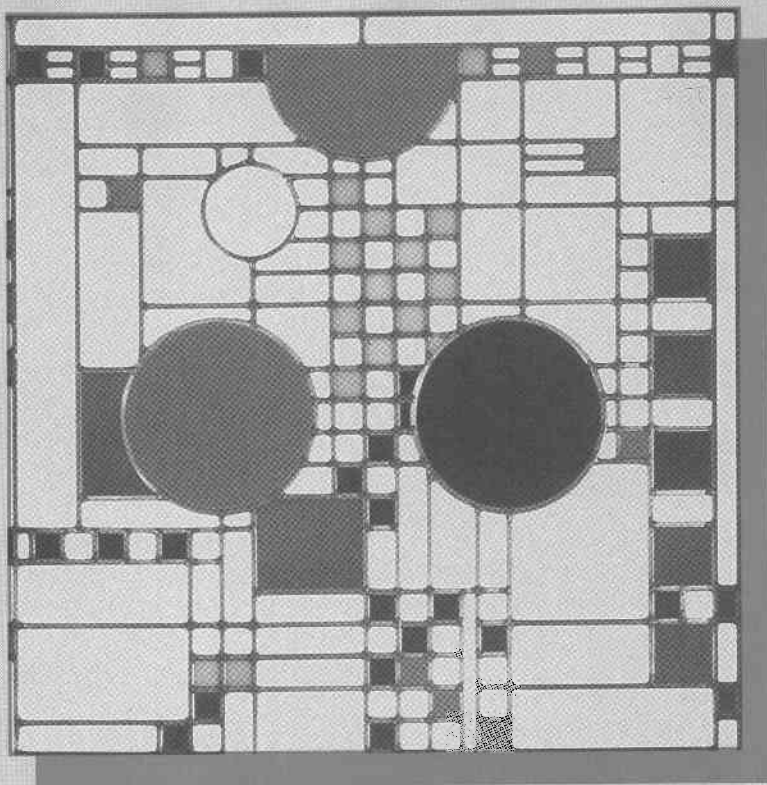
4

Solving Linear Equations and Inequalities



After completing this chapter, you should

Section 4.1 Solving Equations
- be able to identify various types of equations
- understand the meaning of solutions and equivalent equations
- be able to solve equations of the form $x + a = b$ and $x - a = b$
- be familiar with and able to solve literal equations

Section 4.2 Solving Equations of the Form $ax = b$ and $\frac{x}{a} = b$
- understand the equality property of addition and multiplication
- be able to solve equations of the form $ax = b$ and $x/a = b$

Section 4.3 Further Techniques in Equation Solving
- be comfortable with combining techniques in equation solving
- be able to recognize identities and contradictions

Section 4.4 Applications I – Translating from Verbal to Mathematical Expressions
- be able to translate from verbal to mathematical expressions

Section 4.5 Applications II – Solving Problems
- be able to solve various applied problems

Section 4.6 Linear Inequalities in One Variable
- understand the meaning of inequalities
- be able to recognize linear inequalities
- know, and be able to work with, the algebra of linear inequalities and with compound inequalities

Section 4.7 Linear Equations in Two Variables
- be able to identify the solution of a linear equation in two variables
- know that solutions to linear equations in two variables can be written as ordered pairs

4.1 Solving Equations

Section Overview

- ☐ TYPES OF EQUATIONS
- ☐ SOLUTIONS AND EQUIVALENT EQUATIONS
- ☐ LITERAL EQUATIONS
- ☐ SOLVING EQUATIONS OF THE FORM $x + a = b$ AND $x - a = b$

☐ TYPES OF EQUATIONS

Identity

Some equations are always true. These equations are called identities. **Identities** are equations that are true for all acceptable values of the variable, that is, for all values in the domain of the equation.

$5x = 5x$ is true for all acceptable values of x.
$y + 1 = y + 1$ is true for all acceptable values of y.
$2 + 5 = 7$ is true, and no substitutions are necessary.

Contradiction

Some equations are never true. These equations are called contradictions. **Contradictions** are equations that are never true regardless of the value substituted for the variable.

$x = x + 1$ is never true for any acceptable value of x.
$0 \cdot k = 14$ is never true for any acceptable value of k.
$2 = 1$ is never true.

Conditional Equation

The truth of some equations is conditional upon the value chosen for the variable. Such equations are called conditional equations. **Conditional equations** are equations that are true for at least one replacement of the variable and false for at least one replacement of the variable.

$x + 6 = 11$ is true only on the condition that $x = 5$.
$y - 7 = -1$ is true only on the condition that $y = 6$.

☐ SOLUTIONS AND EQUIVALENT EQUATIONS

Solutions and Solving an Equation

The collection of values that make an equation true are called **solutions** of the equation. An equation is **solved** when all its solutions have been found.

Equivalent Equations

Some equations have precisely the same collection of solutions. Such equations are called **equivalent equations.** The equations

$2x + 1 = 7, \qquad 2x = 6, \qquad \text{and} \qquad x = 3$

are equivalent equations because the only value that makes each one true is 3.

☆ SAMPLE SET A

Tell why each equation is an identity, a contradiction, or conditional.

1. The equation $x - 4 = 6$ is a conditional equation since it will be true only on the condition that $x = 10$.
2. The equation $x - 2 = x - 2$ is an identity since it is true for all values of x. For example,

   if $x = 5$, $\quad 5 - 2 = 5 - 2$ is true
   if $x = -7$, $\quad -7 - 2 = -7 - 2$ is true

3. The equation $a + 5 = a + 1$ is a contradiction since every value of a produces a false statement. For example,

if $a = 8$, $8 + 5 = 8 + 1$ is false
if $a = -2$, $-2 + 5 = -2 + 1$ is false

★ PRACTICE SET A

For each of the following equations, write "identity," "contradiction," or "conditional." If you can, find the solution by making an educated guess based on your knowledge of arithmetic.

1. $x + 1 = 10$

2. $y - 4 = 7$

3. $5a = 25$

4. $\frac{x}{4} = 9$

5. $\frac{18}{b} = 6$

6. $y - 2 = y - 2$

7. $x + 4 = x - 3$

8. $x + x + x = 3x$

9. $8x = 0$

10. $m - 7 = -5$

❑ LITERAL EQUATIONS

Literal Equations

Some equations involve more than one variable. Such equations are called **literal equations.**

An equation is solved for a particular variable if that variable alone equals an expression that does not contain that particular variable.

The following equations are examples of literal equations.

1. $y = 2x + 7$. It is solved for y.
2. $d = rt$. It is solved for d.
3. $I = prt$. It is solved for I.
4. $z = \frac{x - u}{s}$. It is solved for z.
5. $y + 1 = x + 4$. This equation is not solved for any particular variable since no variable is isolated.

❑ SOLVING EQUATIONS OF THE FORM $x + a = b$ AND $x - a = b$

Recall that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side.

This number	is the same as	this number
↓	↓	↓
x	$=$	6
$x+2$	$=$	8
$x-1$	$=$	5

This suggests the following procedures:

1. We can obtain an equivalent equation (an equation having the same solutions as the original equation) by *adding* the *same number* to *both sides* of the equation.
2. We can obtain an equivalent equation by *subtracting* the *same number* from *both sides* of the equation.

We can use these results to isolate x, thus solving for x.

Solving $x + a = b$ for x

$x + a = b$	The a is associated with x by addition. Undo the association by subtracting a from *both* sides.
$x + a - a = b - a$	
$x + 0 = b - a$	$a - a = 0$ and 0 is the additive identity. $x + 0 = x$.
$x = b - a$	This equation is equivalent to the first equation, and it is solved for x.

Solving $x - a = b$ for x

$x - a = b$	The a is associated with x by subtraction. Undo the association by adding a to *both* sides.
$x - a + a = b + a$	
$x + 0 = b + a$	$-a + a = 0$ and 0 is the additive identity. $x + 0 = x$.
$x = b + a$	This equation is equivalent to the first equation, and it is solved for x.

Method for Solving $x + a = b$ and $x - a = b$ for x

To solve the equation $x + a = b$ for x, *subtract a* from *both* sides of the equation.

To solve the equation $x - a = b$ for x, *add a* to *both* sides of the equation.

☆ SAMPLE SET B

1. Solve $x + 7 = 10$ for x.

$x + 7 = 10$	**7 is associated with x by addition. Undo the association by subtracting 7 from *both* sides.**
$x + 7 - 7 = 10 - 7$	
$x + 0 = 3$	**$7 - 7 = 0$ and 0 is the additive identity. $x + 0 = x$.**
$x = 3$	**x is isolated, and the equation $x = 3$ is equivalent to the original equation $x + 7 = 10$. Therefore, these two equations have the same solution. The solution to $x = 3$ is clearly 3. Thus, the solution to $x + 7 = 10$ is also 3.**

Check: Substitute 3 for x in the original equation.

$x + 7 = 10$
$3 + 7 \stackrel{?}{=} 10$
$10 \stackrel{\checkmark}{=} 10$

2. Solve $m - 2 = -9$ for m.

$m - 2 = -9$

$m - 2 + 2 = -9 + 2$

$m + 0 = -7$

$m = -7$

2 is associated with m by subtraction. Undo this association by adding 2 to *both* sides.
$-2 + 2 = 0$ and 0 is the additive identity. $m + 0 = m$.

Check: Substitute -7 for m in the original equation.

$m - 2 = -9$

$-7 - 2 \stackrel{?}{=} -9$

$-9 \stackrel{\checkmark}{=} -9$

3. Solve $y - 2.181 = -16.915$ for y.

$y - 2.181 = -16.915$

$y - 2.181 + 2.181 = -16.915 + 2.181$

$y = -14.734$

On the Calculator

Type	16.915
Press	$\boxed{+/-}$
Press	$\boxed{+}$
Type	2.181
Press	$\boxed{=}$
Display reads:	-14.734

4. Solve $y + m = s$ for y.

$y + m = s$

$y + m - m = s - m$

$y + 0 = s - m$

$y = s - m$

m is associated with y by addition. Undo the association by subtracting m from *both* sides.
$m - m = 0$ and 0 is the additive identity. $y + 0 = y$.

Check: Substitute $s - m$ for y in the original equation.

$y + m = s$

$s - m + m \stackrel{?}{=} s$

$s \stackrel{\checkmark}{=} s$ True

5. Solve $k - 3h = -8h + 5$ for k.

$k - 3h = -8h + 5$

$k - 3h + 3h = -8h + 5 + 3h$

$k + 0 = -5h + 5$

$k = -5h + 5$

$3h$ is associated with k by subtraction. Undo the association by adding $3h$ to *both* sides.
$-3h + 3h = 0$ and 0 is the additive identity. $k + 0 = k$.

★ PRACTICE SET B

1. Solve $y - 3 = 8$ for y.

2. Solve $x + 9 = -4$ for x.

3. Solve $m + 6 = 0$ for m.

4. Solve $g - 7.2 = 1.3$ for g.

5. Solve $f + 2d = 5d$ for f.

6. Solve $x + 8y = 2y - 1$ for x.

7. Solve $y + 4x - 1 = 5x + 8$ for y.

Answers to Practice Sets are on p. 139.

Section 4.1 EXERCISES

For problems 1–6, classify each of the equations as an identity, contradiction, or conditional equation.

1. $m + 6 = 15$
2. $y - 8 = -12$
3. $x + 1 = x + 1$
4. $k - 2 = k - 3$
5. $g + g + g + g = 4g$
6. $x + 1 = 0$

For problems 7–13, determine which of the literal equations have been solved for a variable. Write "solved" or "not solved."

7. $y = 3x + 7$
8. $m = 2k + n - 1$
9. $4a = y - 6$
10. $hk = 2k + h$
11. $2a = a + 1$
12. $5m = 2m - 7$
13. $m = m$

For problems 14–31, solve each of the conditional equations.

