

$$A1 \text{ score} = 2x$$

only accuracy can be misleading.

covid 19

| | <u>Actual</u> | <u>Predicted</u> |
|---------------------------|---------------|------------------|
| | 0 | 0 |
| | 0 | 0. |
| | 01 | 0 |
| | 0 | 0 |
| | 0 | 0 |
| MT2 syllabus | | |
| A* search | 0 | 6 |
| Game playing | 1 | 0 |
| Regression | 0 | 0 |
| Evaluation | 0 | 0 |
| Metrics | 0 | 0 |
| upto <u>this</u> syllabus | 0 | 0 |

| | |
|--------|-----------------|
| TP^0 | FN^2 |
| FP^0 | ϵ_{TN} |

$$\text{Accuracy} = \frac{0.8}{10} = 0.8 \\ 80\%$$

But here it is not valid.

Recall = $\frac{o}{o+g}$ undefined

~~to~~ mathematically
not correct

$$\text{precision} = \frac{0}{0}$$

Classification Algorithm

① Decision Tree.

Data:

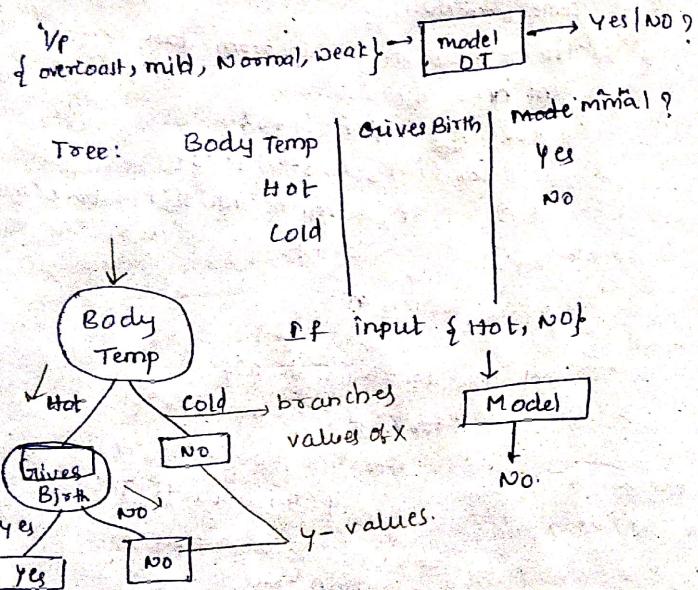
| Day | Outlook | Temperature | Humidity | Wind | play Golf? |
|-----|---------|-------------|----------|------|------------|
| | | | | | N |
| 1 | S | H | H | S | N |
| 2 | S | H | H | W | T |
| 3 | O | H | H | W | Y |
| 4 | R | M | N | W | Y |
| 5 | R | C | P | S | N |
| 6 | R | C | P | S | Y |
| 7 | O | C | H | W | N |
| 8 | S | M | N | W | Y |
| 9 | S | C | N | S | Y |
| 10 | R | M | H | S | Y |
| 11 | S | M | N | W | Y |
| 12 | O | H | H | S | N |
| 13 | O | M | N | W | Y |
| 14 | R | M | H | S | N |

classification:

decision Tree:

| outlook | Temp | Humidity | wind | Pby Golf? |
|---------|------|----------|------|--------------|
| | | | | |

After M7
17/11/2023
Friday



Entropy: measure of randomness.

more Random ⇒ less certainty ⇒ more entropy.

Less Randomness ⇒ more certainty ⇒ Less entropy.

eg: {0.5, 0.5} → less certainty → High Randomness High Entropy

{0.9, 0.1}

↓ low low
{1.0, 0.0}

→ We have total certainty, AND will win

From previous table Y, N pgoof?

$$\left\{ \frac{9}{10}, \frac{5}{10} \right\}$$

yes's no's
prob prob.

