F Test

F Distribution

F-Distribution

- We may be interested in knowing whether the population variances are equal or not, based on the response of the random samples.
- The F-distribution was developed by Fisher to study the behavior of two variances from random samples taken from two independent normal populations.
- Let U_1 and U_2 be chi-square random variables with d_1 and d_2 degrees of freedom, respectively. Then if U_1 and U_2 are independent

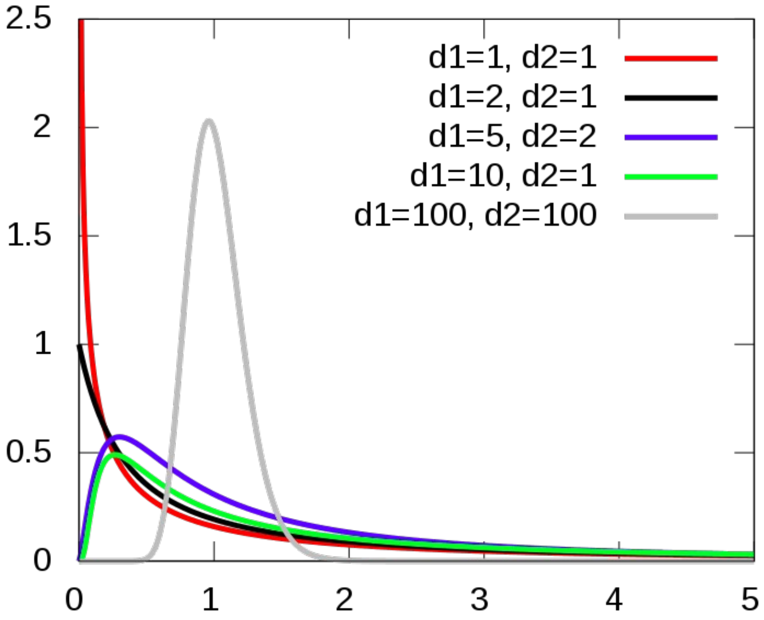
$$X = \frac{U_1 / d_1}{U_2 / d_2}$$

- Ran dom variable X is said to have an F-distribution with d_1 numerator degrees of freedom and d_2 denominator degrees of freedom. We denote this by $X \sim F(d_1, d_2)$.

F-Distribution

- Then the probability density function (pdf) for X is given by

$$f(x)=\begin{cases} \dfrac{\Gamma\left(\frac{d_1+d_2}{2}\right)\left(\frac{d_1}{d_2}\right)^{\frac{d_1}{2}}}{\Gamma\left(\frac{d_1}{2}\right)\Gamma\left(\frac{d_2}{2}\right)}\, x\, \dfrac{X^{\frac{d_1}{2}-1}}{\left(1+\frac{d_1 x}{d_2}\right)^{\frac{d_1+d_2}{2}}}, & x>0 \\ 0, & otherwise \end{cases}$$



- Using the above formula the critical value is found but we do not integrate the complex functions insted uses the F table.

F-Distribution

- The random variable of the F-distribution may also be written

$$X = \frac{s_1^2}{\sigma_1} \div \frac{s_2^2}{\sigma_2}$$

- Where $s_1^2 = S_1^2/d_1$ and $s_2^2 = S_2^2/d_2$, variances of the random samples
- s_1^2 is the sum of squares of d_1 random variables from normal distribution N (0 , σ_1^2)
- s_2^2 is the sum of squares of d_2 random variables from normal distribution N (0 , σ_2^2)
- F distribution is non symetrical and the shape of the distribution depends upon the values of d_1 and d_2
- F distribution is used in analysis of varience test to test mean values of multiple groups

F Test

The F-test can be used to know two samples come from populations with equal variancess or from populations with different variancess

F-Test

- The data set contains 480 ceramic strength measurements for two batches of material. The summary statistics for each batch are shown below.

Batch1	Batch2
No of observations = 240	No of observations = 240
Mean = 688.99	Mean = 611.15
Standard Deviation = 65.5	Standard Deviation = 61.8

test the variances for the two batches are equal.

F-Test

Step1: Formulate Null and Alternate Hypothesis

$$H_0: \quad \sigma_1^2 = \sigma_2^2$$
$$H_1: \quad \sigma_1^2 \neq \sigma_2^2$$

Step 2: Choose level of significance(α)

Significance level: $\alpha = 0.05$

F-Test

Step 3: Determine appropriate test to use and find the test statistic

Here we want know two samples come from populations with equal variancess or from populations with different variancess

F test is appropriate

Assume Populations follow normal Distribution

$$F\ statistc = \frac{s_1^2}{\sigma_1} \div \frac{s_2^2}{\sigma_2}$$

$$F\ statistc = \frac{varience_1}{varience_2}$$

Since null hypothesis is two variances are equal.

$$F\ statistc = \frac{s_1^2}{s_2^2} = \frac{65.5^2}{61.8^2} = \frac{4290.25}{3819.24} = 1.1233$$

F-Test

Step 4: Determine if we need a 1 tailed or a 2 tailed t-critical value and find the t-critical value for desired confidence level

