` as per the instructions below:

```

def fit_(x, y, theta, alpha, max_iter):
    """
    Description:
        Fits the model to the training dataset contained in x and y.
    Args:
        x: has to be a numpy.array, a vector of shape m * 1: (number of training examples, 1).
        y: has to be a numpy.array, a vector of shape m * 1: (number of training examples, 1).
        theta: has to be a numpy.array, a vector of shape 2 * 1.
        alpha: has to be a float, the learning rate
        max_iter: has to be an int, the number of iterations done during the gradient descent
    Return:
        new_theta: numpy.array, a vector of shape 2 * 1.
        None if there is a matching shape problem.
        None if x, y, theta, alpha or max_iter is not of the expected type.
    Raises:
        This function should not raise any Exception.
    """
    ... your code ...

```

Hopefully, you have already written a function to calculate the linear gradient.

## Examples

```
import numpy as np
x = np.array([[12.4956442], [21.5007972], [31.5527382], [48.9145838], [57.5088733]])
y = np.array([[37.4013816], [36.1473236], [45.7655287], [46.6793434], [59.5585554]])
theta = np.array([[1], [ 1]])

# Example 0:
theta1 = fit_(x, y, theta, alpha=5e-6, max_iter=15000)
# Output:
array([[1.40709365],
       [1.1150909 ]])

# Example 1:
predict(x, theta1)
# Output:
array([[15.3408728 ],
       [25.38243697],
       [36.59126492],
       [55.95130097],
       [65.53471499]])
```




- You can create more training data by generating an  $x$  array with random values and computing the corresponding  $y$  vector as a linear expression of  $x$ . You can then fit a model on this artificial data and find out if it comes out with the same  $\theta$  coefficients that first you used.
- It is possible that  $\theta_0$  and  $\theta_1$  become "nan". In that case, it means you probably used a learning rate that is too large.



# Chapter IV

## Exercise 03

	Exercise : 03
Linear Regression with Class	
Turn-in directory : <i>ex03/</i>	
Files to turn in : <i>my_linear_regression.py</i>	
Forbidden functions : any functions from sklearn	

### Objective

Write a class that contains all methods necessary to perform linear regression.

### Instructions

In this exercise, you will not learn anything new but don't worry, it's for your own good. You are expected to write your own `MyLinearRegression` class which looks similar to the class available in Scikit-learn: `sklearn.linear_model.LinearRegression`

```
class MyLinearRegression():
    """
    Description:
        My personnal linear regression class to fit like a boss.
    """
    def __init__(self, thetas, alpha=0.001, max_iter=1000):
        self.alpha = alpha
        self.max_iter = max_iter
        self.thetas = thetas
        ... Your code ...

    ... other methods ...
```

You will add the following methods:

- `fit_(self, x, y),`
- `predict_(self, x),`
- `loss_elem_(self, y, y_hat),`
- `loss_(self, y, y_hat).`

You have already implemented these functions, you just need a few adjustments so that they all work well within your `MyLinearRegression` class.

## Examples

```
import numpy as np
from mylinearregression import MyLinearRegression as MyLR
x = np.array([[12.4956442], [21.5007972], [31.5527382], [48.9145838], [57.5088733]])
y = np.array([[37.4013816], [36.1473236], [45.7655287], [46.6793434], [59.5585554]])

lr1 = MyLR([2, 0.7])

# Example 0.0:
lr1.predict_(x)
# Output:
array([[10.74695094],
       [17.05055804],
       [24.08691674],
       [36.24020866],
       [42.25621131]])

# Example 0.1:
lr1.loss_elem_(lr1.predict(x),y)
# Output:
array([[710.45867381],
       [364.68645485],
       [469.96221651],
       [108.97553412],
       [299.37111101]])

# Example 0.2:
lr1.loss_(lr1.predict(x),y)
# Output:
195.34539903032385

# Example 1.0:
lr2 = MyLR([1, 1], 5e-8, 1500000)
lr2.fit_(x, y)
lr2.thetas
# Output:
array([[1.40709365],
       [1.1150909 ]])


# Example 1.1:
lr2.predict_(x)
# Output:
array([[15.3408728 ],
       [25.38243697],
       [36.59126492],
       [55.95130097],
       [65.53471499]])

# Example 1.2:
MyLR.loss_elem_(lr2.predict(x),y)
# Output:
array([[486.66604863],
       [115.88278416],
       [ 84.16711596],
       [ 85.96919719],
       [ 35.71448348]])

# Example 1.3:
MyLR.loss_(lr2.predict(x),y)
# Output:
80.83996294128525
```

