# PROBABILITY TOPICS: HOMEWORK

## EXERCISE 1

Suppose that you have 8 cards. 5 are green and 3 are yellow. The 5 green cards are numbered 1, 2, 3, 4, and 5. The 3 yellow cards are numbered 1, 2, and 3. The cards are well shuffled. You randomly draw one card. Consider the following events:

G = card drawn is green; E = card drawn is even-numbered

a. List the sample space.
b. P(G) =
c. P(G|E) =
d. P(G and E) =
e. P(G or E) =
f. Are G and E mutually exclusive? Justify your answer numerically.

## EXERCISE 2

Refer to problem (1) above. Suppose that this time you randomly draw two cards, one at a time, and with replacement.

$G_1$ = first card is green; $G_2$ = second card is green.

a. Draw a tree diagram of the situation.
b. P( $G_1$ and $G_2$) =
c. P(at least one green) =
d. P($G_2$|$G_1$) =
e. Are $G_2$ and $G_1$ independent events? Explain why or why not.

## EXERCISE 3

Refer to problem (1) above. Suppose that this time you randomly draw two cards, one at a time, and without replacement.

$G_1$ = first card is green; $G_2$ = second card is green.

a. Draw a tree diagram of the situation.
b. P( $G_1$ and $G_2$) =
c. P(at least one green) =

d. $P(G_2|G_1)$ =
e. Are $G_2$ and $G_1$ independent events? Explain why or why not.

**EXERCISE 4**

Roll two fair dice. Each die has 6 faces.

a. List the sample space.
b. Let A be the event that either a 3 or 4 is rolled first, followed by an even number. Find P(A).
c. Let B be the event that the sum of the two rolls is at most 7. Find P(B).
d. In words, explain what "P(A|B)" represents. Find P(A|B).
e. Are A and B mutually exclusive events? Explain your answer in 1 - 3 complete sentences, including numerical justification.
f. Are A and B independent events? Explain your answer in 1 - 3 complete sentences, including numerical justification.

**EXERCISE 5**

A special deck of cards has 10 cards. Four are green, three are blue, and three are red. When a card is picked, the color of it is recorded. An experiment consists of first picking a card and then tossing a coin.

a. List the sample space.
b. Let A be the event that a blue card is picked first, followed by landing a head on the coin toss. Find P(A).
c. Let B be the event that a red or green is picked, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.
d. Let C be the event that a red or blue is picked, followed by landing a head on the coin toss. Are the events A and C mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.

**EXERCISE 6**

An experiment consists of first rolling a die and then tossing a coin.

a. List the sample space.
b. Let A be the event that either a 3 or 4 is rolled first, followed by landing a head on the coin toss. Find P(A).
c. Let B be the event that a number less than 2 is rolled, followed by landing a head on the coin toss. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including numerical justification.

## EXERCISE 7

An experiment consists of tossing a nickel, a dime and a quarter. Of interest is the side the coin lands on.

a. List the sample space.
b. Let A be the event that there are at least two tails. Find P(A).
c. Let B be the event that the first and second tosses land on heads. Are the events A and B mutually exclusive? Explain your answer in 1 - 3 complete sentences, including justification.

## EXERCISE 8

Let $P(C) = 0.4$; $P(D) = 0.5$; $P(C|D) = 0.6$ .

a. Find P(C and D) .
b. Are C and D mutually exclusive? Why or why not?
c. Are C and D independent events? Why or why not?
d. Find P(C or D) .
e. Find $P(D|C)$ .

## EXERCISE 9

E and F are mutually exclusive events. $P(E) = 0.4$; $P(F) = 0.5$ . Find $P(E|F)$ .

## EXERCISE 10

J and K are independent events. $P(J|K) = 0.3$ . $P(K) = 0.5$ . Find $P(J)$ .

## EXERCISE 11

U and V are mutually exclusive events. $P(U) = 0.26$; $P(V) = 0.37$ . Find:

a. P(U and V)
b. P(U | V)
c. P(U or V)

**EXERCISE 12**

Q and R are independent events. P(Q) = 0.4; P(Q and R) = 0.10 . Find P(R).