Numerator degrees of freedom: $d_1 = 240-1 = 239$

Denominator degrees of freedom: $d_2 = 240-1 = 239$

Significance level: $\alpha = 0.05$

$F_C(1-\alpha/2, d_1, d_2) = 0.7756$

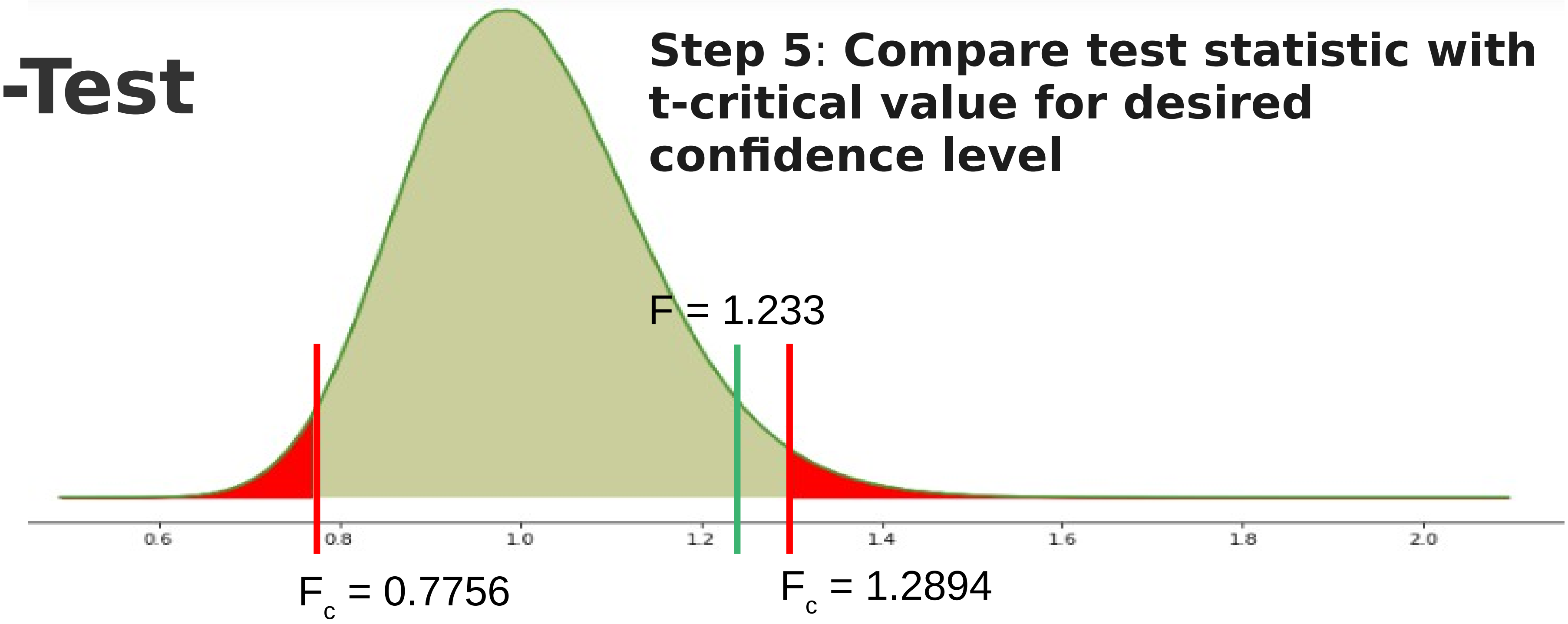
$F_C(\alpha/2, d_1, d_2) = 1.2894$

F - Distribution ($\alpha = 0.025$ in the Right Tail)

Denominator Degrees of Freedom	Numerator Degrees of Freedom								
df_2 \ df_1	1	2	3	4	5	6	7	8	9
1	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
2	38.506	39.000	39.165	39.248	39.298	39.331	39.335	39.373	39.387
3	17.443	16.044	15.439	15.101	14.885	14.735	14.624	14.540	14.473
4	12.218	10.649	9.9792	9.6045	9.3645	9.1973	9.0741	8.9796	8.9047
5	10.007	8.4336	7.7636	7.3879	7.1464	6.9777	6.8531	6.7572	6.6811
6	8.8131	7.2599	6.5988	6.2272	5.9876	5.8198	5.6955	5.5996	5.5234
7	8.0727	6.5415	5.8898	5.5226	5.2852	5.1186	4.9949	4.8993	4.8232
8	7.5709	6.0595	5.4160	5.0526	4.8173	4.6517	4.5286	4.4333	4.3572
9	7.2093	5.7147	5.0781	4.7181	4.4844	4.3197	4.1970	4.1020	4.0260
10	6.9367	5.4564	4.8256	4.4683	4.2361	4.0721	3.9498	3.8549	3.7790
11	6.7241	5.2559	4.6300	4.2751	4.0440	3.8807	3.7586	3.6638	3.5879
12	6.5538	5.0959	4.4742	4.1212	3.8911	3.7283	3.6065	3.5118	3.4358
13	6.4143	4.9653	4.3472	3.9959	3.7667	3.6043	3.4827	3.3880	3.3120
14	6.2979	4.8567	4.2417	3.8919	3.6634	3.5014	3.3799	3.2853	3.2093
15	6.1995	4.7650	4.1528	3.8043	3.5764	3.4147	3.2934	3.1987	3.1227
16	6.1151	4.6867	4.0768	3.7294	3.5021	3.3406	3.2194	3.1248	3.0488
17	6.0420	4.6189	4.0112	3.6648	3.4379	3.2767	3.1556	3.0610	2.9849
18	5.9781	4.5597	3.9539	3.6083	3.3820	3.2209	3.0999	3.0053	2.9291
19	5.9216	4.5075	3.9034	3.5587	3.3327	3.1718	3.0509	2.9563	2.8801
20	5.8715	4.4613	3.8587	3.5147	3.2891	3.1283	3.0074	2.9128	2.8365
21	5.8266	4.4199	3.8188	3.4754	3.2501	3.0895	2.9686	2.8740	2.7977
22	5.7863	4.3828	3.7829	3.4401	3.2151	3.0546	2.9338	2.8392	2.7628
23	5.7498	4.3492	3.7505	3.4083	3.1835	3.0232	2.9023	2.8077	2.7313
24	5.7166	4.3187	3.7211	3.3794	3.1548	2.9946	2.8738	2.7791	2.7027
25	5.6864	4.2909	3.6943	3.3530	3.1287	2.9685	2.8478	2.7531	2.6766
26	5.6586	4.2655	3.6697	3.3289	3.1048	2.9447	2.8240	2.7293	2.6528
27	5.6331	4.2421	3.6472	3.3067	3.0828	2.9228	2.8021	2.7074	2.6309
28	5.6096	4.2205	3.6264	3.2863	3.0626	2.9027	2.7820	2.6872	2.6106
29	5.5878	4.2006	3.6072	3.2674	3.0438	2.8840	2.7633	2.6686	2.5919
30	5.5675	4.1821	3.5894	3.2499	3.0265	2.8667	2.7460	2.6513	2.5746
40	5.4239	4.0510	3.4633	3.1261	2.9037	2.7444	2.6238	2.5289	2.4519
60	5.2856	3.9253	3.3425	3.0077	2.7863	2.6274	2.5068	2.4117	2.3344
120	5.1523	3.8046	3.2269	2.8943	2.6740	2.5154	2.3948	2.2994	2.2217
∞	5.0239	3.6889	3.1161	2.7858	2.5665	2.4082	2.2875	2.1918	2.1136

F-Test

Step 5: Compare test statistic with t-critical value for desired confidence level



Rejection region: Reject H_0 if $F < 0.7756$ or $F > 1.2894$

$0.7756<1.233 <1.2894$

The F test indicates that there is not enough evidence to reject the null hypothesis

The two batch variancess are equal at the 0.05 significance level.

Analysis Of Variance(ANOVA)

Types of ANOVA

1. One-way ANOVA
2. Two-way ANOVA

- One-way ANOVA is a hypothesis test in which onlysingle factor is taken into consideration.
- Two-way ANOVA examines the effect of two independent factors

ANOVA

- One-way analysis of variance (ANOVA) is a technique that can be used to compare means of two or more samples
- This technique can be used only for one factor(variable) hence "one-way"
- If we want to study the effect of three levels(a_1,a_2,a_3) of a fertilizer on plant growth
- where a1, a2, and a3 are the three levels of the factor being studied.
- One factor but three levels.
- when comparing more than two groups apply the t test by implementing multiple t tests on multiple pairs of means.
- It is inappropriate because the repetition of the multiple tests may repeatedly add multiple chances of error, which may result in a larger Type I error(α) level than the pre-set α level.

Multiple t tests to compare more than two groups

- When we try to compare means of three groups, A, B, and C, using the t test, we need to implement 3 pairwise tests
- A vs B, A vs C, and B vs C.
- Similarly if comparisons are repeated k times in an experiment and the α level 0.05 was set for each comparison, an unacceptably increased total error rate of $1-(0.95)^k$ may be expected for the total comparison procedure in the experiment.
- The probability of one or more heads
- For two coin flips, the probability of all tails is $0.50 \times 0.50 = 0.25$.
- if the coin flipped three times, the probability of one or more heads is $1 – (0.50 \times 0.50 \times 0.50) = 1 – (0.50)^3 = 1 – 0.125 = 0.875$
- you will get one or more heads in about 94% of sets of "four coin flips".

