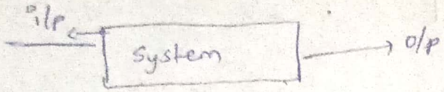
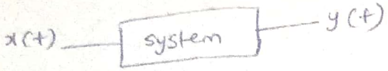
Systems



Excitation signal (i/p)

process



we have two types of systems.

1) Static system

2) Dynamic system

$y(t) = x(t)$

output response with respect to $x(t)$ at the time period of '0' sec.

$y(0) = x(0)$

$t=1$

$y(t) = x(t)$

$y(1) = x(1)$

$t=2$ $y(2) = x(2)$

$t=-1$

$y(-1) = x(-1)$

present time output is completely depends upon present input signal.

Static ~~system~~

The complete output response is depends on complete present input signal.

$y(t) = x(t)$

→ Dynamic system

The output response of present signal is depends on ~~present~~ past and future input signal.

$y(t) = x(t+1) + x(t-1)$

$y(0) = x(0+1) + x(0-1)$

$\underbrace{y(0)}_{\text{present time}} = \underbrace{x(1) + x(-1)}_{\text{past \& future.}}$

Eg: $y(t) = x(\sin t)$

$y(0) = x(\sin 0)$

$y(0) = x(0)$

$y(1) = x(\sin 1)$

$y(1) = x(0.84)$

$y(2) = x(\sin 2)$

$y(2) = x(0.91)$

Static system

$y(t) = \sin t \, x(t)$
$y(1) = \sin 1 \, x(1)$
$y(2) = \sin 2 . x(2)$
$y(3) = \sin 3 . x(3)$

present output completely depends on present input.

→ memoryless system :-
which doesn't require memory. So, It is Static system

Eg: Image u want to see in mobile.

→ memory system :-
which requires memory. So, It is dynamic system.

Eg: Image u wat to store in mobile requires memory (past & Future)

$y(t) = \text{even } x(t)$

The response which is given by a input response is Dynamic

$$y(t) = \frac{x(t) + x(-t)}{2}$$

$$y(0) = \frac{x(0) + x(0)}{2}, \quad y(1) = \frac{x(1) + x(-1)}{2}$$

∴ Dynamic system.

$y(t) = \text{real } x(t)$

$x(t) = A + i\beta$

$x^*(t) = A - i\beta$

$x(t) + x^*(t) = 2A$

$$A = \frac{x(t) + x^*(t)}{2}$$

$$y(t) = \frac{x(t) + x^*(t)}{2}$$

$$y(0) = \frac{x(0) + x^*(0)}{2}$$

$$y(1) = \frac{x(1) + x^*(1)}{2}$$

$$y(2) = \frac{x(2) + x^*(2)}{2}$$

∴ It will be static system. The response will only depends on the present input signal.

$$y(0) = \int_{-\infty}^{2} x(t)\, dt$$

It is a dynamic system.

→ present Output response is depends on past & future response

Causal system and non-causal system

↓

o/p → past and present i/p

$y(t) = x(t) + x(t-1)$

$y(0) = x(0) + x(-1)$

$y(1) = x(1) + x(0)$

o/p → future i/p

$y(t) = x(t)^2$

$y(0) = x(0)$

$y(1) = x(1)$

$y(2) = x(4)$

$y(3) = x(9)$

$y(-2) = x(4)$

we have to check from $-\infty$ to ∞ for any system.

i) $y(t) = x(3t)$ $t < 0$

ii) $y(t) = x(t-1)$ $t \geq 0$

$y(t) = x(3t)$ $t < 0$

$y(-2) = x(3(-2))$

$y(-2) = x(-6)$

$y(-3) = x(3(-3))$

$y(-3) = x(-9)$

~~non-cas~~

~~causal~~

causal system

$y(t) = x(t-1)$

$y(0) = x(0-1)$

$y(0) = x(-1)$

$y(1) = x(1-1)$

$y(1) = x(0)$

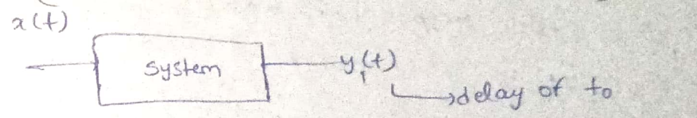
causal system

→ If a system changes with time is Time variant. and

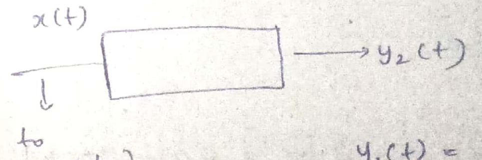
If a system doesn't changes with time is Time Invariant

Time variant and Time Invariant

(dependent) (independent)



$y_1(t) = y(t-t_0)$



$x(t+t_0)$

$y_1(t) = y_2(t)$

→ If we change the output after the delay or before the delay, the time doesn't changes. That respons, that system is time variant system.

$y_1(t) = y(t-t_0)$

$x_2(t) = x_2(t+t_0)$

$y(t) = 2x(t)$ Time Invariant

$y(t-t_0)$

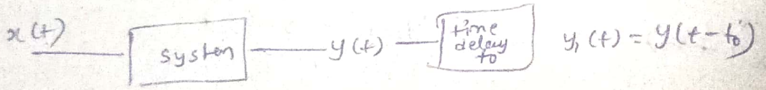
$\Rightarrow y(t) = 2x(t-t_0)$

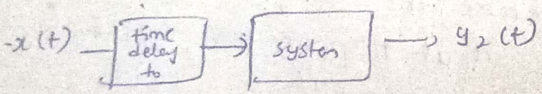
$y_1(t) = 2x(t-t_0)$ $y_2(t) = x(t-t_0) \Rightarrow 2x(t-t_0)$

delay of t_0 $y_1(t) = y_2(t)$

→ If a system response is varies with time, It is Time variat.

→ If a system response is not varies with time variation. It is Time Invariant.





$x(t-t_0)$

$y_1(t) = y_2(t)$ = Time Invariat

$y_1(t) \neq y_2(t)$ – Time variat

$\Rightarrow y(t) = t \cdot x(t)$

$y_1(t) = y(t-t_0) = (t-t_0)\,x(t-t_0)$

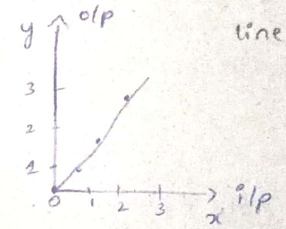
$y_2(t) = x(t-t_0) = t\,x(t-t_0)$

$y_1(t) \neq y_2(t)$

$\therefore$ It is time variant

10/10/2023

linear System and non-linear system



linear

Law of Additivity

$\boxed{\text{Sum of inputs} = \text{sum of outputs}}$

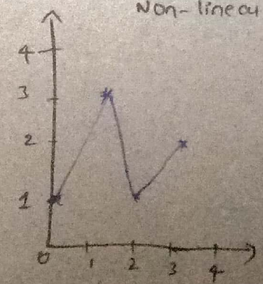
Eg:- $x_1 + x_2 = y_1 + y_2$

$0+1 = 0+1$

$1 = 1$

$x_1 = 0$	$y_1 = 0$
$x_2 = 1$	$y_2 = 1$
$x_3 = 2$	$y_3 = 2$
$x_4 = 3$	$y_4 = 3$

Non-linear



$x_1 - 0$	$y_1 - 1$
$x_2 - 1$	$y_2 - 3$
$x_3 - 2$	$y_3 - 1$
$x_4 - 3$	$y_4 - 2$

$\boxed{\text{Sum of inputs} \neq \text{sum of outputs}}$

→ Non-linear doesn't hold the Law of Additivity.

Linear:

considering any 'k' value

$k = 1, 2, 3, 4, 5, 6 \ldots$

Law of Homogenity [product of input → product of output]

$\boxed{k x_1 = k y_1}$ $(\because k = 1, 2, 3, 4 \ldots)$

Eg: $2(0) = 2(0) \Rightarrow 0 = 0$ ✓

$2(1) = 2(1) \Rightarrow 2 = 2$ ✓

Non-Linear:

$\boxed{k x_1 \neq k y_1}$

Non-Linear system does not hold the Law of homogenity.

Law of Additivity

→ $y(t) = x(t)$

Given, $y(t) = x(t)$

$x_1(t)$ — system — $y_1(t)$

$x_2(t)$ — system — $y_2(t)$

$y(t) = y_1(t) + y_2(t)$

$x_1(t) + x_2(t)$ — system — $y'(t) = x_1(t) + x_2(t)$

$y(t) = y'(t)$

satisfying the LOA.

Law of Homogenity

$x_1(t)$ — system — $y_1(t)$ — product (k)

↳ $y''(t) = k \cdot x_1(t)$

$x_1(t)$ — product (k) — system →

↳ $k \cdot x_1(t)$ $\quad y'''(t) = k \cdot x_1(t)$

$\boxed{y''(t) = y'''(t)}$

→ It is satisfying LOH.

$\therefore$ The system is Linear.

→ $$\frac{d^2 y(t)}{dt^2} + 2t \frac{dy(t)}{dt} = t^2 x(t)$$

$$\frac{d^2 y_1(t)}{dt^2} + 2t \frac{dy_1(t)}{dt} = t^2 x_1(t) \quad \text{—— (1)}$$

$$\frac{d^2 y_2(t)}{dt^2} + 2t \frac{dy_2(t)}{dt} = t^2 x_2(t) \quad \text{—— (2)}$$

By Adding (1) & (2), we get

$$\frac{d^2}{dt^2}\left[y_1(t) + y_2(t)\right] + 2t \frac{d}{dt}\left[y_2(t) + y_1(t)\right]$$
$$= t^2\left[x_1(t) + x_2(t)\right]$$

Holds, LOA.

$$a\left(\frac{d^2y_1(t)}{dt^2} + 2 + \frac{dy_1(t)}{dt^2}\right) \pm \left(t^2\, x_1(t)\right) a$$

$$b\left(\frac{d^2y_2(t)}{dt^2} + 2 + \frac{dy_2(t)}{dt^2}\right) = \left(t^2 x_2(t)\right) b$$

$$\frac{d^2}{dt^2}\left[ay_1(t) + by_2(t)\right] + 2 + \frac{d}{dt}\left[by_2(t) + y_1(t)\right] =$$
$$t^2\left[ax_1(t) + bx_2(t)\right]$$

→ It is Holding LOH.

∴ It is a Linear.

$$\rightarrow \frac{d^2y(t)}{dt^2} = x_1^2(t)\,t^2$$

It is a non-linear system.

Stable system and unstable system.

BIBO :- Bounded input and bounded output

Bounded :- finite

→ If we are responding finite output to the finite input.

→ It is stable system.

Eg: $y(t) = x(t)$

$x(t) = 2$

$y(t) = 2$

$$\boxed{\begin{array}{l} u(t) \\ t \geq 0 = 1 \\ t < 0 \quad 0 \end{array}}$$

→ $x(t) = u(t)$

$\boxed{y(t) = u(t)}$ → stable system.

unstable system:

If the output is undefined or infinite for the given Input.