Entropy of a distribution = $-\sum_{i=1}^k p(x_i) \log_2 p(x_i)$

$$\{0.5, 0.5\} = -\left(\frac{1}{2} \log_2 \frac{1}{2} + \frac{1}{2} \log_2 \frac{1}{2}\right)$$

$$= -\frac{1}{2} \log_2 \frac{1}{2} - \frac{1}{2} \log_2 \frac{1}{2}$$

$$= -\log_2 \frac{1}{2}$$

$$= -\log_2 \frac{1}{2} + \log_2 \frac{1}{2}$$

$$= 1$$

$$\{1.0, 0.0\} = -\left(1 \cdot \log_2 1 + 0 \cdot \log_2 0\right)$$

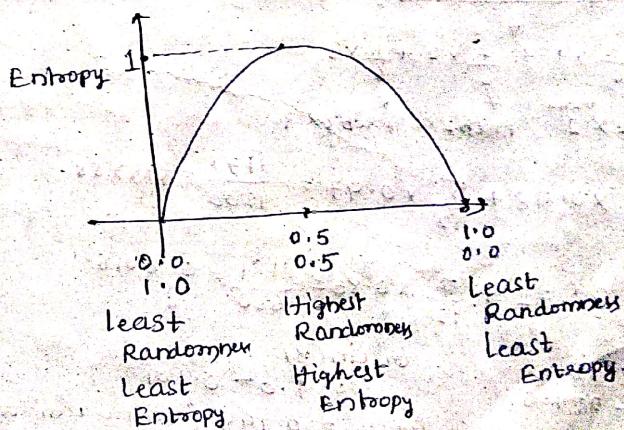
$$= 0$$

$$\{0.9, 0.1\} = -\left(\frac{9}{10} \log_2 \frac{9}{10} + \frac{1}{10} \log_2 \frac{1}{10}\right)$$

$$= 0.03118$$

$$= +ve.$$

Entropy plot

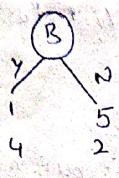
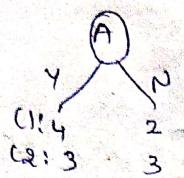


Using Entropy to select attribute

| A | B | clay |
|---|---|-------|
| Y | Y | C1: 6 |
| N | N | C2: 6 |



Scanned with OKEN Scanner



Entropy for A

$$\begin{aligned} & 6 \quad 6 \\ & -\frac{6}{12} \log_2 \frac{6}{12} - \frac{6}{12} \log_2 \frac{6}{12} \\ & = 1. \end{aligned}$$

$$\left\{ \frac{4}{7}, \frac{3}{7} \right\} \left\{ \frac{1}{5}, \frac{4}{5} \right\}$$

more certainty.

For children entropy calculation

| A | B | C |
|--|--|--|
| $c_1: 4$ | $c_1: 2$ | $c_1: 5$ |
| $c_2: 3$ | $c_2: 3$ | $c_2: 4$ |
| $-\frac{4}{7} \log_2 \frac{4}{7} - \frac{3}{7} \log_2 \frac{3}{7}$ | $-\frac{2}{3} \log_2 \frac{2}{3} - \frac{2}{3} \log_2 \frac{2}{3}$ | $-\frac{5}{7} \log_2 \frac{5}{7} - \frac{2}{7} \log_2 \frac{2}{7}$ |
| $= 0.98$ | $= 0.97$ | $= 0.72$ |

Average entropy

Expectation

$$\begin{aligned} & 1, 1, 1, 2, 2, 3 \\ & = \frac{1+1+1+2+2+3}{6} \\ & = \frac{3 \times 1 + 2 \times 2 + 1 \times 3}{6} \end{aligned}$$

$$\begin{aligned} & = \frac{3}{6} \times 1 + \frac{2}{6} \times 2 + \frac{1}{6} \times 3 \\ & \downarrow \text{value} \quad \downarrow \text{prob of } 2 \times 1 \quad \downarrow \text{prob of } 2 \times 2 \quad \downarrow \text{prob of } 3 \times 3 \end{aligned}$$

$$\frac{7}{12} \times 0.98 + \frac{5}{12} \times 0.97 = \frac{1171}{1200} = 0.9758$$

$$\frac{5}{12} \times 0.72 + \frac{7}{12} \times 0.86 = \frac{481}{600} = 0.8016$$

