Data set

The following data set might be familiar with. We've used similar data set in our previous experiments but that one denotes golf playing decision based on some factors. In other words, golf playing decision was nominal target consisting of true or false values. Herein, the target column is number of golf players and it stores real numbers. We have counted the number of instances for each class when the target was nominal. I mean that we can create branches based on the number of instances for true decisions and false decisions. Here, we cannot count the target values because it is continuous. Instead of counting, we can handle regression problems by switching the metric to standard deviation.

Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
3	Overcast	Hot	High	Weak	46
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
7	Overcast	Cool	Normal	Strong	43
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38
10	Rain	Mild	Normal	Weak	46

11	Sunny	Mild	Normal	Strong	48
12	Overcast	Mild	High	Strong	52
13	Overcast	Hot	Normal	Weak	44
14	Rain	Mild	High	Strong	30

Standard deviation

Golf players = {25, 30, 46, 45, 52, 23, 43, 35, 38, 46, 48, 52, 44, 30}

Average of golf players = (25 + 30 + 46 + 45 + 52 + 23 + 43 + 35 + 38 + 46 + 48 + 52 + 44 + 30)/14 = 39.78

Standard deviation of golf players = $\sqrt{[((25 – 39.78)2 + (30 – 39.78)2 + (46 – 39.78)2 + … + (30 – 39.78)2)/14]}$ = 9.32

Outlook

Outlook can be sunny, overcast and rain. We need to calculate standard deviation of golf players for all of these outlook candidates.

Sunny outlook

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38
11	Sunny	Mild	Normal	Strong	48

Golf players for sunny outlook = {25, 30, 35, 38, 48}

Average of golf players for sunny outlook = (25+30+35+38+48)/5 = 35.2

Standard deviation of golf players for sunny outlook = $\sqrt{(((25 – 35.2)2 + (30 – 35.2)2 + …)/5)}$ = 7.78

Overcast outlook

Day	Outlook	Temp.	Humidity	Wind	Golf Players
3	Overcast	Hot	High	Weak	46
7	Overcast	Cool	Normal	Strong	43
12	Overcast	Mild	High	Strong	52
13	Overcast	Hot	Normal	Weak	44

Golf players for overcast outlook = {46, 43, 52, 44}

Average of golf players for overcast outlook = (46 + 43 + 52 + 44)/4 = 46.25

Standard deviation of golf players for overcast outlook = √(((46-46.25)2+(43-46.25)2+…)= 3.49

Rainy outlook

Day	Outlook	Temp.	Humidity	Wind	Golf Players
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
10	Rain	Mild	Normal	Weak	46
14	Rain	Mild	High	Strong	30

Golf players for overcast outlook = {45, 52, 23, 46, 30}

Average of golf players for overcast outlook = (45+52+23+46+30)/5 = 39.2

Standard deviation of golf players for rainy outlook = √(((45 – 39.2)2+(52 – 39.2)2+…)/5)=10.87

Summarizing standard deviations for the outlook feature

Outlook	Stdev of Golf Players	Instances
Overcast	3.49	4
Rain	10.87	5
Sunny	7.78	5

Weighted standard deviation for outlook = (4/14)x3.49 + (5/14)x10.87 + (5/14)x7.78 = 7.66

You might remember that we have calculated the global standard deviation of golf players 9.32 in previous steps. Standard deviation reduction is difference of the global standard deviation and standard deviation for current feature. In this way, maximized standard deviation reduction will be the decision node.

Standard deviation reduction for outlook = 9.32 – 7.66 = 1.66

Temperature

Temperature can be hot, cool or mild. We will calculate standard deviations for those candidates.

Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
3	Overcast	Hot	High	Weak	46
13	Overcast	Hot	Normal	Weak	44

Golf players for hot temperature = {25, 30, 46, 44}

Standard deviation of golf players for hot temperature = 8.95

Cool temperature

Day	Outlook	Temp.	Humidity	Wind	Golf Players
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
7	Overcast	Cool	Normal	Strong	43
9	Sunny	Cool	Normal	Weak	38

Golf players for cool temperature = {52, 23, 43, 38}

Standard deviation of golf players for cool temperature = 10.51

Mild temperature

Day	Outlook	Temp.	Humidity	Wind	Golf Players
4	Rain	Mild	High	Weak	45
8	Sunny	Mild	High	Weak	35
10	Rain	Mild	Normal	Weak	46
11	Sunny	Mild	Normal	Strong	48
12	Overcast	Mild	High	Strong	52
14	Rain	Mild	High	Strong	30