# Chapter V

## Exercise 04

	Exercise : 04
Practicing Linear Regression	
Turn-in directory : <code>ex04/</code>	
Files to turn in : <code>linear_model.py</code> , <code>are_blue_pills_magics.csv</code>	
Forbidden functions : <code>sklearn</code>	

### Objective

Evaluate a linear regression model on a very small dataset, with a given hypothesis function  $h$ . Manipulate the loss function  $J$ , plot it, and briefly analyze the plot.

### Instructions

You can find in the `resources` folder a tiny dataset called `are_blue_pills_magics.csv` which gives you the driving performance of space pilots as a function of the quantity of the "blue pills" they took before the test. You have a description of the data in the file named `are_blue_pills_magics.txt`. As your hypothesis function  $h$ , you will take:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Where  $x$  is the variable, and  $\theta_0$  and  $\theta_1$  are the coefficients of the hypothesis. The hypothesis is a function of  $x$ .

**You are strongly encouraged to use the class you have implement in the previous exercise.**

Your code has to:

- Read the dataset from the csv file,
- perform a linear regression,

Then you will model the data and plot 2 different graphs:

- A graph with the data and the hypothesis you get for the spacecraft piloting score versus the quantity of "blue pills" (see figure V.1)

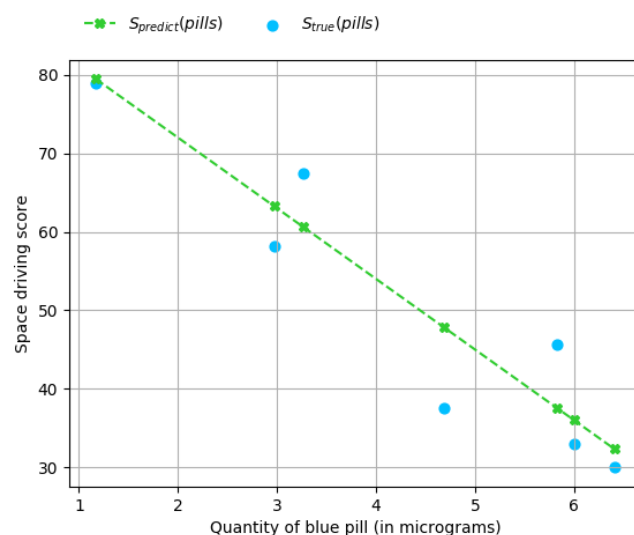


Figure V.1: Space driving score as a function of the quantity of blue pill (in micrograms). In blue the real values and in green the predicted values.

- The loss function  $J(\theta)$  in function of the  $\theta$  values (see figure V.2),

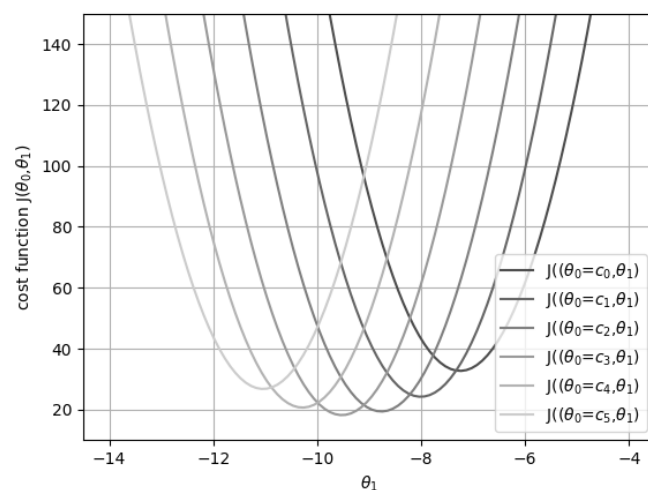


Figure V.2: Evolution of the loss function  $J$  as a function of  $\theta_1$  for different values of  $\theta_0$ .