Eg: $x(t) = t$

$y(t) = x(t)$

$y(t) = 2t$

Invertible and Non-Invertible system

$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = 2 \cdot x(t)$

Here, the system is multiplying by 2; that '2' is call it as $h(t)$

$2 ;- h(t)$

$\frac{1}{h(t)} \rightarrow h^{-1}(t)$ (Inverting)

$y(t) = 2 \cdot x(t) \times h^{-1}(t)$

$= \cancel{2}\, x(t) \times \frac{1}{\cancel{2}}$

$= x(t)$

Here, we are getting back $x(t)$ by doing Inverse function, so It is Invertible system.

Eg: $y(t) = x^2(t)$ (Non-Invertible)

$x(t)$ 2 $\quad y(t) = 4$

$x(t) = \sqrt{4} = 2$

$x(t)$ -2 $\quad y(t) = 4$

$x(t) = \cancel{2}$

many inputs — one outputs (same output)

→ not getting back $x(t)$.

→ $y(t) = 3x(t) + 2$

$3x(t)+2 \Rightarrow -2$

$3x$

$x(t) = 1 \quad y(t) = 3(1) + 2 = 5$

$x(t) = 2 \quad y(t) = 3(2) + 2 = 8$

$x(t) = -2 \quad y(t) = 3(-2) + 2 = -6+2 = -4$

Inverting

$y(t) = 5 - 2 \Rightarrow \frac{3}{3} = 1$

$y(t) = 8 - 2 \Rightarrow \frac{6}{3} = 2$

$y(t) = -4 - 2 \Rightarrow \frac{-6}{3} = -2$

For one i/p → one o/p → Invertible

For many i/ps → one o/p → Non-Invertible.

Here we are getting back $x(t)$

one inputs maps with one output.

→ Find whether the following systems are dynamic (or) not.

i) $y(t) = x(t-3)$ – D

ii) $y(t) = x(2t)$ → D

iii) $y(t) = x(t-2) + x(t)$

iv) $y(t) = \frac{d^2 x(t)}{dt^2} + 2x(t)$ → D

→ Find whether the following are casual / non-causual.

i) $y(t) = x^2(t) + x(t-4)$ → casu

ii) $y(t) = x(2-t) + x(t-4)$ → casual

iii) $y(t) = x(t/2)$ → Non-casual.

iv) $y(t) = x(\sin 2t)$ → Non-Casual.

Problems

Static & Dynamic

i) $y(t) = x(t-3)$

$y(1) = x(1-3) = x(-2)$

$y(2) = x(-1)$

$y(3) = x(0)$

$y(-1) = x(-4)$

$y(4) = x(1)$

$\therefore$ The o/p is depends upon past & future

so, It is Dynamic system.

ii) $y(t) = x(2t)$

$y(1) = x(2)$

$y(2) = x(4)$

$y(-1) = x(-2)$

$y(-2) = x(-4)$

$y(0) = x(0)$

$\therefore$ The o/p is depends on past & future

So, It is dynamic.

iii) $y(t) = x(t-2) + x(t)$

$y(1) = x(-1) + x(1)$

$y(0) = x(-2) + x(0)$

$y(-1) = x(-3) + x(-1)$

$y(-2) = x(-4) + x(-2)$

$y(2) = x(0) + x(2)$

It depends on past & future

so, It is dynamic.

iv) $y(t) = \dfrac{d^2x(t)}{dt^2} + 2x(t)$

$t = 0,\quad y(0) = \dfrac{d^2x(0)}{dt^2} + 2x(0)$

$\dfrac{d}{dt}(x(t)) = \dfrac{x(t) - x(t-\Delta t)}{\Delta t}$ changing

$\therefore$ It is Dynamic

Causal & Non-Causal

i) $y(t) = x^2(t) + x(t-4)$

$y(-1) = x^2(-1) + x(-1-4)$

$y(0) = x^2(0) + x(-4)$

$y(1) = x^2(1) + x(-3)$

It depends on past & present.

$\therefore$ It is causal.

ii) $y(t) = x(2-t) + x(t-4)$

$y(-1) = x(3) + x(-5)$

$y(0) = x(2) + x(-4)$

$y(1) = x(1) + x(-3)$

$y(2) = x(0) + x(-2)$

It depends on past & present

$\therefore$ It is causal.

iii) $y(t) = x(t/2)$

$t=0, \quad y(0) = x(0/2) = x(0)$

$t=1 \quad y(1) = x(1/2) = x(0.5)$

$t=-1 \quad y(-1) = x(-1/2) = x(-0.5)$ future

$t=-2 \quad y(-2) = x(-2/2) = x(-1)$ future

$\therefore$ It is non-causal.

iv) $y(t) = x(\sin 2t)$

$t=-\pi, \quad y(-\pi) = x(\sin 2(-\pi))$

$= x(0) \rightarrow$ future

$\therefore$ non-causal.

v) $y(t) = \int_{-\infty}^{3t} x(t)\,dt$

$y(t) = \int_{-\infty}^{3t} x(t)\,dt = x(3t) - x(-\infty)$

$t=0, \quad y(0) = x(0)$

$t=1, \quad y(1) = x(3) \rightarrow$ future ✓

$t=-1, \quad y(-1) = x(-3)$ (past)

$t=(-2) \quad y(-2) = x(-6)$ (past

$\therefore$ It is non-causal.

Linear / Non-Linear

i) $\frac{dy(t)}{dt} + y(t) = x(t) + \frac{dx(t)}{dt}$

$x_1 - y_1 \qquad a \times x_1(t) \cdot \frac{dx_1(t)}{dt} = \frac{dy_1(t)}{dt} + y_1(t) \times a$

$x_2 - y_2 \qquad b \times x_2(t) \cdot \frac{dx_2(t)}{dt} = \frac{dy_2(t)}{dt} + y_2(t) \times b$

Sum

$\frac{d}{dt}\left[a x_1^2(t) + x_2^2(t) b\right] = \frac{d}{dt}\left[a y_1(t) + b y_2(t)\right] +$

$a y_1(t) + b y_2(t).$

$\therefore$ Non-Linear response.

It is a non-linear system.

ii) $y(t) = x(t^2)$

$y_1(t) = x_1(t^2)$

$y_2(t) = x_2(t^2)$

$y_1(t) + y_2(t) = x_1(t^2) + x_2(t^2)$

It satisfies of LOA

$a.y_1(t) = a.x_1(t^2)$

$b.y_2(t) = b.x_2(t^2)$

$a.y_1(t) + b.y_2(t) = a.x_1(t^2) + b x_2(t^2)$

It satisfies LOH (outside output should be equal)

$\therefore$ It is a Linear system.

iii) $y(t) = \int_{-\infty}^{t} x(t)\,dt$

$y_1(t) = \int_{-\infty}^{t} x_1(t)\,dt$

$y_2(t) = \int_{-\infty}^{t} x_2(t)\,dt$

$y_1(t) + y_2(t) = \int_{-\infty}^{t} x_1(t)\,dt + \int_{-\infty}^{t} x_2(t)\,dt$

$y_1(t) + y_2(t) = x_1(t^2) + x_2(t^2)$

It satisfies LOH & LOA.

$\therefore$ Linear system.

iv) $y(t) = 2x^2(t)$

$y_1(t) = 2x_1^2(t)$

$y_2(t) = 2x_2^2(t)$ $\neq x_1(t)$

$y_1(t) + y_2(t) = 2x_1^2(t) + 2x_2^2(t)$

It not satisfies LOH, LOA.

$\therefore$ It is non-Linear system.

v) $y(t) = e^{x(t)}$

$y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y_1(t) + y_2(t) = e^{x_1(t)} + e^{x_2(t)}$

$a.y_1(t) + b.y_2(t) = a.e^{x_1(t)} + b.e^{x_2(t)}$

It is not satisfying the LOH & LOA.

$\therefore$ It is a Non-Linear system.

Time variant & Time Invariant

i) $y(t) = t^2 x(t)$

$y_1(t) = y(t-t_0) = (t-t_0)^2 x(t-t_0)$

$y_{11}(t) = x(t-t_0) = t^2 x(t-t_0)$

$y_1(t) = (t-t_0)^2 x(t-t_0)$

$y_{11}(t) = t^2 x(t-t_0)$

$y_1(t) \neq y_{11}(t)$

$\therefore$ It is Time variant.

ii) $y(t) = x(t) \sin 10\pi t$

$y_1(t) = y(t-t_0) = x(t-t_0) \sin 10\pi (t-t_0)$

$y_{11}(t) = x(t-t_0) = x(t-t_0) \sin 10\pi t$

$y_1(t) \neq y_{11}(t)$

$\therefore$ It is Time variant.

iii) $y(t) = x(t^2)$

$y_1(t) = y(t-t_0) \Rightarrow x((t-t_0)^2)$

$y_{11}(t) = x(t-t_0)$

$y_1 \neq y_{11}(t)$

$\therefore$ It is Time variant

$y(t) = x(t) \rightarrow$ Time Invariant

$y(t) = x(t^2) \rightarrow$ Time varied

$y(t) = x(2t) \rightarrow$ Time varied

iv) $y(t) = e^{2x(t)}$

$y_1 = y(t-t_0) = e^{2x(t-t_0)}$

$y_{11} = x(t-t_0) = e^{2x(t-t_0)}$ } Equal.

$y_1 = y_{11}$

$\therefore$ It is Time Invariant

v) $y(t) = x(-2t)$

$y(t-t_0) = x(-2t-t_0)$

$x(t-t_0) = x(-2(t-t_0))$

$\therefore$ Time variant.

Stable & unstable

i) $y(t) = e^{x(t)}$, $|x(t)| \leq 8$.

$x(t) = 0$

$y(t) = e^0 = 1$

$x(t) = 1, \Rightarrow y(t) = e^1 = e$

$x(t) = 2, \Rightarrow y(t) = e^2 = e^2$

$x(t) = 8, \Rightarrow y(t) = e^8 = e^8$

$\therefore$ It is stable.

ii) $h(t) = (2+e^{-3t})\, \underset{1\ (\text{unit step signal})}{\underbrace{u(t)}}$

$$\int_{-\infty}^{\infty} (2+e^{-3t})$$

$\therefore$ unstable.

$$\Rightarrow \int_{0}^{\infty} (2+e^{-3t}) \Rightarrow \infty$$

iii) $y(t) = (t+5)\, \underset{1\ (\text{unit step signal})}{\underbrace{u(t)}}$

when $t \to \infty$

$y(t) = \infty$

$\therefore$ It is unstable.

iv) $h(t) = e^{2t} \cdot \underset{1}{\underbrace{u(t)}}$

$$h(t) = \int_{-\infty}^{\infty} e^{2t}\, dt = \infty$$

It is unstable.

v) $y(t) = \int_{-\infty}^{t} x(t) \cdot dt$

It is unstable.