Multiple t tests to compare more than two groups

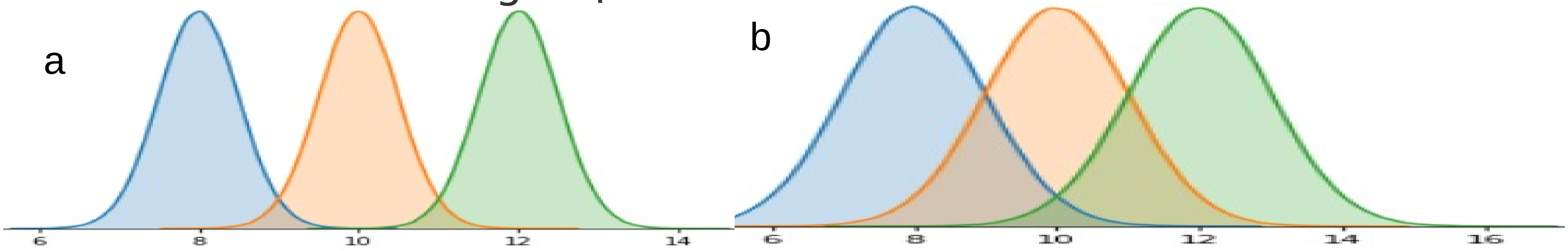
- Similarly, for a statistical test (such as a t test) with α= 0.05, if the null hypothesis is true then the probability of not obtaining a significant result is 1 – 0.05 = 0.95.
- Multiply 0.95 by the number of tests to calculate the probability of not obtaining one or more significant results across all tests
- Here 0.95 × 0.95 x 0.95 = 0.857375
- Subtract that result from 1.00 to calculate the probability of making at least one type I error with multiple tests: 1 – 0.857375 = 0.142625.
- means your chances of incorrectly rejecting the null hypothesis (a type I error) is 0.14 instead of 0.05
- For a comparison of more than two group means the one-way analysis of variance (ANOVA) is the appropriate method instead of the t test

ANOVA

- Then why are we using the 'analysis of variance’ to compare means?
- It is because that the relative location of the several group means can be more conveniently identified by variance among the group means than comparing many group means directly when number of means are large
- The ANOVA method assesses the relative size of variance among group means (between group variance) compared to the average variance within groups (within group variance).

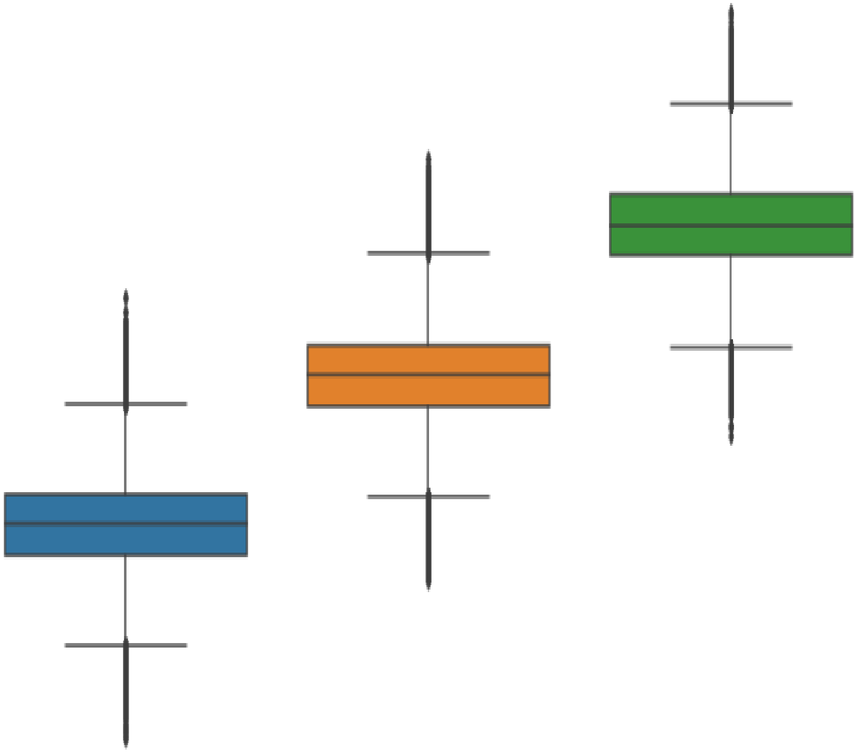
ANOVA

- Figure shows two comparative cases which have similar 'between group variances' (the same distance among three group means) but have different 'within group variances'.

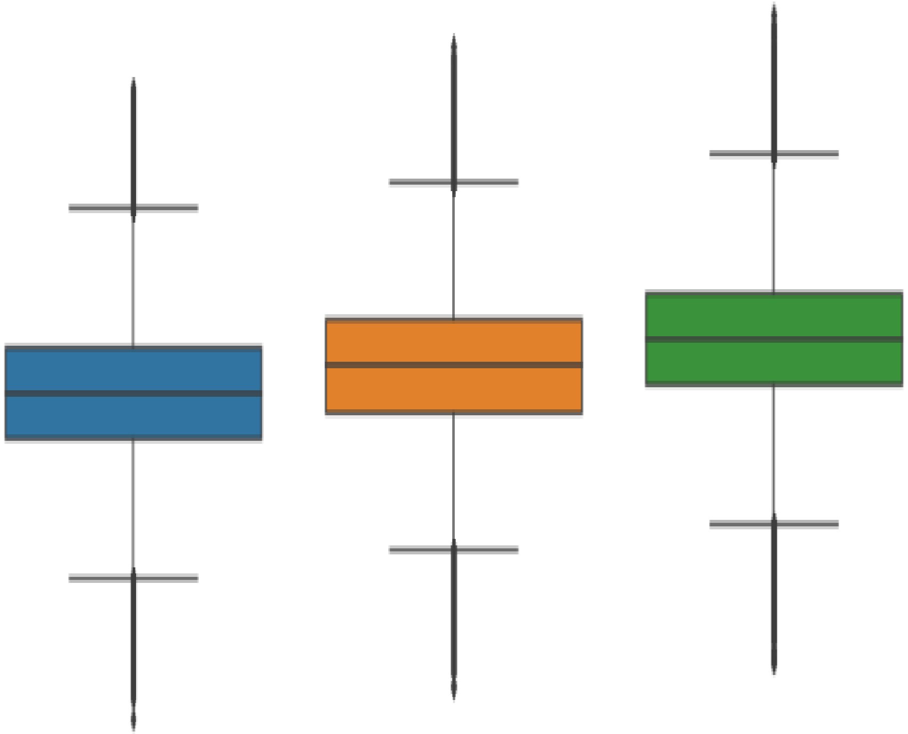


- When the between group variances are the same, mean differences among groups seem more distinct in the distributions with smaller within group variances (a) compared to those with larger within group variances (b).
- Therefore the ratio of between group variance to within group variance is of the main interest in the ANOVA.