- You will calculate the MSE of the hypothesis you chose (you know how to do it already).

## Examples

```
import pandas as pd
import numpy as np
from sklearn.metrics import mean_squared_error
from mylinearregression import MyLinearRegression as MyLR

data = pd.read_csv("are_blue_pills_magics.csv")
Xpill = np.array(data[Micrograms]).reshape(-1,1)
Yscore = np.array(data[Score]).reshape(-1,1)

# Example 1:
linear_model1 = MyLR(np.array([[89.0], [-8]]))
Y_model1 = linear_model1.predict_(Xpill)
print(linear_model1.mse_(Yscore, Y_model1))
print(mean_squared_error(Yscore, Y_model1))
# Output:
57.60304285714282
57.60304285714282

# Example 2:
linear_model2 = MyLR(np.array([[89.0], [-6]]))
Y_model2 = linear_model2.predict_(Xpill)
print(linear_model2.mse_(Yscore, Y_model2))
print(mean_squared_error(Yscore, Y_model2))
# Output:
232.16344285714285
232.16344285714285
```

Here, the use of scikit learn is to ensure that our code is performing as we expected. The use of scikit learn is forbidden in the code you will turn-in.



There is no method named `.mse_` in sklearn's `LinearRegression` class, but there is also a method named `.score`. The `.score` method corresponds to the  $R^2$  score. The metric MSE is available in the `sklearn.metrics` module.

# Chapter VI

## Exercise 05

### Interlude - Normalization


The values inside the  $x$  vector can vary quite a lot in magnitude, depending on the type of data you are working with. For example, if your dataset contains distances between planets in km, the numbers will be huge. On the other hand, if you are working with planet masses expressed as a fraction of the solar system's total mass, the numbers will be very small (between 0 and 1). Both cases may slow down convergence in Gradient Descent (or even sometimes prevent convergence at all). To avoid that kind of situation, normalization is a very effective way to proceed.

The idea behind this technique is straightforward: **scaling the data**.

With normalization, you can transform your  $x$  vector into a new  $x'$  vector whose values range between  $[-1, 1]$  more or less. Doing this allows you to see much more easily how a training example compares to the other ones:

- If an  $x'$  value is close to 1, you know it's among the largest in the dataset
- If an  $x'$  value is close to 0, you know it's close to the median
- If an  $x'$  value is close to  $-1$ , you know it's among the smallest

So with the upcoming normalization techniques, you'll be able to map your data to two different value ranges:  $[0, 1]$  or  $[-1, 1]$ . Your algorithm will like it and thank you for it.

	Exercise : 05
Normalization I: Z-score Standardization	
Turn-in directory : <i>ex05/</i>	
Files to turn in : <b>z-score.py</b>	
Forbidden functions : <b>None</b>	

## Objective

Introduction to standardization/normalization methods. You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - \frac{1}{m} \sum_{i=1}^m x^{(i)}}{\sqrt{\frac{1}{m} \sum_{i=1}^m (x^{(i)} - \frac{1}{m} \sum_{i=1}^m x^{(i)})^2}} \quad \text{for } i \text{ in } 1, \dots, m$$

Where:

- $x$  is a vector of dimension  $m$ ,
- $x^{(i)}$  is the  $i^{\text{th}}$  component of the  $x$  vector,
- $x'$  is the normalized version of the  $x$  vector.

The equation is much easier to understand in the following form:

$$x'^{(i)} = \frac{x^{(i)} - \mu}{\sigma} \quad \text{for } i \text{ in } 1, \dots, m$$

This should remind you something from **TinyStatistician...**

Nope?