Invertible or non-Invertible

i) $y(t) = x(t) + 2$

$x(t) = 1 \quad y(t) = 3-2 = 1$

$x(t) = 2 \quad y(t) = 4-2 = 2$

$x(t) = 3 \quad y(t) = 5-2 = 3$

$x(t) = 4 \quad y(t) = 6-2 = 4$

one to one

$\therefore$ It is invertible

ii) $y(t) = x(t-2)$

$x(1) \Rightarrow y(t) = x(-1) = x(1)$

$x(2) \Rightarrow y(t) = x(0) = x(2)$

$x(3) \Rightarrow y(t) = x(1) = x(3)$

$x(4) \Rightarrow y(t) = x(2) = x(4)$

$\therefore$ It is Invertible.

iii) $y(t) = \sin(x(t))$

$\therefore$ It is non- Invertible.

iv) $y(t) = \sin t \cdot x(t)$

$x(t) = 1 \Rightarrow y(t) = \sin t \rightarrow \sin t$

$x(t) = 2 \Rightarrow y(t) = 2\sin t \rightarrow 2\sin t/_2 = \sin t$

$x(t) = 3 \Rightarrow y(t) = 3\sin t \rightarrow 3\sin t/_3 = \sin t.$

$\therefore$ It is non- Invertible.

v) $y(t) = x^2(t)$

$x(t) = 2 \Rightarrow y(t) = 4$

$x(t) = -2 \Rightarrow y(t) = 4$

many to one

Non - Invertible.

vi) $y(t) = \frac{d}{dt} x(t)$

$t=0, \; y(0) = 0$

$t=1, \; y(1) = 0$

$t=2, \; y(2) = 0$

for constants, it will be zero.

many i/ps → one o/p.

$\therefore$ Non - Invertible.

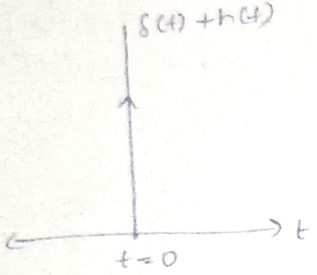
Impulse (or) Delta signal :-
↓
dirac

→ It is also known as dirac Delta signal

→ It is developed by dirac

→ It is represented as $\delta(t)$ or $h(t)$.

Graphical



Equation :-

$\delta(t) = \infty \quad t = 0$

$\delta(t) = 0 \quad t \neq 0$

→ It is Even signal.

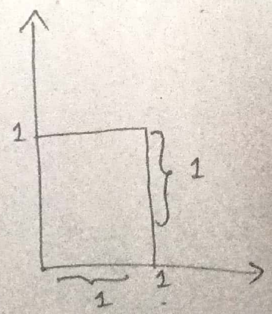
→ It is neither energy nor power signal

Area of Impulse

$$\int_{0}^{\infty} \delta(t) = 1$$

⇒ Area = 1.

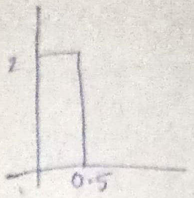
It is called as unit Impulse signal.



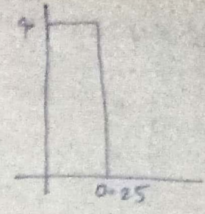
Area = width × height.
= 1×1
= 1

→ As time gets minimum, signal goes to infinite. $t \to 0$, Amplitude → ∞.

$\boxed{\therefore \text{ Area} = 1}$



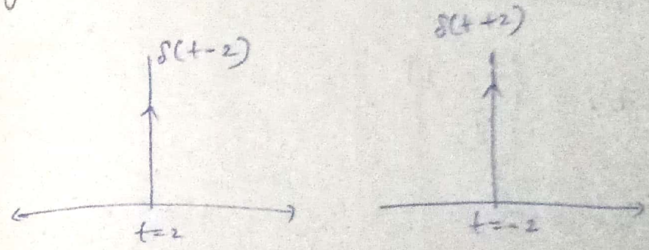
$2 \times \frac{1}{2} = 1$



$4 \times \frac{1}{4} = 1$

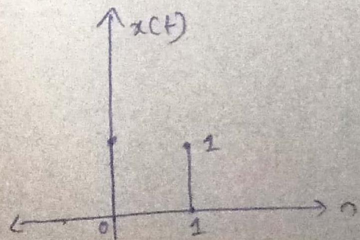
⇒ Signal will be always upward arrow mark.

Eg: $\delta(t-2)$



Discrete Time Signals :-

1) Graphical Representation :-



2) Tabular Representation.

n	x(n)
0	1
1	1

3) Functional Representation.

$$x(n) = \begin{cases} 1 & n=0 \\ 1 & n=1 \end{cases}$$

4) Sequence Representation.

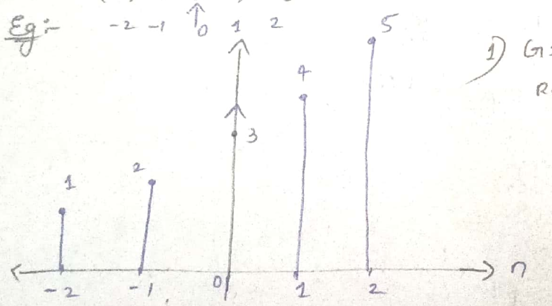
→ Here, sequence of Amplitudes are represented.

→ Represented as { }.

$$x(n) = \{1, 1\}$$
$\uparrow_0$

⇒ ↑ for n = 0.

Eg:- $\{1, 2, \underset{\uparrow}{3}, 4, 5\}$
-2 -1 0 1 2



1) Graphical Representation.

2) Tabular Representation.

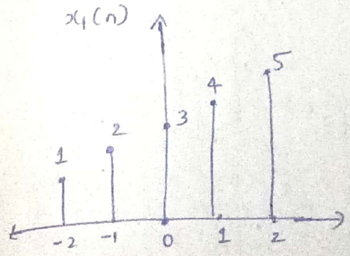
n	x(n)
-2	1
-1	2
0	3
1	4
2	5

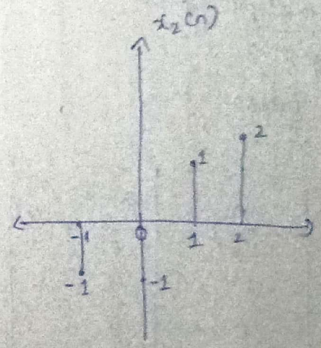
3) Functional Representation :-

$$x(n) = \begin{cases} 1 & n=-2 \\ 2 & n=-1 \\ 3 & n=0 \\ 4 & n=1 \\ 5 & n=2 \end{cases}$$

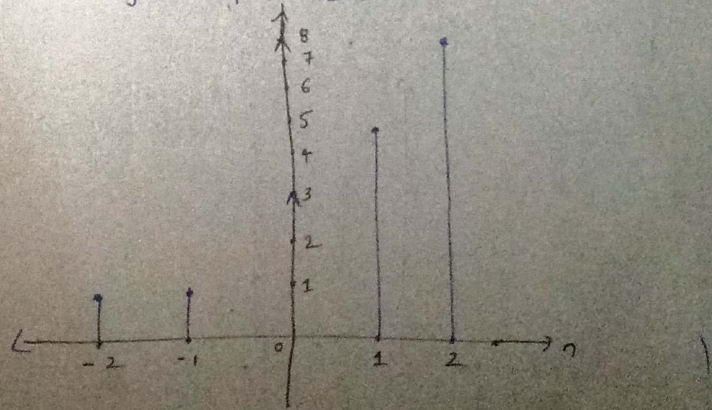
Operations of DT Signals :-

1) Addition :-





$$y(n) = x_1(n) + x_2(n)$$



→ Tabular Representation

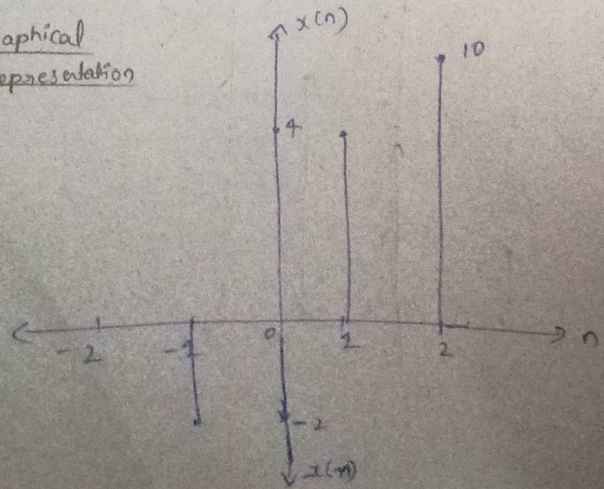
n	$y(n) = x_1(n) + x_2(n)$
$n=-2$	1
$n=-1$	1
$n=0$	3
$n=1$	5
$n=2$	~~4~~

2) multiplication :-

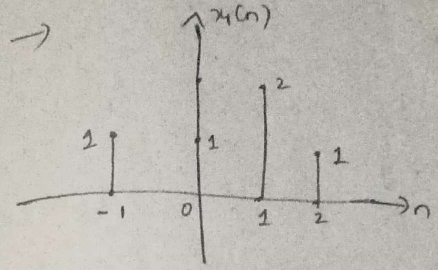
Eg:
1) Table:-

n	$y(n) = x_1(n) * x_2(n)$
-2	0
-1	-2
0	0
1	4
2	10

Graphical Representation

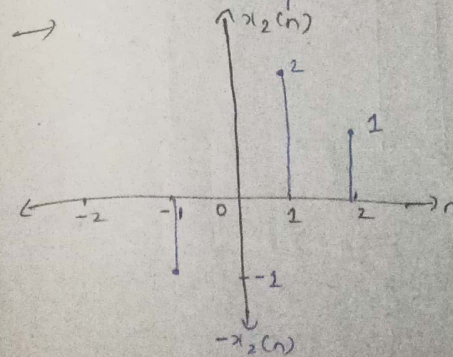


⇒ multiplication of two signals:

