UNIT-III

(Production and Cost Analysis)

MODULE- 5: Returns to Scale and Linear Programming in Production

CONTENTS

5.0: Objectives:

The objective of this module is to discuss the concepts of returns to scale, linear programming in production and types of production functions used for estimation. After reading this module you should be able to understand the :

Different types of returns to scale

Linear programming and production analysis

Estimation of production function

5.01: Meaning of Returns to Scale:

The proportionate change in output as a result of given proportionate change in input is called returns to scale. This is simply the input elasticity of output. We can define returns to scale in terms of a ratio between percentage change in output to percentage change in input as shown below.

$$\text{Returns to scale} = \frac{\text{Proportionate change in output}}{\text{Proportionate change in input}}$$

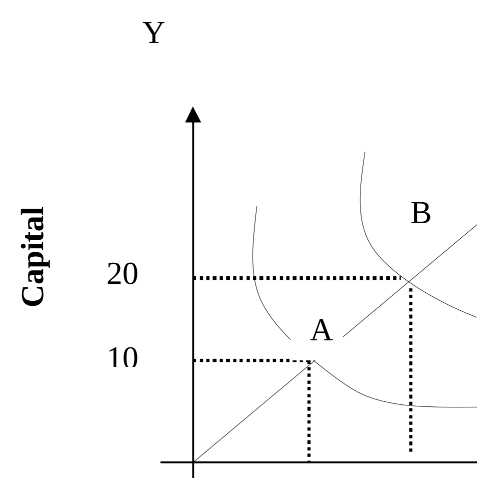
5.02: Types of Returns to scale:

There are three different types of returns to scale. They are (1) Increasing returns to scale (2) constant returns to scale (3) decreasing returns to scale. Now we shall discuss these three types of returns to scale.

Increasing Returns to Scale:

If the proportionate change in output is more than the proportionate change in input, it is called as increasing returns to scale. If the firm experiences increasing returns to scale, then one percent increase in input causes more than one percent increase in output. In the same way 100 percent increase in input leads to more than 100 percent increase in output. For example: A firm employed 5 units of labour &10 units of capital and produced 100 units of output. Suppose the same firm doubled the input use i.e it employed 10 units of labour & 20 units of capital,and could produce 300 units of output. The increase in input is 100 percent where as the increase in output is more than hundred percent. Under these conditions if the firm doubles the employment, output gets more than doubled.

GRAPH-1



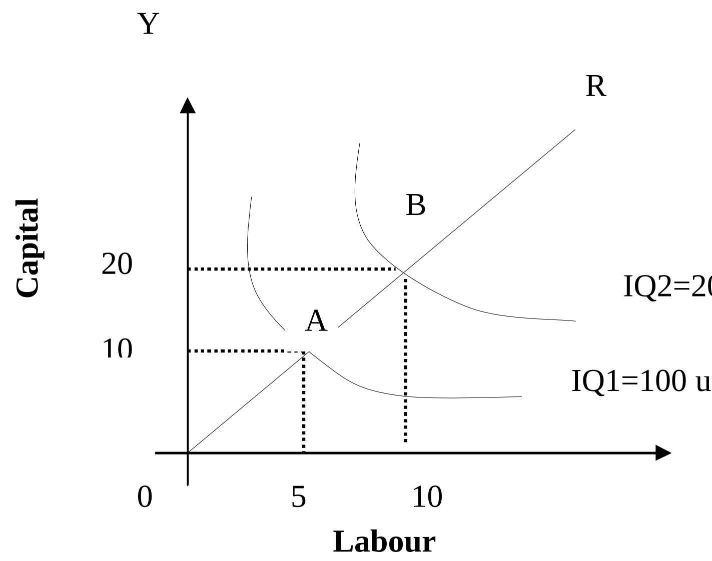
In the above graph OR is the scale line. It indicates that, every time, the increase in inputs is 100 percent. But increase in out is more than 100 percent i.e increased from 100 units to 300 units. On scale line OA = AB.