14. $h - 8 = 14$

15. $k + 10 = 1$

16. $m - 2 = 5$

17. $y + 6 = -11$

18. $y - 8 = -1$

19. $x + 14 = 0$

20. $m - 12 = 0$

21. $g + 164 = -123$

22. $h - 265 = -547$

23. $x + 17 = -426$

24. $h - 4.82 = -3.56$

25. $y + 17.003 = -1.056$

26. $k + 1.0135 = -6.0032$

27. Solve $n + m = 4$ for n.

28. Solve $P + 3Q - 8 = 0$ for P.

29. Solve $a + b - 3c = d - 2f$ for b.

30. Solve $x - 3y + 5z + 1 = 2y - 7z + 8$ for x.

31. Solve $4a - 2b + c + 11 = 6a - 5b$ for c.

EXERCISES FOR REVIEW

(1.6) 32. Simplify $(4x^5y^2)^3$.

(2.6) 33. Write $\frac{20x^3y^7}{5x^5y^3}$ so that only positive exponents appear.

(3.1) 34. Write the number of terms that appear in the expression $5x^2 + 2x - 6 + (a + b)$, and then list them.

(3.6) 35. Find the product. $(3x - 1)^2$.

(3.7) 36. Specify the domain of the equation $y = \frac{5}{x-2}$.

★ Answers to Practice Sets (4.1)

A. 1. conditional, $x = 9$ 2. conditional, $y = 11$ 3. conditional, $a = 5$ 4. conditional, $x = 36$ 5. conditional, $b = 3$ 6. identity 7. contradiction 8. identity 9. conditional, $x = 0$ 10. conditional, $m = 2$

B. 1. $y = 11$ 2. $x = -13$ 3. $m = -6$ 4. $g = 8.5$ 5. $f = 3d$ 6. $x = -6y - 1$ 7. $y = x + 9$

4.2 Solving Equations of the Form $ax = b$ and $\frac{x}{a} = b$

Section Overview

- ❑ EQUALITY PROPERTY OF DIVISION AND MULTIPLICATION
- ❑ SOLVING $ax = b$ AND $\frac{x}{a} = b$ FOR x

❑ EQUALITY PROPERTY OF DIVISION AND MULTIPLICATION

Recalling that the equal sign of an equation indicates that the number represented by the expression on the left side is the same as the number represented by the expression on the right side suggests the equality property of division and multiplication, which states:

1. We can obtain an equivalent equation by *dividing both sides* of the equation by the same nonzero number, that is, if $c \neq 0$, then $a = b$ is equivalent to $\frac{a}{c} = \frac{b}{c}$.
2. We can obtain an equivalent equation by *multiplying both sides* of the equation by the same nonzero number, that is, if $c \neq 0$, then $a = b$ is equivalent to $ac = bc$.

We can use these results to isolate x, thus solving the equation for x.

Solving $ax = b$ for x

$ax = b$ — a is associated with x by multiplication. Undo the association by dividing *both* sides by a.

$$\frac{ax}{a} = \frac{b}{a}$$

$$\frac{\cancel{a}x}{\cancel{a}} = \frac{b}{a}$$

$1 \cdot x = \frac{b}{a}$ — $\frac{a}{a} = 1$ and 1 is the multiplicative identity. $1 \cdot x = x$.

Solving $\frac{x}{a} = b$ for x

$x = \frac{b}{a}$ This equation is equivalent to the first and is solved for x.

$\frac{x}{a} = b$ a is associated with x by division. Undo the association by multiplying *both* sides by a.

$a \cdot \frac{x}{a} = a \cdot b$

$\cancel{a} \cdot \frac{x}{\cancel{a}} = ab$

$1 \cdot x = ab$ $\frac{a}{a} = 1$ and 1 is the multiplicative identity. $1 \cdot x = x$.

$x = ab$ This equation is equivalent to the first and is solved for x.

❑ SOLVING $ax = b$ AND $\frac{x}{a} = b$ FOR x

Method for Solving $ax = b$ and $\frac{x}{a} = b$

To solve $ax = b$ for x, *divide both sides* of the equation by a.

To solve $\frac{x}{a} = b$ for x, *multiply both sides* of the equation by a.

☆ SAMPLE SET A

1. Solve $5x = 35$ for x.

$5x = 35$ **5 is associated with x by multiplication. Undo the association by dividing *both* sides by 5.**

$\frac{5x}{5} = \frac{35}{5}$

$\frac{\cancel{5}x}{\cancel{5}} = 7$

$1 \cdot x = 7$ **$\frac{5}{5} = 1$ and 1 is the multiplicative identity. $1 \cdot x = x$.**

$x = 7$

Check: $5(7) \stackrel{?}{=} 35$
$35 \stackrel{\checkmark}{=} 35$

2. Solve $\frac{x}{4} = 5$ for x.

$\frac{x}{4} = 5$ **4 is associated with x by division. Undo the association by multiplying *both* sides by 4.**

$4 \cdot \frac{x}{4} = 4 \cdot 5$

$\cancel{4} \cdot \frac{x}{\cancel{4}} = 4 \cdot 5$

$1 \cdot x = 20$ — **$\frac{4}{4} = 1$ and 1 is the multiplicative identity. $1 \cdot x = x$.**

$x = 20$

Check: $\frac{20}{4} \stackrel{?}{=} 5$

$5 \stackrel{\checkmark}{=} 5$

3. Solve $\frac{2y}{9} = 3$ for y.

Method (1) (Use of cancelling):

$\frac{2y}{9} = 3$ — **9 is associated with y by division. Undo the association by multiplying *both* sides by 9.**

$(\cancel{9})\left(\frac{2y}{\cancel{9}}\right) = (9)(3)$

$2y = 27$ — **2 is associated with y by multiplication. Undo the association by dividing *both* sides by 2.**

$\frac{\cancel{2}y}{\cancel{2}} = \frac{27}{2}$

$y = \frac{27}{2}$

Check: $\frac{\cancel{2}\left(\frac{27}{\cancel{2}}\right)}{9} \stackrel{?}{=} 3$

$\frac{27}{9} \stackrel{?}{=} 3$

$3 \stackrel{\checkmark}{=} 3$

Method (2) (Use of reciprocals):

$\frac{2y}{9} = 3$ — **Since $\frac{2y}{9} = \frac{2}{9}y$, $\frac{2}{9}$ is associated with y by multiplication. Then, since $\frac{9}{2} \cdot \frac{2}{9} = 1$, the multiplicative identity, we can undo the association by multiplying *both* sides by $\frac{9}{2}$.**

$\left(\frac{9}{2}\right)\left(\frac{2y}{9}\right) = \left(\frac{9}{2}\right)(3)$

$\left(\frac{9}{2} \cdot \frac{2}{9}\right)y = \frac{27}{2}$

$1 \cdot y = \frac{27}{2}$

$y = \frac{27}{2}$

Continued

4. Solve the literal equation $\frac{4ax}{m} = 3b$ for x.

$\frac{4ax}{m} = 3b$

m is associated with *x* by division. Undo the association by multiplying *both* sides by *m*.

$\cancel{m}\left(\frac{4ax}{\cancel{m}}\right) = m \cdot 3b$

$4ax = 3bm$

4a is associated with *x* by multiplication. Undo the association by multiplying *both* sides by *4a*.

$\frac{\cancel{4a}x}{\cancel{4a}} = \frac{3bm}{4a}$

$x = \frac{3bm}{4a}$

Check: $\frac{4a\left(\frac{3bm}{4a}\right)}{m} \stackrel{?}{=} 3b$

$\frac{\cancel{4a}\left(\frac{3bm}{\cancel{4a}}\right)}{m} \stackrel{?}{=} 3b$

$\frac{3b\cancel{m}}{\cancel{m}} \stackrel{?}{=} 3b$

$3b \stackrel{\checkmark}{=} 3b$

★ PRACTICE SET A

1. Solve $6a = 42$ for a.

2. Solve $-12m = 16$ for m.

3. Solve $\frac{y}{8} = -2$ for y.

4. Solve $6.42x = 1.09$ for x.
   Round the result to two decimal places.

5. Solve $\frac{5k}{12} = 2$ for k.

6. Solve $\frac{-ab}{2c} = 4d$ for b.

7. Solve $\frac{3xy}{4} = 9xh$ for y.

8. Solve $\frac{2k^2mn}{5pq} = -6n$ for m.

Answers to the Practice Set are on p. 145.

Section 4.2 EXERCISES

In problems 1–43, solve each of the conditional equations.

1. $3x = 42$

2. $5y = 75$

3. $6x = 48$

4. $8x = 56$

5. $4x = 56$

6. $3x = 93$

7. $5a = -80$

8. $9m = -108$

9. $6p = -108$

10. $12q = -180$

11. $-4a = 16$

12. $-20x = 100$

13. $-6x = -42$

14. $-8m = -40$

15. $-3k = 126$

16. $-9y = 126$

17. $\frac{x}{6} = 1$

18. $\frac{a}{5} = 6$

19. $\frac{k}{7} = 6$

20. $\frac{x}{3} = 72$

21. $\frac{x}{8} = 96$

22. $\frac{y}{-3} = -4$

23. $\frac{m}{7} = -8$

24. $\frac{k}{18} = 47$

25. $\frac{f}{-62} = 103$

26. $3.06m = 12.546$

27. $5.012k = 0.30072$

28. $\frac{x}{2.19} = 5$

29. $\frac{y}{4.11} = 2.3$

30. $\frac{4y}{7} = 2$

31. $\frac{3m}{10} = -1$

32. $\frac{5k}{6} = 8$

33. $\frac{8h}{-7} = -3$

34. $\frac{-16z}{21} = -4$

35. Solve $pq = 7r$ for p.

36. Solve $m^2n = 2s$ for n.

37. Solve $2.8ab = 5.6d$ for b.

38. Solve $\frac{mnp}{2k} = 4k$ for p.

39. Solve $\frac{-8a^2b}{3c} = -5a^2$ for b.

40. Solve $\frac{3pcb}{2m} = 2b$ for pc.

41. Solve $\frac{8rst}{3p} = -2prs$ for t.

42. Solve $\dfrac{\square \cdot \star}{\triangle} = \diamond$ for $\square$.

43. Solve $\dfrac{3\square\triangle\nabla}{2\nabla} = \triangle\nabla$ for $\square$.

EXERCISES FOR REVIEW

(1.6) 44. Simplify $\left(\dfrac{2x^0y^0z^3}{z^2}\right)^5$.

(3.3) 45. Classify $10x^3 - 7x$ as a monomial, binomial, or trinomial. State its degree and write the numerical coefficient of each item.

(3.4) 46. Simplify $3a^2 - 2a + 4a(a + 2)$.

(3.7) 47. Specify the domain of the equation $y = \dfrac{3}{7 + x}$.

(4.1) 48. Solve the conditional equation $x + 6 = -2$.

★ Answers to Practice Set (4.2)

A. 1. $a = 7$ 2. $m = -\dfrac{4}{3}$ 3. $y = -16$ 4. $x = 0.17$ (rounded to two decimal places) 5. $k = \dfrac{24}{5}$

6. $b = \dfrac{-8cd}{a}$ 7. $y = 12h$ 8. $m = \dfrac{-15pq}{k^2}$

4.3 Further Techniques in Equation Solving

Section Overview

- ❑ COMBINING TECHNIQUES IN EQUATION SOLVING
- ❑ RECOGNIZING IDENTITIES AND CONTRADICTIONS

❑ COMBINING TECHNIQUES IN EQUATION SOLVING

In Sections 4.1 and 4.2 we worked with techniques that involved the use of addition, subtraction, multiplication, and division to solve equations. We can combine these techniques to solve more complicated equations. To do so, it is helpful to recall that an equation is solved for a particular variable when all other numbers and/or letters have been disassociated from it and it is alone on one side of the equal sign. We will also note that

> To associate numbers and letters we use the order of operations.
>
> 1. Multiply/divide
> 2. Add/subtract
>
> To undo an association between numbers and letters we use the order of operations in reverse.
>
> 1. Add/subtract
> 2. Multiply/divide

☆ SAMPLE SET A

1. Solve $4x - 7 = 9$ for x.

$4x - 7 = 9$

First, undo the association between x and 7. The 7 is associated with x by subtraction. Undo the association by adding 7 to *both* sides.

