CONTINUOUS RANDOM VARIABLES: HOMEWORK

For each probability and percentile problem, DRAW THE PICTURE!

EXERCISE 1

Consider the following experiment. You are one of 100 people enlisted to take part in a study to determine the percent of nurses in America with an R.N. (registered nurse) degree. You ask nurses if they have an R.N. degree. The nurses answer "yes" or "no." You then calculate the percentage of nurses with an R.N. degree. You give that percentage to your supervisor.

a. What part of the experiment will yield discrete data?
b. What part of the experiment will yield continuous data?

EXERCISE 2

When age is rounded to the nearest year, do the data stay continuous, or do they become discrete? Why?

EXERCISE 3

Births are approximately uniformly distributed between the 52 weeks of the year. They can be said to follow a Uniform Distribution from 1 – 53 (spread of 52 weeks).

a. $X \sim$
b. Graph the probability distribution.
c. $f(x) =$
d. $\mu =$
e. $\sigma =$
f. Find the probability that a person is born at the exact moment week 19 starts. That is, find $P(X = 19)$.
g. $P(2 < X < 31) =$
h. Find the probability that a person is born after week 40.
i. $P(12 < X \mid X < 28) =$
j. Find the 70th percentile.
k. Find the minimum for the upper quarter.

EXERCISE 4

A random number generator picks a number from 1 to 9 in a uniform manner.

a. $X \sim$
b. Graph the probability distribution.
c. $f(x) =$
d. $\mu =$
e. $\sigma =$
f. $P(3.5 < X < 7.25) =$
g. $P(X > 5.67) =$
h. $P(X > 5 \mid X > 3) =$
i. Find the 90th percentile.

EXERCISE 5

The speed of cars passing through the intersection of Blossom Hill Road and the Almaden Expressway varies from 10 to 35 mph and is uniformly distributed. None of the cars travel over 35 mph through the intersection.

a. $X =$
b. $X \sim$
c. Graph the probability distribution.
d. $f(x) =$
e. $\mu =$
f. $\sigma =$
g. What is the probability that the speed of a car is at most 30 mph?
h. What is the probability that the speed of a car is between 16 and 22 mph?
i. $P(20 < X < 53) =$
j. State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.
k. Find the 90th percentile. This means that 90% of the time, the speed is less than _____ mph while passing through the intersection per minute.
l. Find the 75th percentile. In a complete sentence, state what this means. (See **j**.)
m. Find the probability that the speed is more than 24 mph given (or knowing that) it is at least 15 mph.

EXERCISE 6

According to a study by Dr. John McDougall of his live-in weight loss program at St. Helena Hospital, the people who follow his program lose between 6 and 15 pounds a month until they approach trim body weight. Let's suppose that the weight loss is uniformly distributed. We are interested in the weight loss of a randomly selected individual following the program for one month. (Source*: The McDougall Program for Maximum Weight Loss* by John A. McDougall, M.D.)

a. X =
b. X ~
c. Graph the probability distribution.
d. f(x) =
e. μ =
f. σ =
g. Find the probability that the individual lost more than 10 pounds in a month.
h. Suppose it is known that the individual lost more than 10 pounds in a month. Find the probability that he lost less than 12 pounds in the month.
i. P(7 < X < 13 | X > 9) = ? State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.

EXERCISE 7

A subway train on the Red Line arrives every 8 minutes during rush hour. We are interested in the length of time a commuter must wait for a train to arrive. The time follows a uniform distribution.

a. X =
b. X ~
c. Graph the probability distribution.
d. f(x) =
e. μ =
f. σ =
g. Find the probability that the commuter waits less than one minute.
h. Find the probability that the commuter waits between three and four minutes.
i. 60% of commuters wait more than how long for the train? State this in a probability question (similar to **g** and **h**), draw the picture, and find the probability.

EXERCISE 8

The age of a first grader on September 1 at Garden Elementary School is uniformly distributed from 5.8 to 6.8 years. We randomly select one first grader from the class.

a. X =
b. X ~
c. Graph the probability distribution.
d. f(x) =
e. μ =
f. σ =
g. Find the probability that she is over 6.5 years.

h. Find the probability that she is between 4 and 6 years.
i. Find the 70th percentile for the age of first graders on September 1 at Garden Elementary School.