→


n	$x_1(n)$
-1	1
0	1
1	2
2	1

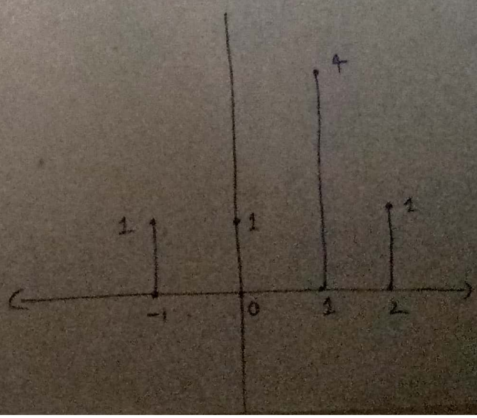
$$x_1(n) = \{1, 1, 2, 1\}$$
-1 0 1 2
↑

→


n	$x_2(n)$
-1	-1
0	0
1	2
2	1

$$x_2(n) = \{-1, 0, 2, 1\}$$
-1 0 1 2
↑

$$y(n) = x_1(n) * x_2(n)$$



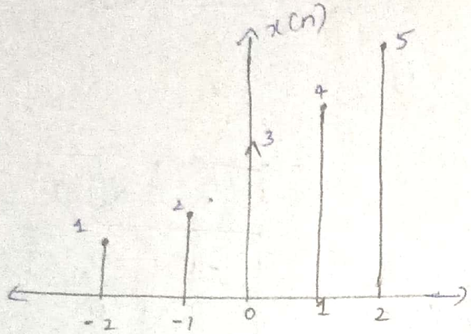
n	$x_1(n)*x_2(n)$
-1	1
0	1
1	4
2	1

Time Scaling :-

→ $k > 1$ → compressed

→ $k < 1$ → expanded

$x(n) = \{1, 2, 3, 4, 5\}$
↑



Sol: $x(n) = \{1 \quad 2 \quad 3 \quad 4 \quad 5\}$
$\quad -2 \quad -1 \quad 0 \quad 1 \quad 2$
↑

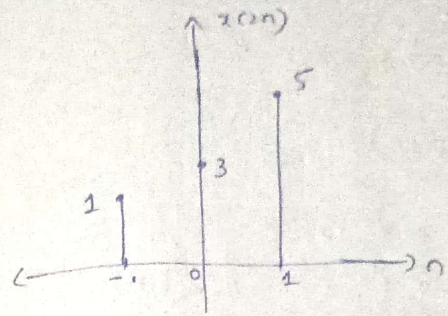
i) $x(2n)$

-3	-2	-1	0	1	2	3
-6	-4	-2	0	2	4	6

→ As we don't have signal at -4 & +4 in $x(n)$, so, we don't take it.

$x(2n) = \{1, 3, 5\}$
$\quad -2 \quad 0 \quad 2$

Graphical representation



ii) $x(n/2)$

$\{1, 2, 3, 4, 5\}$

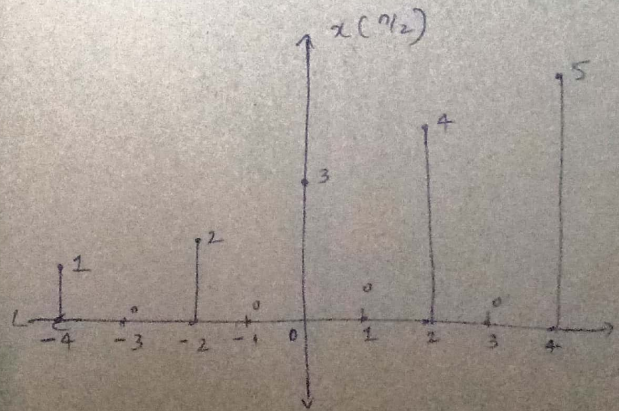
$-4/2 \quad -3/2 \quad -2/2 \quad -1/2 \quad 0/2 \quad 1/2 \quad 2/2 \quad 3/2 \quad 4/2$

$-2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2$

whenever, decimals come, make it as zero.

$x(n/2) = \{1, 0, 2, 0, 3, 0, 4, 0, 5\}$
$\quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

Graphical Representation.



Another example of Time Scaling

1) $x(n) = \{7, 6, 8, 10\}$

find $x(n/2)$, $x(2n)$

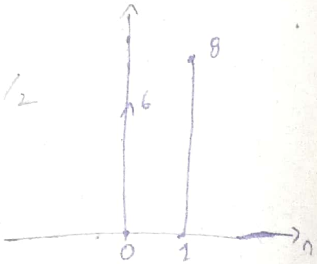
Given, $x(n) = \{7, \underset{\uparrow}{6}, 8, 10\}$

n: -1, 0, 1, 2

$\left(-\frac{1}{2}, \frac{0}{2}, \frac{1}{2}, \frac{2}{2}\right) /2$

-0.5, 0, 0.5, 1

$\cancel{x}$, 0, $\cancel{x}$, 1



$x(2n) = \{6, 8\}$

$\rightarrow x(n/2) = \{7, 6, 8, 10\}$

$\Rightarrow \{-1, 0, 1, 2\}$

$(-2\ 0\ 2\ 4) \times 2$

$x(n/2) = \{7, 0, 6, 0, 8, 0, 10\}$

Amplitude Scaling

$x(n): \{1, 2, \underset{\uparrow}{3}, 4, 5\}$, $2x(n)$, $x(n)/2$

n: -2, -1, 0, 1, 2

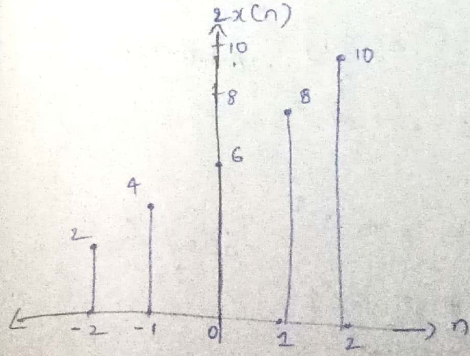
$2x(n) = \{2, 4, \underset{\uparrow}{6}, 8, 10\}$ → Amplified

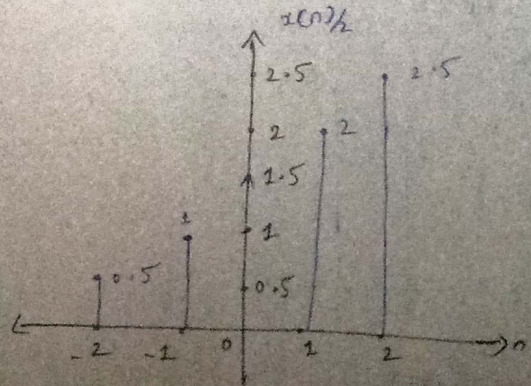
n: -2, -1, 0, 1, 2

$\frac{x(n)}{2} = \{0.5, 1, \underset{\uparrow}{1.5}, 2, 2.5\}$ → Attenuation

n: -2, -1, 0, 1, 2

Graphical Representation





Time shifting

$x(n \pm k)$ $+k \rightarrow$ Advanced
$-k \rightarrow$ delayed

$\rightarrow x(n+2)$ $n+2$ -2 Advanced.
$n-2$ $+2$ delayed.

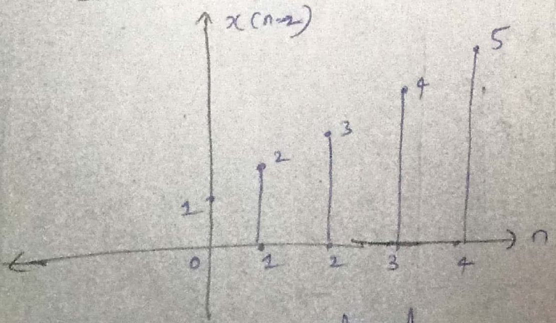
$n+\cancel{2}-\cancel{2} = n.$

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$
$-2 \quad -1 \quad 0 \quad 1 \quad 2$

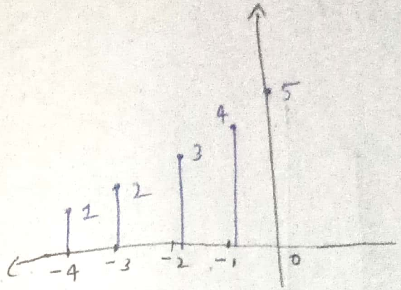
$x(n+2)$ $\{1, 2, 3, 4, \underset{\uparrow}{5}\}$ Advanced
$-4 \quad -3 \quad -2 \quad -1 \quad 0$

$x(n-2)$ $\{\underset{\uparrow}{1}, 2, 3, 4, 5\}$ delayed.
$0 \quad 1 \quad 2 \quad 3 \quad 4$

Graphical Representation



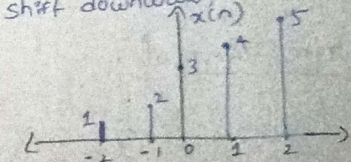
delayed



Advanced.

Amplitude shifting

$x(n) \pm k$ $+k \rightarrow$ shift upward
$-k \rightarrow$ shift downward

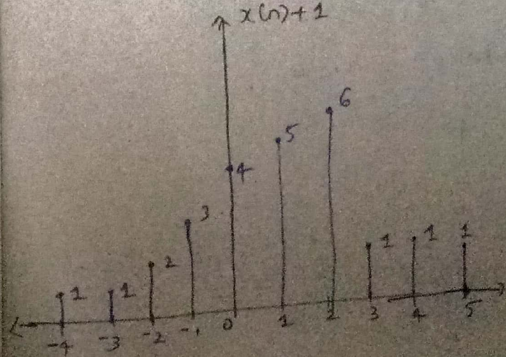


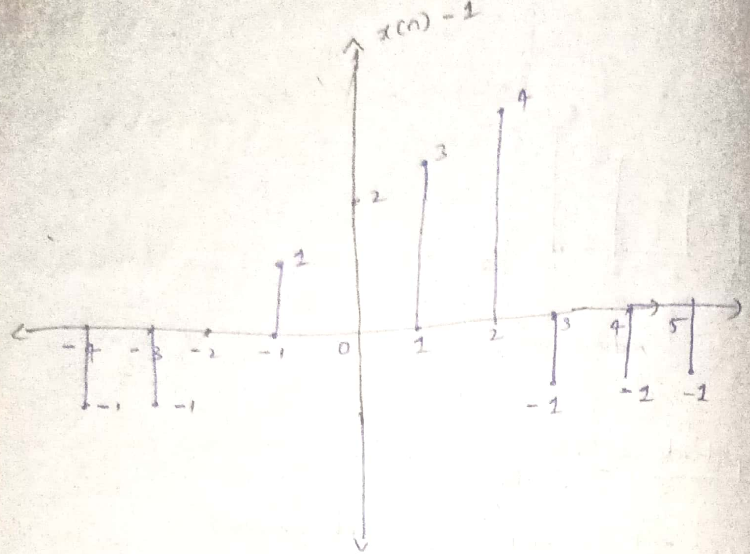
1) $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$
$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$x(n)+1 = \{\ldots +1, +1, +1, +1, 2, 3, 4, 5, 6, 1, 1, 1, \ldots\}$
$-6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

$x(n)-1 = \{\ldots -1, -1, -1, 0, 1, 2, 3, 4, -1, -1, -1 \ldots\}$
$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