Gain(A)

$$\begin{aligned} & = 1.0 - 0.9758 \\ & = 0.025 \end{aligned}$$

Gain(B)

$$\begin{aligned} & = 1.0 - 0.8016 \\ & = 0.199 \quad (\text{reduce randomness}) \\ & \text{B is selected because it is reducing the randomness.} \end{aligned}$$

| Decision Tree: | | | | |
|----------------|------|----------|------|------------|
| outlook | Temp | Humidity | wind | play golf! |
| | | | | Wednesday |

Gain = Entropy (parent) - Average Entropy of children

$Y: 9 \quad N: 5$ parent

$$\text{Entropy} = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

for children.

$$-\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$-\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$-\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97$$

$$\text{Average children Entropy} = \frac{5}{14} \times 0.97 + \frac{4}{14} \times 0 + \frac{5}{14} \times 0.97 = 0.69$$

parent [Temp] same

Temp

$$H \Rightarrow \frac{2}{2} = 1$$

$$m \Rightarrow -\frac{4}{6} \log_2 \frac{4}{6} - \frac{2}{6} \log_2 \frac{2}{6} = 0.92$$

$$C \Rightarrow -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.81$$

$$\text{Average children Entropy} = \frac{4}{14} \times 1 + \frac{6}{14} \times 0.92 + \frac{4}{14} \times 0.81 = 0.91$$

$$H \Rightarrow -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.98$$

$$N \Rightarrow -\frac{6}{7} \log_2 \frac{6}{7} - \frac{1}{7} \log_2 \frac{1}{7} = 0.59$$

$$\text{Average children Entropy} = \frac{7}{14} \times 0.98 + \frac{7}{14} \times 0.59 = 0.79$$

Humidity

$$H \Rightarrow 0$$

$$N \Rightarrow 1$$

$$W \Rightarrow -\frac{6}{8} \log_2 \frac{6}{8} - \frac{2}{8} \log_2 \frac{2}{8} = 0.81$$

$$G \Rightarrow -\frac{3}{6} \log_2 \frac{3}{6} - \frac{3}{6} \log_2 \frac{3}{6} = 1$$

$$S \Rightarrow \frac{8}{14} \times 0.81 + \frac{6}{14} \times 1 = 0.89$$

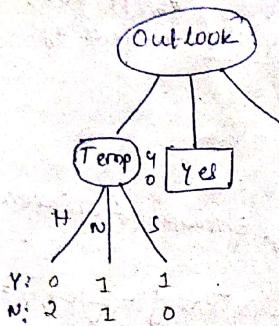
$$\text{Gain}(outlook) = 0.94 - 0.69 = 0.25 \quad \text{selected}$$

$$\text{Gain(Temp)} = 0.94 - 0.91 = 0.03 \quad \text{not}$$

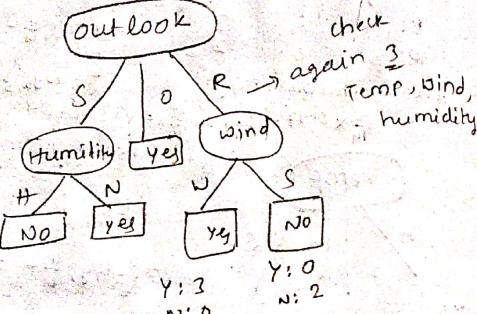
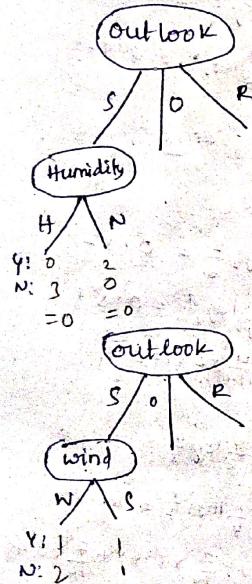
$$\text{Gain(Humidity)} = 0.94 - 0.79 = 0.15$$

$$\text{Gain(Wind)} = 0.94 - 0.89 = 0.05$$





final model.



e.g.: day = {sunny, high, normal, weak}.
O/p: Yes
H → S → N (at Humidity)
day = {Rainy, cold, high, strong}
O/p: NO.
R → S (at wind).

