Confidence Intervals: Homework

NOTE: If you are using a student-t distribution for a homework problem below, you may assume that the underlying population is normally distributed. (In general, you must first prove that assumption, though.)

EXERCISE 1

Among various ethnic groups, the standard deviation of heights is known to be approximately 3 inches. We wish to construct a 95% confidence interval for the mean height of male Swedes. 48 male Swedes are surveyed. The sample mean is 71 inches. The sample standard deviation is 2.8 inches.

   a.

        i. $\bar{}=$_____

        ii. $\sigma=$_____

        iii. $S_x=$_____

        iv. $n=$_____

        v. $n-1=$_____

   b. Define the Random Variables X and $\bar{X}$ in words.

   c. Which distribution should you use for this problem? Explain your choice.

   d. Construct a 95% confidence interval for the population mean height of male Swedes.

        i. State the confidence interval.

        ii. Sketch the graph.

        iii. Calculate the margin of error.

   e. What will happen to the level of confidence obtained if 1000 male Swedes are surveyed instead of 48? Why?

EXERCISE 2

In six packages of "The Flintstones® Real Fruit Snacks" there were 5 Bam-Bam snack pieces. The total number of snack pieces in the six bags was 68. We wish to calculate a 96% confidence interval for the population proportion of Bam-Bam snack pieces.

   a. Define the Random Variables X and $\hat{p}$, in words.

   b. Which distribution should you use for this problem? Explain your choice

c. Calculate $\hat{p}$.

d. Construct a 96% confidence interval for the population proportion of Bam-Bam snack pieces per bag.

> **i.** State the confidence interval.
>
> **ii.** Sketch the graph.
>
> **iii.** Calculate the margin of error.

e. Do you think that six packages of fruit snacks yield enough data to give accurate results? Why or why not?

EXERCISE 3

A random survey of enrollment at 35 community colleges across the United States yielded the following figures (source: *Microsoft Bookshelf*): 6414; 1550; 2109; 9350; 21828; 4300; 5944; 5722; 2825; 2044; 5481; 5200; 5853; 2750; 10012; 6357; 27000; 9414; 7681; 3200; 17500; 9200; 7380; 18314; 6557; 13713; 17768; 7493; 2771; 2861; 1263; 7285; 28165; 5080; 11622. Assume the underlying population is normal.

> **a.**
>
> > **i.** $X=$_____
> >
> > **ii.** $S_x=$_____
> >
> > **iii.** $n=$_____
> >
> > **iv.** $n-1=$_____
>
> **b.** Define the Random Variables X and X, in words.
>
> **c.** Which distribution should you use for this problem? Explain your choice.
>
> **d.** Construct a 95% confidence interval for the population mean enrollment at community colleges in the United States.
>
> > **i.** State the confidence interval.
> >
> > **ii.** Sketch the graph.
> >
> > **iii.** Calculate the margin of error.
>
> **e.** What will happen to the margin of error and confidence interval if 500 community colleges were surveyed? Why?

EXERCISE 4

From a stack of *IEEE Spectrum* magazines, announcements for 84 upcoming engineering conferences were randomly picked. The mean length of the conferences was 3.94 days, with a standard deviation of 1.28 days. Assume the underlying population is normal.

   a. Define the Random Variables X and $\bar{X}$, in words.

   b. Which distribution should you use for this problem? Explain your choice.

   c. Construct a 95% confidence interval for the population mean length of engineering conferences.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

EXERCISE 5

Suppose that a committee is studying whether or not there is waste of time in our judicial system. It is interested in the mean amount of time individuals waste at the courthouse waiting to be called for service. The committee randomly surveyed 81 people. The sample mean was 8 hours with a sample standard deviation of 4 hours.

   a.

   i. $X =$_____

   ii. $S_x =$_____

   iii. $n =$_____

   iv. $n-1 =$_____

   b. Define the Random Variables X and $\bar{X}$, in words.

   c. Which distribution should you use for this problem? Explain your choice.

   d. Construct a 95% confidence interval for the population mean time wasted.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

   e. Explain in a complete sentence what the confidence interval means.