$4x - 7 + 7 = 9 + 7$

$4x = 16$

Now, undo the association between x and 4. The 4 is associated with x by multiplication. Undo the association by dividing *both* sides by 4.

$$\frac{\cancel{4}x}{\cancel{4}} = \frac{16}{4}$$

$16 - 7 \stackrel{?}{=} 9$

$x = 4$

Check: $4(4) - 7 \stackrel{?}{=} 9$

$9 \stackrel{\checkmark}{=} 9$

2. Solve $\dfrac{3y}{4} - 5 = -11$.

$$\frac{3y}{4} - 5 = -11$$

-5 is associated with y by subtraction. Undo the association by adding 5 to *both* sides.

$$\frac{3y}{4} - 5 + 5 = -11 + 5$$

$$\frac{3y}{4} = -6$$

4 is associated with y by division. Undo the association by multiplying *both* sides by 4.

$$4 \cdot \frac{3y}{4} = 4(-6)$$

$$\cancel{4} \cdot \frac{3y}{\cancel{4}} = 4(-6)$$

$3y = -24$

3 is associated with y by multiplication. Undo the association by dividing *both* sides by 3.

$$\frac{3y}{3} = \frac{-24}{3}$$

$$\frac{\cancel{3}y}{\cancel{3}} = -8$$

$y = -8$

Check: $\dfrac{3(-8)}{4} - 5 \stackrel{?}{=} -11$

$$\frac{-24}{4} - 5 \stackrel{?}{=} -11$$

$-6 - 5 \stackrel{?}{=} -11$

$-11 \stackrel{\checkmark}{=} -11$

3. Solve $\frac{8a}{3b} + 2m = 6m - 5$ for a.

$\frac{8a}{3b} + 2m = 6m - 5$

$2m$ is associated with a by addition. Undo the association by subtracting $2m$ from *both* sides.

$\frac{8a}{3b} + 2m - 2m = 6m - 5 - 2m$

$\frac{8a}{3b} = 4m - 5$

$3b$ is associated with a by division. Undo the association by multiplying *both* sides by $3b$.

$(3b)\left(\frac{8a}{3b}\right) = 3b(4m - 5)$

$8a = 12bm - 15b$

8 is associated with a by multiplication. Undo the multiplication by dividing *both* sides by 8.

$\frac{\cancel{8}a}{\cancel{8}} = \frac{12bm - 15b}{8}$

$a = \frac{12bm - 15b}{8}$

★ PRACTICE SET A

1. Solve $3y - 1 = 11$ for y.

2. Solve $\frac{5m}{2} + 6 = 1$ for m.

3. Solve $2n + 3m = 4$ for n.

4. Solve $\frac{9k}{2h} + 5 = p - 2$ for k.

Sometimes when solving an equation it is necessary to simplify the expressions composing it.

☆ SAMPLE SET B

1. Solve $4x + 1 - 3x = (-2)(4)$ for x.

$4x + 1 - 3x = (-2)(4)$
$x + 1 = -8$
$x = -9$

Check: $4(-9) + 1 - 3(-9) \stackrel{?}{=} -8$
$-36 + 1 + 27 \stackrel{?}{=} -8$
$-8 \stackrel{\checkmark}{=} -8$

Continued

2. Solve $3(m-6)-2m=-4+1$ for m.

$3(m-6)-2m=-4+1$
$3m-18-2m=-3$
$m-18=-3$
$m=15$

Check: $3(15-6)-2(15)\stackrel{?}{=}-4+1$
$3(9)-30\stackrel{?}{=}-3$
$27-30\stackrel{?}{=}-3$
$-3\stackrel{\checkmark}{=}-3$

★ PRACTICE SET B

Solve and check each equation.

1. $16x-3-15x=8$ for x.

2. $4(y-5)-3y=-1$ for y.

3. $-2(a^2+3a-1)+2a^2+7a=0$ for a.

4. $5m(m-2a-1)-5m^2+2a(5m+3)=10$ for a.

Often the variable we wish to solve for will appear on both sides of the equal sign. We can isolate the variable on either the left or right side of the equation by using the techniques of Sections 4.1 and 4.2.

☆ SAMPLE SET C

1. Solve $6x - 4 = 2x + 8$ for x.

$6x - 4 = 2x + 8$ — **To isolate x on the left side, subtract $2x$ from both sides.**

$6x - 4 - 2x = 2x + 8 - 2x$

$4x - 4 = 8$ — **Add 4 to both sides.**

$4x - 4 + 4 = 8 + 4$

$4x = 12$ — **Divide both sides by 4.**

$\frac{\cancel{4}x}{\cancel{4}} = \frac{12}{4}$

$x = 3$

Check: $6(3) - 4 \stackrel{?}{=} 2(3) + 8$

$18 - 4 \stackrel{?}{=} 6 + 8$

$14 \stackrel{\checkmark}{=} 14$

2. Solve $6(1 - 3x) + 1 = 2x - [3(x - 7) - 20]$ for x.

$6(1 - 3x) + 1 = 2x - [3(x - 7) - 20]$

$6 - 18x + 1 = 2x - [3x - 21 - 20]$

$-18x + 7 = 2x - [3x - 41]$

$-18x + 7 = 2x - 3x + 41$

$-18x + 7 = -x + 41$ — **To isolate x on the right side, add $18x$ to both sides.**

$-18x + 7 + 18x = -x + 41 + 18x$

$7 = 17x + 41$ — **Subtract 41 from both sides.**

$7 - 41 = 17x + 41 - 41$

$-34 = 17x$ — **Divide both sides by 17.**

$\frac{-34}{17} = \frac{\cancel{17}x}{\cancel{17}}$

$-2 = x$ — **Since the equation $-2 = x$ is equivalent to the equation $x = -2$, we can write the answer as $x = -2$.**

$x = -2$

Check: $6(1 - 3(-2)) + 1 \stackrel{?}{=} 2(-2) - [3(-2 - 7) - 20]$

$6(1 + 6) + 1 \stackrel{?}{=} -4 - [3(-9) - 20]$

$6(7) + 1 \stackrel{?}{=} -4 - [-27 - 20]$

$42 + 1 \stackrel{?}{=} -4 - [-47]$

$43 \stackrel{?}{=} -4 + 47$

$43 \stackrel{\checkmark}{=} 43$

★ PRACTICE SET C

1. Solve $8a + 5 = 3a - 5$ for a.

2. Solve $9y + 3(y + 6) = 15y + 21$ for y.

3. Solve $3k + 2[4(k - 1) + 3] = 63 - 2k$ for k.

RECOGNIZING IDENTITIES AND CONTRADICTIONS

As we noted in Section 4.1, some equations are identities and some are contradictions. As the problems of Sample Set D will suggest,

Recognizing an Identity

Recognizing a Contradiction

1. If, when solving an equation, all the variables are eliminated and a true statement results, the equation is an **identity.**
2. If, when solving an equation, all the variables are eliminated and a false statement results, the equation is a **contradiction.**

SAMPLE SET D

1. Solve $9x + 3(4 - 3x) = 12$ for x.

$9x + 3(4 - 3x) = 12$

$9x + 12 - 9x = 12$
$12 = 12$

The variable has been eliminated and the result is a true statement. The original equation is an *identity.*

2. Solve $-2(10 - 2y) - 4y + 1 = -18$ for y.

$-2(10 - 2y) - 4y + 1 = -18$

$-20 + 4y - 4y + 1 = -18$
$-19 \neq -18$

The variable has been eliminated and the result is a false statement. The original equation is a *contradiction.*

PRACTICE SET D

Classify each equation as an identity or a contradiction.

1. $6x + 3(1 - 2x) = 3$

2. $-8m + 4(2m - 7) = 28$

3. $3(2x - 4) - 2(3x + 1) + 14 = 0$

4. $-5(x + 6) + 8 = 3[4 - (x + 2)] - 2x$

Answers to Practice Sets are on p. 155.

Section 4.3 EXERCISES

For problems 1–37, solve each conditional equation. If the equation is not conditional, identify it as an identity or a contradiction.

1. $3x + 1 = 16$

2. $6y - 4 = 20$

3. $4a - 1 = 27$

4. $3x + 4 = 40$

5. $2y + 7 = -3$

6. $8k - 7 = -23$

7. $5x + 6 = -9$

8. $7a + 2 = -26$

9. $10y - 3 = -23$

10. $14x + 1 = -55$

11. $\frac{x}{9} + 2 = 6$

12. $\frac{m}{7} - 8 = -11$

13. $\frac{y}{4} + 6 = 12$

14. $\frac{x}{8} - 2 = 5$

15. $\frac{m}{11} - 15 = -19$

16. $\frac{k}{15} + 20 = 10$

17. $6 + \frac{k}{5} = 5$

18. $1 - \frac{n}{2} = 6$

19. $\frac{7x}{4} + 6 = -8$

20. $\frac{-6m}{5} + 11 = -13$

21. $\frac{3k}{14} + 25 = 22$

22. $3(x - 6) + 5 = -25$

23. $16(y - 1) + 11 = -85$

24. $6x + 14 = 5x - 12$

25. $23y - 19 = 22y + 1$

26. $-3m + 1 = 3m - 5$

27. $8k + 7 = 2k + 1$

28. $12n + 5 = 5n - 16$

29. $2(x - 7) = 2x + 5$

30. $-4(5y + 3) + 5(1 + 4y) = 0$

31. $3x + 7 = -3 - (x + 2)$

32. $4(4y + 2) = 3y + 2[1 - 3(1 - 2y)]$

33. $5(3x - 8) + 11 = 2 - 2x + 3(x - 4)$

34. $12 - (m - 2) = 2m + 3m - 2m + 3(5 - 3m)$

35. $-4 \cdot k - (-4 - 3k) = -3k - 2k - (3 - 6k) + 1$

36. $3[4 - 2(y + 2)] = 2y - 4[1 + 2(1 + y)]$

37. $-5[2m - (3m - 1)] = 4m - 3m + 2(5 - 2m) + 1$

For problems 38–50, solve the literal equations for the indicated variable. When directed, find the value of that variable for the given values of the other variables.

38. Solve $I = \frac{E}{R}$ for R. Find the value of R when $I = 0.005$ and $E = 0.0035$.

39. Solve $P = R - C$ for R. Find the value of R when $P = 27$ and $C = 85$.

40. Solve $z = \dfrac{x - \bar{x}}{s}$ for x. Find the value of x when $z = 1.96$, $s = 2.5$, and $\bar{x} = 15$.

41. Solve $F = \dfrac{S_x^2}{S_y^2}$ for S_x^2. S_x^2 represents a single quantity. Find the value of S_x^2 when $F = 2.21$ and $S_y^2 = 3.24$.

42. Solve $p = \dfrac{nRT}{V}$ for R.

43. Solve $x = 4y + 7$ for y.

44. Solve $y = 10x + 16$ for x.

45. Solve $2x + 5y = 12$ for y.

46. Solve $-9x + 3y + 15 = 0$ for y.

47. Solve $m = \dfrac{2n - h}{5}$ for n.

48. Solve $t = \dfrac{Q + 6P}{8}$ for P.

49. Solve $\star = \dfrac{\square + 9j}{\triangle}$ for j.

50. Solve $\diamond = \dfrac{\triangle + \star\square}{2\triangledown}$ for $\star$.

EXERCISES FOR REVIEW

(1.5) 51. Simplify $(x+3)^2(x-2)^3(x-2)^4(x+3)$.
(3.6) 52. Find the product. $(x-7)(x+7)$.
(3.6) 53. Find the product. $(2x-1)^2$.
(4.1) 54. Solve the equation $y-2=-2$.
(4.2) 55. Solve the equation $\frac{4x}{5}=-3$.

