fourier transform: continuous spectrum Representation

Spectrum = signal.

$t \rightarrow \omega_0$

In discrete spectrum

trigonometric	exponential
a_1	c_1
a_2	c_2
a_3	c_3

$x(t) = x(\omega)$

$x(t) = x(j\omega)$

$x(t) = x(f)$

Missed class.
1/11/2023

⇒ Transformers.

(i) fourier series
(ii) fourier transform
(iii) laplace transform
} Techniques.

(iv) Z-transform

→ converting one domain to another domain

time domain ⇒ frequency domain

Using Amplitude is converting from one domain to another domain.

$\omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{\omega_0}$

$x(t) \rightarrow$ we are writing it in the form of frequency

Time domain ⟶ frequency domain.

why Time domain ⟶ frequency domain

In both the cases, we are giving as voltage.

Fourier Series:- These are applicable for periodic signals.

trigonometric
exponential
} periodic

eg:- class, voice has frequency.

→ we are defining/determining the voltage using frequency.

Trigonometric form

$x(t) = A \sin \omega_0 t$

$x(t) = A \cos \omega_0 t$

Exponential form

$x(t) = A e^{j\omega_0 t}$

$e^{j\theta} = \cos\theta + j\sin\theta$

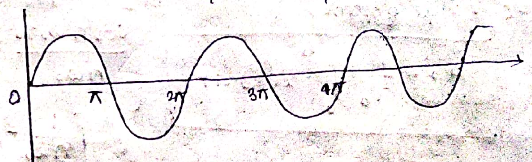
⇒ Representing in Sin & Cos will become trigonometric fourier series.

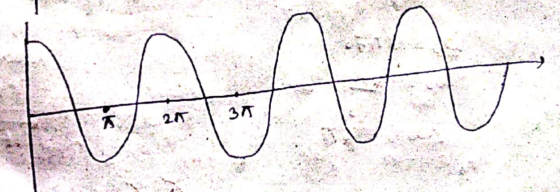
$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \cdots + \cdots a_n \cos n\omega_0 t \longrightarrow (1)$

$x(t) = b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \cdots + b_n \sin n\omega_0 t \longrightarrow (2)$

Adding (1) & (2)

$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \cdots + a_n \cos n\omega_0 t + b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \cdots + \cdots$



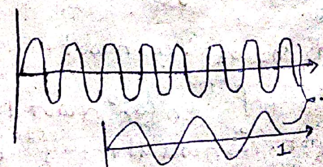


$x(t) = a_0 + a_1 \cos 1\omega_0 t + \cdots + a_n \cos n\omega_0 t + \cdots b_1 \sin 1\omega_0 t + \cdots + b_n \sin n\omega_0 t$

Trigonometric fourier Series:

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$a_0, b_n, a_n \longrightarrow$ are called fourier coefficient/DC values.



→ High frequency signal is reached fastly.

→ frequency analysis.

low frequency

⇒ Easy analysis in our signal will give Amplitude

$$\int_0^{T_0} x(t)dt = \int_0^{T_0} a_0 dt + \int_0^{T_0} \sum_{n=1}^{\infty} a_n \cos n\omega_0 t\, dt + \int_0^{T_0} \sum_{n=1}^{\infty} b_n \sin n\omega_0 t\, dt$$

$$\int_0^{T_0} x(t)dt = a_0 T_0 + \sum_{n=1}^{\infty} a_n \left.\frac{\sin n\omega_0 t}{n\omega_0}\right|_0^{T_0} - b_n \sum_{n=1}^{\infty} \frac{\cos\omega_0 t}{n\omega_0}$$

$$\int_0^{T_0} x(t)dt = a_0 T_0 + \left[\frac{\sin n\omega_0 T_0}{n\omega_0} - \frac{\sin n\omega_0}{n\omega_0}\right]$$

$$\frac{\sin 2\pi n}{n\omega_0} - 0$$

$$0 - 0 = 0.$$

DC value become,

$$\boxed{a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\cdot dt}$$

$$\left[\frac{\cos n\omega_0 T_0}{n\omega_0} - \frac{\cos n_0 \omega_0}{n\omega_0}\right]$$

$$\left[\frac{1}{n\omega_0} - \frac{1}{n\omega_0}\right] = 0$$

$\omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T_0}$

$\omega_0 T_0 = 2\pi$

$$\boxed{\int_0^{T_0} \cos n\omega_0 t\, dt = 0. \qquad \int_0^{T_0} \sin n\omega_0 t\, dt = 0.}$$

To find coefficients

$$\int_0^{T_0} x(t) \cos n\omega_0 t\, dt = \int_0^{T_0} a_0 \cos n\omega_0 t\, dt + \int_0^{T_0} a_1 \cos\omega_0 t \cdot \cos n\omega_0 t + \cdots + \int_0^{T_0} a_n \cos^2 n\omega_0 t\, dt + \int_0^{T_0} b_1 \sin\omega_0 t \cdot \cos n\omega_0 t\, dt + \cdots \int_0^{T_0} b_n \sin n\omega_0 t \cos n\omega_0 t\, dt$$

$$\int_0^{T_0} x(t)\cdot \cos n\omega_0 t\, dt = \int_0^{T_0} a_n \cos^2 n\omega_0 t\, dt$$

$$= \int_0^{T_0} a_n \cdot \frac{1+\cos 2n\omega_0 t}{2}\, dt$$

$$\int_0^{T_0} x(t)\cdot \cos n\omega_0 t\, dt = \int_0^{T_0} \frac{a_n}{2}\, dt + \int_0^{T_0} \frac{\cos 2n\omega_0 t}{2}$$

$$\int_0^{T_0} x(t)\cdot \cos n\omega_0 t\, dt = \int_0^{T_0} \frac{a_n}{2}\, dt + \int_0^{T_0} \frac{\cos 2n\omega_0 t}{2}$$

$$\boxed{a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot \cos n\omega_0 t\, dt.}$$

→ Sine.

