Signals and Systems

* Signals ÷ Passing information from one place to another place.

Signals divided into two types.

i) i/p signal ÷ Accross initial point / source.

ex ÷ traffic lights.

ii) O/P Signal ÷ Accross destination point / destination.

ex ÷ Moment of vehicles with respect to traffic signals.

* System ÷ Which does some process in a systematic way.

Units

1) Signals
2) Systems
3) & 4) Transformers
5) Sampling & Reconstruction

There are two types of currents.

1) AC : Alternating Current

• It follows in two directions.

2) DC : Direct Current

• Flows in one direction.

Signals

1) Mathematical Operations
2) Classification of Signals
3) Properties of Signals

* Parameters of Signals

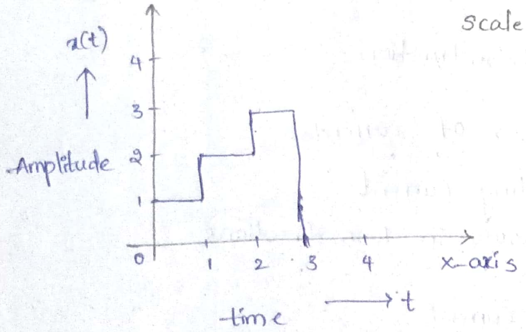
1) Amplitude = $x(t)$ = function of time period.
2) Time = t.

* Three types of Signal representation.

1) Graphical representation
2) Tabular representation
3) Mathematical representation.

1) Graphical representation :- Signals represented in a graphs

Scale :- x-axis : 1 unit = 1 sec
Y-axis : 1 unit = 1 V



2) Tabular representation :-

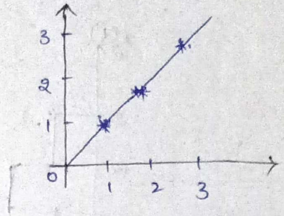
t	$x(t)$
$0<t<1$	1 V
$1<t<2$	2 V
$2<t<3$	3 V

3) Mathematical representation :-

$y = x$
$y = 2x+1$

Equation representation = $\boxed{x(t) = t}$

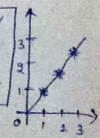
$x(t) = 2t + 1$



* Classification of Signals.

1) CT [continuous Time] signal → Ex:-

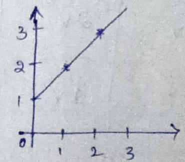
$0<t<1$	1V
$1<t<2$	2V
$2<t<3$	3V



2) DT [Discrete Time] signal

Ex:-

$t=0$	1V
$t=1$	2V
$t=2$	3V



* CT Signals

Mathematical Operations on CT Signals

1) Addition of CT Signals
2) Multiplication of CT signals
3) Amplitude Scaling of CT signals
4) Time Scaling of CT Signals
5) Amplitude Shifting of CT Signals
6) Time Shifting of CT Signals
7) Amplitude reverse of CT Signals
8) Time reverse of CT Signals
9) Multiple Transformation of CT Signals.

1) Addition of two CT signals

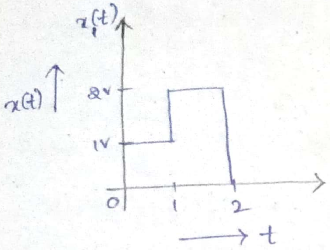
ex:- 1)

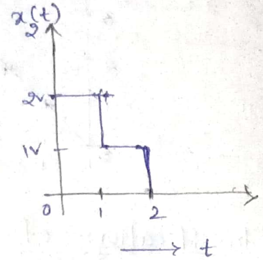
t	$x(t)$
$0<t<1$	1 V
$1<t<2$	2 V

2)

t	$x(t)$
$0<t<1$	2 V
$1<t<2$	1 V

Graphical representation.

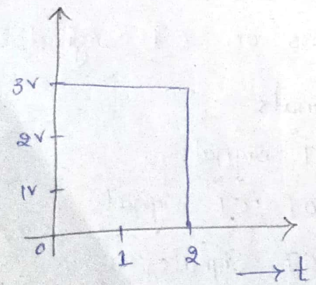




Tabulation of resultant.

t	$x_1(t)$	$x_2(t)$	$x_1(t)+x_2(t)=x(t)$
$0<t<1$	1 V	2 V	3 V
$1<t<2$	2 V	1 V	3 V

Resultant Graphical representation

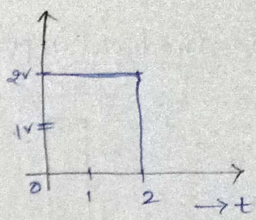


2) Multiplication of two CT Signals.

$x_1(t) * x_2(t) = x(t)$

$0<t<1 = (1*2) = 2\,V$

$1<t<2 = (2*1) = 2\,V$



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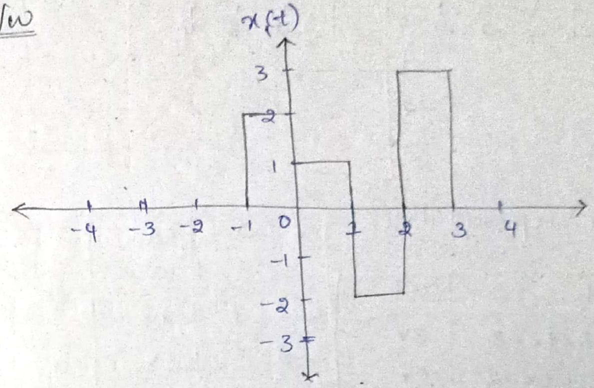
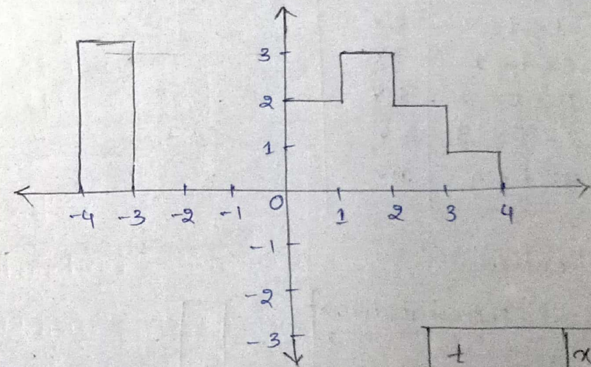




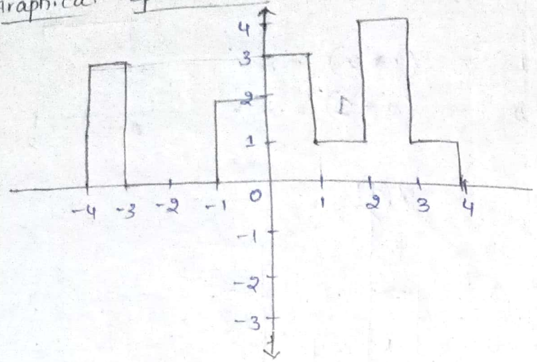
Tabular forms

t	$x_1(t)$
$-1<t<0$	2 V
$0<t<1$	1 V
$1<t<2$	−2 V
$2<t<3$	3 V

t	$x_2(t)$
$-4<t<-3$	3 V
$-3<t<-2$	0 V
$-2<t<-1$	0 V
$-1<t<0$	0 V
$0<t<1$	2 V
$1<t<2$	3 V
$2<t<3$	2 V
$3<t<4$	1 V

i) Addition

Graphical representation.

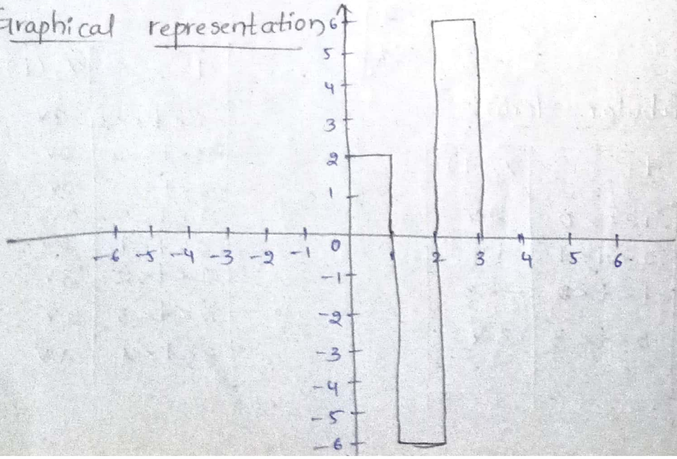


Tabular representation.

t	x(t)
$-4<t<-3$	3V
$-3<t<-2$	0V
$-2<t<-1$	0V
$-1<t<0$	2V
$0<t<1$	3V
$1<t<2$	1V
$2<t<3$	5V
$3<t<4$	1V

ii) Multiplication.

Graphical representation



Tabular representation

t	x(t)
$-4<t<-3$	0V
$-3<t<-2$	0V
$-2<t<-1$	0V
$-1<t<0$	0V
$0<t<1$	2V
$1<t<2$	-6V
$2<t<3$	6V
$3<t<4$	0V

3) Time Scaling of CT signal

Scaling factor :

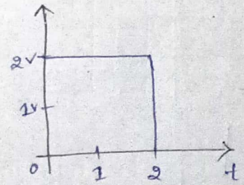
Denoted by 'α'

$x(t)$ = Initial Signal

After scaling, the resultant signal is $\boxed{y(t) = x(\alpha t)}$.

case (i) :- $\alpha > 1$.

Ex



t	x(t)
$t<0$	0V
$0<t<1$	2V
$1<t<2$	0V

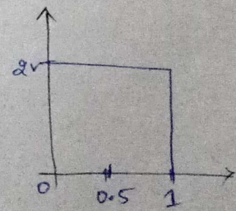
consider $\alpha = 2$.

for $\alpha > 1$:

$2t = 0$; $t = 0$

$2t = 1$; $t = 0.5$

$2t = 2$; $t = 1$



t	y(t)
$t<0$	0V
$0<t<0.5$	2V
$t>1$	0V

For $\alpha > 1$, Signal time period has been compressed.

Case (ii) : $\alpha < 1$

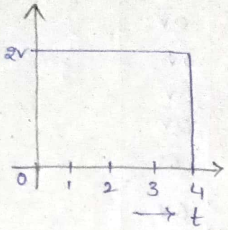
$y(t) = x(\alpha t)$ $\alpha = 1/2$

$y(t) = x(t/2)$

$t/2 = 0 \;;\; t = 0$

$t/2 = 1 \;;\; t = 2$

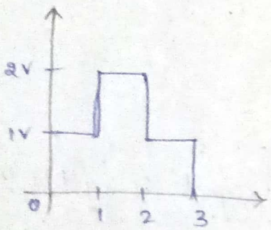
$t/2 = 2 \;;\; t = 4$



t	$y(t)$
$t<0$	0
$0<t<1$	2V
$1<t<2$	2V
$2<t<3$	2V
$3<t<4$	2V
$t>4$	0V

For $\alpha < 1$, Signal time period has been Expanded.

H/w



t	$x(t)$
$t<0$	0
$0<t<1$	1V
$1<t<2$	2V
$2<t<3$	1V
$t>3$	0V

Case (i) :

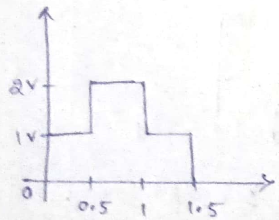
$x(2t)$

$2t = 0 \;;\; t = 0$

$2t = 1 \;;\; t = 0.5$

$2t = 2 \;;\; t = 1$

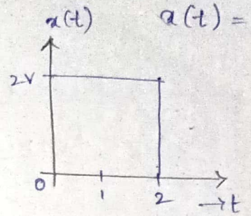
$2t = 3 \;;\; t = 1.5$



4) Amplitude Scaling of CT Signal.