★ Answers to Practice Sets (4.3)

A. 1. $y=4$ 2. $m=-2$ 3. $n=\frac{4-3m}{2}$ 4. $k=\frac{2hp-14h}{9}$

B. 1. $x=11$ 2. $y=19$ 3. $a=-2$ 4. $a=\frac{10+5m}{6}$

C. 1. $a=-2$ 2. $y=-1$ 3. $k=5$

D. 1. identity, $3=3$ 2. contradiction, $-28=28$ 3. identity, $0=0$ 4. contradiction, $-22=6$

4.4 Applications I – Translating from Verbal to Mathematical Expressions

Section Overview

❑ TRANSLATING FROM VERBAL TO MATHEMATICAL EXPRESSIONS

❑ TRANSLATING FROM VERBAL TO MATHEMATICAL EXPRESSIONS

To solve a problem using algebra, we must first express the problem algebraically. To express a problem algebraically, we must scrutinize the wording of the problem to determine the variables and constants that are present and the relationships among them. Then we must translate the verbal phrases and statements to algebraic expressions and equations.

To help us translate verbal expressions to mathematics, we can use the following table as a mathematics dictionary.

MATHEMATICS DICTIONARY

Word or Phrase	Mathematical Operation
Sum, sum of, added to, increased by, more than, plus, and	$+$
Difference, minus, subtracted from, decreased by, less, less than	$-$
Product, the product of, of, multiplied by, times	$\cdot$
Quotient, divided by, ratio	$\div$
Equals, is equal to, is, the result is, becomes	$=$
A number, an unknown quantity, an unknown, a quantity	x (or any symbol)

☆ SAMPLE SET A

Translate the following phrases or sentences into mathematical expressions or equations.

1. Six more than a number.

$$\underbrace{\underbrace{\text{Six}}_{6}\ \underbrace{\text{more than}}_{+}\ \underbrace{\text{a number}}_{x}}_{6+x}$$

2. Fifteen minus a number.

$$\underbrace{\underbrace{\text{Fifteen}}_{15}\ \underbrace{\text{minus}}_{-}\ \underbrace{\text{a number}}_{x}}_{15-x}$$

3. A quantity less eight.

$$\underbrace{\underbrace{\text{A quantity}}_{y}\ \underbrace{\text{less}}_{-}\ \underbrace{\text{eight}}_{8}}_{y-8}$$

4. Twice a number is ten.

$$\underbrace{\underbrace{\text{Twice}}_{2\cdot}\ \underbrace{\text{a number}}_{x}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{ten}}_{10}}_{2x=10}$$

5. One half of a number is twenty.

$$\underbrace{\underbrace{\text{One half}}_{\frac{1}{2}}\ \underbrace{\text{of}}_{\cdot}\ \underbrace{\text{a number}}_{z}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{twenty}}_{20}}_{\frac{1}{2}z=20}$$

6. Three times a number is five more than twice the same number.

$$\underbrace{\underbrace{\text{Three}}_{3}\ \underbrace{\text{times}}_{\cdot}\ \underbrace{\text{a number}}_{y}\ \underbrace{\text{is}}_{=}\ \underbrace{\text{five}}_{5}\ \underbrace{\text{more than}}_{+}\ \underbrace{\text{twice}}_{2\cdot}\ \underbrace{\text{the same number}}_{y}}_{3y=5+2y}$$

★ PRACTICE SET A

Translate the following phrases or sentences into mathematical expressions or equations.

1. Eleven more than a number.

2. Nine minus a number.

3. A quantity less twenty.

4. Four times a number is thirty two.

5. One third of a number is six.

6. Ten times a number is eight more than five times the same number.

Sometimes the structure of the sentence indicates the use of grouping symbols.

☆ SAMPLE SET B

Translate the following phrases or sentences into mathematical expressions or equations.

1. A number divided by five, minus ten, is fifteen.

$$\underbrace{\underbrace{\text{A number divided by five,}}_{(x \ \div \ 5)} \ \underbrace{\text{minus}}_{-} \ \underbrace{\text{ten,}}_{10} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{fifteen.}}_{15}}_{\frac{x}{5} - 10 = 15}$$

Commas set off terms.

2. Eight divided by five more than a number is ten.

$$\underbrace{\underbrace{\text{Eight}}_{8} \ \underbrace{\text{divided by}}_{\div} \ \underbrace{\text{five more than a number}}_{(5 \ + \ x)}} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{ten.}}_{10}$$

The wording indicates this is to be considered as a single quantity.

$$\frac{8}{5+x} = 10$$

3. A number multiplied by ten more than itself is twenty.

$$\underbrace{\underbrace{\text{A number}}_{x} \ \underbrace{\text{multiplied by}}_{\cdot} \ \underbrace{\text{ten more than itself}}_{(10 \ + \ x)} \ \underbrace{\text{is}}_{=} \ \underbrace{\text{twenty.}}_{20}}_{x(10 + x) = 20}$$

4. A number plus one is divided by three times the number minus twelve and the result is four.

$(x + 1) \div (3 \cdot x - 12) = 4$

$$\frac{x+1}{3x-12} = 4$$

Notice that since the phrase "three times the number minus twelve" does not contain a comma, we get the expression $3x - 12$. If the phrase had appeared as "three times the number, minus twelve," the result would have been

$$\frac{x+1}{3x} - 12 = 4$$

5. Some phrases and sentences do not translate directly. We must be careful to read them properly. The word *from* often appears in such phrases and sentences. The word *from* means "a point of departure for motion." The following translation will illustrate this use.

$$\underbrace{\text{Fifteen}}_{15} \ \underbrace{\text{is subtracted } from}_{-} \ \underbrace{\text{some quantity.}}_{x}$$

$$x \quad - \quad 15$$

The word *from* indicates the motion (subtraction) is to begin at the point of "some quantity."

6. Eight less than some quantity. Notice that *less than* could be replaced with *from*.

$x - 8$

★ **PRACTICE SET B**

Translate the following phrases and sentences into mathematical expressions or equations.

1. A number divided by sixteen, plus one, is five.

2. Seven times two more than a number is twenty-one.

3. A number divided by two more than itself is zero.

4. A number minus five is divided by twice the number plus three and the result is seventeen.

5. Fifty-two is subtracted from some quantity.

6. An unknown quantity is subtracted from eleven and the result is five less than the unknown quantity.

Answers to Practice Sets are on p. 160.

Section 4.4 EXERCISES

For problems 1 – 50, translate the following phrases or sentences into mathematical expressions or equations.

1. A quantity less four.

2. Eight more than a number.

3. A number plus seven.

4. A number minus three.

5. Negative five plus an unknown quantity.

6. Negative sixteen minus some quantity.

7. Fourteen added to twice a number.

8. Ten added to three times some number.

9. One third minus an unknown quantity.

10. Twice a number is eleven.

11. Four ninths of a number is twenty-one.

12. One third of a number is two fifths.

13. Three times a number is nine more than twice the number.

14. Five times a number is that number minus two.

15. Twice a number added to six results in thirty.

16. Ten times a number less four results in sixty-six.

17. A number less twenty-five is equal to 3.019.

18. Seven more than some number is five more than twice the number.

19. When a number is divided by four, the result is sixty-eight.

20. Eleven fifteenths of two more than a number is eight.

21. One tenth of a number is that number less one.

22. Two more than twice a number is one half the number less three.

23. A number is equal to itself plus four times itself.

24. Three fifths of a quantity added to the quantity itself is thirty-nine.

25. A number plus seven is divided by two and the result is twenty-two.

26. Ten times a number minus one is divided by fourteen and the result is one.

27. A number is added to itself then divided by three. This result is then divided by three. The entire result is fifteen.

28. Ten divided by two more than a number is twenty-one.

29. Five divided by a number plus six is fourteen.

30. Twelve divided by twice a number is fifty-five.

31. Twenty divided by eight times a number added to one is nine.

32. A number divided by itself, plus one, results in seven.

33. A number divided by ten, plus four, results in twenty-four.

34. A number plus six, divided by two, is seventy-one.

35. A number plus six, divided by two, plus five, is forty-three.

36. A number multiplied by itself added to five is thirty-one.

37. A quantity multiplied by seven plus twice itself is ninety.

38. A number is increased by one and then multiplied by five times itself. The result is eighty-four.

39. A number is added to six and that result is multiplied by thirteen. This result is then divided by six times the number. The entire result is equal to fifty-nine.

40. A number is subtracted from ten and that result is multiplied by four. This result is then divided by three more than the number. The entire result is equal to six.

41. An unknown quantity is decreased by eleven. This result is then divided by fifteen. Now, one is subtracted from this result and five is obtained.

42. Ten less than some number.

43. Five less than some unknown number.

44. Twelve less than a number.

45. One less than an unknown quantity.

46. Sixteen less than some number is forty-two.

47. Eight less than some unknown number is three.

48. Seven is added to ten less than some number. The result is one.

49. Twenty-three is divided by two less than twice some number and the result is thirty-four.

50. One less than some number is multiplied by three less than five times the number and the entire result is divided by six less than the number. The result is twenty-seven less than eleven times the number.

EXERCISES FOR REVIEW

(1.2) 51. Supply the missing word. The point on a line that is associated with a particular number is called the ________ of that number.

(1.4) 52. Supply the missing word. An exponent records the number of identical ________ in a multiplication.

(2.2) 53. Write the algebraic definition of the absolute value of the number a.

(4.3) 54. Solve the equation $4y + 5 = -3$.

(4.3) 55. Solve the equation $2(3x + 1) - 5x = 4(x - 6) + 17$.

★ Answers to Practice Sets (4.4)

A. 1. $11 + x$ 2. $9 - x$ 3. $x - 20$ 4. $4x = 32$ 5. $\frac{x}{3} = 6$ 6. $10x = 8 + 5x$

B. 1. $\frac{x}{16} + 1 = 5$ 2. $7(2 + x) = 21$ 3. $\frac{x}{2 + x} = 0$ 4. $\frac{x - 5}{2x + 3} = 17$ 5. $x - 52$ 6. $11 - x = x - 5$

4.5 Applications II – Solving Problems

Section Overview

☐ SOLVING APPLIED PROBLEMS

☐ SOLVING APPLIED PROBLEMS

Let's study some interesting problems that involve linear equations in one variable. In order to solve such problems, we apply the following five-step method:

Five-Step Method for Solving Word Problems

1. Let x (or some other letter) represent the unknown quantity.
2. Translate the words to mathematical symbols and form an equation.
3. Solve this equation.
4. Ask yourself, "Does this result seem reasonable?" Check the solution by substituting the result into the original statement of the problem.
   If the answer doesn't check, you have either solved the equation incorrectly, or you have developed the wrong equation. Check your method of solution first. If the result does not check, reconsider your equation.
5. Write the conclusion.

If it has been your experience that word problems are difficult, then follow the five-step method carefully. Most people have difficulty because they neglect step 1.

Always start by INTRODUCING A VARIABLE!

Keep in mind what the variable is representing throughout the problem.

☆ SAMPLE SET A

This year an item costs \$44, an increase of \$3 over last year's price. What was last year's price?

Step 1: Let $x =$ last year's price.
Step 2: $x + 3 = 44$. **$x + 3$ represents the \$3 increase in price.**

Step 3:
$$x + 3 = 44$$
$$x + 3 - 3 = 44 - 3$$
$$x = 41$$

Step 4: $41 + 3 \stackrel{?}{=} 44$.
Step 5: Last year's price was \$41.

★ PRACTICE SET A

This year an item costs \$23, an increase of \$4 over last year's price. What was last year's price?

Step 1: Let $x =$
Step 2:

Step 3:

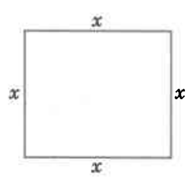
Step 4:

Step 5: Last year's price was ________.

☆ SAMPLE SET B

The perimeter (length around) of a square is 60 cm (centimeters). Find the length of a side.

Step 1: Let $x =$ length of a side.
Step 2: We can draw a picture.



$$x + x + x + x = 60$$

Continued

Step 3: $x + x + x + x = 60$
$4x = 60$ **Divide both sides by 4.**
$x = 15.$

Step 4: $4(15) \stackrel{\checkmark}{=} 60.$
Step 5: The length of a side is 15 cm.