**EXERCISE 13**

Y and Z are independent events.

a. Rewrite the basic Addition Rule ( P(Y or Z) = P(Y) + P(Z) - P(Y and Z) ) using the information that Y and Z are independent events.
b. Use the rewritten rule to find P(Z) if P(Y or Z) = 0.71 and P(Y) = 0.42 .

**EXERCISE 14**

G and H are mutually exclusive events. P(G) = 0.5; P(H) = 0.3.

a. Explain why the following statement MUST be false: P(H|G) = 0.4 .
b. Find: P(H or G).
c. Are G and H independent or dependent events? Explain in a complete sentence.

## EXERCISE 15

The following are real data from Santa Clara County, CA. As of March 31, 2000, there was a total of 3059 documented cases of AIDS in the county. They were grouped into the following categories (Source: Santa Clara County Public H.D.):

**Risk Factors**

| **Gender** | **Homosexual/ Bisexual** | **IV Drug User *** | **Heterosexual Contact** | **Other** |
|---|---|---|---|---|
| female | 0 | 70 | 136 | 49 |
| male | 2146 | 463 | 60 | 135 |

* includes homosexual/bisexual IV drug users

Suppose one of the persons with AIDS in Santa Clara County is randomly selected. Compute the following:

a. P(person is female) = _____
b. P(person has a risk factor heterosexual contact) = _____
c. P(person is female OR has a risk factor of IV Drug User) = _____
d. P(person is female AND has a risk factor of homosexual/bisexual) = _____
e. P(person is male AND has a risk factor of IV Drug User) = _____
f. P(female GIVEN person got the disease from heterosexual contact) = _____
g. Construct a Venn Diagram. Make one group females and the other group heterosexual contact. Fill in all values as integers.

## EXERCISE 16

Solve these questions using probability rules. Do NOT use the contingency table above. 3059 cases of AIDS had been reported in Santa Clara County, CA, through March 31, 2000. Those cases will be our population. Of those cases, 6.4% obtained the disease through heterosexual contact and 7.4% are female. Out of the females with the disease, 53.3% got the disease from heterosexual contact.

a. P(person is female) = _____
b. P(person obtained the disease through heterosexual contact) = _____
c. P(female GIVEN person got the disease from heterosexual contact) = _____

d. Construct a Venn Diagram. Make one group females and the other group heterosexual contact. Fill in all values as probabilities.

## EXERCISE 17

The following table identifies a group of children by one of four hair colors, and by type of hair.

**Hair color**

| Hair Type | Brown | Blond | Black | Red | Totals |
|---|---|---|---|---|---|
| **Wavy** | 20 | | 15 | 3 | 43 |
| **Straight** | 80 | 15 | | 12 | |
| **Totals** | | 20 | | | 215 |

a. Complete the table above.
b. What is the probability that a randomly selected child will have wavy hair?
c. What is the probability that a randomly selected child will have either brown or blond hair?
d. What is the probability that a randomly selected child will have wavy brown hair?
e. What is the probability that a randomly selected child will have red hair, given that he has straight hair?
f. If B is the event of a child having brown hair, find the probability of the complement of B.
g. In words, what does the complement of B represent?

## EXERCISE 18

A previous year, the weights of the members of the San Francisco *49ers* and the Dallas *Cowboys* were published in the *San Jose Mercury News*. The factual data are compiled into the following table.