Scaling factor = β

$x(t) = \beta$

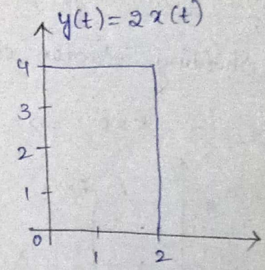


t	$x(t)$
$t<0$	0
$0<t<1$	2V
$1<t<2$	2V
$t>2$	0V

Case (i) :

$\beta > 1$. $(\beta = 2)$

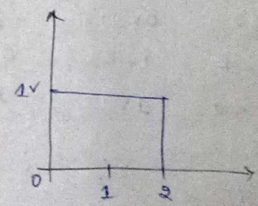
t	$x(t)$	$2x(t)$
$t<0$	0	0
$0<t<1$	2V	4V
$1<t<2$	2V	4V
$t>2$	0V	0V



Amplification

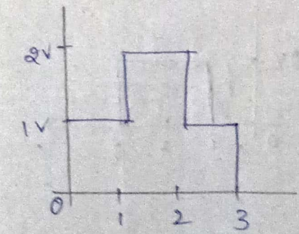
Case (ii) :

t	$x(t)$	$\frac{x(t)}{2}$
$t<0$	0	0
$0<t<1$	2V	1V
$1<t<2$	2V	1V
$t>2$	0V	0V



This is called Attenuation.

H/w



t	$x(t)$
$t<0$	0
$0<t<1$	2V
$1<t<2$	2V
$t>2$	0V

Case (i) :-

$\beta = 2$ $\quad 2x(t)$

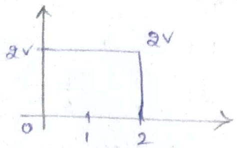
t	$x(t)$	$2x(t)$
$t<0$	0	0V
$0<t<1$	2V	4V
$1<t<2$	2V	4V
$t>0$	0	0V

5) Time Shifting of CT Signals

Shifting factor denoted by 'k'.

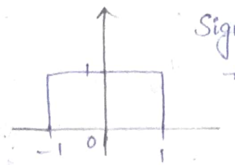
Case (i) :- $k>0 \Rightarrow x(t+k)$

ex:-



t	$x(t)$
$t<0$	0V
$0<t<1$	2V
$1<t<2$	2V
$t>2$	0V

t	$x(t)$	$x(t+k)$
$t<0$	0V	−1
$0<t<1$	1V	0
$1<t<2$	1V	1

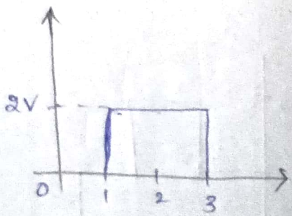


Signal get Advanced.

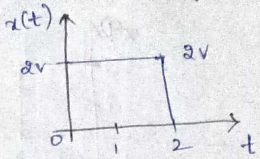
Case (ii) :-

$t<0 \Rightarrow x(t-k)$

t	$x(t)$	$x(t-k)$
$t<1$	0V	0V
$1<t<2$	1V	2V
$2<t<3$	0V	2V

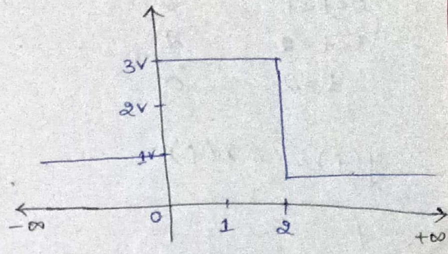


6) Amplitude Shifting of CT Signals



(i) $y(t) = x(t) + 1$.

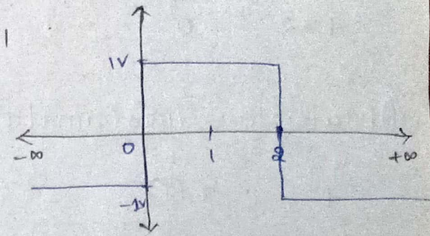
t	$x(t)$	$x(t)+1$
$t<0$	0	1V
$0<t<1$	2V	3V
$1<t<2$	2V	3V
$t>2$	0V	1V



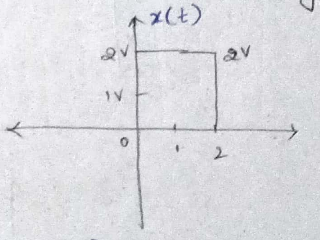
Shifted

(ii) $y(t) = x(t) - 1$.

t	$x(t)$	$x(t)-1$
$t<0$	0V	−1V
$0<t<1$	2V	1V
$1<t<2$	2V	1V
$t>2$	0V	−1V



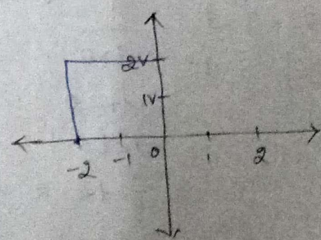
7) Time reversal of CT signals (Folding / Reflection)



t	$x(t)$
$t<0$	0
$0<t<1$	2
$1<t<2$	2
$t>2$	0

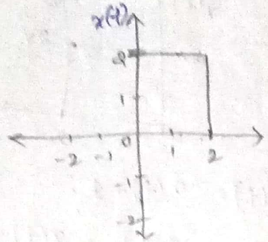
$y(t) = x(-t)$

t	~~x(t)~~ $y(t)$
$t>0$	0
$-1<t<0$	2
$-2<t<-1$	2
$t<-2$	0



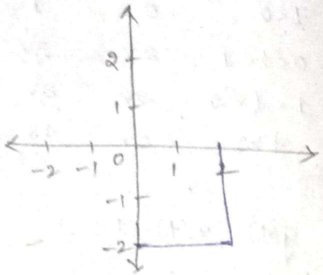
8) Amplitude reversal of CT Signals (Folding, Reflection)

t	x(t)
t<0	0
0<t<1	2
1<t<2	2
t>2	0



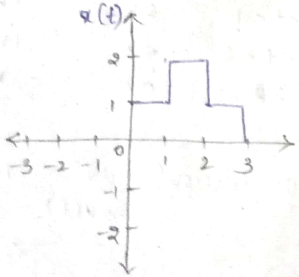
$y(t) = -x(t)$

t	y(t)
t<0	0
0<t<1	-2
1<t<2	-2
t>2	0



9) Multiple Transformation

t	x(t)
t<0	0V
0<t<1	1V
1<t<2	2V
2<t<3	1V
t>3	0V



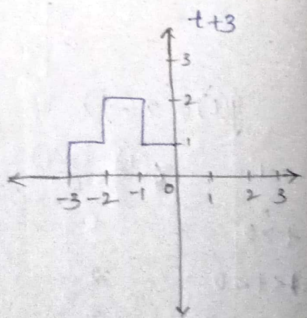
$y(t) = x(2t+3)$

2t+3 (After shifting)

$\Rightarrow 2t+3-3$

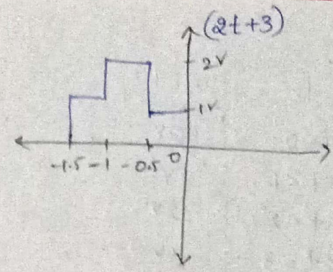
$\Rightarrow \frac{2t}{2} = t.$

t	x(t)
t>0	0
-1<t<0	1V
-2<t<-1	2V
-3<t<-2	1V
t<-3	0V



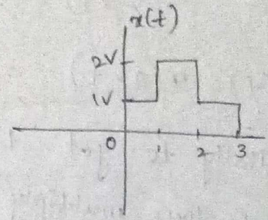
(After Scaling)

t	x(t)
t>0	0V
-0.5<t<0	1V
-1<t<-0.5	2V
-1.5<t<-1	1V
t<-1.5	0V



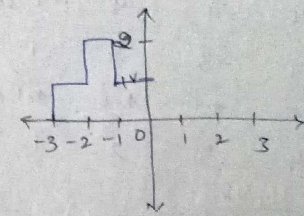
Ex2.

t	x(t)
t<0	0V
1<t<0	1V
2<t<1	2V
3<t<2	1V
t>3	0V



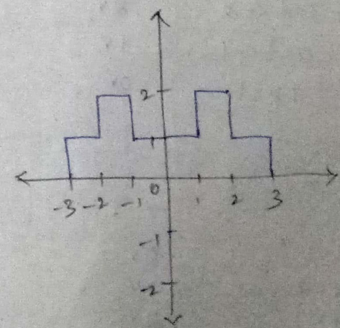
$y(t) = x(-t)$

t	x(-t)
t>0	0V
-1<t<-0	1V
-2<t<-1	2V
-3<t<-2	1V
t<-3	0V



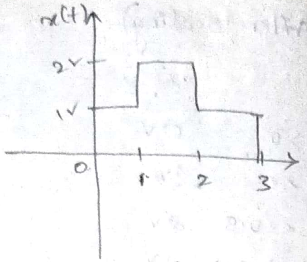
$y(t) = x(t) + x(-t)$

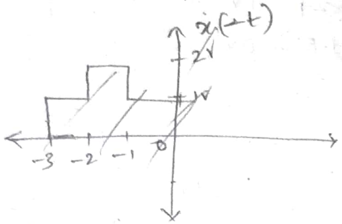
t	x(t)+x(-t)
t<-3	0V
-3<t<-2	1V
-2<t<-1	2V
-1<t<0	1V
0<t<1	1V
1<t<2	2V
2<t<3	1V
t>3	0V



Ex 3

t	$x(t)$
$t<0$	0V
$0<t<1$	1V
$1<t<2$	2V
$2<t<3$	1V
$t>3$	0V



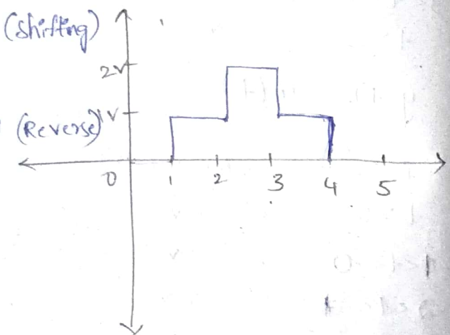


t $\quad x(-t)$

$y(t) = x(4-t)$

Initially to get 't' by we have to subtract 4 & then multiply by -1. then we get,

0	1	2	3	
→ -4	-4	-4	-4	(shifting)
-4	-3	-2	-1	
→ *-1	*-1	*-1	*-1	(Reverse)
4	3	2	1	



Shifting

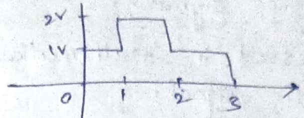
t	$x(t)$
$t<-4$	0V
$-4<t<-3$	1V
$-3<t<-2$	2V
$-2<t<-1$	1V
$t>-1$	0V

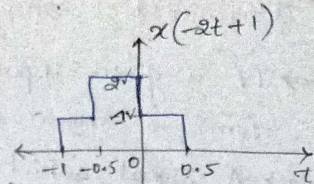
Scaling

t	$x(t)$
$t<1$	0V
$1<t<2$	1V
$2<t<3$	2V
$3<t<4$	1V
$t>4$	0V

Ex 4

$y(t) = x(-2t+1)$





-1	-1	-1	-1	(shifting)
-1	0	1	2	
$\times \frac{1}{2}$	$\times \frac{1}{2}$	$\times \frac{1}{2}$	$\times \frac{1}{2}$	(scaling)
-0.5	0	0.5	1	
*-1	*-1	*-1	*-1	(Reverse)
0.5	0	-0.5	-1	

Final table

t	$x(t)$
$t<-1$	0V
$-1<t<-0.5$	1V
$-0.5<t<0$	2V
$0<t<0.5$	1V
$t>0.5$	0V

Shifting

t	$x(t)$
$t<-1$	0V
$-1<t<0$	1V
$0<t<1$	2V
$1<t<2$	1V
$t>2$	0V

Scaling

t	$x(t)$
$t<-0.5$	0V
$-0.5<t<0$	1V
$0<t<0.5$	2V
$0.5<t<1$	1V
$t>1$	0V

* Classification of CT Signals.

1) Deterministic signal and non-determinstic Signal
2) Even Signal and odd signal.
   Symmetric signal & Assymetric Signal.
3) Periodic and Aperiodic Signal.
4) Causal Signal, anticausal and non causal signal.
5) Power Signal and Energy Signal.

1) Deterministic and Non-Deterministic Signal.

⇒ If a given input is expressed in a mathematical form is known as deterministic signal.

⇒ A signal which cannot expressed in mathematical form is called Non-deterministic signal.