Graphical Representation





Time Reversal

$x(n) = x(-n)$

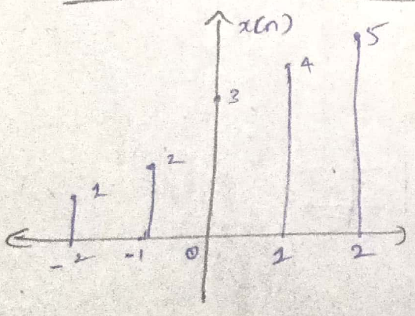
$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

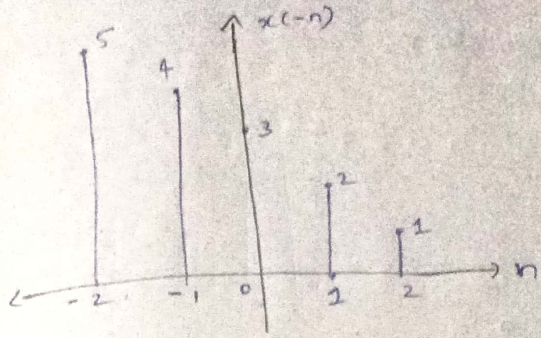
$\quad\;\; -2 \; -1 \; 0 \; 1 \; 2$

$x(-n) = \{5, 4, \underset{\uparrow}{3}, 2, 1\}$

$\quad\;\; -2 \; -1 \; 0 \; 1 \; 2$

Graphical representation





Amplitude Reversal

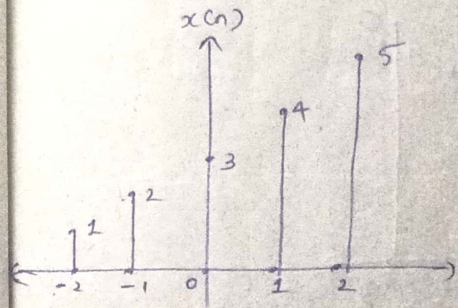
$x(n) = -x(n)$

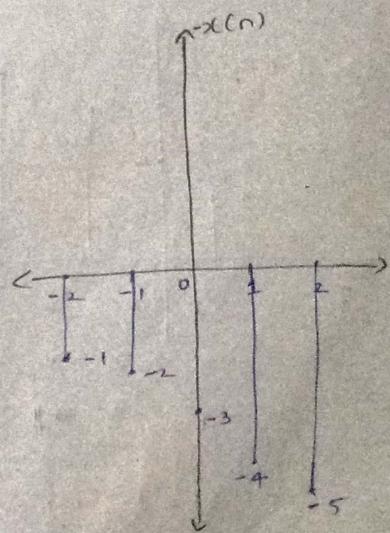
1) $x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$

$\quad\;\; -2 \; -1 \; 0 \; 1 \; 2$

$-x(n) = \{-1, -2, -3, -4, -5\}$

Graphical representation





Scanned with Oken Scanner

Multiple Transformation

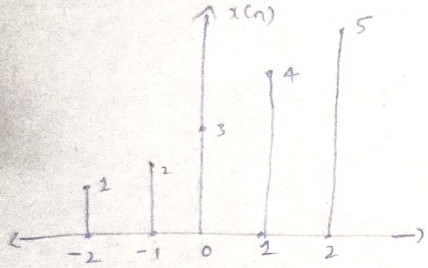
find $x(-2n-1)$

$-2n-1$

$-2n+\cancel{1}-\cancel{1}$

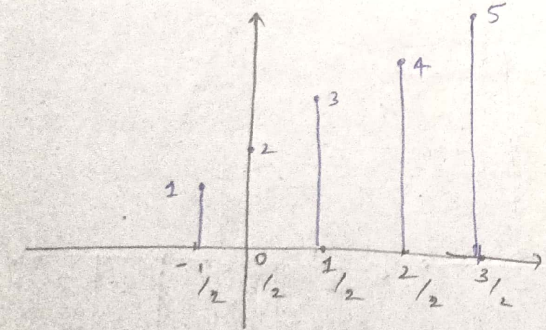
$-2n/2 \quad -nx-1 = n.$

$x(n) = \{1, 2, 3, 4, 5\}$



+1 +1 +1 +1 +1

-1 0 +1 2 3



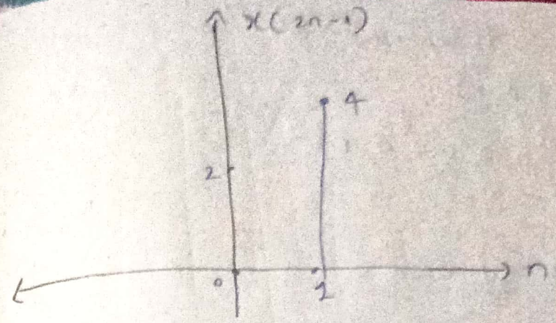
X 0 X 1 X

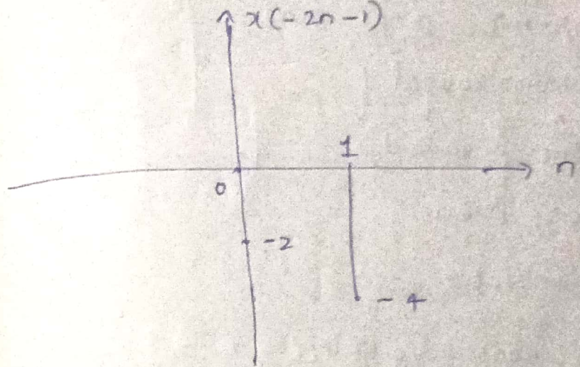
-1 -0 ~~-1~~ 1 2 3

{-1 0 1 2 3}

1 2 3 4 5

↑

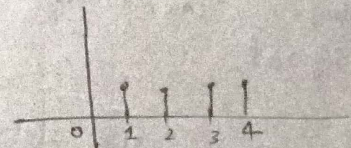




find i) $3n/2$ ii) $2n/3$

Discrete time signals

Graphical



Tabular

n	x(n)
0	1
1	1
2	1
3	1

Functional

$x(n)$ = 1, n = 0; 1, n = 1; 1, n = 2; 1, n = 3

Sequence

$x(n) = \{1, 1, 1, 1\}$

↑

(2-1) = 1 → It represents '0'

$\frac{n}{2} = \{1, 0, 1, 0, 1, 0, 1\}$

$\frac{n}{3} = \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1\}$

(3-1 = 2)

classification of systems

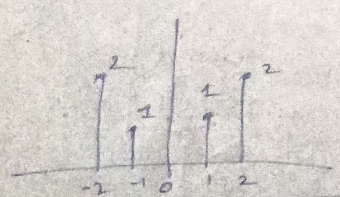
1) static & Dynamic $x(n) = y(n)$
2) causal & non-causal
3) Time & Time Invariant
4) Linear & Non-Linear.
5) stable & unstable
6) Invertible & non-Invertible.

classification of signals

1) deterministic (mathematical value) & non-deterministic (no mathematical value)
2) Even & odd signal.

even signal

$x(n) = x(-n)$



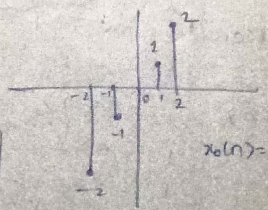
$x_e(n) = \frac{x(n)+x(-n)}{2}$ $\boxed{x_e(n)+x_o(n)}$

odd signal

$x(n) = -x(-n)$

(or)

$-x(n) = x(-n)$

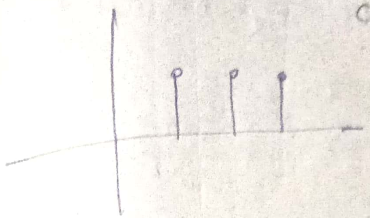


$x_o(n) = \frac{x(n)-x(-n)}{2}$

3) causal, Non-causal & Anti-causal.

$x(n) \neq 0, n > 0$ $x(n) = 0$ $n < 0$

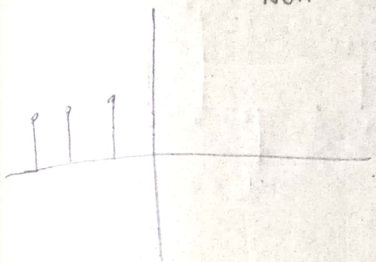
Causal



Non-causal

$x(n) \neq 0$ $n > 0$

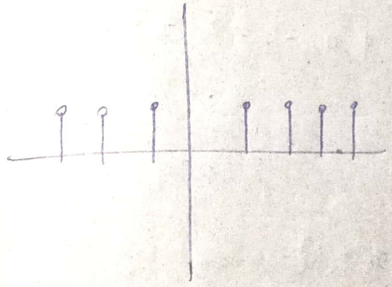
$x(n) \neq 0$ $n < 0$



Anti-causal.

$x(n) \neq 0$ $n > 0$

$x(n) \neq 0$ $n < 0$



4) periodic & non-periodic.

The same signal should be repeated for a fixed fundamental time period.

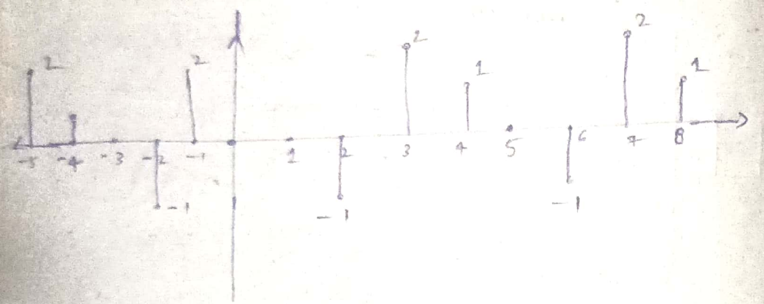
→ It is from $-\infty$ to ∞.

F.T.P :- N_0

$x(n) = x(n + N_0)$

Non-periodic :

The signal should not be repeated for a same fixed time period.



Fundamental time period

$\{0, -1, +2, 1\}$

$N_0 = N_2 - N_1 + 1$

$2 - 1 + 1$

2

$\therefore 3 - 0 + 1 \qquad (N_2 - N_1 + 1)$

4

$N_2 - N_1 + 4$

$-1 - (-4) + 4$

$\cancel{-1} + 4 + \cancel{1}$

4

$x(n) = x(n + N_0)$

$x(2) = x(2+4)$

$x(2) = x(6)$ ($\because$ from Graph)

$-1 = -1$ ✓

It satisfies.

Formula

$\omega_0 = \frac{2\pi}{T_0}$

$\omega_0 = \frac{2\pi}{N_0}$

$N_0 = \frac{2\pi}{\omega_0}$

→ If we are having composite signals,

1) Determine fundamental time period of individual signals.

$N_1 \; N_2 \; N_3 \; \ldots$

2) Ratio of first signal to each & every signal fundamental time period.

$\frac{N_1}{N_2}, \frac{N_1}{N_3}, \frac{N_1}{N_4}, \ldots$

ratio - rational - periodic.

Irrational - non-periodic.

3) Resultant F.T.P = LCM (N_1, N_2, N_3).