**Weight (in pounds)**

| Shirt # | ≤ 210 | 211 - 250 | 251 - 290 | 291 ≤ |
|---|---|---|---|---|
| **1 - 33** | 21 | 5 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| **34 - 66** | 6 | 18 | 7 | 4 |
| **66 - 99** | 6 | 12 | 22 | 5 |

For (a) - (e), suppose that you randomly select one player from the 49ers or Cowboys.

a. Find the probability that his shirt number is from 1 to 33.
b. Find the probability that he weighs at most 210 pounds.
c. Find the probability that his shirt number is from 1 to 33 AND he weighs at most 210 pounds.
d. Find the probability that his shirt number is from 1 to 33 OR he weighs at most 210 pounds.
e. Find the probability that his shirt number is from 1 to 33 GIVEN that he weighs at most 210 pounds.
f. If having a shirt number from 1 to 33 and weighing at most 210 pounds were independent events, then what should be true about P(shirt # 1 - 33 | $\leq$ 210 pounds)?

**EXERCISE 19**

Approximately 249,000,000 people live in the United States. Of these people, 31,800,000 speak a language other than English at home. Of those who speak another language at home, over 50 percent speak Spanish. (Source: U.S. Bureau of the Census, 1990 Census)

Let: E = speak English at home; E' = speak another language at home; S = speak Spanish at home

Finish each probability statement by matching the correct answer.

a. P(E' ) = i. 0.8723
b. P(E) = ii. > 0.50
c. P(S) = iii. 0.1277
d. P(S | E' ) = iv. > 0.0639

**EXERCISE 20**

The probability that a male develops some form of cancer in his lifetime is 0.4567 (Source: American Cancer Society). The probability that a male has at least one false positive test result (meaning the test comes back for cancer when the man does not have it) is 0.51 (Source: *USA Today*). Some of the questions below do not have enough information for you to answer them. Write “not enough information” for those answers.

Let: C = a man develops cancer in his lifetime; P = man has at least one false positive

a. Construct a tree diagram of the situation.
b. P(C) = __________
c. P(P|C) = __________
d. d. P(P|C' ) = __________
e. If a test comes up positive, based upon numerical values, can you assume that man has cancer? __________ Justify numerically and explain why or why not.

## Exercise 21

In 1994, the U.S. government held a lottery to issue 55,000 Green Cards (permits for non-citizens to work legally in the U.S.). Renate Deutsch, from Germany, was one of approximately 6.5 million people who entered this lottery. Let G = won Green Card

a. What was Renate's chance of winning a Green Card? Write your answer as a probability statement.
b. In the summer of 1994, Renate received a letter stating she was one of 110,000 finalists chosen. Once the finalists were chosen, assuming that each finalist had an equal chance to win, what was Renate's chance of winning a Green Card? Let F = was a finalist. Write your answer as a conditional probability statement.
c. Are G and F independent or dependent events? Justify your answer numerically and also explain why.
d. Are G and F mutually exclusive events? Justify your answer numerically and also explain why.

P.S. Amazingly, on 2/1/95, Renate learned that she would receive her Green Card -- true story!

## Exercise 22

Three professors at George Washington University did an experiment to determine if economists are more selfish than other people. They dropped 64 stamped, addressed envelopes with $10 cash in different classrooms on the George Washington campus. 44% were returned overall. From the economics classes 56% of the envelopes were returned. From the business, psychology, and history classes 31% were returned. (Source: *Wall Street Journal*) Let: R = money returned; E = economics classes; O = other classes

a. Write a probability statement for the overall percent of money returned.
b. Write a probability statement for the percent of money returned out of the economics classes.

c. Write a probability statement for the percent of money returned out of the other classes.
d. Is money being returned independent of the class? Justify your answer numerically and explain it.
e. Based upon this study, do you think that economists are more selfish than other people? Explain why or why not. Include numbers to justify your answer.

