fourier transform

$t-\omega_0$

continous spectrum

$x(t) = x(\omega)$

$x(t) = x(j\omega)$

$x(t) = x(f)$

difference b/w fourier series & fourier transform is in F.S we represent frequency in discrete and in FT we represent in continous

here we represent

$t \to \omega_0$

n is for discrete values

in fourier series we represent $t \to n\omega_0$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n . e^{jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \underbrace{\frac{1}{T}\int_{-\infty}^{\infty} x(t) . e^{-jn\omega_0 t} dt}_{c_n} . e^{jn\omega_0 t}$$

$\frac{1}{T} = \Delta f$

$$= \sum_{n=-\infty}^{\infty} \Delta f \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt . e^{j\omega t}$$

continou dis

$t \to n\omega \to \omega$

f.s f.t

$$x(t) = \int_{-\infty}^{\infty} df . \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt . e^{j\omega t}$$

$\omega = 2\pi f$

$d\omega = 2\pi df$

$df = \frac{d\omega}{2\pi}$

$$= \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \underbrace{x(t) . e^{-j\omega t} dt}_{x(\omega)} e^{j\omega t}$$

f.T.

$t \to \omega$

$t \leftarrow \omega$ reverse of f.T it is Inverse fourier trans.

$x(t) \xrightarrow{F.T} x(\omega)$

$x(t) \xleftarrow{I.F.T} x(\omega)$

$$x(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} x(\omega) .. e^{j\omega t}$$

$$x(t) = \int_{-\infty}^{\infty} x(\omega) . e^{j\omega t} . \frac{d\omega}{2\pi}$$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega) . e^{j\omega t} d\omega}$$

$x(\omega) = \underbrace{\text{oper } f x(t)}$

so, by t we finding ω

F.T

t-ω

ω-t

by ω we getting t it is Inverse fourier transform.

$x(t) = \underbrace{\text{oper}(x(\omega))}$

I.F.S

$$\boxed{x(\omega) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt}$$

* formulas imp

by t we getting ω it's Fourier transform.

fourier transform properties.

1) linearity

$x(t) \xrightarrow{F.T} X(\omega)$

$X(\omega) \xrightarrow[I.F.T]{} x(t)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega).e^{j\omega t} d\omega \qquad IFT$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt \qquad FT$$

let two signals

$x_1(t) \rightarrow X_1(\omega)$.

$x_2(t) - X_2(\omega)$

$$\alpha x_1(t) + \beta x_2(t) \underset{IFT}{\overset{F.T}{\rightleftharpoons}} \alpha X_1(\omega) + \beta X_2(\omega)$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{+j\omega t} dt.$$

$$X^*(\omega) = \int_{-\infty}^{\infty} (\alpha x_1(t) + \beta x_2(t))e^{-j\omega t} dt.$$ satisfies linearity

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) .e^{j\omega t} d\omega.$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} (\alpha X_1(\omega) + \beta X_2(\omega)) e^{j\omega t} d\omega.$$

2) Time scaling

$x(t) - X(\omega)$

$x(at) - X^*(\omega)$

$\tau = at \qquad t = -\infty \quad \tau = -\infty \qquad t = \frac{\tau}{a}$

$\qquad t = \infty \quad \tau = \infty \qquad t = \tau . \frac{1}{a}$

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt \qquad dt = \frac{d\tau}{a}$$

$$X^*(\omega) = \int_{-\infty}^{\infty} x(at).e^{-j\omega \frac{\tau}{a}} . \frac{d\tau}{a}$$

$$\boxed{X^*(\omega) = \frac{1}{a}\int_{-\infty}^{\infty} x(\tau). e^{-j\frac{\omega}{a}\tau} . d\tau}$$

where $\left(X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt\right.$

$$\boxed{X^*(\omega) = \frac{1}{a} . X\left(\frac{\omega}{a}\right)}$$

3) Time shifting

$x(t) - x(\omega)$

$x(t+a) - x^*(\omega)$

$\tau = t+a$ $t = -\infty$ $\tau = -\infty$ ~~$d\tau$~~ $\tau = t+a$

$t = \infty$ $\tau = \infty$ $d\tau = dt + a$

$dt = d\tau - a.$

$x(\omega) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt$

$x^*(\omega) = \int_{-\infty}^{\infty} x(t+a) e^{-j\omega(\tau-a)} d\tau.$

$x^*(\omega) = \int_{-\infty}^{\infty} x(\tau) . e^{-j\omega\tau} . e^{j\omega a} . d\tau.$

$$\boxed{x^*(\omega) = x(\omega) . e^{j\omega a}.}$$

$x - a.$

$x^*(\omega) = x(\omega) . e^{-j\omega a}.$

4) Convolution in time

$x(t) - x(\omega).$

$x_1(t) * x_2(t) \quad - x^*(\omega)$

$x(\omega) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt.$

$x^*(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) . x_2(t-\tau) d\tau \right] e^{-j\omega t} dt.$

$x^*(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) . e^{-j\omega\tau} \, e^{-j\omega\tau} . x_2(t-\tau) d\tau \right] e^{-j\omega t} dt$

$x^*(\omega) = \int_{-\infty}^{\infty} x_1(\omega) . x_2(t-\tau) \frac{e^{-j\omega t}}{e^{-j\omega\tau}} dt$

$x^*(\omega) = x_1(\omega) \int_{-\infty}^{\infty} x_2(t-\tau) . e^{-j\omega(t-\tau)} dt$

$\beta = t - \tau \qquad d\beta = dt.$

$$\boxed{x^*(\omega) = x_1(\omega) . x_2(\omega)}$$

i/p. o/p

convolution - multipl

multipl ication - convolute.

5)

Parsuel's Energy theorem

$x(t) \longrightarrow X(\omega)$

$$E_x(t) = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{+j\omega t} \, d\omega$$

$$x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \, e^{-j\omega t} \, d\omega$$

it canbe written likell

$$E_x(t) = \int_{-\infty}^{\infty} x(t) \cdot x^*(t) \, dt$$

$$E_x(t) = \int_{-\infty}^{\infty} x(t) \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \cdot e^{-j\omega t} \, d\omega \cdot dt$$

$$E_x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot X^*(\omega) \, d\omega$$

$$\boxed{E_x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega}$$

differentiation

$x(t) - X(\omega)$

$\frac{d}{dt} x(t) - X(\omega)$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \, d\omega$$

$$\boxed{\frac{d^k}{dt^k} x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} \, d\omega \, (j\omega)^k}$$

Modulation : adding additional (more) signals to original signal to use communication is modulation. if we want original signal we remove so it is demodulation.

frequency shifting

$x(t) \longrightarrow X(\omega)$

$X(\omega - \omega_0)$

$X(\omega + \omega_0)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt$$

$$X(\omega-\omega_0) = \int_{-\infty}^{\infty} x(t) . e^{-j(\omega-\omega_0)t} dt$$

$$\boxed{X(\omega-\omega_0) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} . e^{j\omega_0 t} dt}$$

$$\boxed{x(\omega-\omega_0) = x(\omega) . e^{j\omega_0 t}}$$

$x(t) . e^{-j\omega_0 t}$ if $(\omega+\omega_0)$

Modulation

$x(t) \longrightarrow X(\omega)$

$x(t) . \sin\omega_0 t - X(\omega)$

$x(t) . \cos\omega_0 t - X(\omega)$

$x(t) . \cos\omega t \longrightarrow X(\omega)$

$$x(t) . \left[\frac{e^{j\omega t} + e^{-j\omega t}}{2} \right]$$

$$= \frac{1}{2}\left[x(t) . e^{j\omega t} + x(t) e^{-j\omega t} \right]$$

$$\boxed{x(t) . \cos t = \frac{1}{2}[X(\omega-\omega_0) + X(\omega+\omega_0)]}$$

$$x(t) . \sin\omega t = x(t) . \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right]$$

$$= \frac{1}{2j}\left[x(t) . e^{j\omega t} - x(t) e^{-j\omega t} \right]$$

$$= -\frac{j}{2}\left[X(\omega-\omega_0) - X(\omega+\omega_0) \right] = \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0) \right]$$

$$\boxed{x(t) . \sin\omega t = \frac{j}{2}[X(\omega+\omega_0) - X(\omega-\omega_0)]}$$

Duality property in F.T.