$$\int_0^{T_0} x(t) \sin n\omega_0 t\, dt = \int_0^{T_0} a_0 \sin n\omega_0 t\, dt + \int_0^{T_0} a_1 \cos\omega_0 t \cdot \sin n\omega_0 t + \cdots + \int_0^{T_0} a_n \cos n\omega_0 t \sin n\omega_0 t + \int_0^{T_0} b_1 \sin\omega_0 t + \sin n\omega_0 t\, dt + \cdots + \int_0^{T_0} b_n \sin^2 n\omega_0 t\, dt$$

$$\int_0^{T_0} x(t)\cdot \sin n\omega_0 t\, dt = \int_0^{T_0} b_n \sin^2 n\omega_0 t\, dt$$

$$= \int_0^{T_0} b_n \frac{1-\cos 2n\omega_0 t}{2}\, dt$$

$$\boxed{b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot \sin n\omega_0 t\, dt}$$

$$\int_0^{T_0} x(t) \sin n\omega_0 t\, dt = \int_0^{T_0} \frac{a_n}{2}\, dt + \int_0^{T_0} \frac{\cos 2n\omega_0 t}{2}$$

$$\int_0^{T_0} x(t) \sin n\omega_0 t\, dt = \frac{a_n}{2}\cdot T_0$$

Exponential fourier series.

$$x(t) = C_0 e^{j0\omega_0 t} + C_1 e^{j1\omega_0 t} + \ldots \ldots + C_n e^{jn\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_2 e^{-j2\omega_0 t} + \ldots + C_{-n} e^{-jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

$$\boxed{C_n = \frac{1}{T_0}\int_0^{T_0} x(t) \cdot e^{-jn\omega_0 t} dt}$$

. C_n is the coefficient of exponential fourier series.

08/11/2023
Wednesday.

Fourier transform:

t - time domain. frequency domain.

⇒ replacing t with only ω.

$t \to \omega$: fourier transform

$\omega \to t$: Inverse fourier transform

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T}\int_{-\infty}^{\infty} x(t) \cdot e^{-jn\omega_0 t} dt \qquad x(t) \underset{IFT}{\overset{FT}{\rightleftharpoons}} x(\omega)$$

$$x(t) = \sum_{n=-\infty}^{\infty} \Delta f \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \cdot e^{j\omega t}$$

F.T: $x(\omega)$ = oper $x(t)$

IFT: $x(t)$ = operation($x(\omega)$)

$$x(t) = \sum_{n=-\infty}^{\infty} df \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \, e^{j\omega t}$$

$t \to n\omega_0 \to \omega$

$\to \omega_0$

$\omega = 2\pi f$

$d\omega = 2\pi df$

$\frac{d\omega}{2\pi} = df$

$$x(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \cdot e^{j\omega t}$$

$$x(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} x(\omega) e^{j\omega t}$$

$$x(t) = \int_{-\infty}^{\infty} x(\omega) e^{j\omega t} \frac{d\omega}{2\pi}$$

* $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) \cdot e^{j\omega t} d\omega$

↳ Inverse fourier transform

* $x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$

↳ fourier transform

Fourier transform properties

$x(t)$

(i) linearity. $x(t) \underset{IF\cdot T}{\overset{F\cdot T}{\rightleftharpoons}} x(\omega)$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega) e^{j\omega t} \cdot d\omega$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$\alpha x_1(t) + \beta x_2(t) \underset{IFT}{\overset{F\cdot T}{\rightleftharpoons}} \alpha x_1(\omega) + \beta x_2(\omega)$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} (\alpha x_1(t) + \beta x_2(t)) e^{-j\omega t} dt.$$

using

$x_1(t) — x_1(\omega)$

$x_2(t) — x_2(\omega)$

$$x^*(\omega) = \alpha x_1(\omega) + \beta x_2(\omega)$$

$$\therefore \int_{-\infty}^{\infty} x_1(t) e^{-j\omega t} dt = x_1(\omega).$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega) e^{j\omega t} d\omega.$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} (\alpha x_1(\omega) + \beta x_2(\omega)) e^{j\omega t} d\omega.$$

$$x^*(t) = \alpha x_1(t) + \beta x_2(t).$$

∴ obeys linearity property.

Time scaling: $x(t) — x(\omega)$

$x(at) — x^*(\omega)$

$\tau = at$ $t = -\infty$ $\tau = -\infty$ $t = \frac{\tau}{a}$

$t = \infty$ $\tau = \infty$ $t = \tau \cdot \frac{1}{a}$

$dt = \frac{d\tau}{a}$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} x(at)\, e^{-j\omega \frac{\tau}{a}} \frac{d\tau}{a}$$

$$x^*(\omega) = \frac{1}{a} \int_{-\infty}^{\infty} x(\tau) \cdot e^{-j\frac{\omega}{a}\tau} d\tau$$

$$x^*(\omega) = \frac{1}{a}\, x\left(\frac{\omega}{a}\right)$$

time shifting:

$x(t) \longrightarrow x(\omega)$

$x(t+a) \longrightarrow x^*(\omega)$

$\tau = t+a \quad t = -\infty \quad \tau = -\infty \quad dt = d\tau$

$t = +\infty \quad \tau = +\infty \quad t = \tau - a$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} x(t+a)\, e^{-j\omega(\tau - a)} d\tau$$

$$x^*(\omega) = \int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, e^{j\omega a} d\tau$$

$$\boxed{x^*(\omega) = x(\omega) \cdot e^{j\omega a}}$$

$\tau = t - a \quad t = -\infty \quad \tau = -\infty \quad dt = d\tau$

$t = +\infty \quad \tau = +\infty \quad t = \tau + a$

$$x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} x(t-a)\, e^{-j\omega(\tau + a)} d\tau$$

$$\int_{-\infty}^{\infty} x(\tau)\, e^{-j\omega\tau}\, e^{-j\omega a} d\tau$$

$$\boxed{x^*(\omega) = x(\omega) \cdot e^{-j\omega a}}$$

convolution in time.