**EXERCISE 23**

The chart below gives the number of suicides comparing blacks and whites estimated in the U.S. for a recent year by age, race and sex. We are interested possible relationships between age, race, and sex. We will let suicide victims be our population. (Source: The National Center for Health Statistics, U.S. Dept. of Health and Human Services)

**Age**

| **Race and Sex** | **1 - 14** | **15 - 24** | **25 - 64** | **over 64** | **TOTALS** |
|---|---|---|---|---|---|
| **white, male** | 210 | 3360 | 13,610 | | 22,050 |
| **white, female** | 80 | 580 | 3380 | | 4930 |
| **black, male** | 10 | 460 | 1060 | | 1670 |
| **black, female** | 0 | 40 | 270 | | 330 |
| **all others** | | | | | |
| **TOTALS** | 310 | 4650 | 18,780 | | 29,760 |

*NOTE: Do not include "all others" for (f), (g), and (i).*

a. Fill in the column for the suicides for individuals over age 64.
b. Fill in the row for all other races.
c. Find the probability that a randomly selected individual was a white male.
d. Find the probability that a randomly selected individual was a black female.
e. Find the probability that a randomly selected individual was black.
f. Find the probability that a randomly selected individual was male.
g. Out of the individuals over age 64, find the probability that a randomly selected individual was a black or white male.
h. Comparing "Race and Sex" to "Age," which two groups are mutually exclusive? How do you know?
i. Are being male and committing suicide over age 64 independent events? How do you know?

**The following refers to questions (24) and (25):** The percent of licensed U.S. drivers (from a recent year) that are female is 48.60. Of the females, 5.03% are age 19 and under; 81.36% are age 20 - 64; 13.61% are age 65 or over. Of the licensed U.S. male drivers, 5.04% are age 19 and under; 81.43% are age 20 - 64; 13.53% are age 65 or over. (Source: Federal Highway Administration, U.S. Dept. of Transportation)

### EXERCISE 24

Complete the following:

a. Construct a table or a tree diagram of the situation.
b. P(driver is female) = __________
c. P(driver is age 65 or over | driver is female) = __________
d. P(driver is age 65 or over AND female) = __________
e. In words, explain the difference between the probabilities in part (c) and part (d).
f. P(driver is age 65 or over) = __________
g. Are being age 65 or over and being female independent events? How do you know?
h. Are being age 65 or over and being female mutually exclusive events? How do you know?

### EXERCISE 25

Suppose that 10,000 U.S. licensed drivers are randomly selected.

a. How many would you expect to be male?
b. Using the table or tree diagram from problem (21), construct a contingency table of gender versus age group.
c. Using the contingency table, find the probability that out of the age 20 - 64 group, a randomly selected driver is female.

### EXERCISE 26

Approximately 86.5% of Americans commute to work by car, truck or van. Out of that group, 84.6% drive alone and 15.4% drive in a carpool. Approximately 3.9% walk to work and approximately 5.3% take public transportation. (Source: Bureau of the Census, U.S. Dept. of Commerce. Disregard rounding approximations.)

a. Construct a table or a tree diagram of the situation. Include a branch for all other modes of transportation to work.
b. Assuming that the walkers walk alone, what percent of all commuters travel alone to work?

c. Suppose that 1000 workers are randomly selected. How many would you expect to travel alone to work?
d. Suppose that 1000 workers are randomly selected. How many would you expect to drive in a carpool?