Constant Returns to Scale:

If the proportionate change in output is equal to the proportionate change in input, it is called as constant returns to scale. If the firm experiences constant returns to scale, then one percent increase in input causes exactly one percent increase in output. In the same way 100 percent increase in input leads to 100 percent increase in output. For example: A firm employed 5 units of labour &10 units of capital and produced 100 units of output. Suppose the same firm doubled the input use i.e it employed 10 units of labour & 20 units of capital, and could produce 200 units of output. The increase in input is 100 percent and also the increase in output is 100 percent.

Under these conditions if the firm doubles the employment, output also gets doubled.

GRAPH-2

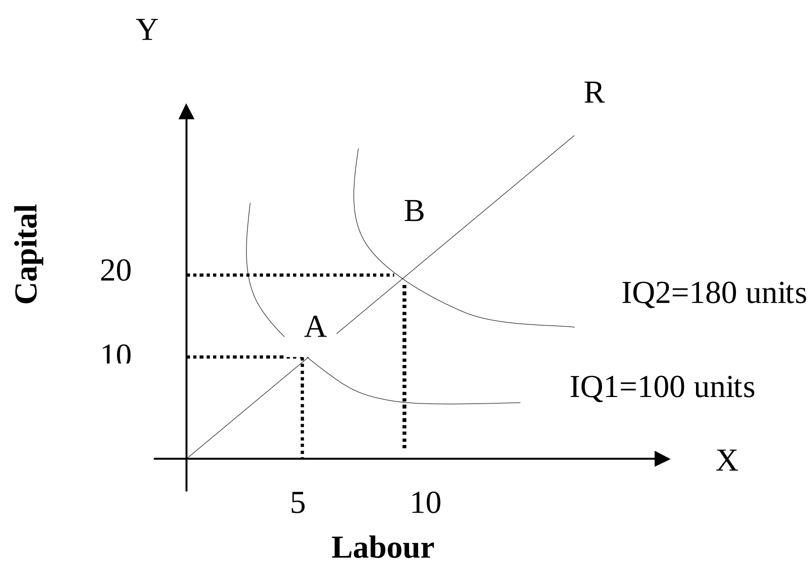


In the above graph OR is the scale line. It indicates that, every time, the increase in inputs is 100 percent. But increase in out is also 100 percent i.e increased from 100 units to 200 units. On scale line OA = AB.

Decreasing Returns to Scale:

If the proportionate change in output is less than the proportionate change in input, it is called as decreasing returns to scale. If the firm experiences decreasing returns to scale, then one percent increase in input causes less than one percent increase in output. In the same way 100 percent increase in input leads to less than 100 percent increase in output. For example: A firm employed 5 units of labour &10 units of capital and produced 100 units of output. Suppose the same firm doubled the input use i.e it employed 10 units of labour & 20 units of capital, and could produce 180 units of output. The increase in input is 100 percent and where as the increase in output is less than 100i.e 80 percent. Under these conditions if the firm doubles the employment, output gets less than doubled.

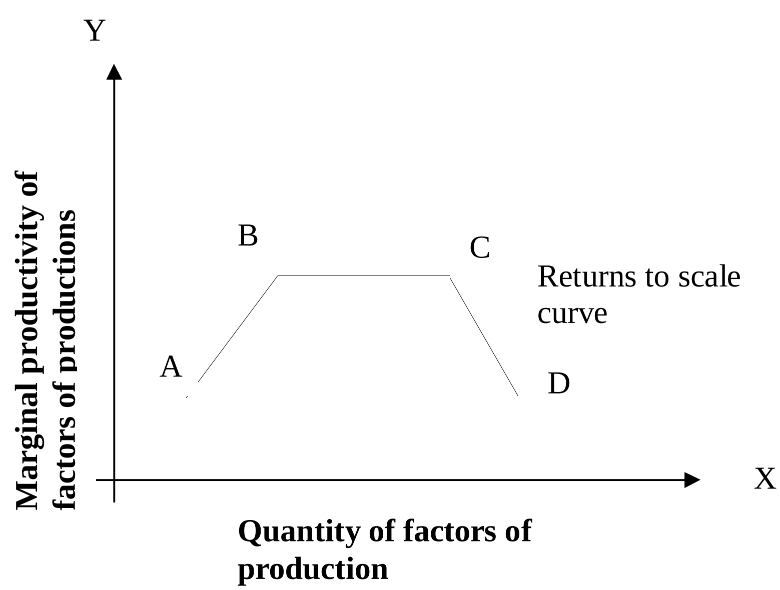
GRAPH-3



In the above graph OR is the scale line. It indicates that, every time, the increase in inputs is 100 percent. But increase in out is less than 100 percent i.e increased from 100 units to 180 units. On scale line OA = AB.