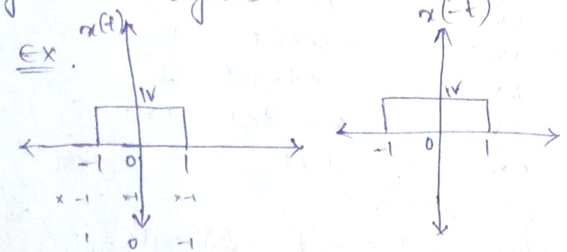
Ex: Noise signal.

→ Non-deterministic signal is also called as random signal.

2) Even Signal and Odd Signal:

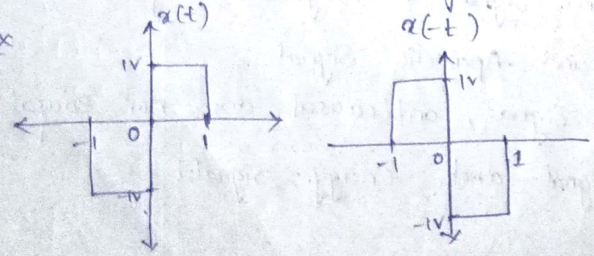
⇒ If any signal which satisfies $\boxed{x(t) = x(-t)}$ then those signals are called as Even (or) Symmetric Signals

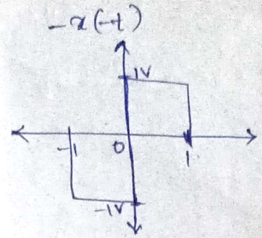
Ex.



⇒ If any signal which does ~~not~~ satisfies the condition $\boxed{x(t) = -x(-t)}$ then those signals are classified as odd (or) Assymetric signal.

Ex





→ Odd Signal always passes through origin.

Ex2: let;

$x(t) = \sin t$.

$x(-t) = \sin(-t)$

$= -\sin(t)$.

$-x(-t) = -(-\sin(t))$

$-x(-t) = \sin(t)$.

Example

1) $x(t) = t^2$

$x(t) = t^2$

$x(-t) = (-t)^2$

$x(-t) = t^2$.

$\boxed{x(t) = x(-t)}$

It is an even signal.

2) $x(t) = t^3$

$x(t) = t^3$

$x(-t) = (-t)^3$

$x(-t) = -t^3$.

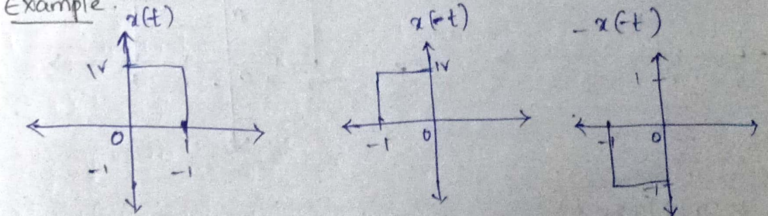
$-x(-t) = -(-t^3)$

$-x(-t) = t^3$

$\boxed{x(t) = -x(-t)}$

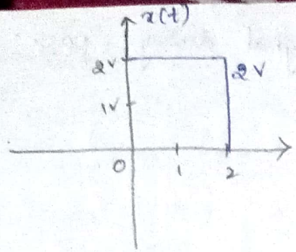
It is an odd signal.

Example.



⇒ It is neither even nor odd signal.

⇒ Any signal which is classified as neither even nor odd, that can be retrieved by by adding even signal & odd signal.



$$x(t) = x_e(t) + x_o(t) \quad — (1)$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \quad — (2)$$

add (1) & (2).

$$x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} \rightarrow \text{Even part}$$

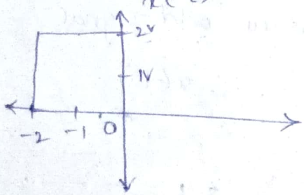
sub (1) & (2)

$$x(t) - x(-t) = 2x_o(t)$$

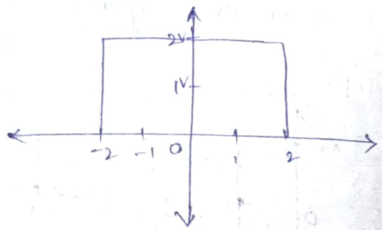
$$x_o(t) = \frac{x(t) - x(-t)}{2} \rightarrow \text{odd part.}$$

Reversal. ($x(-t)$)

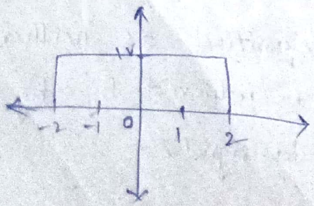
$x(-t)$



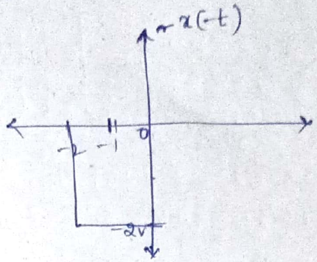
$\Rightarrow x(t) + x(-t)$.



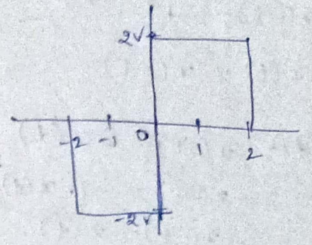
$$\frac{x(t)+x(-t)}{2}$$



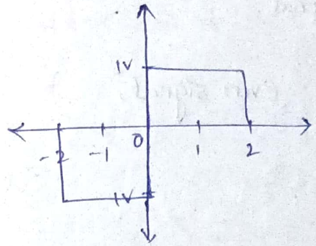
$-x(-t)$



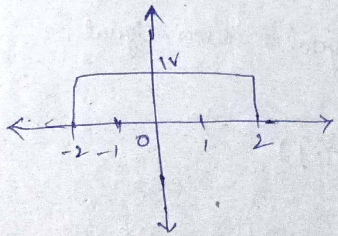
$x(t) + (-x(-t))$



$$\frac{x(t) + (-x(-t))}{2}$$



$$\frac{x(t)+x(-t)}{2} + \frac{x(t) + (-x(-t))}{2}$$



* Properties of an even & odd signal.

i) $x(t) = t^2$

$x(-t) = (-t)^2$

$x(-t) = t^2$

$x(t) = x(-t) \rightarrow$ Even signal.

ii) $x(t) = t^3$

$x(t) = (-t)^3$

$x(-t) = -t^3$

$x(t) \neq x(-t)$.

$\cancel{\text{Ex}}$ $x(-t) = -x(t)$.

$-x(-t) = -(-x(t))$

$-x(-t) = x(t)$.

$-(-t^3) = t^3$

$x(t) = t^3$.

$x(t) = -x(-t) \longrightarrow$ odd signal.

1) Even signal + Even signal = Even signal.

$t^2 + \cos t = x(t)$

$x(-t) = (-t)^2 + \cos(-t)$

$= t^2 + \cos t$

$x(t) = x(-t)$.

2) Even signal * Even signal = even signal

$t^2 * \cos t = x(t)$

$x(-t) = (-t)^2 * \cos(-t)$

$= t^2 * \cos t$.

$x(t) = x(-t)$.

3) Odd signal * odd signal = even signal.

$t^3 * \sin t = x(t)$

$x(-t) = (-t)^3 * \sin(-t)$

$= -t^3 * -\sin t$

$= t^3 * \sin t$

$x(t) = x(-t)$.

4) Even signal * odd signal = odd signal.

$t^2 * \sin t = x(t)$

$x(-t) = (-t)^2 * \sin(-t)$

$= t^2 * -\sin t$

$x(-t) = -t^2 * \sin t$

$x(t) \neq x(-t)$

$-x(-t) = -(-t^2 * \sin t)$

$-x(-t) = t^2 * \sin t$

$x(t) = -x(-t)$.

5) odd signal + odd signal = odd signal.

$t^3 + \sin t = x(t)$

$x(-t) = -t^3 + \sin(-t)$

$= -t^3 - \sin t$.

$x(t) \neq x(-t)$

$-x(-t) = -(-t^3 - \sin t)$

$-x(-t) = t^3 + \sin t$.

$x(t) = -x(-t)$

6) Even signal + odd signal = ~~odd~~ sig NENO

$t^2 + \sin t = x(t)$.

$x(-t) = (-t)^2 + \sin(-t)$

$x(-t) = t^2 - \sin t$

$x(t) \neq x(-t)$

$-x(-t) = -(t^2 - \sin t)$

$-x(-t) = -t^2 + \sin t$

$\cancel{-x(-t) =}$

$x(t) \neq -x(-t)$.

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7) $\frac{d}{dt}$ (even signal) = odd signal.

$$\frac{d}{dt}(t^2) \Rightarrow \cancel{2t} \quad x(t) = 2t$$
$$x(-t) = -2t$$
$$x(t) \neq x(-t)$$
$$-x(-t) = -(-2t)$$
$$-x(-t) = 2t$$
$$x(t) = -x(-t).$$

8) $\frac{d}{dt}$ (odd signal) = Even signal.

$$\frac{d}{dt}(t^3) \Rightarrow x(t) = 3t^2$$
$$x(-t) = 3(-t)^2$$
$$x(-t) = 3t^2$$
$$x(t) = x(-t).$$

Even + Even = Even

Even * Even = Even

Odd + Odd = Odd

Odd * Odd = Even

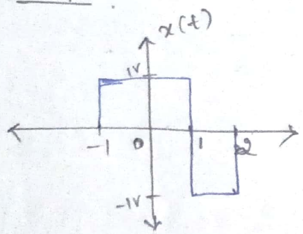
Odd * Even = Odd

Odd + Even = NENO

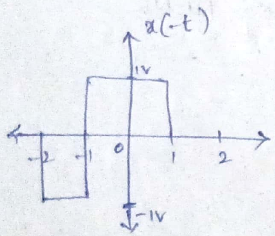
d/dt(Even) = Odd

d/dt(Odd) = Even

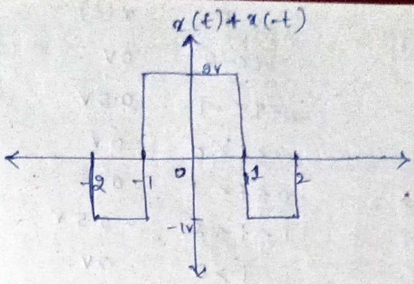
Example



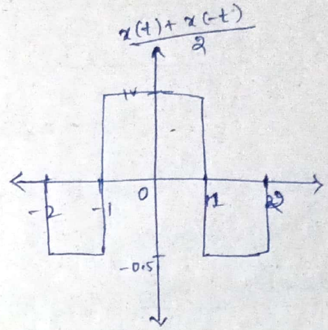
t	x(t)
t < -1	0 V
$-1 \le t \le 0$	1 V
$0 \le t \le 1$	1V
$1 \le t \le 2$	-1V
t > 2	0 V



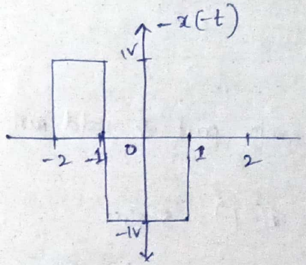
t	x(-t)
t < -2	0 V
$-2 \le t \le -1$	-1V
$-1 \le t \le 0$	1 V
$0 \le t \le 1$	1V
t > 1	0 V



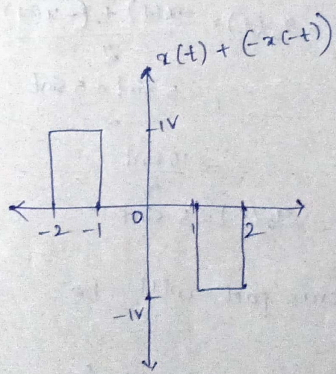
t	x(t)
t < -2	0
$-2 \le t \le -1$	-1V
$-1 \le t \le 0$	2V
$0 \le t \le 1$	2V
$1 \le t \le 2$	-1V
t > 2	0V



t	x(t)
t < -2	0
$-2 \le t \le -1$	-0.5 V
$-1 \le t \le 0$	1 V
$0 \le t \le 1$	1 V
$1 \le t \le 2$	-0.5 V
t > 2	0 V

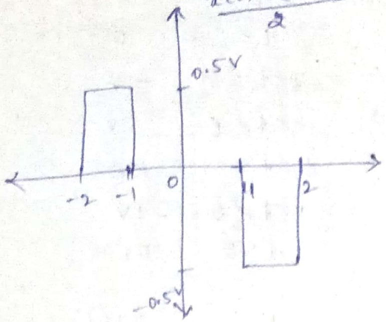


t	x(t)
t < -2	0V
$-2 \le t \le -1$	1 V
$-1 \le t \le 0$	-1V
$0 \le t \le 1$	-1V
t > 1	0V

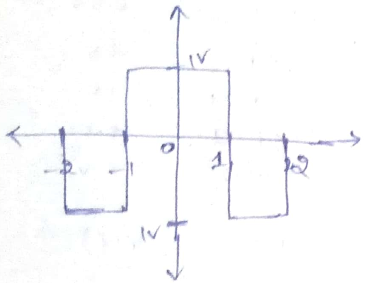


t	x(t)
t < -2	0V
$-2 \le t \le -1$	1V
$-1 \le t \le 0$	0V
$0 \le t \le 1$	0V
$1 \le t \le 2$	-1V
t > 2	0 V

$\frac{x(t)+(-x(-t))}{2}$



t	$x(t)$
$t<-2$	0V
$-2\le t\le -1$	0.5V
$-1\le t\le 0$	0V
$0\le t\le 1$	0V
$1\le t\le 2$	-0.5V
$t>2$	0V.