★ PRACTICE SET B

The perimeter of a triangle is 54 inches. If each side has the same length, find the length of a side.

Step 1: Let $x =$
Step 2:

Step 3:

Step 4:

Step 5: The length of a side is ________ inches.

☆ SAMPLE SET C

Six percent of a number is 54. What is the number?

Step 1: Let $x =$ the number
Step 2: We must convert 6% to a decimal.

$6\% = .06$
$.06x = 54$ **$.06x$ occurs because we want 6% of x.**

Step 3: $.06x = 54.$ **Divide both sides by .06.**

$$x = \frac{54}{.06}$$

$x = 900$

Step 4: $.06(900) \stackrel{\checkmark}{=} 54.$
Step 5: The number is 900.

★ PRACTICE SET C

Eight percent of a number is 36. What is the number?

Step 1: Let $x =$
Step 2:

Step 3:

Step 4:

Step 5: The number is ________.

☆ SAMPLE SET D

An astronomer notices that one star gives off about 3.6 times as much energy as another star. Together the stars give off 55.844 units of energy. How many units of energy does each star emit?

Step 1: In this problem we have two unknowns and, therefore, we might think, two variables. However, notice that the energy given off by one star is given in terms of the other star. So, rather than introducing two variables, we introduce only one. The other unknown(s) is expressed in terms of this one. (We might call this quantity the base quantity.)

Let x = number of units of energy given off by the less energetic star. Then, $3.6x$ = number of units of energy given off by the more energetic star.

Step 2: $x + 3.6x = 55.844$.

Step 3: $x + 3.6x = 55.844$

$4.6x = 55.844$ **Divide both sides by 4.6. A calculator would be useful at this point.**

$$x = \frac{55.844}{4.6}$$

$x = 12.14$ **The wording of the problem implies *two* numbers are needed for a complete solution. We need the number of units of energy for the other star.**

$$3.6x = 3.6(12.14) = 43.704$$

Step 4: $12.14 + 43.704 \stackrel{?}{=} 55.844$.

Step 5: One star gives off 12.14 units of energy and the other star gives off 43.704 units of energy.

★ PRACTICE SET D

Garden A produces 5.8 times as many vegetables as garden B. Together the gardens produce 102 pounds of vegetables. How many pounds of vegetables does garden A produce?

Step 1: Let x =

Step 2:

Step 3:

Step 4:

Step 5:

☆ SAMPLE SET E

Two consecutive even numbers sum to 432. What are the two numbers?

Step 1: Let $x =$ the smaller even number. Then $x + 2 =$ the next (consecutive) even number since consecutive even numbers differ by 2 (as do consecutive odd numbers).

Step 2: $x + x + 2 = 432$.

Step 3:
$$x + x + 2 = 432$$
$$2x + 2 = 432$$
$$2x = 430$$
$x = 215$. Also, since $x = 215$, $x + 2 = 217$.

Step 4: $215 + 217 = 432$, but 215 and 217 are odd numbers and we are looking for even numbers. Upon checking our method of solution and reexamining our equation, we find no mistakes.

Step 5: We must conclude that this problem has no solution. There are no two consecutive *even* numbers that sum to 432.

★ PRACTICE SET E

The sum of two consecutive even numbers is 498. What are the two numbers?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Answers to Practice Sets are on p. 170.

Section 4.5 EXERCISES

Solve problems 1–33. Note that some of the problems may seem to have no practical applications and may not seem very interesting. They, along with the other problems, will, however, help to develop your logic and problem-solving ability.

1. If eighteen is subtracted from some number the result is fifty-two. What is the number?

   Step 1: Let $x =$

   Step 2: The equation is

   Step 3: (Solve the equation.)

   Step 4: (Check)

   Step 5: The number is ________.

2. If nine more than twice a number is forty-six, what is the number?

Step 1: Let $x =$
Step 2: The equation is

Step 3: (Solve the equation.)

Step 4: (Check)

Step 5: The number is ________.

3. If nine less than three eighths of a number is two and one fourth, what is the number?

Step 1: Let $x =$
Step 2:

Step 3:

Step 4:

Step 5: The number is ________.

4. Twenty percent of a number is 68. What is the number?

Step 1: Let $x =$
Step 2:

Step 3:

Step 4:

Step 5: The number is ________.

5. Eight more than a quantity is 37. What is the original quantity?

Step 1: Let $x =$
Step 2:

Step 3:

Step 4:

Step 5: The original quantity is ________.

6. If a quantity plus 85% more of the quantity is 62.9, what is the original quantity?

Step 1: Let $x =$ original quantity.
Step 2: $\underbrace{x}_{\text{original quantity}} + \underbrace{.85x}_{\text{85\% more}} = 62.9$

Step 3:

Step 4:

Step 5: The original quantity is ________.

7. A company must increase production by 12% over last year's production. The new output will be 56 items. What was last year's output?

Step 1: Let $P =$
Step 2:

Step 3:

Step 4:

Step 5: Last year's output was ________ items.

8. A company has determined that it must increase production of a certain line of goods by $1\frac{1}{2}$ times last year's production. The new output will be 2885 items. What was last year's output?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: Last year's output was ________ items.

9. A proton is about 1837 times as heavy as an electron. If an electron weighs 2.68 units, how many units does a proton weigh?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: A proton weighs ________ units.

10. Neptune is about 30 times as far from the sun as is the Earth. If it takes light 8 minutes to travel from the sun to the Earth, how many minutes does it take to travel to Neptune?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: Light takes ________ minutes to reach Neptune.

11. The radius of the sun is about 695,202 km (kilometers). That is about 109 times as big as the radius of the Earth. What is the radius of the earth?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: The radius of the earth is ________ km.

12. The perimeter of a triangle is 105 cm. If each of the two legs is exactly twice the length of the base, how long is each leg?

Step 1: Let x =
Draw a picture.

Step 2:

Step 3:

Step 4:

Step 5: Each leg is ________ cm long. The base is ________.

13. A lumber company has contracted to cut boards into two pieces so that one piece is three times the length of the other piece. If a board is 12 feet long, what is the length of each piece after cutting?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: The length of the shorter piece is ________ feet, and the length of the longer piece is ________ feet.

14. A student doing a chemistry experiment has a beaker that contains 84 ml (milliliters) of an alcohol and water solution. Her lab directions tell her that there is 4.6 times as much water as alcohol in the solution. How many milliliters of alcohol are in the solution? How many milliliters of water?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: There are ________ ml of alcohol in the solution. There are ________ ml of water in the solution.

15. A statistician is collecting data to help him estimate the average income of accountants in California. He needs to collect 390 pieces of data and he is $\frac{2}{3}$ done. How many pieces of data has the statistician collected?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: The statistician has collected ________ pieces of data.

Suppose the statistician is 4 pieces of data short of being $\frac{2}{3}$ done. How many pieces of data has he collected?

16. A television commercial advertises that a certain type of battery will last, on the average, 20 hours longer than twice the life of another type of battery. If consumer tests show that the advertised battery lasts 725 hours, how many hours must the other type of battery last for the advertiser's claim to be valid?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: The other type of battery must last ________ hours for the advertiser's claim to be valid.

17. A 1000-ml flask containing a chloride solution will fill 3 beakers of the same size with 210 ml of the solution left over. How many milliliters of the chloride solution will each beaker hold?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: Each beaker will hold ________ ml of the chloride solution.

18. A star burns $\frac{2}{9}$ of its original mass then blows off $\frac{3}{7}$ of the remaining mass as a planetary nebula. If the final mass is 3 units of mass, what was the original mass?

Step 1:
Step 2:

Step 3:

Step 4:

Step 5: The original mass was ________ units of mass.

Continue using the five-step procedure for problems 19–33.

19. The sum of a number and sixteen is forty-two. What is the number?

20. When eleven is subtracted from a number, the result is 85. What is the number?

21. Three times a number is divided by 6 and the result is 10.5. What is the number?

22. When a number is multiplied by itself, the result is 144. What is the number?

23. A number is tripled, then increased by seven. The result is 48. What is the number?

24. Eight times a number is decreased by three times the number, giving a difference of 22. What is the number?

25. One number is fifteen more than another number. The sum of the two numbers is 27. What are they?

26. The length of a rectangle is 6 meters more than three times the width. The perimeter of the rectangle is 44 meters. What are the dimensions of the rectangle?

27. Seven is added to the product of 41 and some number. The result, when divided by four, is 63. What is the number?

28. The second side of a triangle is five times the length of the smallest side. The third is twice the length of the second side. The perimeter of the triangle is 48 inches. Find the length of each side.

29. Person A is four times as old as person B, who is six times as old as person C, who is twice as old as person D. How old is each person if their combined ages are 189 months?

30. Two consecutive odd integers sum to 151. What are they?

31. Three consecutive integers sum to 36. What are they?

32. Three consecutive even integers add up to 131. What are they?

33. As a consequence of Einstein's theory of relativity, the rate of time passage is different for a person in a stationary position and a person in motion. (Hard to believe, but true!) To the moving observer, the rate of time passage is slower than that of the stationary observer, that is, the moving person ages slower than the stationary observer. (This fact has been proven many times by experiments with radioactive materials.) The effect is called "time dilation" and is really only noticeable when an object is traveling at near the speed of light (186,000 miles per second). Considering these ideas, try to solve the following problems:

Two people have identical clocks. One is standing on the earth and the other is moving in a spacecraft at 95% the speed of light, 176,700 miles per second. The moving person's rate of time passage at this speed is about 0.31 times as fast as the person standing on earth.

(a) If two days of earth time pass, how many days actually pass on the spacecraft?

(b) If 30 years of earth time pass, how many years have actually passed on the spacecraft?

Step 1:

Step 2:

Step 3:

Step 4:

Step 5: ________ years have passed on the spacecraft.

(c) If 30 years have passed on the spacecraft, how many years have passed on the earth?

(d) A space traveler makes a round-trip voyage to the star Capella. The trip takes her 120 years (traveling at 176,000 miles per second). If it is the year 2000 on earth when she leaves, what earth year will it be when she returns?

EXERCISES FOR REVIEW

(3.7) 34. Specify the domain of the equation $y = \frac{x-1}{x+4}$.

(4.1) 35. Classify the equation $x + 4 = 1$ as an identity, a contradiction, or a conditional equation.

(4.1) 36. Classify the equation $2x + 3 = 2x + 3$ as an identity, a contradiction, or a conditional equation.

(4.3) 37. Solve the equation $4(x - 1) + 12 = -3(2x + 4)$.

(4.4) 38. Translate the following sentence to a mathematical equation. Three less than an unknown number is multiplied by negative four. The result is two more than the original unknown number.

★ Answers to Practice Sets (4.5)

A. Last year's price was $19

B. The length of a side is 18 inches.

C. The number is 450.

D. Garden A produces 87 pounds of vegetables.

E. The two numbers are 248 and 250.

4.6 Linear Inequalities in One Variable

Section Overview

- INEQUALITIES
- LINEAR INEQUALITIES
- THE ALGEBRA OF LINEAR INEQUALITIES
- COMPOUND INEQUALITIES

INEQUALITIES

Relationships of Inequality

We have discovered that an equation is a mathematical way of expressing the relationship of equality between quantities. Not all relationships need be relationships of equality, however. Certainly the number of human beings on earth is greater than 20. Also, the average American consumes less than 10 grams of vitamin C every day. These types of relationships are not relationships of equality, but rather, relationships of **inequality.**

LINEAR INEQUALITIES

Linear Inequality

A **linear inequality** is a mathematical statement that one linear expression is greater than or less than another linear expression.

Inequality Notation

The following notation is used to express relationships of inequality:

$>$ Strictly greater than
$<$ Strictly less than
$\geq$ Greater than or equal to
$\leq$ Less than or equal to

Note that the expression $x > 12$ has infinitely many solutions. Any number strictly greater than 12 will satisfy the statement. Some solutions are 13, 15, 90, 12.1, 16.3, and 102.51.

☆ SAMPLE SET A

The following *are* linear inequalities in one variable.