We can understand the concept of returns to scale with the help of marginal productivities. In case of increasing returns to scale, the marginal productivity of factors of production increases. In case of constant returns to scale the marginal productivity of factors of production remains constant. In case of decreasing returns to scale, the marginal productivity of factors of production decreases.

GRAPH-4



In the above graph, we measured the quantity of factors of production on horizontal axis and marginal productivity on vertical axis. From point A to B on returns to scale line represents increasing returns to scale. From point B to C represents constant returns to scale. From point C to D represents decreasing returns to scale.

ACTIVITY-1

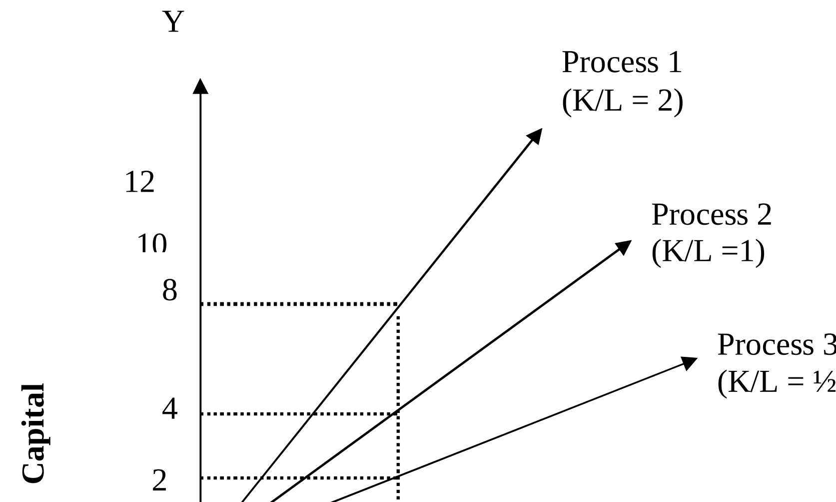
1. Define the concept of returns to scale.
2. Explain the nature of marginal products under different types of returns to scale.

5.03: Linear programming and production analysis:

Linear programming is a mathematical technique for solving constrained optimization and minimization problems, when there are many constraints and the objective function to be optimized, as well as constraints faced are linear i.e. represented by straight lines. The usefulness of linear programming arises because firms and other organizations face many constraints in achieving their goals of profit maximization or cost minimization or other objectives. For example: The optimization problem of a business firm is maximization of output subject to given cost constraint. In order to maximize output, the firm should produce at the point where isocost is tangent to its isoquant.