Ok let's do a quick recap:

- $\mu$  is the mean of  $x$ ,
- $\sigma$  is the standard deviation of  $x$ .

## Instructions

In the `zscore.py` file, write the `zscore` function as per the instructions given below:

```
def zscore(x):
    """Computes the normalized version of a non-empty numpy.array using the z-score standardization.
    Args:
        x: has to be a numpy.array, a vector.
    Return:
        x' as a numpy.array.
        None if x is a non-empty numpy.array or not a numpy.array.
        None if x is not of the expected type.
    Raises:
        This function shouldn't raise any Exception.
    """
    ... Your code ...
```




## Examples

```
# Example 1:
X = numpy.array([[0],[ 15],[ -9],[ 7],[ 12],[ 3],[ -21]])
zscore(X)
# Output:
array([-0.08620324,  1.2068453 , -0.86203236,  0.51721942,  0.94823559,
        0.17240647, -1.89647119])

# Example 2:
Y = numpy.array([[2],[ 14],[ -13],[ 5],[ 12],[ 4],[ -19]])
zscore(Y)
# Output:
array([ 0.11267619,  1.16432067, -1.20187941,  0.37558731,  0.98904659,
        0.28795027, -1.72770165])
```

# Chapter VII

## Exercise 06

	Exercise : 06
Normalization II: Min-max Standardization	
Turn-in directory : <i>ex06/</i>	
Files to turn in : <b>minmax.py</b>	
Forbidden functions : <b>None</b>	

### Objective

Introduction to standardization/normalization methods. Implement another normalization method.

You must implement the following formula as a function:

$$x'^{(i)} = \frac{x^{(i)} - \min(x)}{\max(x) - \min(x)} \quad \text{for } i = 1, \dots, m$$

Where:

- $x$  is a vector of dimension  $m$ ,
- $x^{(i)}$  is the  $i^{\text{th}}$  component of vector  $x$ ,
- $\min(x)$  is the minimum value found among the components of vector  $x$ ,
- $\max(x)$  is the maximum value found among the components of vector  $x$ .

You will notice that this min-max standardization doesn't scale the values to the  $[-1, 1]$  range. What do you think the final range will be?

### Instructions

In the `minmax.py` file, create the `minmax` function as per the instructions given below:

```
def minmax(x):
    """Computes the normalized version of a non-empty numpy.array using the min-max standardization.
    Args:
        x: has to be an numpy.array, a vector.
    Return:
        x' as a numpy.array.
        None if x is a non-empty numpy.array or not a numpy.array.
        None if x is not of the expected type.
    Raises:
        This function shouldn't raise any Exception.
    """
    ... Your code ...
```

## Examples

```
# Example 1:
X = np.array([[0],[ 15],[ -9],[ 7],[ 12],[ 3],[ -21]])
minmax(X)
# Output:
array([0.58333333, 1., 0.33333333, 0.77777778, 0.91666667,
       0.66666667, 0.])

# Example 2:
Y = np.array([[2],[ 14],[ -13],[ 5],[ 12],[ 4],[ -19]])
minmax(Y)
# Output:
array([0.63636364, 1., 0.18181818, 0.72727273, 0.93939394,
       0.6969697 , 0.])
```

# Chapter VIII

## Conclusion - What you have learned

The exercises serie is finished, well done! Based on all the knowledges tackled today, you should be able to discuss and answer the following questions:

1. What is a hypothesis and what is its goal?
2. What is the loss function and what does it represent?
3. What is Linear Gradient Descent and what does it do? (hint: you have to talk about  $J$ , its gradient and the theta parameters...)
4. What happens if you choose a learning rate that is too large?
5. What happens if you choose a very small learning rate, but still a sufficient number of cycles?
6. Can you explain MSE and what it measures?

## Contact

You can contact 42AI association by email: [contact@42ai.fr](mailto:contact@42ai.fr)

You can join the association on [42AI slack](#) and/or posutale to [one of the association teams](#).

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