Golf players for mild temperature = {45, 35, 46, 48, 52, 30}

Standard deviation of golf players for mild temperature = 7.65

Summarizing standard deviations for temperature feature

Temperature	Stdev of Golf Players	Instances
Hot	8.95	4
Cool	10.51	4
Mild	7.65	6

Weighted standard deviation for temperature = (4/14) x8.95 + (4/14)x10.51 + (6/14)x7.65 = 8.84

Standard deviation reduction for temperature = 9.32 – 8.84 = 0.47

Humidity

Humidity is a binary class. It can either be normal or high.

High humidity

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
3	Overcast	Hot	High	Weak	46
4	Rain	Mild	High	Weak	45
8	Sunny	Mild	High	Weak	35
12	Overcast	Mild	High	Strong	52
14	Rain	Mild	High	Strong	30

Golf players for high humidity = {25, 30, 46, 45, 35, 52, 30}

Standard deviation for golf players for high humidity = 9.36

Normal humidity

Day	Outlook	Temp.	Humidity	Wind	Golf Players
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
7	Overcast	Cool	Normal	Strong	43
9	Sunny	Cool	Normal	Weak	38
10	Rain	Mild	Normal	Weak	46
11	Sunny	Mild	Normal	Strong	48
13	Overcast	Hot	Normal	Weak	44

Golf players for normal humidity = {52, 23, 43, 38, 46, 48, 44}

Standard deviation for golf players for normal humidity = 8.73

Summarizing standard deviations for humidity feature

Humidity	Stdev of Golf Player	Instances
High	9.36	7
Normal	8.73	7

Weighted standard deviation for humidity = (7/14)x9.36 + (7/14)x8.73 = 9.04

Standard deviation reduction for humidity = 9.32 – 9.04 = 0.27

Wind

Wind is a binary class, too. It can either be Strong or Weak.

Strong Wind

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
2	Sunny	Hot	High	Strong	30
6	Rain	Cool	Normal	Strong	23
7	Overcast	Cool	Normal	Strong	43
11	Sunny	Mild	Normal	Strong	48
12	Overcast	Mild	High	Strong	52
14	Rain	Mild	High	Strong	30

Golf players for strong wind= {30, 23, 43, 48, 52, 30}

Standard deviation for golf players for strong wind = 10.59

Weak Wind

Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
3	Overcast	Hot	High	Weak	46
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38

10	Rain	Mild	Normal	Weak	46
13	Overcast	Hot	Normal	Weak	44

Golf players for weakk wind= {25, 46, 45, 52, 35, 38, 46, 44}

Standard deviation for golf players for weak wind = 7.87

Summarizing standard deviations for wind feature

Wind	**Stdev of Golf Player**	**Instances**
Strong	10.59	6
Weak	7.87	8

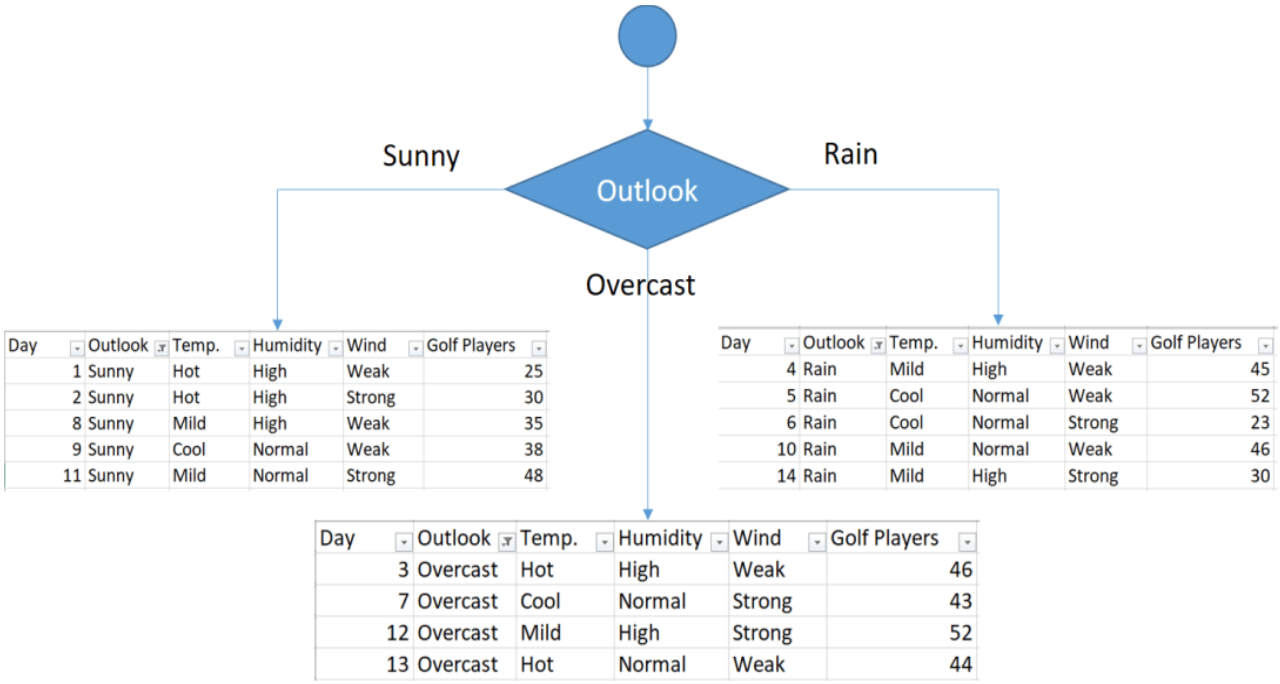
Weighted standard deviation for wind = (6/14)x10.59 + (8/14)x7.87 = 9.03