$x(t) \longrightarrow x(\omega)$

$x_1(t) * x_2(t) \longrightarrow x^*(\omega)$

$$x(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau \right] e^{-j\omega t} dt$$

$$x^*(\omega) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x_1(\tau) \cdot \frac{e^{-j\omega\tau}}{e^{-j\omega\tau}} \cdot x_2(t-\tau) \cdot d\tau \right] e^{-j\omega t} dt$$

$x_1(\omega)$

$$x^*(\omega) = \int_{-\infty}^{\infty} x_1(\omega) \cdot x_2(t-\tau) \cdot \frac{e^{-j\omega t}}{e^{-j\omega\tau}} dt.$$

$$x^*(\omega) = x_1(\omega) \int_{-\infty}^{\infty} x_2(t-\tau) \cdot e^{-j\omega(t-\tau)} dt.$$

let $\beta = t - \tau \quad d\beta = dt.$

$$x_2(\omega) = \int_{-\infty}^{\infty} x_2(\beta) \cdot e^{-j\omega\beta} d\beta.$$

$$\boxed{x^*(\omega) = x_1(\omega) \cdot x_2(\omega)}$$

try for time

Parseval's energy theorem

$x(t) \longrightarrow X(\omega)$

$E\, x(t) = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} d\omega$

conjugate.

conjugation

$x^*(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \cdot e^{-j\omega t} d\omega$

$E\, x(t) = \int_{-\infty}^{\infty} x(t) \cdot x^*(t) \cdot dt$

$E\, x(t) = \int_{-\infty}^{\infty} x(t) \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \cdot e^{-j\omega t} d\omega\, dt$

can be written as $X(\omega)$

$E\, x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot X^*(\omega) d\omega$

$$\boxed{E\, x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega}$$

Differentiation in time:

$x(t) \longrightarrow X(\omega)$

$\frac{d}{dt}(x(t)) \longrightarrow X(\omega)$

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \cdot e^{j\omega t} d\omega$

$\frac{d^k}{dt^k} x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega (j\omega)^k$

Modulation: Adding more signals will increase the probability of communication.

eg:- paper throwing to destination.

=> If we Remove signals: demodulation.

Frequency shifting: $x(t) \longrightarrow X(\omega)$

$x(t) e^{j\omega_0 t} \longrightarrow X(\omega - \omega_0)$

$x(t) \cdot e^{-j\omega_0 t} \longrightarrow X(\omega + \omega_0)$

$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt$

$X(\omega - \omega_0) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j(\omega - \omega_0) t} dt$

$X(\omega - \omega_0) = \int_{-\infty}^{\infty} \underbrace{x(t) \cdot e^{j\omega_0 t}}_{\text{input}} e^{-j\omega t} dt$

If $X(\omega + \omega_0) = \int_{-\infty}^{\infty} \underbrace{x(t) \cdot e^{-j\omega_0 t}}_{\text{input}} e^{-j\omega t} dt$

Modulation property.

$x(t) \longrightarrow X(\omega)$

$x(t) \cdot \sin\omega_0 t \longrightarrow X(\omega)$

$x(t) \cdot \cos\omega_0 t \longrightarrow X(\omega)$

Linear or not

$[a y_1(t) + b y_2(t)] = a\,T[x_1(t)] + b\,T[x_2(t)]$

eg:- $\frac{d^2 y(t)}{dt^2} + 3t\,y(t) = t^2 x(t)$

$\frac{d^2 y_1(t)}{dt^2} + 3t\,y_1(t) = t^2 x_1(t)$ —— ①

$\frac{d^2 y_2(t)}{dt^2} + 3t\,y_2(t) = t^2 x_2(t)$ —— ②

$a \times ① + b \times ②$

$\Rightarrow a.\frac{d^2 y_1(t)}{dt^2} + a\cdot 3t\,y_1(t) = a\cdot t^2 x_1(t)$

$+$

$b.\frac{d^2 y_2(t)}{dt^2} + b\cdot 3t\,y_2(t) = b\cdot t^2 x_2(t)$

$\frac{d^2}{dt^2}[a\cdot y_1(t) + b\,y_2(t)] + 3t[a\cdot y_1(t) + b\cdot y_2(t)] = t^2(a.x_1(t) + b.x_2(t))$

weighted sum of i/ps = weighted sum of o/ps.

$\therefore$ Given system is Linear.

Causal and Non-causal:

present & past but not on future — Future values of the input.

eg:- (1) $y(t) = x^2(t) + x(t-3)$

$y(0) = \underbrace{x^2(0)}_{\text{present}} + \underbrace{x(-3)}_{\text{past}}$

$\therefore$ causal

(2) $y(t) = x(3-t) + x(t-2)$

$y(0) = \underbrace{x(3)}_{\text{future}} + \underbrace{x(-2)}_{\text{past}}$

$\therefore$ non-causal

③ $y(n) = x(2n)$

$n=0 \quad x(0)$

$n=1 \quad \underbrace{x(2)}_{\text{future}}$

$\therefore$ NC

④ $y(n) = \sin[x(n)]$

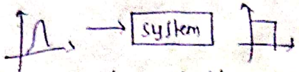
$y(0) = \sin[x(0)]$

$\therefore$ Causal.

⑤ $y(t) = \int_{-\infty}^{2t} x(u)\,du$

$t=0 \; \int_{-\infty}^{0} x(u)\,du = [x(0) - x(-\infty)]$ present, past

$t=2 = \int_{-\infty}^{4} x(u)\,du = [x(4) - x(-\infty)]$ future $\therefore$ NC



Stable and unstable → bounded i/p → unbounded o/p

bounded i/p → bounded o/p

eg:- 1) $y(t) = e^{x(t)}$ for $|x(t)| \le 6$.

$e^{-6} \le y(t) \le e^{6}$ bounded o/p $\Rightarrow \therefore$ stable.

2) $h(t) = \frac{1}{RC} e^{-\frac{1}{RC}} u(t)$

1.

$\therefore$ Stable.