ANOVA

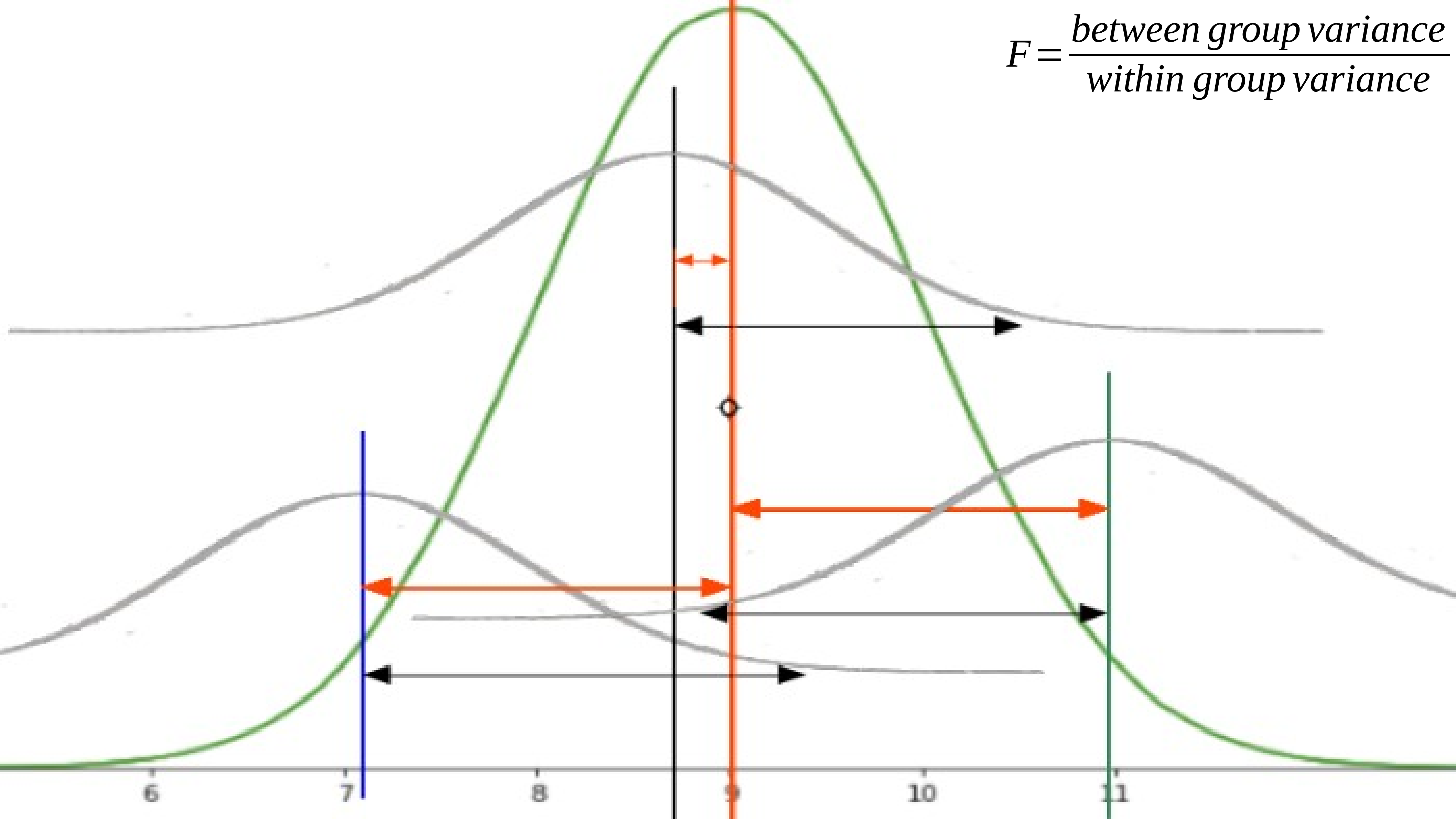


a



b

$$F = \frac{between\ group\ variance}{within\ group\ variance}$$



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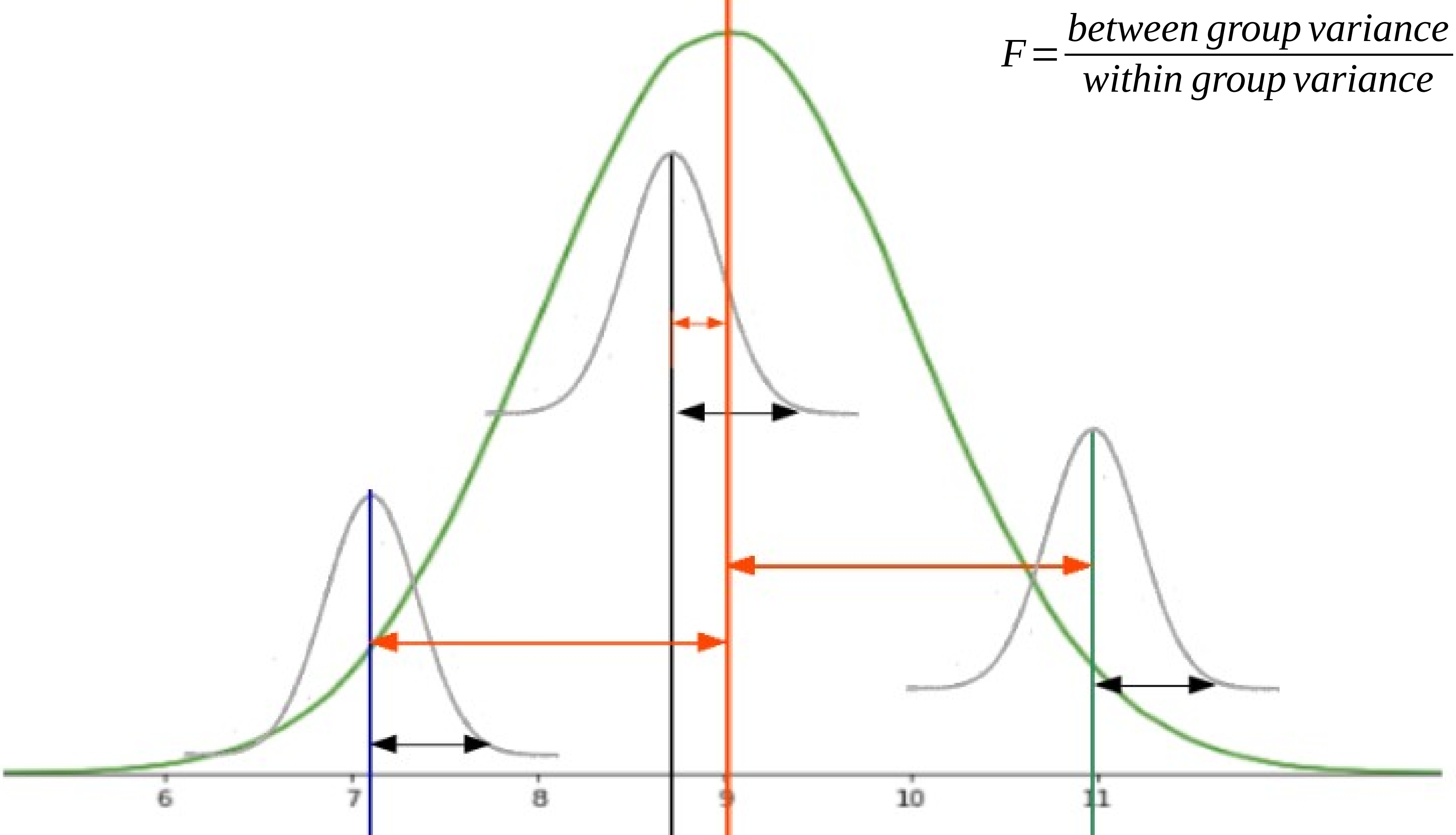