EXERCISE 9

Let X ~ Exp(0.1)

a. decay rate =
b. μ =
c. Graph the probability distribution function.
d. On the above graph, shade the area corresponding to P(X < 6) and find the probability.
e. Sketch a new graph, shade the area corresponding to P(3< X < 6) and find the probability.
f. Sketch a new graph, shade the area corresponding to P(X >7) and find the probability.
g. Sketch a new graph, shade the area corresponding to the 40th percentile and find the value.
h. Find the average value of X.

EXERCISE 9

Suppose that the length of long distance phone calls, measured in minutes, is known to have an exponential distribution with the average length of a call equal to 8 minutes.

a. X =
b. Is X continuous or discrete?
c. X ~
d. μ =
e. σ =
f. Draw a graph of the probability distribution. Label the axes.
g. Find the probability that a phone call lasts less than 9 minutes.
h. Find the probability that a phone call lasts more than 9 minutes.
i. Find the probability that a phone call lasts between 7 and 9 minutes.
j. If 25 phone calls are made one after another, on average, what would you expect the total to be? Why?

EXERCISE 11

Suppose that the useful life of a particular car battery, measured in months, decays with parameter 0.025. We are interested in the life of the battery.

a. X =
b. Is X continuous or discrete?
c. X ~
d. On average, how long would you expect 1 car battery to last?
e. On average, how long would you expect 9 car batteries to last, if they are used one after another?
f. Find the probability that a car battery lasts more than 36 months.
g. 70% of the batteries last at least how long?

EXERCISE 12

The percent of persons (ages 5 and older) in each state who speak a language at home other than English is approximately exponentially distributed with a mean of 9.848 . Suppose we randomly pick a state. (Source: Bureau of the Census, U.S. Dept. of Commerce)

a. X =
b. Is X continuous or discrete?
c. X ~
d. μ =
e. σ =
f. Draw a graph of the probability distribution. Label the axes.
g. Find the probability that the percent is less than 12.
h. Find the probability that the percent is between 8 and 14.
i. The percent of all individuals living in the United States who speak a language at home other than English is 13.8 .
j. Why is this number different from 9.848%?
k. What would make this number higher than 9.848%?

EXERCISE 13

The time (in years) **after** reaching age 60 that it takes an individual to retire is approximately exponentially distributed with a mean of about 5 years. Suppose we randomly pick one retired individual. We are interested in the time after age 60 to retirement.

a. X =
b. Is X continuous or discrete?
c. X ~
d. μ =
e. σ =
f. Draw a graph of the probability distribution. Label the axes.

g. Find the probability that the person retired after age 70.
h. Do more people retire before age 65 or after age 65?
i. In a room of 1000 people over age 80, how many do you expect will NOT have retired yet?

EXERCISE 14

The cost of all maintenance for a car during its first year is approximately exponentially distributed with a mean of $150.

a. X =
b. X ~
c. μ =
d. σ =
e. Draw a graph of the probability distribution. Label the axes.
f. Find the probability that a car required over $300 for maintenance during its first year.

Try these multiple choice problems.

Questions 15 – 17 refer to the following: The average lifetime of a certain new cell phone is 3 years. The manufacturer will replace any cell phone failing within 2 years of the date of purchase. The lifetime of these cell phones is known to follow an exponential distribution.

EXERCISE 15

15. The decay rate is

A. 0.3333
B. 0.5000
C. 2.0000
D. 3.0000

EXERCISE 16

16. What is the probability that a phone will fail within 2 years of the data of purchase?

A. 0.8647
B. 0.4866
C. 0.2212
D. 0.9997

EXERCISE 17

What is the median lifetime of these phones (in years)?

A. 0.1941
B. 1.3863
C. 2.0794
D. 5.5452

Questions 18 – 20 refer to the following: The Sky Train from the terminal to the rental car and long term parking center is supposed to arrive every 8 minutes. The waiting times for the train are known to follow a uniform distribution.

EXERCISE 18

What is the average waiting time (in minutes)?

A. 0.0000
B. 2.0000
C. 3.0000
D. 4.0000

EXERCISE 19

Find the 30^{th} percentile for the waiting times (in minutes).

a. 2.0000
b. 2.4000
c. 2.750
d. 3.000

EXERCISE 20

The probability of waiting more than 7 minutes given a person has waited more than 4 minutes is?

A. 0.1250
B. 0.2500
C. 0.5000
D. 0.7500