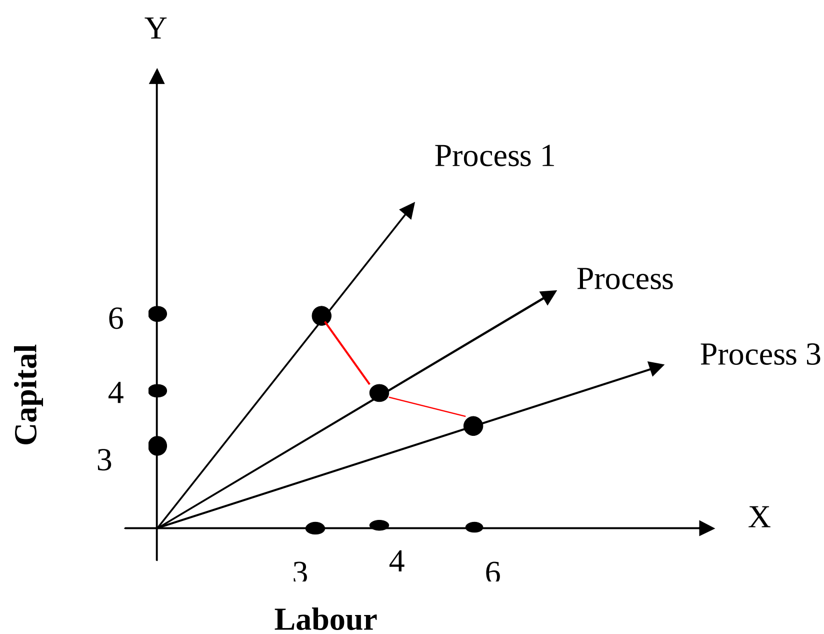
One of the basic assumptions of linear programming is that a particular commodity can be produced with only a limited number of input combinations. Each of these input combinations or ratios is called a production process or activity and can be represented by a straight line ray from the origin in input space. For example a particular commodity can be produced with three different processes, each utilizing a particular combination of labour (L) and capital(K).These are : process 1 with K/L =2. Process 2with K/L= 1, and process 3 with K/L = ½. Each of these processes is represented by the ray from the origin with slope equal to the particular K/L ratio used.

GRAPH- 5



In the above graph process 1 uses 2 units of capital for each unit of labour used, process 2 uses 1 unit of capital for each unit of labour and process 3 uses ½ unit of capital for each unit of labour. By joining points of equal output on the rays or processes, we define isoquant for the particular level of output of the commodity. The process of derivation of isoquants is shown in the following graph.

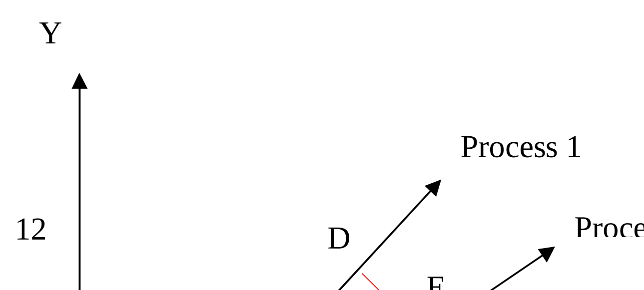
GRAPH-6



In the above graph, the isoquants are straight line segments and have kinks. Point A on process 1 shows that 100 units of output can be produced by using 3 units of labour and 6 units of capital. Point B on process 2 shows that 100 units of output can be produced with 4 units of labour and 4 units of capital. Point C on process 3 shows that 100 units of output can be produced with 6 units of labour and 3 units of capital. By joining, points A,B, C we get the isoquant for 100 units of output. Further, since we have constant returns to scale, the isoquant for twice as much output i.e 200 units is determined by using twice as much of each input with each process. This defines the isoquant for 200Q with kinks at points D(6L,12K), E(8L,8K), and F(12L,6K).

If the firm faced only one constraint, such as isocost line whose level is determined by volume of investment and the prices of factors of production. Assume the isocost line of firm is GH as shown below.