1. $x \leq 12$

2. $x + 7 > 4$

3. $y + 3 \geq 2y - 7$

4. $P + 26 < 10(4P - 6)$

5. $\dfrac{2r - 9}{5} > 15$

The following *are not* linear inequalities in one variable.

6. $x^2 < 4$. The term x^2 is quadratic, not linear.

7. $x \leq 5y + 3$. There are two variables. This is a linear inequality in two variables.

8. $y + 1 \neq 5$. Although the symbol $\neq$ certainly expresses an inequality, it is customary to use only the symbols $<, >, \leq, \geq$.

★ PRACTICE SET A

A linear equation, we know, may have exactly one solution, infinitely many solutions, or no solution. Speculate on the number of solutions of a linear inequality. (*Hint:* Consider the inequalities $x < x - 6$ and $x \geq 9$.)

A linear inequality may have ________ solutions, or no solutions.

☐ THE ALGEBRA OF LINEAR INEQUALITIES

Inequalities can be solved by basically the same methods as linear equations. There is one important exception that we will discuss in item 3 of the algebra of linear inequalities.

THE ALGEBRA OF LINEAR INEQUALITIES

Let a, b, and c represent real numbers and assume that

$a < b$ (or $a > b$)

Then, if $a < b$,

1. $a + c < b + c$ and $a - c < b - c$.

   If any real number is added to or subtracted from both sides of an inequality, the sense of the inequality remains unchanged.
2. If c is a *positive* real number, then if $a < b$,

   $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

   If both sides of an inequality are multiplied or divided by the same positive number the sense of the inequality remains unchanged.
3. If c is a *negative* real number, then if $a < b$,

   $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.

   If both sides of an inequality are multiplied or divided by the same *negative* number, **the inequality sign must be reversed** (change direction) in order for the resulting inequality to be equivalent to the original inequality. (See problem 4 in the next set of examples.)

For example, consider the inequality $3 < 7$.

1. For $3 < 7$, if 8 is added to both sides, we get

   $3 + 8 < 7 + 8$.
   $11 < 15$ True
2. For $3 < 7$, if 8 is subtracted from both sides, we get

   $3 - 8 < 7 - 8$
   $-5 < -1$ True
3. For $3 < 7$, if both sides are multiplied by 8 (a positive number), we get

   $8(3) < 8(7)$
   $24 < 56$ True
4. For $3 < 7$, if both sides are multiplied by -8 (a negative number), we get

   $(-8)3 > (-8)7$

Notice the change in direction of the inequality sign.

$-24 > -56$ True

If we had forgotten to reverse the direction of the inequality sign we would have obtained the incorrect statement $-24 < -56$.

5. For $3 < 7$, if both sides are divided by 8 (a positive number), we get

$\frac{3}{8} < \frac{7}{8}$ True

6. For $3 < 7$, if both sides are divided by -8 (a negative number), we get

$\frac{3}{-8} > \frac{7}{-8}$ True (since $-.375 > -.875$)

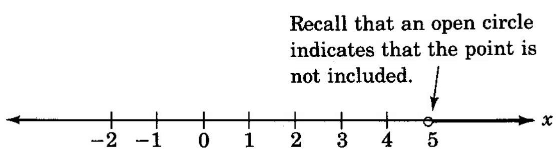
☆ SAMPLE SET B

Solve the following linear inequalities. Draw a number line and place a point at each solution.

1. $3x > 15$ **Divide both sides by 3. The 3 is a positive number, so we need not reverse the sense of the inequality.**

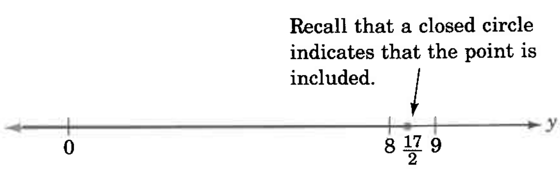
$x > 5$

Thus, all numbers strictly greater than 5 are solutions to the inequality $3x > 15$.



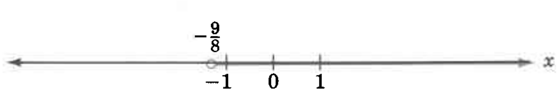
2. $2y - 1 \le 16$ **Add 1 to both sides.**
$2y \le 17$ **Divide both sides by 2.**

$y \le \frac{17}{2}$



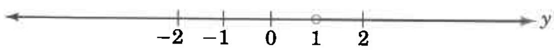
3. $-8x + 5 < 14$ **Subtract 5 from both sides.**
$-8x < 9$ **Divide both sides by -8. We must reverse the sense of the inequality since we are dividing by a negative number.**

$x > -\frac{9}{8}$



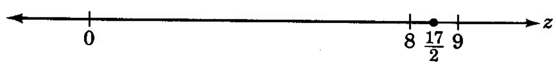
Continued

4. $5 - 3(y + 2) < 6y - 10$
$5 - 3y - 6 < 6y - 10$
$-3y - 1 < 6y - 10$
$-9y < -9$
$y > 1$



5. $\dfrac{2z + 7}{-4} \ge -6$ **Multiply by −4**

$2z + 7 \le 24$ **Notice the change in the sense of the inequality.**
$2z \le 17$
$z \le \dfrac{17}{2}$



★ PRACTICE SET B

Solve the following linear inequalities.

1. $y - 6 \le 5$

2. $x + 4 > 9$

3. $4x - 1 \ge 15$

4. $-5y + 16 \le 7$

5. $7(4s - 3) < 2s + 8$

6. $5(1 - 4h) + 4 < (1 - h)2 + 6$

7. $18 \ge 4(2x - 3) - 9x$

8. $-\dfrac{3b}{16} \le 4$

9. $\dfrac{-7z + 10}{-12} < -1$

10. $-x - \dfrac{2}{3} \le \dfrac{5}{6}$

❑ COMPOUND INEQUALITIES

Compound Inequality

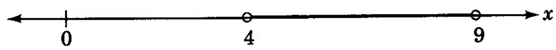
Another type of inequality is the *compound inequality.* A compound inequality is of the form:

$a < x < b$

There are actually two statements here. The first statement is $a < x$. The next statement is $x < b$. When we read this statement we say "a is less than x," then continue saying "and x is less than b."

Just by looking at the inequality we can see that the number x is between the numbers a and b. The compound inequality $a < x < b$ indicates "betweenness." Without changing the meaning, the statement $a < x$ can be read $x > a$. (Surely, if the number a is less than the number x, the number x must be greater than the number a.) Thus, we can read $a < x < b$ as "x is greater than a and at the same time is less than b." For example:

1. $4 < x < 9$. The letter x is some number strictly between 4 and 9. Hence, x is greater than 4 and, at the same time, less than 9. The numbers 4 and 9 are not included so we use open circles at these points.



2. $-2 < z < 0$. The z stands for some number between -2 and 0. Hence, z is greater than -2 but also less than 0.

3. $1 < x + 6 < 8$. The expression $x + 6$ represents some number strictly between 1 and 8. Hence, $x + 6$ represents some number strictly greater than 1, but less than 8.

4. $\frac{1}{4} \le \frac{5x-2}{6} \le \frac{7}{9}$. The term $\frac{5x-2}{6}$ represents some number between and including $\frac{1}{4}$ and $\frac{7}{9}$. Hence, $\frac{5x-2}{6}$ represents some number greater than or equal to $\frac{1}{4}$ but less than or equal to $\frac{7}{9}$.

Consider problem 3 above, $1 < x + 6 < 8$. The statement says that the quantity $x + 6$ is between 1 and 8. This statement will be true for only certain values of x. For example, if $x = 1$, the statement is true since $1 < 1 + 6 < 8$. However, if $x = 4.9$, the statement is false since $1 < 4.9 + 6 < 8$ is clearly not true. The first of the inequalities is satisfied since 1 is less than 10.9, but the second inequality is not satisfied since 10.9 is not less than 8.

We would like to know for exactly which values of x the statement $1 < x + 6 < 8$ is true. We proceed by using the properties discussed earlier in this section, but now we must apply the rules to all *three* parts rather than just the two parts in a regular inequality.

☆ SAMPLE SET C

1. Solve $1 < x + 6 < 8$.

$1 - 6 < x + 6 - 6 < 8 - 6$ **Subtract 6 from all three parts.**
$-5 < x < 2$

Thus, if x is any number strictly between -5 and 2, the statement $1 < x + 6 < 8$ will be true.

Continued

2. Solve $-3 < \frac{-2x-7}{5} < 8$.

$-3(5) < \frac{-2x-7}{5}(5) < 8(5)$	**Multiply each part by 5.**
$-15 < -2x - 7 < 40$	**Add 7 to all three parts.**
$-8 < -2x < 47$	**Divide all three parts by -2.**
$4 > x > -\frac{47}{2}$	**Remember to reverse the direction of the inequality signs.**
$-\frac{47}{2} < x < 4$	**It is customary (but not necessary) to write the inequality so that inequality arrows point to the left.**

Thus, if x is any number between $-\frac{47}{2}$ and 4, the original inequality will be satisfied.

★ PRACTICE SET C

Find the values of x that satisfy the given continued inequality.

1. $4 < x - 5 < 12$
2. $-3 < 7y + 1 < 18$
3. $0 \le 1 - 6x \le 7$
4. $-5 \le \frac{2x+1}{3} \le 10$
5. $9 < \frac{-4x+5}{-2} < 14$
6. Does $4 < x < -1$ have a solution?

Answers to Practice Sets are on p. 179.

Section 4.6 EXERCISES

For problems 1–50, solve the inequalities.

1. $x + 7 < 12$
2. $y - 5 \le 8$
3. $y + 19 \ge 2$
4. $x - 5 > 16$
5. $3x - 7 \le 8$
6. $9y - 12 \le 6$
7. $2z + 8 < 7$
8. $4x - 14 > 21$
9. $-5x \le 20$
10. $-8x < 40$
11. $-7z < 77$
12. $-3y > 39$

13. $\frac{x}{4} \geq 12$

14. $\frac{y}{7} > 3$

15. $\frac{2x}{9} \geq 4$

16. $\frac{5y}{2} \geq 15$

17. $\frac{10x}{3} \leq 4$

18. $\frac{-5y}{4} < 8$

19. $\frac{-12b}{5} < 24$

20. $\frac{-6a}{7} \leq -24$

21. $\frac{8x}{-5} > 6$

22. $\frac{14y}{-3} \geq -18$

23. $\frac{21y}{-8} < -2$

24. $-3x + 7 \leq -5$

25. $-7y + 10 \leq -4$

26. $6x - 11 < 31$

27. $3x - 15 \leq 30$

28. $-2y + \frac{4}{3} \leq -\frac{2}{3}$

29. $5(2x - 5) \geq 15$

30. $4(x + 1) > -12$

31. $6(3x - 7) \geq 48$

32. $3(-x + 3) > -27$

33. $-4(y + 3) > 0$

34. $-7(x - 77) \leq 0$

35. $2x - 1 < x + 5$

36. $6y + 12 \leq 5y - 1$

37. $3x + 2 \leq 2x - 5$

38. $4x + 5 > 5x - 11$

39. $3x - 12 \geq 7x + 4$

40. $-2x - 7 > 5x$

41. $-x - 4 > -3x + 12$

42. $3 - x \geq 4$

43. $5 - y \leq 14$

44. $2 - 4x \leq -3 + x$

45. $3[4 + 5(x + 1)] < -3$

46. $2[6 + 2(3x - 7)] \geq 4$

47. $7[-3-4(x-1)] \leq 91$

48. $-2(4x-1) < 3(5x+8)$

49. $-5(3x-2) > -3(-x-15)+1$

50. $-.0091x \geq 2.885x - 12.014$

51. What numbers satisfy the condition: twice a number plus one is greater than negative three?

52. What numbers satisfy the condition: eight more than three times a number is less than or equal to fourteen?

53. One number is five times larger than another number. The difference between these two numbers is less than twenty-four. What are the largest possible values for the two numbers? Is there a smallest possible value for either number?