Example.

$x(t) = 5\sin t$ find its even part & odd part.

$x_e(t) = \frac{x(t)+x(-t)}{2}$.

$x(t) = 5\sin t$

$x(-t) = -5\sin t$

$x_e(t) = \frac{5\sin t - 5\sin t}{2} = 0$

$-x(-t) = 5\sin t$

$x_o(t) = \frac{x(t) + (-x(-t))}{2}$

$= \frac{5\sin t + 5\sin t}{2}$

$= \frac{10\sin t}{2}$

$x_o(t) = 5\sin t$

→ In an odd signal, even part will be zero.

→ In an even signal, odd part will be zero.

2) $x(t) = \sin t + \cos t + \sin t\cos t$

Even part.

$x(-t) = -\sin t + \cos t - \sin t\cos t$

$\frac{x(t)+x(-t)}{2} = \frac{\cancel{\sin t} + \cos t + \cancel{\sin t\cos t} - \cancel{\sin t} + \cos t - \cancel{\sin t\cos t}}{2}$

$\boxed{x_e(t) = \cos t}$

Odd part.

$-x(-t) = -(-\sin t + \cos t - \sin t\cos t)$

$= \sin t - \cos t + \sin t\cos t$

$x_o(t) = \frac{\sin t + \cancel{\cos t} + \sin t\cos t + \sin t - \cancel{\cos t} + \sin t\cos t}{2}$

$= \frac{2(\sin t + \sin t\cos t)}{2}$

$\boxed{x_o(t) = \sin t + \sin t\cos t}$

3) $x(t) = t + 2t^2 + 3t^3 + 1$.

Even part.

$x(-t) = -t + 2t^2 - 3t^3 + 1$

$x_e(t) = \frac{\cancel{t} + 2t^2 + \cancel{3t^3} + 1 \cancel{-t} + 2t^2 - \cancel{3t^3} + 1}{2}$

$= \frac{4t^2+2}{2}$

$\boxed{x_e(t) = 2t^2+1}$

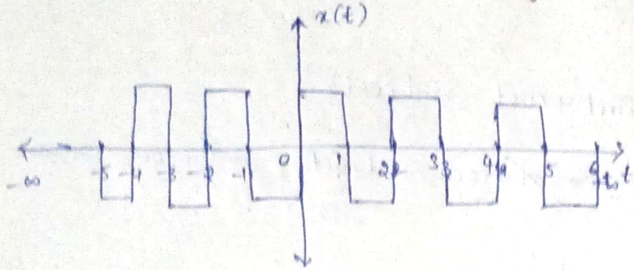
Odd part.

$-x(-t) = t - 2t^2 + 3t^3 - 1$

$x_o(t) = \frac{t + \cancel{2t^2} + 3t^3 + \cancel{1} + t - \cancel{2t^2} + 3t^3 - \cancel{1}}{2}$

$\boxed{x_o(t) = t + 3t^2}$

3) Periodic and Non-Periodic Signal.



→ At an initial time period where the signal repeats is known as fundamental time period. ~~(from -∞ to ∞)~~

Here at an initial time period = 2s, the signal repeats

⟹ If a signal repeats in a fundamental time period from -∞ to +∞ is known as "Periodic Signal".

→ Fundamental time period is denoted as T_0.

→ If $x(t) = x(t+T_0)$ then it is known as periodic Signal.

Periodicity doesn't changes with below operations.

1) Amplitude scaling
2) Amplitude shifting
3) Amplitude reversal
4) Time reversal.
5) Time shifting.

⟹ If a signal fails to satisfy the condition $x(t) = x(t+T_0)$ is known as Aperoidic (or) Non-periodic signal.

$x(t) \neq x(t+T_0)$ → Aperiodic signal.

$f_0 = \frac{1}{T_0}$

$\boxed{\omega_0 = 2\pi f_0}$

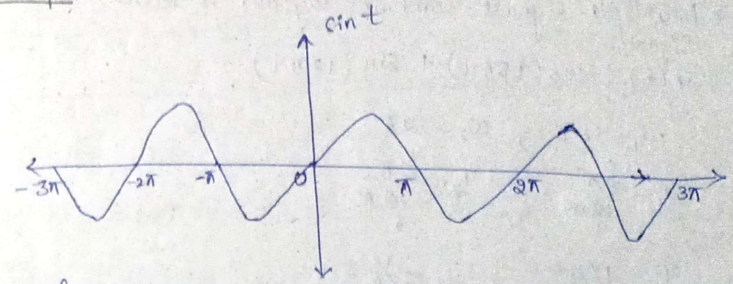
$\omega_0 = 2\pi \times \frac{1}{T_0}$

$\boxed{T_0 = \frac{2\pi}{\omega_0}}$

f_0 = fundamental frequency
T_0 = fundamental time period
ω_0 = angular frequency.

(Units of ω_0 = rad/sec).

Example



fundamental time period = 2π.

It is a periodic signal

$x(t) = A \sin \omega_0 t$

$A = 1$

$x(t) = \sin t$

$\omega_0 = 1$ rad/sec.

$T_0 = \frac{2\pi}{\omega_0} = 2\pi$

$x(t) = x(t+T_0)$

$x(t) = \sin t$

$x(t+T_0) = \sin(t+T_0)$

$= \sin(2\pi + t)$

$= \sin t$.

Example 2.

$x(t) = je^{j\frac{\pi}{4}t}$

$x(t) = je^{j\frac{\pi}{4}t}$

$x(t) = Aje^{j\omega_0 t}$

$\omega_0 = \frac{\pi}{4}$.

$T_0 = \frac{2\pi}{\pi/4}$

$T_0 = 8$ sec.

$x(t) = je^{j\frac{\pi}{4}t}$.

$x(t+T_0) = je^{j\frac{\pi}{4}(t+8)}$

$= je^{\frac{\pi}{4}jt} \cdot e^{j\frac{\pi}{4} \times \cancel{8}^{2}}$

$= je^{\frac{\pi}{4}jt} \cdot e^{2\pi j}$

$\cancel{= e_0}$.

$(e^{j\theta} = \cos\theta + j\sin\theta)$

$$\boxed{\begin{aligned} e^{2\pi j} &= \cos 2\pi + j\sin 2\pi \\ e^{2\pi j} &= 1 + j(0) \Rightarrow 1 \end{aligned}}$$

$$x(t+8) = j e^{j\frac{\pi}{4}t} \times 1$$

$$x(t+8) = j e^{j\frac{\pi}{4}t}$$

4) Combination / composite periodic signals.

$\Rightarrow$ Multiple signals combine together is known as Composite Signal.

$$x(t) = \cos(18\pi t) + \sin(12\pi t)$$

$$\omega_1 = 18\pi \quad , \quad \omega_2 = 12\pi$$

$$T_1 = \frac{2\pi}{18\pi} \qquad T_2 = \frac{2\pi}{12\pi}$$

$$T_1 = 1/9 \text{ sec} \qquad T_2 = 1/6 \text{ sec.}$$

Ratio of 1st fundamental time period to every signal fundamental time period.

Step 1 :- Find $T_1, T_2, T_3, T_4, \ldots$

Step 2 :- Find ratio of 1st fundamental time period to every signal fundamental time period.

Step 3 :- If $\frac{T_1}{T_2}, \frac{T_1}{T_3}, \frac{T_1}{T_4}, \ldots$ are rational numbers then it is a periodic signal.

Here,

$$\frac{T_1}{T_2} = \frac{1/9}{1/6} \Rightarrow \frac{6}{9} \Rightarrow 0.6666$$

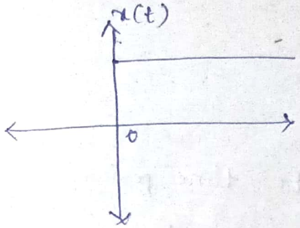
~~LCM [1/9, 1/6]~~

Resultant time period = LCM $[T_1, T_2, T_3, \ldots]$

$$W_0 = \underset{\text{HCF}}{\text{GCD (or)}} [\omega_1, \omega_2, \omega_3, \omega_4, \ldots]$$

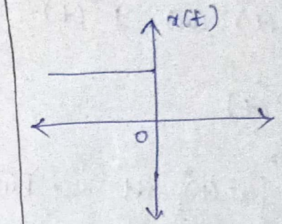
5) Causal, Anti Causal and Non Causal Signals.

Causal



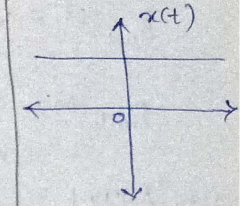
t	x(t)
t < 0	0

Anticausal



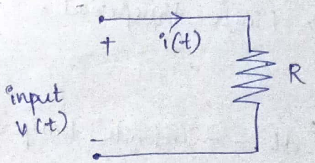
t	x(t)
t > 0	0

Non-Causal



t	x(t)
t > 0	≠ 0
t < 0	≠ 0

6) Energy and Power Signal.



$$P_{instant} = V_{instant}(t) * I_{instant}(t)$$

(power signal)

→ Energy = ∞

→ In Energy signal power is zero.

* $0 < E < \infty$ — energy finite.
* $0 < P < \infty$ — power finite.

$P_{inst}(t) = v(t) \times i(t)$

$= v(t) \times \frac{v(t)}{R}$

$P_{inst}(t) = \frac{v^2(t)}{R}$ (or) $P_{inst}(t) = I^2(t) \times R$

If $R = 1$.

$P_{inst}(t) = v^2(t)$ (or) $I^2(t)$.

$P_{inst}(t) = x^2(t)$

$$\text{Energy} = \int_{-T/2}^{T/2} |x^2(t)|\, dt \rightarrow \text{Finite time period.}$$

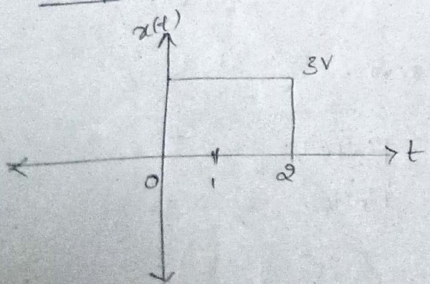
$$= \int_{-\infty}^{\infty} |x^2(t)|\, dt \rightarrow \text{Infinite time period.}$$

$$\boxed{\text{Power} = \frac{\text{Energy}}{T}}$$

$$\text{Power} = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt \rightarrow \text{Finite timeperiod.}$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt \rightarrow \text{Infinite timeperiod.}$$

Example



Find Energy & power.