EXERCISE 6

Suppose that an accounting firm does a study to determine the time needed to complete one person's tax forms. It randomly surveys 100 people. The sample mean is 23.6 hours. There is a known standard deviation of 7.0 hours. The population distribution is assumed to be normal.

a.

    i. $X=$ _____

    ii. $\sigma=$ _____

    iii. $S_x=$ _____

    iv. $n=$ _____

    v. $n-1=$ _____

b. Define the Random Variables X and $\overline{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population mean time to complete the tax forms.

    i. State the confidence interval.

    ii. Sketch the graph.

    iii. Calculate the margin of error.

e. If the firm wished to increase its level of confidence and keep the margin of error the same by taking another survey, what changes should it make?

f. If the firm did another survey, kept the margin of error the same, and only surveyed 49 people, what would happen to the level of confidence? Why?

g. Suppose that the firm decided that it needed to be at least 96% confident of the population mean length of time to within 1 hour. How would the number of people the firm surveys change? Why?

EXERCISE 7

A sample of 16 small bags of the same brand of candies was selected. Assume that the population distribution of bag weights is normal. The weight of each bag was then recorded. The mean weight was 2 ounces with a standard deviation of 0.12 ounces. The population standard deviation is known to be 0.1 ounce.

a.

    i. $X=$ _____

    ii. $\sigma=$ _____

iii. $S_x=$_____

iv. $n=$_____

v. $n-1=$_____

b. Define the Random Variable X, in words.

c. Define the Random Variable X, in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 90% confidence interval for the population mean weight of the candies.

  i. State the confidence interval.

  ii. Sketch the graph.

  iii. Calculate the margin of error.

f. Construct a 98% confidence interval for the population mean weight of the candies.

  i. State the confidence interval.

  ii. Sketch the graph.

  iii. Calculate the margin of error.

g. In complete sentences, explain why the confidence interval in (f) is larger than the confidence interval in (e).

h. In complete sentences, give an interpretation of what the interval in (f) means.

EXERCISE 8

A pharmaceutical company makes tranquilizers. It is assumed that the distribution for the length of time they last is approximately normal. Researchers in a hospital used the drug on a random sample of 9 patients. The effective period of the tranquilizer for each patient (in hours) was as follows: 2.7; 2.8; 3.0; 2.3; 2.3; 2.2; 2.8; 2.1; and 2.4 .

  a.

  i. $X=$_____

  ii. $S_x=$_____

  iii. $n=$_____

  iv. $n-1=$_____

b. Define the Random Variable X, in words.

c. Define the Random Variable X, in words.

d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 95% confidence interval for the population mean length of time.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

f. What does it mean to be "95% confident" in this problem?

EXERCISE 9

Suppose that 14 children were surveyed to determine how long they had to use training wheels. It was revealed that they used them an average of 6 months with a sample standard deviation of 3 months. Assume that the underlying population distribution is normal.

   a.

   i. $X=$_____
   ii. $S_x=$_____
   iii. $n=$_____
   iv. $n-1=$_____

b. Define the Random Variable X, in words.
c. Define the Random Variable X, in words.
d. Which distribution should you use for this problem? Explain your choice.

e. Construct a 99% confidence interval for the population mean length of time using training wheels.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

f. Why would the margin of error change if the confidence level was lowered to 90%?

EXERCISE 10

Suppose that the insurance companies did do a survey. They randomly surveyed 400 drivers and found that 320 claimed to always buckle up. We are interested in the population proportion of drivers who claim to always buckle up.

a.

   i. $x=$_____

   ii. $n=$_____

   iii. $\hat{p}=$_____

b. Define the Random Variables X and $\hat{p}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population proportion that claim to always buckle up.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

e. If this survey were done by telephone, list 3 difficulties the companies might have in obtaining random results.

EXERCISE 11

Unoccupied seats on flights cause airlines to lose revenue. Suppose a large airline wants to estimate its mean number of unoccupied seats per flight over the past year. To accomplish this, the records of 225 flights are randomly selected and the number of unoccupied seats is noted for each of the sampled flights. The sample mean is 11.6 seats and the sample standard deviation is 4.1 seats.

a.

   i. $\bar{X}=$_____

   ii. $S_x=$_____

   iii. $n=$_____

   iv. $n-1=$_____

b. Define the Random Variables X and $\bar{X}$, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 92% confidence interval for the population mean number of unoccupied seats per flight.

 i. State the confidence interval.

 ii. Sketch the graph.

 iii. Calculate the margin of error.

EXERCISE 12

According to a recent survey of 1200 people, 61% feel that the president is doing an acceptable job. We are interested in the population proportion of people who feel the president is doing an acceptable job.

 a. Define the Random Variables X and $\hat{p}$, in words.

 b. Which distribution should you use for this problem? Explain your choice.

 c. Construct a 90% confidence interval for the population proportion of people who feel the president is doing an acceptable job.

 i. State the confidence interval.

 ii. Sketch the graph.

 iii. Calculate the margin of error.