54. The area of a rectangle is found by multiplying the length of the rectangle by the width of the rectangle. If the length of a rectangle is 8 feet, what is the largest possible measure for the width if it must be an integer (positive whole number) and the area must be less than 48 square feet?

EXERCISES FOR REVIEW

(1.6) 55. Simplify $(x^2y^3z^2)^5$.
(2.2) 56. Simplify $-[-(-|-8|)]$.
(3.5) 57. Find the product. $(2x-7)(x+4)$.
(4.5) 58. Twenty-five percent of a number is 12.32. What is the number?
(4.5) 59. The perimeter of a triangle is 40 inches. If the length of each of the two legs is exactly twice the length of the base, how long is each leg?

★ Answers to Practice Sets (4.6)

A. infinitely many

B. 1. $y \leq 11$ 2. $x > 5$ 3. $x \geq 4$ 4. $y \geq \frac{9}{5}$ 5. $s < \frac{29}{2}$ 6. $h > \frac{1}{18}$ 7. $x \geq -30$
8. $b \geq \frac{-64}{3}$ 9. $z < -\frac{2}{7}$ 10. $x \geq \frac{-3}{2}$

C. 1. $9 < x < 17$ 2. $-\frac{4}{7} < y < \frac{17}{7}$ 3. $-1 \leq x \leq \frac{1}{6}$ 4. $-8 \leq x \leq \frac{29}{2}$ 5. $\frac{23}{4} < x < \frac{33}{4}$ 6. no

4.7 Linear Equations in Two Variables

Section Overview

- ❑ SOLUTIONS TO LINEAR EQUATIONS IN TWO VARIABLES
- ❑ ORDERED PAIRS AS SOLUTIONS

❑ SOLUTIONS TO LINEAR EQUATIONS IN TWO VARIABLES

Solution to an Equation in Two Variables

We have discovered that an equation is a mathematical way of expressing the relationship of equality between quantities. If the relationship is between two quantities, the equation will contain two variables. We say that an equation in two variables has a solution if an ordered *pair* of values can be found such that when these two values are substituted into the equation a true statement results. This is illustrated when we observe some solutions to the equation $y = 2x + 5$.

1. $x = 4, y = 13$; since $13 = 2(4) + 5$ is true.
2. $x = 1, y = 7$; since $7 = 2(1) + 5$ is true.
3. $x = 0, y = 5$; since $5 = 2(0) + 5$ is true.
4. $x = -6, y = -7$; since $-7 = 2(-6) + 5$ is true.

❑ ORDERED PAIRS AS SOLUTIONS

It is important to keep in mind that a solution to a linear equation in two variables is an ordered pair of values, one value for each variable. A solution is not completely known until the values of *both* variables are specified.

Independent and Dependent Variables

Recall that, in an equation, any variable whose value can be freely assigned is said to be an **independent variable.** Any variable whose value is determined once the other value or values have been assigned is said to be a **dependent variable.** If, in a linear equation, the independent variable is x and the dependent variable is y, and a solution to the equation is $x = a$ and $y = b$, the solution is written as the

ORDERED PAIR	(a, b)

Ordered Pair

In an **ordered pair,** (a, b), the first component, a, gives the value of the independent variable, and the second component, b, gives the value of the dependent variable.

(a, b)

Value of the independent variable — First component

Value of the dependent variable — Second component

We can use ordered pairs to show some solutions to the equation $y = 6x - 7$.

1. $(0, -7)$. If $x = 0$ and $y = -7$, we get a true statement upon substitution and computation.

$$
\begin{aligned}
y &= 6x - 7 \\
-7 &\stackrel{?}{=} 6(0) - 7 \\
-7 &\stackrel{\checkmark}{=} -7 \quad \text{True}
\end{aligned}
$$

2. (8, 41). If $x = 8$ and $y = 41$, we get a true statement upon substitution and computation.

$$\begin{aligned} y &= 6x - 7 \\ 41 &\overset{?}{=} 6(8) - 7 \\ 41 &\overset{?}{=} 48 - 7 \\ 41 &\overset{\checkmark}{=} 41 \qquad \text{True} \end{aligned}$$

3. (−4, −31). If $x = -4$ and $y = -31$, we get a true statement upon substitution and computation.

$$\begin{aligned} y &= 6x - 7 \\ -31 &\overset{?}{=} 6(-4) - 7 \\ -31 &\overset{?}{=} -24 - 7 \\ -31 &\overset{\checkmark}{=} -31 \qquad \text{True} \end{aligned}$$

These are only three of the infinitely many solutions to this equation.

☆ SAMPLE SET A

Find a solution to each of the following linear equations in two variables and write the solution as an ordered pair.

1. $y = 3x - 6$, if $x = 1$

Substitute 1 for x, compute, and solve for y.

$$\begin{aligned} y &= 3(1) - 6 \\ &= 3 - 6 \\ &= -3 \end{aligned}$$

Hence, one solution is $(1, -3)$.

2. $y = 15 - 4x$, if $x = -10$

Substitute −10 for x, compute, and solve for y.

$$\begin{aligned} y &= 15 - 4(-10) \\ &= 15 + 40 \\ &= 55 \end{aligned}$$

Hence, one solution is $(-10, 55)$.

3. $b = -9a + 21$, if $a = 2$

Substitute 2 for a, compute, and solve for b.

$$\begin{aligned} b &= -9(2) + 21 \\ &= -18 + 21 \\ &= 3 \end{aligned}$$

Hence, one solution is $(2, 3)$.

4. $5x - 2y = 1$, if $x = 0$

Substitute 0 for x, compute, and solve for y.

$$\begin{aligned} 5(0) - 2y &= 1 \\ 0 - 2y &= 1 \\ -2y &= 1 \\ y &= -\frac{1}{2} \end{aligned}$$

Hence, one solution is $\left(0, -\frac{1}{2}\right)$.

★ **PRACTICE SET A**

Find a solution to each of the following linear equations in two variables and write the solution as an ordered pair.

1. $y = 7x - 20$, if $x = 3$
2. $m = -6n + 1$, if $n = 2$
3. $b = 3a - 7$, if $a = 0$
4. $10x - 5y - 20 = 0$, if $x = -8$
5. $3a + 2b + 6 = 0$, if $a = -1$

Answers to the Practice Set are on page 186.

Section 4.7 EXERCISES

For problems 1–38, solve the linear equations in two variables.

1. $y = 8x + 14$, if $x = 1$
2. $y = -2x + 1$, if $x = 0$
3. $y = 5x + 6$, if $x = 4$
4. $x + y = 7$, if $x = 8$
5. $3x + 4y = 0$, if $x = -3$
6. $-2x + y = 1$, if $x = \frac{1}{2}$
7. $5x - 3y + 1 = 0$, if $x = -6$
8. $-4x - 4y = 4$, if $y = 7$
9. $2x + 6y = 1$, if $y = 0$
10. $-x - y = 0$, if $y = \frac{14}{3}$

11. $y = x$, if $x = 1$

12. $x + y = 0$, if $x = 0$

13. $y + \frac{3}{4} = x$, if $x = \frac{9}{4}$

14. $y + 17 = x$, if $x = -12$

15. $-20y + 14x = 1$, if $x = 8$

16. $\frac{3}{5}y + \frac{1}{4}x = \frac{1}{2}$, if $x = -3$

17. $\frac{1}{5}x + y = -9$, if $y = -1$

18. $y + 7 - x = 0$, if $x = ☆$

19. $2x + 31y - 3 = 0$, if $x = a$

20. $436x + 189y = 881$, if $x = -4231$

21. $y = 6(x - 7)$, if $x = 2$

22. $y = 2(4x + 5)$, if $x = -1$

23. $5y = 9(x - 3)$, if $x = 2$

24. $3y = 4(4x + 1)$, if $x = -3$

25. $-2y = 3(2x - 5)$, if $x = 6$

26. $-8y = 7(8x + 2)$, if $x = 0$

27. $b = 4a - 12$, if $a = -7$

28. $b = -5a + 21$, if $a = -9$

29. $4b - 6 = 2a + 1$, if $a = 0$

30. $-5m + 11 = n + 1$, if $n = 4$

31. $3(t + 2) = 4(s - 9)$, if $s = 1$

32. $7(t - 6) = 10(2 - s)$, if $s = 5$

33. $y = 0x + 5$, if $x = 1$

34. $2y = 0x - 11$, if $x = -7$

35. $-y = 0x + 10$, if $x = 3$

36. $-5y = 0x - 1$, if $x = 0$

37. $y = 0(x - 1) + 6$, if $x = 1$

38. $y = 0(3x + 9) - 1$, if $x = 12$

Calculator Problems

39. An examination of the winning speeds in the Indianapolis 500 automobile race from 1961 to 1970 produces the equation $y = 1.93x + 137.60$, where x is the number of years from 1960 and y is the winning speed. Statistical methods were used to obtain the equation, and, for a given year, the equation gives only the approximate winning speed. Use the equation $y = 1.93x + 137.60$ to find the approximate winning speed in

(a) 1965 (b) 1970
(c) 1986 (d) 1990

40. In electricity theory, Ohm's law relates electrical current to voltage by the equation $y = 0.00082x$, where x is the voltage in volts and y is the current in amperes. This equation was found by statistical methods and for a given voltage yields only an approximate value for the current. Use the equation $y = 0.00082x$ to find the approximate current for a voltage of

(a) 6 volts (b) 10 volts

41. Statistical methods have been used to obtain a relationship between the actual and reported number of German submarines sunk each month by the U.S. Navy in World War II. The equation expressing the approximate number of actual sinkings, y, for a given number of reported sinkings, x, is $y = 1.04x + 0.76$. Find the approximate number of actual sinkings of German submarines if the reported number of sinkings is

(a) 4 (b) 9 (c) 10

42. Statistical methods have been used to obtain a relationship between the heart weight (in milligrams) and the body weight (in milligrams) of 10-month-old diabetic offspring of crossbred male mice. The equation expressing the approximate body weight for a given heart weight is $y = 0.213x - 4.44$. Find the approximate body weight for a heart weight of

(a) 210 mg (b) 245 mg

43. Statistical methods have been used to produce the equation $y = 0.176x - 0.64$. This equation gives the approximate red blood cell count (in millions) of a dog's blood, y, for a given packed cell volume (in millimeters), x. Find the approximate red blood cell count for a packed cell volume of

(a) 40 mm (b) 42 mm

44. An industrial machine can run at different speeds. The machine also produces defective items, and the number of defective items it produces appears to be related to the speed at which the machine is running. Statistical methods found that the equation $y = 0.73x - 0.86$ is able to give the approximate number of defective items, y, for a given machine speed, x. Use this equation to find the approximate number of defective items for a machine speed of

(a) 9 (b) 12

45. A computer company has found, using statistical techniques, that there is a relationship between the aptitude test scores of assembly line workers and their productivity. Using data accumulated over a period of time, the equation $y = 0.89x - 41.78$ was derived. The x represents an aptitude test score and y the approximate corresponding number of items assembled per hour. Estimate the number of items produced by a worker with an aptitude score of

(a) 80 (b) 95

46. Chemists, making use of statistical techniques, have been able to express the approximate weight of potassium bromide, W, that will dissolve in 100 grams of water at T degrees centigrade. The equation expressing this relationship is $W = 0.52T + 54.2$. Use this equation to predict the potassium bromide weight that will dissolve in 100 grams of water that is heated to a temperature of

(a) 70 degrees centigrade
(b) 95 degrees centigrade

47. The marketing department at a large company has been able to express the relationship between the demand for a product and its price by using statistical techniques. The department found, by analyzing studies done in six different market areas, that the equation giving the approximate demand for a product (in thousands of units) for a particular price (in cents) is $y = -14.15x + 257.11$. Find the approximate number of units demanded when the price is

(a) \$0.12 (b) \$0.15

48. The management of a speed-reading program claims that the approximate speed gain (in words per minute), G, is related to the number of weeks spent in its program, W, is given by the equation $G = 26.68W - 7.44$. Predict the approximate speed gain for a student who has spent

(a) 3 weeks in the program
(b) 10 weeks in the program

EXERCISES FOR REVIEW

(3.5) **49.** Find the product. $(4x-1)(3x+5)$.
(3.6) **50.** Find the product. $(5x+2)(5x-2)$.
(4.4) **51.** Solve the equation $6[2(x-4)+1] = 3[2(x-7)]$.
(4.6) **52.** Solve the inequality $-3a-(a-5) \geq a+10$.
(4.6) **53.** Solve the compound inequality $-1 < 4y + 11 < 27$.