Energy and power signal

Energy :- $\lim_{n \to \infty} \sum_{n=-\infty}^{\infty} |x(n)|^2$

Infinite case

finite case

$\sum_{n=-\infty}^{\infty} |x(n)|^2$

power:-

$$T_2 - T_1 + 1$$
$$N - (-N) + 1$$
$$2N + 1$$

$$\left.\begin{array}{l} \frac{1}{N_0}\sum_{n=0}^{N_0-1} |x(n)|^2 \\ \\ \frac{1}{2N+1}\sum_{n=-N}^{+N} |x(n)|^2 \end{array}\right\} \text{finite.}$$

Infinite

$$\underset{n\to\infty}{Lt} \frac{1}{N_0}\sum_{n=0}^{N_0-1} |x(n)|^2$$

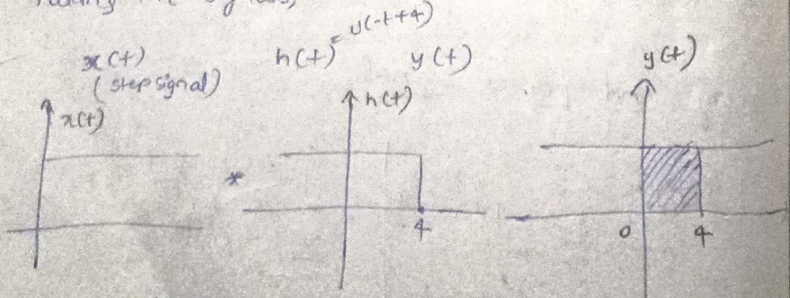
$$\underset{n\to\infty}{Lt} \frac{1}{2N+1}\sum_{n=-N}^{+N} |x(n)|^2$$

Convolution :-

overlapping of two signals is convolution.
whenever, we are having two signals, (overlap)
If we want to produce third signals, It is by overlapping.
→ It is represented by * (convolution)

continuous convolution $\int_{\infty}^{\infty} x(t) * h(-t+4) = y(t)$

Taking the signals,



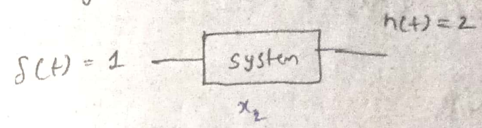
Here, $x(t) * h(-t+4) = y(t)$

~~Discrete convolution~~

Linear Time Invariant system will gives the same response.

LTI.

If we give impulse signal to system.



The above systems are Linear Time Invariant systems, which will help us to convolution.

formula for convolution

$$y(t) = \int_{-\infty}^{\infty} x(t) * h(-t+4)$$

$$y(t) = \int_{-\infty}^{\infty} x(-t+4) * h(t)$$

first step:

1) Actual Representation 'τ'

$$y(\tau) = \int_{-\infty}^{\infty} x(\tau) * h(-\tau+t)\, d\tau$$

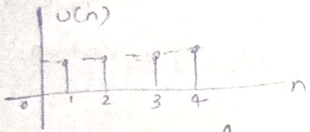
Discrete convolution

$x(n)$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k) * h(-k+n)}$$

Special signals of Discrete signals

1) unit step signal.



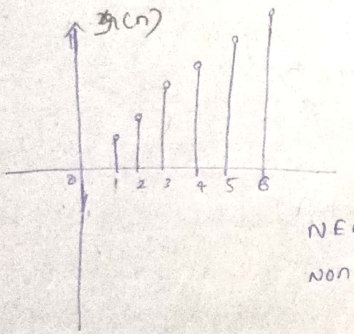
$u(n) = 1 \quad n > 1$

$u(n) = 0 \quad n < 0$

NENO

Non-periodic

2) unit ramp signal



$r(n) = n \quad n > 0$

$r(n) = 0 \quad n < 0$

NENO

Non-periodic.

3) unit Impulse signal

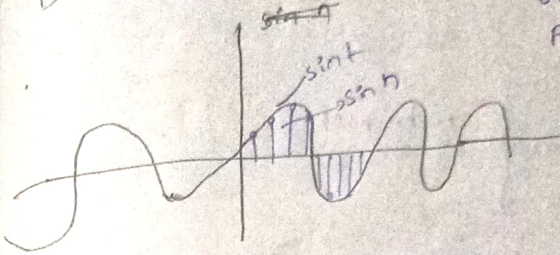
$\delta(n) = 1 \quad n = 0$

$\delta(n) = 0 \quad n \neq 0$

4) sine signal

odd signal

periodic signal



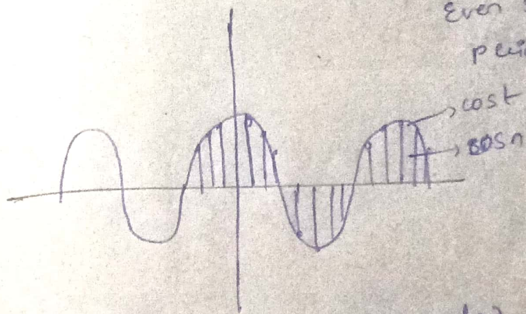
Inside → sin n (discrete)

outside → sin t (continuous)

5) cos signal

Even signal.

periodic signal.



Inside → cos n (discrete)

outside → cos t (continuous)

1/11/2023

→ Transformers
(person name)
i) fourier series
ii) fourier transform
iii) laplace transform
iv) z-transform

} techniques.

→ converting one domain to another domain.

time domain → frequency domain.

using Amplitude is converting from one domain to another domain.

$\omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{\omega_0}$

→ $\boxed{x(t) \rightarrow \text{we are writing it in the form of frequency. Time domain} \rightarrow \text{frequency domain}}$

why Time domain → frequency domain

In both the cases, we are giving as voltage.

fourier series — These are applicable for periodic signals.

trigonometric
Exponential } → periodic.

→ eg: class, voice has frequency

→ we are defining/determing the voltage using frequency.

Trigonometric form

$x(t) = A \sin \omega_0 t$

$x(t) = A \cos \omega_0 t$

Exponential form.

$x(t) = A e^{j\omega_0 t}$

$e^{j\theta} = \cos\theta + i\sin\theta$

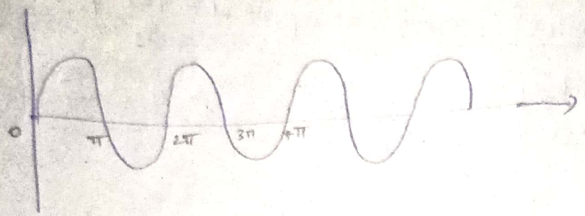
→ Representing in sin & cos will become trigonometric fourier series.

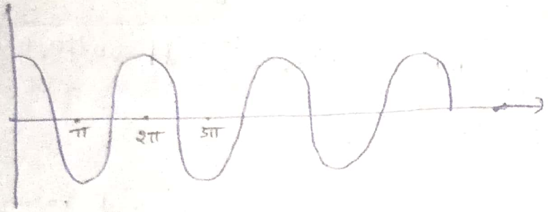
$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \text{———}$
$+ \text{——} a_n \cos n\omega_0 t \rightarrow ①$

$x(t) = b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \text{---} b_n \sin n\omega_0 t \rightarrow ②$

Adding ① & ②

$x(t) = a_0 \cos 0\omega_0 t + a_1 \cos 1\omega_0 t + a_2 \cos 2\omega_0 t + \text{--}$
$+ a_n \cos n\omega_0 t + b_0 \sin 0\omega_0 t + b_1 \sin 1\omega_0 t + \text{--}$
$+ \text{--} b_n \sin n\omega_0 t.$





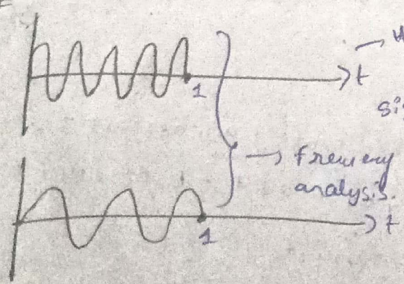
$$x(t) = a_0 + a_1 \cos\omega_0 t + \dots + a_n \cos n\omega_0 t + \dots$$
$$b_1 \sin\omega_0 t + \dots + b_n \sin n\omega_0 t$$

Trigonometric fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

a_0, b_n, a_n → are called fourier coefficients/ Dc values.

Eg:



→ High frequency signal is reached fastly

→ frequency analysis.

Low frequency.

→ Easy analysis in our signal will give Amplitude.

$$\int_0^{T_0} x(t)\,dt = \int_0^{T_0} a_0\,dt + \int_0^{T_0} \sum_{n=1}^{\infty} a_n \cos n\omega_0 t\,dt + \int_0^{T_0} \sum_{n=1}^{\infty} b_n \sin n\omega_0 t\,dt$$

$$\int_0^{T_0} x(t)\,dt = a_0 T_0 + \sum_{n=1}^{\infty} a_n \cancel{\frac{\sin n\omega_0 t}{n\omega_0}}\Bigg|_0^{T_0} - b_n \sum_{n=1}^{\infty} \cancel{\frac{\cos n\omega_0 t}{n\omega_0}}\Bigg|_0^{T_0}$$

$$\left[\frac{\sin n\omega_0 T_0}{n\omega_0} - \frac{\sin n 0 \omega_0}{n\omega_0}\right]$$
$$\frac{\sin 2\pi n}{n\omega_0} - 0$$
$$0 - 0 = 0$$

$$\int_0^{T_0} x(t)\,dt = a_0 T_0$$

Dc value become,

$$\boxed{a_0 = \frac{1}{T_0}\int_0^{T_0} x(t).dt}$$

$$\left(\frac{\cos n\omega_0 T_0}{n\omega_0} - \frac{\cos n 0 \omega_0}{n\omega_0}\right)$$
$$\left(\frac{1}{n\omega_0} - \frac{1}{n\omega_0}\right] = 0$$

$\omega_0 = 2\pi f_0$

$\omega_0 = \frac{2\pi}{T_0}$

$\omega_0 T_0 = 2\pi$

$$\boxed{\int_0^{T_0} \cos n\omega_0 t\,dt = 0 \qquad \int_0^{T_0} \sin n\omega_0 t\,dt = 0}$$

To find coefficients: → cosine

$$\int_0^{T_0} x(t)\cos n\omega_0 t\,dt = \int_0^{T_0} a_0 \cancel{\cos n\omega_0 t}\,dt + \int_0^{T_0} a_1 \cos\omega_0 t\cdot \cos n\omega_0 t + \cdots$$

$$+ \int_0^{T_0} a_n \cos^2 n\omega_0 t\,dt +$$

$$+ \int_0^{T_0} b_1 \cancel{\sin \omega_0 t}\cdot\cos n\omega_0 t\,dt + \cdots \int_0^{T_0} b_n \sin n\omega_0 t\cdot\cos n\omega_0 t\,dt.$$

$$\int_0^{T_0} x(t)\cdot\cos n\omega_0 t\,dt = \int_0^{T_0} a_n \cos^2 n\omega_0 t\,dt$$

$$= \int_0^{T_0} a_n\cdot\frac{1+\cos 2n\omega_0 t}{2}\,dt$$

$$\int_0^{T_0} x(t)\cdot\cos n\omega_0 t\,dt = \int_0^{T_0}\frac{a_n}{2}\,dt + \int_0^{T_0}\cancel{\frac{\cos 2n\omega_0 t}{2}}$$

$$= \frac{a_n}{2}\cdot T_0$$

$$\boxed{a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot\cos n\omega_0 t\,dt}$$