Standard deviation reduction for wind = 9.32 – 9.03 = 0.29

So, we've calculated standard deviation reduction values for all features. The winner is outlook because it has the highest score.

Feature	**Standard Deviation Reduction**
Outlook	1.66
Temperature	0.47
Humidity	0.27
Wind	0.29

We'll put outlook decision at the top of decision tree. Let's monitor the new sub data sets for the candidate branches of outlook feature.



Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38
11	Sunny	Mild	Normal	Strong	48

Day	Outlook	Temp.	Humidity	Wind	Golf Players
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
10	Rain	Mild	Normal	Weak	46
14	Rain	Mild	High	Strong	30

Day	Outlook	Temp.	Humidity	Wind	Golf Players
3	Overcast	Hot	High	Weak	46
7	Overcast	Cool	Normal	Strong	43
12	Overcast	Mild	High	Strong	52
13	Overcast	Hot	Normal	Weak	44

Fig. 1. Putting outlook at the top of the tree

Sunny Outlook

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38
11	Sunny	Mild	Normal	Strong	48

Golf players for sunny outlook = {25, 30, 35, 38, 48}

Standard deviation for sunny outlook = 7.78

Notice that we will use this standard deviation value as global standard deviation for this sub data set.

Sunny outlook and Hot Temperature

Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30

Standard deviation for sunny outlook and hot temperature = 2.5

Sunny outlook and Cool Temperature

Day	Outlook	Temp.	Humidity		Wind
9	Sunny	Cool	Normal		Weak

Standard deviation for sunny outlook and cool temperature = 0

Sunny outlook and Mild Temperature

Day	Outlook	Temp.	Humidity	Wind	Golf Players
8	Sunny	Mild	High	Weak	35
11	Sunny	Mild	Normal	Strong	48

Standard deviation for sunny outlook and mild temperature = 6.5

Summary of standard deviations for temperature feature when outlook is sunny

Temperature	Stdev for Golf Players	Instances
Hot	2.5	2
Cool	0	1

Mild	6.5	2

Weighted standard deviation for sunny outlook and temperature = (2/5)x2.5 + (1/5)x0 + (2/5)x6.5 = 3.6

Standard deviation reduction for sunny outlook and temperature = 7.78 – 3.6 = 4.18

Sunny outlook and high humidity

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
1	Sunny	Hot	High	Weak	25
2	Sunny	Hot	High	Strong	30
8	Sunny	Mild	High	Weak	35

Standard deviation for sunny outlook and high humidity = 4.08

Sunny outlook and normal humidity

Day	**Outlook**	**Temp.**	**Humidity**	**Wind**	**Golf Players**
9	Sunny	Cool	Normal	Weak	38
11	Sunny	Mild	Normal	Strong	48

Standard deviation for sunny outlook and normal humidity = 5

Summarizing standard deviations for humidity feature when outlook is sunny

Humidity	**Stdev for Golf Players**	**Instances**
High	4.08	3
Normal	5.00	2

Weighted standard deviations for sunny outlook and humidity = (3/5)x4.08 + (2/5)x5 = 4.45