(4) $h(t) = e^{3t}u(t)$

$\int_{-\infty}^{\infty}|h(t)|\,dt = \int_{-\infty}^{\infty} e^{3t}u(t)\,dt$

$= \int_{-\infty}^{\infty} e^{3t}\,dt = \frac{e^{\infty}}{3} - \frac{e^{0}}{3}$

$= \infty$

$\therefore$ unstable

3) $y(t) = (t+7)u(t)$

$y(t) = t+7 \;;\; t \ge 0.$

o/p of the system ↑ without any bound

$\therefore$ unstable.

Invertible and Non-invertible:

eg:- 1) $y(t) = 5x(t)$ ✓

2) $y(t) = 3 + x(t)$ ✓

③ $y(t) = 5x^2(t)$

$x(t) = 1$

$y(t) = 5\times1 = 5$

$x(t) = -1$

$y(t) = 5\times1 = 5$

Time variant and time invariant:

System change with time — system does not change with time.

eg:- ① $y(t) = 2t^2 x(t)$

$y(t-t_0)$ ✗

$2(t-t_0)^2 x(t-t_0) \rightarrow \boxed{\text{system}}$

$y(t) \rightarrow \boxed{\text{system}} \rightarrow 2t^2 x(t)$

$2(t-t_0)^2 x(t-t_0).$

$y(t) = T[x(t)] = 2t^2 x(t).$

o/p of system for the delayed t_0 input secs:

$= 2t^2 x(t-t_0).$

output of the system delayed by t_0 seconds is

$= 2(t-t_0)^2 x(t-t_0).$

$\therefore$ So Time variant

② $y(t) = 3e^{3x(t)}$

Time invariant

Static and dynamic

present — past, future

eg:- ① $y(t) = x(t-4)$

$y(0) = x(0-4)$

past

∴ Dynamic

② $y(n) = x(6n)$

Dynamic

Ans: Static

doubt

③ $y(t) = \frac{d^2x(t)}{dt^2} + 3x(t)$

Dynamic

After MTE

17/11/2023.

Friday.

$$\int_{-\infty}^{\infty} x(t)\,dt = x(\omega) \Rightarrow F\cdot T$$

$$\int_{\infty}^{\infty} x(\omega)\,d\omega = x(t) \Rightarrow \text{Inverse } F\cdot T.$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt \Rightarrow F\cdot T$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)\cdot e^{j\omega t}\,d\omega \Rightarrow I\cdot F\cdot T.$$

Modulation property in F·T: Increasing signal Strength.

⇒ If noice ↑ probability to reach destination from source is less.

Demodulation:- it gives your original signal to output.

$x(t)\cdot \cos\omega t$ — $x(\omega) = ?$ ⟶ when the signal is modulated with $\cos\omega t$ and $\sin\omega t$

$x(t)\cdot \sin\omega t$ — $x(\omega) = ?$

$$x(t)\cdot\left[\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right]$$

$$= \frac{1}{2}\left[x(t)\,e^{j\omega t} + x(t)\cdot e^{-j\omega t}\right]$$

$$= \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$x(t)\cdot\sin\omega t = x(t)\cdot\left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right]$$

$\frac{1}{j}\times\frac{j}{j}$

$$= \frac{1}{2j}\left[x(t)\cdot e^{j\omega t} - x(t)\cdot e^{-j\omega t}\right]$$

$$= \frac{-j}{2}\left[x(\omega-\omega_0) - x(\omega+\omega_0)\right]$$

$$= \frac{j}{2}\left[X(\omega+\omega_0) - X(\omega-\omega_0)\right]$$

duality property in F·T: applying 2 times F·T

$x(t)$ — $x(\omega)$ F·T

$x(t)$ $t \to \omega$ F·T ✓

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)\cdot e^{j\omega t}\,d\omega$$

$$x(\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt.$$

replacing,

$$x(-t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)\cdot e^{-j\omega t}\,d\omega$$

$t \to \omega$ & $\omega \to t$

$$x(-\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt.$$

$$2\pi x(-\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt.$$

∴ $x(t)$ — $X(\omega)$ F·T

$x(t)$ — $2\pi X(-\omega)$

Basic signal in Fourier Transformation.

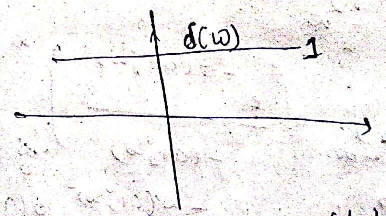
1) Impulse signal.

$\delta(t) = 1$ $t = 0$

$\delta(t) = 0$ $t \neq 0$

If do represention in $\delta(\omega)$ by then it is fourier transformation

Area $\int_{-\infty}^{\infty} \delta(t)\,dt = 0.$



In ω

$$\delta(\omega) = \int_{-\infty}^{\infty} \delta(t)\cdot e^{-j\omega t}\,dt$$

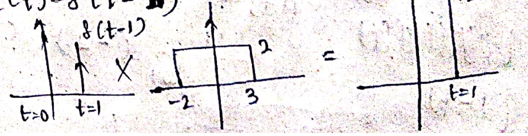
if $t = 0$

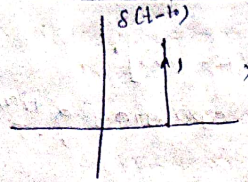
becomes $\int_{-\infty}^{\infty} \delta(t)\cdot e^{0}\,dt = 1$

$1 \times 1 = 1$ area.