$x(t) \text{---} X(\omega)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) . e^{j\omega t} d\omega$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) . e^{-j\omega t} dt$$

(1) replace $t \to -t$.

$$x(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) . e^{-j\omega t} d\omega .$$

$t \to \omega$ and $\omega \to t$

$$x(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$\boxed{2\pi \cdot x'(-\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt.}$$

Basic signal in Fourier transform

1) Impulse signal

$\delta(t) = 1 \quad t = 0$

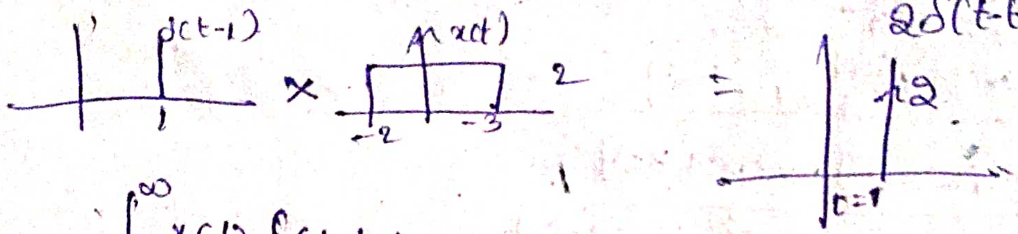
$\delta(t) = 0 \quad t \neq 0$

$\int_{-\infty}^{\infty} \delta(t) \cdot dt = 1$

$$\delta(\omega) = \int_{-\infty}^{\infty} \delta(t) \cdot e^{-j\omega t} dt.$$

$$\boxed{\delta(\omega) = 1}$$

$x(t) \cdot \delta(t-t_0) = x(t_0)\delta(t-t_0)$



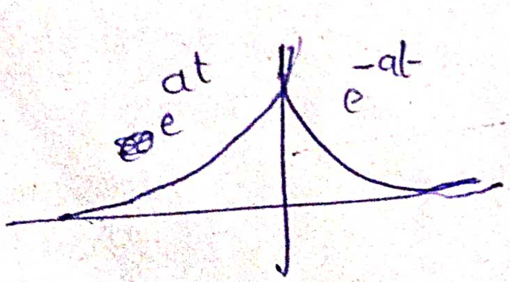
$$\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0)\, dt = x(t_0)$$

$$\int_{-\infty}^{\infty} 2 \cdot \delta(t-t_0) dt.$$

$$x(t_0)\int_{-\infty}^{\infty} \delta(t-t_0) dt = x(t_0)$$

$$\boxed{x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)}$$

Exponential



$x(t) = e^{-at}$ $\quad x(\omega)$

$x(t) = e^{at}$

$x(t) = e^{-at} u(t)$

$$x(\omega) = \int_{-\infty}^{\infty} e^{-at} \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-t(a+j\omega)} \cdot dt$$

$$= \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)}\right]_{-\infty}^{\infty}$$

$$= -\frac{1}{a+j\omega}\left[e^{-\infty(a+j\omega)} - e^{\infty(a+j\omega)}\right]$$

$t=0$

$\sigma = -at$

$t=0$

$$= -\frac{1}{a+j\omega}\left[e^{-\infty} - e^{\infty}\right]$$

$$= -\frac{1}{a+j\omega}\left[\right.$$

$$= \frac{-1}{(a+j\omega)}\left[e^{-t(a+j\omega)}\right]_0^{\infty}$$

$$= -\frac{1}{a+j\omega}\left[e^{-\infty(a+j\omega)} - e^{0}\right]$$

$$= -\frac{1}{a+j\omega}(-1) = \frac{1}{a+j\omega}.$$

$e^{\infty a} \cdot e^{-tj\omega}$

$e^{\infty} \cdot e^{-\infty} \to 0 = 0$

$$x(t) = e^{at}.$$

$$x(\omega) = \int_{-\infty}^{\infty} e^{at} \cdot e^{-j\omega t}\, dt$$

$$= \left[e^{t(a-j\omega)}\, dt\right]_0^{\infty}$$

$$= \left[\frac{e^{t(a-j\omega)}}{a-j\omega}\right]_0^{\infty}$$

$$= \frac{1}{a-j\omega}[0-1] = \frac{-1}{a-j\omega}.$$

$e^{-a|t|}$ $t>0$, $t<0$

(need to write one class)

Laplace transform

$x(t) \overset{L.T}{\rightleftharpoons} x(S)$ (or) $L\{x(t)\}$.

Formula

$$\boxed{x(S) = \int_{-\infty}^{\infty} x(t) \cdot e^{-St}\, dt}$$

where $S = \sigma + j\omega$.

σ = Damping factor

ω = Angular freq.

Relation b/w Laplace transform & F.T

$x(t) \overset{F.T}{\rightleftharpoons} x(j\omega)$

$x(t) \overset{L.T}{\rightleftharpoons} x(S)$ $\quad S = \sigma + j\omega$

$$LT \Rightarrow x(S) = \int_{-\infty}^{\infty} x(t) e^{-St}\, dt = \int_{-\infty}^{\infty} x(t) e^{-(\sigma+j\omega)t}\, dt = \int_{-\infty}^{\infty} x(t) e^{-\sigma t} \cdot e^{-j\omega t}\, dt.$$

$$F.T\{x(t) \cdot e^{-\sigma t}\} = x(S)$$

If $\sigma = 0$

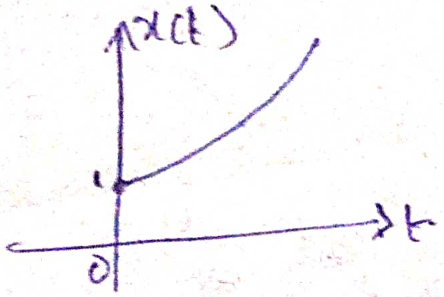
$$FT\{x(t)\} = L.T\{x(t)\} = x(S) = x(j\omega)$$

where range of σ will be:
Region of convergence (ROC)

$$x(t) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$$

$$x(s) = L \cdot T = \int_{-\infty}^{\infty} x(s) \cdot e^{-st}\, dt$$

① Find ROC of the signal $x(t) = e^{2t} u(t)$



laplace transform.

$$\int_0^{\infty} e^{2t} \cdot e^{-\sigma t}\, dt = \int_0^{\infty} e^{(2-\sigma)t}\, dt.$$

$$= \left[\frac{e^{(2-\sigma)t}}{2-\sigma} \right]_0^{\infty} = \left[\frac{e^{(2-\sigma)\infty} - e^{(2-\sigma)0}}{2-\sigma} \right]$$

$2-\sigma$ value should

$2 - \sigma < 0$

$-(2-\sigma) > 0$

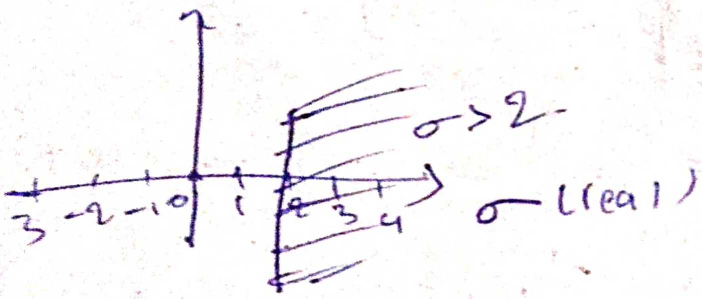
$-2 + \sigma > 0$

$\boxed{\sigma > 2}$

$$= \left[\frac{0-1}{2-\sigma} \right] = \frac{1}{\sigma - 2}.$$

finite value

ROC
$j\omega$ (img) splane.