Sol: $\int_{-\infty}^{0} 0^2 dt + \int_{0}^{2} 3^2 dt + \int_{2}^{\infty} 0^2 dt$

$\Rightarrow 9t\Big|_0^2$

$\Rightarrow 18.$

$\boxed{E = 18J}$

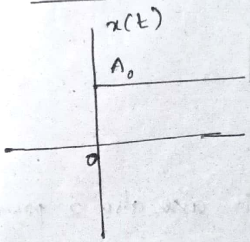
$\text{Power} = \lim_{T\to\infty} \frac{1}{T} \times 18J$

$= \frac{18J}{\infty}$

$\boxed{\text{Power} = 0}$

* If $\lim_{t\to\infty} x(t) = 0 \rightarrow$ then it is an Energy signal.
* If $\lim_{t\to\infty} x(t) \neq 0 \rightarrow$ then it is a power signal.

Example2.



Power

$\lim_{T\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} dt$

$\Rightarrow \lim_{T\to\infty} \frac{1}{T_0}\int_{-T_0/2}^{0} 0\, dt + \int_{0}^{T_0/2} (A_0)^2 dt$

$\Rightarrow \lim_{T\to\infty} \frac{1}{T_0} A_0^2\left[\frac{T_0}{2} - 0\right]$

$\Rightarrow \lim_{T\to\infty} \frac{A_0^2}{2}$

$$\boxed{P = \frac{A_0^2}{2} \text{ watts}}$$

$$E = \lim_{T\to\infty}\int_{-T_0/2}^{0} 0\, dt + \int_{0}^{T_0/2} A_0\, dt$$

$$\Rightarrow \lim_{T\to\infty} A_0^2\left[\frac{T_0}{2} - 0\right]$$

$$\boxed{E \Rightarrow \infty}$$

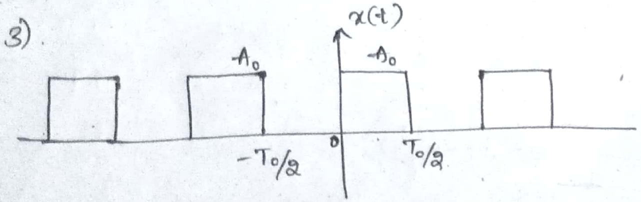
So it is a power signal.

→ $\lim_{T\to\infty} x(t) = 0$ (Energy signal)

→ $\lim_{T\to\infty} x(t) \neq 0$ (Power signal)

→ $\lim_{T\to\infty} x(t) = \infty$ (Neither energy nor power signal) NENP.

3).



→ If it is a periodic then it is ~~also~~ also a power signal.

→ All power signals are not periodic.

* Special Signals.

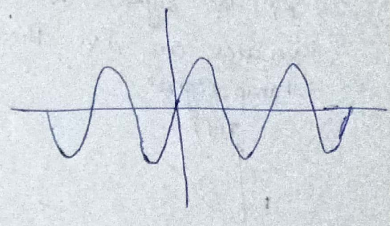
i) Sinusoidal Signal

→ If the signal is in sinusoidal shape then that signal is known as sine signal.

→ It is an odd signal.

→ Periodic signal

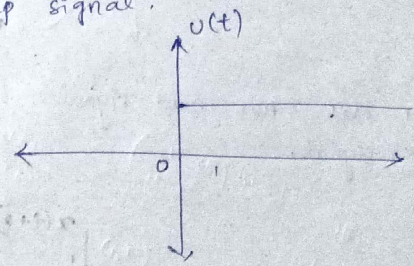
→ Power signal.

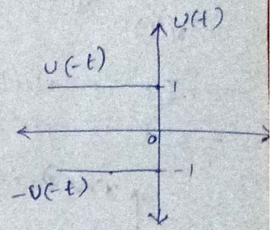


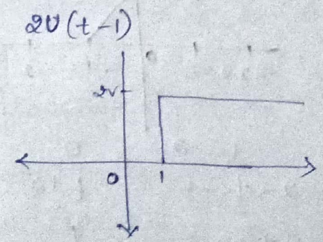
ii) Unit step signal.

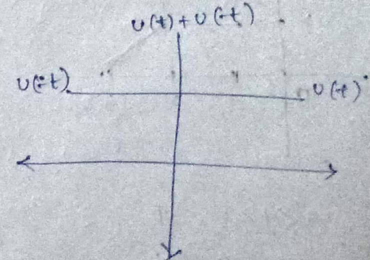
If the signal is in the form of flat line then that signal is called as step signal.

→ If the values are in positive & in the form of step then that signal is known as unit step signal.









→ It is NENO.

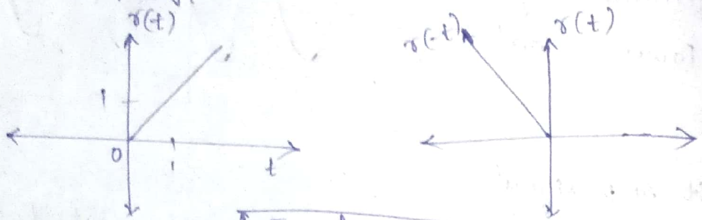
→ It Non periodic

→ Power Signal.

iii) Unit Ramp Signal. ($r(t)$)

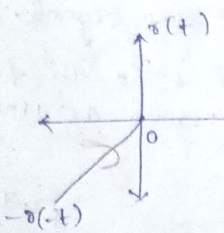
→ If the signal is in the shape of ramp then that signal is called as ramp signal.

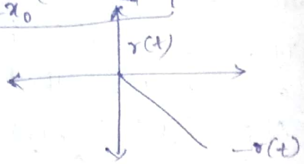
→ If the signal is in the shape of ramp & the values are +ve then it is known as unit ramp signal.



Formula

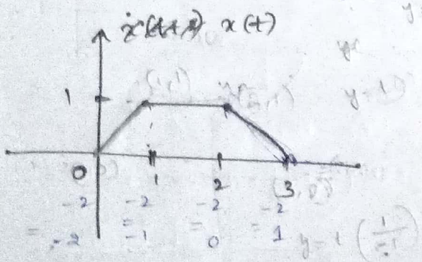
$$(y-y_0) = \frac{y_1-y_0}{x_1-x_0}(x-x_0)$$

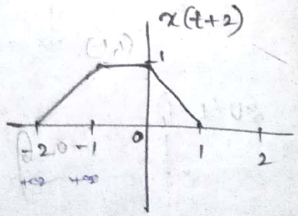




→ It is Neither even nor odd signal.

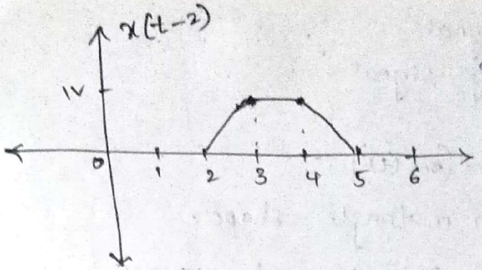
→ It is NENP signal.





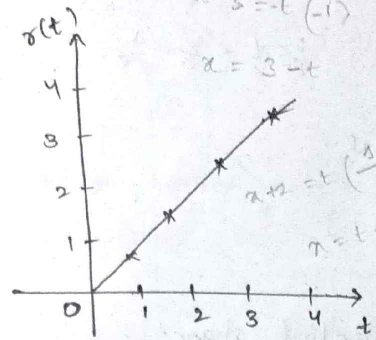
$0<t<1$	t
$1<t<2$	1
$2<t<3$	$3-t$
$t>3$	0

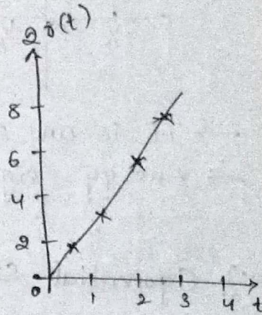
$t<-2$	0
$-2<t<-1$	$t+2$
$-1<t<0$	1
$0<t<1$	$3-(t+2) = 1-t$
$t>1$	0

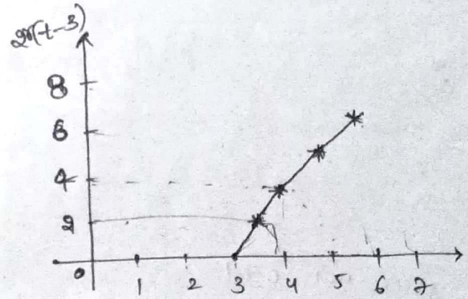


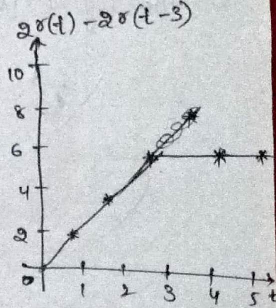
$t<2$	0
$2<t<3$	$t-2$
$3<t<4$	1
$4<t<5$	$3-(t-2) = (5-t)$
$t>5$	0

Example 2



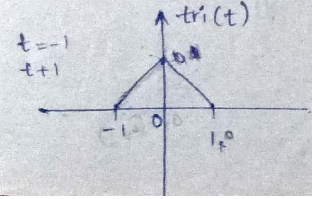






iv) Triangular Signal. ($tri(t)$)

→ If the signal is in triangle shape then it is known as triangular signal.



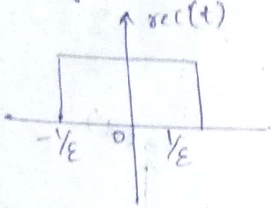
$-1<t<0$	$t+1$
$0<t<1$	$1-t$

Scanned with Oken Scanner

→ It is an even signal.
→ It is an energy signal.

v) Rectangular Signal. (rec (t))

→ The signal is in rectangle shape.



$t > 1/\varepsilon \quad 0$

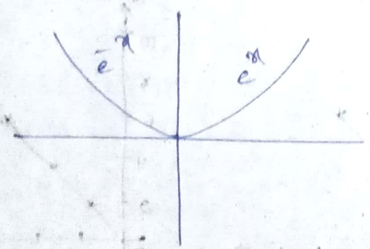
$t < -1/\varepsilon \quad 0$

$-1/\varepsilon < t < 1/\varepsilon \quad 1/\varepsilon$

→ It is an Even signal.
→ Energy signal.

vi) Exponential Signal

→ The signal is in exponential shape.

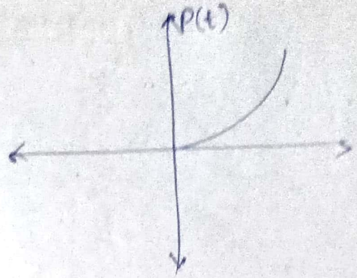


→ It is Neither even nor odd
→ It is Neither energy nor power signal.

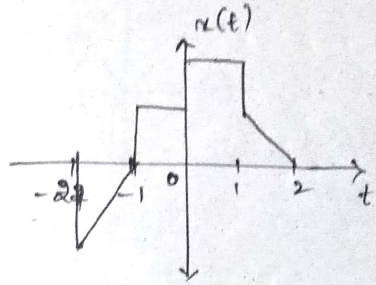
vii) ~~Hyperbolic~~ Parabolic Signal.

→ It is in the form of ~~hyperbola~~. Parabola.

→ It is NENO signal
→ It is NENP signal.