Stopping criteria:

① All examples belongs to same class.

$$Y: 0 \quad N: 7 \Rightarrow \boxed{\text{No}}$$

② Reached maximum depth, assign majority label.

$$Y: 9 \quad N: 1 \Rightarrow \boxed{\text{Yes}}$$

Bayesian classifier:
↳ Naive Bayes classification.
probability - Marginal
- Joint
- conditional.
e.g.: Bank employee.

| | Rank | | | Total |
|--------|------|-----|-----|-------|
| Gen | R1 | R2 | R3 | |
| Male | 30 | 80 | 90 | 200 |
| Female | 20 | 40 | 40 | 100 |
| Total | 50 | 120 | 130 | 300 |

Marginal: $P(\text{Gen} = \text{male}) = \frac{200}{300}$
 $P(\text{Rank} = R_2) = \frac{120}{300}$

Joint: $P(\text{Gen} = \text{male} \text{ and } \text{Rank} = R_2) = \frac{80}{300}$

Marginal using Joint: $P(\text{Gen} = \text{male}) =$
 $P(\text{Gen} = M \text{ and } \text{Rank} = R_1) + P(\text{Gen} = M \text{ and }$
 $\text{Rank} = R_2) + P(\text{Gen} = M \text{ and rank} = R_3)$.
 $= \frac{30}{300} + \frac{80}{300} + \frac{90}{300}$
 $= \frac{200}{300}$

conditional: $P(\text{Rank} = R_2 | \text{Gen} = M) = \frac{80}{200}$.

↳ can be expressing using Joint and probability.

$$= \frac{P(\text{Rank} = R_2 \text{ and Gen} = m)}{P(\text{Gen} = m)}$$

$$= \frac{\frac{80}{300}}{\frac{200}{300}} = \frac{80}{200}$$

Formulae:

$$P(Y/x) = \frac{P(x \text{ and } Y)}{P(x)}$$



$$P(X \text{ and } Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

$$P(X|Y) = \frac{P(X \text{ and } Y)}{P(Y)}$$

$$P(X \text{ and } Y) = P(X|Y)P(Y)$$

$$P(Y|X) \cdot P(X) = P(X|Y)P(Y)$$

$$P(Y|X) = \frac{P(X|Y)P(Y)}{P(X)}$$

Bayes theorem

e.g.: Day = {sunny, cold, High, weak}

$$P(\text{Play} = \text{Yes} | \text{Day})$$

$$P(\text{Play} = \text{No} | \text{Day})$$

$$P(Y = \text{Yes} | \text{Day}) = \frac{P(\text{Day} | \text{Yes})P(\text{Yes})}{P(\text{Day})}$$

Bayesian classification

- Naive Bayes classifier.

- Marginal - $P(X)$

- Joint - $P(X \text{ and } Y)$

- conditional - $P(Y|X)$

$X = \{\text{Rainy, Cold, High, Strong}\}$

$$P(Y = \text{Yes} | X) = ?$$

$$P(Y = \text{No} | X) = ?$$

$$\text{From } P(Y = \text{Yes} | X) = \frac{P(Y = \text{Yes} \text{ and } X)}{P(X)}$$

$$= \frac{P(X | Y = \text{Yes})P(Y = \text{Yes})}{P(X)}$$

$$= \frac{P(X | Y = \text{Yes})P(Y = \text{Yes})}{P(X \text{ and } Y = \text{Yes}) + P(X \text{ and } Y = \text{No})}$$

$$= \frac{P(X | Y = \text{Yes})P(Y = \text{Yes})}{P(X | Y = \text{Yes})P(Y = \text{Yes}) + P(X | Y = \text{No})P(Y = \text{No})}$$

↑ Bayes
↓ confirmation
This is Bayes' thm.