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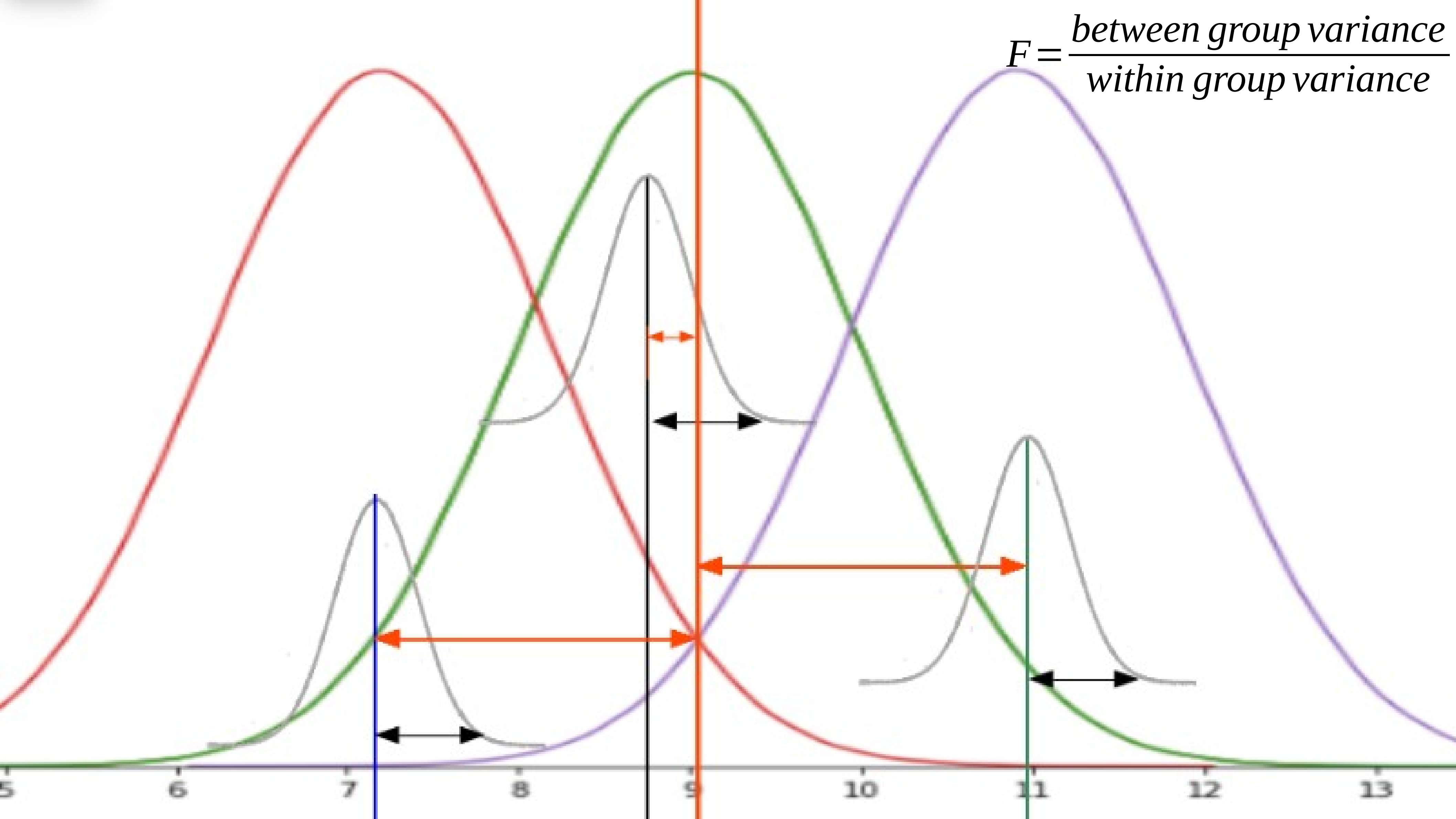
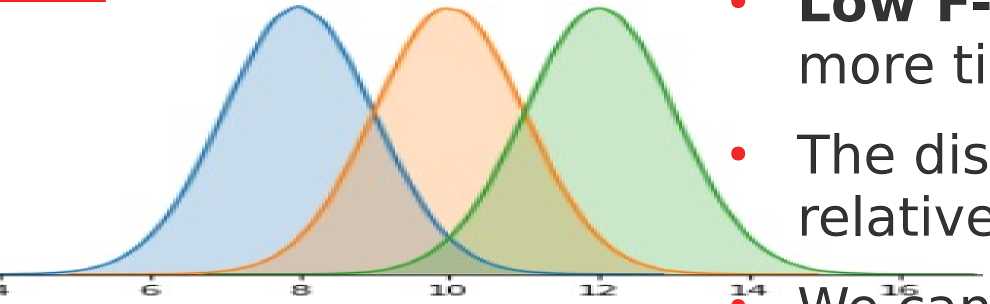
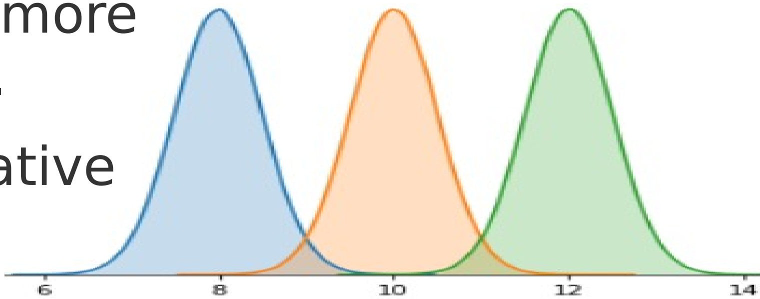


ANOVA



- **Low F-value:** The group means cluster together more tightly than the within-group variability.
- The distance between the means is small relative to the random error within each group.
- We can't conclude that these groups are truly different at the population level.

- **High F-value:** The group means spread out more than the variability of the data within groups.
- The distance between the means is large relative to the random error within each group.
- We can conclude that the observed differences between group means reflect differences at the population level.



ANOVA

- Consider an experiment to study the effect of three different levels of a factor on a response (e.g. three levels of a fertilizer on plant growth). If we had 6 observations for each level, we could write the outcome of the experiment in a table like this, where a1, a2, and a3 are the three levels of the factor being studied.

a1	a2	a3
6	8	13
8	12	9
4	9	11
5	11	8
3	6	7
4	8	12

ANOVA

- .

$$H_0 : \mu_1 = \mu_2 = \mu_3$$

$$H_1 : Not\,all\,means\,same \ \left(atleast \ \mu_1 \neq \mu_2 \vee \mu_2 \neq \mu_3 \vee \mu_1 \neq \mu_3 true \right)$$

Step 1: Calculate the mean within each group:

a1	a2	a3
6	8	13
8	12	9
4	9	11
5	11	8
3	6	7
4	8	12
5	**9**	**10**

ANOVA

- **Step2:** Calculate the overall mean:

  (5+9+10)/3 = 8

- **Step 3:** Calculate the "**between-group" mean square** value:
- Between-group sum of squared difference:

$$SS_B = 6(5-8)^2 + 6(9-8)^2 + 6(10-8)^2 = 84$$

- The between-group degrees of freedom is one less than the number of groups

  $df_b = 3 - 1 = 2$

- so the between-group mean square value is

$$MS_B = SS_B / df_b = 84/2 = 42$$

ANOVA

- **Step 4:** Calculate the **"within-group" mean square** value.

a1	a2	a3
6 – 5 = 1	8 – 9 = -1	13– 10 = 3
8 – 5 = 3	12 – 9 = 3	9 – 10 = -1
4 – 5 = -1	9 – 9 = 0	11– 10 = 1
5 – 5 = 0	11 – 9 = 2	8– 10 = -2
3 – 5 = -2	6 – 9 = -3	7– 10 = -3
4 – 5 = -1	8 – 9 = -1	12– 10 = 2