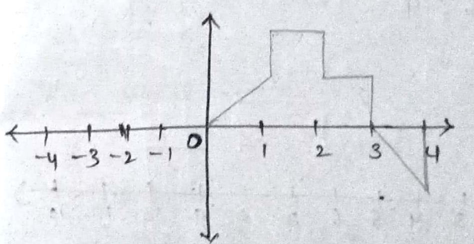


Example:



i) $x(2-t)$
ii) $x(2t+1)$
iii) $x(4-t/2)$

i) $x(2-t)$



$(2-t) = 2-t-2$
$= -t$

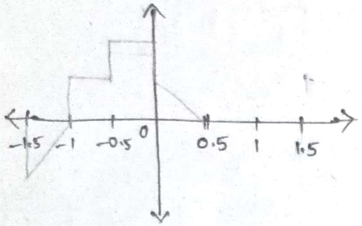
Reversal

4

i)	-2	-1	0	1	2
	-2	-2	-2	-2	-2
	-4	-3	-2	-1	0
	x(-1)	x(-1)	x(-1)	x(-1)	x(-1)
	4	3	2	1	0

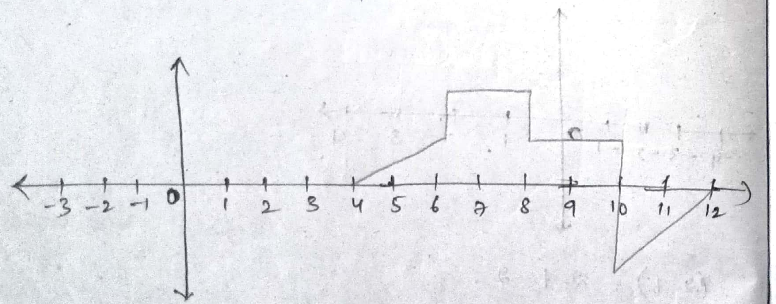
ii) $x(2t+1)$

$t \rightarrow$	-2	-1	0	1	2
	$+(-1)$	$+(-1)$	$+(-1)$	$+(-1)$	$+(-1)$
	-3	-2	-1	0	1
	$*(1/2)$	$*(1/2)$	$*(1/2)$	$*(1/2)$	$*(1/2)$
	-1.5	-1	-0.5	0	0.5

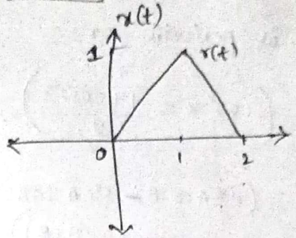


iii) $x(4 - t/2)$

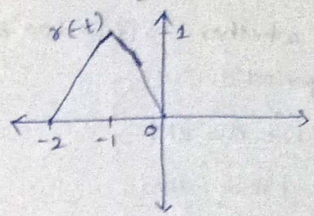
$t \Rightarrow$	-2	-1	0	1	2
	$+(-4)$	$+(-4)$	$+(-4)$	$+(-4)$	$+(-4)$
	-6	-5	-4	-3	-2
	$*(-1)$	$*(-1)$	$*(-1)$	$*(-1)$	$*(-1)$
	6	5	4	3	2
	$*(2)$	$*(2)$	$*(2)$	$*(2)$	$*(2)$
	12	10	8	6	4



Example 1

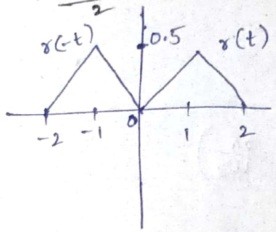


$x(-t)$



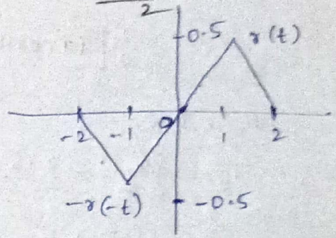
Even signal

$\frac{x(t) + x(-t)}{2}$

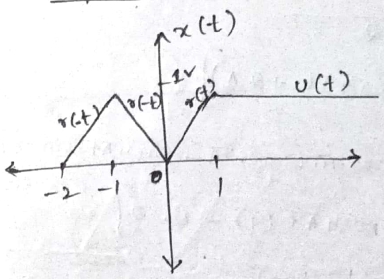


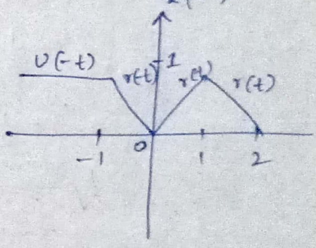
odd signal

$\frac{x(t) - x(-t)}{2}$



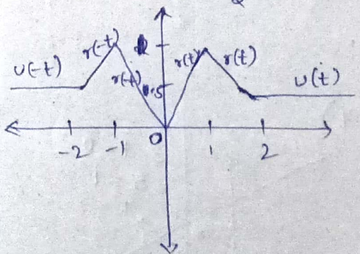
Example 2





Even signal

$\frac{x(t) + x(-t)}{2}$



odd signal

Example.

1) Find whether $x(t) = \cos^2 2\pi t$ is periodic or non-periodic.

Sol:- $x(t) = \cos^2 2\pi t$

$\left(\cos^2 x = \frac{1+\cos 2x}{2} \right)$

We know that,

$x(t) = A \cos \omega_0 t$.

$\left(\cos A \cos B - \sin A \sin B = \cos(A+B) \right)$

$\cos^2 2\pi t = \frac{1}{2}\left[1+\cos 2(2\pi t)\right]$

$= \frac{1}{2}\left[1+\cos 4\pi t\right]$.

$\omega_0 = 4\pi$

$T_0 = \frac{2\pi}{4\pi} = 0.5$ $\left(\because T_0 = \frac{2\pi}{\omega_0} \right)$.

$x(t) = x(t+T_0)$

$\frac{1+\cos 4\pi t}{2} = \frac{1+\cos 4\pi(t+0.5)}{2}$

$= \frac{1}{2}\left[1+\cos(4\pi t + 2\pi)\right]$

$= \frac{1}{2}\left[1+\cos 4\pi t \cdot \cos 2\pi - \sin 4\pi t \cdot \sin 2\pi\right]$

$= \frac{1}{2}\left[1+\cos 4\pi t\,(1) - 0 \cdot 0\right]$

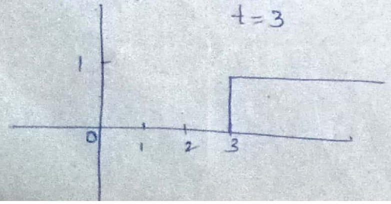
$= \frac{1}{2}\left[1+\cos 4\pi t\right]$.

$\frac{1+\cos 4\pi t}{2} = \cos^2 2\pi t$.

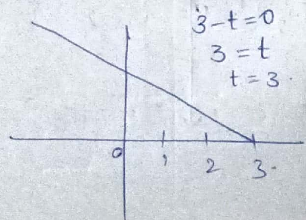
$\therefore$ It is a periodic signal.

2) Sketch $U(t-3)$ & $R(3-t)$.

$U(t-3)$ $\quad t-3=0$, $t=3$

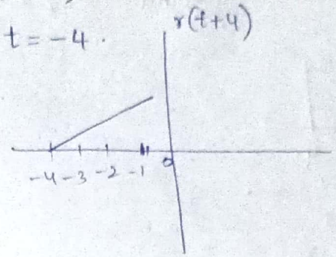


$3-t=0$, $3=t$, $t=3$.

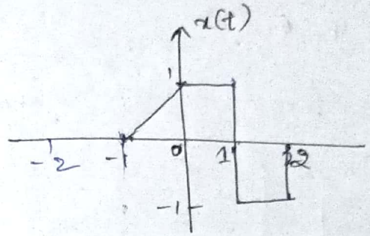


3) $r(t+4)$

$t=-4$.



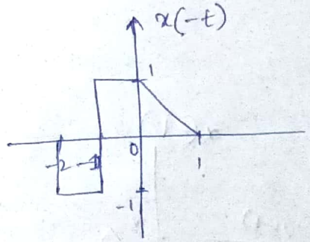
4)

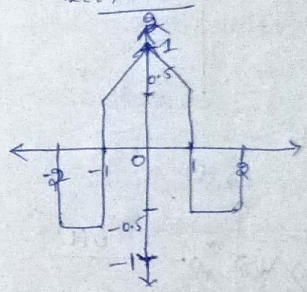


Sketch whether it is even or odd.

Even ~~sign~~ part

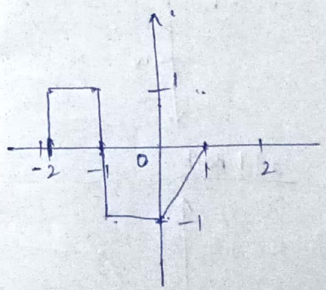
$\frac{x(t)+x(-t)}{2}$ $\quad \frac{x(t)-x(-t)}{2}$

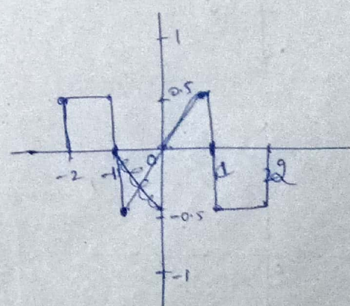




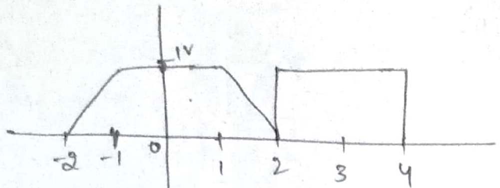
odd part.

$\frac{x(t)-x(-t)}{2}$

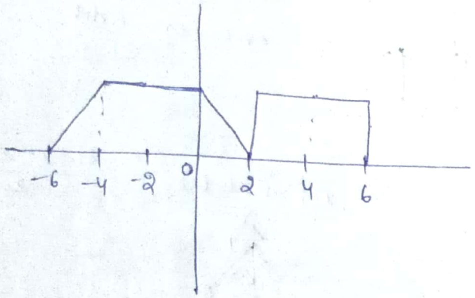




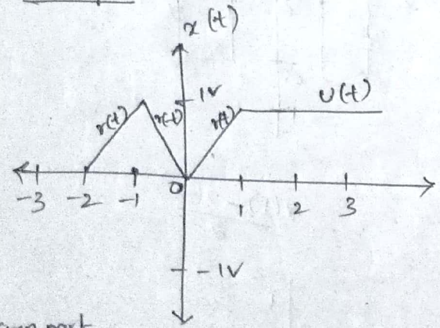
4) find $x\left(\frac{t}{2}+1\right)$

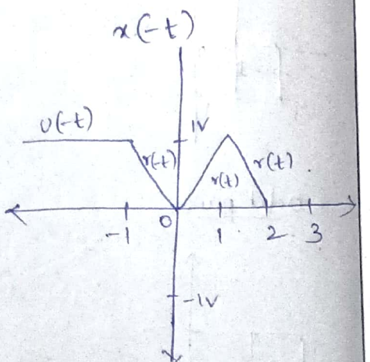


-1	-1	-1	-1	-1	-1	-1
-3	-2	-1	0	1	2	3
(x2)	(x2)	(x2)	(x2)	(x2)	(x2)	(x2)
-6	-4	-2	0	2	4	6

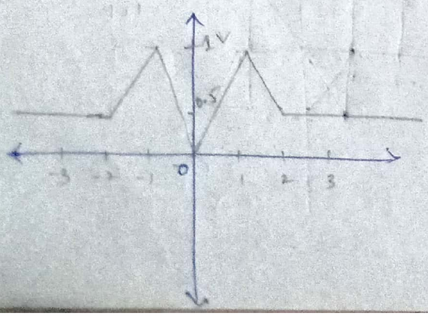


Example

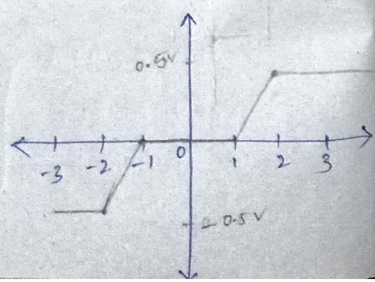




Even part

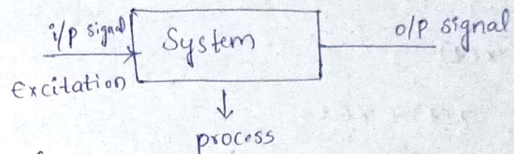


odd part



System

A device which do some process is known as System.



Types of Systems

1) Dynamic System and Static System.

→ If o/p response depends on either past or ~~futhus~~ future input ~~response~~ is known as dynamic system.

→ If o/p response depends on present input then that system is known as static system.

Ex: $y(t) = \sin t \cdot x(t)$

$$\left.\begin{aligned} y(0) &= \sin 0 \cdot x(0) \\ y(-1) &= \sin(-1)\, x(-1) \\ y(2) &= \sin 2\, x(2) \end{aligned}\right\} \text{Static.}$$

2) $y(t) = e^{-3x^2(t)}$

$$\left.\begin{aligned} y(0) &= e^{-3x^2(0)} \\ y(1) &= e^{-3x^2(1)} \\ y(-1) &= e^{-3x^2(-1)} \\ y(2) &= e^{-3x^2(2)} \\ y(-2) &= e^{-3x^2(-2)} \end{aligned}\right\} \text{Static}$$

3) $y(t) = x(\sin t)$

$$\left.\begin{aligned} y(0) &= x(\sin 0) \Rightarrow x(0) \\ y(1) &= x(\sin 1) = x(0.84) \end{aligned}\right\} \text{dynamic.}$$

4) $y(t) = \text{even } x(t)$

$$\text{even } x(t) = \frac{x(t)+x(-t)}{2}$$

$$\left.\begin{aligned} y(t) &= \frac{x(t)+x(-t)}{2} \\ y(0) &= \frac{x(0)+x(0)}{2} \\ y(1) &= \frac{x(1)+x(-1)}{2} \\ y(2) &= \frac{x(2)+x(-2)}{2} \end{aligned}\right\} \text{dynamic.}$$

2) Causal and Non-Causal System.