Standard deviation reduction for sunny outlook and humidity = 7.78 – 4.45 = 3.33

Sunny outlook and Strong Wind

Day	Outlook	Temp.	Humidity	Wind	Golf Players
2	Sunny	Hot	High	Strong	30
11	Sunny	Mild	Normal	Strong	48

Standard deviation for sunny outlook and strong wind = 9

Sunny outlook and Weak Wind

Day	Outlook	Temp.	Humidity	Wind	Golf Players
1	Sunny	Hot	High	Weak	25
8	Sunny	Mild	High	Weak	35
9	Sunny	Cool	Normal	Weak	38

Standard deviation for sunny outlook and weak wind = 5.56

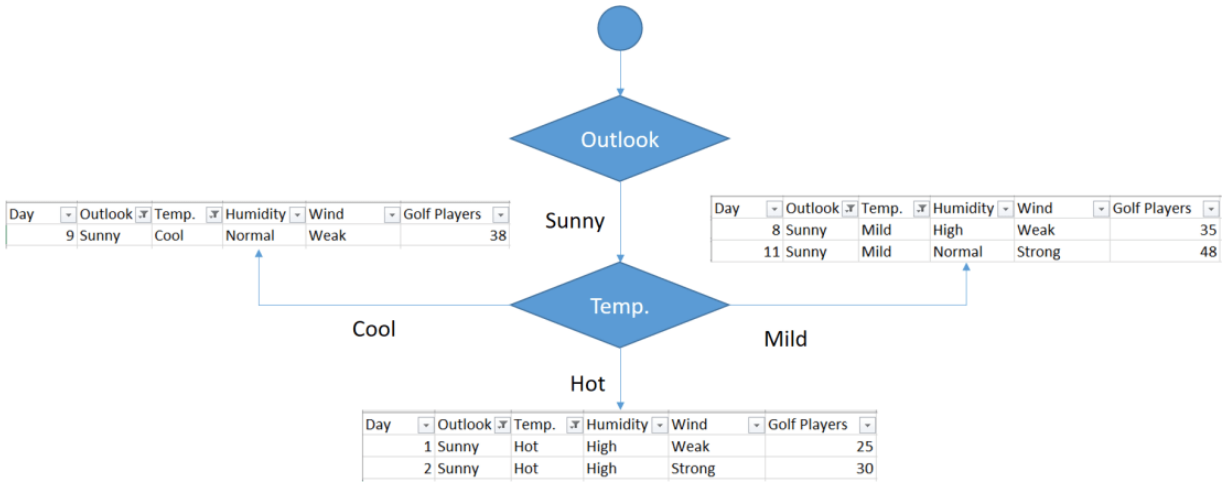
Wind	Stdev for Golf Players	Instances
Strong	9	2
Weak	5.56	3

Weighted standard deviations for sunny outlook and wind = (2/5)x9 + (3/5)x5.56 = 6.93

Standard deviation reduction for sunny outlook and wind = 7.78 – 6.93 = 0.85

We've calculated standard deviation reductions for sunny outlook. The winner is temperature.

Feature	Standard Deviation Reduction
Temperature	4.18
Humidity	3.33
Wind	0.85



Putting temperature decision at the bottom of sunny outlook
Pruning

Cool branch has one instance in its sub data set. We can say that if outlook is sunny and temperature is cool, then there would be 38 golf players. But what about hot branch? There are still 2 instances. Should we add another branch for weak wind and strong wind? No, we should not. Because this causes over-fitting. We should terminate building branches, for example if there are less than five instances in the sub data set. Or standard deviation of the sub data set can be less than 5% of the entire data set. I prefer to apply the first one. I will terminate the branch if there are less than 5 instances in the current sub data set. If this termination condition is satisfied, then I will calculate the average of the sub data set. This operation is called as pruning in decision tree trees.

Overcast outlook

Overcast outlook branch has already 4 instances in the sub data set. We can terminate building branches for this leaf. Final decision will be average of the following table for overcast outlook.