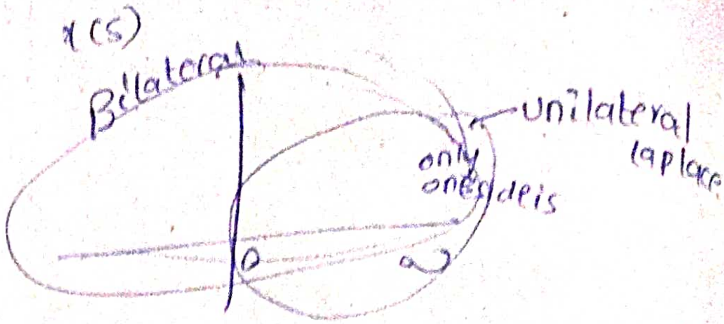
$\frac{1}{s+2}$ ROC not include any poles

$-2+2=\infty$

laplace transform

$x(t) \rightarrow x(s)$

$s = \sigma + j\omega$



properties

linearity.

timeshifting.

time scaling

convolution.

differentiation in time.

initial value theorem

final " "

gain = o/p / i/p

initial value theorem

final value theorem

$x(\infty) - x(0)$

$x(t) \rightarrow x(s)$

$x(0) \rightarrow$ L.T.

differentiation

$$L.T\left\{\frac{d}{dt}x(t)\right\} = \int_0^{\infty} \frac{d}{dt}x(t).e^{-st}\,dt.$$

applying limit $s \to \infty$

$$\lim_{s\to\infty} sx(s) - x(0) = \lim_{s\to\infty}\int_0^{\infty}\frac{d}{dt}x(t).e^{-st}\,dt$$

∞ then $\rightarrow 0$

$$\lim_{s\to\infty} sx(s) - x(0) = 0$$

$$\boxed{x(0) = \lim_{s\to\infty} sx(s)}$$

Final value theorem

$x(t) \rightarrow x(s)$

$x(\infty) \rightarrow$ L.T.

$$L.T\left\{\frac{d}{dt}x(t)\right\} = \int_0^{\infty}\frac{d}{dt}x(t).e^{-st}\,dt.$$

applylimit $s \to 0$.

$$\lim_{s\to 0} sx(s) - x(0) = \lim_{s\to\infty}\int_0^{\infty}\frac{d}{dt}x(t).e^{-st}\,dt.$$

$$\lim_{s\to 0} sx(s) - x(0) = \int_0^{\infty}\frac{d}{dt}x(t)\,dt$$

$$\lim_{s\to 0} sx(s) - x(0) = x(t)\Big]_0^{\infty}$$

$$\lim_{s\to 0} sx(s) - \cancel{x(0)} = x(\infty) - \cancel{x(0)}$$

$$\boxed{x(\infty) = \lim_{s\to 0} sx(s)}$$

LT of $\boxed{e^{-at}u(t) = \frac{1}{s+a} \quad \text{where } \sigma > -a}$

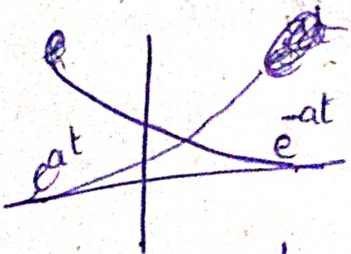
$\boxed{e^{at}u(t) = \frac{1}{s-a} \quad \text{where } \sigma > a}$

$\boxed{e^{-at}u(-t) = \frac{-1}{s+a} \quad \sigma < -a}$

$\boxed{e^{at}u(-t) = -\frac{1}{s-a} \quad \sigma < a}$

$x(t) = -e^{-at}u(-t) = \frac{1}{s+a}$

$\sigma < -a.$



$x(t) = e^{-at}u(-t)$

$x(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$

$x(s) = \int_{-\infty}^{0} e^{-at} \cdot e^{-st} dt.$

$x(s) = \int_{-\infty}^{0} e^{-t(a+s)} dt$

$= \left. \frac{e^{-t(a+s)}}{-(a+s)} \right]_{-\infty}^{0}$

$x(s) = \frac{1}{-(a+s)} \left[e^{-0(a+s)} - e^{\infty(a+s)} \right]$

$e^{-\infty(a+\sigma+j\omega)}$

$= \frac{1}{-(a+s)} [1 - 0]$

$\sigma + a < 0$

$\sigma < -a$

$x(s) = -\frac{1}{a+s}.$

Find L.T of

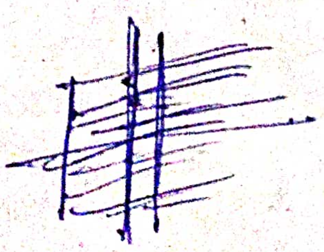
$x(t) = e^{-3t}u(t) + e^{-2t}u(t)$ find L.T ROC.

$x(s) = \frac{1}{s+3} + \frac{1}{s+2}$

ROC.

$\sigma > -3$

$\sigma > -2$



$\boxed{\sigma > -2.}$

- Impulse
- step
- Exponential

Z-transform

is to repent discrete signal in frequency.

$n \to f$

$$\boxed{Z.T\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n).e^{-j\omega n}}$$

$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt$

- two sided. $L.T\{x(t)\} = \int_{-\infty}^{\infty} x(t).e^{-st} d$

Z trans

$$\boxed{Z.T\{x(n)\} = \sum_{n=0}^{\infty} x(n).e^{-j\omega n}}$$ - one sided Z transform.

$z = e^{j\omega}$

$$\boxed{Z.T\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n}}$$

$x(n) = \{1, 2, 3, 4, 5\}$
$\quad\ \ \uparrow$
$\quad\ \ 0 \ \ 1 \ \ 2 \ \ 3 \ \ 4$

one sided.. because +ve all.

$X(z) = \sum_{n=0}^{4} x(n).z^{-n}$

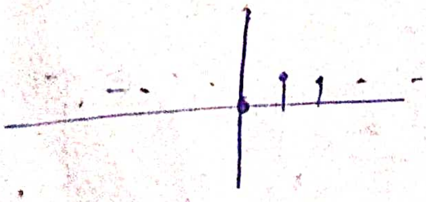
$= x(0).z^{-0} + x(1).z^{-1} + x(2).z^{-2} + x(3)z^{-3} + x(4)z^{-4}$

$= 1 + 2.\frac{1}{z} + 3.\frac{1}{z^2} + 4.\frac{1}{z^3} + 5.\frac{1}{z^4}$

$X(z) = 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} + \frac{5}{z^4}$

what should z for ROC.

except $z=0$ all values give defined value

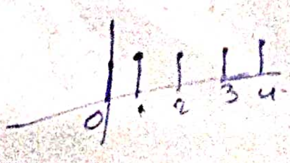


$1 + \frac{2}{0} + \frac{3}{0} + \to \infty$

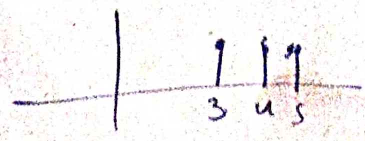
$1 + \frac{2}{-1} + \frac{3}{(-1)}$. defined

$x(n) = u(n) - u(n-3)$

$u(n)$



$u(n-3)$



$u(n) - u(n-3)$

$x(n) = \{1, 1, 1\}$
$\qquad\quad\ \uparrow$

$$X(z) = \sum_{n=0}^{2} x(n) \cdot z^{-n}$$

$$= x(0) \cdot \frac{1}{z^0} + x(1) \cdot \frac{1}{z^1} + x(2) \cdot z^{-2}$$

$$= 1 \cdot 1 + 1 \cdot \frac{1}{z} + 1 \cdot \frac{1}{z^2} = 1 + \frac{1}{z} + \frac{1}{z^2}$$

except $z=0$ we get ROC.