→ If a system o/p depends on past & present input then that system is known as Causal System.

→ If a system o/p depends on future i/p then that system is known as Non-Causal System.

Ex: $y(t) = x(t^2)$

$$\left.\begin{aligned} y(0) &= x(0) \\ y(1) &= x(1) \\ y(2) &= x(4) \\ y(3) &= x(9) \\ y(-1) &= x(1) \\ y(-2) &= x(4) \end{aligned}\right\} \text{Non-causal System response}$$

2) $y(t) = x(3t) \quad t<0$

$$\left.\begin{aligned} y(-1) &= x(-3) \\ y(-2) &= x(-6) \\ y(-3) &= x(-9) \\ y(-4) &= x(-16) \end{aligned}\right\} \text{Causal}$$

3) $y(t) = x(t-1) \quad t \geq 0$

$$\left.\begin{aligned} y(0) &= x(0-1) = x(-1) \\ y(1) &= x(0) \\ y(2) &= x(1) \\ y(3) &= x(2) \end{aligned}\right\} \text{Causal.}$$

3) Time variant System and Time-invariant System

→ If a system depends on time then that system is known as time variant System.

→ If a system does not depends on time then the System is known as time invariant system.

Let, $t = t - t_0$.

$y(t) = t \cdot x(t)$.

$y(t-t_0) = (t-t_0)\, x(t-t_0)$

$y_1(t) = (t-t_0)\, x(t-t_0)$

$y(t-t_0) = t\, x(t-t_0)$

$y_2(t) = t \cdot x(t-t_0)$.

$y_1(t) \neq y_2(t)$ hence, it is time-variant.

Examples

1) $y(t) = \int_{-\infty}^{2} x(t)^2 dt$ S.S or d.S.

2) $y(t) = \text{Real } x(t)$ S.S or d.S

3) $y(t) = 3x(-t^2)$ C.S or n.CS

4) $y(t) = x(e^t)$ T.V or T.I V

5) $y(t) = x(\cos(t))$ T.V or T.IV

6) $y(t) = x(t)+2$ T.V or T.IV.

7) $y(t) = \sin t \cdot x(t)$

3) $y(t) = 3x(-t^2)$

$y(0) = 3x(0)$

$y(1) = 3x(-1)$ Causal System.

$y(2) = 3x(-4)$

$y(-1) = 3x(-1)$

$y(-2) = 3x(-4)$

4) $y(t) = x(e^t)$

$y(t-t_0) = x(e^{t-t_0})$

$y_1(t) = x(e^{t-t_0})$

$y(t-t_0) = x(e^t - t_0)$ Time Invariant.

$y_2(t) = x(e^t - t_0)$

$y_1(t) = y_2(t)$.

5) $y(t) = x(\cos t)$

$y(t-t_0) = x(\cos(t-t_0))$

$y_1(t) = x(\cos(t-t_0))$ Time Invariant

$y(t-t_0) = x(\cos(t-t_0))$

$y_2(t) = x(\cos(t-t_0))$

$y_1(t) = y_2(t)$

6) $y(t) = x(t)+2$

$y(t-t_0) = x(t-t_0)+2$ Time Invariant

$y_1(t) = x(t-t_0)+2$

$y(t-t_0) = x(t-t_0)+2$

$y_2(t) = x(t-t_0)+2$

$y_1(t) = y_2(t)$.

7) $y(t) = \sin t \cdot x(t)$

$y(t-t_0) = \sin(t-t_0) \cdot x(t-t_0)$

$y_1(t) = \sin(t-t_0)\, x(t-t_0)$

$y(t-t_0) = \sin(t)\cancel{-t}\; x(t-t_0)$ Time variant.

$y_2(t) = \sin t \cdot x(t-t_0)$

$y_1(t) \neq y_2(t)$

2) $y(t) = \text{Real } x(t)$.

$\cancel{x}y(t) = t \cdot x(t)$

$y(0) = 0 \cdot x(0)$

$y(1) = 1 \cdot x(1)$

$y(2) = 2 \cdot x(2)$

$y(-1) = -1 \cdot x(-1)$

$\times\; y(-2) = -2 \cdot x(-2)$

$x(t) = A + jB$

$x^*(t) = A - jB$

~~Static~~

$2A = x(t) + x^*(t)$

$A = \frac{x(t)+x^*(t)}{2}$

$y(t) = \frac{x(t)+x^*(t)}{2}$

$y(0) = 0$

$y(1) = \frac{x(1)+x^*(1)}{2}$

$y(2) = \frac{x(2)+x^*(2)}{2}$

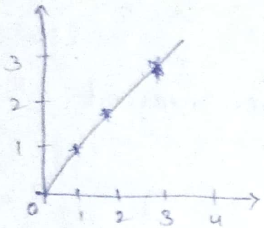
Static.

4) Linear and Non-Linear System.

→ If a system response is linear then that system is known as Linear System.

→ If a system response is Non-Linear then that system is known as Non-Linear System.

Linear

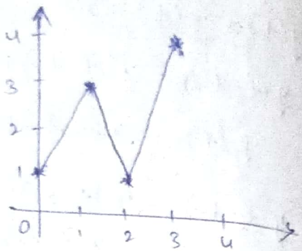


$x_1 - 0$	$y_1 - 0$
$x_2 - 1$	$y_2 - 1$
$x_3 - 2$	$y_3 - 2$
$x_4 - 3$	$y_4 - 3$

Sum of i/ps &
sum of o/ps

Sum of i/ps = sum of o/ps

Non-Linear.



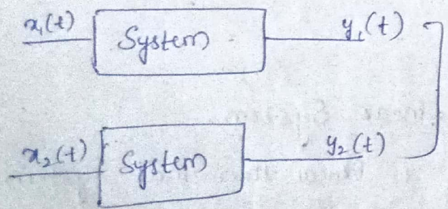
$x_1 - 0$	$y_1 - 1$
$x_2 - 1$	$y_2 - 3$
$x_3 - 2$	$y_3 - 1$
$x_4 - 3$	$y_4 - 4$

Sum of i/ps ≠ sum of o/ps

⇒ If sum of i/ps = sum of o/ps then that law is known as Law of Additivity (LOA)

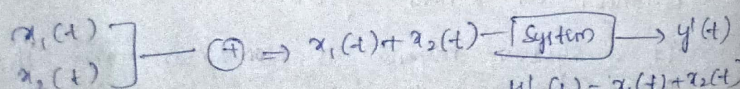
⇒ If product of o/p = product of i/p then the law is known as Law of Homogenity (LOH).

$y(t) = x(t)$.



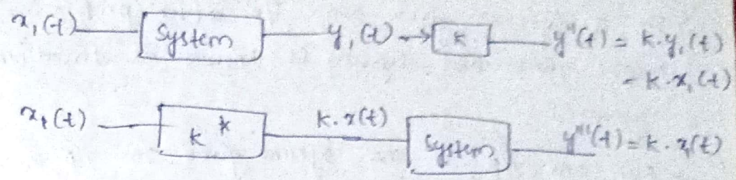
$\rightarrow y_1(t) + y_2(t) = y(t)$

$x_1(t) + x_2(t) = y_1(t) + y_2(t)$



$y'(t) = x_1(t) + x_2(t)$

$y(t) = y'(t) \rightarrow$ LOA



LOH $\quad y''(t) = y'''(t)$

5) B.O Stable and Unstable system.

→ If a system response is finite for finite i/p then that is known as stable system

→ If a system response is undefine / infinite o/p then that is known as Unstable system

Ex:-

$y(t) = u(t) \rightarrow$ stable

$y(t) = 2 \rightarrow$ stable

Ex2

$y(t) = t\,x(t)$

$x(t) = 2.$

$x(t) = u(t)$

i) $x(t) = 2$

$y(t) = 2t$.

ii) $x(t) = u(t)$

$y(t) = t$

} Undefine o/p
Unstable.

6) Invertible and Non-Invertible System:

→ If a system gives same i/p after performing reverse then that system is known as Invertible System.

→ For one i/p, if the system gives one o/p then that system is known as Invertible System.

→ If a system gives different i/p after performing reverse then that system is known as Non-invertible system.

→ ~~For~~ If multiple i/ps gives same o/p then that System is known as Non-invertible system.

→ classify the following systems are dynamic (or) not.

1. $y(t) = x(t-3) \rightarrow$ Dynamic
2. $y(t) = x(2t) \rightarrow$ Dynamic
3. $y(t) = \frac{d^2x(t)}{dt^2} + 2x(t) \rightarrow$ Dynamic

→ classify the following systems are causal or not

1. $y(t) = x^2(t) + x(t-4) \rightarrow$ Causal
2. $y(t) = x(2-t) + x(t-4) \rightarrow$ Causal
3. $y(t) = x\left(\frac{t}{2}\right) \rightarrow$ ~~Causal~~ Non-causal
4. $y(t) = x(\sin 2t) \rightarrow$ Non-causal
5. $y(t) = \int_{-\infty}^{3t} x(t)\, dt. \rightarrow$ Non-causal

→ classify the following systems are linear or Non-linear.

1. $\frac{d^2y(t)}{dt^2} + 2t\,y(t) = t^2 x(t) \rightarrow$ Linear
2. $2\frac{dy(t)}{dt} + 5y(t) = x^2(t) \rightarrow$ Non-linear
3. $\frac{dy(t)}{dt} + y(t) = x(t)\cdot\frac{dx(t)}{dt} \rightarrow$ Non-Linear
4. $y(t) = 2x^2(t) \rightarrow$ Non-Linear
5. $y(t) = x(t^2) \rightarrow$ Linear
6. $y(t) = e^{x(t)} \rightarrow$ Non-linear
7. $y(t) = \int_{-\infty}^{t} x(\tau)\cdot d\tau \rightarrow$ Linear

→ classify the following systems are Time variant or Time invariant.

1. $y(t) = t^2 x(t) \rightarrow$ Time variant
2. $y(t) = x(t^2) \rightarrow$ Time variant
3. $y(t) = x(t)\cdot \sin 10\pi t \rightarrow$ Time variant
4. $y(t) = x(-2t) \rightarrow$ Time variant
5. $y(t) = e^{2x(t)} \rightarrow$ Time invariant.

→ classify the following Systems are Stable or not.

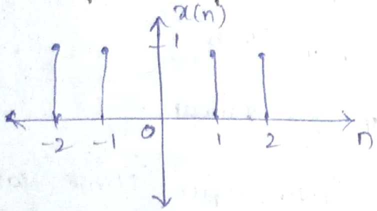
1. $y(t) = (t+5)\cdot u(t) \rightarrow$ Unstable
2. $h(t) = e^{2t}.u(t) \rightarrow$ Unstable
3. $h(t) = (2+e^{-3t})\,u(t) \rightarrow$ Unstable
4. $h(t) = e^{2t}.u(t) \rightarrow$ Unstable

→ Classify the following systems are Invertible or Non-Invertible.

1. $y(t) = x(t-2)$ → Invertible
2. $y(t) = \sin(x(t))$ → Non invertible
3. $y(t) = \sin t \cdot x(t)$ → Non invertible.
4. $y(t) = x^2(t)$. → Non invertible

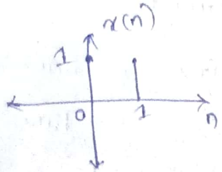
* Discrete Signal.

i) Graphic Representation



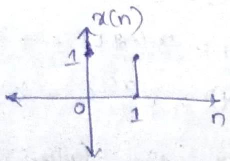
ii) Tabular Representation

n	x(n)
0	1
1	1



iii) Functional Representation

$$x(n) = \begin{cases} 1 & n=0 \\ 1 & n=1 \end{cases}$$

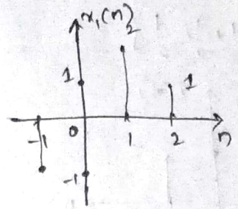


iv) Sequential Representation

$$x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$$

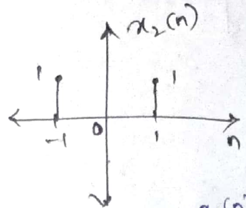
→ Keeping arrow at n=0.