To simplify, we use Naive Bayes Assumption: Input variables are independent (given class label). If independent variables something you are oversimplifying.
 $P(X_1, X_2) = P(X_1) \cdot P(X_2)$.

$$P(X = \{x_1, x_2, x_3, x_4\} | Y = \text{Yes})$$

$$= P(X = x_1 | Y = \text{Yes}) \cdot P(X = x_2 | Y = \text{Yes}) \cdot P(X = x_3 | Y = \text{Yes})$$

$$P(X = x_4 | Y = \text{Yes})$$

Assumption: Input variables does not depend on each other, they are independent.

Rewrite using Naive Bayes Assumption.

↳ continuation.

$$= \frac{P(X = x_1 | Y = \text{Yes}) \cdot P(X = x_2 | Y = \text{Yes}) \cdot P(X = x_3 | Y = \text{Yes}) \cdot P(X = x_4 | Y = \text{Yes})P(Y = \text{Yes})}{P(X = x_1 | Y = \text{Yes}) \cdot P(X = x_2 | Y = \text{Yes}) \cdot P(X = x_3 | Y = \text{Yes}) \cdot P(X = x_4 | Y = \text{Yes})P(Y = \text{Yes})}$$

$$+ P(X = x_1 | Y = \text{No}) \cdot P(X = x_2 | Y = \text{No}) \cdot P(X = x_3 | Y = \text{No}) \cdot P(X = x_4 | Y = \text{No})P(Y = \text{No}).$$

↓ This is Naive Bayes theorem.

from $X = \{\text{Rainy, Cold, High, Strong}\}$

$$P(Y = \text{Yes} | X = \{\text{Rainy, Cold, High, Strong}\})$$

$$= \frac{P(\text{Rainy} | \text{Yes}) \cdot P(\text{Cold} | \text{Yes}) \cdot P(\text{High} | \text{Yes}) \cdot P(\text{Strong} | \text{Yes})}{P(\text{Rainy} | \text{Yes}) \cdot P(\text{Cold} | \text{Yes}) \cdot P(\text{High} | \text{Yes}) \cdot P(\text{Strong} | \text{Yes}) + P(\text{Rainy} | \text{No}) \cdot P(\text{Cold} | \text{No}) \cdot P(\text{High} | \text{No}) \cdot P(\text{Strong} | \text{No})}$$

$$P(Y = \text{Yes}) = \frac{9}{14}$$

$$P(\text{No}) = \frac{5}{14}$$

$$P(\text{outlook} | Y)$$

| | yes | no | |
|----------|-------------------------|------------------------|-----|
| | $P(\cdot \text{Yes})$ | $P(\cdot \text{No})$ | |
| sunny | 2/3 | 2/9 | 3/5 |
| overcast | 4/9 | 4/9 | 0/5 |
| rainy | 3/8 | 3/9 | 2/5 |



$P(\text{Temp} | Y)$

| | Yes | No | $P(\cdot \text{Yes})$ | $P(\cdot \text{No})$ |
|------|-----|----|-------------------------|------------------------|
| Hot | 2 | 2 | 2/9 | 2/5 |
| mild | 4 | 2 | 4/9 | 2/5 |
| cold | 3 | 1 | 3/9 | 1/5 |

$P(\text{Humidity} | Y)$

| | Yes | No | $P(\cdot \text{Yes})$ | $P(\cdot \text{No})$ |
|--------|-----|----|-------------------------|------------------------|
| High | 3 | 4 | 3/9 | 4/5 |
| Normal | 6 | 1 | 6/9 | 1/5 |