Properties

Linearity

$$x(n) \rightarrow X(z)$$

$$\alpha x_1(n) + \beta x_2(n) \rightarrow X^*(z)$$

$$X(n) = \sum_{n=-\infty}^{\infty} [\alpha x_1(n) + \beta x_2(n)] \cdot z^{-n}$$

$$X^*(z) = \alpha X_1(z) + \beta X_2(z)$$

Timeshifting

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(n) \rightarrow X(z)$$

$$x(n-k) \rightarrow ?$$

$m = n-k$

$m = -\infty \quad n = -\infty$

$m - \infty \quad n = +\infty$

$$X^*(z) = \sum_{m=-\infty}^{\infty} x(n-k) z^{-(m+k)} \quad n = m+k$$

$$= \sum_{m=-\infty}^{\infty} x(m) \cdot z^{-m} \cdot z^{-k}$$

$$\boxed{X^*(z) = X(z) \cdot z^{-k}}$$

$m = n+k$

$n = m-k$

if $n+k$

$$\boxed{X^*(z) = X(z) \cdot z^{k}}$$

Convolution

$$x(n) \rightarrow X(z)$$

$$x_1(n) * x_2(n) \rightarrow ?$$

$t - \tau$

$n-k$

$$\sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k)$$

$$\boxed{X(z) = \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k) \cdot z^{-n}}$$

$$X(z) = \sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n} \cdot \sum_{k=-\infty}^{\infty} x_1(k)$$

$$= X_2(z) z^{-k} \sum_{k=\infty}^{\infty} x_1(k)$$

$$= X_2(z) \sum_{k=-\infty}^{\infty} x_1(k) \cdot z^{-k}$$

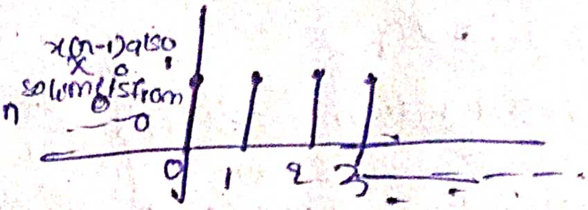
$$\boxed{X(z) = X_2(z)\cdot X_1(k)}$$

1) $x(n) = x(n-1)\,u(n)$ z-transform representation?

$n \to z$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\cdot z^{-n}$$

$U(n)$



$$X(z) = \sum_{n=0}^{\infty} x(n-1)\cdot u(n)\, z^{-n}$$

$$\sum_{m=-\infty}^{\infty} x(n-1)\,u(n)\; z^{-(m+1)}$$

$m = n-1$ $n=0$ $m=-1$

$n=\infty$ $m=\infty$

$$= \sum_{m=-1}^{\infty} x(m)\,\underbrace{u(n)}_{1}\, z^{-m}\cdot z^{-1}$$

$$= \underbrace{\sum_{m=-1}^{\infty} x(m)\, z^{-m}}\cdot z^{-1}$$

$$= \left[x(-1)\cdot z^{-(-1)} + \sum_{m=0}^{\infty} x(m)\, z^{-m}\right)\cdot z^{-1}$$

$$= x(-1) + \underbrace{\sum_{m=0}^{\infty} x(m)\, z^{-m}}_{X(z)}\cdot z^{-1}$$

$$= x(-1) + \sum_{m=0} X(z)\cdot z^{-n}$$

2) $x(n) = x(n+1)\,u(n)$

$$X(z) = \sum_{n=0}^{\infty} x(n+1)\,u(n)\cdot z^{-n}$$

$m = n+1$ $n=0$ $m=1$

$n = m-1$ $n=\infty$

$n=\infty$

$$= \sum_{m=1}^{\infty} x(m)\,\underbrace{u(n)}_{1}\cdot z^{-(m-1)}$$

$$= \sum_{m=1}^{\infty} x(m)\cdot z^{-m}\cdot z$$

$$= \left[\sum_{m=0}^{\infty} x(m)\cdot z^{-m} - x(0)\right] z$$

$$= X(z)\cdot z - x(0)\, z$$

3) $x(n) = x(n+2)\,u(n)$.

$$X(z) = \sum_{n=-\infty}^{\infty} x(z) \cdot z^{-n}.$$

$$X(z) = \sum_{n=0}^{\infty} x(n+2) \cdot u(n) \cdot z^{-n}.$$

$m = n+2$ $n=0$, $m=2$. $n=\infty$, $m=\infty$

$$X(z) = \sum_{m=2}^{\infty} x(m) \cdot z^{-(m-2)}$$

$$X(z) = \sum_{m=2}^{\infty} x(m) \cdot z^{-m} \cdot z^{2}.$$

$$X(z) = \sum_{m=2}^{\infty} x(m) \cdot z^{-m} - x(0) z^{0} - x(1) \cdot z^{-1}] \, z^{2}$$

$$X(z) = X(z) z^{2} - x(0) z^{2} - x(1) z.$$

Initial value theorem

$x(n) \rightarrow X(z)$

$x(0) \rightarrow ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

if unilateral +ve side

$$X(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

$$X(z) = x(0) \cdot z^{-0} + x(1) \cdot z^{-1} + x(2) \cdot z^{-2} + \cdots$$

apply limit $z \to \infty$ bothsides.

$$\lim_{z \to \infty} X(z) = \lim_{z \to 0} x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^{2}} + \cdots$$

$$\boxed{\lim_{z \to \infty} X(z) = x(0)}$$

Final value

$x(n) \rightarrow X(z)$

$x(\infty) - ?$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}.$$

$x(n+1) - x(n)$

$$z[x(n+1) - x(n)] = \sum_{n=0}^{\infty} [x(n+1) - x(n)] \cdot z^{-n}.$$

$$zX(z) - zx(0) - z[x(n)] = \sum_{n=0}^{\infty} [x(n+1) - x(n)] \cdot z^{-n}.$$

$$z \cdot x(z) - z\,x(0) - x(z) = \sum_{n=0}^{\infty} [x(n+1) - x(n)] \cdot z^{-n}$$

$$x(z)\,[z-1] - z\,x(0) = \sum_{n=0}^{\infty} [x(n+1) - x(n)] \cdot z^{-n}$$

apply limit $z \to 1$ on Bothsides

$$\lim_{z\to 1} x(z)[z-1] - z\,x(0) = \lim_{z\to 1} \sum_{n=0}^{\infty} [x(n+1) - x(n)] \cdot z^{-n}$$

$$\lim_{z\to 1} x(z)(z-1) - x(0) = \sum_{n=0}^{\infty} [x(n+1) - x(n)]$$

$$= x(0+1) - x(0) + x(1+1) - x(1) + x(2) + x(3+1) - x(3) + \ldots$$

$$+ x(\infty+1) - x(\infty)$$

$$= x(\infty+1) + x(0)$$

$$\boxed{\lim_{z\to 1} x(z) \cdot (z-1) = x(\infty)}$$

Basic signals z-transform.

$$\delta(z) = 1$$

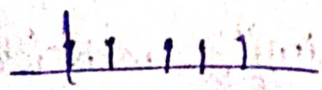
$$\sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n}$$

$$1 \cdot z^{-0} = 1 \cdot 1 = 1.$$

unit step signal.

$u(n) = 1 \quad n > 0$

$u(n) = 0 \quad n < 0$



$$x(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n)(z^{-1})^{n}$$

$$= x(0) \cdot (z^{-1})^{0} + x(1) \cdot (z^{-1})^{1} + x(2) \cdot (z^{-1})^{2} + \ldots$$

$$= 1 + 1 \cdot z^{-1} + 1 (z^{-1})^{2} + \ldots$$

$x = z^{-1}$

$$= 1 + x + x^2 + \ldots = \frac{1}{1-x}$$

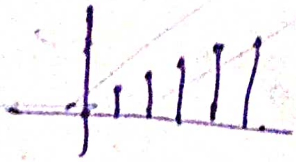
$x = z^{-1}$

$$\frac{1}{1-z^{-1}} = \frac{1}{1-\frac{1}{z}} = \frac{z}{z-1} = x(z)$$

ramp signal

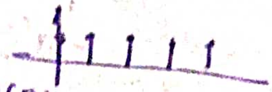
$r(n) = n\,u(n) \quad n \geq 0$

$r(n) = 0 \quad n < 0$



$x(z) = \sum_{n=0}^{\infty} n\,u(n) . z^{-n}$

$= \sum_{n=0}^{\infty} n . (z^{-1})^n$



$u(n)$

$= 0.(z^{-1})^0 + 1.(z^{-1})^1 + 2(z^{-1})^2 + \dots$ $n x(n)$

$n=0$; $1x(1) = 1.1$; $2x(2)$ $2.1 = 2$

$= 0 + z^{-1} + 2(z^{-1})^2 + \dots$

$= z^{-1}[1 + 2z^{-1} + 3(z^{-1})^2 + \dots]$

$= z^{-1}[\quad \dots \quad]$

$x = z^{-1}$

$1 + 2x + 3x^2 + \dots = \frac{1}{(1-x)^2}$

$= z^{-1}\left(\frac{1}{(1-z^{-1})^2}\right) = z^{-1}\left(\frac{1}{(1-\frac{1}{z})^2}\right) = z^{-1} . \frac{z^2}{(z-1)^2}$

$x(z) = \frac{z}{(z-1)^2}$

Q) $x(n) = a^n u(n)$ in z-transform?