1) Addition of two DT signals.



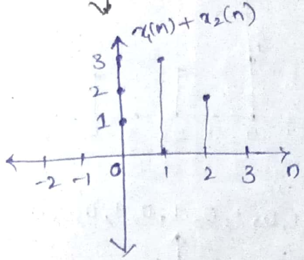
n	x(n)
−1	−1
0	1
1	2
2	1

$x(n) = \{-1, \underset{\uparrow}{1}, 2, 1\}$



n	x(n)
−1	1
0	0
1	1

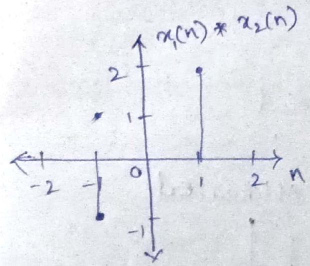
$x(n) = \{1, \underset{\uparrow}{0}, 1\}$



n	x(n)
−1	0
0	1
1	3
2	1

$x(n) = \{0, \underset{\uparrow}{1}, 3, 1\}$

2) Multiplication of two DT Signals.



n	x(n)
−1	−1
0	0
1	2
2	0

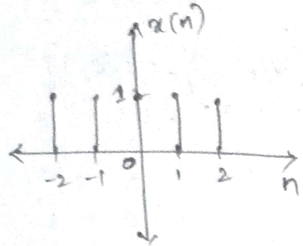
$x(n) = \{-1, \underset{\uparrow}{0}, 2, 0\}$

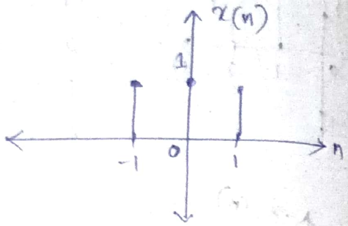
3) Time scaling

i) $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$x(2n) = \{1, 1, 1\}$

-2   0   +2





ii) $x(n/2)$

$n=-4 \quad x(-2)=1$

$n=-3 \quad x(-1.5)=0$

$n=-2 \quad x(-1)=1$

$n=-1 \quad x(-0.5)=0$

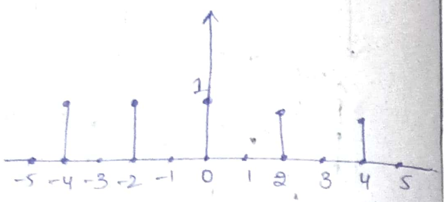
$n=0 \quad x(0)=1$

$n=1 \quad x(0.5)=0$

$n=2 \quad x(1)=1$

$n=3 \quad x(1.5)=0$

$n=4 \quad x(2)=1$



$x\left(\frac{n}{2}\right) = \{1,0,1,0,\underset{\uparrow}{1},0,1,0,1,0\}$

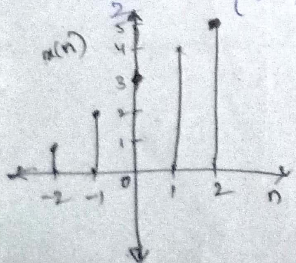
0

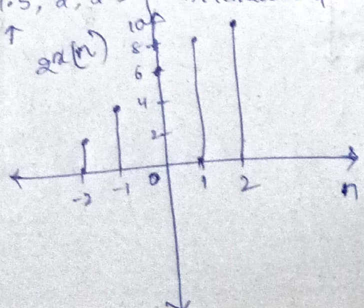
4) Amplitude scaling

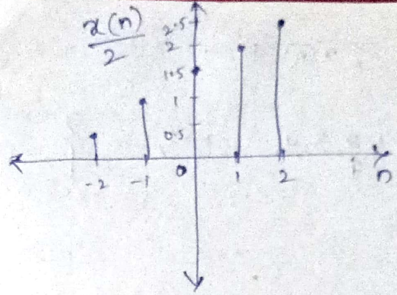
$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$2x(n) = \{2, 4, \underset{\uparrow}{6}, 8, 10\}$ amplified

$\frac{x(n)}{2} = \{0.5, 1, \underset{\uparrow}{1.5}, 2, 2.5\}$ Attenuated.







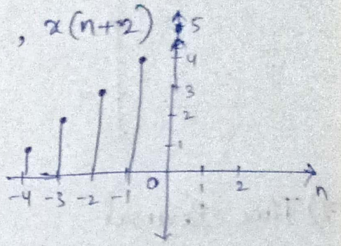
5) Time shifting

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$, $x(n+2)$

-2 -1 0 1 2

-2 -2 -2 -2 -2

-4 -3 -2 -1 0



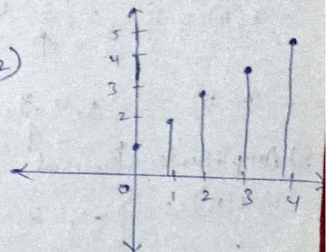
$x(n+2) = \{1, 2, 3, 4, \underset{\uparrow}{5}\}$

$x(n) = \{1, 2, 3, 4, 5\}$, $x(n-2)$

+2 +1 0 +1 +2

+2 +2 +2 +2 +2

0 1 2 3 4

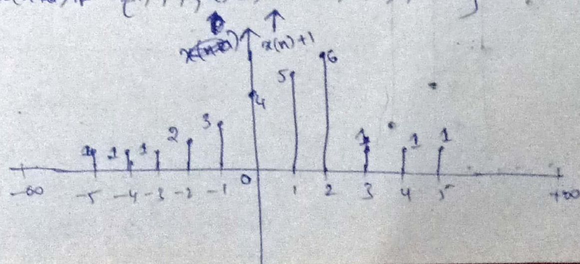


$x(n-2) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$

6) Amplitude shifting

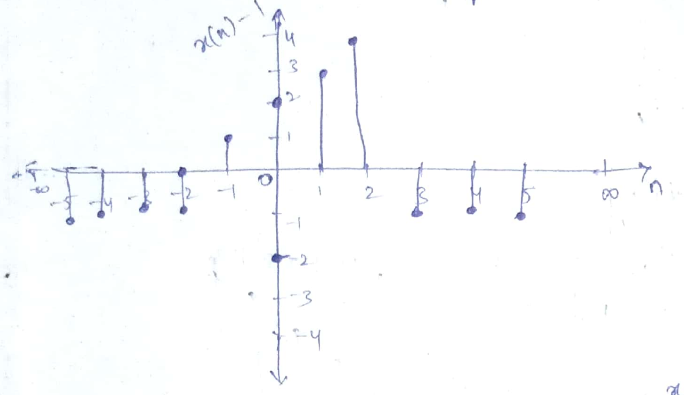
$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$ $\quad x(n)+1$

$x(n)+1 = \{1, 1, 1, 2, 3, 4, 5, 6, 1, 1, 1, \ldots\}$



$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$, $x(n) - 1$

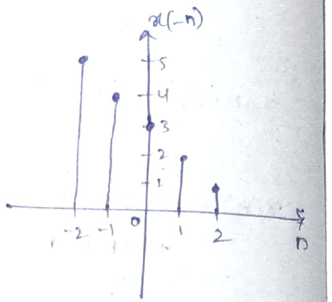
$x(n) - 1 = \{\ldots -1, -1, -1, 0, 1, \underset{\uparrow}{2}, 3, 4, -1, -1, -1, \ldots\}$



7) Time Reversal

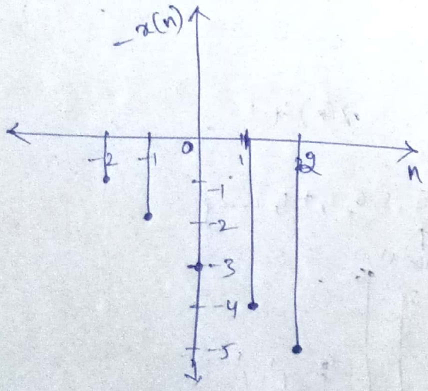
$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$x(-n) = \{5, 4, \underset{\uparrow}{3}, 2, 1\}$



8) Amplitude Reversal

$-x(n) = \{-1, -2, \underset{\uparrow}{-3}, -4, -5\}$



9) Multiple Transformation

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

1) $x(-2n-1)$

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$$\begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \\ +1 & +1 & +1 & +1 & +1 \\ -1 & 0 & 1 & 2 & 3 \\ \times(1/2) & \times(1/2) & \times(1/2) & \times(1/2) & \times(1/2) \\ -0.5 & 0 & 0.5 & 1 & 1.5 \end{array}$$

$\{\underset{\uparrow}{2}, 4\}$

$x(-2n-1) = \{4, \underset{\uparrow}{2}\}$

2) $x\left(\frac{2n}{3}\right)$

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$$\begin{array}{ccccc} -2 & -1 & 0 & 1 & 2 \\ \times\left(\frac{3}{2}\right) & \times\left(\frac{3}{2}\right) & \times\left(\frac{3}{2}\right) & \times\left(\frac{3}{2}\right) & \times\left(\frac{3}{2}\right) \\ -3 & \times & 0 & \times & 3 \end{array}$$

$x\left(\frac{2n}{3}\right) = \{1, 0, 0, \underset{\uparrow}{3}, 0, 0, 5\}$

* Classification of DT Systems:

~~i) Static~~ System :

i) Static System & Dynamic System

ii) Causal & Non-causal

iii) Time variant & Time invariant

iv) Linear & Non-linear

v) Stable & Unstable

1) Classify whether the system is static or dynamic

i) $y(n) = x^2(n) \rightarrow$ Static

ii) $y(n) = x(n+2) \rightarrow$ Dynamic

iii) $y(n) = x(n-2) + x(n) \rightarrow$ Dynamic

2) Classify causal or Non-causal

i) $y(n) = x(2n) \rightarrow$ Non-causal

ii) $y(n) = x(-n) \rightarrow$ Causal

iii) $y(n) = x(n) + x(n-2) \rightarrow$ Causal

iv) $y(n) = \sin[x(n)]$

3) Classify linear or Non-linear.

i) $y(n) = n^2 x(n) \rightarrow$ linear

ii) $y(n) = x(n) + \frac{1}{2x(n-2)}$

iii) $y(n) = 2x(n) + 4 \rightarrow$ linear

iv) $y(n) = x(n) \cdot \cos \omega n \rightarrow$ linear

4) Classify Time variant or Time invariant

i) $y(n) = x\left(\frac{n}{2}\right)$

ii) $y(n) = x(n)$

iii) $y(n) = x(n) + n x(n-2)$

iv) $y(n) = x^2(n-2)$

* Classification of DT Signals.

i) Deterministic & Non-deterministic

ii) Even & odd signal

Ext

$x(n) = \{-4-5j, \underset{\uparrow}{1+2j}, 4\}$

Even

$x(n) = x(-n)$

odd

$x(n) = -x(n)$ (or)

$-x(n) = x(-n)$

Even part

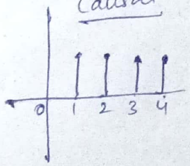
$\frac{x(n) + x(-n)}{2}$

odd

$\frac{x(n) - x(-n)}{2}$

iii) Causal and Anticausal.

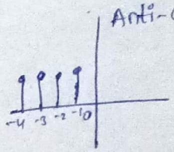
Causal



$x(n) = 0 \quad n<0$

$x(n) \neq 0 \quad n>0$

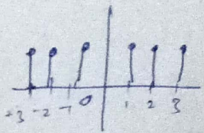
Anti-causal



$x(n) = 0 \quad n>0$

$x(n) \neq 0 \quad n<0$

Non-causal



$x(n) \neq 0 \quad n>0$

$x(n) \neq 0 \quad n<0$