Day	Outlook	Temp.	Humidity	Wind	Golf Players
3	Overcast	Hot	High	Weak	46
7	Overcast	Cool	Normal	Strong	43
12	Overcast	Mild	High	Strong	52
13	Overcast	Hot	Normal	Weak	44

If outlook is overcast, then there would be (46+43+52+44)/4 = 46.25 golf players

Rainy Outlook

Day	Outlook	Temp.	Humidity	Wind	Golf Players
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
6	Rain	Cool	Normal	Strong	23
10	Rain	Mild	Normal	Weak	46
14	Rain	Mild	High	Strong	30

We need to find standard deviation reduction values for the rest of the features in same way for the sub data set above.

Standard deviation for rainy outlook = 10.87

Notice that we will use this value as global standard deviation for this branch in reduction step.

Rainy outlook and temperature

Temperature	Standard deviation for golf players	instances
Cool	14.50	2
Mild	7.32	3

Weighted standard deviation for rainy outlook and temperature = (2/5)x14.50 + (3/5)x7.32 = 10.19

Standard deviation reduction for rainy outlook and temperature = 10.87 – 10.19 = 0.67

Rainy outlook and humidity

Humidity	**Standard deviation for golf players**	**instances**
High	7.50	2
Normal	12.50	3

Weighted standard deviation for rainy outlook and humidity = (2/5)x7.50 + (3/5)x12.50 = 10.50

Standard deviation reduction for rainy outlook and humidity = 10.87 – 10.50 = 0.37

Rainy outlook and wind

Wind	**Standard deviation for golf players**	**instances**
Weak	3.09	3
Strong	3.5	2

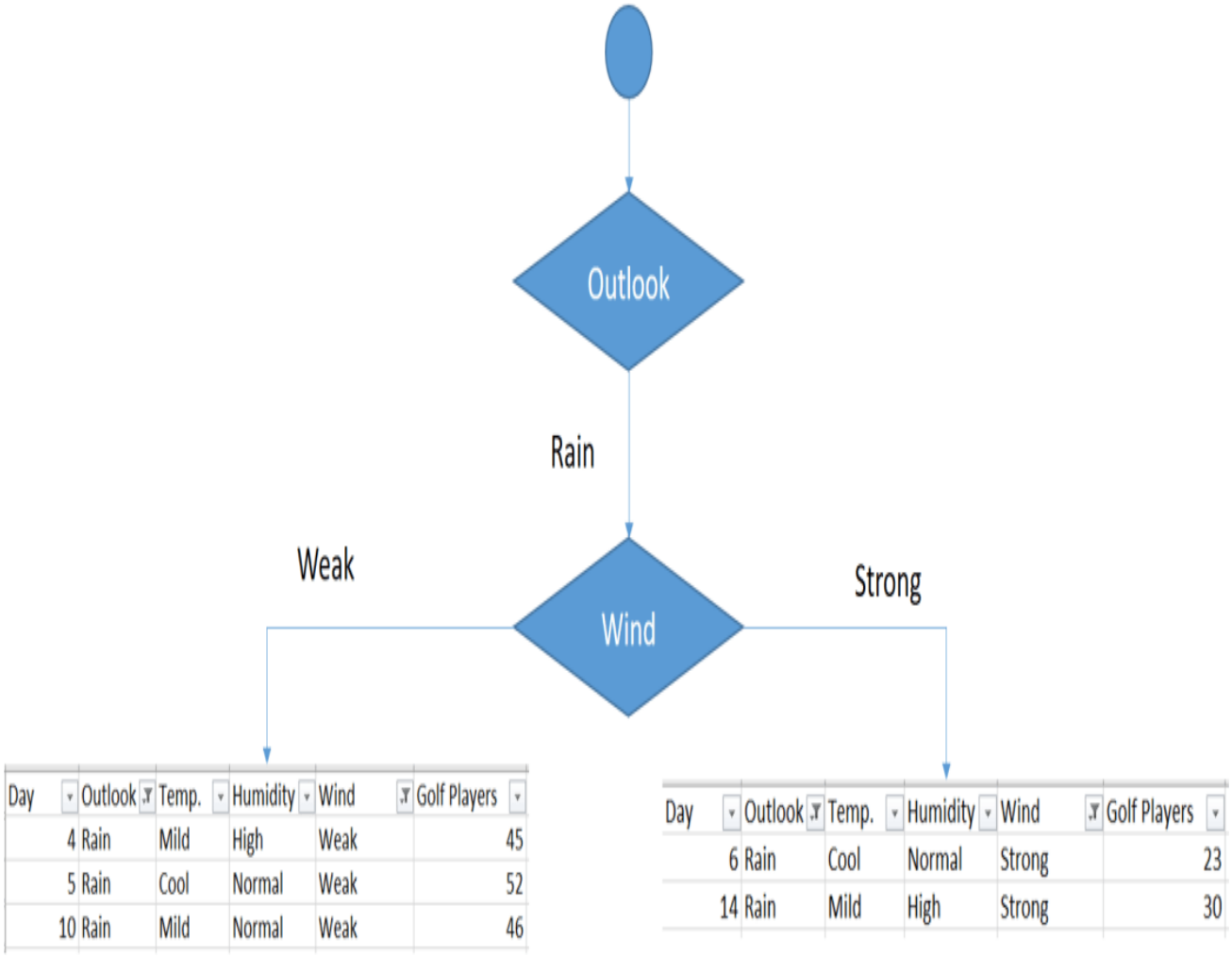
Weighted standard deviation for rainy outlook and wind = (3/5)x3.09 + (2/5)x3.5 = 3.25