$X(z) = \sum_{n=-\infty}^{\infty} x(n) . z^{-n}$

$= \sum_{n=0}^{\infty} a^n u(n) . z^{-n}$

$= \sum_{n=0}^{\infty} a^n . z^{-n} = \sum_{n=0}^{\infty} (a z^{-1})^n$

$\cancel{x(z) = a^0 . z^{-0} + a . z^{-1} +}$

$= (az^{-1})^0 + (a.z^{-1})^1 + (a.z^{-1})^2 + \dots$

$= 1 + a.z^{-1} + a^2 . z^{-2} + \dots$

$= 1 + (az^{-1}) + (a.z^{-1})^2 + \dots$

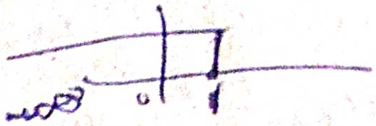
$1 + x + x^2 + \dots = \frac{1}{1-x}$

$x = az^{-1}$

$x(z) = \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$

$x(n) = -a^n u(-n-1)$ give its z-transform?

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$ $\quad u(-n-1) \quad -u(n+1) \quad n+1$



$= \sum_{n=-1} -a^n$

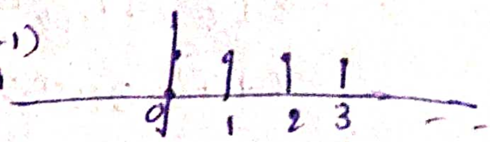
$= \sum_{n=-\infty}^{-1} -a^n . z^{-n}$ $\quad u(-n-1)$

$+1$

$n = -n$

-1

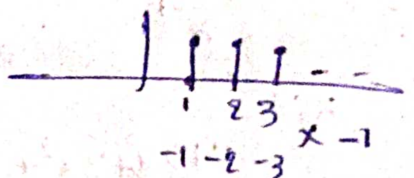
$= -\sum_{n=1}^{\infty} a^{-n} . z^n$ $\quad n=-1, \; n=-(-1) = 1$



$= -\sum_{n=1}^{\infty} (a^{-1}.z)^n$ $\quad n=\infty, \; n=-(-\infty) = \infty$

$1 \; 2 \; 3 \; 4 \; +1$

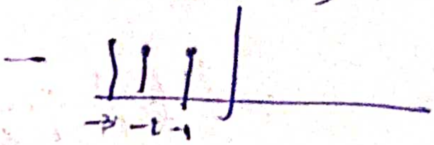
$= -[(a^{-1}.z)^1 + (a^{-1}.z)^2 + (a^{-1}.z)^3 + ...]$



$= -(a^{-1}z)[1+(a^{-1}z)+(a^{-1}z)^2+ ...]$

$= -(a^{-1}.z)(1+x+x^2+ ...)$

$= -(a^{-1}.z)\left(\frac{1}{1-(a^{-1}z)}\right)$



$= -(a^{-1}.z)\left(\frac{1}{1-\frac{z}{a}}\right)$

$x(z) = -(a^{-1}.z)\left(\frac{a}{a-z}\right) = \frac{-1}{\cancel{a}}.z.\frac{\cancel{a}}{a-z} = \frac{-z}{a-z} = \frac{z}{z-a}$

Multiplication with n

$x(n) \longrightarrow x(z)$

$n x(n) \longrightarrow ?$

$x(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n}$

diff w.r.to 'z'

$\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} x(n). \frac{d}{dz}.z^{-n}$

$\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} x(n).(-n).z^{-n}$

$-\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} n x(n).z^{-n}$

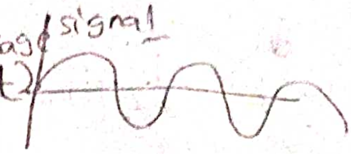
$$\boxed{-z\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} n x(n).z^{-n}}$$

$x(t) = \sum_{n=-\infty}^{\infty}$

Sampling and reconstruction

1) sampling theorem
2) feed Back.
3) filters.
4) control systems

message signal $(m(t))$



Sampler → (taking sample from original signal)

sampling signal

$s(t) = m(t)$ ⇒ samples are nothing but parts from original signal

⇒ if more no. of samples we can construct original signal easily.

$s(t) = m(t) * \delta(t)$

$S(t) = m(t) * \delta(t)$

using F.S

$\sum_{n=-\infty}^{\infty} \delta(t - nt_0)$

$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{b=1}^{\infty} b_n \sin n\omega_0 t$

$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt = \frac{1}{TS}\int_{-TS/2}^{TS/2} \delta(t).dt$

$a_0 = \frac{1}{TS}$

$a_n = \frac{2}{T}\int_{-T}^{T} x(t)\cos n\omega_0 t\,dt = \frac{2}{TS}\int_{-TS/2}^{TS/2} \delta(t).\cos n\omega_0 t\,dt$ $\quad t=0$

$a_n = \frac{2}{TS}$

$b_n = \frac{2}{TS}\int_{-TS/2}^{TS/2} \delta(t).\sin n\omega_0 t\,dt = 0$ $\quad t=0$

$\delta(t) = \frac{1}{TS} + \sum_{n=1}^{\infty} \frac{2}{TS}\cos n\omega_s t + 0$

$\delta(t) = \frac{1}{TS} + \frac{2}{TS}[\cos\omega_s t + \cos 2\omega_s t + \ldots]$

$= \frac{1}{TS}[1 + 2\cos\omega_s t + 2\cos 2\omega_s t + \ldots]$

$= \frac{1}{TS}\left[1 + 2\left[\frac{e^{\omega_s t} + e^{-\omega_s t}}{2}\right] + 2\left[\frac{e^{2\omega_s t} + e^{-2\omega_s t}}{2}\right] + \ldots\right]$

$S(t) = m(t) \times \delta(t)$

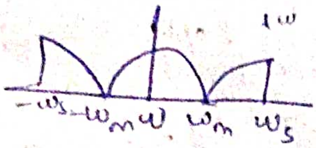
$= m(t) \cdot \left[\frac{1}{TS}[1 + e^{j\omega_s t} + e^{-j\omega_s t} + e^{2j\omega_s t} + e^{-2j\omega_s t} + \ldots]\right]$

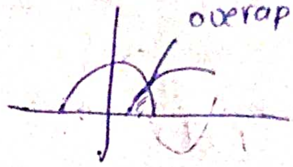
$s(t) = \frac{1}{TS}\left[m(t) + m(t).e^{j\omega_s t} + m(t).e^{-j\omega_s t} + m(t)e^{-j2\omega_s t} + \right.$

$m(t) - m(\omega)$

$m(\omega+\omega_s)$
$-m(m+\omega_s)$

$S(\omega) = \frac{1}{TS}\left[m(\omega) + m(\omega-\omega_s) + m(\omega+\omega_s) + m(\omega-2\omega_s) + m(\omega+2\omega_s) + \right]$





overapi's called Aliasing effect or undersampling

i) $\omega_m = \omega_s - \omega_m$.

$\omega_s = 2\omega_m$.

$\cancel{2\pi} f_s = \cancel{2\pi} 2f_m$.

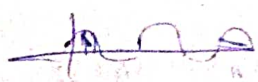
$\boxed{f_s = 2f_m}$

ii) $\omega_m < \omega_s - \omega_m$

$f_s < 2f_m$.

iii) $\omega_m > \omega_s - \omega_m$

$f_s > 2f_m$.



$U(t)$

$S(t-1)$ $\int S(t)$

a ~~signal can be~~

ω_1

Sampling theorem :-

a CT signal can be reconstructed or reproduced. if its sampling frequency is greater than or equal to ~~Sampling~~ twice of its message signal frequency.

$\boxed{f_s \geq 2f_m}$

feedback :- (giving i/p as o/p)

two types of Feedback :-

i) +ve feedback.

ii) -ve feedback.

amplifier

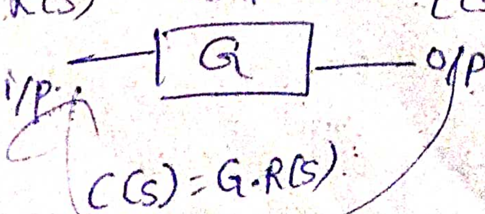
i/p — o/p.