$P(\text{wind} | Y)$

| | Yes | No | $P(\cdot \text{Yes})$ | $P(\cdot \text{No})$ |
|--------|-----|----|-------------------------|------------------------|
| Strong | 3 | 3 | 3/9 | 3/5 |
| weak | 6 | 2 | 6/9 | 2/5 |

$$= \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{9}{14}$$

$$\frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{3}{9} \times \frac{9}{14} + \frac{2}{5} \times \frac{1}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{5}{14}$$

$$= 0.37$$

$$P(Y=\text{no} | X) = 1 - P(Y=\text{yes} | X) = 1 - 0.37$$

$$= 0.63$$

Output: $P(Y=\text{no} | X) > P(Y=\text{yes} | X)$
 "no" → will not play the golf

Email classification.

| Set of words | |
|---|---------------|
| content | label |
| w ₁ , w ₂ , w ₃ , w ₄ | no → not spam |
| w ₅ , w ₆ , w ₇ | yes |

$$\text{new email} = \{w_1, w_2, w_3, w_4\}$$

$$= \{\text{you}, \text{won}, \text{RS}, \text{10L}\}$$

$$\begin{aligned} \text{90% if will correctly implement} &= P(\text{yes} | \text{you won RS 10L}) \\ \rightarrow \text{you need data for tables programs} &= \frac{P(\text{you yes}) \cdot P(\text{won yes}) \cdot P(\text{RS yes}) \cdots P(\text{yes})}{P(\text{you yes}) \cdots P(\text{yes}) + P(\text{you no}) \cdots P(\text{no})} \end{aligned}$$



$$L = \{a^m b^n c^n \mid m, n \geq 1\}$$

aa bcc

$$L = \{a b c, a a b c c, a a b b c c, a a a b c c c, \dots\}$$

$$\begin{aligned} \delta(q_0, a, z_0) &= (q_0, a, z_0) \\ \delta(q_0, a, a) &= (q_0, a a, z_0) \\ \delta(q_0, b, a) &= (q_1, a a, z_0) \\ \delta(q_1, c, a) &= (q_2, \epsilon, z_0) \\ \delta(q_2, \epsilon, z_0) &= (q_3, \epsilon) \end{aligned} \quad \left| \begin{array}{l} (q_0, a a b c c, z_0) \xrightarrow{} (q_0, b c c, a a z_0) \\ \xrightarrow{} (q_1, c c, b a z_0) \\ \xrightarrow{} (q_2, c, a z_0) \\ \xrightarrow{} (q_2, \epsilon, z_0) \end{array} \right.$$

Accepted $\boxed{F'(q_3, \epsilon)}$

23/11/23

Pumping Lemma for Regular languages

Let L be a regular language. There will be a natural no. 'n' (pumping lemma constant) for every $\bar{z} \in L$ & $|z| \geq n$, \bar{z} can be written as uvw , satisfying

i) $|v| \geq 1$
ii) ~~uv~~ $|uv| \leq n$

iii) $uv^i w \in L$ for all $i \geq 0$.

\Rightarrow Pumping Lemma is necessary but not sufficient condition for RLs.

\Rightarrow So this can be used to prove a language as

Non-regular if it doesn't satisfy pumping lemma.

* If RL only pumping lemma, we can not say it is compulsory RL

Pumping Lemma for context free languages

Let L be a context-free language. There will be a Natural n (pumping lemma constant depending only on L , for every $z \in L$ & $|z| \geq n$, z can be written as $uvwxy$ satisfying

$$i) |vxi| \geq 1$$

$$ii) |uvw| \leq n$$

$$iii) uv^iw^xv^y \in L \text{ for all } i \geq 0.$$

→ Pumping Lemma is necessary but not sufficient condition for CFLs.

→ If, this can be used to prove a language is NonCFL if it doesn't satisfy Pumping Lemma.

- * Both 'v' and 'w' cannot be epsilon at a time.
- * (v) any one can be epsilon.

Equivalence b/w CFG & PDA

$$\text{CFG} \Rightarrow \text{PDA}$$

$$\text{PDA} \Rightarrow \text{CFG}$$

- ⇒ If CFG exists, then PDA will exist.
- ⇒ If PDA exists, then CFG will exist.

Conversion of CFG to PDA

⇒ Given a stack.