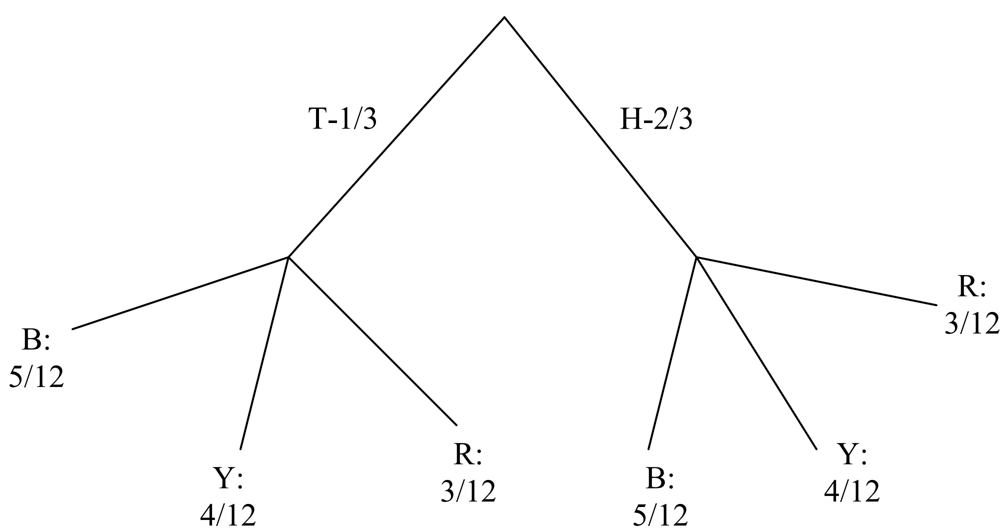
**EXERCISE 27**

Explain what is wrong with the following statements. Use complete sentences.

a. If there's a 60% chance of rain on Saturday and a 70% chance of rain on Sunday, then there's a 130% chance of rain over the weekend.
b. The probability that a baseball player hits a home run is greater than the probability that he gets a successful hit.

Try these multiple choice questions.

**Questions 28 – 29 refer to the following** probability tree diagram which shows tossing an unfair coin FOLLOWED BY drawing one bead from a cup containing 3 red (R), 4 yellow (Y) and 5 blue (B) beads. For the coin, P(H) = 2/3 and P(T) = 1/3 where H = "heads" and T = "tails."




**EXERCISE 28**

Find P(tossing a **Head** on the coin AND a **Red** bead)

A. $\frac{2}{3}$

B. $\frac{5}{15}$

C. $\frac{6}{36}$

D. $\frac{5}{36}$

**EXERCISE 29**

Find P(**Blue** bead)

A. $\frac{15}{36}$

B. $\frac{10}{36}$

C. $\frac{10}{12}$

D. $\frac{6}{36}$

Questions 30 – 32 refer to the following table of data obtained from **www.baseball-almanac.com** showing hit information for 4 well known baseball players.

**Type of Hit**

| **NAME** | **Single** | **Double** | **Triple** | **Home Run** | **TOTAL HITS** |
|---|---|---|---|---|---|
| **Babe Ruth** | 1517 | 506 | 136 | 714 | 2873 |
| **Jackie Robinson** | 1054 | 273 | 54 | 137 | 1518 |
| **Ty Cobb** | 3603 | 174 | 295 | 114 | 4189 |
| **Hank Aaron** | 2294 | 624 | 98 | 755 | 3771 |
| **TOTAL** | 8471 | 1577 | 583 | 1720 | 12351 |

**EXERCISE 30**

Find P(hit was made by Babe Ruth)

A. $\frac{1518}{2873}$

B. $\frac{2873}{12351}$

C. $\frac{583}{12351}$

D. $\frac{4189}{12351}$

**EXERCISE 31**

Find P(hit was made by Ty Cobb | the hit was a Home Run)

A. $\frac{4189}{12351}$

B. $\frac{114}{1720}$

C. $\frac{1720}{4189}$

D. $\frac{114}{12351}$

**EXERCISE 32**

Are the hit being made by Hank Aaron and the hit being a double independent?

A. Yes, because P(hit by Hank Aaron | hit is a double) = P(hit by Hank Aaron)
B. No, because P(hit by Hank Aaron | hit is a double) ≠ P(hit is a double)
C. No, because P(hit by Hank Aaron | hit is a double) ≠ P(hit by Hank Aaron)
D. Yes, because P(hit by Hank Aaron | hit is a double) = P(hit is a double)

**EXERCISE 33**

Given events G and H: $P(G) = 0.43$ ; $P(H) = 0.26$ ; $P(H \text{ and } G) = 0.14$

A. Find P(H or G)

B. Find the probability of the complement of event (H and G)
C. Find the probability of the complement of event (H or G)

### EXERCISE 34

Given events J and K: P(J) = 0.18 ; P(K) = 0.37 ; P(J or K) = 0.45

A. Find P(J and K)
B. Find the probability of the complement of event (J and K)
C. Find the probability of the complement of event (J or K)

### EXERCISE 35

United Blood Services is a blood bank that serves more than 500 hospitals in 18 states. According to their website, http://www.unitedbloodservices.org/humanbloodtypes.html, a person with type O blood and a negative Rh factor (Rh−) can donate blood to any person with any bloodtype. Their data show that 43% of people have type O blood and 15% of people have Rh− factor; 52% of people have type O or Rh− factor.