$x(t)\cdot\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

$x(t) = \delta(t-1)$





$\times 2 \quad 2\delta(t-t_0)$

$$\boxed{x(t)\cdot\delta(t-t_0)=x(t_0)\delta(t-t_0)}$$

$$2\delta(t-t_0)=2\delta(t-t_0)$$

$x(t)=2$

$x(t_0)$

$x(1)=2$ ✓

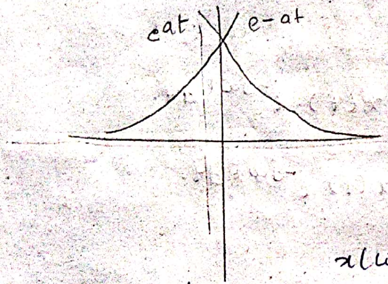
$$\int_{-\infty}^{\infty} x(t)\cdot\delta(t-t_0)\,dt = \frac{x(t_0)}{2}$$

$$\int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0) = x(t_0)\int_{-\infty}^{\infty}\underbrace{\delta(t-t_0)\,dt}_{1} = x(t_0)$$

because area $=1$

$= x(t_0)$

exponential:



$x(t)=e^{-at}$

$x(t)=e^{at}$

$x(t)=e^{-at}u(t)$ no change

$x(t)=e^{at}u(-t)$ no change.

$$X(\omega)=\int_{-\infty}^{\infty} e^{-at}e^{-j\omega t}\,dt$$

$$\int_{0}^{\infty} e^{-at-j\omega t}\,dt$$

limit change

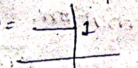
$$\int_{0}^{\infty} e^{-t(a+j\omega)}\,dt$$

$$\left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)}\right]_0^{\infty}$$

$$\frac{1}{-(a+j\omega)}\left[\underbrace{e^{-\infty(a+j\omega)}}_{0}-\underbrace{e^{0(a+j\omega)}}_{1}\right]$$

$$=\frac{1}{-(a+j\omega)}(-1) \quad (\because e^0=1)$$

$$\boxed{X(\omega)=\frac{1}{a+j\omega}}$$

$u(-t)=$ 

For $x(t)=e^{at}$

$$X(\omega)=\int_{-\infty}^{\infty} e^{at}e^{-j\omega t}\,dt$$

$$=\int_{-\infty}^{0} e^{t(a-j\omega)}\,dt$$

$$=\left[\frac{e^{t(a-j\omega)}}{a-j\omega}\right]_{-\infty}^{0}$$

$$=\frac{1}{a-j\omega}\left[e^{0(a-j\omega)}-e^{-\infty(a-j\omega)}\right]$$

$$=\frac{1}{a-j\omega}\left[e^{+0(j\omega-a)}-e^{-\infty(a-j\omega)}\right]$$

$$=\frac{1}{a-j\omega}[\;]$$

$$=\frac{1}{a-j\omega}[1-0]$$

$$\boxed{X(\omega)=\frac{1}{a-j\omega}}$$

$e^{-a|t|}$ $t>0$

$t<0$

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Laplace transform

time domain → frequency domain.

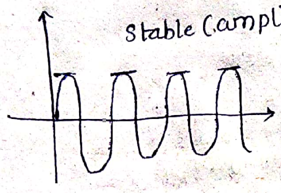
$x(t) = x(n\omega_0)$ $t \to n\omega_0$ - freq fourier series.

$x(t) = x(\omega)$ $t \to \omega \to$ fourier transform.

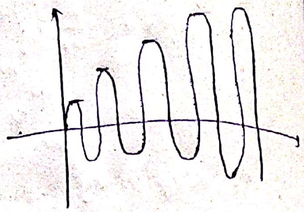
$x(t) = x(s)$ $t \to s \to$ laplace transform.

$s = \sigma + j\omega$

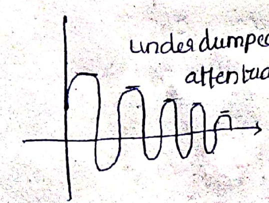
Stable (amplitude is constant here)



overdumped



underdumped attentuation



$$x(s) = \int_{-\infty}^{\infty} \boxed{x(t) \cdot e^{-\sigma t}} \cdot e^{-j\omega t} dt.$$

$$x(s) = F \cdot T[\, x(t) \cdot e^{-\sigma t}]$$

defined

If F.T is defined then $x(s)$ is defined

If F.T is undefined then $x(s)$ is undefined.

$$\int_{-\infty}^{\infty} |x(t) e^{-\sigma t}| dt < \infty$$

$x(s) = 0$

$x(s) = \infty$] undefined.

$x(s) = -\infty$

$x(s) = 1$ ✓

Impulse:

$\delta(t) = 1 \quad t = 0$

$\delta(t) = 0 \quad t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

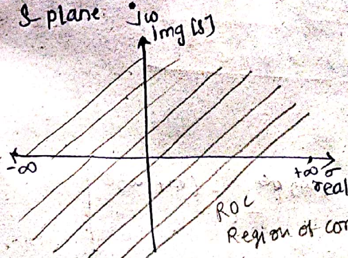
$\delta(s) = ?$

$$\delta(s) = \int_{-\infty}^{\infty} \underbrace{\delta(t)}_{1} \cdot e^{-st} dt.$$

$s = \sigma + j\omega$

$$\delta(s) = \int_{-\infty}^{\infty} 1 \cdot e^{-st} dt$$

s plane



ROC

Region of convergence.

$$x(s) = \int_{0}^{\infty}$$

$$\delta(s) = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt$$

δ(S) ఎక్కడ define అవుతుందో ఆ region ను ROC.

$$= \int_{-\infty}^{\infty} \delta(t) \cdot e^{-(\sigma+j\omega)t} dt$$

$$= \int_{-\infty}^{\infty} \underline{\delta(t) e^{-(\sigma+j\omega)t} dt}$$

$$\delta(s) = 1 \cdot e^{-(\sigma+j\omega)t}$$

$$\delta(s) = 1 \cdot e^{-(\sigma+j\omega)0}$$

$$\boxed{\delta(s) = 1}$$

$\sigma = -\infty \Rightarrow \delta(s) = 1$ σ is independent here.

$\sigma = +\infty \Rightarrow \delta(s) = 1$

$\sigma = 0 \Rightarrow \delta(s) = 1$.