→ sine

$$\int_0^{T_0} x(t)\sin n\omega_0 t\,dt = \int_0^{T_0} a_0 \sin n\omega_0 t\,dt + \int_0^{T_0} a_1\cos\omega_0 t\cdot\sin n\omega_0 t + \cdots$$

$$+ \int_0^{T_0} a_n \cos n\omega_0 t\,\sin n\omega_0 t\cdot dt$$

$$+ \int_0^{T_0} b_1 \sin\omega_0 t + \sin n\omega_0 t\,dt + \cdots + \int_0^{T_0} b_n \sin^2 n\omega_0 t\cdot dt$$

$$\int_0^{T_0} x(t)\sin n\omega_0 t\,dt = \int_0^{T_0} b_n \sin^2 n\omega_0 t\,dt$$

$$= \int_0^{T_0} b_n\,\frac{1-\cos 2n\omega_0 t}{2}\,dt$$

$$\boxed{b_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cdot\sin n\omega_0 t\,dt}$$

$$\int_0^{T_0} x(t)\sin n\omega_0 t\,dt = \int_0^{T_0}\frac{a_n}{2}\cdot dt + \int_0^{T_0}\cancel{\frac{\cos 2n\omega_0 t}{2}}$$

$$\int_0^{T_0} x(t)\sin n\omega_0 t\,dt = \frac{a_n}{2}\cdot T_0$$

Exponential fourier series

$$x(t) = C_0 e^{j0\omega_0 t} + C_1 e^{j1\omega_0 t} + \cdots$$
$$\cdots + C_n e^{jn\omega_0 t} + C_{-1} e^{-j\omega_0 t} + C_{-2} e^{-j2\omega_0 t} + \cdots$$
$$+ C_{-n} e^{-jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}}$$

$$\boxed{C_n = \frac{1}{T_0}\int_0^{T_0} x(t) \cdot e^{-jn\omega_0 t}\, dt}$$

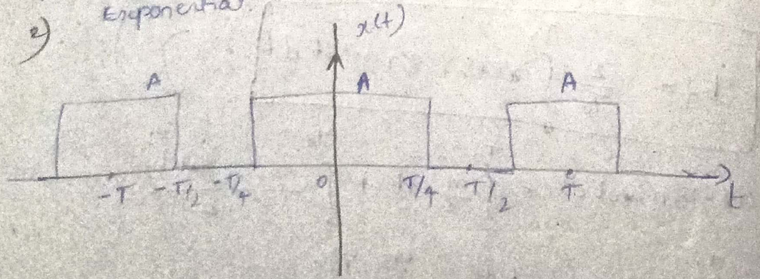
⇒ C_n is the cofficient of exponential fourier series.

Analysis

frequency → { Amplitude, Time

Fourier series

1) Trigonometric
2) Exponential



$x(t) = A$

$-T/4 < t < T/4$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$a_0 = \frac{1}{T}\int_{-T/4}^{T/4} A \cdot dt$$

$$a_0 = \frac{A}{T}\left[T/4 + T/4\right]$$

$$a_0 = \frac{A}{T}\left[\frac{2T}{4}\right]$$

$$\boxed{a_0 = \frac{A}{2}}$$

$$a_n = \frac{2}{T}\int_{-T/4}^{T/4} A \cdot \cos n\omega_0 t\, dt$$

$$a_n = \frac{2A}{T}\left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-T/4}^{T/4}$$

$$a_n = \frac{2A}{n\omega_0 T}\left[\sin \omega_0 (T/4) - \sin n\omega_0 (-T/4)\right]$$

$$a_n = \frac{A}{n\pi}\left[\sin n\pi/4 + \sin n\pi/2\right]$$

$$a_n = \frac{2A}{n\pi}\left[\sin \frac{n\pi}{2}\right]$$

$$a_1 = \frac{2A}{\pi}\left[\sin \frac{\pi}{2}\right]$$

$$\boxed{a_1 = \frac{2A}{2\pi}} \quad \left(\because \sin \frac{\pi}{2} = 1\right)$$

$\boxed{a_2 = 0}$

$\boxed{a_3 = \frac{-2A}{3\pi}}$

$\boxed{a_4 = 0}$

$a_5 = \frac{A}{5\pi}\left[\sin\frac{5\pi}{2}\right]$

$\boxed{a_5 = \frac{2A}{5\pi}}$

$$b_n = \frac{2}{T}\int_{-T/4}^{T/4} A.\sin n\omega_0 t \, .dt$$

$$b_n = \frac{-2A}{T n\omega_0}\left[\cos n\omega_0 t\right]_{-T/4}^{T/4}$$

$\boxed{b_n = 0}$

$\boxed{b_1 = 0}$ $\boxed{b_2 = 0}$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n.\cos n\omega_0 t + \cancel{\sum_{n=1}^{\infty} b_n.\sin n\omega_0 t}^{\,0}$$

$b_n = 0$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n.\cos n\omega_0 t$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\cos\omega_0 t - \frac{2A}{3\pi}\cos 3\omega_0 t + \frac{2A}{5\pi}\cos 5\omega_0 t + \cdots$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\left[\cos\omega_0 t - \frac{\cos 3\omega_0 t}{3} + \frac{\cos 5\omega_0 t}{5} + \cdots\right]$$

$x(t)$ is a Trigonometric fourier series.

Exponential fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$C_n = \frac{1}{T}\int_{-T/4}^{T/4} A.e^{-jn\omega_0 t} dt$$

$$C_n = \frac{A}{T}\left[\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\right]_{-T/4}^{T/4}$$

$$C_n = \frac{-A}{Tjn\omega_0}\left[e^{-jn\omega_0 T/4} - e^{-jn\omega_0(-T/4)}\right]$$

$$C_n = \frac{-A}{2\pi nj}\left[e^{-jn\pi/2} - e^{jn\pi/2}\right]$$

$$C_n = \frac{A}{n\pi}\left[\frac{e^{+jn\pi/2} - e^{-jn\pi/2}}{2j}\right]$$

$$C_n = \frac{A}{n\pi}\left[\sin n\frac{\pi}{2}\right]$$

$\boxed{\sin x = \frac{e^{ix} - e^{-ix}}{2i}}$

$\boxed{C_1 = \frac{A}{\pi}}$ $(\because \sin\frac{\pi}{2} = 1)$

$\boxed{C_{-1} = \frac{A}{\pi}}$

$C_2 = \frac{A}{2\pi}\left[\sin\frac{\cancel{2}\pi}{\cancel{2}}\right]$

$\boxed{C_2 = 0}$ $\boxed{C_{-2} = 0}$

$$C_3 = \frac{A}{3\pi}\left[\sin\frac{3\pi}{2}\right]$$

$$\boxed{C_3 = \frac{-A}{3\pi}} \qquad \boxed{C_{-3} = \frac{-A}{3\pi}}$$

$$C_0 = \lim_{n\to 0} \frac{2A}{n\pi} \cdot \sin n\frac{\pi}{2}$$

$$\boxed{\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

$$C_0 = \lim_{n\to 0} \frac{A}{2} \cdot \frac{2}{n\pi} \sin\frac{n\pi}{2}$$

$$C_0 = \frac{A}{2} \lim_{n\to 0} \frac{\sin\frac{n\pi}{2}}{n\pi/2} \to 1$$

$$\boxed{C_0 = \frac{A}{2}}$$

$$x(t) = \cdots + \frac{A}{5\pi} e^{-j5\omega_0 t} - \frac{A}{3\pi} e^{-j3\omega_0 t} + \frac{A}{\pi} e^{-j\omega_0 t} + \frac{A}{2} + \frac{A}{\pi} \cdot e^{j\omega_0 t} + \frac{A}{3\pi} e^{j3\omega_0 t}$$

$x(t)$ is Exponential fourier series.

$$x(t) = \frac{A}{2} + \frac{A}{\pi}\left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] - \frac{A}{3\pi}\left[e^{j3\omega_0 t} + e^{-j3\omega_0 t}\right] + \frac{A}{5\pi}\left[e^{+j5\omega_0 t} + e^{-j5\omega_0 t}\right] + \cdots$$

for above expression we are multiplying & dividing with 2

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] - \frac{2A}{3\pi}\left[\frac{e^{j3\omega_0 t} + e^{-j3\omega_0 t}}{2}\right] + \frac{2A}{5\pi}\left[\frac{e^{j5\omega_0 t} + e^{-j5\omega_0 t}}{2}\right] + \cdots$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}[\cos\omega_0 t] - \frac{2A}{3\pi}[\cos 3\omega_0 t] + \frac{2A}{5A}[\cos 5\omega_0 t] + \cdots$$

$$x(t) = \frac{A}{2} + \frac{2A}{\pi}\left[\cos\omega_0 t - \frac{\cos 3\omega_0 t}{3} + \frac{\cos 5\omega_0 t}{5} + \cdots\right]$$

It means,

Exponential & Trigonometric fourier series are same.

<u>Relation b/w the Fourier (Trigonomic & Exponential) series</u>

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n\left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right] + b_n\left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2}\right]\right]$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} \left[e^{jn\omega_0 t}\left[\frac{a_n}{2} + \frac{b_n}{2j}\right] + e^{-jn\omega_0 t}\left[\frac{a_n}{2} - \frac{b_n}{2j}\right]\right] \qquad \frac{1}{j} = -j$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[e^{jn\omega_0 t}\left[\frac{a_n}{2} - \frac{jb_n}{2}\right] + e^{-jn\omega_0 t}\left(\frac{a_n}{2} + \frac{jb_n}{2}\right)\right]$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\underbrace{\left[\frac{a_n}{2} - \frac{jb_n}{2}\right]}_{c_n} e^{jn\omega_0 t} + \sum_{n=1}^{\infty}\underbrace{\left[\frac{a_n}{2} + \frac{jb_n}{2}\right]}_{c_{-n}} e^{-jn\omega_0 t}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty} c_n e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} c_n e^{jn\omega_0 t}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e}$$

Now, the Relation becomes,

$$c_n = \frac{a_n}{2} - \frac{jb_n}{2} \longrightarrow ①$$

$$c_{-n} = \frac{a_n}{2} + \frac{jb_n}{2} \longrightarrow ②$$

Adding ① + ②, we get

Relation c_n, b_n, a_n, a_0

$$c_n + c_{-n} = \frac{2a_n}{2} = a_n$$

$$\boxed{c_n + c_{-n} = a_n}$$

By subtracting ① − ②, we get

$$c_n - c_{-n} = -jb_n$$

$$j(c_n - c_{-n}) = b_n$$

$$\text{Real } c_n = \frac{a_n}{2}$$

$$a_n = 2\,\text{real}\,[c_n]$$

$$b_n = -2\,\text{img}\,[c_n]$$

$$\left(\because \text{img}\,[c_n] = \frac{-b_n}{2}\right.$$

$$\left. b_n = -2\,\text{img}\,[c_n]\right)$$

$$c_n = \cos\theta + j\sin\theta$$

$$\boxed{\begin{aligned}\sin\theta &= b_n \\ \cos\theta &= a_n\end{aligned}}$$

real $c_n = c_n$

img $c_n = j\sin\theta$.