A. Find the probability that a person has both type O blood and the Rh− factor
B. Find the probability that a person does NOT have both type O blood and the Rh− factor.

### EXERCISE 36

At a college, 72% of courses have final exams and 46% of courses require research papers. Suppose that 32% of courses have a research paper and a final exam. Let F be the event that a course has a final exam.

A. Let R be the event that a course requires a research paper.
B. Find the probability that a course has a final exam or a research project.
C. Find the probability that a course has NEITHER of these two requirements.

### EXERCISE 37

In a box of assorted cookies, 36% contain chocolate and 12% contain nuts. Of those, 8% contain both chocolate and nuts. Sean is allergic to both chocolate and nuts.

A. Find the probability that a cookie contains chocolate or nuts (he can't eat it).
B. Find the probability that a cookie does not contain chocolate or nuts (he can eat it).

### Exercise 38

A college finds that 10% of students have taken a distance learning class and that 40% of students are part time students. Of the part time students, 20% have taken a distance learning class. Let D = event that a student takes a distance learning class and E = event that a student is a part time student

A. Find P(D and E).
B. Find P(E | D).
C. Find P(D or E).
D. Using an appropriate test, show whether D and E are independent.
E. Using an appropriate test, show whether D and E are mutually exclusive.

### Exercise 39

At a certain store the manager has determined that 30% of customers pay cash and 70% of customers pay by debit card. (No other method of payment is accepted.)
Let M = event that a customer pays cash and D= event that a customer pays by debit card. Suppose two customers (Al and Betty) come to the store. Explain why it would be reasonable to assume that their choices of payment methods are independent of each other.

A. Draw the tree that represents the all possibilities for the 2 customers and their methods of payment. Write the probabilities along each branch of the tree.
B. For each complete path through the tree, write the event it represents and find the probability.
C. Let S be the event that both customers use the same method of payment. Find P(S).
D. Let T be the event that both customers use different methods of payment. Find P(T) by two different methods: by using the complement rule and by using the branches of the tree. Your answers should be the same with both methods.
E. Let U be the event that the second customer uses a debit card. Find P(U).

### Exercise 40

A box of cookies contains 3 chocolate and 7 butter cookies. Miguel randomly selects a cookie and eats it. Then he randomly selects another cookie and eats it also. (How many cookies did he take?)

A. Are the probabilities for the flavor of the SECOND cookie that Miguel selects independent of his first selection, or do the probabilities depend on the type of cookie that Miguel selected first? Explain.
B. Draw the tree that represents the possibilities for the cookie selections. Write the probabilities along each branch of the tree.

C. For each complete path through the tree, write the event it represents and find the probabilities.
D. Let S be the event that both cookies selected were the same flavor. Find P(S).
E. Let T be the event that both cookies selected were different flavors. Find P(T) by two different methods: by using the complement rule and by using the branches of the tree. Your answers should be the same with both methods.
F. Let U be the event that the second cookie selected is a butter cookie. Find P(U).

#### EXERCISE 41

When the Euro coin was introduced in 2002, two math professors had their statistics students test whether the Belgian 1 Euro coin was a fair coin. They spun the coin rather than tossing it, and it was found that out of 250 spins, 140 showed a head (event H) while 110 showed a tail (event T). Therefore, they claim that this is not a fair coin.

A. Based on the data above, find P(H) and P(T).
B. Use a tree to find the probabilities of each possible outcome for the experiment of tossing the coin twice.
C. Use the tree to find the probability of obtaining exactly one head in two tosses of the coin.
D. Use the tree to find the probability of obtaining at least one head.