Step signal: $u(t) = 1 \quad t > 0$

$u(t) = 0 \quad t < 0$

$$x(s) = \int_0^{\infty} x(t) \cdot e^{-st} dt$$

$$x(s) = \int_0^{\infty} 1 \cdot e^{-st} dt$$

$$x(s) = \int_0^{\infty} e^{-st} dt$$

$$x(s) = \left[\frac{e^{-st}}{-s}\right]_0^{\alpha}$$

$$x(s) = \frac{1}{-s}\left[e^{-(\sigma+j\omega)\infty} - e^{-(\sigma+j\omega)0}\right]$$

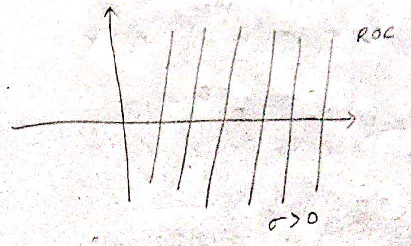
$$= \frac{1}{-s}\left[e^{-(\sigma+j\omega)\infty} - 1\right]$$

✗ if $\sigma = 0$ then $\frac{1}{-s}\left[e^{-(0+j\omega)\infty} - 1\right]$

✗ if $\sigma = -\infty$ then $\frac{1}{-s}\left[\frac{e^{-(-\infty+j\omega)\infty}}{e^{\infty}} - 1\right]$

✓ if $\sigma = +\infty$ then $\frac{1}{-s}\left[e^{\overset{=\infty}{-(\infty+j\omega)\infty}} - 1\right]$ (with $=0$ under the exponential)

$= \frac{1}{s}$



Exponential: $x(t) = e^{-at} \cdot u(t)$.

$$x(s) = \int_0^{\infty} e^{-at} \cdot e^{-st} dt$$

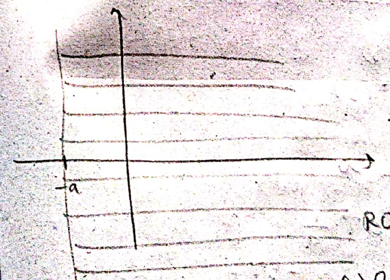
$$x(s) = \int_0^{\infty} e^{-t(a+s)} dt$$

$$= \left[\frac{e^{-(a+s)t}}{-(a+s)}\right]_0^{\infty}$$

$$= \frac{1}{-(a+s)}\left[e^{-(a+s)\infty} - e^{-(a+s)0}\right]$$

$$= \frac{1}{-(a+s)}\left[e^{-(a+s)\infty} - e^{-(a+\sigma+j\omega)0}\right]$$

$$= \frac{1}{-(a+s)}\left[e^{-(\overset{a+}{\sigma}+j\omega)\infty} - 1\right]$$



$\sigma + a > 0$

$\sigma > -a$

Properties :

$x(t) \longrightarrow x(s)$

$x(s) = \int_{-\infty}^{\infty}$

$x(t+t_0) \longrightarrow x(s)$

$x(t-t_0) \longrightarrow x(s)$

$dt = d\tau$

$\tau = t+t_0 \quad t = \tau - t_0$

$\tau = +\infty \quad \tau = -\infty$

$x^*(s) = \int_{-\infty}^{\infty} x(t+t_0) e^{-s(\tau - t_0)} d\tau$

why ?

$x^*(s) = \int_{-\infty}^{\infty} x(\tau). e^{-s\tau} e^{st_0} d\tau$

$\boxed{x^*(s) = x(s). e^{st_0}}$

for $x(t-t_0)$ $\boxed{x^*(s) = x(s). e^{-st_0}}$

Scaling : $x(t) \longrightarrow x(s)$

$x(at) \longrightarrow x^*(s)$

$\tau = at$

$\tau = -\infty \quad \tau = \infty$

$t = \frac{\tau}{a} \quad dt = d\tau . \frac{1}{a}$

$x^*(s) = \int_{-\infty}^{\infty} x(at). e^{-\frac{s\tau}{a}} . d\tau . \frac{1}{a}$

$x^*(s) = \frac{1}{a}\int_{-\infty}^{\infty} x(\tau). e^{-\frac{s}{a}\tau} d\tau$

$x^*(s) = \frac{1}{a} x(s/a)$.

if ROC $\sigma > 0$

$x^*(s) = \frac{1}{a}\left(\frac{\sigma + j\omega}{a}\right)$ find it?

Differentiation in time.

$L.T[x(t)] = \int_{-\infty}^{\infty} x(t). e^{-st} dt$.

$L.T\left[\frac{d}{dt}(x(t))\right] = \int_{-\infty}^{\infty} \frac{d}{dt} x(t). e^{-st} dt$

$L.T\left[\frac{d}{dt} x(t)\right] = \int_{+0}^{\infty} e^{-st} \frac{d}{dt} x(t). dt$

$L.T\left[\frac{d}{dt} x(t)\right] = e^{-st} \int_{+0}^{\infty} \frac{d}{dt} x(t). dt$

$- \int_{0}^{\infty} \frac{d}{dt} e^{-st} dt \int_{0}^{\infty} \frac{d}{dt} x(t) dt$

$\boxed{\because \int u.v = u\int v - \int du \int v}$

$= e^{-st} x(t)\Big]_0^{\infty} - \int_0^{\infty} e^{-st} (-s) dt . x(t)\Big]_0^{\infty}$

$= e^{-s\infty} x(\infty) - e^{-s(0)} x(0) + s x(s)$

$= 0 - x(0) + s x(s)$

$= s(x(s)) - x(0)$.

24/11/2023
Friday
(See missed notes in oops notes middle)

Time shifting

$n \longrightarrow z$

$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$\Rightarrow x(n) \longrightarrow x(z)$

$x(n-k) \longrightarrow ?$

$x(n+k) \longrightarrow ?$

$n = m+k$

$m = n-k$

$m = -\infty \quad n = -\infty$

$m = +\infty \quad n = +\infty$

$$x'(z) = \sum_{n=-\infty}^{\infty} x(n-k) \cdot z^{-(m+k)}$$

$$= \boxed{\sum_{m=-\infty}^{+\infty} x(m) \cdot z^{-m}} \cdot z^{-k}$$

$$= x(z) \cdot z^{-k}$$

Convolution:

$x(n) \longrightarrow x(z)$

$x_1(n) * x_2(n) \longrightarrow ?$ there t as τ, here n as k.

$$\sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k)$$

$$x(z) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k) * x_2(n-k) \cdot z^{-n}$$

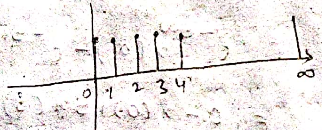
$$x(z) = \underbrace{\sum_{n=-\infty}^{\infty} x_2(n-k) z^{-n}}_{\text{time shifting}} \sum_{k=-\infty}^{\infty} x_1(k)$$

$$= x_2(z) \sum_{k=-\infty}^{\infty} x_1(k) z^{-k}$$

$$= x_2(z)\, x_1(z).$$

eg: $x(n) = x(n-1)\, u(n)$.