- The within-group sum of squares is the sum of squares of all 18 values in this table

$$SS_W = 1+9+1+0+4+1+1+9+0+4+9+1+9+1+1+4+9+4 = 68$$

- The within-group degrees of freedom is

$$df_w = a(n-1) = 3(6 - 1) = 15$$

ANOVA

- Thus the within-group mean square value is

$$MS_W = SS_W / df_w = 68/15 = 4.5$$

- **Step 5:** The F-ratio is

$$F\ statistc = \frac{MS_B}{MS_W} = 42/4.5 = 9.3$$

- In this case, Fcrit(2,15) = 3.68 at α = 0.05.

of the *F* Distribution

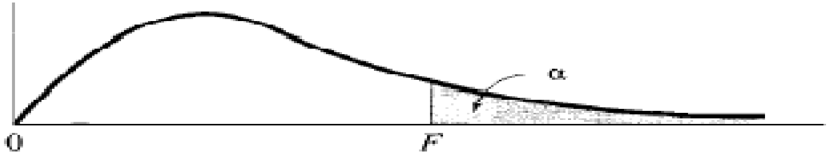
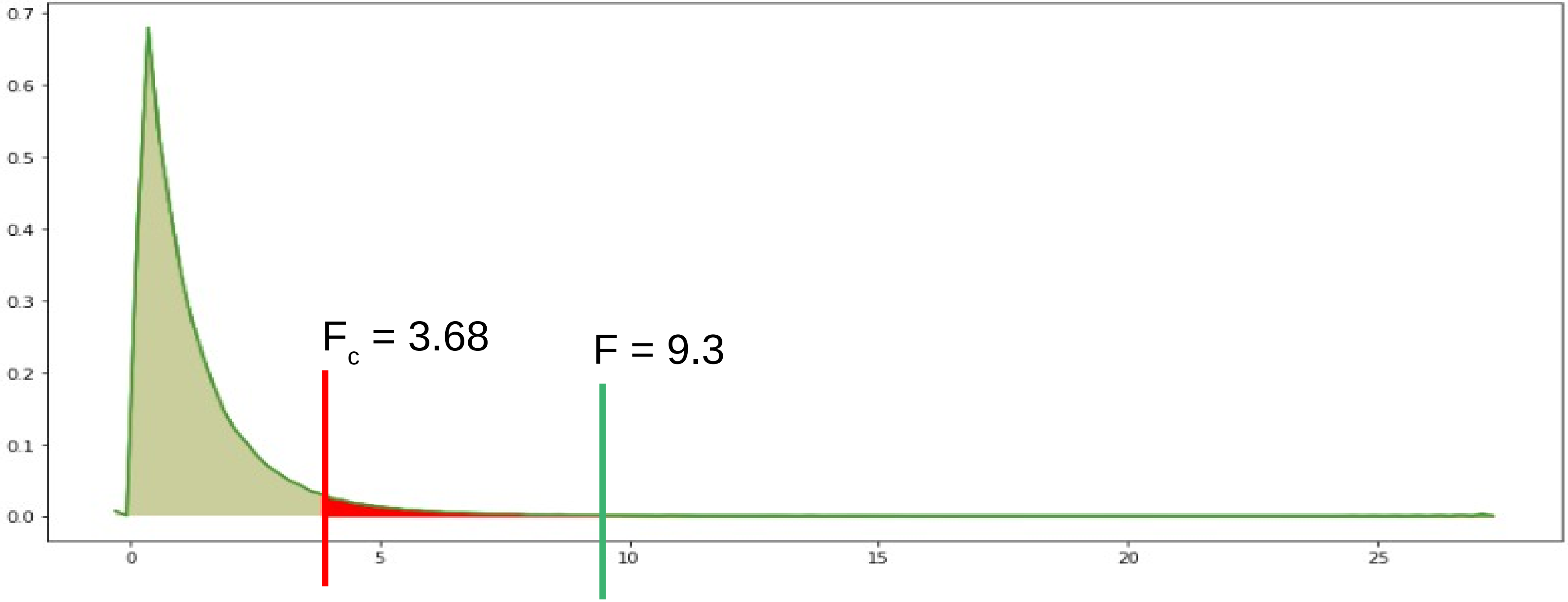


Table 1 α = 0.05

Degrees of Freedom for Denominator	Degrees of Freedom for Numerator															
	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50
1	161.4	199.5	215.8	224.8	230.0	233.8	236.5	238.6	240.1	242.1	245.2	248.4	248.9	250.5	250.8	252.6
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.44	19.46	19.47	19.48	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	1.97
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	1.86
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	1.82
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	1.79
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	1.76
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	1.66

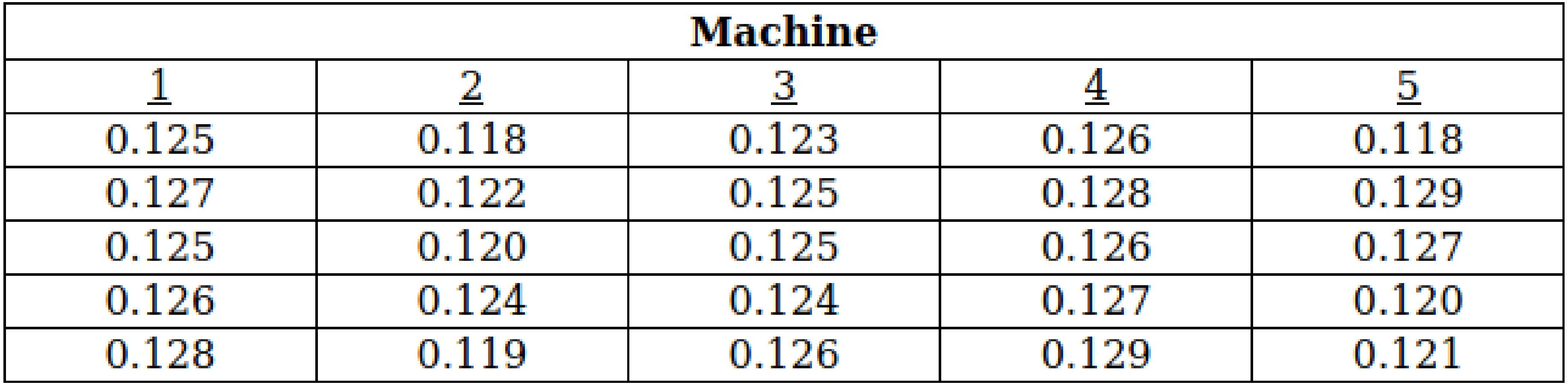
ANOVA



- Since F=9.3 > 3.68, the results are significant at the 5% significance level.
- We can reject the null hypothesis, concluding that there is strong evidence that the expected values in the three groups differ.