Gain = $\frac{O/P}{i/P}$.

if i connect o/p to i/p

$\boxed{O/P = G.i/P}$

$R(S)$ amplifier. $C(S)$.



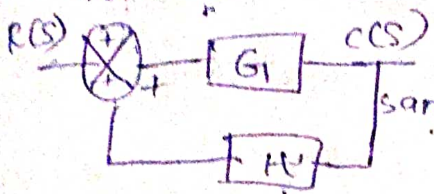
$C(s) = G.R(S)$.

$\boxed{V = iR}$ $\boxed{I = \frac{V}{R}}$

$\Rightarrow$ voltage.

calculated R.B high voltage in Resistor.

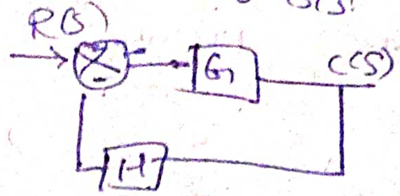
+ve feedBack (we have noise)

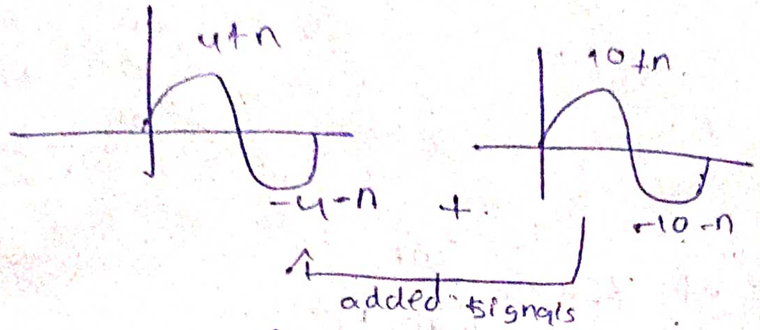


stability ↓ses

-ve feedBack (Noise reduction)

⇒ Stability ↑ses

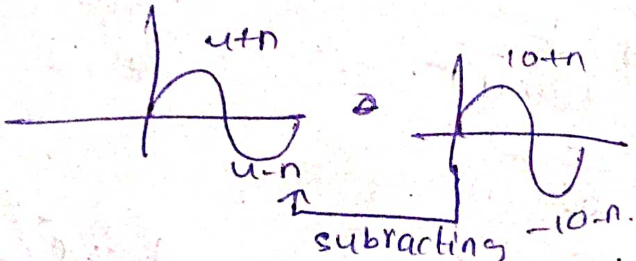




+ve feedBack

+2n

-2n · noise ↑se then stability ↓ses

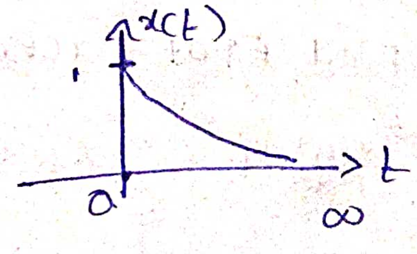


-ve FB.

noise ↓ses then stability ↑ses.

Problems on FT

1) Find FT of $x(t) = e^{-at}u(t)$. Plot its magnitude and phase spectrum.



$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt.$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt.$$

$$= \int_{0}^{\infty} e^{-t(a+j\omega)} dt.$$

$$= \left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^{\infty}$$

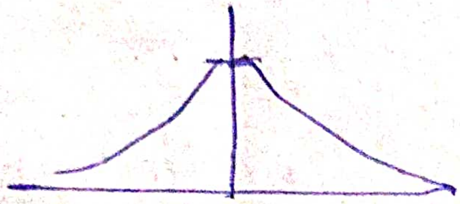
$$= \frac{1}{-(a+j\omega)}[0-1] = \frac{-1}{a+j\omega}(-1) = \frac{1}{a+j\omega}$$

$$|x(\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}.$$

ω	$\lvert x(\omega)\rvert$
0.	$\frac{1}{a}$
$-\infty$.	0.
∞.	0.

$e^{-at}u(t) = \frac{1}{a+j\ldots}$

$t^n e^{-at} u(t) = \frac{\ldots}{(j\omega\ldots}$



1) Find Inverse F.T of $x(\omega) = \frac{j\omega+3}{(j\omega+1)^2}$

$$x(\omega) = \frac{j\omega+1+2}{(j\omega+1)^2}$$

$$= \frac{(j\omega+1)}{(j\omega+1)^2} + \frac{2}{(j\omega+1)^2}$$

$$= \frac{1}{j\omega+1} + \frac{2}{(j\omega+1)^2}$$

$$x(t) = F^{-1}\left[\frac{1}{j\omega+1} + \frac{2}{(j\omega+1)^2}\right]$$

$\frac{1}{j\omega+1} = e^{-t}u(t)$

$\frac{1}{(j\omega+1)^2} = t\,e^{-t}u(t)$

$$= e^{-t}u(t) + 2t\,e^{-t}u(t).$$

$$x(t) = (1+2t)\,e^{-t}u(t)$$

3) Find FT of $x(t) = \begin{cases} 1, & |t|<1 \\ 0, & |t|>1 \end{cases}$ Hence, evaluate.

$\int_0^\infty \frac{\sin x}{x}\,dx.$

A:

$x(t) = e^{-at^2}, a>0. \quad X(\omega) = ?$

$x(t) = \frac{1}{e^{at^2}}$

$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-at^2} \cdot e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} e^{-(at^2 + j\omega t)} dt$

Laplace transform

⇒ Find the LT and ROC of $e^{-2t}u(t) + e^{-t/2}u(-t)$

compare

$e^{-2t}u(t)$ with $e^{-at}u(t) = \frac{1}{s+a}$

$= \frac{1}{s+2}$

wkt,

$e^{-at}u(t) = \frac{1}{s+a} \quad \sigma > -a$

$e^{-at}u(-t) = \frac{-1}{s+a} \quad \sigma < -a$

$e^{-t/2}u(-t)$ with $e^{-at}u(-t) = \frac{-1}{s+a} \quad \sigma < -a$

$= \frac{-1}{s+\frac{1}{2}}$

$e^{-2t}u(t) + e^{-t/2}u(-t)$

$\frac{1}{s+2} - \frac{1}{s+\frac{1}{2}}$

$-2 < Re\{s\} < -\frac{1}{2}$

Real part

while comparing denominators with 0.

$s+2=0$

$s=-2$

$\sigma > -2$

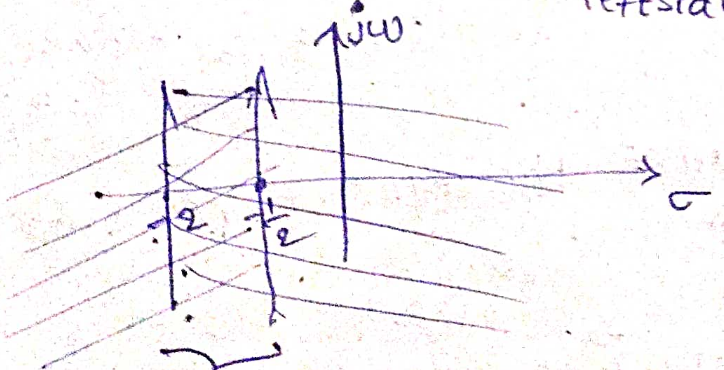
Right sided signal

$s+\frac{1}{2}=0$

$s = -\frac{1}{2}$

$Real\{s\} < -\frac{1}{2}$

leftsided.



real imagin

$s = \sigma + j\omega$

Total ROC. ROC : ~~Real~~

$-2 < Re\{s\} < -\frac{1}{2}$

⇒ Find L.T and ROC of $e^{-2t}u(-t)+e^{5t}u(t)$?

ROC

$e^{-at}u(t)=\frac{1}{s+a}$ $\sigma>-a$

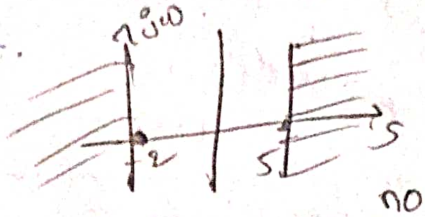
$e^{at}u(t)=\frac{1}{s-a}$ $\sigma>a$

$e^{-at}u(-t)=\frac{-1}{s+a}$ $\sigma<-a$

$$L.T\left[e^{-2t}u(-t)+e^{5t}u(t)\right]$$

$$=\frac{-1}{s+2}+\frac{1}{s-5}$$

$\sigma<-2.$ $\quad$ $\sigma>5.$



no ROC.