$n \longrightarrow z$



$$x(z) = \sum_{n=\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n-1)\, u(n)\, z^{-n}$$

let $m = n-1$ $\quad n=0 \Rightarrow m=-1$
$\quad n=\infty \Rightarrow m=\infty$

$$= \sum_{m=-1}^{\infty} x(m) \cdot \underset{1}{u(n)} \cdot z^{-(m+1)}$$

$$= \sum_{m=-1}^{\infty} x(m) \cdot z^{-m} z^{-1}$$ we have 0 to ∞ only so $x(-1)$ split it

$$= \left(\overset{z^{-(-1)}}{x(-1) z^{1}} + \sum_{m=0}^{\infty} x(m) \cdot z^{-m} \right) z^{-1}$$

$$x(z) = x(-1) \underset{1}{z z^{-1}} + \underbrace{\sum_{m=0}^{\infty} x(m) \cdot z^{-m}}_{x(z)} z^{-1}$$

$$x(z) = \left[x(-1) + x(z) \cdot z^{-1} \right]$$

if $x(n) = x(n+1)\, u(n)$

$m = n+1 \Rightarrow n=0,\ m=1$
$n = m-1 \Rightarrow n=\infty,\ m=\infty.$

$$= \sum_{m=0}^{\infty} x(n+1)\, u(n) \cdot z^{-(m-1)}$$

$$= \sum_{m=1}^{\infty} x(m) \cdot z^{-m} \cdot z^{1}$$

$$= \sum_{m=1}^{\infty} x(m) \cdot z^{-m} z$$

$$= \left[\sum_{m=0}^{\infty} x(m) \cdot z^{-m} - x(0) \right] z$$

$x(0) + x(1) \cdots + x(\infty) - x(0)$

$$= x(z) \cdot z - x(0)\, z.$$

2) $x(n) = x(n+2)\, u(n)$

Given

$m = n+2 \quad n=0 \quad m=2$
$n = m-2 \quad n=\infty \quad m=\infty.$

$$= \sum_{m=2}^{\infty} x(n+2)\, u(n) \cdot z^{-(m-2)}$$

$$= \sum_{m=2}^{\infty} x(m) \cdot \underset{1}{u(n)} \cdot z^{-m} z^{2}$$

$$= \sum_{m=2}^{\infty} x(m) z^{-m} \cdot z^{2}$$

$$= \left[\sum_{m=0}^{\infty} x(m) \cdot z^{-m} - x(0) \right] z^{2}$$

$$= \left[\sum_{m=0}^{\infty} x(m) \cdot z^{-m} - x(0) - x(1) z^{-1} \right] z^{2}$$

$$\left[x(z) - x(0) - x(1) z^{-1} \right] z^{2}$$

$$x(z) = x(z) z^{2} - x(0) z^{2} - x(1) z$$

Initial value & final value.

$x(0)$ — $x(z)$

$x(\infty)$ — ?

Initial value theorem:

$x(n)$ — $x(t)$

$x(0)$ — ?

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$x(z) = x(0) z^{-0} + x(1) \cdot z^{-1} + x(2) \cdot z^{-2} \cdots\cdots x(\infty) \cdot z^{-\infty}$$

apply $\lim_{z\to\infty}$ on both sides.

$$\lim_{z\to\infty} x(z) = \lim_{z\to\infty} x(0) + \frac{x(1)}{z} + \frac{x(2)}{z^2} \cdots\cdots$$

$$\boxed{\lim_{z\to\infty} x(z) = x(0)}$$

Final value theorem:

$x(n)$ — $x(t)$

$x(\infty)$ — ?

$$x(n+1) - x(n)$$

$$Z[x(n+1) - x(n)] = \sum_{n=0}^{\infty} [x(n+1) - x(n)] z^{-n}$$

$$zx(z) - zx(0) - z[x(n)] = \sum_{n=0}^{\infty} [x(n+1) - x(n)] z^{-n}$$

$$zx(z) - zx(0) - x(z) = \sum_{n=0}^{\infty} [x(n+1) - x(n)] z^{-n}$$

$$x(z)[z-1] - zx(0) = \sum_{n=0}^{\infty} [x(n+1) - x(n)] z^{-n}$$

apply $\lim_{z\to 1}$ on Both sides

$$\lim_{z\to 1} [x(z)[z-1] - zx(0)] = \lim_{z\to 1} \sum_{n=0}^{\infty} [x(n+1) - x(n)] z^{-n}$$

$$\lim_{z\to 1} x(z)(z-1) - x(0) = \sum_{n=0}^{\infty} [x(n+1) - x(n)]$$

$$\cancel{x(0)} + x(1\cancel{+1}) - x(\cancel{1}) + \cancel{x(2)} - \cancel{x(2)} \cdots x(\infty+1) - \cancel{x(\infty)}$$

$-x(0)$

$$x(\infty+1) - x(0)$$

$$x(\infty) - x(0)$$

here $x(\infty+1) = x(\infty)$

$$\lim_{z\to 1} x(z)(z-1) = x(\infty) - \cancel{x(0)}$$

$-x(0)$

$$\boxed{x(\infty) = \lim_{z\to 1} x(z)(z-1)}$$

Unit step. $\delta(z) = 1$

$$\sum_{n=-\infty}^{\infty} \delta(n) z^{-n} \Rightarrow \delta(z) = 1 \cdot 1 \cdot 1$$