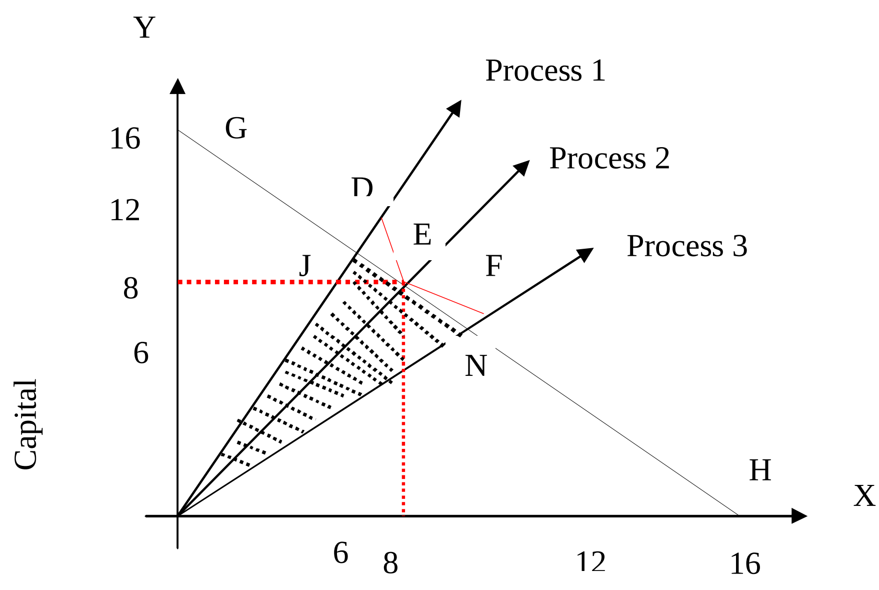
GRAPH-7



Feasible Region and Optimal Solution

With isocost line GH the feasible region is shaded triangle OJN and the optimal solution is at point E where the firm uses 8L and 8K and produces 200 units of output.

GRAPH-8



ACTIVITY-2

1. Identify feasible set of inputs with linear programming technique.

5.04: Types of production functions:

Several types of mathematical functions are commonly employed in the measurement of production function. But in applied research four types have had the widest use. These are:

Linear Function:

A linear production function would take the form Y= a + b X. Here Y is the total product. X is the input.

$$\text{Average product} = \frac{Y}{X} = \frac{a}{X} + \frac{bX}{X}$$

$$= \frac{a}{X} + b$$

Power function:

The production function most commonly used in empirical estimation is the power function of the form:

A power function expresses output Y, as a function of input X in the form:

$Y = aX^b$.

The exponents are the elasticities of output. Power function as it is can not be used for estimation. It is to be written in the log linear form as:

Log Y = log a + b log X. The best example of power function is the cobb-douglas production function of the form:

$Q = A\ K^a\ L^b$. Where Q is the output, K is the capital and L is the labour. **'a'** and **'b'** are the parameters to be estimated empirically. This production function is often referred to as cobb –douglas production function. In this production function the exponents **'a'** and **‘b’** represents output elasticity of capital and labour respectively. The sum exponents **‘a’+ ‘b’ = 1.** The sum of exponents represents returns to scale. If **‘a’+ ‘b’ = 1** it represents constant returns to scale. If **‘a’+ ‘b’** is more than **1,** it represents increasing returns to scale. If **‘a’+ ‘b’** is less than **1,** it represents decreasing returns to scale.

Quadratic production function:

$Y = a + b X – c X^2$. In this function Y is the total output, and X is the input. a, b, c are the parameters.

The minus sign in the last term denotes diminishing marginal returns. The equation allows for diminishing marginal product but not for increasing and decreasing marginal products. The elasticity of output is not constant at all points along the curve as in power function but declines with input magnitude.

Cubic production function

$Y = a + b X + c X^{2} - d X^3$.

It allows for increasing and decreasing marginal productivity. The elasticity of output varies at each point along the curve. Marginal productivity decreases at an increasing rate in the later stages.

ACTIVITY-3

1. Bring out the main features of power function.

5.05: Summary:

In this module we discussed at length the types of returns to scale, linear programming in production and types of production functions used in the estimation of input output relationship. The ratio between the proportionate changes in output to proportionate change input is called returns to scale. Thus it is the degree of responsiveness in output as a result of given percentage change in input. Linear programming technique is used to find out solution to optimization problem. In the production analysis, the optimization problem is maximization of output with minimum cost. Using linear programming technique we can find out optimum input combination to produce given level of maximum output. Business firms' and researchers can adopt different types of production function to estimate input output relationship.

5.06: References:

1. P.L.Mehta : *Managerial Economics- Analysis,Problems and Cases.*

2. Dominick Salvatore: *Managerial Economics in a global economy*

3. R.L Varshney and Maheswari : *Managerial Economics.*

4. H.Craig Petersen and Cris Lewis: *Managerial Economics*

5.07: Self Assessment Test:

1. Discuss different types of returns to scale.
2. Discuss the importance of linear programming in production analysis.
3. Spell out different types of production functions used to estimate input out relationship.