1) Z-transform

Find the Z-transform of the sequence $x(n)=a^{-n}u(-n-1)$

Sol: $x(n)=a^{-n}u(-n-1)$

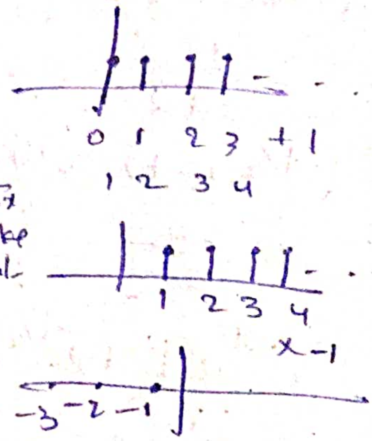
$-n-1$

$-n-1+1$ shift

$-n\times-1=n$ reversal

$$x(z)=\sum_{-\infty}^{-1}a^{-n}z^{-n}.$$

$$=\sum_{-\infty}^{-1}(az)^{-n}$$



$$=\sum_{n=1}^{\infty}(az)^{n}.$$

we have formula for $1+x+x^2+\ldots=\frac{1}{1-x}$ convert like that

$$=\sum_{n=0}^{\infty}(az)^{n}-(az)^{0}$$

$$=\underbrace{(az)^{0}+(az)^{1}+(az)^{2}+\cdots}_{x}-1$$

$$=1+x+x^{2}+\cdots-1$$

$$=\frac{1}{1-x}-1=\frac{1}{1-az}-1=\frac{1-(1-az)}{1-az}=\frac{az}{1-az}$$

Ex: if given like these $\left(\frac{1}{3}\right)^{-n}u(-n-1)$ use these formula

$$=\frac{\frac{z}{3}}{1-\frac{z}{3}}=\frac{z}{3}\times\frac{3}{3-z}=\frac{z}{3-z}$$

2). Find the z-transform of the sequence $x(n)=-a^{n}u(-n-1)$

$$x(z)=\sum_{n=-\infty}^{\infty}x(n).z^{-n}$$

$$=\sum_{-\infty}^{-1}-a^{n}u(-n-1).z^{-n}.$$

$$=\sum_{-\infty}^{-1}-a^{n}z^{-n}.$$

$$=-\sum_{-\infty}^{-1}a^{n}z^{-n}.$$

$$=-\cancel{\sum_{n=1}^{\infty}a^{-n}z^{-n}}=-\sum_{n=1}^{\infty}\cancel{(az)^{-n}}.$$

$$= -\sum_{-\infty}^{-1} (a^{-1}z)^{n}$$ reverse limits

$$= -\sum_{n=1}^{\infty} ((a^{-1}z)^{-n})^{-1}$$

$$= -\sum_{n=1}^{\infty} (a^{-1}z)^{n}$$

$$\sum_{n=0}^{\infty} a^{n} = \frac{1}{1-a}$$

$$= -\left[\sum_{n=0}^{\infty} (a^{-1}z)^{n} - (a^{-1}z)^{0}\right]$$

$$= -[(a^{-1}z)^{0} + (a^{-1}z)^{1} + (a^{-1}z)^{2} + \cdots - 1]$$

$$= -\left[\frac{1}{1-a^{-1}z} - 1\right] = -\left[\frac{1}{1-a^{-1}z} - 1\right]$$

$$= -\left[\frac{1-1+a^{-1}z}{1-a^{-1}z}\right] = -\left[\frac{\frac{z}{a}}{\frac{a-z}{a}}\right] = -\left[\frac{z}{a} \times \frac{a}{a-z}\right]$$

$$= -\left[\frac{z}{a-z}\right] = \frac{z}{z-a}.$$

if given $-(\frac{1}{2})^{n} u(-n-1)$

substitute in $\frac{z}{z-a} = \frac{z}{z-\frac{1}{2}}$

3) Find the z-transform of the signals $a^{n}u(n)$ and $a^{-n}u(n)$

Sol:- $x(n) = a^{n}u(n)$

$$z.T[x(n)] = X(z)$$

$$= \sum_{n=-\infty}^{\infty} x(n).z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n).z^{-n} = \sum_{n=0}^{\infty} a^{n}.z^{-n} = \sum_{n=0}^{\infty} (az^{-1})^{n}$$

$$= (az^{-1})^{0} + (az^{-1})^{1} + (az^{-1})^{2} + \cdots$$

$$= 1 + x + x^{2} + \cdots = \frac{1}{1-x}$$

$$= \frac{1}{1-az^{-1}} = \frac{1}{\frac{z-a}{z}} = \frac{z}{z-a}.$$

$x(n) = a^{-n}u(n)$

$$z.T[x(n)] = \sum_{n=-\infty}^{\infty} x(n).z^{-n}$$

$$= \sum_{n=0}^{\infty} a^{-n}.z^{-n} = \sum_{n=0}^{\infty} (az)^{-n} = \sum_{n=0}^{\infty} (a^{-1}z^{-1})^{n}$$

$$= (a^{-1}z^{-1})^{0} + (a^{-1}z^{-1})^{1} + (a^{-1}z^{-1})^{2} + \cdots$$

$$= \frac{1}{1-a^{-1}z^{-1}} = \frac{1}{\frac{az-1}{az}} = \frac{az}{az-1}$$

4) A finite sequence $x(n)$ is defined as $x(n) = \{5, 3, -3, 0, 4, -2\}$
Find $X(z)$.

if arrow not given then assume it as right sided then initial time value is 0.

$$x(n) = \{\underset{\uparrow}{5}, 3, -3, 0, 4, -2\}$$

0 1 2 3 4 5

$$X(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n} = \sum_{n=0}^{5} x(n).z^{-n}$$

$$x(z) = x(0).z^{-0} + x(1).z^{-1} + x(2).z^{-2} + x(3).z^{-3} + x(4).z^{-4} + x(5).z^{-5}$$

$$= 5.(1) + \frac{3}{z} + \frac{(-3)}{z^2} + \frac{0}{z^3} + \frac{4}{z^4} + \frac{(-2)}{z^5}$$

$$= 5 + \frac{3}{z} - \frac{3}{z^2} + \frac{4}{z^4} - \frac{2}{z^5}$$

∴ A finite duration sequence $x(n) = \{5, 3, 0, \underset{\uparrow}{1}, 2, 4\}$. Find its z-transform.

$-3, -2, -1, 0, 1, 2$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n).z^{-n}$$

$$= \sum_{-3}^{2} x(n).z^{-n} = x(-3).z^{-(-3)} + x(-2).z^{-(-2)} + x(-1).z^{-(-1)} + x(0).z^{0} + x(1).z^{-1} + x(2).z^{-2}$$

$$= 5.z^{3} + 3.z^{2} + (0).z + (1).(1) + 2.z^{-1} + 4.z^{-2}$$

$$x(z) = 5z^3 + 3z^2 + 0 + 1 + \frac{2}{z} + \frac{4}{z^2}$$

Find the signal $x(n)$ for which the z-transform is

(i) $x(z) = 4z^4 - z^3 - 3z + 4z^{-1} + 3z^{-2}$

$-4 \quad -3 \quad -1 \quad 1 \quad 2$ $\qquad x(n).z^{-n}$

$$= \sum_{n=-4}^{2} x(n).z^{-n}$$

$\{4, -1, -3, 4, 3$

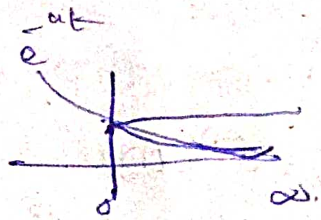
$\{4, -1, 0, -3, \underset{\uparrow}{0}, 4, 3\}$

Laplace transform

① Find the L.T of $e^{-at} u(t)$ and determine the Roc

$$x(s) = LT[x(t)] = \int_{-\infty}^{\infty} x(t).e^{-st}dt.$$

$$x(s) = \int_{0}^{\infty} e^{-at} u(t).e^{-st}dt$$

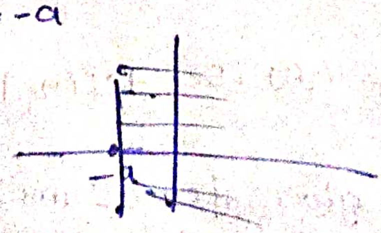


$$= \int_0^{\infty} e^{-at}.e^{-st}dt.$$

$$= \int_0^{\infty} e^{-t(a+s)}dt = \left[\frac{e^{-t(a+s)}}{-(a+s)}\right]_0^{\infty} = \frac{-1}{a+s}\left[e^{-\infty(a+s)} - e^{0}\right]$$

$$= \frac{-1}{a+s}[0-1] = \frac{1}{s+a}$$

Roc: $Re\{s\} > -a$



② Find the laplace transform and Roc of the signal $-e^{-at} u(-t)$?