EXERCISE 13

A survey of the mean amount of cents off that coupons give was done by randomly surveying one coupon per page from the coupon sections of a recent San Jose Mercury News. The following data were collected: 20¢; 75¢; 50¢; 65¢; 30¢; 55¢; 40¢; 40¢; 30¢; 55¢; $1.50; 40¢; 65¢; 40¢. Assume the underlying distribution is approximately normal.

 a.

 i. $X=$_____

 ii. $S_x=$_____

 iii. $n=$_____

 iv. $n-1=$_____

 b. Define the Random Variables X and $\overline{X}$, in words.

 c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 95% confidence interval for the population mean worth of coupons.

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

e. If many random samples were taken of size 14, what percent of the confident intervals constructed should contain the population mean worth of coupons? Explain why.

EXERCISE 14

An article regarding interracial dating and marriage recently appeared in the *Washington Post*. Of the 1709 randomly selected adults, 315 identified themselves as Latinos, 323 identified themselves as blacks, 254 identified themselves as Asians, and 779 identified themselves as whites. In this survey, 86% of blacks said that their families would welcome a white person into their families. Among Asians, 77% would welcome a white person into their families, 71% would welcome a Latino, and 66% would welcome a black person.

a. We are interested in finding the 95% confidence interval for the percent of all black families that would welcome a white person into their families. Define the Random Variables X and $\hat{p}$, in words.

b. Which distribution should you use for this problem? Explain your choice.

c. Construct a 95% confidence interval

   i. State the confidence interval.

   ii. Sketch the graph.

   iii. Calculate the margin of error.

EXERCISE 15

Refer to the problem above.

a. Construct three 95% confidence intervals.

   i: Percent of all Asians that would welcome a white person into their families.

   ii: Percent of all Asians that would welcome a Latino into their families.

   iii: Percent of all Asians that would welcome a black person into their families.

b. Even though the three point estimates are different, do any of the confidence intervals overlap? Which?

c. For any intervals that do overlap, in words, what does this imply about the significance of the differences in the true proportions?

d. For any intervals that do not overlap, in words, what does this imply about the significance of the differences in the true proportions?

EXERCISE 16

A camp director is interested in the mean number of letters each child sends during his/her camp session. The population standard deviation is known to be 2.5. A survey of 20 campers is taken. The mean from the sample is 7.9 with a sample standard deviation of 2.8.

a.

i. $X=$_____
ii. $\sigma=$_____
iii. $S_x=$_____
iv. $n=$_____
v. $n-1=$_____

b. Define the Random Variables X and X, in words.

c. Which distribution should you use for this problem? Explain your choice.

d. Construct a 90% confidence interval for the population mean number of letters campers send home.

i. State the confidence interval.

ii. Sketch the graph.

iii. Calculate the margin of error.

e. What will happen to the margin of error and confidence interval if 500 campers are surveyed? Why?

EXERCISE 17

Stanford University conducted a study of whether running is healthy for men and women over age 50. During the first eight years of the study, 1.5% of the 451 members of the 50-Plus Fitness Association died. We are interested in the proportion of people over 50 who ran and died in the same eight–year period.

    a. Define the Random Variables X and $\hat{p}$, in words.

    b. Which distribution should you use for this problem? Explain your choice.

    c. Construct a 97% confidence interval for the population proportion of people over 50 who ran and died in the same eight–year period.

        i. State the confidence interval.

        ii. Sketch the graph.

        iii. Calculate the margin of error.

    d. Explain what a "97% confidence interval" means for this study.

EXERCISE 18

In a recent sample of 84 used cars sales costs, the sample mean was $6425 with a standard deviation of $3156. Assume the underlying distribution is approximately normal.

    a. Which distribution should you use for this problem? Explain your choice.

    b. Define the Random Variable X, in words.

    c. Construct a 95% confidence interval for the population mean cost of a used car.

        i. State the confidence interval.

        ii. Sketch the graph.

        iii. Calculate the margin of error.

    d. Explain what a "95% confidence interval" means for this study.

EXERCISE 19

A telephone poll of 1000 adult Americans was reported in an issue of *Time Magazine*. One of the questions asked was "What is the main problem facing the country?" 20% answered "crime". We are interested in the population proportion of adult Americans who feel that crime is the main problem.

   a. Define the Random Variables X and $\hat{p}$, in words.

   b. Which distribution should you use for this problem? Explain your choice.

   c. Construct a 95% confidence interval for the population proportion of adult Americans who feel that crime is the main problem.

       i. State the confidence interval.

       ii. Sketch the graph.

       iii. Calculate the margin of error.

   d. Suppose we want to lower the sampling error. What is one way to accomplish that?

   e. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In 1-3 complete sentences, explain what the ± 3% represents.