$u(n) = 1 \quad n \geq 0$

$u(n) = 0 \quad n < 0$

$$x(z) = \sum_{n=0}^{\infty} x(n) \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} x(n) \cdot (z^{-1})^n$$

$$= x(0) \cdot (z^{-1})^0 + x(1) \cdot (z^{-1})^1 + x(z^{-1})^2 + \cdots$$

$$= 1 \cdot 1 + \frac{1 \cdot z^{-1}}{x} + \frac{1 \cdot z^{-2}}{x^2} \cdots\cdots$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} \cdots\cdots$$

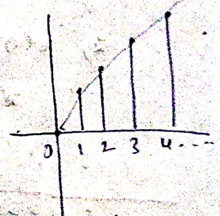
$$= 1 + x + x^2 \cdots\cdots$$

$$= \frac{1}{1-x} = \frac{1}{1-\frac{1}{z}} = \boxed{\frac{z}{z-1} = x(z)}$$

Ramp signal.

$x(n) = n u(n) \quad n \geq 0$

$= 0 \quad n < 0$



$$x(z) = \sum_{n=0}^{\infty} n(u(n)) \cdot z^{-n}$$

$$= 0 \cdot (z^{-1})^0 + 1 \cdot (z^{-1})^1 + 2 \cdot (z^{-1})^2 \cdots\cdots$$

$$= 0 + z^{-1} + 2 \cdot (z^{-1})^2 + 3 (z^{-1})^3 \cdots$$

$$= \frac{z^{-1}}{x} (1 + 2 \cdot z^{-1} + 3 \cdot (z^{-1})^2 \cdots\cdots)$$

$$= \frac{z^{-1}}{x} [1 + 2x + 3x^2 \cdots\cdots]$$

$$= \frac{1}{(1-x)^2} z^{-1}$$

$$= \frac{1}{(1-z^{-1})^2} z^{-1} \Rightarrow z^{-1} \left(\frac{1}{(1-\frac{1}{z})^2} \right) = \frac{z^2 z^{-1}}{(z-1)^2}$$

$$\boxed{x(z) = \frac{z}{(z-1)^2}}$$

① eg: $x(n) = a^n u(n)$ → Give its z-transform

$$x(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

$u(n)$ limits are 0 to ∞

$$= \sum_{n=0}^{\infty} a^n \underbrace{u(n)}_{1} \cdot z^{-n}$$

$$= \sum_{n=0}^{\infty} a^n z^{-n}$$

$$= \sum_{n=0}^{\infty} (az^{-1})^n$$

$$= (az^{-1})^0 + (az^{-1})^1 + (az^{-1})^2 \ldots\ldots$$

$$= 1 + \underbrace{(az^{-1})}_{x} + \underbrace{(az^{-1})^2}_{x^2} \ldots$$

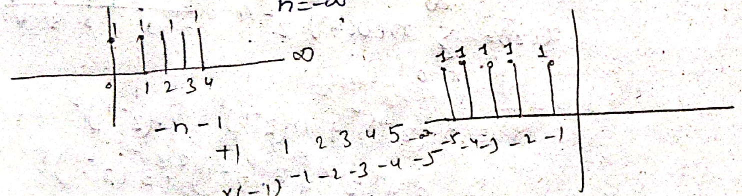
$$= 1 + x + x^2 \ldots$$

$$= \frac{1}{1-x}$$

$$= \frac{1}{1-az^{-1}} = \frac{1}{1-\frac{a}{z}} = \frac{z}{z-a}$$

2) $x(n) = -a^n u(-n-1)$ → Give its z-transform.

$$x(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}.$$



$$= -\sum_{n=-\infty}^{-1} a^n z^{-n}$$

$n = -n \Rightarrow 1.$

$$= -\sum_{n=1}^{\infty} a^{-n} z^{n}$$ because $n = -n$.

$$= -\sum_{n=1}^{\infty} (a^{-1}z)^n.$$

channel Behaviour. 07/11/2023 Tuesday.

⇒ dealer behaviour → eg:- many will come and buy.

channel conflict.

- Horizontal conflict ↓ same level
- Vertical conflict ↓ another level

⇒ channel design decisions.
- prompt delivery.
- convenience

short channel: 0 or 1 level   long channel: >1 level.

perishable goods: short channels

⇒ 1. Intensive distribution: అన్నిచోట్లా ఆ వస్తువు products

2. Exclusive distribution: ఒకే వ్యక్తికే product ని అమ్ముకొనేందుకు permit చేయటం.

3. Selective " ; neither ID nor ED. more than one or a few.

⇒ Evaluating the major alternatives:-

$$= -\left[(a^{-1}z)^1 + (a^{-1}z)^2 + (a^{-1}z)^3 \ldots\right]$$

$$= -[a^{-1}z]\left[1 + \underbrace{(a^{-1}z)}_{x} + \underbrace{(a^{-1}z)^2}_{x^2} + \ldots\right]$$

$$= 1 + x + x^2 \ldots\ldots$$

$$= \frac{1}{1-x}$$

$$= -(a^{-1}z)\left[\frac{1}{1-a^{-1}z}\right]$$

$$= -(a^{-1}z)\left[\frac{a}{a-z}\right]$$

$$\boxed{x(z) = \frac{-z}{a-z}}$$

or.

$$\boxed{x(z) = \frac{z}{z-a}}$$

Property: multiplication 'n'.

$x(n)$ — $x(z)$

$n \cdot x(n)$ — ? $\quad -z\frac{d}{dz}x(z)$

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n}$$

diff w.r.t z.

$$\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot \frac{d}{dz} z^{-n}$$

$$\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} x(n)\left[-n z^{-n-1}\right]$$

$$-\frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} n \cdot x(n) \cdot z^{-n} \cdot z^{-1}$$

$$-z \frac{d}{dz} x(z) = \sum_{n=-\infty}^{\infty} n \cdot x(n) \cdot z^{-n}$$

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