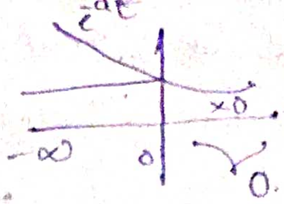
$$x(s)=\int_{-\infty}^{\infty} x(t).e^{-st}dt$$

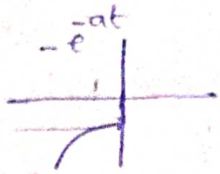
$$=\int_{-\infty}^{0} -e^{-at}.e^{-st}\,dt.$$

$$=-\int_{-\infty}^{0} e^{-t(a+s)}\,dt.$$

$$=\cancel{-}\left[\frac{e^{-t(a+s)}}{\cancel{-}(a+s)}\right]_{-\infty}^{0}.$$

$$=\frac{1}{a+s}.\left[e^{-0(a+s)} - e\right]$$

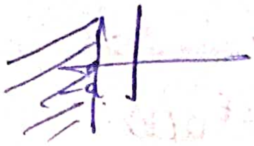




changelimits.

$$=+(-)\int_{0}^{\infty} e^{-t(a+s)}\,dt.=\left[\frac{e^{-t(a+s)}}{-(a+s)}\right]_{0}^{\infty}=\frac{-1}{a+s}[0-1]=\frac{1}{a+s}$$

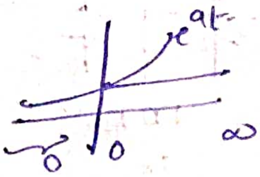
ROC: $Re\{s\} < -a$ $\quad s+a=0,\ s=-a$



Find laplace transform and Roc of $e^{at}u(t)$ and $e^{at}u(-t)$.

i) $x(t)=e^{at}u(t)$

$$x(s)=\int_{0}^{\infty} e^{at}.e^{-st}dt$$



$$=\int_{0}^{\infty} e^{t(a-s)}\,dt.$$

$$=\left[\frac{e^{t(a-s)}}{a-s}\right]_{0}^{\infty}=\frac{1}{a-s}\left[e^{(\infty)a}.e^{(\infty)(-s)}-1\right]=\frac{1}{a-s}[0-1]=\frac{-1}{a-s}$$

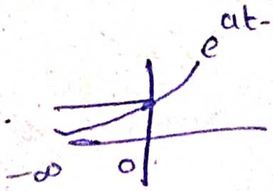
$$=\frac{1}{s-a}.$$

$s-a=0$
$s=a.$

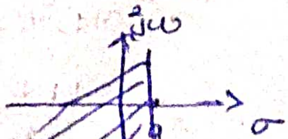
ROC: $Re\{s\}>a$

ii) $e^{at}u(-t)$

$$x(s)=\int_{-\infty}^{0} e^{at}.e^{-st}dt.$$



$$=\int_{-\infty}^{0} e^{t(a-s)}dt.=\frac{1}{a-s}\left[e^{t(a-s)}\right]_{-\infty}^{0}=\frac{1}{a-s}[1-0]=\frac{1}{a-s}=\frac{-1}{s-a}$$



Roc
$Re\{s\}<a$

find the LT & Roc of the signal. $e^{-t}u(t)+e^{-2t}u(t)$

$$LT[e^{-t}u(t)]=\frac{1}{s+a}=\frac{1}{s+1}\qquad LT[e^{-2t}u(t)]=\frac{1}{s+2}.$$

$$=\frac{1}{s+1}+\frac{1}{s+2}$$

$s=-1$ $\quad s=-2$

$Re\{s\}>-1$ $\quad Re\{s\}>-2.$

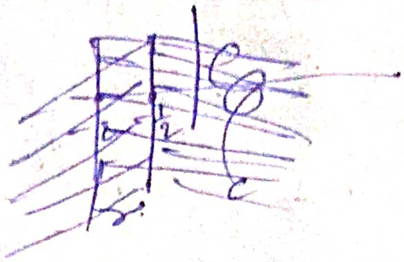
ROC: $Re\{s\}>-1$

Find the L.T and ROC of $e^{-2t}u(t) + e^{-(t/2)}u(-t)$

$L.T[e^{-2t}u(t)] = \frac{1}{s+2}$. $L.T[e^{-\frac{t}{2}}u(-t)] = -\frac{1}{s+\frac{1}{2}}$

$\frac{1}{s+2} + \frac{1}{s+\frac{1}{2}}$

$Re\{s\} > -2$ $Re\{s\} < -\frac{1}{2}$



ROC: $-2 < Re\{s\} < -\frac{1}{2}$

$\Rightarrow$ L.T & ROC of $e^{-2t}u(t) + e^{5t}u(t)$

$\frac{1}{s+2} + \frac{1}{s-5}$

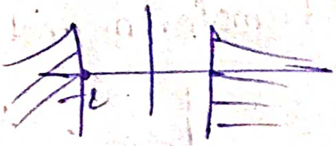
$Re\{s\} > -2$ $Re\{s\} > 5$

ROC: $Re\{s\} > 5$.

$e^{-2t}u(-t) + e^{5t}u(t)$

$\frac{-1}{s+2} + \frac{1}{s-5}$.

$\sigma < -2$ $\sigma > 5$



no ROC.

Inverse Laplace transform

$x(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} dt$.

① determine the signal $x(t)$, if $x(s) = \frac{s+2}{s^2+2s+10}$.

$x(s) = \frac{s+2}{s^2+2s+10}$

$= \frac{s+2}{s^2+2s+1+9}$

$= \frac{s+2}{(s+1)^2+3^2}$

$= \frac{s+1+1}{(s+1)^2+3^2} = \frac{s+1}{(s+1)^2+3^2} + \frac{1}{(s+1)^2+3^2}$

$e^{-at}\cos\omega t\, u(t) \rightarrow \frac{(s+a)}{(s+a)^2+\omega^2}$

$e^{-at}\sin\omega t\, u(t) \rightarrow \frac{\omega}{(s+a)^2+\omega^2}$

$= L.T^{-1}\left[\frac{s+1}{(s+1)^2+3^2}\right] + L.T^{-1}\left[\frac{1}{(s+1)^2+3^2}\right]$ $\frac{1}{3} \cdot 3$ multiply

$x(t) = e^{-(1)t}\cos 3t\, u(t) + \frac{1}{3}e^{-t}\sin 3t\, u(t)$

Find inverse L.T of $x(s) = \frac{2s+4}{s^2+4s+3}$

$$x(s) = \frac{2s+4}{s^2+4s+3} = \frac{2s+4}{s^2+3s+s+3} = \frac{2s+4}{s(s+3)+1(s+3)}$$

$$= \frac{2s+4}{(s+1)(s+3)}$$

$$= \frac{s+s+3+1}{(s+1)(s+3)}$$

$$= \frac{(s+3)}{(\cancel{s+3})(s+1)} + \frac{(\cancel{s+1})}{(\cancel{s+1})(s+3)} = \frac{1}{s+1} + \frac{1}{s+3}.$$

$$x(t) = e^{-t}u(t) + e^{-3t}u(t)$$

$$= [e^{-t} + e^{-3t}]u(t)$$

$e^{-at}u(t) = \frac{1}{s+a}$

$e^{-t}u(t) = \frac{1}{s+1}$

$e^{-3t}u(t) = \frac{1}{s+3}$