ANOVA

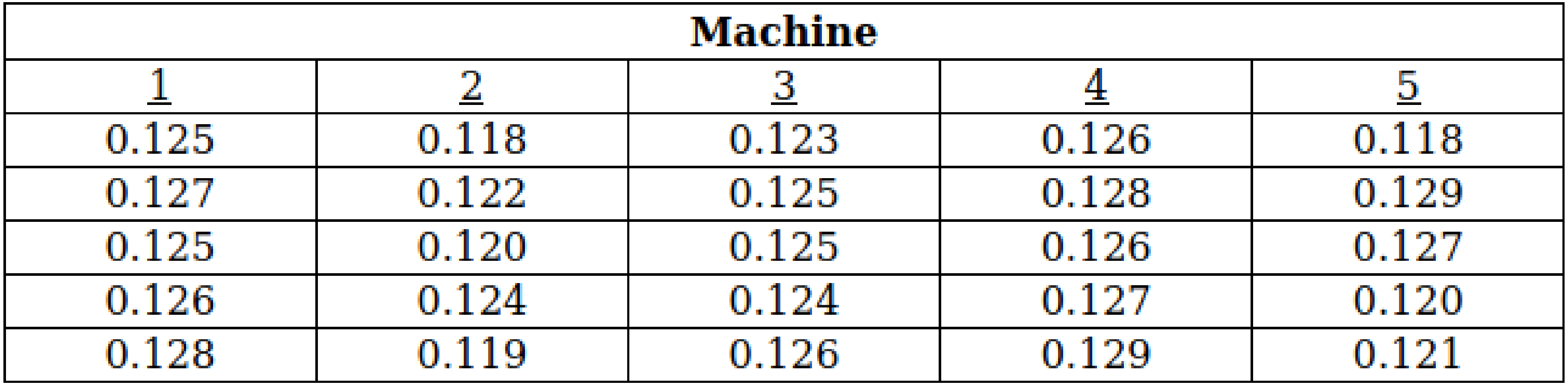
- we have five different machines making the same part and we take five random samples from each machine to obtain the following diameter data:



Machine				
1	2	3	4	5
0.125	0.118	0.123	0.126	0.118
0.127	0.122	0.125	0.128	0.129
0.125	0.120	0.125	0.126	0.127
0.126	0.124	0.124	0.127	0.120
0.128	0.119	0.126	0.129	0.121

ANOVA

- we have five different machines making the same part and we take five random samples from each machine to obtain the following diameter data:



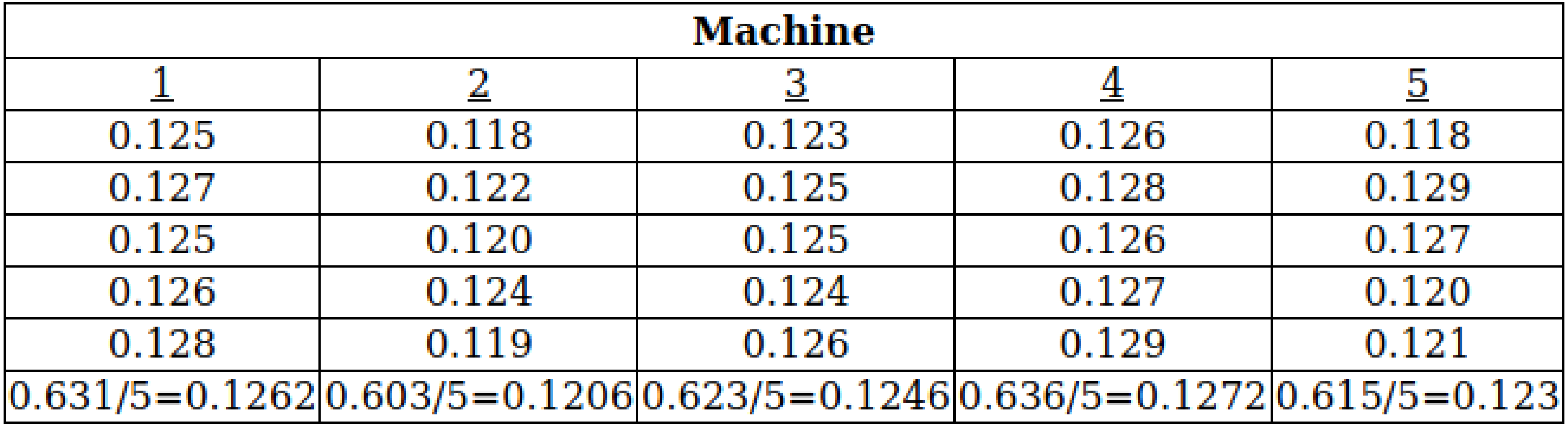
Machine				
1	2	3	4	5
0.125	0.118	0.123	0.126	0.118
0.127	0.122	0.125	0.128	0.129
0.125	0.120	0.125	0.126	0.127
0.126	0.124	0.124	0.127	0.120
0.128	0.119	0.126	0.129	0.121

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

$$H_1 : Not\ all\ means\ same$$

ANOVA

- **Step 1:** Calculate the mean within each group:



Machine				
1	2	3	4	5
0.125	0.118	0.123	0.126	0.118
0.127	0.122	0.125	0.128	0.129
0.125	0.120	0.125	0.126	0.127
0.126	0.124	0.124	0.127	0.120
0.128	0.119	0.126	0.129	0.121
0.631/5=0.1262	0.603/5=0.1206	0.623/5=0.1246	0.636/5=0.1272	0.615/5=0.123

- **Step2:** Calculate the overall mean:

  (0.1262+0.1206+0.1246+0.1272+0.123)/5 = 0.12432

ANOVA

- **Step 3:** Calculate the "**between-group" mean square** value:
- Between-group sum of squared difference:

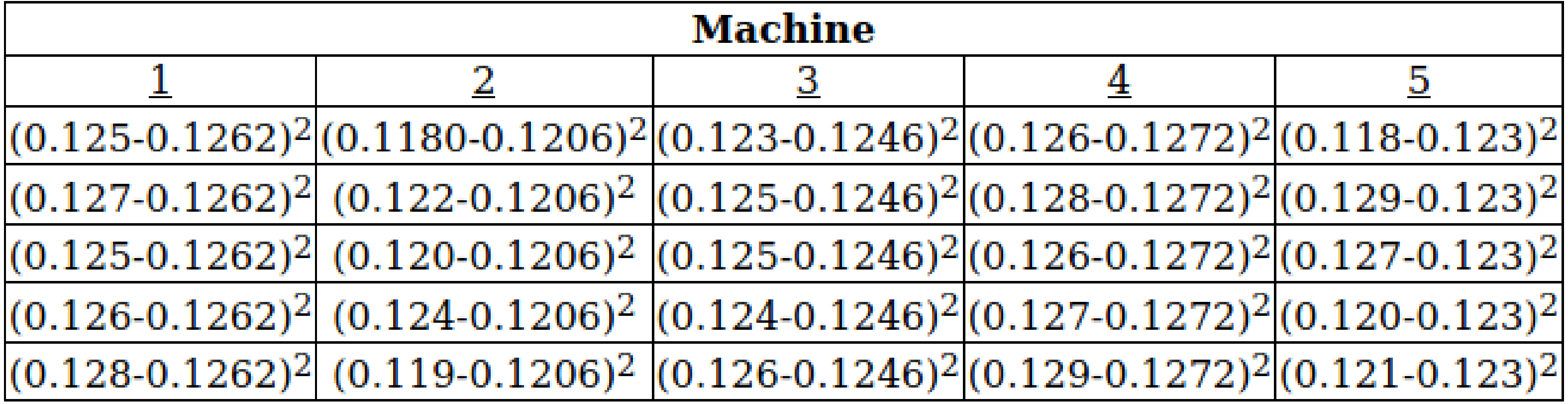
$$SS_B = 5(0.1262-0.12432)^2+5(0.1206-0.12432)^2+5(0.1246-0.12432)^2$$
$$+5(0.1272-0.12432)^2+5(0.123-0.12432)^2$$
$$SS_B = 5*0.0000035344+5*0.0000138384+5*0.0000000784$$
$$+5*0.0000082944+5*0.0000017424$$
$$SS_B = 0.000017672+0.000069192+0.000000392+0.000041472+0.000008712$$
$$SS_B = 0.00013744$$

- The between-group degrees of freedom is one less than the number of groups $df_b = 5 - 1 = 4$
- so the between-group mean square value is

$$MS_B=SS_B/df_b=0.00013744/4=0.00003436$$

ANOVA

- **Step 4:** Calculate the **"within-group" mean square** value.
- Within-group sum of squares.