Fourier series properties

→ Linearity

$$x_1(t) \xrightarrow{F.S} F_{x_1}(t)$$

$$x_2(t) \xrightarrow{F.S} F_{x_2}(t)$$

$$\left.\begin{aligned} x_1(t) + x_2(t) &= F_{x_1}(t) + F_{x_2}(t) \\ \alpha x_1(t) + \beta x_2(t) &= \alpha F_{x_1}(t) + \beta F_{x_2}(t)\end{aligned}\right\} \text{LOH \& LOA}$$

→ If the signal holds LOA, LOH then the signal is Linearity.

$$\boxed{\begin{aligned} c_n &= \frac{1}{T}\int_0^T x_1(t)\cdot e^{-jn\omega_0 t}\,dt \\ c_n &= \frac{1}{T}\int_0^T x_2(t)\cdot e^{-jn\omega_0 t}\,dt\end{aligned}}$$

$$c_n = \frac{1}{T}\int_0^T x(t)\cdot e^{-jn\omega_0 t}\,dt$$

$$c_n = \frac{1}{T}\int_0^T [\alpha x_1(t) + \beta x_2(t)]\, e^{-jn\omega_0 t}$$

$$C_n = \alpha\left[\frac{1}{T}\int_0^T x_1(t).e^{-jn\omega_0 t}\,dt\right] + \beta\left[\frac{1}{T}\int_0^T x_2(t).e^{-jn\omega_0 t}.dt\right]$$

$$\boxed{C_n = \alpha F x_1(t) + \beta F x_2(t)}$$

→ Linearity will holds the property of fourier series.
i.e, fourier series holds Linearity.

Time Scaling

$x(t) \longrightarrow C_n$

$x(at) \longrightarrow C_n^*$

$\tau = at \qquad d\tau = a\,dt$

$$C_n = \frac{1}{T}\int_0^{T_0} x(t).e^{-jn\omega_0 t}\,dt$$

$$C_n^* = \frac{1}{\frac{T_0}{a}}\int_0^{T_0} x(at).e^{-jn\frac{2\pi}{\frac{T_0}{a}}.\frac{\tau}{a}}\,dt$$

$\omega_0 = \frac{2\pi}{T}$

$\omega_0' = \frac{2\pi}{\frac{T_0}{a}}$

$t=0 \quad L.T \Rightarrow \tau = 0$

$t = T_0 \quad U.P \Rightarrow \tau = a.\frac{T_0}{a}$

$\boxed{\tau = T_0}$

$$C_n^* = \frac{a}{T_0}\int_0^{T_0} x(\tau)e^{-jn\frac{2\pi}{T_0}.\cancel{a}.\frac{\tau}{a}}\,dt$$

$$C_n^* = \frac{a}{T_0}\int_0^{T_0} x(\tau)\,e^{-jn\omega_0\tau}\,a.dt$$

$$\boxed{C_n^* = \frac{1}{T_0}\int_0^{T_0} x(\tau).e^{-jn\omega_0\tau}\,d\tau}$$

Time Shifting

$x(t) \text{ --- } C_n$

$x(t \pm a) \text{ --- } C_n^*$

$d\tau = dt$

(i) $\tau = t + a$

L.T $t=0 \quad \tau = a$

U.P $t = T_0 \quad \tau = T_0 + a$

$T_0 + \cancel{a} - \cancel{a}$

T_0

$$C_n = \frac{1}{T}\int_0^{T_0} x(t)\,e^{-jn\omega_0 t}$$

$$C_n^* = \frac{1}{T}\int_0^{T_0} x(t+a)\,e^{-jn\omega_0(t+a)}\,dt$$

$$C_n^* = \frac{1}{T}\int_0^{T_0} x(\tau)\,e^{-jn\omega_0 t}\,e^{-jn\omega_0 a}\,d\tau$$

$$\boxed{C_n = \frac{1}{T}\int_0^{T_0} x(\tau)\,e^{-jn\omega_0 t}\,d\tau \text{ is a coefficient}}$$

$$C_n^* = C_n.e^{-jn\omega_0 a}$$

ii) $\tau = t - a$

L.T $t=0 \quad \tau = -a$

U.P $t = T_0 \quad \tau = T_0 - a$

$T_0 \cancel{-a} \cancel{+a}$

T_0

$$C_n = \frac{1}{T}\int_0^{T_0} x(t)\,e^{-jn\omega_0 t}$$

$$C_n^* = \frac{1}{T}\int_0^{T_0} x(t-a)\,e^{-jn\omega_0(t-a)}\,dt$$

$$C_n^* = \frac{1}{T}\int_0^{T_0} x(\tau).e^{-jn\omega_0 t}\,e^{+jn\omega_0 a}\,d\tau$$

$$\boxed{C_n^* = C_n.e^{jn\omega_0 a}}$$

Imp (Time domain to fourier series power)

→ **parseval's power theorem** :-

$$x(t) \text{ — } p_{avg} = \frac{1}{T}\int_0^T |x(t)|^2\, dt$$

$x(t) = a - ib$ $\qquad |x(t)| = \sqrt{a^2+b^2}$

$x^*(t) = a + ib$ $\qquad |x(t)|^2 = a^2+b^2$

$x(t).x^*(t) = a^2 + b^2$ $\qquad |x(t)|^2 = x(t).x^*(t)$

$$P_{avg} = \frac{1}{T}\int_0^{T_0} x(t) \,.\, x^*(t) \,.\, dt$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n . e^{jn\omega_0 t}$$

$$x^*(t) = \sum_{n=-\infty}^{\infty} c_n^* \, e^{-jn\omega_0 t}$$

$$P_{avg} = \frac{1}{T}\int_0^T x(t) \sum_{n=-\infty}^{\infty} c_n^* \, e^{-jn\omega_0 t}\, dt$$

$$\left(\because c_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt\right)$$

$$P_{avg} = \sum_{n=-\infty}^{\infty} c_n^* c_n$$

$$\boxed{P_{avg} = \sum_{n=-\infty}^{\infty} |c_n|^2}$$

Convolution in Time :-

$x_1(t)$

$x_2(t)$

$$x_1(t) * x_2(t) = \int_0^{T_0} x_1(\tau) \cdot x_2(t-\tau) \cdot d\tau$$

$$c_n = \frac{1}{T_0}\int_0^{T_0} x(t) \,.\, e^{-jn\omega_0 t}\, dt$$

$$c_n^* = \frac{1}{T_0}\int_0^{T_0}\left[\int_0^{T_0} x(\tau).x(t-\tau)\, d\tau\right] e^{-jn\omega_0 t}\, dt.$$

$$c_n^* = \frac{1}{T_0}\int_0^{T_0}\left[\int_0^{T_0} x(\tau) \cdot \frac{e^{-jn\omega_0\tau}}{e^{-jn\omega_0\tau}} \cdot x(t-\tau)\, d\tau\right] e^{-jn\omega_0 t}\, dt$$

$$\boxed{c_n = \frac{1}{T_0}\int_0^{T_0} x(\tau) . e^{-jn\omega_0\tau}\, d\tau}$$

$$c_n^* = \int_0^{T_0} c_n \cdot x(t-\tau) \cdot \frac{e^{-jn\omega_0 t}}{e^{-jn\omega_0\tau}}\, dt$$

$$c_n^* = T_0 \cdot \frac{1}{T_0}\int_0^{T_0} c_n \cdot x(t-\tau)\, e^{-jn\omega_0(t-\tau)} \,.\, dt$$

$$\left[\frac{e^{-jn\omega_0 t}}{e^{-jn\omega_0\tau}} = e^{-jn\omega_0(t-\tau)}\right]$$

$$\left(\because \frac{a^m}{a^n} = a^{m-n}\right)$$

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$$C_n^{+} = T_0 \cdot \frac{1}{T_0} \int_0^{T_0} x(t-\tau) \cdot e^{-jn\omega_0 (t-\tau)} \, dt \cdot C_{1n}$$

$$C_n^{+} = T_0 * C_{2n} \cdot C_{1n}$$

$$\boxed{C_n^{+} = C_{1n} \cdot C_{2n} \cdot T_0}$$

Differentiation in Time :-

when we do differentiation in time domain, fourier series property.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$$

$$\frac{d}{dt} x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \cdot (jn\omega_0)$$

$$\frac{d^2}{dt^2} x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \, (jn\omega_0)(jn\omega_0)$$

k^{th} derivation becomes,

$$\frac{d^k}{dt^k} x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \, (jn\omega_0)^k$$

differentiation in time

Multiplication in Time :- (convolution of the signal)

Instead of n, we are using p & k.

$$x_1(t) \cdot x_2(t)$$

$$x_1(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{jk\omega_0 t}$$

$$x_2(t) = \sum_{p=-\infty}^{\infty} B_p \cdot e^{jp\omega_0 t}$$

$$x_1(t) \cdot x_2(t)$$

$$\sum_{k=-\infty}^{\infty} C_k \cdot e^{jk\omega_0 t} \cdot \sum_{p=-\infty}^{\infty} B_p \cdot e^{jp\omega_0 t}$$

$p + k = n$

$p = n - k$

$$\sum_{k=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} C_k \cdot B_p \cdot e^{j\omega_0 t (p+k)}$$

$$\sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k B_{n-k} e^{jn\omega_0 t}$$

$$\sum_{p=-\infty}^{\infty} C_n * B_n \, e^{jn\omega_0 t}$$

$$\sum_{n=-\infty}^{\infty} C_n * B_n \cdot e^{jn\omega_0 t}$$

Convolution
continuous
$\int_0^{T} x(\tau) \, x(t-\tau) \, d\tau$
Discrete
$\sum_{k=-\infty}^{\infty} C_k \cdot B_{n-k}$
$C_n * B_n$

Fourier Transform

$t \longrightarrow \omega_0$

Continuous Spectrum (signal)

$x(t) = x(\omega)$

$x(t) = x(j\omega)$

$x(t) = x(f)$

Discrete spectrum	
tri	ex
a_1	C_1
a_2	C_2
a_3	C_3