★ Answers to Practice Set (4.7)

A. **1.** $(3, 1)$ **2.** $(2, -11)$ **3.** $(0, -7)$ **4.** $(-8, -20)$ **5.** $\left(-1, \frac{-3}{2}\right)$

Chapter 4 SUMMARY OF KEY CONCEPTS

Identity **(4.1)**	An equation that is true for all acceptable values of the variable is called an *identity*. $x+3=x+3$ is an identity.
Contradiction **(4.1)**	*Contradictions* are equations that are never true regardless of the value substituted for the variable. $x+1=x$ is a contradiction.
Conditional Equation **(4.1)**	An equation whose truth is conditional upon the value selected for the variable is called a *conditional equation.*
Solutions and Solving an Equation **(4.1)**	The collection of values that make an equation true are called the *solutions* of the equation. An equation is said to be *solved* when all its solutions have been found.
Equivalent Equations **(4.1, 4.2)**	Equations that have precisely the same collection of solutions are called *equivalent equations.* An equivalent equation can be obtained from a particular equation by applying the *same* binary operation to *both* sides of the equation, that is, 1. adding or subtracting the *same* number to or from *both* sides of that particular equation. 2. multiplying or dividing *both* sides of that particular equation by the *same non-zero* number.
Literal Equation **(4.1)**	A *literal equation* is an equation that is composed of more than one variable.
Recognizing an Identity **(4.3)**	If, when solving an equation, all the variables are eliminated and a true statement results, the equation is an *identity.*
Recognizing a Contradiction **(4.3)**	If, when solving an equation, all the variables are eliminated and a false statement results, the equation is a *contradiction.*
Translating from Verbal to Mathematical Expressions **(4.4)**	When solving word problems it is absolutely necessary to know how certain words translate into mathematical symbols.
Five-Step Method for Solving Word Problems **(4.5)**	1. Let x (or some other letter) represent the unknown quantity. 2. Translate the words to mathematics and form an equation. A diagram may be helpful. 3. Solve the equation. 4. Check the solution by substituting the result into the original statement of the problem. 5. Write a conclusion.
Linear Inequality **(4.6)**	A *linear inequality* is a mathematical statement that one linear expression is greater than or less than another linear expression.
Inequality Notation **(4.6)**	$>$ Strictly greater than $<$ Strictly less than $\geq$ Greater than or equal to $\leq$ Less than or equal to
Compound Inequality **(4.6)**	An inequality of the form $a<x<b$ is called a *compound inequality.*
Solution to an Equation in Two Variables and Ordered Pairs **(4.7)**	A pair of values that when substituted into an equation in two variables produces a true statement is called a solution to the equation in two variables. These values are commonly written as an *ordered pair.* The expression (a, b) is an ordered pair. In an ordered pair, the independent variable is written first and the dependent variable is written second.

EXERCISE SUPPLEMENT

Sections 4.1 – 4.3

Solve the equations for problems 1–45.

1. $y + 3 = 11$
2. $a - 7 = 4$
3. $r - 1 = 16$
4. $a + 2 = 0$
5. $x + 6 = -4$
6. $x - 5 = -6$
7. $x + 8 = 8$
8. $y - 4 = 4$
9. $2x = 32$
10. $4x = 24$
11. $3r = -12$
12. $6m = -30$
13. $-5x = -30$
14. $-8y = -72$
15. $-x = 6$
16. $-y = -10$
17. $3x + 7 = 19$
18. $6x - 1 = 29$
19. $4x + 2 = -2$
20. $6x - 5 = -29$
21. $8x + 6 = -10$
22. $9a + 5 = -22$
23. $\frac{m}{6} + 4 = 8$
24. $\frac{b}{5} - 2 = 5$
25. $\frac{y}{9} = 54$
26. $\frac{a}{-3} = -17$
27. $\frac{c}{6} = 15$
28. $\frac{3a}{4} = 9$
29. $\frac{4y}{5} = -12$
30. $\frac{r}{4} = 7$
31. $\frac{6a}{-5} = 11$
32. $\frac{9x}{7} = 6$
33. $\frac{c}{2} - 8 = 0$
34. $\frac{m}{-5} + 4 = -1$
35. $\frac{x}{7} - 15 = -11$
36. $\frac{3x}{4} + 2 = 14$
37. $\frac{3r + 2}{5} = -1$
38. $\frac{6x - 1}{7} = -3$
39. $\frac{4x - 3}{6} + 2 = -6$
40. $\frac{y - 21}{8} = -3$
41. $4(x + 2) = 20$
42. $-2(a - 3) = 16$
43. $-7(2a - 1) = 63$
44. $3x + 7 = 5x - 21$
45. $-(8r + 1) = 33$
46. Solve $I = prt$ for t. Find the value of t when $I = 3500$, $p = 3000$, and $r = 0.05$.
47. Solve $A = LW$ for W. Find the value of W when $A = 26$ and $L = 2$.
48. Solve $\rho = mv$ for m. Find the value of m when $\rho = 4240$ and $v = 260$.
49. Solve $P = R - C$ for R. Find the value of R when $P = 480$ and $C = 210$.
50. Solve $P = \frac{nRT}{V}$ for n.
51. Solve $y = 5x + 8$ for x.
52. Solve $3y - 6x = 12$ for y.
53. Solve $4y + 2x + 8 = 0$ for y.
54. Solve $k = \frac{4m + 6}{7}$ for m.

55. Solve $t = \dfrac{10a - 3b}{2c}$ for b.

Section 4.4

For problems 56–70, translate the phrases or sentences to mathematical expressions or equations.

56. A quantity less eight.
57. A number, times four plus seven.
58. Negative ten minus some number.
59. Two fifths of a number minus five.
60. One seventh of a number plus two ninths of the number.
61. Three times a number is forty.
62. Twice a quantity plus nine is equal to the quantity plus sixty.
63. Four times a number minus five is divided by seven. The result is ten more than the number.
64. A number is added to itself five times, and that result is multiplied by eight. The entire result is twelve.
65. A number multiplied by eleven more than itself is six.
66. A quantity less three is divided by two more than the quantity itself. The result is one less than the original quantity.
67. A number is divided by twice the number, and eight times the number is added to that result. The result is negative one.
68. An unknown quantity is decreased by six. This result is then divided by twenty. Ten is subtracted from this result and negative two is obtained.
69. One less than some number is divided by five times the number. The result is the cube of the number.
70. Nine less than some number is multiplied by the number less nine. The result is the square of six times the number.

Section 4.5

For problems 71–80, find the solution.

71. This year an item costs \$106, an increase of \$10 over last year's price. What was last year's price?
72. The perimeter of a square is 44 inches. Find the length of a side.
73. Nine percent of a number is 77.4. What is the number?
74. Two consecutive integers sum to 63. What are they?
75. Four consecutive odd integers add to 56. What are they?
76. If twenty-one is subtracted from some number and that result is multiplied by two, the result is thirty-eight. What is the number?
77. If 37% more of a quantity is 159.1, what is the quantity?
78. A statistician is collecting data to help her estimate the number of pickpockets in a certain city. She needs 108 pieces of data and is $\frac{3}{4}$ done. How many pieces of data has she collected?
79. The statistician in problem 78 is eight pieces of data short of being $\frac{5}{6}$ done. How many pieces of data has she collected?
80. A television commercial advertises that a certain type of light bulb will last, on the average, 200 hours longer than three times the life of another type of bulb. If consumer tests show that the advertised bulb lasts 4700 hours, how many hours must the other type of bulb last for the advertiser's claim to be valid?

Section 4.6

Solve the inequalities for problems 81–100.

81. $y + 3 < 15$
82. $x - 6 \geq 12$
83. $4x + 3 > 23$
84. $5x - 14 < 1$
85. $6a - 6 \leq -27$
86. $-2y \geq 14$
87. $-8a \leq -88$
88. $\dfrac{x}{7} > -2$
89. $\dfrac{b}{-3} \leq 4$
90. $\dfrac{2a}{7} < 6$

91. $\frac{16c}{3} \geq -48$

92. $-4c + 3 \leq 5$

93. $-11y + 4 > 15$

94. $3(4x - 5) > -6$

95. $-7(8x + 10) + 2 < -32$

96. $5x + 4 \geq 7x + 16$

97. $-x - 5 < 3x - 11$

98. $4(6x + 1) + 2 \geq -3(x - 1) + 4$

99. $-(5x + 6) + 2x - 1 < 3(1 - 4x) + 11$

100. What numbers satisfy the condition: nine less than negative four times a number is strictly greater than negative one?

Section 4.7

Solve the equations for problems 101–110.

101. $y = -5x + 4$, if $x = -3$

102. $y = -10x + 11$, if $x = -1$

103. $3a + 2b = 14$, if $b = 4$

104. $4m + 2k = 30$, if $m = 8$

105. $-4r + 5s = -16$, if $s = 0$

106. $y = -2(7x - 4)$, if $x = -1$

107. $-4a + 19 = 2(b + 6) - 5$, if $b = -1$

108. $6(t + 8) = -(a - 5)$, if $a = 10$

109. $-(a + b) = 5$, if $a = -5$

110. $-a(a + 1) = 2b + 1$, if $a = -2$

Chapter 4 PROFICIENCY EXAM

Solve the equations and inequalities for problems 1–16.

1. ______

1. **(4.1)** $x + 8 = 14$

2. ______

2. **(4.1)** $6a + 3 = -10$

3. ______

3. **(4.2)** $\frac{-3a}{8} = 6$

4. ______

4. **(4.3)** $\frac{x}{-2} + 16 = 11$

5. ______

5. **(4.2)** $\frac{y-9}{4} + 6 = 3$

6. ______

6. **(4.3)** $5b - 8 = 7b + 12$

7. ______

7. **(4.3)** $3(2a + 4) = 2(a + 3)$

8. ______

8. **(4.3)** $5(y + 3) - (2y - 1) = -5$

9. ______

9. **(4.2)** $\frac{-(4x + 3 - 5x)}{3} = 2$

10. ______

10. **(4.3)** Solve $2p - 6q + 1 = -2$ for p.

11. ______

11. **(4.2)** Solve $p = \frac{nRT}{V}$ for T.

12. ______

12. **(4.3)** Solve $\dfrac{\triangle + \square}{\star} = \nabla$ for $\triangle$.

13. ______

13. **(4.6)** $a - 8 \geq 4$

14. ______

14. **(4.6)** $-3a + 1 < -5$

15. ______

15. **(4.6)** $-2(a + 6) \leq -a + 11$

16. ______

16. **(4.6)** $\dfrac{-4x - 3}{3} > -9$

17. ______

18. ______

Translate the phrases or sentences into mathematical expressions or equations for problems 17–21.

17. **(4.4)** Three added to twice a number.

19. ______

18. **(4.4)** Eight less than two thirds of a number.

20. ______

19. **(4.4)** Two more than four times a number.

20. **(4.4)** A number is added to itself and this result is multiplied by the original number cubed. The result is twelve.

21. ______

21. **(4.4)** A number is decreased by five and that result is divided by ten more than the original number. The result is six times the original number.

22. ______

Solve problems 22–25.

22. **(4.5)** Eight percent of a number is 1.2. What is the number?

23. ______

23. **(4.5)** Three consecutive odd integers sum to 38. What are they?

24. ______

24. **(4.5)** Five more than three times a number is strictly less than seventeen. What is the number?

25. ______

25. **(4.7)** Solve $y = 8x - 11$ for y if $x = 3$, and write the solution as an ordered pair.