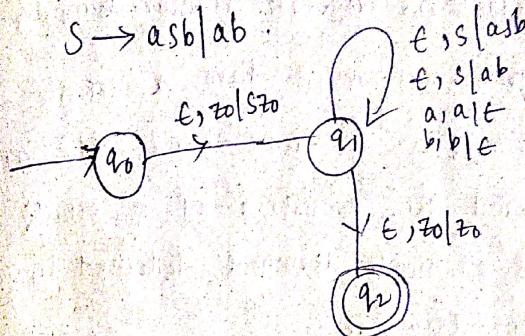
⇒ we push into stack

⇒ If top of the stack is Non-terminal then we (push) substitute the right hand side. of the production

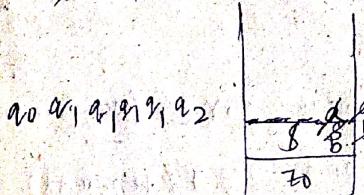
⇒ If top of the stack is terminal and if it matches the string then we pop.

$$L = \{a^m b^m \mid m \geq 1\}$$

$$S \rightarrow asb \mid ab$$

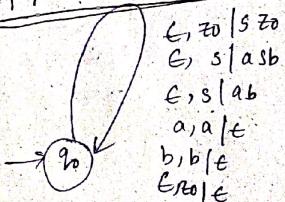


Let, aabb



(i)

Empty stack acceptance



⇒ A CFG can be converted into PDA with only single state by empty stack acceptance.



① Type-3 grammar :-

⇒ Type-3 grammars generate regular languages.

⇒ Type-3 grammars must have a single non-terminal on the left-hand side and right-hand side consisting of a single terminal or single terminal followed by a single non-terminal.

⇒ The productions must be in the form

$$X \rightarrow a$$

(or)

$$X \rightarrow aY$$

where $X, Y \in N$ (Non-Terminal)

and $a \in T$ (Terminal).

∴ The rule $S \rightarrow e$ is allowed if S does not appear on the right side of any rule.

Eg :-

$$X \rightarrow e$$

$$X \rightarrow a/aY$$

$$X \rightarrow b$$

② Type-2 grammar :-

⇒ Type-2 grammars generate Context-free languages.

⇒ The productions must be in the form $A \rightarrow \gamma$ where $A \in N$ (Non-terminal)

and $\gamma \in (T \cup N)^*$ (String of terminal and non-terminals).

⇒ These languages generated by these grammars are recognized by a non-deterministic pushdown automation.

Eg :-

$$S \rightarrow Xa$$

$$X \rightarrow a$$

$$X \rightarrow aX$$

$$X \rightarrow abc$$

$$X \rightarrow e$$

③ Type-1 grammar :-

⇒ Type-1 grammars generate Context-sensitive languages.

⇒ The productions must be in the form

$$\alpha A \beta \rightarrow \alpha \gamma \beta$$

where $A \in N$ (Non-Terminal)

and $\alpha, \beta, \gamma \in (T \cup N)^*$ (Strings of terminals and Non-terminals)

⇒ The strings α and β may be empty, but γ must be non-empty.

⇒ The rule $S \rightarrow e$ is allowed if S does not appear on the right side of any rule. The languages generated by these grammars are recognized by a linear bounded automaton.

Eg :-

$$AB \rightarrow ABBc$$

$$A \rightarrow bcA$$

$$B \rightarrow b$$



④ Type -0 grammars

- ⇒ Type-0 grammars generate recursively enumerable languages.
- ⇒ The productions have no restrictions.
- ⇒ They are any phrase structure grammar including all formal grammars.
- ⇒ They generate the languages that are recognized by a Turing Machine.
- ⇒ The productions can be in the form of $\alpha \rightarrow \beta$ where α is a string of terminals and non-terminals with atleast one non-terminal and α cannot be null.
 β is a string of terminals and non-terminal

Eg:-

$$S \rightarrow ACAB$$

$$Bc \rightarrow acB$$

$$CB \rightarrow DB$$

$$\alpha D \rightarrow Db$$