Machine				
1	2	3	4	5
$(0.125-0.1262)^2$	$(0.1180-0.1206)^2$	$(0.123-0.1246)^2$	$(0.126-0.1272)^2$	$(0.118-0.123)^2$
$(0.127-0.1262)^2$	$(0.122-0.1206)^2$	$(0.125-0.1246)^2$	$(0.128-0.1272)^2$	$(0.129-0.123)^2$
$(0.125-0.1262)^2$	$(0.120-0.1206)^2$	$(0.125-0.1246)^2$	$(0.126-0.1272)^2$	$(0.127-0.123)^2$
$(0.126-0.1262)^2$	$(0.124-0.1206)^2$	$(0.124-0.1246)^2$	$(0.127-0.1272)^2$	$(0.120-0.123)^2$
$(0.128-0.1262)^2$	$(0.119-0.1206)^2$	$(0.126-0.1246)^2$	$(0.129-0.1272)^2$	$(0.121-0.123)^2$

- The within-group sum of squares is the sum of squares of all 25 values in this table

$$SS_W = 0.000132$$

- The within-group degrees of freedom is

$$df_w = a(n-1) = 5(5-1) = 20$$

ANOVA

- Thus the within-group mean square value is

$$MS_W = SS_W / df_w = 0.000132/20 = 0.000007$$

- **Step 5:** The F-ratio is

$$F\ statistc = \frac{MS_B}{MS_W} = 0.000034/0.000007 = 4.86$$

- In this case, Fcrit(4,20) = 2.87 at α = 0.05.

of the *F* Distribution

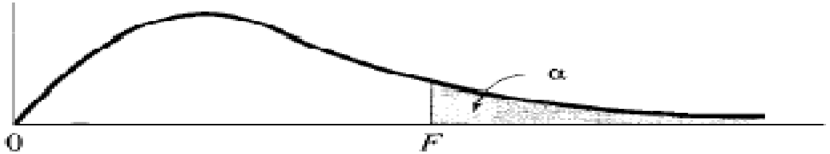


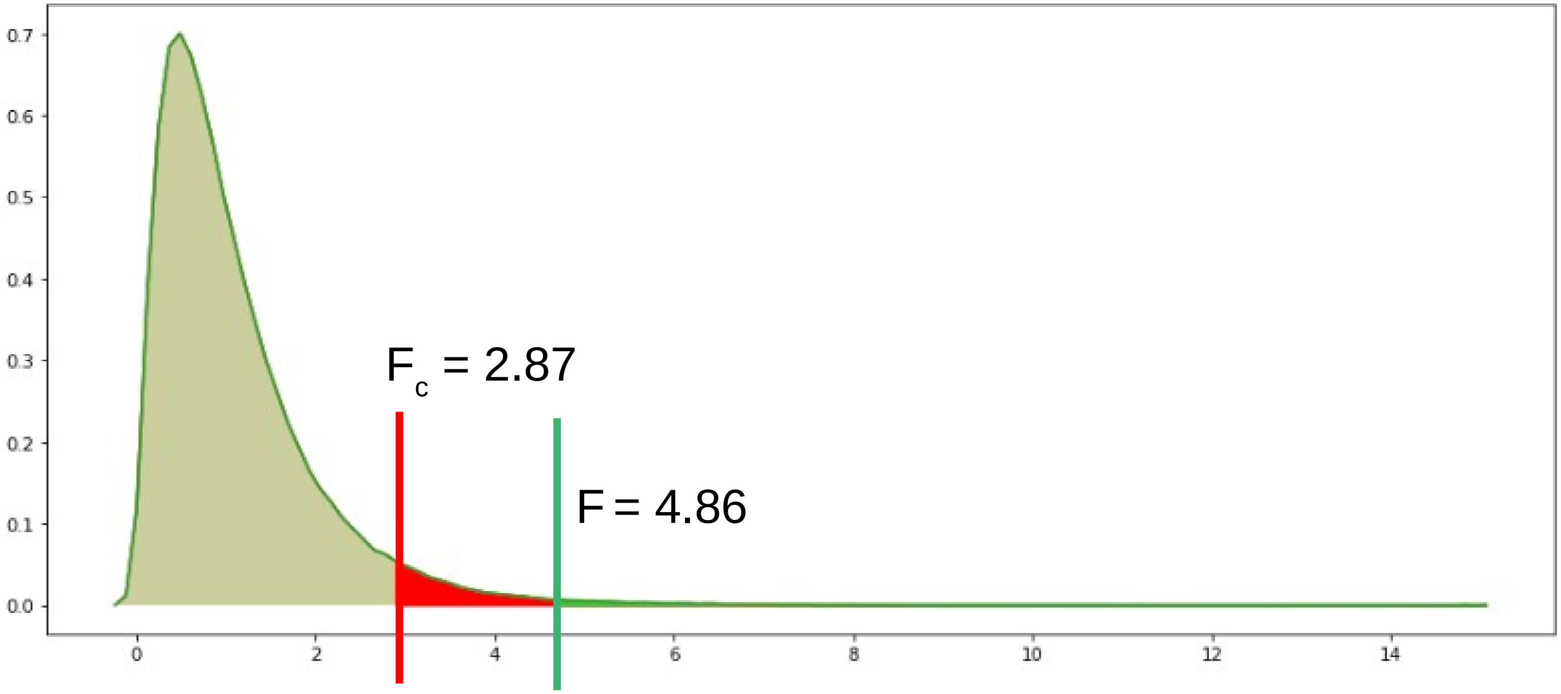
Table 1 α = 0.05

Degrees of Freedom for Numerator

Degrees of Freedom for Denominator

	1	2	3	4	5	6	7	8	9	10	15	20	25	30	40	50
1	161.4	199.5	215.8	224.8	230.0	233.8	236.5	238.6	240.1	242.1	245.2	248.4	248.9	250.5	250.8	252.6
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.43	19.44	19.46	19.47	19.48	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.70	8.66	8.63	8.62	8.59	8.58
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.86	5.80	5.77	5.75	5.72	5.70
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.62	4.56	4.52	4.50	4.46	4.44
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	3.94	3.87	3.83	3.81	3.77	3.75
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.51	3.44	3.40	3.38	3.34	3.32
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.22	3.15	3.11	3.08	3.04	3.02
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.01	2.94	2.89	2.86	2.83	2.80
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.85	2.77	2.73	2.70	2.66	2.64
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.72	2.65	2.60	2.57	2.53	2.51
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.62	2.54	2.50	2.47	2.43	2.40
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.53	2.46	2.41	2.38	2.34	2.31
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.46	2.39	2.34	2.31	2.27	2.24
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.40	2.33	2.28	2.25	2.20	2.18
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19	2.15	2.12
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15	2.10	2.08
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11	2.06	2.04
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07	2.03	2.00
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04	1.99	1.97
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98	1.94	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94	1.89	1.86
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90	1.85	1.82
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.04	1.96	1.91	1.87	1.82	1.79
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.01	1.93	1.88	1.84	1.79	1.76
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	1.92	1.84	1.78	1.74	1.69	1.66

ANOVA



- Since F=4.86 > 2.87, the results are significant at the 5% significance level.
- We can reject the null hypothesis, concluding that there is strong evidence that the expected values in the three groups differ.