EXERCISE 20

Refer to the above problem. Another question in the poll was "[How much are] you worried about the quality of education in our schools?" 63% responded "a lot". We are interested in the population proportion of adult Americans who are worried a lot about the quality of education in our schools.

   a. Define the Random Variables X and $\hat{p}$, in words.

   b. Which distribution should you use for this problem? Explain your choice.

   c. Construct a 95% confidence interval for the population proportion of adult Americans worried a lot about the quality of education in our schools.

       i. State the confidence interval.

       ii. Sketch the graph.

       iii. Calculate the margin of error.

   d. The sampling error given by Yankelovich Partners, Inc. (which conducted the poll) is ± 3%. In 1-3 complete sentences, explain what the ± 3% represents.

EXERCISE 21

Six different national brands of chocolate chip cookies were randomly selected at the supermarket. The grams of fat per serving are as follows: 8; 8; 10; 7; 9; 9. Assume the underlying distribution is approximately normal.

a. Calculate a 90% confidence interval for the population mean grams of fat per serving of chocolate chip cookies sold in supermarkets.

  i. State the confidence interval.

  ii. Sketch the graph.

  iii. Calculate the margin of error.

b. If you wanted a smaller margin of error while keeping the same level of confidence, what should have been changed in the study before it was done?

c. Go to the store and record the grams of fat per serving of six brands of chocolate chip cookies.

d. Calculate the mean.

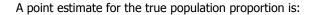**e.** Is the mean within the interval you calculated in part (a)? Did you expect it to be? Why or why not?

EXERCISE 22

A confidence interval for a proportion is given to be (– 0.22, 0.34). Why doesn't the lower limit of the confidence interval make practical sense? How should it be changed? Why?

Try these multiple choice questions.

The next three problems refer to the following: According to a Field Poll, 79% of California adults (actual results are 400 out of 506 surveyed) feel that "education and our schools" is one of the top issues facing California. We wish to construct a 90% confidence interval for the true proportion of California adults who feel that education and the schools is one of the top issues facing California. (Source: http://field.com/fieldpollonline/subscribers/)

EXERCISE 23

A point estimate for the true population proportion is:

A. 0.90

B. 1.27

C. 0.79

D. 400

EXERCISE 24

A 90% confidence interval for the population proportion is:

A. (0.761, 0.820)

B. (0.125, 0.188)

C. (0.755, 0.826)

D. (0.130, 0.183)

EXERCISE 25

The margin of error is approximately

A. 1.581

B. 0.791

C. 0.059

D. 0.030

The next two problems refer to the following:

A quality control specialist for a restaurant chain takes a random sample of size 12 to check the amount of soda served in the 16 oz. serving size. The sample mean is 13.30 with a sample standard deviation of 1.55. Assume the underlying population is normally distributed.

EXERCISE 26

Find the 95% Confidence Interval for the true population mean for the amount of soda served.

    A. (12.42, 14.18)

    B. (12.32, 14.29)

    C. (12.50, 14.10)

    D. Impossible to determine

EXERCISE 27

What is the margin of error?

    A. 0.87

    B. 1.98

    C. 0.99

    D. 1.74

EXERCISE 28

What is meant by the term "90% confident" when constructing a confidence interval for a mean?

    A. If we took repeated samples, approximately 90% of the samples would produce the same confidence interval.

    B. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the sample mean.

    C. If we took repeated samples, approximately 90% of the confidence intervals calculated from those samples would contain the true value of the population mean.

D. If we took repeated samples, the sample mean would equal the population mean in approximately 90% of the samples.

The next two problems refer to the following:

Five hundred and eleven (511) homes in a certain southern California community are randomly surveyed to determine if they meet minimal earthquake preparedness recommendations. One hundred seventy-three (173) of the homes surveyed met the minimum recommendations for earthquake preparedness and 338 did not.

EXERCISE 29

Find the Confidence Interval at the 90% Confidence Level for the true population proportion of southern California community homes meeting at least the minimum recommendations for earthquake preparedness.

A. (0.2975, 0.3796)

B. (0.6270, 6959)

C. (0.3041, 0.3730)

D. (0.6204, 0.7025)

EXERCISE 30

The point estimate for the population proportion of homes that do not meet the minimum recommendations for earthquake preparedness is:

A. 0.6614

B. 0.3386

C. 173

D. 338