Standard deviation reduction for rainy outlook and wind = 10.87 – 3.25 = 7.62

Summarizing rainy outlook

As illustrated below, the winner is wind feature.

Feature	**Standard deviation reduction**
Temperature	0.67
Humidity	0.37
Wind	7.62



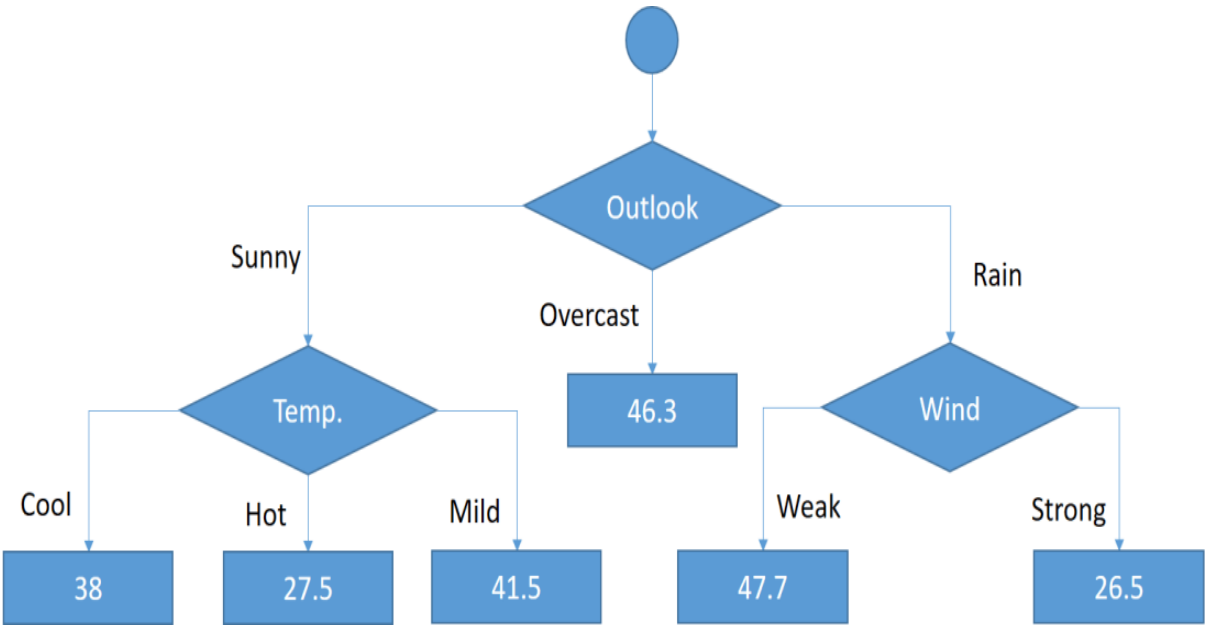
Day	Outlook	Temp.	Humidity	Wind	Golf Players
4	Rain	Mild	High	Weak	45
5	Rain	Cool	Normal	Weak	52
10	Rain	Mild	Normal	Weak	46

Day	Outlook	Temp.	Humidity	Wind	Golf Players
6	Rain	Cool	Normal	Strong	23
14	Rain	Mild	High	Strong	30

Sub data set for rainy outlook

As seen, both branches have items less than 5. Now, we can terminate these leafs based on the termination rule.

So, Final form of the decision tree is demonstrated below.


Outlook
Sunny
Overcast
Rain
Temp.
46.3
Wind
Cool
Hot
Mild
Weak
Strong
38
27.5
41.5
47.7
26.5